CS 581 – Advanced Artificial Intelligence

Topic: Decision Making Under Uncertainty

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UNCERTAINTY

- The agent needs reason in an uncertain world
- Uncertainty can be due to
  - Noisy sensors (e.g., temperature, GPS, camera, etc.)
  - Imperfect data (e.g., low resolution image)
  - Missing data (e.g., lab tests)
  - Imperfect knowledge (e.g., medical diagnosis)
  - Exceptions (e.g., all birds fly except ostriches, penguins, birds with injured wings, dead birds, …)
  - Changing data (e.g., flu seasons, traffic conditions during rush hour, etc.)
  - …
- The agent still must act (e.g., step on the breaks, diagnose a patient, order a lab test, …)

A Few Examples

- Spam filtering
- Medical diagnosis
- Loan approval
- Automated driving
- …

Rational Agent

- Given
  - World states, a utility function, actions, transitions, evidence, and probabilities
- A rational agent chooses the action that maximizes expected utility

$$action = \underset{a}{\operatorname{argmax}}\, EU(a|\mathrm{e})$$

Utility Theory

- Lottery: n possible outcomes with probabilities
  - $[p_1, S_1; p_2, S_2; \ldots p_n, S_n]$
  - Each S_i can be an atomic state or another lottery
- **Expected utility of a lottery**
  - $EU([p_1, S_1;\ p_2, S_2;\ \ldots\ p_n, S_n])\ = \sum_{i=1}^{n} p_i S_i$

Utility ≠ Money

- Most agents prefer more money to less money,
  - But this does not mean money behaves as a utility function
- For example, which lottery would you prefer
  - L_1: [1, \$1 Million]
  - L_2: [0.5, \$0; 0.5, \$2.5 Million]
- If money served as a utility function, then you'd prefer L_2 no matter what, but the answer *often* depends on how much money you currently have
  - The utility of money depends on what you *prefer*
    - If you are short on cash, a little more *certain* money can help
    - If you are already billionaire, you might take the risk
    - Or if you are swimming in debt, you might like to gamble

Utility ≠ Money

- Let's say you currently have $k and let S_k represent the state of having $k
- $EU(L_1) = U(S_{k+1M})$
- $EU(L_2) = 0.5*U(S_k) + 0.5*U(S_{k+2.5M})$
- The rational choice depends on your preferences for S_k, S_{k+1M}, and $S_{k+2.5M}$
  - i.e, it depends on the values of $U(S_k)$, $U(S_{k+1M})$, and $U(S_{k+2.5M})$
- $U(S_i)$ does not have to be a linear function of i, and for people it often is not

We'll Cover

- Single decision
  - Influence diagrams
- Sequence of actions
  - Markov decision processes

INFLUENCE DIAGRAMS

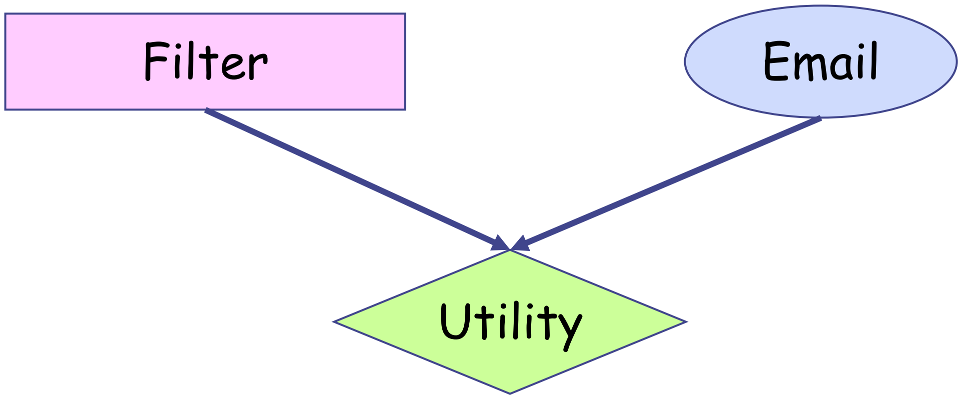
Influence Diagrams / Decision Networks

- Builds on Bayesian networks
- In addition to the chance nodes (ovals), decision networks have
  - Decision nodes – square
    - Represents actions
  - Utility nodes – diamond
    - Represents utilities for possible states and actions

Spam Filtering

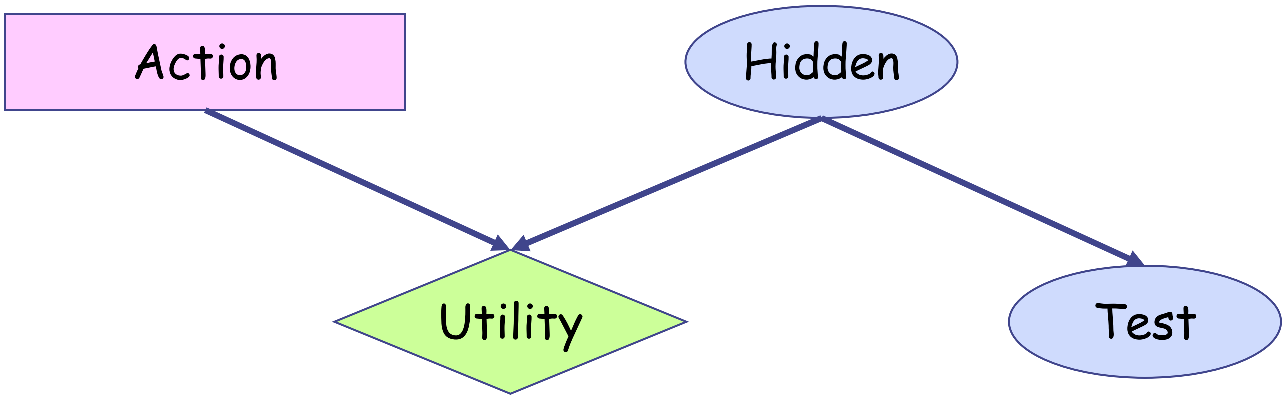
inbox/junkfolder

P(spam) = 0.60



U(inbox, spam) = -300
U(inbox, ~spam) = 100
U(junkfolder, spam) = 200
U(junkfolder, ~spam) = -400

A Common Pattern



Decision networks - applications

- Used for
  - What action to take
  - What information to gather
  - How much to pay for a piece of information
- For example:
  - Medical diagnosis: which test to perform, which treatment to prescribe, …
  - Marketing: which project to invest in, how much to spend on marketing, how much to spend on user surveys, …

Evaluating decision networks

- Set evidence nodes **E** to their values **e**
- For each choice **a** of action **A**
  - Set **A=a**
  - Compute the posterior probability of the parent chance nodes of the utility node; i.e., compute P(Pa(Utility) | **e, a**)
  - Compute expected utility using the utility node and the probability distribution P(Pa(Utility) | **e, a**)
- Choose action **a** with the maximum expected utility

Examples

- See OneNote

Value of Information

- If I am allowed to observe the value of a chance node, how much valuable is that information to me?
- Value of information
  - Expected utility after the information is acquired
    - Minus
  - Expected utility before the information is acquired
- There is one catch: we do not know the content of the information before we acquire it
  - Solution: take an expectation over the possible outcomes