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Risk Factors Impact on the P&L

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Summary

Profit and Loss (P&L) explain analysis is an income statement produced by Product Control Team for traders to control the daily fluctuation in the value of a portfolio of trades to the root causes of the changes. This daily income provides users with a coherent breakdown of the drivers of P&L movements between two points in time with reference to a selected number of easily understandable pricing factors.
P&L Attribution (also called P&L explain) can be calculated in two ways, either the risk based method or step re-evaluation method.
This paper aims at understanding both methodologies from a theoretical point of view and shows the differences of both calculations methods and how they are interdependent in the daily work of a trader in the sense that both methods give a rational to the P&L from different perspectives. The risk based method involves the calculation of the trades sensitivities (also known as the Greeks) and then using them to predict the expected change in the P&L from one period to the next by using the actual market changes in the factors driving the transaction price over the same period and the transaction's sensitivity to those factors. Whereas the re-evaluation method is calculated by aggregating the impact of different valuation scenarios and not on fixed sensitivities.

Keywords: Profit and Loss (P&L), risk factors, Greeks.

Riskfaktorers inverkan på balansräkning

Sammanfattning

Analys och rapportering av balansräkning ger användarna en sammanhängande uppdelning av drivkrafterna för P&L rörelser mellan två punkter i tid med hänvisning till ett urval av lättförståeliga prisfaktorer. P&L attribution kan beräknas på två sätt, känslighet och scenariobaserad metoder.
Detta arbete syftar till att förstå båda metoderna ur ett teoretiskt perspektiv och visar skillnaderna och hur de beror på varandra. Känslighetsmetoden innebär beräkning av en handels känslighet (även känd som beräkning av grekerna) och sedan använda dem för att förutsäga den förväntade förändringen i P&L från en period till nästa genom att använda de faktiska marknadsförändringarna i de faktorer som driver transaktionspriset under samma period och transaktionens känslighet för dessa faktorer. Omvärderingsmetoden beräknas genom att olika värderingsscenariers inverkan sammanställs och inte på fasta känsligheter.

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khawla Gougas

Vocabulary:

P&L	The profit and loss (P&L) statement is a financial statement that summarizes the revenues, costs, and expenses incurred during a specified period, usually a fiscal quarter or year.
P&L explain	an income statement produced by Product Control Team for traders to control the daily fluctuation in the value of a portfolio of trades to the root causes of the changes.
SOD	Start of day (business day)
COB	Close of business for the trader
IR	Interest rate
FX	Foreign exchange rate
Greeks	Trades sensitivities: "Greeks" is a term used in the options market to describe the different dimensions of risk involved in taking an options position.
risk-based method	a method of calculation of p&L explain based on trades sensitivities
Intraday result	Intraday means "within the day." In the financial world, the term is shorthand used to describe securities that trade on the markets during regular business hours. These securities include stocks and exchange-traded funds (ETFs).
Step re-evaluation method	It is a method of calculation p&L explain based on
Taylor Expansion	The Taylor expansion is the standard technique used to obtain a linear or a quadratic approximation of a function of one variable.
Mark to market	Mark to market (MTM) is a method of measuring the fair value of accounts that can fluctuate over time, such as assets and liabilities.
CVA	Credit Value Adjustment (CVA) is new risk measure that offers an opportunity for banks to move beyond the system control of limits and to price dynamically counterparty credit risk of new trades.
FSA	Financial services authority

Contents

1 Introduction **7**

2 Methodology **10**
- 2.1 P&L explain . . . 10
- 2.2 Types of P&L: . . . 11
- 2.3 Types risk factors: . . . 12
- 2.4 Other risk factors: . . . 14
- 2.5 Risk generation . . . 16
  - 2.5.1 First Order risks calculation . . . 16
  - 2.5.2 IR Delta . . . 16
  - 2.5.3 IR Spread . . . 17
  - 2.5.4 Inflation Delta . . . 18
  - 2.5.5 Vega . . . 18
- 2.6 Second Order risks calculation . . . 18
  - 2.6.1 IR Gamma . . . 18
  - 2.6.2 IR Volga . . . 18
- 2.7 Calculation methodologies . . . 19

3 Step re-evaluation P&L **19**
- 3.1 Path dependent Step re-evaluation P&L . . . 19
- 3.2 Path independent Step re-evaluation P&L . . . 21
  - 3.2.1 Single factor step re-evaluation P&L . . . 21
  - 3.2.2 Cross risk factors step re-evaluation P&L . . . 22

4 Risk based P&L calculation **23**

5 Results **24**
- 5.1 Step revaluation Limitations . . . 24
- 5.2 Comparison of the P&L explain calculation methods . . . 28
  - 5.2.1 Empirical result . . . 29
  - 5.2.2 Comparison conclusion . . . 30

6 Conclusion & Extension **32**

7 References **34**

1 Introduction

Financial risk management evolved throughout the past decades. The financial crisis in 2008 proved that insufficient management of financial risk can cause negative waves throughout global markets. Thus after the crisis, a lot of regulations were imposed in the capital markets to have a conform risk measurement reference across banks and financial institutions, these references would help regulation systems such as the CEB (Central European Bank) to track the risk management and raise potential anomalies. In the context of industrialisation of risk inside the banks, P&L explain methodology provides a risk management tool across Global Markets business lines to produce comparable figures and improve risk measurements to help manage financial risk.

Financial crises contributed to the evolution of risk management practices, and regulatory actions. In the 1970s, research lay the intellectual foundations for the risk management practices that were systematically implemented in the 1980s as bond trading revolutionized Wall Street. Quants developed dynamic hedging, Value-at-Risk, and credit risk models based on the insights of financial economics.
In parallel, the Basel I framework which is the round of deliberations by central bankers from around the world in 1988, created a level playing field among banks across countries. Following the 1987 stock market crash, the near failure of Salomon Brothers, and the failure of Drexel Burnham Lambert, in 1996 the Basel Committee on Banking Supervision published the Market Risk Amendment to the Basel I Capital Accord; the amendment went into effect in 1998. It led to a migration of bank risk management practices toward market risk regulations. The framework was further developed in the Basel II Accord. Indeed, the failure to measure and manage risk adequately can be viewed as a key contributor to the 2008 global financial crisis. Subsequent innovations in risk management practices have been dominated by regulatory innovations, including capital and liquidity stress testing, macroprudential surcharges, resolution regimes, and countercyclical capital requirements.

