Competitive Programming Library

rwx 2025/2026

Contents

1 Templates

1.1 Setup

1.1.1 IO Manipulation

Input/Output

```cpp
#include <bits/stdc++.h>

freopen("input.txt", "r", stdin);
freopen("output.txt", "w", stdout);

#define fastIO          \
  ios_base::sync_with_stdio(false), cin.tie(nullptr), cout.tie(nullptr);
```

1.1.2 GCC Compiler Optimization (Vectorization)

GCC Opt

```cpp
// Ref: USACO guide
// will make GCC auto-vectorize for loops and optimizes floating points
    better (assumes associativity and turns off denormals).
#pragma GCC optimize ("Ofast")
// can double performance of vectorized code, but causes crashes on old
    machines.
#pragma GCC target ("avx,avx2,fma")

// slows down run time but throws a Runtime Error if an overflow occured
#pragma GCC optimize("trapv")
```

1.2 MOD Template

```cpp
constexpr int MOD = 1e9+7; // must be a prime number

int add(int a, int b) {
    int res = a+b;
    if(res >= MOD) return res -= MOD;
}

int sub(int a, int b) {
    int res = a-b;
    if(res < 0) return res += MOD;
}

int power(int a, int e) {
    int res = 1;
    while(e) {if(e & 1) res = res * a % MOD; a = a * a % MOD;
    e >>= 1;}
    return res;
}


-----------------------------------------------------------
                                                          |
// note -> from Fermat little theorem, we can deduce that|
// a^(BIG) % prime = (a ^ (BIG%(prime - 1)))%prime        |
// because every a^(prime - 1) is equal to 1              |
// from the identity says a^(prime - 1) % prime = 1       |
                                                          |
-----------------------------------------------------------

int inverse(int a) {
    return power(a, MOD-2);
}

int div(int a, int b) {
    return a * inverse(b) % MOD;
}
```

MOD Template

1.3 Macros

Macros

```cpp
#define getBit(n, k) (n >> k) // & 1
#define ON(n, idx) (n | (1ll << idx))
#define OFF(n, idx) (n & ~(1ll << idx))
#define toggle(n, idx) ((n) ^ (1ll<<(idx)))
#define gray(n) (n ^ (n >> 1))
#define bitCount(x) (__builtin_popcountll(x))
```

```
#define clz(x) (__builtin_clzll(x))
#define ctz(x) (__builtin_ctzll(x))
#define uniq(x) x.resize(unique(x.begin(), x.end())-x.begin());

#define angle(a) (atan2((a).imag(), (a).real()))
//#define vec(a, b) ((b)-(a))
#define same(v1, v2) (dp(vec(v1,v2),vec(v1,v2)) < EPS)
#define dotProduct(a, b) ((conj(a)*(b)).real()) // a*b cos(T), if zero ->
    prep
#define crossProduct(a, b) ((conj(a)*(b)).imag()) // a*b sin(T), if zero
    -> parallel
//#define length(a) (hypot((a).imag(), (a).real()))
#define normalize(a) ((a)/length(a))
#define rotateO(v, ang) ((v)*exp(point(0,ang)))
#define rotateA(p, ang, about) (rotateO(vec(about,p),ang)+about)
#define reflectO(v, m) (conj((v)/(m))*(m))
#define ceil_i(a, b) (((ll)(a)+(ll)(b-1))/(ll)(b))
#define floor_i(a, b) (a/b)
#define round_i(a, b) ((a+(b/2))/b) // if a>0
#define round_m(a, b) ((a-(b/2))/b) // if a<0
#define round_multiple(n, m) round_i(n,m)*m // round to multiple if
    specified element

const double PI = acos(-1.0);
```

1.4 Grid Navigation

Grid Nav

```
// knight moves on a chess board
int dx[] = { -2, -1, 1, 2, -2, -1, 1, 2 };
int dy[] = { -1, -2, -2, -1, 1, 2, 2, 1 };

// Grid up, down, right, left (Moves for Chess Rook)
int dx[4] = {1, -1, 0, 0};
int dy[4] = {0, 0, 1, -1};

// Grid cell all neighbours
const int dx[8] = {1, 0, -1, 0, 1, 1, -1, -1}
const int dy[8] = {0, 1, 0, -1, -1, 1, -1, 1};
// Grid Diagonal (Moves for Chess Bishop)
int dx[] = {1, 1, -1, -1};
int dy[] = {1, -1, 1, -1};
```

1.5 Integer 128

i128

```
typedef __int128 i128;

__int128 read() {
    __int128 x = 0, f = 1;
    char ch = getchar();
    while (ch < '0' || ch > '9') {
        if (ch == '-') f = -1;
        ch = getchar();
    }
    while (ch >= '0' && ch <= '9') {
        x = x * 10 + ch - '0';
        ch = getchar();
    }
    return x * f;
}

void print(__int128 x) {
    if (x < 0) {
        putchar('-');
        x = -x;
    }
    if (x > 9) print(x / 10);
    putchar(x % 10 + '0');
}

bool cmp(__int128 x, __int128 y) { return x > y; }
```

1.6 Matrix Expo

MExpo

```
constexpr int MAX_N = 201, MOD = 1e9+7;
```

```
struct Matrix { int mat[MAX_N][MAX_N]; }; // we return a 2D array
ll mod(ll a, ll m) { return ((a%m)+m) % m; } // ensure positive answer
Matrix matMul(Matrix a, Matrix b) { // normally O(n^3)
    Matrix ans{}; // but O(1) as n = 2
    for (auto & i : ans.mat)
        for (int & j : i)
            j = 0;

    for (int i = 0; i < MAX_N; ++i)
        for (int k = 0; k < MAX_N; ++k) {
            if (a.mat[i][k] == 0) continue; // optimization
            for (int j = 0; j < MAX_N; ++j) {
                ans.mat[i][j] += mod(a.mat[i][k], MOD) * mod(b.mat[k][j],
                    MOD);
                ans.mat[i][j] = mod(ans.mat[i][j], MOD);
            }
        }
    return ans;
}
Matrix matPow(Matrix base, int p) { // normally O(n^3 log p)
    Matrix ans{}; // but O(log p) as n = 2
    for (int i = 0; i < MAX_N; ++i)
        for (int j = 0; j < MAX_N; ++j)
            ans.mat[i][j] = (i == j); // prepare identity matrix
    while (p) { // iterative D&C version
        if (p&1) // check if p is odd
            ans = matMul(ans, base); // update ans
        base = matMul(base, base); // square the base
        p >>= 1; // divide p by 2
    }
    return ans;
}
Matrix matMul(Matrix a, Matrix b, int p, int q, int r) { // O(pqr)
    Matrix c{};
    for (int i = 0; i < p; ++i)
        for (int j = 0; j < r; ++j) {
            c.mat[i][j] = 0;
            for (int k = 0; k < q; ++k)
                c.mat[i][j] += a.mat[i][k] + b.mat[k][j];
        }
    return c;
}
```

2 Dynamic Programming

2.1 Some dp patterns

Maximumu/Minimum path cost

```
const int MAX = 21;
int grid[MAX][MAX];
int mem[MAX][MAX];
int n = 20;
bool valid(int r, int c){
  return r >= 0 && r < n && c >= 0 && c < n;
}

int maxPathSum(int r, int c){
    if(!valid(r,c)){
        return 0;
    }

    if(r == n-1 && c == n-1){
        return mem[r][c] = grid[r][c];
    }
    // available moves
    int path1 = maxPathSum(r+1,c);
    int path2 = maxPathSum(r,c+1);

    return grid[r][c] + max(path1,path2);
}
```

add operators between numbers to get max prod/sum

```
// put +, -, between sequence of numbers such that the sum is divisible by
     k, and maximum as possible
const int MAX = 21;
long long mem[MAX][MAX];
const int n = 20;
int k = 4; // example
int v[20];
int fix(int a){
  return (a % k + k) % k;
}
long long tryAll(int pos, int mod){
```

```
    long long &ret = mem[pos][mod];
    if(ret != -1){
        return ret;
    }
    if(pos == n){
        return ret = mod == 0;
    }
    if(tryAll(pos+1,fix(mod + v[pos])) || tryAll(pos+1,fix(mod-v[pos]))){
        return ret = 1;
    }
    return ret = 0;
}
```

pick choices with no two similar consecutive choices

```
// pick minimum of choinces costs with no two similar consecutive choices
const int choices = 4;
const int n = 20;
int MAX = n;
int mem[MAX][choices];
const int OO = 1e6+1;
int minCost(int pos, int lastChoice){
    if(pos == n){
        return 0; // invalid move
    }
    int &ret = mem[pos][lastChoice];

    if(ret != -1){
        return ret;
    }

    ret = OO; // want to minimze
    // let choices are 0, 1, 2
    if(lastChoice != 0){
        ret = min(ret, minCost(pos+1,0));
    }
    if(lastChoice != 1){
        ret = min(ret, minCost(pos+1,1));
    }
    if(lastChoice != 2){
        ret = min(ret, minCost(pos+1,2));
    }
    return ret;
}
```

sum S and max/min Product

```
int maxK;

ll mem[21][101]; // k, and s

// You are given an integer s and an integer k. Find k positive integers
    a1, a2, ..., ak
// such that their sum is equal to s and their product is the maximal
    possible. Return their product.

ll maxProd(int k, int rem)
{
 if(k == maxK){
  // base case
  if(rem == 0)
   return 1;
  return 0;
 }

 if(rem == 0) // invalid case
  return 0;

 ll &ret = mem[k][rem];

 if(ret != -1)
  return ret;

 ret = 0;

 for (int i = 1; i <= rem; ++i) {
  ll sol = maxProd(k+1, rem - i) * i;
  ret = max(ret, sol);
 }

 return ret;
}
```

2.2 DP solutions

2.2.1 Max Subarray sum (Kadane's Algorithm)

Max Subarray sum

```
int maxSubarraySum(vector<int>& arr, int len) {
    int ans = INT_MIN, cur = 0;

    for (int i = 0; i < len; i++) {
        cur = cur + arr[i];
        if (ans < cur)
            ans = cur;

        if (cur < 0)
            cur = 0;
    }

    return ans;
}
```

2.2.2 Maximum Subarray Alternating Sum

Maximum Subarray Alternating Sum

```
/* REF: GeeksForGeeks
 Input: arr[] = {-4, -10, 3, 5}
 Output: 9
 Explanation: Subarray {arr[0], arr[2]} = {-4, -10, 3}. Therefore, the sum
      of this subarray is 9.
*/
int maxSubarraySumALT(vector<int>& a, int len) {
    int ans = INT_MIN, cur = 0;
    for (int i = 0; i < len; i++) {
        if (i % 2 == 0)
            cur = max(cur + a[i], a[i]);
        else
            cur = max(cur - a[i], -a[i]);

        ans = max(ans, cur);
    }

    cur = 0;
    for (int i = 0; i < len; i++) {
        if (i % 2 == 1)
            cur = max(cur + a[i], a[i]);
        else
            cur = max(cur - a[i], -a[i]);

        ans = max(ans, cur);
    }

    return ans;
}
```

