

Competitive Programming Library

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1 Templates

1.1 Setup

1.1.1 IO Manipulation

Input/Output

```
#include <bits/stdc++.h>

freopen("input.txt", "r", stdin);
freopen("output.txt", "w", stdout);

#define fastIO \
    ios_base::sync_with_stdio(false), cin.tie(nullptr), cout.tie(nullptr);
```

1.1.2 GCC Compiler Optimization (Vectorization)

GCC Opt

```
// Ref: USACO guide
// will make GCC auto-vectorize for loops and optimizes floating points
// better (assumes associativity and turns off denormals).
#pragma GCC optimize ("Ofast")
// can double performance of vectorized code, but causes crashes on old
// machines.
#pragma GCC target ("avx,avx2,fma")

// slows down run time but throws a Runtime Error if an overflow occurred
#pragma GCC optimize("trapv")
```

1.2 MOD Template

```
constexpr int MOD = 1e9+7; // must be a prime number

int add(int a, int b) {
    int res = a+b;
    if(res >= MOD) return res -= MOD;
}
```

```
int sub(int a, int b) {
    int res = a-b;
    if(res < 0) return res += MOD;
}

int power(int a, int e) {
    int res = 1;
    while(e) {if(e & 1) res = res * a % MOD; a = a * a % MOD;
    e >>= 1;}
    return res;
}

int inverse(int a) {
    return power(a, MOD-2);
}

int div(int a, int b) {
    return a * inverse(b) % MOD;
}
```

MOD Template

1.3 Macros

Macros

```
#define getBit(n, k) (n >> k) // & 1
#define ON(n, idx) (n | (1ll << idx))
#define OFF(n, idx) (n & ~(1ll << idx))
#define toggle(n, idx) ((n) ^ (1ll<<(idx)))
#define gray(n) (n ^ (n >> 1))
#define bitCount(x) (__builtin_popcountll(x))
#define clz(x) (__builtin_clzll(x))
#define ctz(x) (__builtin_ctzll(x))
#define uniq(x) x.resize(unique(x.begin(), x.end())-x.begin());

#define angle(a) (atan2((a).imag(), (a).real()))
// #define vec(a, b) ((b)-(a))
#define same(v1, v2) (dp(vec(v1,v2),vec(v1,v2)) < EPS)
#define dotProduct(a, b) ((conj(a)*(b)).real()) // a*b cos(T), if zero ->
// prep
```

```

#define crossProduct(a, b) ((conj(a)*(b)).imag()) // a*b sin(T), if zero
    -> parallel
//#define length(a) (hypot((a).imag(), (a).real()))
#define normalize(a) ((a)/length(a))
#define rotate0(v, ang) ((v)*exp(point(0,ang)))
#define rotateA(p, ang, about) (rotate0(vec(about,p),ang)+about)
#define reflect0(v, m) (conj((v)/(m))*(m))
#define ceil_i(a, b) (((ll)(a)+(ll)(b-1))/(ll)(b))
#define floor_i(a, b) (a/b)
#define round_i(a, b) ((a+(b/2))/b) // if a>0
#define round_m(a, b) ((a-(b/2))/b) // if a<0
#define round_multiple(n, m) round_i(n,m)*m // round to multiple if
    specified element

const double PI = acos(-1.0);

```

1.4 Grid Navigation

Grid Nav

```

// knight moves on a chess board
int dx[] = { -2, -1, 1, 2, -2, -1, 1, 2 };
int dy[] = { -1, -2, -2, -1, 1, 2, 2, 1 };

// Grid up, down, right, left (Moves for Chess Rook)
int dx[4] = {1, -1, 0, 0};
int dy[4] = {0, 0, 1, -1};

// Grid cell all neighbours
const int dx[8] = {1, 0, -1, 0, 1, 1, -1, -1}
const int dy[8] = {0, 1, 0, -1, -1, 1, -1, 1};

// Grid Diagonal (Moves for Chess Bishop)
int dx[] = {1, 1, -1, -1};
int dy[] = {1, -1, 1, -1};

```

1.5 Integer 128

i128

```

typedef __int128 i128;

__int128 read() {
    __int128 x = 0, f = 1;
    char ch = getchar();
    while (ch < '0' || ch > '9') {
        if (ch == '-') f = -1;
        ch = getchar();
    }
    while (ch >= '0' && ch <= '9') {
        x = x * 10 + ch - '0';
        ch = getchar();
    }
    return x * f;
}

void print(__int128 x) {
    if (x < 0) {
        putchar('-');
        x = -x;
    }
    if (x > 9) print(x / 10);
    putchar(x % 10 + '0');
}

bool cmp(__int128 x, __int128 y) { return x > y; }

```

1.6 Matrix Expo

MExpo

```

constexpr int MAX_N = 201, MOD = 1e9+7;
struct Matrix { int mat[MAX_N][MAX_N]; }; // we return a 2D array
ll mod(ll a, ll m) { return ((a%m)+m) % m; } // ensure positive answer
Matrix matMul(Matrix a, Matrix b) { // normally O(n^3)
    Matrix ans{}; // but O(1) as n = 2
    for (auto & i : ans.mat)
        for (int & j : i)
            j = 0;

    for (int i = 0; i < MAX_N; ++i)