Thus one of the most important risk regulations was imposing an explication

of the daily P&l of the financial institutions.

The Profit and Loss (P&L) explained aims at giving a rational to the profit and loss made between the value of the portfolio on the close of business (COB) and the one of the start of day (STO) using these risk factors and mainly and market risk which we can quantify. A P&L explained analysis attributes the daily change in the value – ie the mark-to-market (MTM) – of a portfolio of deals to its root causes. Risk managers use this knowledge of the source of trading profits to act more effectively. For example, they will investigate if they discover that options desk's profits are primarily due to commodity prices changes instead of volatility changes. Traders use this report as a diagnostic tool to help them reconcile at end-of-day if their hand-calculated P&L estimate doesn't match the value produced by their trading system.

The risk computation have to be updated on a regular basis to reflect changes in the environment in which we operate as well as in our underlying models having an impact on the risk. This data is then used to calculate the P&L explain and manage risk. The users range from traders to global risk managers or P&L control teams.

An important thing to mention about this income statement "P&L explanation" is that despite trying to explain fully the P&L it would always envolve an unexplained factor that should theoreticly tend to zero but it it's not null it doesn't mean that our report is incorrect. In fact, the P&L number is always considered correct in a P&L explained report, although it certainly makes risk managers uncomfortable if the value in the 'unexplained' column on the report is not zero. Therefore, when you do find the cause of the unexplained P&L, which is usually done manually after the fact, the total P&L number never changes due to any reallocation, even a reallocation from 'unexplained' to one of the explanatory columns.

Besides the explanatory benefit of the P&L explain, it can also catch errors on the current trades since it puts the trade's P&L in context with other trades and with the same trade done the previous day. Various mistakes in booking will be detected by putting a trade in context with its peers. For example, if a

new trade was erroneously entered with 10 or 100 times more notional, then it will have an abnormally large P&L compared to similar trades. In context with previous day: suppose a piece of market data was incorrectly used in the P&L calculation today. Then the P&L between today and yesterday would show a much higher or lower jump than expected, alerting the reader to a possible problem.

The importance of P&L fluctuation explanation in the financial markets motivated me to work on this subject and try to understand the P&L explain theory and calculation methods.
This project took place at BNP Paribas Arbitrage which is a subsidiary of the French international banking group specialised in the management of the trading activity on derivative products. BNP Paribas is the largest French banking group and the largest bank in the Eurozone. It became one of the five largest banks in the world following the 2008 financial crisis.

2 Methodology

To achieve the goal mentioned in the introduction, I started gathering different documentation sources to have a theoretical background vis-à-vis the P&L explain, then I started training data on both models to compute the P&L explain to finally conclude the differences and also interdependence of both method.