2.2.3 Count number of DISTINCT ordered ways to produce coins sums to x

Count distinct

```
/*
For example, if the coins are \{2,3,5\} and the desired sum is 9, there
    are 3 ways:

2+2+5
3+3+3
2+2+2+3
*/
int n, x;
cin >> n >> x;
vector<int> coins(n);
read(coins);

vector dp(x + 1, 0);

dp[0] = 1;
for (int i = 0; i < n; ++i) {
 for (int j = coins[i]; j <= x; ++j) {
  dp[j] = add(dp[j], dp[j - coins[i]]);
 }
}

cout << dp[x] << el;
```

2.2.4 Min absolute difference between 2 elements from (L, R) (DP Ranges)

Min absolute difference

```
const int N = 1e4 + 1;

int dp[N][N];

int n;
cin >> n;
vector<int> a(n);
read(a);

for (int i = 0; i < n; ++i) dp[i][i] = 1e6; // INF, you can't take the
    element with it self
for (int i = 1; i < n; ++i) dp[i - 1][i] = abs(a[i] - a[i - 1]);

for (int len = 3; len <= n; ++len) {
 for (int l = 0, r = len - 1; r < n; ++l, ++r) {
  dp[l][r] = min(dp[l][r - 1], dp[l + 1][r]);
  dp[l][r] = min(dp[l][r], abs(a[l] - a[r]));
 }
}

int q;
cin >> q;
while (q--) {
 int l, r;
 cin >> l >> r;
 --l, --r;

 cout << dp[l][r] << el;
}
```

2.2.5 Longest common subsequence between 2 Strings

$$\text{dp}[i][j] = \begin{cases} \max(\text{dp}[i-1][j], \text{dp}[i][j-1]) & \text{if } A_i \neq B_j \\ \text{dp}[i-1][j-1] + 1 & \text{if } A_i = B_j \end{cases}$$

LIS 2 Strings

```
// REF: USACO guide
int longestCommonSubsequence(string a, string b) {
 int dp[a.size()][b.size()];
 for (int i = 0; i < a.size(); i++) { fill(dp[i], dp[i] + b.size(), 0); }
 for (int i = 0; i < a.size(); i++) {
  if (a[i] == b[0]) dp[i][0] = 1;
  if (i != 0) dp[i][0] = max(dp[i][0], dp[i - 1][0]);
 }
 for (int i = 0; i < b.size(); i++) {
  if (a[0] == b[i]) dp[0][i] = 1;
  if (i != 0) dp[0][i] = max(dp[0][i], dp[0][i - 1]);
 }
 for (int i = 1; i < a.size(); i++) {
  for (int j = 1; j < b.size(); j++) {
   if (a[i] == b[j]) {
    dp[i][j] = dp[i - 1][j - 1] + 1;
   } else {
    dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
   }
  }
 }
 return dp[a.size() - 1][b.size() - 1];
}
```

2.2.6 Longest common subsequence $\mathcal{O}(n^2)$

LIS

```
// REF: cp-algorithms
int lis(vector<int> const& a) {
    int n = a.size();
    vector<int> d(n, 1);
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < i; j++) {
            if (a[j] < a[i])
                d[i] = max(d[i], d[j] + 1);
        }
    }

    int ans = d[0];
    for (int i = 1; i < n; i++) {
```

```cpp
        ans = max(ans, d[i]);
    }
    return ans;
}

// Restoring
vector<int> lis(vector<int> const& a) {
    int n = a.size();
    vector<int> d(n, 1), p(n, -1);
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < i; j++) {
            if (a[j] < a[i] && d[i] < d[j] + 1) {
                d[i] = d[j] + 1;
                p[i] = j;
            }
        }
    }

    int ans = d[0], pos = 0;
    for (int i = 1; i < n; i++) {
        if (d[i] > ans) {
            ans = d[i];
            pos = i;
        }
    }

    vector<int> subseq;
    while (pos != -1) {
        subseq.push_back(a[pos]);
        pos = p[pos];
    }
    reverse(subseq.begin(), subseq.end());
    return subseq;
}
```

2.2.7 Longest common subsequence Binary Search $\mathcal{O}(n + \log N)$

LIS

```cpp
int lisBS(vector<int> const& a) {
    int n = a.size();
    const int INF = 1e9;
    vector<int> d(n+1, INF);
    d[0] = -INF;

    for (int i = 0; i < n; i++) {
        int l = upper_bound(d.begin(), d.end(), a[i]) - d.begin();
        if (d[l-1] < a[i] && a[i] < d[l])
            d[l] = a[i];
    }

    int ans = 0;
    for (int l = 0; l <= n; l++) {
        if (d[l] < INF)
            ans = l;
    }
    return ans;
}
```

3 Bit Manipulation

3.1 Subset Operations

count subsets with give sum

```cpp
int countDistinctSubsetsWithSum(vector<int>& arr, int n, int k) {
    // Count distinct subsets of array arr that sum up to k
    vector<int> dp(k + 1, 0);
    dp[0] = 1;
    for (int i = 0; i < n; ++i) {
        for (int j = k; j >= arr[i]; --j) {
            dp[j] += dp[j - arr[i]];
        }
    }
    return dp[k]; // Number of distinct subsets with sum k
}
```

max xor of any subset of elements in the array

```cpp
int maximalSubsetXOR(vector<int>& arr, int n) {
    // Find the maximum XOR of any subset of elements in array arr
    int maxXor = 0;
    for (int mask = 0; mask < (1 << n); ++mask) {
```

```
        int xorSum = 0;
        for (int i = 0; i < n; ++i) {
            if (mask & (1 << i)) {
                xorSum ^= arr[i];
            }
        }
        maxXor = max(maxXor, xorSum);
    }
    return maxXor;
}
```

min xor of any subset

```
int minimumSubsetXOR(vector<int>& arr, int n) {
    // Find the minimum XOR of any pair of elements in array arr
    int minSubsetXor = INT_MAX;
    for (int mask = 0; mask < (1 << n); ++mask) {
        int xorSum = 0;
        for (int i = 0; i < n; ++i) {
            if (mask & (1 << i)) {
                xorSum ^= arr[i];
            }
        }
        minSubsetXor = min(minSubsetXor, xorSum);
    }
    return minSubsetXor;
}
```

subset generation

```
void subsetGeneration(int x, int n) {
    // Generate all non-empty subsets of a set represented by an integer x
    for (int subset = x; subset > 0; subset = (subset - 1) & x) {
        // Process subset
        cout << subset << endl;
    }
}
```

check if subset of elements in the array sum up to k

```
void subsetSumCheck(vector<int>& arr, int n, int k) {
    // Check if a subset of elements in array arr sums up to k
    for (int subset = 0; subset < (1 << n); ++subset) {
        int sum = 0;
        for (int i = 0; i < n; ++i) {
            if (subset & (1 << i)) {
                sum += arr[i];
            }
        }
        if (sum == k) {
            // Found subset with sum k
            cout << "Subset with sum " << k << ": " << subset << endl;
        }
    }
}
```

max subset sum mod m

```
int subsetWithMaxSumModuloM(vector<int>& arr, int n, int m) {
    // Find the maximum subset sum modulo m
    vector<int> dp(m, -1);
    dp[0] = 0;
    int currentMod = 0;
    for (int i = 0; i < n; ++i) {
        currentMod = (currentMod + arr[i]) % m;
        for (int j = 0; j < m; ++j) {
            if (dp[j] != -1) {
                dp[(j + currentMod) % m] = max(dp[(j + currentMod) % m], dp
                    [j] + arr[i]);
            }
        }
        dp[currentMod] = max(dp[currentMod], arr[i]);
    }
    return dp[0]; // Maximum subset sum modulo m
}
```

iterate over all supersets represented by x

```
void iterateOverSupersets(int x, int n) {
    // Iterate over all supersets of a set represented by x
    int subset = x;
    do {
        // Process subset
```

```
        cout << subset << endl;
        subset = (subset + 1) | x;
    } while (subset <= (1 << n) - 1);
}
```

4 Algorithms

4.1 FFT

FFT Algorithm

```
constexpr ll mod = 998244353, root = 3;

ll modpow(ll b, ll e, ll m) {
    ll ans = 1;
    for (; e; b = b * b % m, e /= 2)
        if (e & 1) ans = ans * b % m;
    return ans;
}

// Primitive Root of the mod of form 2^a * b + 1
int generator () {
    vector<int> fact;
    int phi = mod-1, n = phi;
    for (int i=2; i*i<=n; ++i)
        if (n % i == 0) {
            fact.push_back (i);
            while (n % i == 0)
                n /= i;
        }
    if (n > 1)
        fact.push_back (n);

    for (int res=2; res<=mod; ++res) {
        bool ok = true;
        for (size_t i=0; i<fact.size() && ok; ++i)
            ok &= modpow(res, phi / fact[i], mod) != 1;
        if (ok) return res;
    }
    return -1;
}

void ntt(vector<ll> &a) {
    int n = a.size(), L = 31 - __builtin_clz(n);
    static vector<ll> rt(2, 1); // erase the static if you want to use two
         moduli;
    for (static int k = 2, s = 2; k < n; k *= 2, s++) { // erase the
        static if you want to use two moduli;
        rt.resize(n);
        ll z[] = {1, modpow(root, mod >> s, mod)};
        for (int i = k; i < 2*k; ++i) rt[i] = rt[i / 2] * z[i & 1] % mod;
    }
    vector<int> rev(n);
    for (int i = 0; i < n; ++i) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    for (int i = 0; i < n; ++i) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2) {
        for (int i = 0; i < n; i += 2 * k) {
            for (int j = 0; j < k; ++j) {
                ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
                a[i + j + k] = ai - z + (z > ai ? mod : 0);
                ai += (ai + z >= mod ? z - mod : z);
            }
        }
    }
}

vector<ll> multi(const vector<ll> &a, const vector<ll> &b) {
    if (a.empty() || b.empty()) return {};
    int s = static_cast<ll>(a.size()) + static_cast<ll>(b.size()) - 1, B =
         32 - __builtin_clz(s), n = 1 << B;
    int inv = modpow(n, mod - 2, mod);
    vector<ll> L(a), R(b), out(n);
    L.resize(n), R.resize(n);
    ntt(L), ntt(R);
    for (int i = 0; i < n; ++i) out[-i & (n - 1)] = L[i] * R[i] % mod *
        inv % mod;
    ntt(out);
    return {out.begin(), out.begin() + s};
}

vector<int> poly_pow(vector<int> poly, int p, int limit = 1e9) {
    vector<int> ans{1};
    while (p) {
        if(p&1) ans = multi(ans, poly);
        poly = multi(poly, poly);
```

```
        ans.resize(limit + 1);
        poly.resize(limit + 1);
        p >>= 1;
    }
    return ans;
}
```

4.2 MO

MO Algorithm

```
// MO      -> O(N+Q SQRT(N)) <= 10^5

const int N = 1e5+5, M = 1e5+5;
int n, m;
int nums[N], q_ans[M];

struct query {
    int idx, block_idx, l, r;

    query() = default;
    query(int _l, int _r, int _idx) {
        idx = _idx;
        r = _r - 1;
        l = _l - 1;
        block_idx = _l / sqrt(n);
    }

    bool operator <(const query & y) const {
        if(y.block_idx == block_idx) return r < y.r;
        return block_idx < y.block_idx;
    }
};

int freq[N], ans;

void add(int idx) {
    freq[nums[idx]]++;
    if (freq[nums[idx]] == 2) ans++;
}

void remove(int idx) {
    freq[nums[idx]]--;
    if (freq[nums[idx]] == 1) ans--;
}

cin >> n >> m;
for (int i = 0; i < n; ++i) cin >> nums[i];

vector<query> Query(m);
for (int i = 0; i < m; ++i) {
    int l, r; cin >> l >> r;
    Query[i] = query(l, r, i);
}

sort(Query.begin(), Query.end());
int l0 = 1, r0 = 0;
for (int i = 0; i < m; ++i) {
    while (l0 < Query[i].l) remove(l0++);
    while (l0 > Query[i].l) add(--l0);
    while (r0 < Query[i].r) add(++r0);
    while (r0 > Query[i].r) remove(r0--);
    q_ans[Query[i].idx] = ans;
}
for (int i = 0; i < m; ++i) {
    cout << q_ans[i] << '\n';
}
```