```

```

    for (int k = 0; k < MAX_N; ++k) {
        if (a.mat[i][k] == 0) continue; // optimization
        for (int j = 0; j < MAX_N; ++j) {
            ans.mat[i][j] += mod(a.mat[i][k], MOD) * mod(b.mat[k][j],
                MOD);
            ans.mat[i][j] = mod(ans.mat[i][j], MOD);
        }
    }
    return ans;
}
Matrix matPow(Matrix base, int p) { // normally  $O(n^3 \log p)$ 
    Matrix ans{}; // but  $O(\log p)$  as  $n = 2$ 
    for (int i = 0; i < MAX_N; ++i)
        for (int j = 0; j < MAX_N; ++j)
            ans.mat[i][j] = (i == j); // prepare identity matrix
    while (p) { // iterative D&C version
        if (p&1) // check if p is odd
            ans = matMul(ans, base); // update ans
        base = matMul(base, base); // square the base
        p >>= 1; // divide p by 2
    }
    return ans;
}
Matrix matMul(Matrix a, Matrix b, int p, int q, int r) { //  $O(pqr)$ 
    Matrix c{};
    for (int i = 0; i < p; ++i)
        for (int j = 0; j < r; ++j) {
            c.mat[i][j] = 0;
            for (int k = 0; k < q; ++k)
                c.mat[i][j] += a.mat[i][k] + b.mat[k][j];
        }
    return c;
}

```

2 Dynamic Programming

2.1 Some dp patterns

Maximumu/Minimum path cost

```

const int MAX = 21;
int grid[MAX][MAX];
int mem[MAX][MAX];
int n = 20;
bool valid(int r, int c){
    return r >= 0 && r < n && c >= 0 && c < n;
}

int maxPathSum(int r, int c){
    if(!valid(r,c)){
        return 0;
    }

    if(r == n-1 && c == n-1){
        return mem[r][c] = grid[r][c];
    }
    // available moves
    int path1 = maxPathSum(r+1,c);
    int path2 = maxPathSum(r,c+1);

    return grid[r][c] + max(path1,path2);
}

```

add operators between numbers to get max prod/sum

// put +, -, between sequence of numbers such that the sum is divisible by k, and maximum as possible

```

const int MAX = 21;
long long mem[MAX][MAX];
const int n = 20;
int k = 4; // example
int v[20];
int fix(int a){
    return (a % k + k) % k;
}
long long tryAll(int pos, int mod){
    long long &ret = mem[pos][mod];
    if(ret != -1){
        return ret;
    }
    if(pos == n){
        return ret = mod == 0;
    }
}

```

```

    if(tryAll(pos+1,fix(mod + v[pos])) || tryAll(pos+1,fix(mod-v[pos]))){
        return ret = 1;
    }
    return ret = 0;
}

```

pick choices with no two similar consecutive choices

```

// pick minimum of choinces costs with no two similar consecutive choices
const int choices = 4;
const int n = 20;
int MAX = n;
int mem[MAX][choices];
const int OO = 1e6+1;
int minCost(int pos, int lastChoice){
    if(pos == n){
        return 0; // invalid move
    }
    int &ret = mem[pos][lastChoice];

    if(ret != -1){
        return ret;
    }

    ret = OO; // want to minimize
    // let choices are 0, 1, 2
    if(lastChoice != 0){
        ret = min(ret, minCost(pos+1,0));
    }
    if(lastChoice != 1){
        ret = min(ret, minCost(pos+1,1));
    }
    if(lastChoice != 2){
        ret = min(ret, minCost(pos+1,2));
    }
    return ret;
}

```

sum S and max/min Product

```
int maxK;
```

```
ll mem[21][101]; // k, and s
```

// You are given an integer s and an integer k. Find k positive integers a1, a2, ..., ak such that their sum is equal to s and their product is the maximal possible. Return their product.

```
ll maxProd(int k, int rem)
```

```

{
    if(k == maxK){
        // base case
        if(rem == 0)
            return 1;
        return 0;
    }

    if(rem == 0) // invalid case
        return 0;

```

```
ll &ret = mem[k][rem];
```

```

if(ret != -1)
    return ret;

```

```
ret = 0;
```

```

for (int i = 1; i <= rem; ++i) {
    ll sol = maxProd(k+1, rem - i) * i;
    ret = max(ret, sol);
}

```

```
return ret;
```

```
}
```

2.2 DP solutions

2.2.1 Max Subarray sum (Kadane's Algorithm)

Max Subarray sum

```
int maxSubarraySum(vector<int>& arr, int len) {
```

```
int ans = INT_MIN, cur = 0;

for (int i = 0; i < len; i++) {
    cur = cur + arr[i];
    if (ans < cur)
        ans = cur;

    if (cur < 0)
        cur = 0;
}

return ans;
}
```

2.2.2 Maximum Subarray Alternating Sum

Maximum Subarray Alternating Sum

/* REF: GeeksForGeeks

Input: arr[] = {-4, -10, 3, 5}

Output: 9

Explanation: Subarray {arr[0], arr[2]} = {-4, -10, 3}. Therefore, the sum of this subarray is 9.

*/

```
int maxSubarraySumALT(vector<int>& a, int len) {
    int ans = INT_MIN, cur = 0;
    for (int i = 0; i < len; i++) {
        if (i % 2 == 0)
            cur = max(cur + a[i], a[i]);
        else
            cur = max(cur - a[i], -a[i]);

        ans = max(ans, cur);
    }

    cur = 0;
```

```
for (int i = 0; i < len; i++) {
    if (i % 2 == 1)
        cur = max(cur + a[i], a[i]);
    else
        cur = max(cur - a[i], -a[i]);
```

```
        ans = max(ans, cur);
    }

    return ans;
}
```

2.2.3 Count number of DISTINCT ordered ways to produce coins sums to x

Count distinct

/*

For example, if the coins are {2,3,5} and the desired sum is 9, there are 3 ways:

2+2+5

3+3+3

2+2+2+3

*/

```
int n, x;
cin >> n >> x;
vector<int> coins(n);
read(coins);

vector dp(x + 1, 0);

dp[0] = 1;
for (int i = 0; i < n; ++i) {
    for (int j = coins[i]; j <= x; ++j) {
        dp[j] = add(dp[j], dp[j - coins[i]]);
    }
}

cout << dp[x] << endl;
```