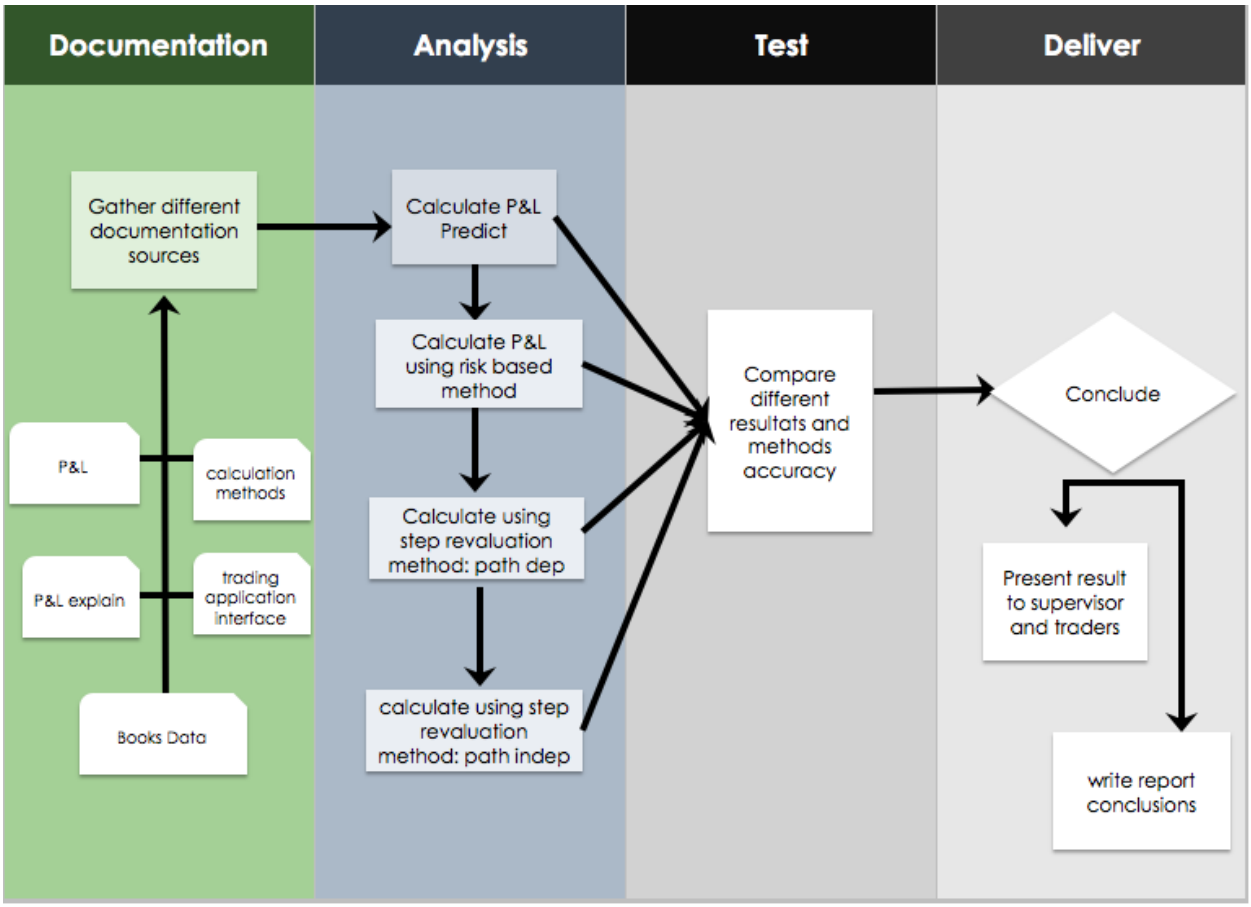

Figure 1: Methodology of work

2.1 P&L explain

The P&L explain aims at justifying the P&L from a set of observable market

parameters. Being able to find a rational to the P&L from the risk figures is a way to validate that the risk positions are correct. Risk, Actual P&L and P&L explained are interdependent legs. Risk is the starting point, both from a timing perspective as well as in causality and deserves primary attention.
Risk production involves the generation and reporting of the Greeks which represent the sensitivities that the value of a portfolio of positions in risky derivatives has to change in the underlying variables and the parameters on which the derivative value is dependent. These underlying variables and parameters are the risk factors for that portfolio.
All risk factors are not created equal even for any one given portfolio of positions. Some risk factors disproportionately affect the change in the value of the portfolio depending on the type of derivative instruments in the portfolio and the current state and value of all variables.

Literature on the P&L explain is very limited since the concept is very new in the banking system and recently imposed by the Financial authority to have a detailed view on the profit and loss to track that the P&L adjustments are regulated. So I constructed the theoretical part from a set of references.
First of all since the P&L discussed in this paper is focused on the interest rate products, I used the model theory used in [1] and [2] enriched with Piterbarg papers [9] and [10] that relate the models to risk management and discusses risk factors more in details. For the P&L explain risk factors, Acuity and derivatives LLC paper [7] discuss different risk effects that constitute the P&L attribution from quantified to non quantified ones (as seen in Figure 3).
Finally the other papers provide a broader vision on the P&L risk management for different products from equity to fixed income ones.

2.2 Types of P&L:

- **Official P&L:** It is the P&L reported directly by the bank. It is computed using one of the following resources:

  Central close markets: Data obtained directly from exchanges when available.

  Multi Cut Off (MCO): It uses either market close data or snapshot of live data time consistent with market close.

- **P&L Predict:** The P&L as it can be explained at Close Of Business (COB) by traders in various regions. (It is approximated since the official P&L may rely on some market data published later by another region).
- **Official P&L Explain:** It is the P&L as it can be explained when all the market data is available. It is generally computed on $T + 1$.

2.3 Types risk factors:

- **Market Effect:** This is the core of the P&L explanation process. It aims at explaining the impact on the P&L of the changes in the market parameters happening during the day. From the various P&L explain steps, the market effect is the only one which can be explained from the market risk sensitivities such as volatility, movement of spot, movement of interest rate.

- **Environment Effect:** Environment effects refers to all the non-trading related parameters (excluded from the VaR) which are updated either overnight or at COB and have a P&L impact. This includes updates in static data, changes in the CSA agreements (collateral agreements), model changes, analytics upgrades.

- **Time Effect:** From a high level point of view, time effect can be assimilated to the cost of running a trading book. This document takes a broad definition of the time effect which includes all the P&L effects associated to the switch from COB_{t-1} to Start of Day SOD_t. Depending on the position this could generate a positive P&L (in this case the book is often said to have a "positive carry") or a negative P&L ("negative carry").

- **Non-Trading P&L:** Non-trading P&L actually refers to the P&L elements which do not make part of the daily traders' activity but still have an overall impact on the P&L and potentially the cash balance. As such,

and in order to match the official P&L a P&L explain will have to receive a feed of those non-trading P&L effects which can typically be fees, brokers fees etc. By their nature, those effects cannot usually be computed from front office system and hence must be fed from official P&L systems.

- **Liquidity Risk:** Liquidity risk is normally characterised in two forms. Asset liquidity risk represents the risk that a transaction cannot be executed at market prices, perhaps due to the size of the position and/or relative illiquidity of the underlying. Funding liquidity risk refers to the inability to fund contractual payments or collateral requirements, potentially forcing an early liquidation of assets and crystallisation of losses.

  Note That this paper will only cover the market effect that can be quantified through the market risk sensitivities, most of other risk factors are quantified for specific financial products once the trade happens and generates fixed numbers that middle officers take into account in their P&L explain analysis. Whereas time effect, needs a lot of assumptions on the opening market structure to conclude the close of business one such as if forwards are realised or not at the end of the day, if not we are dealing with a carry market that needs more analysis and doesn't enter the scope of this paper in terms of quantification.

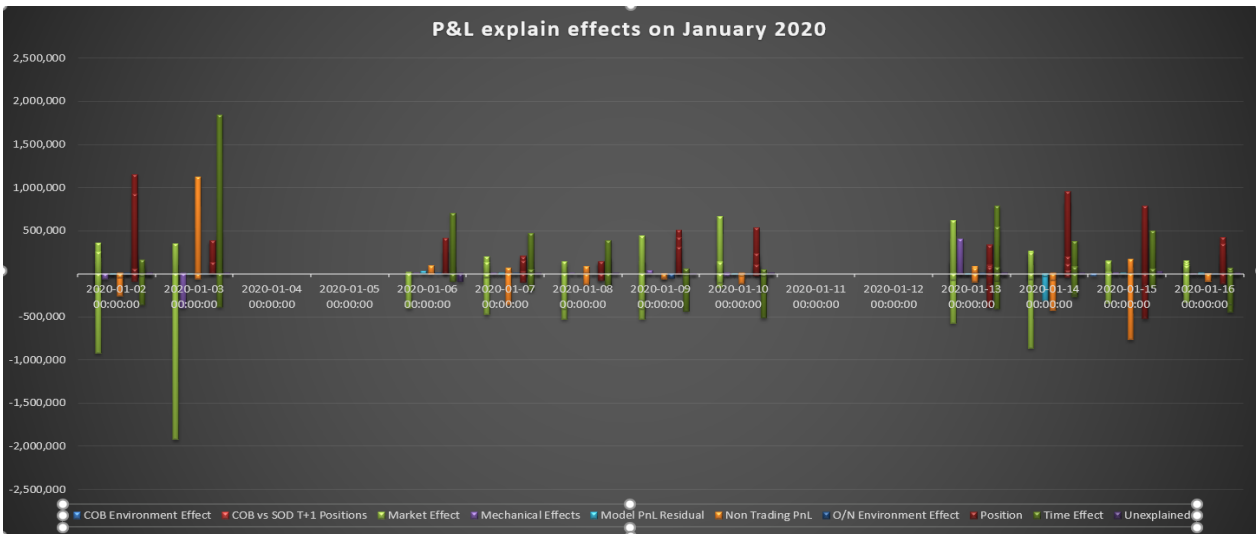

Figure 2: P&L explained effects on BNP Data january 2020

This graph based on BNP Paribas CIB data (time series of P&L explain effects) shows the contribution of different risk factors to the explanation of the P&L during january month 2020. We can see that Time effect and Market effect contribute the most since they're related to the every trades price fluctuation.