4.3 Intervals

4.3.1 Prefix Sum (L, R) intervals

Prefix Sum (L, R) intervals

```
// NOTE: works fine with small n or with large memory

int main() {
    int n, k;
    cin >> n >> k;

    vector<int> a(n + 1);
    vector<vector<int>> rangesPrefix(n + 1, vector<int>(n + 1, 0));
    for (int i = 1; i <= n; ++i)
```

```
        cin >> a[i];

    int l = 1, r = 1, sum = 0;
    // validate your intervals
    // here the intervals are the ones that have a sum of k
    while (r <= n) {
        sum += a[r];

        while (sum > k) {
            sum -= a[l];
            ++l;
        }

        while (l <= r && a[l] == 0) {
            if (sum != k)
                break;

            rangesPrefix[r][l]++;

            ++l;
        }

        if (sum == k) {
            rangesPrefix[r][l]++;
        }

        ++r;
    }


    // prefix sum the columns
    for (int i = 1; i <= n; ++i) {
        for (int j = n - 1; j >= 0; --j) {
            rangesPrefix[i][j] += rangesPrefix[i][j + 1];
        }
    }

    // prefix sum the rows
    for (int i = 0; i <= n; ++i) {
        for (int j = 1; j <= n; ++j) {
            rangesPrefix[j][i] += rangesPrefix[j - 1][i];
        }
    }
    int q; cin >> q;

    while (q--) {
        cin >> l >> r;
        // answer the number of intervals (X, Y) X <= Y that are included
            between L, R
        cout << rangesPrefix[r][l] - rangesPrefix[l - 1][l] << el;
    }
}
```

4.3.2 Find subarrays intervals that sum to K Using Map

Find subarray intervals that sum to K Using Map

```
    int n, k;
    cin >> n >> k;

    vector<int> a(n + 1);
    vector<pair<int, int>> rng;
    for (int i = 1; i <= n; ++i)
        cin >> a[i];

    map<int, set<int>> prev;
    int currSum = 0;

    for (int i = 1; i <= n; ++i) {
        currSum += a[i];
        if (currSum == k) {
            rng.push_back({1, i});
        }
        if (prev.find(currSum - k) != prev.end()) {
            for (auto &j : prev[currSum - k]) {
                rng.push_back({j + 1, i});
            }
        }
        prev[currSum].insert(i);
    }
```

4.4 Ad-hoc

4.4.1 Find duplicate

Find duplicate using XOR

```
int findDuplicate(int arr[] , int n)
{
    int answer=0;
      //XOR all the elements with 0
    for(int i=0; i<n; i++){
       answer=answer^arr[i];
    }
      //XOR all the elements with no from 1 to n
      // i.e  answer^0 = answer
    for(int i=1; i<n; i++){
       answer=answer^i;
    }
    return answer;
}
```

4.5 Sorting Algorithms

4.5.1 Radix Sort

Radix Sort

```
// O(n + b), where n is the number of elements and b is the base of the
    number system
// A function to do counting sort of arr[] according to the digit
    represented by exp.
void countingSort(vector<int>& arr, int exp) {
    int n = arr.size();
    vector<int> output(n); // output array
    int count[10] = {0};

    // Store count of occurrences in count[]
    for (int i = 0; i < n; i++)
        count[(arr[i] / exp) % 10]++;

    // Change count[i] so that count[i] now contains the actual
    // position of this digit in output[]
    for (int i = 1; i < 10; i++)
        count[i] += count[i - 1];

    // Build the output array
    for (int i = n - 1; i >= 0; i--) {
        output[count[(arr[i] / exp) % 10] - 1] = arr[i];
        count[(arr[i] / exp) % 10]--;
    }

    // Copy the output array to arr[], so that arr now
    // contains sorted numbers according to the current digit
    for (int i = 0; i < n; i++)
        arr[i] = output[i];
}

// The main function to that sorts arr[] of size n using Radix Sort
void radixSort(vector<int>& arr) {
    // Find the maximum number to know the number of digits
    int mx = *max_element(arr.begin(), arr.end());

    // Do counting sort for every digit. Note that instead
    // of passing the digit number, exp is passed. exp is 10^i
    // where i is the current digit number
    for (int exp = 1; m / exp > 0; exp *= 10)
        countingSort(arr, exp);
}
```

4.5.2 Counting Sort

Counting Sort

```
// O(N+M), where N and M are the size of inputArray[] and countArray[]
// The main function that sorts arr[] of size n using Counting Sort
void countingSort(vector<int>& arr) {
    int maxElement = *max_element(arr.begin(), arr.end());
    int minElement = *min_element(arr.begin(), arr.end());
    int range = maxElement - minElement + 1;

    vector<int> count(range), output(arr.size());
    for (int i = 0; i < arr.size(); i++)
        count[arr[i] - minElement]++;

    for (int i = 1; i < count.size(); i++)
        count[i] += count[i - 1];
```

```
    for (int i = arr.size() - 1; i >= 0; i--) {
        output[count[arr[i] - minElement] - 1] = arr[i];
        count[arr[i] - minElement]--;
    }

    for (int i = 0; i < arr.size(); i++)
        arr[i] = output[i];
}
```

4.6 Apply permutation k times

permutation k times

```
// Applying a permutation k times
// n log k

vector<int> applyPermutation(vector<int> sequence, vector<int> permutation
    ) {
    vector<int> newSequence(sequence.size());
    for(int i = 0; i < sequence.size(); i++) {
        newSequence[i] = sequence[permutation[i]];
    }
    return newSequence;
}

vector<int> permute(vector<int> sequence, vector<int> permutation, long
    long k) {
    while (k > 0) {
        if (k & 1) {
            sequence = applyPermutation(sequence, permutation);
        }
        permutation = applyPermutation(permutation, permutation);
        k >>= 1;
    }
    return sequence;
}
```

5 Data Structures

5.1 Monotonuc

5.1.1 Min Aggregation Deque

Min Aggregation Deque

```
 struct minstack {
    stack<pair<int, int>> st;
    int getmin() { return st.top().second; }
    bool empty() { return st.empty(); }
    int size() { return st.size(); }
    void push(int x) {
        int mn = x;
        if (!empty()) mn = min(mn, getmin());
        st.push({x, mn});
    }
    void pop() { st.pop(); }
    int top() { return st.top().first; }
    void swap(minstack &x) { st.swap(x.st); }
};

struct mindeque {
    minstack l, r, t;
    void rebalance() {
        bool f = false;
        if (r.empty()) {
            f = true;
            l.swap(r);
        }
        int sz = r.size() / 2;
        while (sz--) {
            t.push(r.top());
            r.pop();
        }
        while (!r.empty()) {
            l.push(r.top());
            r.pop();
        }
        while (!t.empty()) {
            r.push(t.top());
            t.pop();
        }
```

```
        if (f) l.swap(r);
    }
    int getmin() {
        if (l.empty()) return r.getmin();
        if (r.empty()) return l.getmin();
        return min(l.getmin(), r.getmin());
    }
    bool empty() { return l.empty() && r.empty(); }
    int size() { return l.size() + r.size(); }
    void push_front(int x) { l.push(x); }
    void push_back(int x) { r.push(x); }
    void pop_front() {
        if (l.empty()) rebalance();
        l.pop();
    }
    void pop_back() {
        if (r.empty()) rebalance();
        r.pop();
    }
    int front() {
        if (l.empty()) rebalance();
        return l.top();
    }
    int back() {
        if (r.empty()) rebalance();
        return r.top();
    }
    void swap(mindeque &x) {
        l.swap(x.l);
        r.swap(x.r);
    }
};
```

5.2 Strings

5.2.1 Trie (Prefix Tree)

Basic Implementation

```
#define MAX_CHAR 26

struct TrieNode {
    TrieNode *pTrieNode[MAX_CHAR]{};
    bool isWord;

    TrieNode() {
        isWord = false;
        fill(pTrieNode, pTrieNode + 26, (TrieNode *) NULL);
    }

    virtual ~TrieNode() = default;
};

class Trie {
private:
    TrieNode *root;
public:
    Trie() {
        root = new TrieNode();
    }

    virtual ~Trie() = default;

    TrieNode *getTrieNode() {
        return this->root;
    }

    void insert(const string &word) {
        TrieNode *current = root;
        for (char c: word) {
            int i = c - 'a';
            if (current->pTrieNode[i] == nullptr)
                current->pTrieNode[i] = new TrieNode();
            current = current->pTrieNode[i];
        }
        current->isWord = true;
    }

    bool search(const string &word) {
        TrieNode *current = root;
        int ch = 0;
        for (char c: word) {
            ch = c - 'a';
            if (current->pTrieNode[ch] == nullptr)
                return false;
            current = current->pTrieNode[ch];
```

```
        }
        return current->isWord;
    }

    bool startsWith(const string &prefix) {
        TrieNode *current = root;
        int ch = 0;
        for (char c: prefix) {
            ch = c - 'a';
            if (current->pTrieNode[ch] == nullptr)
                return false;
            current = current->pTrieNode[ch];
        }
        return true;
    }
};
```

5.3 Range Queries

5.3.1 Segment Tree

Basic Implementation

```
struct Node {
    long long val;
};

struct SegTree {
private:
    const Node NEUTRAL = {INT_MIN};

    static Node merge(const Node& x1, const Node& x2) {
        return {x1.val + x2.val};
    }

    void set(const int& idx, const int& val, int x, int lx, int rx) {
        if (rx - lx == 1) return void(values[x].val = val);

        int mid = (rx + lx) / 2;

        if (idx < mid)
            set(idx, val, 2 * x + 1, lx, mid);
        else
            set(idx, val, 2 * x + 2, mid, rx);

        values[x] = merge(values[2 * x + 1], values[2 * x + 2]);
    }

    Node query(const int& l, const int& r, int x, int lx, int rx) {
        if (lx >= r || l >= rx) return NEUTRAL;
        if (lx >= l && rx <= r) return values[x];

        int mid = (rx + lx) / 2;

        return merge(query(l, r, 2 * x + 1, lx, mid), query(l, r, 2 * x +
            2, mid, rx));
    }
    void build(vector<int> &a, int x, int lx, int rx) {
        if (rx - lx == 1) {
            if (lx < a.size()) {
                values[x].val = a[lx];
            }
            return;
        }
        int m = (lx + rx) / 2;
        build(a, 2 * x + 1, lx, m);
        build(a, 2 * x + 2, m, rx);
        values[x] = merge(values[2 * x + 1], values[2 * x + 2]);
    }

     void assign_range(int l, int r, int node, int lx, int rx, int time,
         int val) {
        if (lx > r || l > rx) return;
        if (lx >= l && rx <= r) {
            lazy[node] = {time, val};
            return;
        }
        int mid = (lx+rx) / 2;

        assign_range(l, r, 2*node+1, lx, mid, time, val);
        assign_range(l, r, 2*node+2, mid+1, rx, time, val);
    }

    pair<int, int> point_query(int lx, int rx, int node, int idx) {
        if(rx == lx) return lazy[node];
        int mid = (lx+rx) / 2;
```

```
        if(idx <= mid) {
            auto x = point_query(lx, mid, 2*node+1, idx);
            if(x.first > lazy[node].first) return x;
            return lazy[node];
        }
        auto x = point_query(mid+1, rx, 2*node+2, idx);
        if(x.first > lazy[node].first) return x;
        return lazy[node];
    }

public:
    int size{};
    vector<Node> values;

    void build(vector<int> &a) {
        build(a, 0, 0, size);
    }

    void init(int _size) {
        size = 1;
        while (size < _size) size *= 2;
        values.assign(2 * size, NEUTRAL);
    }

    void set(int idx, int val) {
        set(idx, val, 0, 0, size);
    }

    Node query(const int& l, const int& r) {
        return query(l, r, 0, 0, size);
    }
};
```