2.2.4 Min absolute difference between 2 elements from (L, R) (DP Ranges)

Min absolute difference

```
const int N = 1e4 + 1;
```

```

int dp[N][N];

int n;
cin >> n;
vector<int> a(n);
read(a);

for (int i = 0; i < n; ++i) dp[i][i] = 1e6; // INF, you can't take the
    element with it self
for (int i = 1; i < n; ++i) dp[i - 1][i] = abs(a[i] - a[i - 1]);

for (int len = 3; len <= n; ++len) {
    for (int l = 0, r = len - 1; r < n; ++l, ++r) {
        dp[l][r] = min(dp[l][r - 1], dp[l + 1][r]);
        dp[l][r] = min(dp[l][r], abs(a[l] - a[r]));
    }
}

int q;
cin >> q;
while (q--) {
    int l, r;
    cin >> l >> r;
    --l, --r;

    cout << dp[l][r] << endl;
}

```

2.2.5 Longest common subsequence between 2 Strings

$$dp[i][j] = \begin{cases} \max(dp[i-1][j], dp[i][j-1]) & \text{if } A_i \neq B_j \\ dp[i-1][j-1] + 1 & \text{if } A_i = B_j \end{cases}$$

LIS 2 Strings

```

// REF: USACO guide
int longestCommonSubsequence(string a, string b) {
    int dp[a.size()][b.size()];
    for (int i = 0; i < a.size(); i++) { fill(dp[i], dp[i] + b.size(), 0); }
    for (int i = 0; i < a.size(); i++) {
        if (a[i] == b[0]) dp[i][0] = 1;

```

```

        if (i != 0) dp[i][0] = max(dp[i][0], dp[i - 1][0]);
    }
    for (int i = 0; i < b.size(); i++) {
        if (a[0] == b[i]) dp[0][i] = 1;
        if (i != 0) dp[0][i] = max(dp[0][i], dp[0][i - 1]);
    }
    for (int i = 1; i < a.size(); i++) {
        for (int j = 1; j < b.size(); j++) {
            if (a[i] == b[j]) {
                dp[i][j] = dp[i - 1][j - 1] + 1;
            } else {
                dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
            }
        }
    }
    return dp[a.size() - 1][b.size() - 1];
}

```

2.2.6 Longest common subsequence $\mathcal{O}(n^2)$

LIS

```

// REF: cp-algorithms
int lis(vector<int> const& a) {
    int n = a.size();
    vector<int> d(n, 1);
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < i; j++) {
            if (a[j] < a[i])
                d[i] = max(d[i], d[j] + 1);
        }
    }

    int ans = d[0];
    for (int i = 1; i < n; i++) {
        ans = max(ans, d[i]);
    }
    return ans;
}

```

```

// Restoring
vector<int> lis(vector<int> const& a) {
    int n = a.size();

```



```
vector<int> d(n, 1), p(n, -1);
for (int i = 0; i < n; i++) {
    for (int j = 0; j < i; j++) {
        if (a[j] < a[i] && d[i] < d[j] + 1) {
            d[i] = d[j] + 1;
            p[i] = j;
        }
    }
}

int ans = d[0], pos = 0;
for (int i = 1; i < n; i++) {
    if (d[i] > ans) {
        ans = d[i];
        pos = i;
    }
}

vector<int> subseq;
while (pos != -1) {
    subseq.push_back(a[pos]);
    pos = p[pos];
}
reverse(subseq.begin(), subseq.end());
return subseq;
}
```

2.2.7 Longest common subsequence Binary Search $\mathcal{O}(n + \log N)$

LIS

```
int lisBS(vector<int> const& a) {
    int n = a.size();
    const int INF = 1e9;
    vector<int> d(n+1, INF);
    d[0] = -INF;

    for (int i = 0; i < n; i++) {
        int l = upper_bound(d.begin(), d.end(), a[i]) - d.begin();
        if (d[l-1] < a[i] && a[i] < d[l])
            d[l] = a[i];
    }
}
```

```
int ans = 0;
for (int l = 0; l <= n; l++) {
    if (d[l] < INF)
        ans = l;
}
return ans;
}
```

3 Bit Manipulation

3.1 Subset Operations

count subsets with give sum

```
int countDistinctSubsetsWithSum(vector<int>& arr, int n, int k) {
    // Count distinct subsets of array arr that sum up to k
    vector<int> dp(k + 1, 0);
    dp[0] = 1;
    for (int i = 0; i < n; ++i) {
        for (int j = k; j >= arr[i]; --j) {
            dp[j] += dp[j - arr[i]];
        }
    }
    return dp[k]; // Number of distinct subsets with sum k
}
```

max xor of any subset of elements in the array

```
int maximalSubsetXOR(vector<int>& arr, int n) {
    // Find the maximum XOR of any subset of elements in array arr
    int maxXor = 0;
    for (int mask = 0; mask < (1 << n); ++mask) {
        int xorSum = 0;
        for (int i = 0; i < n; ++i) {
            if (mask & (1 << i)) {
                xorSum ^= arr[i];
            }
        }
        maxXor = max(maxXor, xorSum);
    }
}
```

```
    return maxXor;
}
```

min xor of any subset

```
int minimumSubsetXOR(vector<int>& arr, int n) {
    // Find the minimum XOR of any pair of elements in array arr
    int minSubsetXor = INT_MAX;
    for (int mask = 0; mask < (1 << n); ++mask) {
        int xorSum = 0;
        for (int i = 0; i < n; ++i) {
            if (mask & (1 << i)) {
                xorSum ^= arr[i];
            }
        }
        minSubsetXor = min(minSubsetXor, xorSum);
    }
    return minSubsetXor;
}
```

