Notes: For primes related Challenges

1. Search the internet for the Prime Number Theorem.
2. Suppose p_n is the n^{th} prime. There is no simple formula for p_n.
3. It is known there is a bound M (a positive real number so that)

$$p_{n+1} - p_n \leq M$$

for an infinite number of n.

4. The smallest bound M is not known, but it is conjectured to be 2.
5. p_n and p_{n+1} are said to be twin primes if $p_{n+1} - p_n = 2$.
6. The Prime Number Theorem Challenge is asking the following: Suppose $p(x)$ is the number of primes numbers p so that

$$2 \leq p \leq x$$

Recall from Calculus/Real analysis: the function $f(x)$ is asymptotic to the function $g(x)$ if

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1.$$

This challenge is asking you to determine which function, from the multiple choices, is asymptotic to $p(x)$.

7. The Prime distributions Challenge is asking what function, from the multiple choices, is asymptotic to the sum:

$$\sum_{2\leq p\leq x} \frac{1}{p}$$

as $x \to \infty$. Note that from Calculus we know the partial sums of the Harmonic series:

$$\sum_{k=1}^{n} \frac{1}{k} \text{ is asymptotic to } \ln(n) \text{ as } n \to \infty$$

and so clearly $\sum_{2\leq p\leq x} \frac{1}{p}$ is much smaller than $\ln(x)$.

8. The sum in the Twin primes Challenge is asking if the series

$$\sum_{2\leq p\leq x} \frac{1}{p}$$

converges of diverges, as $x \to \infty$, with the sum being over **only** twin primes.

References: next page

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