5.3.2 Lazy Propegation

Lazy Propegation

```
struct SegTree {
private:
    void propegate(int lx, int rx, int node) {
        if(!lazy[node]) return;
        if(lx != rx) {
            lazy[2*node+1] = lazy[node];
            lazy[2*node+2] = lazy[node];
        }
        values[node] = lazy[node] * (rx - lx + 1);
        lazy[node] = 0;
    }

    // assign val in range [l, r]
    void update_range(int l, int r, int node, int lx, int rx, int val,
        bool f) {
        propegate(lx, rx, node);
        if (lx > r || l > rx) return;
        if (lx >= l && rx <= r) {
            lazy[node] = val;
            propegate(lx, rx, node);
            return;
        }
        int mid = (lx+rx) / 2;

        update_range(l, r, 2*node+1, lx, mid, val, f);
        update_range(l, r, 2*node+2, mid+1, rx, val, f);
        values[node] = values[2*node+1] + values[2*node+2];
    }

    // get sum in range [l, r]
    int range_query(int l, int r, int lx, int rx, int node) {
        propegate(lx, rx, node);
        if (lx > r || l > rx) return 0;
        if (lx >= l && rx <= r) return values[node];

        int mid = (lx+rx) / 2;
        return range_query(l, r, lx, mid, 2*node+1) + range_query(l, r, mid
            +1, rx, 2*node+2);
    }

public:
    int size{};
    vector<int> values, lazy;

    void init(int _size) {
        size = 1;
        while (size < _size) size *= 2;
        values.assign(2 * size, 0);
```

```
        lazy.assign(2 * size, 0);
    }

    void update_range(int l, int r, int v, bool f) {
        update_range(l, r, 0, 0, size-1, v, f);
    }

    int range_query(int l, int r) {
        return range_query(l, r, 0, size-1, 0);
    }
};
```

5.3.3 Fenwick Tree

Fenwick Tree

```
struct Fenwick {
    // One Based
    vector<int> tree;

    explicit Fenwick(int n) {tree.assign(n + 5, {});}

    // Computes the prefix sum from [1, i], O(log(n))
    int query(int i) {
        int res = 0;
        while (i > 0) {
            res += tree[i];
            i &= ~(i & -i);
        }
        return res;
    }

    int query(int l, int r) {
        return query(r) - query(l-1);
    }

    // Get the value at index i
    int get(int i) {
        return query(i, i);
    }

    // Add 'v' to index 'i', O(log(n))
    void update(int i, int v) {
        while (i < tree.size()) {
            tree[i] += v;
            i += (i & -i);
        }
    }

    // Update range, Point query
    // To get(k) do prefix sum [1, k] and in insert update_range(i, i, a[i
        ])
    void update_range(int l, int r, int v) {
        update(l, v);
        update(r+1, -v);
    }
};
```

5.3.4 Fenwick UpdateRange

BIT UpdateRange

```
struct BITUpdateRange {
private:
    int n;
    vector<int> B1, B2;

    void add(vector<int> &b, int idx, int x) {
        while (idx <= n) {
            b[idx] += x;
            idx += idx & -idx;
        }
    }

    int sum(vector<int> &b, int idx) {
        int total = 0;
        while (idx > 0) {
            total += b[idx];
            idx &= ~(idx & -idx);
        }
        return total;
    }

    int prefix(int idx) {
        return sum(B1, idx) * idx - sum(B2, idx);
    }
```

```
public:
    explicit BITUpdateRange(int n) : n(n) {
        B1.assign(n + 1, {});
        B2.assign(n + 1, {});
    }

    void update(int l, int r, int x) {
        add(B1, l, x);
        add(B1, r + 1, -x);
        add(B2, l, x * (l - 1));
        add(B2, r + 1, -x * r);
    }

    int query(int i) {
        return prefix(i) - prefix(i - 1);
    }

    int query(int l, int r) {
        return prefix(r) - prefix(l - 1);
    }
};
```

5.3.5 2D BIT

2D BIT

```
struct BIT2D {
    int n, m;
    vector<vector<int>> bit;

    BIT2D(int n, int m) : n(n), m(m) {
        bit.assign(n + 2, vector<int>(m + 2));
    }

    void update(int x, int y, int val) {
        for (; x <= n; x += x & -x) {
            for (int i = y; i <= m; i += i & -i) {
                bit[x][i] += val;
            }
        }
    }
    int prefix(int x, int y) {
        int res = 0;
        for (; x > 0; x &= ~(x & -x)) {
            for (int i = y; i > 0; i &= ~(i & -i)) {
                res += bit[x][i];
            }
        }
        return res;
    }

    int query(int sx, int sy, int ex, int ey) {
        int ans = 0;
        ans += prefix(ex, ey);
        ans -= prefix(ex, sy - 1);
        ans -= prefix(sx - 1, ey);
        ans += prefix(sx - 1, sy - 1);
        return ans;
    }
};
```

5.3.6 Sparse Table

Impl with the index

```
// storing the index also
struct SNode {
    int val;
    int index;
};

class SparseTable {
private:
    vector<vector<SNode>> table;

    function<SNode(const SNode&, const SNode&)> merge;

    static SNode StaticMerge(const SNode& a, const SNode& b) {
        return a.val < b.val ? a : b;
    }

public:
    explicit SparseTable(const vector<int>& arr, const function<SNode(
        const SNode&, const SNode&)>& mergeFunc = StaticMerge) {
```

```
        int n = static_cast<int>(arr.size());
        int log_n = static_cast<int>(log2(n)) + 1;
        this->merge = mergeFunc;

        table.resize(n, vector<SNode>(log_n));

        for (int i = 0; i < n; i++) {
            table[i][0] = {arr[i], i};
        }

        for (int j = 1; (1 << j) <= n; j++) {
            for (int i = 0; i + (1 << j) <= n; i++) {
                table[i][j] = mergeFunc(table[i][j - 1], table[i + (1 << (j
                     - 1))][j - 1]);
            }
        }
    }

    SNode query(int left, int right) {
        int j = static_cast<int>(log2(right - left + 1));
        return merge(table[left][j], table[right - (1 << j) + 1][j]);
    }

    // query in O(log(n)) if its could't apply to Sparse Table directly
    T query_log(int l, int r){
      int len = r - l + 1;
      T ans;
      for(int i = 0; l <= r; i++){
          if (len & (1 << i)){
              ans = merge(ans, table[i][l]);
              l+= (1 << i);
          }
      }
   }
};


int main(void) {
    int n;
    cin >> n;
    vector<int> arr(n);
    for (auto& element : arr) cin >> element;

    SparseTable minSt(arr, [](const SNode& a, const SNode& b) -> SNode {
        return a.val < b.val ? a : b;
    });

    SparseTable maxSt(arr, [](const SNode& a, const SNode& b) -> SNode {
        return a.val > b.val ? a : b;
    });
}
```

5.3.7 Sparse Table (Shorter Version)

Impl

```
template<class T>
struct Rmq {
 vector<vector<T>> jmp;

   // vector is 0-based indexed
 Rmq(const vector<T>& V) : jmp(1, V) {
  for (int pw = 1, k = 1; pw * 2 <= V.size(); pw *= 2, ++k) {
   jmp.emplace_back(V.size() - pw * 2 + 1);
   for(int j = 0; j < jmp[k].size(); j++)
    jmp[k][j] = (jmp[k - 1][j] | jmp[k - 1][j + pw]);
  }
 }
   // query (a, b] inclusive execlusive
 T query(int a, int b) {
  // assert(a < b);
  int dep = 31 - __builtin_clz(b - a);
  return (jmp[dep][a] | jmp[dep][b - (1 << dep)]);
 }
};
```

5.3.8 Sqrt Decomp

Impl

```
#define OO 1e18


const int sz = 2e5 + 5;
const int sq = 500;
```

```
int arr[sz] = {};
int ans[sq] = {};

void pre() {
    fill(ans, ans + sq, OO);
    for(int i = 0; i < sz; i++) {
        ans[i / sq] = min(ans[i / sq], arr[i]);
    }
}

// can be O(1) with inversable operations, else it's sqrt
void update(int k, int x) {
   arr[k] = x;
   int start = k / sq;
   ans[start] = *min_element(arr + (start * sq), arr + (start + 1) * sq);
}

int query(int l, int r){
    int ret = OO;
    while(l <= r) {
        if(l % sq == 0 && l + sq <= r) {
            ret = min(ret, ans[l / sq]);
            l += sq;
        }else {
            ret = min(ret, arr[l]);
            l++;
        }
    }
    return ret;
}
```

5.4 Ordered Set

Ordered Set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;

template<typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

template <class T>
using ordered_multiset = tree<T, null_type, CUSTUM_COMPARE, rb_tree_tag,
    tree_order_statistics_node_update>;

void erase_set(ordered_set &os, int v) {
    // Number of elements less than v
    int rank = os.order_of_key(v);

    auto it = os.find_by_order(rank);
    os.erase(it);
}

// Returns iterator to 0-th
// largest element in the set
cout << *S.find_by_order(0) << " ";

// Returns iterator to 2-nd
// largest element in the set
cout << *S.find_by_order(2);
```

5.5 Custom Compare Functions

Custom Compare functions

```
template<class T>
struct custom_compare {
    bool operator()(const T& a, const T& b) const {
        if (a == b) return true; // Keep duplicates
        return a > b;
    }
};


//REF: GFG
class CustomComparator {
public:
    CustomComparator(int baseValue) : baseValue_(baseValue) {}

    bool operator()(int a, int b) const {
```

```
        // Custom comparison logic involving state
        return (a % baseValue_) < (b % baseValue_);
    }

private:
    int baseValue_;
};

// OR through capture by reference (capture clauses)
auto compare = [&](char a, char b) { return localStructure[a] >
    localStructure[b]; };
```

6 Counting Principles

6.1 nCr

$$C(n,k) = \frac{n!}{(n-k)!k!} = \frac{n*(n-1)*(n-2)*\ldots*(n-k+1)}{k!}$$

6.1.1 Fast nCr

$$C(n,k) = \frac{n*(n-1)*(n-2)*\ldots*(n-k+1)}{1*2*3*\ldots*k} = \prod_{i=0}^{k-1}\frac{n-i}{i+1} = \prod_{i=0}^{k-1}(n-i)(i+1)^{-1}$$

Fast nCr