subset generation

```
void subsetGeneration(int x, int n) {
    // Generate all non-empty subsets of a set represented by an integer x
    for (int subset = x; subset > 0; subset = (subset - 1) & x) {
        // Process subset
        cout << subset << endl;
    }
}
```

check if subset of elements in the array sum up to k

```
void subsetSumCheck(vector<int>& arr, int n, int k) {
    // Check if a subset of elements in array arr sums up to k
    for (int subset = 0; subset < (1 << n); ++subset) {
        int sum = 0;
        for (int i = 0; i < n; ++i) {
            if (subset & (1 << i)) {
                sum += arr[i];
            }
        }
    }
}
```

```
    }
    if (sum == k) {
        // Found subset with sum k
        cout << "Subset with sum " << k << ": " << subset << endl;
    }
}
```

max subset sum mod m

```
int subsetWithMaxSumModuloM(vector<int>& arr, int n, int m) {
    // Find the maximum subset sum modulo m
    vector<int> dp(m, -1);
    dp[0] = 0;
    int currentMod = 0;
    for (int i = 0; i < n; ++i) {
        currentMod = (currentMod + arr[i]) % m;
        for (int j = 0; j < m; ++j) {
            if (dp[j] != -1) {
                dp[(j + currentMod) % m] = max(dp[(j + currentMod) % m], dp[j] + arr[i]);
            }
        }
        dp[currentMod] = max(dp[currentMod], arr[i]);
    }
    return dp[0]; // Maximum subset sum modulo m
}
```

iterate over all supersets represented by x

```
void iterateOverSupersets(int x, int n) {
    // Iterate over all supersets of a set represented by x
    int subset = x;
    do {
        // Process subset
        cout << subset << endl;
        subset = (subset + 1) | x;
    } while (subset <= (1 << n) - 1);
}
```

4 Algorithms

4.1 FFT

FFT Algorithm

```
constexpr ll mod = 998244353, root = 3;

ll modpow(ll b, ll e, ll m) {
    ll ans = 1;
    for (; e; b = b * b % m, e /= 2)
        if (e & 1) ans = ans * b % m;
    return ans;
}

// Primitive Root of the mod of form 2^a * b + 1
int generator () {
    vector<int> fact;
    int phi = mod-1, n = phi;
    for (int i=2; i*i<=n; ++i)
        if (n % i == 0) {
            fact.push_back (i);
            while (n % i == 0)
                n /= i;
        }
    if (n > 1)
        fact.push_back (n);

    for (int res=2; res<=mod; ++res) {
        bool ok = true;
        for (size_t i=0; i<fact.size() && ok; ++i)
            ok &= modpow(res, phi / fact[i], mod) != 1;
        if (ok) return res;
    }
    return -1;
}

void ntt(vector<ll> &a) {
    int n = a.size(), L = 31 - __builtin_clz(n);
    static vector<ll> rt(2, 1); // erase the static if you want to use two moduli;
    for (static int k = 2, s = 2; k < n; k *= 2, s++) { // erase the static if you want to use two moduli;
        rt.resize(n);
```

```
ll z[] = {1, modpow(root, mod >> s, mod)};
    for (int i = k; i < 2*k; ++i) rt[i] = rt[i / 2] * z[i & 1] % mod;
}

vector<int> rev(n);
for (int i = 0; i < n; ++i) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
for (int i = 0; i < n; ++i) if (i < rev[i]) swap(a[i], a[rev[i]]);
for (int k = 1; k < n; k *= 2) {
    for (int i = 0; i < n; i += 2 * k) {
        for (int j = 0; j < k; ++j) {
            ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
            a[i + j + k] = ai - z + (z > ai ? mod : 0);
            ai += (ai + z >= mod ? z - mod : z);
        }
    }
}

vector<ll> multi(const vector<ll> &a, const vector<ll> &b) {
    if (a.empty() || b.empty()) return {};
    int s = static_cast<ll>(a.size()) + static_cast<ll>(b.size()) - 1, B =
        32 - __builtin_clz(s), n = 1 << B;
    int inv = modpow(n, mod - 2, mod);
    vector<ll> L(a), R(b), out(n);
    L.resize(n), R.resize(n);
    ntt(L), ntt(R);
    for (int i = 0; i < n; ++i) out[-i & (n - 1)] = L[i] * R[i] % mod *
        inv % mod;
    ntt(out);
    return {out.begin(), out.begin() + s};
}

vector<int> poly_pow(vector<int> poly, int p, int limit = 1e9) {
    vector<int> ans{1};
    while (p) {
        if(p&1) ans = multi(ans, poly);
        poly = multi(poly, poly);
        ans.resize(limit + 1);
        poly.resize(limit + 1);
        p >>= 1;
    }
    return ans;
}
```

4.2 MO

MO Algorithm

```
// MO      -> O(N+Q SQRT(N)) <= 10^5

const int N = 1e5+5, M = 1e5+5;
int n, m;
int nums[N], q_ans[M];

struct query {
    int idx, block_idx, l, r;

    query() = default;
    query(int _l, int _r, int _idx) {
        idx = _idx;
        r = _r - 1;
        l = _l - 1;
        block_idx = _l / sqrt(n);
    }

    bool operator <(const query & y) const {
        if(y.block_idx == block_idx) return r < y.r;
        return block_idx < y.block_idx;
    }
};

int freq[N], ans;

void add(int idx) {
    freq[nums[idx]]++;
    if (freq[nums[idx]] == 2) ans++;
}

void remove(int idx) {
    freq[nums[idx]]--;
    if (freq[nums[idx]] == 1) ans--;
}

cin >> n >> m;
for (int i = 0; i < n; ++i) cin >> nums[i];

vector<query> Query(m);
for (int i = 0; i < m; ++i) {
    int l, r; cin >> l >> r;
```

```
    Query[i] = query(l, r, i);
}

sort(Query.begin(), Query.end());
int l0 = 1, r0 = 0;
for (int i = 0; i < m; ++i) {
    while (l0 < Query[i].l) remove(l0++);
    while (l0 > Query[i].l) add(--l0);
    while (r0 < Query[i].r) add(++r0);
    while (r0 > Query[i].r) remove(r0--);
    q_ans[Query[i].idx] = ans;
}
for (int i = 0; i < m; ++i) {
    cout << q_ans[i] << '\n';
}
```