2.4 Other risk factors:

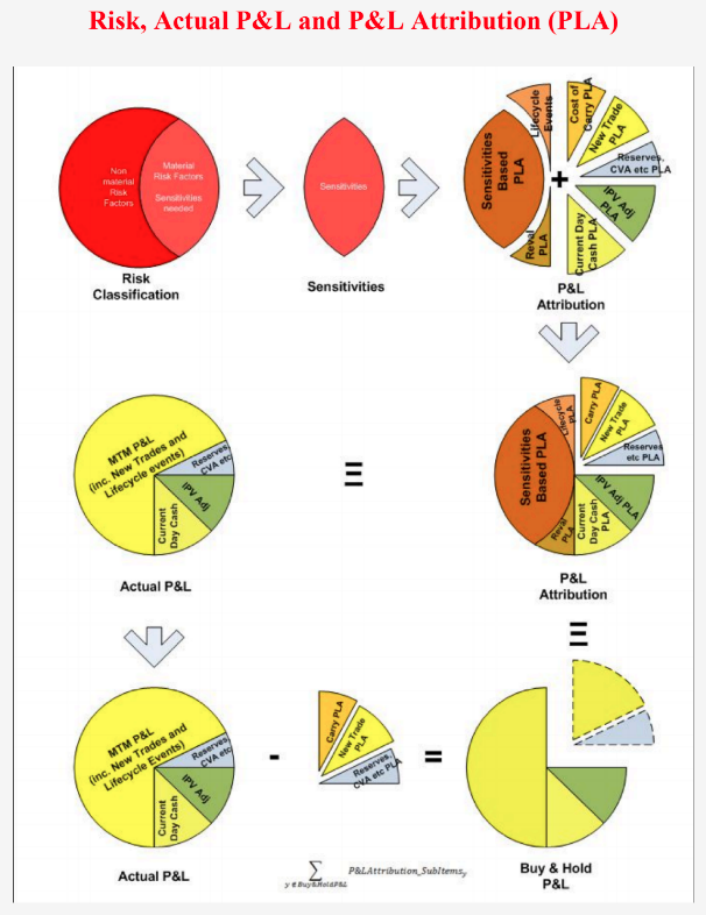

Figure 3: Risk, Actual P&L and P&L explained [7]

This figure shows the risk factors that contribute in the P&L explanation (P&L attribution), and that the aim of both methodologies is to have a number that converge to the actual or official P&L that is calculated differently based on close of business P&L of different business lines.

There are events (other than risk factors market events) that have Mark to Market impact; P&L due to these may be attributed by revaluing affected positions to obtain the isolated P&L impact of the event.

- **CVA P&L explain**
  P&L due to the exposure changers to a counter party's changing default risk.

- **New trade P&L explain**
  P&L due to new trades done that day. New Trade P&L is a critically important metric for P&L control purposes yet the FSA (Financial Services authority)found that more than half of firms it surveyed cannot distinguish between new trade P&L and P&L due to cancels/amends.

- **Trade Life cycle Events P&L explain**
  P&L due to trade lifecycle events e.g. amendments, novations(the act of substituting a valid existing contract with a replacement contract), unwinds(close a position) etc. And P&L due to underlying lifecycle events e.g. option exercises, rate resets, credit events etc.

- **Current day expected Cash**
  Cash flows payable on the valuation day drop off the valuation of the trade, however remain a part of the MTM (mark to market) of the trade and need to be added back.

- **fees, commissions and trader credits**
  P&L Costs due to fees, sales commissions and trader credits earned from the trade.

2.5 Risk generation

2.5.1 First Order risks calculation

2.5.2 IR Delta

The sensitivity of the valuation function to a change in the outright level of the interest rate for a given currency r_{CCY} which represents the interest rate of a

currency CCY.
The value of IR.Delta (interest rate Delta) is as follows:

$$IR.Delta(t_{SOD}, CCY) = \frac{\partial V(P_{mkt}(t_{SOD}))}{\partial r_{CCY}}.$$

with $V(P_{mkt}(t_{SOD})$ the valuation funtion of our portfolio at the Start of Day (SOD) that depends on the different market parameters (time t, interest rate r, stock price if there is an equity s, dividends curve, repo curve...) other parameters might be in included for complex contracts and exotic options.
If our function is differentiable we would apply the derivative otherwise we would use finite differentiation method. The continuity of the price of our portfolio depends on the type of options, stocks, indexes that it includes and the complexity of our pricing function. Here are the practical steps to compute this sensitivity:

- Compute an initial PV (present value) for the considered trade or portfolio of trades.
- Perturb the first tenor of the yield curve and rebuild the pricing market keeping other parameters constant.
- Reprice the considered trade and calculate the difference of the PVs that represents the sensitivity to the first tenor repeat the same for all tenors.

2.5.3 IR Spread

For a given currency, IR basis spread represents the sensitivity of the valuation function V to a change in the spread between the curve used to compute the IR Delta (called the yield curve) given basis curve (for example OIS/IBOR basis: the London Interbank Offered Rate (LIBOR) and the Overnight Indexed Swap (OIS) rate.) The sensitivity of the valuation function to a change in the spread between the curve used to compute IR Delta and a given basis curve is as follows (Inflation delta):

$$IR.Basis.Spread(t_{SOD}, CCY, Basis) = \frac{\partial V(P_{mkt}(t_{SOD}))}{\partial Spd_{CCY}^{Basis}}.$$

2.5.4 Inflation Delta

The sensitivity of the valuation function to a change in the level of an inflation index I.

$$IR.Delta(t_{SOD}, I) = \frac{\partial V(P_{mkt}(t_{SOD}))}{\partial I}.$$

2.5.5 Vega

The sensitivity of the valuation function to a change in the ATM volatility of a given volatility market σ.