```
int nCr(const int& n, const int& r) {
    double res = 1;
    for (int i = 1; i <= r; ++i)
        res = res * (n - r + i) / i;
    return (int)(res + 0.01);
}
```

6.1.2 Method 1: Pascal's Triangle (Dynamic Programming) - $\mathcal{O}(n^2)$

nCk using dp

```
// REF: USACO guide

/** @return nCk mod p using dynamic programming */
int binomial(int n, int k, int p) {
 // dp[i][j] stores iCj
 vector<vector<int>> dp(n + 1, vector<int>(k + 1, 0));

 // base cases described above
 for (int i = 0; i <= n; i++) {
  /*
   * i choose 0 is always 1 since there is exactly one way
   * to choose 0 elements from a set of i elements
   * (don't choose anything)
   */
  dp[i][0] = 1;
  /*
   * i choose i is always 1 since there is exactly one way
   * to choose i elements from a set of i elements
   * (choose every element in the set)
   */
  if (i <= k) { dp[i][i] = 1; }
 }

 for (int i = 0; i <= n; i++) {
  for (int j = 1; j <= min(i, k); j++) {
   if (i != j) { // skips over the base cases
    // uses the recurrence relation above
    dp[i][j] = (dp[i - 1][j - 1] + dp[i - 1][j]) % p;
   }
  }
 }

 return dp[n][k]; // returns nCk modulo p
}
```

6.1.3 Method 2: Factorial Definition (Modular Inverses) - $\mathcal{O}(n + \log MOD)$

nCk using Modular Inverses

```
// REF: USACO guide
```

```cpp
const int MAXN = 1e6;

long long fac[MAXN + 1];
long long inv[MAXN + 1];

/** @return x^n modulo m in O(log p) time. */
long long exp(long long x, long long n, long long m) {
 x %= m; // note: m * m must be less than 2^63 to avoid ll overflow
 long long res = 1;
 while (n > 0) {
  if (n % 2 == 1) { res = res * x % m; }
  x = x * x % m;
  n /= 2;
 }
 return res;
}

/** Precomputes n! from 0 to MAXN. */
void factorial(long long p) {
 fac[0] = 1;
 for (int i = 1; i <= MAXN; i++) { fac[i] = fac[i - 1] * i % p; }
}

/**
 * Precomputes all modular inverse factorials
 * from 0 to MAXN in O(n + log p) time
 */
void inverses(long long p) {
 inv[MAXN] = exp(fac[MAXN], p - 2, p);
 for (int i = MAXN; i >= 1; i--) { inv[i - 1] = inv[i] * i % p; }
}

/** @return nCr mod p */
long long choose(long long n, long long r, long long p) {
 return fac[n] * inv[r] % p * inv[n - r] % p;
}

int main() {
 factorial();
 inverses();
 int n;
 cin >> n;
 for (int i = 0; i < n; i++) {
  int a, b;
  cin >> a >> b;
  cout << choose(a, b) << '\n';
 }
}
```

7 Graph Theory

7.1 Shortest Path algorithms

7.1.1 Dijkstra Algorithm

Dijkstra Implementation

```cpp
#define INF (1e18) // for int defined as ll

int n, m;
vector<vector<pair<int, int>>> adj;
vector<int> cost;
vector<int> parent;

void dijkstra(int startNode = 1) {
    priority_queue<pair<ll, int>, vector<pair<ll, int>>, greater<>> pq;

    cost[startNode] = 0;
    pq.emplace(0, startNode);

    while (!pq.empty()) {
        int u = pq.top().second;
        ll d = pq.top().first;
        pq.pop();

        if (d > cost[u]) continue;

        for (auto &p: adj[u]) {
            int v = p.first;
            int w = p.second;
            if (cost[v] > cost[u] + w) {
                cost[v] = cost[u] + w;
                parent[v] = u;
                pq.emplace(cost[v], v);
```

```
            }
        }
    }
}

void run_test_case(int testNum) {
    cin >> n >> m;

    adj.assign(n + 1, {});
    cost.assign(n + 1, INF);
    parent.assign(n + 1, -1);

    while (m--) {
        // Read Edges
    }

    dijkstra();

    if (cost[n] == INF) {
        cout << -1 << el; // not connected {Depends on you use case}
        return;
    }

    stack<int> ans;
    for (int v = n; v != -1; v = parent[v]) ans.push(v);

    while (!ans.empty()) { // printing the path
        cout << ans.top() << ' ';
        ans.pop();
    }
    cout << el;
}
```

7.1.2 Floyd Warshal Algorithm

FloydWarshal Implementation

```
int main() {
    int n, m; cin >> n >> m;
    vector <vector <int>> adj(n + 1, vector <int> (n + 1, 2e9));
    for (int i = 0; i < n; i++) adj[i][i] = 0;

    while(m--) {
        int u, v, w;
  cin >> u >> v >> w;
        adj[u][v] = min(adj[u][v], w);
        adj[v][u] = min(adj[v][u], w);
    }

    for (int mid = 1; mid <= n; mid++) {
        for (int start = 1; start <= n; start++) {
            for (int end = 1; end <= n; end++) {
                adj[start][end] = min(adj[start][end], adj[start][mid] +
                    adj[mid][end]);
            }
        }
    }

 return 0;
}
```

7.1.3 Bellman Ford Algorithm

BellmanFord Implementation

```
 vector <vector <pair<int, int>>> &adj

vector <long long> BellmanFord(int src) {
    int n = (int)adj.size();
    vector <long long> dist(n, 2e18);

    dist[src] = 0;
    for (int it = 0; it < n-1; it++) {
        bool in = false;
        for (int i = 0; i < n; i++) { // iterate on the edges
            for (auto &[j, w] : adj[i]) {
                if (dist[j] > dist[i] + w) {
                    in = true;
                    dist[j] = dist[i] + w;
                }
            }
        }
        if (!in) return dist;
    }

    for (int i = 0; i < n; i++) {
```

```
        for (auto &[j, w] : adj[i]) {
            if (dist[j] > dist[i] + w) { //negative cycle
                return vector <long long> (n, -1); // or any flag
            }
        }
    }

    return dist;
}
```

7.2 Cycle Detection

7.2.1 DFS Implementation

DFS Implementation

```
// return true with number of nodes in the cycle, either odd cycle or even
bool cycle_detection(unordered_map<int, vector<int>> &graph, int source,
    int par, unordered_map<int,bool> vis, int c){
    if(vis[source]) return true;

   vis[source] = true;

   for(int v: graph[source]){
    if(v != par){
        c++;
        if(dfs(graph,v, source, vis, c)) return true;
    }
   }
   return false;
}
```

7.2.2 Another way for undirected graphs

Another way for undirected graphs

```
// this is true only for undirected graphs
bool dfs1(int cur, int par) {
    bool ret = false;
    vis[cur] = true;
    for (auto &i : adj[cur]) {
        if (!vis[i]) ret|=dfs1(i, cur);
        else if (par != i) ret = true;
    }
    return ret;
}
```

7.2.3 General Way

General Way

```
// general algorithm
vector <bool> cyc;
bool dfs(int cur, int par) {
    bool ret = false;
    vis[cur] = cyc[cur] = true;
    for (auto &i : adj[cur]) {
        if (par == i) continue;
        if (!vis[i]) ret|=dfs(i, cur);
        else if (cyc[i]) ret = true;
    }
    cyc[cur] = false;
    return ret;
}
```

7.2.4 DSU Implementation

DSU Implementation

```
#include <iostream>
#include <vector>

class UnionFind {
public:
    UnionFind(int n) {
        parent.resize(n);
        rank.resize(n, 0);
        for (int i = 0; i < n; ++i) {
            parent[i] = i;
        }
    }
```

```
    int find(int u) {
        if (parent[u] != u) {
            parent[u] = find(parent[u]);
        }
        return parent[u];
    }

    void unionSets(int u, int v) {
        int rootU = find(u);
        int rootV = find(v);

        if (rootU != rootV) {
            if (rank[rootU] > rank[rootV]) {
                parent[rootV] = rootU;
            } else if (rank[rootU] < rank[rootV]) {
                parent[rootU] = rootV;
            } else {
                parent[rootV] = rootU;
                ++rank[rootU];
            }
        }
    }

private:
    std::vector<int> parent;
    std::vector<int> rank;
};

bool detectCycle(const std::vector<std::pair<int, int>>& edges, int n) {
    UnionFind uf(n);

    for (const auto& edge : edges) {
        int u = edge.first;
        int v = edge.second;

        if (uf.find(u) == uf.find(v)) {
            return true;
        }
        uf.unionSets(u, v);
    }

    return false;
}

int main() {
    std::vector<std::pair<int, int>> edges = { {0, 1}, {1, 2}, {2, 3}, {3,
        0} };
    int n = 4; // Number of vertices

    if (detectCycle(edges, n)) {
        std::cout << "Cycle detected" << std::endl;
    } else {
        std::cout << "No cycle detected" << std::endl;
    }

    return 0;
}
```

7.3 Algorithms

7.3.1 Heavy Light Decomposition

Basic HLD Impl

```
struct Node {
    int val;
};

const Node nullNode = {0};

const int N = 2e5 + 5, S = 1 << 19;
int n, q;
int val[N];
int sz[N], par[N], dep[N], id[N], top[N];
vector<int> adj[N];

Node st[S];

Node merge(const Node& a, const Node& b) {
    return {a.val + b.val};
}

void update(int idx, Node val) {
    st[idx += n] = val;
```

```
    for (idx /= 2; idx; idx /= 2) st[idx] = merge(st[idx * 2], st[idx * 2
        + 1]);
}

Node query(int lo, int hi) {
    Node ra = nullNode, rb = nullNode;

    for (lo += n, hi += n + 1; lo < hi; lo /= 2, hi /= 2) {
        if (lo & 1) ra = merge(ra, st[lo++]);
        if (hi & 1) rb = merge(st[--hi], rb);
    }

    return merge(ra, rb);
}

int dfs_size(const int& node, const int& parent) {
    sz[node] = 1;
    par[node] = parent;
    for (const int& ch : adj[node]) {
        if (ch == parent) continue;
        dep[ch] = dep[node] + 1;
        par[ch] = node;
        sz[node] += dfs_size(ch, node);
    }
    return sz[node];
}

int curId = 0;

void dfs_hld(const int& cur, const int& parent, const int& curTop) {
    id[cur] = curId++;
    top[cur] = curTop;
    update(id[cur], {val[cur]});
    int heavyChild = -1, heavyMax = -1;
    for (const int& ch : adj[cur]) {
        if (ch == parent) continue;
        if (sz[ch] > heavyMax) {
            heavyMax = sz[ch];
            heavyChild = ch;
        }
    }

    if (heavyChild == -1) return;
    dfs_hld(heavyChild, cur, curTop);
    for (int ch : adj[cur]) {
        if (ch == parent || ch == heavyChild) continue;

        dfs_hld(ch, cur, ch);
    }
}

Node path(int u, int v) {
    Node ans = nullNode;

    while (top[u] != top[v]) {
        if (dep[top[u]] < dep[top[v]]) swap(u, v);
        ans = merge(ans, query(id[top[u]], id[u]));
        u = par[top[u]];
    }

    if (dep[u] > dep[v]) swap(u, v);
    ans = merge(ans, query(id[u], id[v]));
    return ans;
}

void init() {
    for (int i = 0; i < S; i++) st[i] = nullNode;
    dfs_size(1, 1);
    dfs_hld(1, 1, 1);
}

int main() {
    cin >> n >> q;
    for (int i = 1; i <= n; i++) cin >> val[i];

    int a, b;
    for (int i = 2; i <= n; i++) {
        cin >> a >> b;
        adj[a].pb(b);
        adj[b].pb(a);
    }

    init(); // <---------------- DON'T FORGET TO CALL THIS FUNCTION

    int type;
    while (q--) {
        cin >> type;
        if (type == 1) {
```

```
        cin >> a >> b;
        val[a] = b;
        update(id[a], {val[a]});
      }
      else {
        cin >> a;
        cout << path(1, a).val << el;
      }
   }
}
```

7.3.2 Heavy Light Decomposition with lazy SegTree

Basic HLD Impl