4.3 Intervals

4.3.1 Prefix Sum (L, R) intervals

Prefix Sum (L, R) intervals

// NOTE: works fine with small n or with large memory

```
int main() {
    int n, k;
    cin >> n >> k;

    vector<int> a(n + 1);
    vector<vector<int>> rangesPrefix(n + 1, vector<int>(n + 1, 0));
    for (int i = 1; i <= n; ++i)
        cin >> a[i];

    int l = 1, r = 1, sum = 0;
    // validate your intervals
    // here the intervals are the ones that have a sum of k
    while (r <= n) {
        sum += a[r];

        while (sum > k) {
            sum -= a[l];
```

```

    ++l;
}

while (l <= r && a[l] == 0) {
    if (sum != k)
        break;

    rangesPrefix[r][l]++;

    ++l;
}

if (sum == k) {
    rangesPrefix[r][l]++;
}

++r;
}

// prefix sum the columns
for (int i = 1; i <= n; ++i) {
    for (int j = n - 1; j >= 0; --j) {
        rangesPrefix[i][j] += rangesPrefix[i][j + 1];
    }
}

// prefix sum the rows
for (int i = 0; i <= n; ++i) {
    for (int j = 1; j <= n; ++j) {
        rangesPrefix[j][i] += rangesPrefix[j - 1][i];
    }
}

int q; cin >> q;

while (q--) {
    cin >> l >> r;
    // answer the number of intervals (X, Y) X <= Y that are included
    // between L, R
    cout << rangesPrefix[r][l] - rangesPrefix[l - 1][l] << endl;
}
}

```

4.3.2 Find subarrays intervals that sum to K Using Map

Find subarray intervals that sum to K Using Map

```

int n, k;
cin >> n >> k;

vector<int> a(n + 1);
vector<pair<int, int>> rng;
for (int i = 1; i <= n; ++i)
    cin >> a[i];

map<int, set<int>> prev;
int currSum = 0;

for (int i = 1; i <= n; ++i) {
    currSum += a[i];
    if (currSum == k) {
        rng.push_back({1, i});
    }
    if (prev.find(currSum - k) != prev.end()) {
        for (auto &j : prev[currSum - k]) {
            rng.push_back({j + 1, i});
        }
    }
    prev[currSum].insert(i);
}

```

4.4 Ad-hoc

4.4.1 Find duplicate

Find duplicate using XOR

```

int findDuplicate(int arr[] , int n)
{
    int answer=0;
    //XOR all the elements with 0
    for(int i=0; i<n; i++){
        answer=answer^arr[i];
    }
    //XOR all the elements with no from 1 to n
    // i.e  answer^0 = answer

```

```
for(int i=1; i<n; i++){
    answer=answer^i;
}
return answer;
}
```

4.5 Sorting Algorithms

4.5.1 Radix Sort

Radix Sort

```
// O(n + b), where n is the number of elements and b is the base of the
// number system
// A function to do counting sort of arr[] according to the digit
// represented by exp.
void countingSort(vector<int>& arr, int exp) {
    int n = arr.size();
    vector<int> output(n); // output array
    int count[10] = {0};

    // Store count of occurrences in count[]
    for (int i = 0; i < n; i++)
        count[(arr[i] / exp) % 10]++;

    // Change count[i] so that count[i] now contains the actual
    // position of this digit in output[]
    for (int i = 1; i < 10; i++)
        count[i] += count[i - 1];

    // Build the output array
    for (int i = n - 1; i >= 0; i--) {
        output[count[(arr[i] / exp) % 10] - 1] = arr[i];
        count[(arr[i] / exp) % 10]--;
    }

    // Copy the output array to arr[], so that arr now
    // contains sorted numbers according to the current digit
    for (int i = 0; i < n; i++)
        arr[i] = output[i];
}
```

```
// The main function to that sorts arr[] of size n using Radix Sort
void radixSort(vector<int>& arr) {
    // Find the maximum number to know the number of digits
    int mx = *max_element(arr.begin(), arr.end());

    // Do counting sort for every digit. Note that instead
    // of passing the digit number, exp is passed. exp is 10^i
    // where i is the current digit number
    for (int exp = 1; m / exp > 0; exp *= 10)
        countingSort(arr, exp);
}
```

4.5.2 Counting Sort

Counting Sort

```
// O(N+M), where N and M are the size of inputArray[] and countArray[]
// The main function that sorts arr[] of size n using Counting Sort
void countingSort(vector<int>& arr) {
    int maxElement = *max_element(arr.begin(), arr.end());
    int minElement = *min_element(arr.begin(), arr.end());
    int range = maxElement - minElement + 1;

    vector<int> count(range), output(arr.size());
    for (int i = 0; i < arr.size(); i++)
        count[arr[i] - minElement]++;

    for (int i = 1; i < count.size(); i++)
        count[i] += count[i - 1];

    for (int i = arr.size() - 1; i >= 0; i--) {
        output[count[arr[i] - minElement] - 1] = arr[i];
        count[arr[i] - minElement]--;
    }

    for (int i = 0; i < arr.size(); i++)
        arr[i] = output[i];
}
```