$$Ind.Vega(t_{SOD}, \sigma) = \frac{\partial V(P_{mkt}(t_{SOD}))}{\partial \sigma}.$$

The sensitivity of the valuation function to a change in the forward spread level S_0 (ZWC).

$$ZWC.Spd(t_{SOD}, S_0) = \frac{\partial V(P_{mkt}(t_{SOD}))}{\partial S_0}.$$

2.6 Second Order risks calculation

2.6.1 IR Gamma

The change in the IR Delta for a change in the absolut level of interest rate for a currency.

$$IR.Gamma(t_{SOD}, CCY) = \frac{\partial^2 V(P_{mkt}(t_{SOD}))}{\partial^2 r_{CCY}}.$$

2.6.2 IR Volga

Volatility gamma represents the change in the IR Vega risk for a change in the ATM volatility level.

$$IR.Volga(t_{SOD}, \sigma) = \frac{\partial^2 V(P_{mkt}(t_{SOD}))}{\partial^2 \sigma}.$$

- Compute an IR Vega Risk on the unperturbed market.
- Apply a parallel shift to the ATM volatility.
- Compute an IR Vega on the shifted market.

- Compute the difference between initial IR Vega and the one post perturbation.

2.7 Calculation methodologies

- **Risk based method:** Computes P&L by multiplying risk of a portfolio by the change in the underlying market moves.
- **Step re-evaluation method:** It is based on a sequence of PVs (Present Value) providing a graduate path between two markets which allows to allocate the P&L associated to the various risk factors to the change of a set of parameters. In other words, in each step between two markets we are going to perturb one risk parameter such as the spot and calculate our first P&L step, then we are going to perturb the volatility and so on. This would give us the effect of each risk parameter on our P&L calculation.

3 Step re-evaluation P&L

We assume at any point t time, the value $V(P_{mkt(t)})$ of our portfolio of trades is a function of a set of valuation parameters, These parameters may include future prices, volatility for Market observable data or fixing Leg or payment one for pricing parameters and so on..

$$P_{mkt(t)} = \begin{bmatrix} P_1 \\ \vdots \\ P_n \end{bmatrix},$$

then we will apply a set of perturbations delta

$$\delta = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_n \end{bmatrix},$$

in order to obtain the closing parameters $P_{mkt(t_COB)}$.

3.1 Path dependent Step re-evaluation P&L

The principle of the path dependent step re-evaluation is to sequentially shift the market parameters from their initial value (SOD market: Start of Day market) to their final value (COB market: close of business).
That's why we need to define delta

$$\delta_{SRI} = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_i \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

which corresponds to the i-th perturbation of the market and in this case, the $P\&L_{SRi}$ associated to this step is

$$SRi = V(P_{mkt(t)} + \delta_{SRi}) - V(P_{mkt(t)} - \delta_{SRi}),$$

we then have

$$\sum_{i=1}^{N} SR_i = V(P_{mkt(t_{COB})}) - V(P_{mkt(t_{SOD})}).$$

The advantage of this methodology is that it is able to allocate the realized P&L to the change in each market factor. It is also reasonably cheap to compute as for a step re-evaluation P&L of N steps, there are N + 1 valuations to perform.
On *the downside* the P&L allocated to each of those steps is actually dependent on the ordering on the vector,thus the notion of path dependent step re-evaluation P&L.

Let's take the example of an at-the-money (ATM) option being pushed out of-the-money (OTM) by the changes in the rates during the day. Assuming the Vega step is performed before the interest rate step, the P&L allocated to the Vega step can be quite significant whereas if the Vega step had been performed after the interest rate step, then its P&L would be much smaller.

3.2 Path independent Step re-evaluation P&L

In the path independent method we want to allocate the P&L associated to the change of each market parameter independently.
For each step of the P&L calculation process, we will perturb the market, compute the PV of our portfolio on the the perturbed market.
We are performing the valuation steps in line with the Taylor expansion of a multi-variate function. Thus we will have to introduce an unexplained term.

3.2.1 Single factor step re-evaluation P&L

A simple form of bump and reset explain will capture the P&L associated to each risk factors independently.

$$V(P_{mkt(t_COB)}) = V(P_{mkt(t_SOD)}) + \nabla^T \delta.$$

We then introduce the UnExplained term:

$$V(P_{mkt(t_COB)}) = V(P_{mkt(t_SOD)}) + \nabla^T \delta + UnExplained,$$

with:

$$\nabla_i = \frac{\partial V(P_{mkt(t)})}{\partial P_i}.$$

The P&L associated to each SR_i^* step is:

$$SR_i^* = V(P_{mkt(t)} + \delta^*_{SR_i}) - V(P_{mkt(t)}).$$

The computational cost of this method grows linearly with the number N of steps.

Remark:

The *UnExplained* term is a critical metric that regulators and product control within a bank alike pay attention to.

P&L explained is used to test the hypothesis that the risk factors identified for a risky position are sufficient to materially explain the value change expected from the risky position.
Such that if position sensitivities to those risk factors are calculated, then the value change observed over a day can be attributed to the market price change of those risk factors, with the magnitude of the estimated as a sum product of the risk factor sensitivities and the corresponding daily risk factor price change.
Any residual P&L left unexplained (P&L Unexplained) would be expected to be small if the identified risk factors are indeed sufficient to materially explain the expected value change of the position AND if the models used to calculate sensitivities to these risk factors are correct.
P&L Unexplained is thus a critical metric that when large may highlight instances where the risk factors classified for a risky position are incomplete or the models used for sensitivities calculations are incorrect or inconsistent.

This expression is equivalent to pricing our 20Y interest rate swap with our shifted delta which the first order risk factor calculated as below minus of the present value of our IRS (interest rate rate swap) with a non shifted delta:
1- Compute an initial PV for the considered trade or portfolio of trades
2- Perturb the first tenor of the yield curve and rebuild the pricing market keeping all other pricing parameters in $P_mkt(t_{SOD})$ constant.
3- Re-price the considered trade or portfolio of trade on the perturbed market. PV difference between (1) and (3) is the sensitivity (or risk) to the first tenor
4- Repeat the same sequence for all other tenors in the curve. Note that it is not recommended to reset the perturbation done on (2) before doing the next perturbation.