```
struct Node {
   int val;
};

const Node nullNode = {0};

const int N = 2e5 + 5, S = 1 << 19;
int n, q;
int val[N];
int sz[N], par[N], dep[N], id[N], top[N];
vector<int> adj[N];

Node st[S];
int lazy[S];

Node merge(const Node& a, const Node& b) {
   return {a.val + b.val};
}

void push(int idx, int l, int r) {
   if (lazy[idx] != 0) {
      st[idx].val += lazy[idx] * (r - l + 1);
      if (l != r) {
         lazy[idx * 2] += lazy[idx];
         lazy[idx * 2 + 1] += lazy[idx];
      }
      lazy[idx] = 0;
   }
}

void update_range(int lo, int hi, int l, int r, int idx, int value) {
   push(idx, l, r);
   if (lo > r || hi < l) return;
   if (lo <= l && r <= hi) {
      lazy[idx] += value;
      push(idx, l, r);
      return;
   }
   int mid = (l + r) / 2;
   update_range(lo, hi, l, mid, idx * 2, value);
   update_range(lo, hi, mid + 1, r, idx * 2 + 1, value);
   st[idx] = merge(st[idx * 2], st[idx * 2 + 1]);
}

void update(int idx, Node val) {
   update_range(idx, idx, 0, n - 1, 1, val.val);
}

void update_range(int lo, int hi, int value) {
   update_range(lo, hi, 0, n - 1, 1, value);
}

Node query(int lo, int hi, int l, int r, int idx) {
   push(idx, l, r);
   if (lo > r || hi < l) return nullNode;
   if (lo <= l && r <= hi) return st[idx];
   int mid = (l + r) / 2;
   return merge(query(lo, hi, l, mid, idx * 2), query(lo, hi, mid + 1, r,
        idx * 2 + 1));
}

Node query(int lo, int hi) {
   return query(lo, hi, 0, n - 1, 1);
}

int dfs_size(const int& node, const int& parent) {
   sz[node] = 1;
   par[node] = parent;
   for (const int& ch : adj[node]) {
      if (ch == parent) continue;
      dep[ch] = dep[node] + 1;
      par[ch] = node;
```

```cpp
        sz[node] += dfs_size(ch, node);
    }
    return sz[node];
}

int curId = 0;

void dfs_hld(const int& cur, const int& parent, const int& curTop) {
    id[cur] = curId++;
    top[cur] = curTop;
    update(id[cur], {val[cur]});
    int heavyChild = -1, heavyMax = -1;
    for (const int& ch : adj[cur]) {
        if (ch == parent) continue;
        if (sz[ch] > heavyMax) {
            heavyMax = sz[ch];
            heavyChild = ch;
        }
    }

    if (heavyChild == -1) return;
    dfs_hld(heavyChild, cur, curTop);
    for (int ch : adj[cur]) {
        if (ch == parent || ch == heavyChild) continue;

        dfs_hld(ch, cur, ch);
    }
}

int get(int u) {
    return query(id[u], id[u]).val;
}

void path(int u, int v, int val) {
    // Node ans = nullNode;

    while (top[u] != top[v]) {
        if (dep[top[u]] < dep[top[v]]) swap(u, v);
        // ans = merge(ans, query(id[top[u]], id[u]));
        update_range(id[top[u]], id[u], val);
        u = par[top[u]];
    }

    if (dep[u] > dep[v]) swap(u, v);
    // ans = merge(ans, query(id[u], id[v]));
    update_range(id[u], id[v], val);
    // return ans;
}

void init() {
    for (int i = 0; i < S; i++) st[i] = nullNode;
    memset(lazy, 0, sizeof(lazy));
    dfs_size(1, 1);
    dfs_hld(1, 1, 1);
}

int main(void) {
    cin >> n >> q;
    for (int i = 1; i <= n; i++) val[i] = 0;
    int a, b;
    for (int i = 2; i <= n; i++) {
        cin >> a >> b;
        adj[a].push_back(b);
        adj[b].push_back(a);
    }

    init(); // <---------------- DON'T FORGET TO CALL THIS FUNCTION
    int v;
    while (q--) {
        cin >> a >> b >> v;
        path(a, b, v);
    }

    for (int i = 1; i <= n; i++) {
        cout << get(i) << " ";
    }
    cout << el;
}
```

7.3.3 LCA functions using Binary Lifting

LCA functions using Binary Lifting

```cpp
const int N = 2e5 + 15, M = 23;
int ancestors[N][M], depth[N], parent[N], val[N];
vector<vector<int>> adj;
//int tin[N], tout[N], timer;
```

```cpp
void dfs_LCA(const int &node, const int &par) {
//    tin[node] = timer++;
    parent[node] = par;
    ancestors[node][0] = par;
    depth[node] = depth[par] + 1;

    for (int i = 1; i < M; i++) {
        int p = ancestors[node][i - 1];
        ancestors[node][i] = ancestors[p][i - 1];
    }

    for (const int &v: adj[node]) {
        if (v == par) continue;
        dfs_LCA(v, node);
    }
//    tout[node] = timer++;
}

//bool is_ancestor(int u, int v) {
//    return tin[u] <= tin[v] && tout[u] >= tout[v];
//}

int findKth(int u, int k) {
    if (depth[u] <= k) return -1;
    for (int i = M - 1; i >= 0; i--) {
        if (k & (1 << i)) {
            u = ancestors[u][i];
        }
    }
    return u;
}

int getLCA(int u, int v) {
    if (depth[u] < depth[v])
        swap(u, v);

    u = findKth(u, depth[u] - depth[v]);
    if (u == v) return u;

    for (int i = M - 1; i >= 0; i--) {
        if (ancestors[u][i] == ancestors[v][i]) continue;
        u = ancestors[u][i];
        v = ancestors[v][i];
    }
    return ancestors[u][0];
}

int getDistance(int u, int v) {
    int lca = getLCA(u, v);
    return (depth[u] + depth[v]) - (2 * depth[lca]);
}

int dfs_accumulate(const int &node, const int &par) {

    for (const int& ch: adj[node]) {
        if (ch == par) continue;
        val[node] += dfs_accumulate(ch, node);
    }
    return val[node];
}

void applyOpOnPath(const int a, const int b, const int w) {
    // adding w to each node on the path a to b
    val[a] += w;
    val[b] += w;
    int lca = getLCA(a, b);
    val[lca] -= w;
    val[parent[lca]] -= w;
}

int main(void) {
    int n, q;
    cin >> n >> q;
    adj.resize(n + 1);

    int u, v;
    for (int i = 2; i <= n; ++i) {
        cin >> u >> v;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }

    dfs_LCA(1, 1);
    parent[1] = -1;

    int w;
    for (int i = 0; i < q; ++i) {
```

```
        cin >> u >> v;
        cout << getDistance(u, v) << el;
    }

//    dfs_accumulate(1, 0);
//
//    for (int i = 1; i <= n; i++) {
//        cout << val[i] << " ";
//    }
//    cout << el;
}
```

7.3.4 Topological Sort

Topological Sort Using DFS

```
//REF: USACO Guide
vector<int> top_sort;
vector<vector<int>> graph;
vector<bool> visited;

void dfs(int node) {
 for (int next : graph[node]) {
  if (!visited[next]) {
   visited[next] = true;
   dfs(next);
  }
 }
 top_sort.push_back(node);
}

int main() {
 int n, m; // The number of nodes and edges respectively
 std::cin >> n >> m;

 graph = vector<vector<int>>(n);
 for (int i = 0; i < m; i++) {
  int a, b;
  std::cin >> a >> b;
  graph[a - 1].push_back(b - 1);
 }

 visited = vector<bool>(n);
 for (int i = 0; i < n; i++) {
  if (!visited[i]) {
   visited[i] = true;
   dfs(i);
  }
 }
 std::reverse(top_sort.begin(), top_sort.end());

 vector<int> ind(n);
 for (int i = 0; i < n; i++) { ind[top_sort[i]] = i; }

 // Check if the topological sort is valid
 bool valid = true;
 for (int i = 0; i < n; i++) {
  for (int j : graph[i]) {
   if (ind[j] <= ind[i]) {
    valid = false;
    goto answer;
   }
  }
 }
answer:;

 if (valid) {
  for (int i = 0; i < n - 1; i++) { cout << top_sort[i] + 1 << ' '; }
  cout << top_sort.back() + 1 << endl;
 } else {
  cout << "IMPOSSIBLE" << endl;
 }
}
```

8 Techniques

8.1 Coordinate Compression

```
void coordinate_compress(vector<int> &x, int start=0, int
   step=1) {
 set unique(x.begin(), x.end());
 map<int, int> valPos;

 int idx=0;
```

```
 for (auto i: unique) {
  valPos[i] = start + idx * step;
  ++idx;
 }
 for(auto &i: x) i = valPos[i];

}
```

Coordinate Compression

8.2 Binary to decimal

Binary to decimal

```
// Function to convert binary to decimal
// O(32)
int binaryToDecimal(string str)
{
    int dec_num = 0;
      int power = 0 ;
    int n = str.length() ;

      for(int i = n-1 ; i>=0 ; i--){
      if(str[i] == '1'){
        dec_num += (1<<power) ;
      }
      power++ ;
    }

    return dec_num;
}
```

8.3 Decimal to binary

Decimal to bianry

```
// Function that convert Decimal to binary
// O(32)
void decToBinary(int n)
{
    // Size of an integer is assumed to be 32 bits
    for (int i = 31; i >= 0; i--) {
        int k = n >> i;
        if (k & 1)
            cout << "1";
        else
            cout << "0";
    }
}

// O(logn)
string DecimalToBinary(int num)
{
    string str;
      while(num){
      if(num & 1) // 1
        str+='1';
      else // 0
        str+='0';
      num>>=1; // Right Shift by 1
    }
      return str;
}
```

9 Number Theory

9.1 Divisors

9.1.1 formulas

number of divisors

```
int d(int n){
    unordered_map<int, int> factors = pf(n);
    int c = 1;
    for(const auto& factor: factors){
        c *= (factor.second+1);
    }
    return c;
}
```

```
// range Count Divisors backward thinking MAXN = 2e6
  for(int i=1; i <= n; ++i) {
    for(int j = i; j <= n; j += i) {
      numFactors[j]++;
    }
  }

int countDivisors(int n) {
  int count = 0;
  for (int i = 1; i * i <= n; ++i) {
    if (n % i == 0) {
        if (i == n / i) {
            count++; // Perfect square
        } else {
            count += 2; // Pair of divisors
        }
    }
  }
    return count;
}
```

---

sum of divisors

```
int s(int n){
    unordered_map<int,int> factors = pf(n);
    int sum = 1;
    for(const auto& factor: factors){
        int p = factor.first;
        int exp = factor.second;
        sum *= (pow(p,exp+1)-1)/p-1;
    }
    return sum;
}
```

9.2 Primes

prime factorization

```
void factorize(int x, unordered_map<int, int>& factors) {
    while (x % 2 == 0) {
        factors[2]++;
        x /= 2;
    }
    for (int i = 3; i * i <= x; i += 2) {
        while (x % i == 0) {
            factors[i]++;
            x /= i;
        }
    }
    if (x > 2) factors[x]++;
}
```

---

number of co-primes with n

```
int eulerTotient(int n){
    int result = n;

    for(int i = 2; SQ(i) <= n; i++){
        if(n%i == 0){
            while(n%i == 0){
                n/=i;
            }
            result -= result/i;
        }
    }

    if(n > 1) result -= result/n;
    return result;
}

//Phi(n) = n * (1 - 1/P1) * (1 - 1/P2) * ...