4.6 Apply permutation k times

permutation k times

```
// Applying a permutation k times
// n log k
```

```
vector<int> applyPermutation(vector<int> sequence, vector<int> permutation
) {
    vector<int> newSequence(sequence.size());
    for(int i = 0; i < sequence.size(); i++) {
        newSequence[i] = sequence[permutation[i]];
    }
    return newSequence;
}

vector<int> permute(vector<int> sequence, vector<int> permutation, long
long k) {
    while (k > 0) {
        if (k & 1) {
            sequence = applyPermutation(sequence, permutation);
        }
        permutation = applyPermutation(permutation, permutation);
        k >>= 1;
    }
    return sequence;
}
```

5 Data Structures

5.1 Strings

5.1.1 Trie (Prefix Tree)

Basic Implementation

```
#define MAX_CHAR 26
```

```
struct TrieNode {
    TrieNode *pTrieNode[MAX_CHAR]{};
    bool isWord;
```

```
TrieNode() {
    isWord = false;
    fill(pTrieNode, pTrieNode + 26, (TrieNode *) NULL);
}

virtual ~TrieNode() = default;
};

class Trie {
private:
    TrieNode *root;
public:
    Trie() {
        root = new TrieNode();
    }

    virtual ~Trie() = default;

    TrieNode *getTrieNode() {
        return this->root;
    }

    void insert(const string &word) {
        TrieNode *current = root;
        for (char c: word) {
            int i = c - 'a';
            if (current->pTrieNode[i] == nullptr)
                current->pTrieNode[i] = new TrieNode();
            current = current->pTrieNode[i];
        }
        current->isWord = true;
    }

    bool search(const string &word) {
        TrieNode *current = root;
        int ch = 0;
        for (char c: word) {
            ch = c - 'a';
            if (current->pTrieNode[ch] == nullptr)
                return false;
            current = current->pTrieNode[ch];
        }
        return current->isWord;
    }
}
```

```
bool startsWith(const string &prefix) {
    TrieNode *current = root;
    int ch = 0;
    for (char c: prefix) {
        ch = c - 'a';
        if (current->pTrieNode[ch] == nullptr)
            return false;
        current = current->pTrieNode[ch];
    }
    return true;
};
```

5.2 Range Queries

5.2.1 Segment Tree

Basic Implementation

```
struct Node {
    long long val;
};

struct SegTree {
private:
    const Node NEUTRAL = {INT_MIN};

    static Node merge(const Node& x1, const Node& x2) {
        return {x1.val + x2.val};
    }

    void set(const int& idx, const int& val, int x, int lx, int rx) {
        if (rx - lx == 1) return void(values[x].val = val);

        int mid = (rx + lx) / 2;

        if (idx < mid)
            set(idx, val, 2 * x + 1, lx, mid);
        else
            set(idx, val, 2 * x + 2, mid, rx);
    }
};
```

```
values[x] = merge(values[2 * x + 1], values[2 * x + 2]);
}

Node query(const int& l, const int& r, int x, int lx, int rx) {
    if (lx >= r || l >= rx) return NEUTRAL;
    if (lx >= l && rx <= r) return values[x];

    int mid = (rx + lx) / 2;

    return merge(query(l, r, 2 * x + 1, lx, mid), query(l, r, 2 * x + 2, mid, rx));
}

void build(vector<int> &a, int x, int lx, int rx) {
    if (rx - lx == 1) {
        if (lx < a.size()) {
            values[x].val = a[lx];
        }
        return;
    }
    int m = (lx + rx) / 2;
    build(a, 2 * x + 1, lx, m);
    build(a, 2 * x + 2, m, rx);
    values[x] = merge(values[2 * x + 1], values[2 * x + 2]);
}

void assign_range(int l, int r, int node, int lx, int rx, int time,
    int val) {
    if (lx > r || l > rx) return;
    if (lx >= l && rx <= r) {
        lazy[node] = {time, val};
        return;
    }
    int mid = (lx+rx) / 2;

    assign_range(l, r, 2*node+1, lx, mid, time, val);
    assign_range(l, r, 2*node+2, mid+1, rx, time, val);
}

pair<int, int> point_query(int lx, int rx, int node, int idx) {
    if(rx == lx) return lazy[node];
    int mid = (lx+rx) / 2;

    if(idx <= mid) {
        auto x = point_query(lx, mid, 2*node+1, idx);
```



```

        if(x.first > lazy[node].first) return x;
        return lazy[node];
    }
    auto x = point_query(mid+1, rx, 2*node+2, idx);
    if(x.first > lazy[node].first) return x;
    return lazy[node];
}

public:
    int size{};
    vector<Node> values;

    void build(vector<int> &a) {
        build(a, 0, 0, size);
    }

    void init(int _size) {
        size = 1;
        while (size < _size) size *= 2;
        values.assign(2 * size, NEUTRAL);
    }

    void set(int idx, int val) {
        set(idx, val, 0, 0, size);
    }

    Node query(const int& l, const int& r) {
        return query(l, r, 0, 0, size);
    }
};

```

5.2.2 Lazy Propagation

Lazy Propagation

```

struct SegTree {
private:
    void propagate(int lx, int rx, int node) {
        if(!lazy[node]) return;

        if(lx != rx) {
            lazy[2*node+1] = lazy[node];
            lazy[2*node+2] = lazy[node];