3.2.2 Cross risk factors step re-evaluation P&L

Generally portfolios may exhibit a significant dynamic and thus the previous method won't be valid. This abnormality would manifest in a large Unexplained term.
In this case we need to push Taylor expansion to the second order.

$$V(P_{mkt(t_COB)}) = V(P_{mkt(t_SOD)}) + \nabla^T \delta + \frac{1}{2}\delta^T A^V \delta + UnExplained$$

With A^V is the Hessian matrix:

$$A^V_{i,j} = \frac{\partial^2 V(P_{mkt(t)})}{\partial P_i \partial P_j}$$

with $i, j \in [\![1, N]\!]$.
Practically, we define the new steps $SR^*_{i,i}$ of the bump-and-reset explain capturing the second order and cross risk P&L:

$$SR_{i,j} = V(P_{mkt}(t) + \delta_{SR_{i,j}}) - V(P_{mkt(t)}) - SR_i - SR_j.$$

with:

$$\delta_{SR_{i,j}} = \begin{bmatrix} \\ 0 \\ \vdots \\ \delta_i \\ 0 \\ \vdots \\ \delta_j \\ 0 \end{bmatrix}$$

. The computation cost needs an extra $\binom{N}{2}$ valuations to capture the cross risk P&L so the number of steps is now quadratic.

4 Risk based P&L calculation

Risk factor sensitivities from the prior close (T-1) are used to attribute the P&L expected from changes from the prior market close in the market price of the risk factors.
Once risks have been computed, the P&L can be computed from the definition of the taylor expansion of the valuation function V, generally speaking by multiplying the risk by the change in the underlying market parameters.

The valuation function of a given trades or portfolio of trade can be evaluated at t_{COB} starting from its value at t_{SOD}.

For a linear portfolio a first order theta-delta only approximation is sufficient. However, most derivative instruments are non-linear and have some curvature e.g. interest rate swaps are quasi-linear in the floating leg and have some gamma. So more conservatively, a second order theta-delta-gamma Taylor approximation is more likely. And where the positions include optionality then a second-order theta-delta-gamma-vega Taylor approximation is chosen. Below shows a second-order theta-delta-rho-gamma-vega-volga-vanna Taylor approximation that would be appropriate for a portfolio of complex options on a single factor.

$$V(P_{mkt(t_{COB})}) = \theta\Delta t + \delta\Delta S + \frac{1}{2}\gamma\Delta S^2 + \cdots .$$

with:
θ representing the rate of change between the option price and time, or time sensitivity - sometimes known as an option's time decay.
δ representing the rate of change between the option's price and a 1 unit change in the underlying asset's price.
γ representing the rate of change between an option's delta and the underlying asset's price.
Method's limitations:

- It is not very accurate since some risk factors can be missing and this the P&L won't be fully explained.
- The necessary volume of data to store can be unmanageable.

5 Results

5.1 Step revaluation Limitations

The P&L computation of a given portfolio (ie. trading book) consists in computing its present value in a given market context. The step revaluation approach consists in:

- Classifying the differences between two P&L computations at D-1 and D.
- Computing some intermediary P&L values associated to this classification and then allocate the P&L contribution for each category of difference.

Then we can choose a given level of details for the classification and categorize all the differences between D-1 and D. The P&L contribution for each category can be computed in two ways:
In a cumulative mode (path dependent): i.e. starting from the D-1 P&L computation, we import step by step each category of difference and for each its P&L contribution. After the final step, there may be an unexplained effect when comparing the final importation step and the D P&L value if the classification does not cover all potential differences. Otherwise there should not be any unexplained effect in theory.

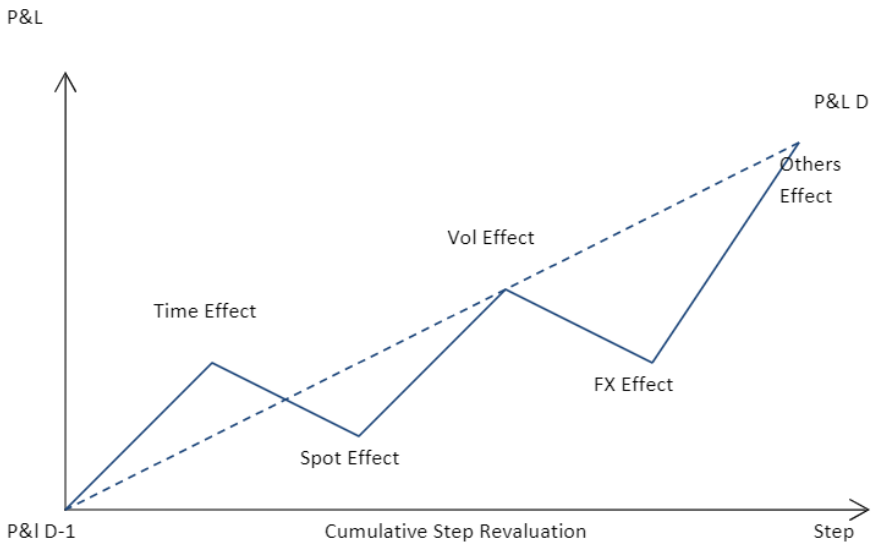

Figure 4: Cumulative step revaluation P&L calculation result based January 2020 Bnp data

In a star mode (path independent): i.e. for each category, the P&L contribution is always computed from the D-1 P&L computation. We may also start in a cumulative way and switch to the star mode. This approach may result in

an unexplained effect even for a complete classification.