//NOTE: summation of Euler function over divisors of n is equal to n


// using seive
void phi_generator() {
    const int MAX = 1000000;
    char primes[MAX];
    int phi[MAX];

    memset(primes, 1, sizeof(primes));
```

```
    for (int k = 0; k < MAX; ++k)
        phi[k] = 1;

    for (int i = 2; i <= MAX; ++i) {
        if (primes[i]) {
            phi[i] = i - 1; // phi(prime) = p-1

            for (int j = i * 2; j <= MAX; j += i) {
                primes[j] = 0;
                int n = j, pow = 1;
                while (n % i == 0) {
                    pow *= i;
                    n /= i;
                }
                phi[j] *= (pow / i) * (i - 1);
            }
        }
    }
}

// phi(N!) = (N is prime ? N-1 : N) * phi((N-1)!)
ll phi_factn(int n) {
    ll ret = 1;
    for (int i = 2; i <= n; ++i)
        ret = ret * (isprime(i) ? i - 1 : i);
    return ret;
}
```

Prime Check

```
vector<bool> isPrime(MAXN, true);

void sieve() {
  isPrime[0] = isPrime[1] = false;

  for (int i=2; i * i <= isPrime.size(); ++i) {
    if(isPrime[i]) {
      for (int j = 2 * i; i <= isPrime.size(); j += i)
        prime[j] = false;
    }
  }
}
```

```
bool Prime(int n) {
  if(n == 2) return true;
  if(n < 2 || n % 2 == 0) return false;

  for(int i=3; i * i <= n; i += 2) {
    if(n % i == 0) return false;
  }
  return true;
}

// Generate Primes
const int sz = sqrt(MAXN);
vector<int> prime;
vector<bool> vis(sz);

void pre() {
    prime.push_back(2);
    for (int j = 4; j < sz; j += 2) vis[j] = true;
    for (int i = 3; i < sz; i += 2) {
        if (vis[i]) continue;
        prime.push_back(i);
        for (int j = i * i; j < sz; j += i) vis[j] = true;
    }
}


// Preprocessing Prime Factorization of range numbers
constexpr int N = 5e6+1;
int a[N];

for(int i=2; i < N; ++i) {
    if(!a[i]) {
        for(int j=1; i*j < N; ++j) {
            for(int k=i*j; k%i==0; k/=i) a[i*j]++;
        }
    }
    a[i] += a[i-1];
}
```

9.3 Math

9.3.1 Vieta's Formula for a Polynomial of Degree n

Problem: Given a polynomial of degree n:

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

with roots $r_1, r_2, \ldots, r_n$, express the sums and products of its roots using Vieta's formulas.

Solution: Using Vieta's formulas, we can relate the coefficients of the polynomial to sums and products of its roots:

- Sum of the roots taken one at a time:

$$r_1 + r_2 + \cdots + r_n = -\frac{a_{n-1}}{a_n}$$

- Sum of the products of the roots taken two at a time:

$$r_1 r_2 + r_1 r_3 + \cdots + r_{n-1} r_n = \frac{a_{n-2}}{a_n}$$

- Sum of the products of the roots taken three at a time:

$$r_1 r_2 r_3 + r_1 r_2 r_4 + \cdots + r_{n-2} r_{n-1} r_n = -\frac{a_{n-3}}{a_n}$$

- Continue this pattern until:
- Product of the roots (for even n):

$$r_1 r_2 \cdots r_n = (-1)^n \frac{a_0}{a_n}$$

Example Problem Using Vieta's Formula

Problem: Given a quadratic equation $x^2 + bx + c = 0$ with roots r_1 and r_2, find $r_1 + r_2$ and $r_1 r_2$.

Solution: Using Vieta's formulas for a quadratic equation $ax^2+bx+c=0$:

- Sum of the roots:

$$r_1 + r_2 = -\frac{b}{a}$$

- Product of the roots:

$$r_1 r_2 = \frac{c}{a}$$

For the given quadratic $x^2 + bx + c = 0$ (where $a = 1$):

Also: To find the roots of the quadratic equation, we can use the discriminant formula, $D = b^2 - 4ac$. The roots will then be $x_1 = \frac{b-\sqrt{D}}{2}$ and $x_2 = \frac{b+\sqrt{D}}{2}$.

- $r_1 + r_2 = -b$
- $r_1 r_2 = c$

Example Vieta's Formula for Cubic Equation

When considering a cubic equation in the form of $f(x) = ax^3 + bx^2 + cx + d$, Vieta's formula states that if the equation $f(x) = 0$ has roots r_1, r_2, and r_3, then:

- $r_1 + r_2 + r_3 = -\frac{b}{a}$
- $r_1 r_2 + r_2 r_3 + r_3 r_1 = \frac{c}{a}$
- $r_1 r_2 r_3 = -\frac{d}{a}$

9.4 Phi Function

- Count integers $i < n$ such that $\gcd(i, n) = 1$
- $\gcd(a, b) = 1 \Rightarrow$ then coprimes: $\gcd(5, 7)$, $\gcd(4, 9)$
- $\gcd(\text{prime}, i) = 1$ for $i <$ prime
- $\varphi(10) = 4 \Rightarrow 1, 3, 7, 9$
- $\varphi(5) = 4 \Rightarrow 1, 2, 3, 4 \ldots \varphi(\text{prime}) = \text{prime} - 1$

- If a, b, c are pairwise coprimes, then

$$\varphi(a \cdot b \cdot c) = \varphi(a) \cdot \varphi(b) \cdot \varphi(c)$$

- If $k \geq 1$

$$\varphi(p^k) = p^k - p^{k-1} = p^{k-1}(p-1) = p^k \left(1 - \frac{1}{p}\right)$$

9.5 Euler's Totient Numbers

Online Sequence

$\varphi(n) = 1, 1, 2, 2, 4, 2, 6, 4, 6, 4, 10, 4, 12, 6, 8, 8, 16, 6, 18, 8, 12, 10, 22, 8, 20, 12, 18, 12, 28, 8, 30, 16, 20, 16, 24, 12, 36, 18, 24, 16, 40, 12$

- $\varphi(1) = \varphi(2) = 1$, $\varphi(5) = 4$
- $\varphi(n)$ is even for $n > 2$
- $\sqrt{n} \leq \varphi(n) \leq n - \sqrt{n}$: Except 2, 6
- $\varphi(n^k) = n^{k-1} \cdot \varphi(n)$
- $n = \sum \varphi(d_i)$ where d are the divisors of n

9.6 Möbius Function

Möbius Function

```
int mobius(int n) {
    int p = 0;
    // Handling 2 separately
    if (n%2 == 0){
        n = n/2;
        p++;

        // If 2^2 also divides N
        if (n % 2 == 0)
           return 0;
    }

    // Check for all other prime factors
    for (int i = 3; i <= sqrt(n); i = i+2) {
        if (n%i == 0){
            n = n/i;
            p++;

            if (n % i == 0) return 0;
        }
    }

    return (p % 2 == 0)? -1 : 1;
}


void mobius_generator() const {
  const int MAX = 1000000;
  int moebius[MAX + 1];
  char prime[MAX + 1];

  for (ll i = 2; i <= MAX; i++)
    moebius[i] = -1, prime[i] = 1;

  for (ll i = 2; i <= MAX; ++i)
    if (prime[i]) {
      moebius[i] = 1;

      for (ll j = 2 * i; j <= MAX; j += i)
        prime[j] = 0, moebius[j] = j % (i * i) == 0 ? -moebius[j] : 1;
    }
}

// Mobius Inclusion Exclusion
// Count triples gcd(a, b, c) = 1
int n = 4;
ll sum = n * n * n;
for (ll i = 2; i <= n; ++i)
    sum -= moebius[i] * (n / i) * (n / i) * (n / i);
```

9.7 Möbius and Inclusion Exclusion

Count the triples (a, b, c) such that $a, b, c \leq n$, and $\gcd(a, b, c) = 1$

- Reverse thinking, total - (# triples gcd > 1)

- How many triples with gcd multiple of 2: $(n/2)^3$
- How many triples with gcd multiple of 3: $(n/3)^3$
- and 4? Ignore any numbers of internal duplicate primes
- and 6? already computed in 2, 3. Remove it: $-(n/6)^3$

9.8 Totient and Möbius Connection

Sum over divisors d of n

$$\sum_d d\mu\left(\frac{n}{d}\right) = \varphi(n)$$

9.9 Lagrange's four-square theorem

Lagrange's four-square theorem states that, every positive integer can be expressed as the sum of the squares of four integers.

Lagrange's four-square theorem

```
#include <cmath>

bool isPerfectSquare(int n) {
    int rt = sqrt(n);
    return rt * rt == n;
}

// Function to compute the minimum number of perfect squares
int numSquares(int n) {
    // Case 1:
    if (isPerfectSquare(n)) return 1;

    // Case 2: Check if n can be expressed as the sum of two perfect
        squares
    for (int i = 1; i * i <= n; i++) {
        int d = n - i * i;
        if (isPerfectSquare(d)) return 2;
    }

    // Case 3:
    // If n can be reduced to the form 4^a * (8b + 7), then it requires 4
        squares
    int m = n;
    while (m % 4 == 0) m /= 4;
    if (m % 8 == 7) return 4;

    // Case 4:
    return 3;
}
```

10 Geometry

10.1 Linearity

10.1.1 co-linear points

check if two points are co-linear

```
bool co_linear(int x1, int y1, int x2, int y2, int x3, int y3){
  int area = x1*(y2-y3) + x2*(y3-y1) + x3*(y1-y2);
  return area == 0;
}
```

10.2 Polygons

10.2.1 Polygon formation

check if can form polygon with given angle

```
bool possible(double angle){
  if(angle <= 0 || angle >= 180) return false;

  double sides = 360.0/(180.0-angle);

  return (sides == static_cast<int>(sides) && sides >= 3);
}
```

10.2.2 Polygon Area

area of any polygon with x vertices

```
double shoelace(vector<pair<double, double>> points) {
 double leftSum = 0.0;
 double rightSum = 0.0;

 for (int i = 0; i < points.size(); ++i) {
  int j = (i + 1) % points.size();
  leftSum += points[i].first * points[j].second;
  rightSum += points[j].first * points[i].second;
 }

 return 0.5 * abs(leftSum - rightSum);
}
```

10.3 Intersections

10.3.1 Rectangle

intersection area between 2 rectangles