```

```

        }
        values[node] = lazy[node] * (rx - lx + 1);
        lazy[node] = 0;
    }

    // assign val in range [l, r]
    void update_range(int l, int r, int node, int lx, int rx, int val,
        bool f) {
        propagate(lx, rx, node);
        if (lx > r || l > rx) return;
        if (lx >= l && rx <= r) {
            lazy[node] = val;
            propagate(lx, rx, node);
            return;
        }
        int mid = (lx+rx) / 2;

        update_range(l, r, 2*node+1, lx, mid, val, f);
        update_range(l, r, 2*node+2, mid+1, rx, val, f);
        values[node] = values[2*node+1] + values[2*node+2];
    }

    // get sum in range [l, r]
    int range_query(int l, int r, int lx, int rx, int node) {
        propagate(lx, rx, node);
        if (lx > r || l > rx) return 0;
        if (lx >= l && rx <= r) return values[node];

        int mid = (lx+rx) / 2;
        return range_query(l, r, lx, mid, 2*node+1) + range_query(l, r, mid
            +1, rx, 2*node+2);
    }

public:
    int size{};
    vector<int> values, lazy;

    void init(int _size) {
        size = 1;
        while (size < _size) size *= 2;
        values.assign(2 * size, 0);
        lazy.assign(2 * size, 0);
    }
}

```

```
void update_range(int l, int r, int v, bool f) {
    update_range(l, r, 0, 0, size-1, v, f);
}

int range_query(int l, int r) {
    return range_query(l, r, 0, size-1, 0);
}
};
```

5.2.3 Fenwick Tree

Fenwick Tree

```
struct Fenwick {
    // One Based
    vector<int> tree;

    explicit Fenwick(int n) {tree.assign(n + 5, {});}

    // Computes the prefix sum from [1, i], O(log(n))
    int query(int i) {
        int res = 0;
        while (i > 0) {
            res += tree[i];
            i &= ~(i & -i);
        }
        return res;
    }

    int query(int l, int r) {
        return query(r) - query(l-1);
    }

    // Get the value at index i
    int get(int i) {
        return query(i, i);
    }

    // Add 'v' to index 'i', O(log(n))
    void update(int i, int v) {
        while (i < tree.size()) {
            tree[i] += v;
            i += (i & -i);
        }
    }
};
```

```
    }
}

// Update range, Point query
// To get(k) do prefix sum [1, k] and in insert update_range(i, i, a[i])
void update_range(int l, int r, int v) {
    update(l, v);
    update(r+1, -v);
}
};
```

5.2.4 Fenwick UpdateRange

BIT UpdateRange

```
struct BITUpdateRange {
private:
    int n;
    vector<int> B1, B2;

    void add(vector<int> &b, int idx, int x) {
        while (idx <= n) {
            b[idx] += x;
            idx += idx & -idx;
        }
    }

    int sum(vector<int> &b, int idx) {
        int total = 0;
        while (idx > 0) {
            total += b[idx];
            idx &= ~(idx & -idx);
        }
        return total;
    }

    int prefix(int idx) {
        return sum(B1, idx) * idx - sum(B2, idx);
    }

public:
    explicit BITUpdateRange(int n) : n(n) {
```

```

        B1.assign(n + 1, {});
        B2.assign(n + 1, {});
    }

    void update(int l, int r, int x) {
        add(B1, l, x);
        add(B1, r + 1, -x);
        add(B2, l, x * (l - 1));
        add(B2, r + 1, -x * r);
    }

    int query(int i) {
        return prefix(i) - prefix(i - 1);
    }

    int query(int l, int r) {
        return prefix(r) - prefix(l - 1);
    }
};

```

5.2.5 2D BIT

2D BIT

```

struct BIT2D {
    int n, m;
    vector<vector<int>> bit;

    BIT2D(int n, int m) : n(n), m(m) {
        bit.assign(n + 2, vector<int>(m + 2));
    }

    void update(int x, int y, int val) {
        for (; x <= n; x += x & -x) {
            for (int i = y; i <= m; i += i & -i) {
                bit[x][i] += val;
            }
        }
    }

    int prefix(int x, int y) {
        int res = 0;
        for (; x > 0; x &= ~(x & -x)) {

```

```

            for (int i = y; i > 0; i &= ~(i & -i)) {
                res += bit[x][i];
            }
        }
        return res;
    }

    int query(int sx, int sy, int ex, int ey) {
        int ans = 0;
        ans += prefix(ex, ey);
        ans -= prefix(ex, sy - 1);
        ans -= prefix(sx - 1, ey);
        ans += prefix(sx - 1, sy - 1);
        return ans;
    }
};

```

5.2.6 Sparse Table

Impl with the index

```

// storing the index also
struct SNode {
    int val;
    int index;
};

class SparseTable {
private:
    vector<vector<SNode>> table;

    function<SNode(const SNode&, const SNode&)> merge;

    static SNode StaticMerge(const SNode& a, const SNode& b) {
        return a.val < b.val ? a : b;
    }

public:
    explicit SparseTable(const vector<int>& arr, const function<SNode(
        const SNode&, const SNode&)>& mergeFunc = StaticMerge) {
        int n = static_cast<int>(arr.size());
        int log_n = static_cast<int>(log2(n)) + 1;
        this->merge = mergeFunc;
    }
};

```

```

    table.resize(n, vector<SNode>(log_n));

    for (int i = 0; i < n; i++) {
        table[i][0] = {arr[i], i};
    }

    for (int j = 1; (1 << j) <= n; j++) {
        for (int i = 0; i + (1 << j) <= n; i++) {
            table[i][j] = mergeFunc(table[i][j - 1], table[i + (1 << (j
                - 1))] [j - 1]);
        }
    }

    SNode query(int left, int right) {
        int j = static_cast<int>(log2(right - left + 1));
        return merge(table[left][j], table[right - (1 << j) + 1][j]);
    }

    // query in O(log(n)) if its could't apply to Sparse Table directly
    T query_log(int l, int r){
        int len = r - l + 1;
        T ans;
        for(int i = 0; l <= r; i++){
            if (len & (1 << i)){
                ans = merge(ans, table[i][1]);
                l+= (1 << i);
            }
        }
    }

};

int main(void) {
    int n;
    cin >> n;
    vector<int> arr(n);
    for (auto& element : arr) cin >> element;

    SparseTable minSt(arr, [](const SNode& a, const SNode& b) -> SNode {
        return a.val < b.val ? a : b;
    });