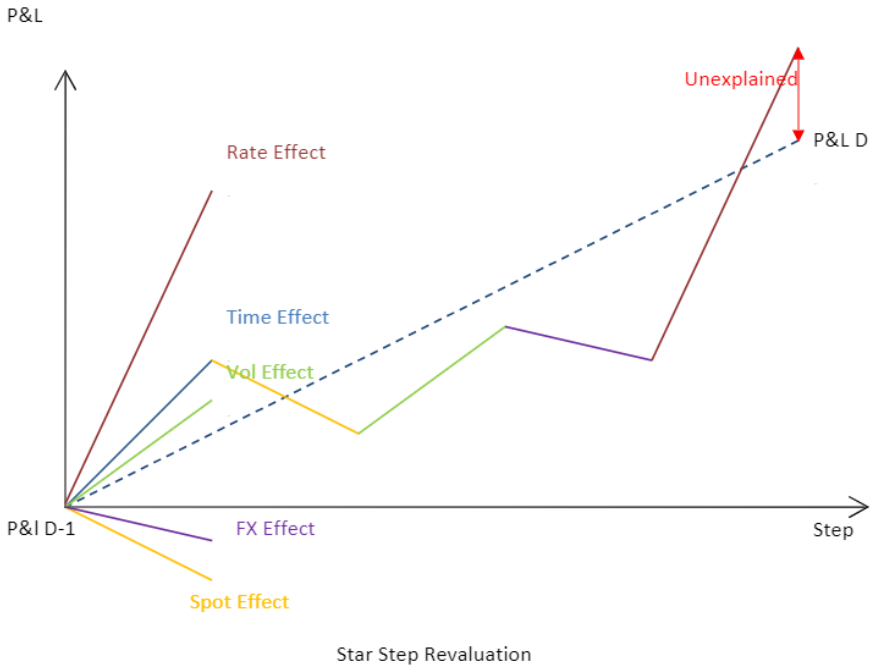

Figure 5: Star step revaluation P&L calculation result

There are difficulties for the stepped revaluation method, since in order to price the contract we generally use Monte Carlo simulation. This method does not provide the true present value of a contract but only an approximated value. As the target of this document is not to present the Monte Carlo simulation, we will consider all the underlying mathematical concepts as known, but we will try to describe their consequences in a way that can be understood by a large audience. Then we can consider that, for most of the stocks books with Monte Carlo calculations, the computed P&L value is in fact the real one plus a "brownian noise" (i.e. a trust interval). And the problem is that this noise depends of the pricing context: pricing a contract alone or with some others but also technical differences in the market data representation (let's think about a null pillar on a dividend curve and the same curve without the pillar). Note that PDE (partial differencial equaltions) computations are not fully exact also

(e.g. grid discretization) but the error does not depend of the pricing context and then there are no painful consequences for the step revaluation.

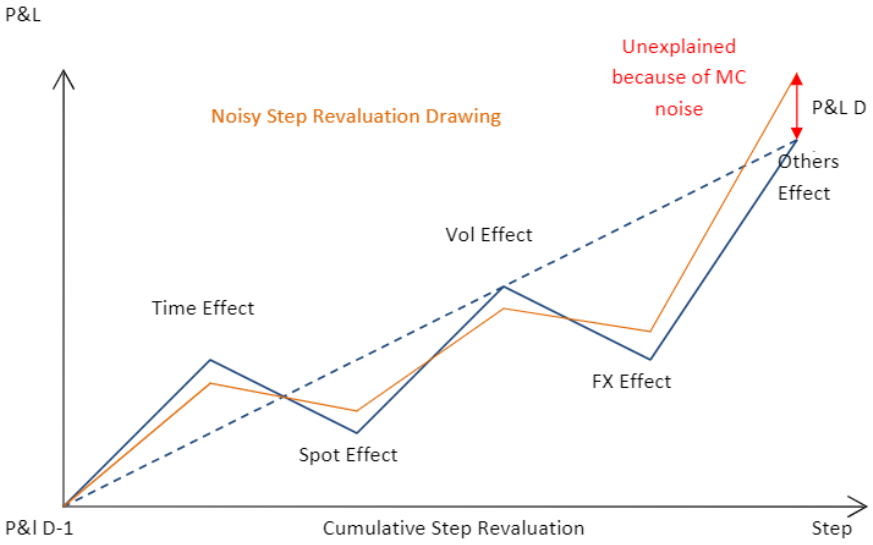

Figure 6: Noisy step revaluation P&L calculation result

For the step revaluation effect computation, this brownian noise is polluting a lot the results because the effect contributions are computed as premium differences and then the noise can be multiplied by two in the worst case. Then even for a cumulative step revaluation computation with a complete classification, there can be some unexplained P&L while the theory says there is no.

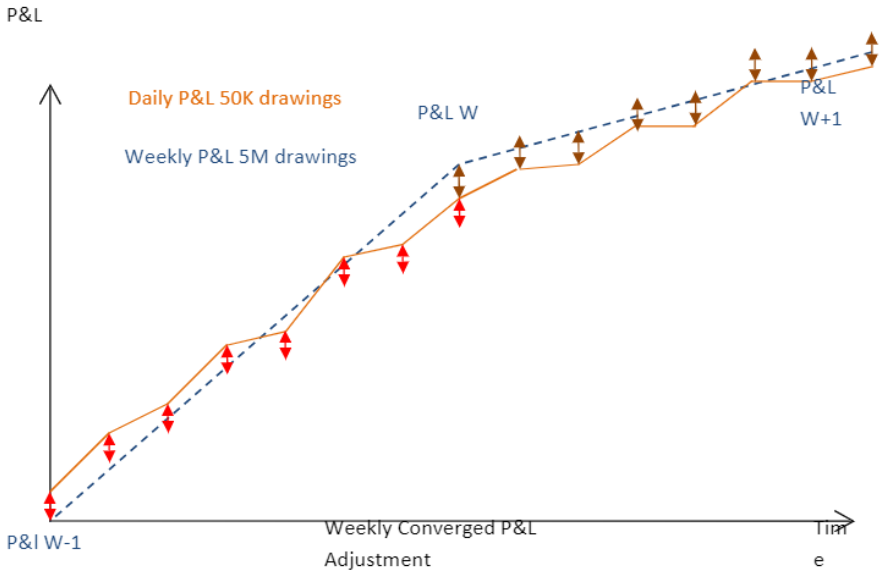

Figure 7: Weekly P&L 5M graph convergeance

5.2 Comparison of the P&L explain calculation methods

To present one of the most risk based method limitations, let's take an extreme shock scenario. Since we focus on the underlying, we can neglect the Theta, Vega and cross-gammas for our P&L prediction. Then we take the delta-gamma expansion of the P&L with s the perturbation of spot we have.

$$V(P_{mkt(t_{COB})}) - V(P_{mkt(t_{COB})}) = \delta s + \frac{1}{2}\gamma s^2$$

Let the valuations function be a simple valuation of a Call option for example. We can replace gamma by her finite difference approximation, which gives:

$$V(C+s) - V(C) = \delta s + \frac{1}{2} * \frac{\delta_{c+s} - \delta_c}{S} s^2$$

$$V(C+s) - V(C) = \delta s + \frac{s}{2} * \delta_{c+s} - \delta_c$$

This is a first order method. For large shocks, the approximation error (hence the unexplained PnL) becomes material, so the method diverges.