```
struct Rectangle {
    int x1, y1; // Bottom-left corner
    int x2, y2; // Top-right corner
};

int intersectionArea(const Rectangle& rect1, const Rectangle& rect2){

    int x_left = max(rect1.x1, rect2.x1);
    int y_bottom = max(rect1.y1, rect2.y1);
    int x_right = min(rect1.x2, rect2.x2);
    int y_top = min(rect1.y2, rect2.y2);

    int intersection_width = x_right - x_left;
    int intersection_height = y_top - y_bottom;

    if (intersection_width > 0 && intersection_height > 0) {
        return intersection_width * intersection_height;
    }

    return 0;
}
```

10.3.2 Circle

intersection area between 2 circles

```
double area(int x0, int y0, int r0, int x1, int y1, int r1){
    const double PI = 3.14159265358979323846;
    double rr0 = r0 * r0;
    double rr1 = r1 * r1;
    double d = sqrt((x1 - x0) * (x1 - x0) + (y1 - y0) * (y1 - y0));
    if(d >= r0 + r1){
        return 0;
    }
    double phiAngle = (rr0 + (d * d) - rr1) / (2 * r0 * d);
    double phi = acos(phiAngle) * 2;
    double thetaAngle = (rr1 + (d * d) - rr0) / (2 * r1 * d);
    double theta = acos(thetaAngle) * 2;
    double area1 = 0.5 * theta * rr1 - 0.5 * rr1 * sin(theta);
    double area2 = 0.5 * phi * rr0 - 0.5 * rr0 * sin(phi);
    return area1+area2;
}
```

10.3.3 Triangle

intersection area between 2 triangles

```
struct Point {
    double x, y;
};

typedef vector<Point> Polygon;

bool inside(const Point &p, const pair<Point, Point> &edge) {
    Point a = edge.first, b = edge.second;
    return (b.x - a.x) * (p.y - a.y) - (b.y - a.y) * (p.x - a.x) >= 0;
}
Point compute_intersection(const Point &p1, const Point &p2, const pair<
    Point, Point> &edge) {
    Point a = edge.first, b = edge.second;
    double A1 = b.y - a.y, B1 = a.x - b.x, C1 = b.x * a.y - a.x * b.y;
```

```
    double A2 = p2.y - p1.y, B2 = p1.x - p2.x, C2 = p2.x * p1.y - p1.x *
        p2.y;
    double det = A1 * B2 - A2 * B1;
    if (det == 0) return {numeric_limits<double>::quiet_NaN(),
        numeric_limits<double>::quiet_NaN()}; // parallel
    double x = (B1 * C2 - B2 * C1) / det;
    double y = (A2 * C1 - A1 * C2) / det;
    return {x, y};
}

Polygon HodgmanClip(const Polygon &subjectPolygon, const Polygon &
    clipPolygon) {
    Polygon outputList = subjectPolygon;
    for (size_t i = 0; i < clipPolygon.size(); ++i) {
        Point a = clipPolygon[i];
        Point b = clipPolygon[(i + 1) % clipPolygon.size()];
        Polygon inputList = outputList;
        outputList.clear();
        if (inputList.empty()) break;
        Point s = inputList.back();
        for (const auto &e : inputList) {
            if (inside(e, {a, b})) {
                if (!inside(s, {a, b})) {
                    Point intersection = compute_intersection(s, e, {a, b});
                    if (!std::isnan(intersection.x)) {
                        outputList.push_back(intersection);
                    }
                }
                outputList.push_back(e);
            } else if (inside(s, {a, b})) {
                Point intersection = compute_intersection(s, e, {a, b});
                if (!std::isnan(intersection.x)) {
                    outputList.push_back(intersection);
                }
            }
            s = e;
        }
    }
    return outputList;
}

// area of polygon
double shoelace(const Polygon &vertices) {
    double area = 0;
    for (size_t i = 0; i < vertices.size(); ++i) {
        Point p1 = vertices[i];
        Point p2 = vertices[(i + 1) % vertices.size()];
        area += p1.x * p2.y - p2.x * p1.y;
    }
    return fabs(area) / 2.0;
}
```

10.3.4 Rectangle & Circle

intersection area between Rectangle and Circle

```
struct Point {
    double x, y;
};

struct Circle {
    Point center;
    double radius;
};

struct Rectangle {
    Point bottomLeft, topRight;
};

const double PI = acos(-1.0);

bool point_inside_circle(const Point &p, const Circle &c) {
    double dx = p.x - c.center.x;
    double dy = p.y - c.center.y;
    return (dx * dx + dy * dy) <= (c.radius * c.radius);
}



double rect_area(const Rectangle &rect) {
    return (rect.topRight.x - rect.bottomLeft.x) * (rect.topRight.y - rect
        .bottomLeft.y);
}

bool rect_in_circle(const Circle &circle, const Rectangle &rect) {
    vector<Point> corners = {
        rect.bottomLeft,
```

```
        {rect.topRight.x, rect.bottomLeft.y},
        rect.topRight,
        {rect.bottomLeft.x, rect.topRight.y}
    };
    for (const auto &corner : corners) {
        if (!isPointInsideCircle(corner, circle)) {
            return false;
        }
    }
    return true;
}

bool circle_in_rect(const Circle &circle, const Rectangle &rect) {
 return circle.center.x - circle.radius >= rect.bottomLeft.x &&
           circle.center.x + circle.radius <= rect.topRight.x &&
           circle.center.y - circle.radius >= rect.bottomLeft.y &&
           circle.center.y + circle.radius <= rect.topRight.y;
}


double intersection(const Circle &circle, const Rectangle &rect) {
    if (rect_in_circle(circle, rect)) {
        return rect_area(rect);
    }

    if (circle_in_rect(circle, rect)) {
        return PI * circle.radius * circle.radius;
    }

    double intersectionArea = 0.0;
    double dx1 = max(rect.bottomLeft.x, circle.center.x - circle.radius);
    double dx2 = min(rect.topRight.x, circle.center.x + circle.radius);
    double dy1 = max(rect.bottomLeft.y, circle.center.y - circle.radius);
    double dy2 = min(rect.topRight.y, circle.center.y + circle.radius);

    for (double x = dx1; x < dx2; x += 0.001) {
        for (double y = dy1; y < dy2; y += 0.001) {
            Point p = {x, y};
            if (point_inside_circle(p, circle)) {
                intersectionArea += 0.001 * 0.001;
            }
        }
    }
    return intersectionArea;
}
```

10.3.5 Line & Circle

intersection points between Line and Circle

```
struct Point {
    double x, y;
};

struct Circle {
    Point center;
    double radius;
};

struct Line {
    double slope;
    double intercept;
};

vector<Point> intersect_points(const Circle &circle, const Line &line) {
    vector<Point> intersections;

    // solve y = mx + b with (x - h)^2 + (y - k)^2 = r^2
    double h = circle.center.x;
    double k = circle.center.y;
    double r = circle.radius;
    double m = line.slope;
    double b = line.intercept;

    // Quadratic coefficients
    double A = 1 + m * m;
    double B = 2 * (m * b - m * k - h);
    double C = k * k - r * r + h * h - 2 * b * k + b * b;

    // Discriminant
    double discriminant = B * B - 4 * A * C;

    if (discriminant < 0) {
        // No intersection
        return intersections;
    } else if (discriminant == 0) {
```

```
        // One intersection (tangent line)
        double x = -B / (2 * A);
        double y = m * x + b;
        intersections.push_back({x, y});
    } else {
        // Two intersections
        double sqrtDiscriminant = sqrt(discriminant);
        double x1 = (-B + sqrtDiscriminant) / (2 * A);
        double y1 = m * x1 + b;
        double x2 = (-B - sqrtDiscriminant) / (2 * A);
        double y2 = m * x2 + b;
        intersections.push_back({x1, y1});
        intersections.push_back({x2, y2});
    }

    return intersections;
}
```

10.4 Linear Operations

10.5 2D Points

Basic Operations

```
#include<bits/stdc++.h>

using namespace std;

#define ftype double

struct Point2D {
    ftype x, y;
    Point2D() {}
    Point2D(ftype x, ftype y): x(x), y(y) {}
    Point2D& operator+=(const Point2D &p2) {
        x += p2.x;
        y += p2.y;
        return *this;
    }
    Point2D& operator-=(const Point2D &p2) {
        x -= p2.x;
        y -= p2.y;
        return *this;
    }

    Point2D& operator*=(const Point2D p2) {
        x *= p2.x;
        y *= p2.y;
        return *this;
    }
    Point2D& operator/=(const Point2D p2) {
        x /= p2.x;
        y /= p2.y;
        return *this;
    }

    Point2D& operator+(const Point2D p2) {
        return Point2D(*this) += p2;
    }
    Point2D& operator-(const Point2D p2) {
        return Point2D(*this) -= p2;
    }
    Point2D& operator/(const Point2D p2) {
        return Point2D(*this) /= p2;
    }
    Point2D& operator*(const Point2D p2) {
        return Point2D(*this) *= p2;
    }


};

Point2D operator*(ftype a, Point2D b) {
        return a * b;
}


int main() {
    Point2D p1(1, 2);
    Point2D p2(3, 4);

    Point2D p3 = p1 * p2;

}
```

10.6 3D Points

Basic Operations

```
#include<bits/stdc++.h>

#define ftype double

struct Point3D {
    ftype x, y, z;
    Point3D() {}
    Point3D(ftype x, ftype y, ftype z): x(x), y(y), z(z) {}
    Point3D& operator+=(const Point3D &p2) {
        x += p2.x;
        y += p2.y;
        z += p2.z;
        return *this;
    }
    Point3D& operator-=(const Point3D &p2) {
        x -= p2.x;
        y -= p2.y;
        z -= p2.z;
        return *this;
    }

    Point3D& operator*=(const Point3D p2) {
        x *= p2.x;
        y *= p2.y;
        z *= p2.z;
        return *this;
    }
    Point3D& operator/=(const Point3D p2) {
        x /= p2.x;
        y /= p2.y;
        z /= p2.z;
        return *this;
    }

    Point3D& operator+(const Point3D p2) {
        return Point3D(*this) += p2;
    }
    Point3D& operator-(const Point3D p2) {
        return Point3D(*this) -= p2;
    }
    Point3D& operator/(const Point3D p2) {
        return Point3D(*this) /= p2;
    }
    Point3D& operator*(const Point3D p2) {
        return Point3D(*this) *= p2;
    }


};

Point3D operator*(ftype a, Point3D b) {
        return a * b;
}

int main() {
    Point3D p1(1, 2);
    Point3D p2(3, 4);

    Point3D p3 = p1 * p2;
}
```

11 Miscellaneous

11.1 Faster implementations

11.1.1 hashes

custom hash

```
#define safe hash unordered_map<type, type, custom_hash> // same for
    gp_hash_table
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
```

```
        static const uint64_t FIXED_RANDOM = chrono::steady_clock::now().
            time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
```

gb hash table

```
//policy based ds (faster hash table)
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
gp_hash_table<int, int> table;
```

11.1.2 Binary Search the value

nearest sqrt

```
long long my_sqrt(long long a)
{
    long long l=0,r=5000000001;
    while(r-l>1)
    {
        long long mid=(l+r)/2;
        if(1ll*mid*mid<=a)l=mid;
        else r=mid;
    }
    return l;
}
```