5.2.1 Empirical result

Let's consider a 1 year maturity Call option with 2 percent strike, with shocks from 10bps to 3 percent. The delta-gamma (second order Taylor expansion) prediction method diverges as the shock increases to 2 percent.

shock	0.001	0.002	0.003	0.004	0.005	0.009	0.011	0.015	0.020
delta-gamma	0.000	0.001	0.006	0.016	0.032	0.131	0.187	0.288	0.395

Figure 8: Empirical result of P&L calculation result

We can see that as the shock increases our delta-gamma P&L increases to diverge so the unexplained factor diverges too. To generalise, Taylor expansion method converges only for large orders, so we need high order sensitivities to be stocked on a daily basis in order to compute our P&L explain. It is very expensive and needs large data-base, that's why the re-evaluation method tend to be more practical since it shocks every risk factor separately with (1bp basis point) for example and evaluates the impact on the P&L of our contract.

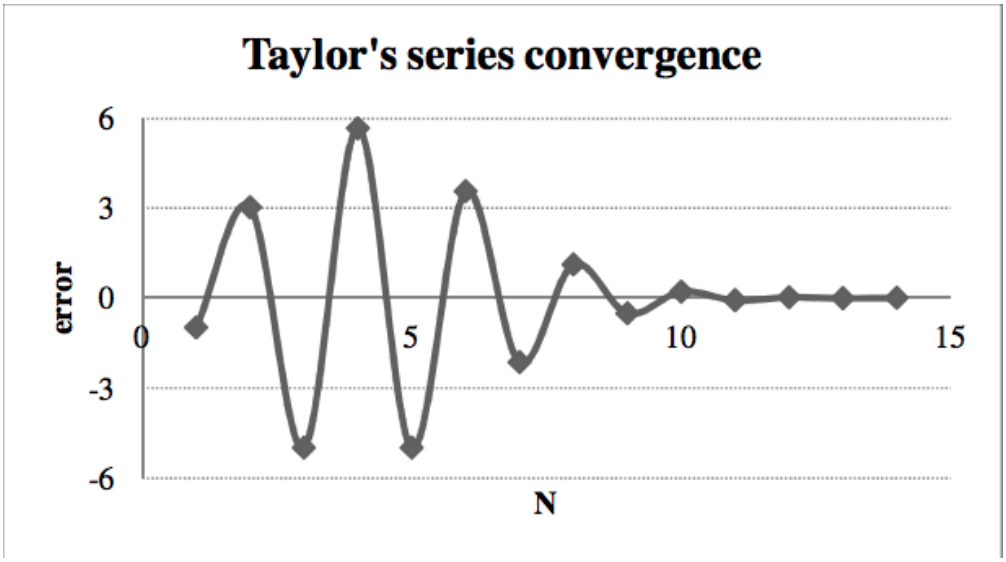

Figure 9: Divergence of Taylor expansion method for small orders

5.2.2 Comparison conclusion

So as we have seen this scenario showcases the divergence of our method for a simple contract and helps us understand one of the method's limitations. At the industrial level, on a big scale portfolios this method still can be used under the convergence condition because it partially helps explain our P&L despite the accuracy problem. For the re-evaluation method, it is an exact method as the theory behind is very precise and doesn't include a lot of approximations. Since it doesn't have to stock all the sensitivities, it is actually faster to compute. The most important limitation of this method is that it is expensive to rerun during the day, because it should import all the changes in the market data and shock our risk factors successively to see the impact on the P&L, that's why it is generally computed at the end of our business day.
So both method present advantages and disadvantages, but we choose the adapted method depending on our financial products and the risk factors they may include.

	Step re-evaluation	**Risk based**
Pros	-Exact methodology. -Faster than risk-based calculation wise.	-Provide link between risk factors and P&L -Fast intraday predict if we assume and risk factors are available
Cons	-Expensive to re-run for intraday result. -Only provides an allocation of the P&L with no risk and market moves.	-Not all P&L effects can be computed risk-based -Accuracy cannot be guaranteed

We need to mention that the previous analysis is based on the market effects as it analyses the P&L explained by the Greeks. In real markets life, other parameters may play a big role in the P&L result fluctuation such as time between the close of business and start of day and other environmental effect that doesn't enter in the scope of this thesis analysis.
I chose quantitative parameters to make the study very concrete using mathe-

matical equations.

6 Conclusion & Extension

Unlike risk sensitivities based explain P&L, re-evaluation method provide an aggregate measure of the P&L impact due to a risk factor's market price change; without differentiating between and attributing to the varying rates of value change embedded in the relationship between portfolio value and the risk factor. It is important to note this distinction between risk based explain and re-evaluation methods' aggregate measures as they are sometimes used interchangeably and sometimes netted together.
Re-evaluation methods may isolate the P&L impact of the change in value of an interest rate swaption to a change in the forward curve but have nothing to say about how much of that change was due to first order effect of that change i.e. delta and how much was due to second order effects i.e. gamma and so on. This is analogous to estimating the velocity of a car driven at varying rates of acceleration between points 1 and 2 simply by dividing the distance covered by the time taken but without attribution to the segments of the journey with different foot-to-pedal action.

In addition to helping to enrich and evolve the bank's models, the P&L explained processes described above help build a rich data set on P&L and its linkages into risk, VAR(Value At Risk) and CVA(credit value adjustment).

A data set that can be mined through analysis for insight on P&L drivers, strategy and trade performance across all components of P&L and much more. Unexplained P&L and VAR breaches also provide information about operational problems and the root cause analysis to close these out can help build a very rich picture of the efficacy of a bank's operational infrastructure and better target remediation efforts. However, a **2010 Ernst Young survey of 14 global financial institutions** found that on average their survey respondents'product controllers spent only 13% of their time on the analysis and explain of P&L; but spent 46% of the time on the production of P&L, and another 22% on reporting and sign off. This distribution of effort is worrying but the relative amount of time taken up by production is likely a good indicator of the weaknesses that often exist and persist in the infrastructure and processes.

Hopefully the degree of focus that regulators have paid to the P&L explained production processes over the last 2 years may encourage progress in that sense that would increase the banks profit and rebalance the efforts.

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TRITA SCI-GRU 2020:335