```

```

    SparseTable maxSt(arr, [](const SNode& a, const SNode& b) -> SNode {
        return a.val > b.val ? a : b;
    });
}

```

5.2.7 Sqrt Decomp

Impl

```

#define OO 1e18

const int sz = 2e5 + 5;
const int sq = 500;

int arr[sz] = {};
int ans[sq] = {};

void pre() {
    fill(ans, ans + sq, OO);
    for(int i = 0; i < sz; i++) {
        ans[i / sq] = min(ans[i / sq], arr[i]);
    }
}

// can be O(1) with inversable operations, else it's sqrt
void update(int k, int x) {
    arr[k] = x;
    int start = k / sq;
    ans[start] = *min_element(arr + (start * sq), arr + (start + 1) * sq);
}

int query(int l, int r){
    int ret = OO;
    while(l <= r) {
        if(l % sq == 0 && l + sq <= r) {
            ret = min(ret, ans[l / sq]);
            l += sq;
        }else {
            ret = min(ret, arr[l]);
            l++;
        }
    }
}

```

```
    return ret;
}
```

5.3 Ordered Set

Ordered Set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;

template<typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

template <class T>
using ordered_multiset = tree<T, null_type, CUSTUM_COMPARE, rb_tree_tag,
    tree_order_statistics_node_update>;

void erase_set(ordered_set &os, int v) {
    // Number of elements less than v
    int rank = os.order_of_key(v);

    auto it = os.find_by_order(rank);
    os.erase(it);
}

// Returns iterator to 0-th
// largest element in the set
cout << *S.find_by_order(0) << " ";

// Returns iterator to 2-nd
// largest element in the set
cout << *S.find_by_order(2);
```

5.4 Custom Compare Functions

Custom Compare functions

```
template<class T>
```

```
struct custom_compare {
    bool operator()(const T& a, const T& b) const {
        if (a == b) return true; // Keep duplicates
        return a > b;
    }
};

//REF: GFG
class CustomComparator {
public:
    CustomComparator(int baseValue) : baseValue_(baseValue) {}

    bool operator()(int a, int b) const {
        // Custom comparison logic involving state
        return (a % baseValue_) < (b % baseValue_);
    }

private:
    int baseValue_;
};

// OR through capture by reference (capture clauses)
auto compare = [&](char a, char b) { return localStructure[a] >
    localStructure[b]; };
```

6 Counting Principles

6.1 nCr

$$C(n, k) = \frac{n!}{(n-k)!k!} = \frac{n * (n-1) * (n-2) * \dots * (n-k+1)}{k!}$$

6.1.1 Fast nCr

$$C(n, k) = \frac{n * (n-1) * (n-2) * \dots * (n-k+1)}{1 * 2 * 3 * \dots * k} = \prod_{i=0}^{k-1} \frac{n-i}{i+1} = \prod_{i=0}^{k-1} (n-i)(i+1)^{-1}$$

Fast nCr

```
int nCr(const int& n, const int& r) {
    double res = 1;
    for (int i = 1; i <= r; ++i)
        res = res * (n - r + i) / i;
    return (int)(res + 0.01);
}
```

6.1.2 Method 1: Pascal's Triangle (Dynamic Programming) - $\mathcal{O}(n^2)$

nCk using dp

// REF: USACO guide

```
/** @return nCk mod p using dynamic programming */
int binomial(int n, int k, int p) {
    // dp[i][j] stores iCj
    vector<vector<int>> dp(n + 1, vector<int>(k + 1, 0));

    // base cases described above
    for (int i = 0; i <= n; i++) {
        /*
         * i choose 0 is always 1 since there is exactly one way
         * to choose 0 elements from a set of i elements
         * (don't choose anything)
         */
        dp[i][0] = 1;
        /*
         * i choose i is always 1 since there is exactly one way
         * to choose i elements from a set of i elements
         * (choose every element in the set)
         */
        if (i <= k) { dp[i][i] = 1; }
    }

    for (int i = 0; i <= n; i++) {
        for (int j = 1; j <= min(i, k); j++) {
            if (i != j) { // skips over the base cases
                // uses the recurrence relation above
                dp[i][j] = (dp[i - 1][j - 1] + dp[i - 1][j]) % p;
            }
        }
    }
}
```

```
}
}

return dp[n][k]; // returns nCk modulo p
}
```

6.1.3 Method 2: Factorial Definition (Modular Inverses) - $\mathcal{O}(n + \log MOD)$

nCk using Modular Inverses

// REF: USACO guide

```
const int MAXN = 1e6;

long long fac[MAXN + 1];
long long inv[MAXN + 1];

/** @return  $x^n$  modulo m in  $\mathcal{O}(\log p)$  time. */
long long exp(long long x, long long n, long long m) {
    x %= m; // note: m * m must be less than  $2^{63}$  to avoid ll overflow
    long long res = 1;
    while (n > 0) {
        if (n % 2 == 1) { res = res * x % m; }
        x = x * x % m;
        n /= 2;
    }
    return res;
}

/** Precomputes  $n!$  from 0 to MAXN. */
void factorial(long long p) {
    fac[0] = 1;
    for (int i = 1; i <= MAXN; i++) { fac[i] = fac[i - 1] * i % p; }
}

/**
 * Precomputes all modular inverse factorials
 * from 0 to MAXN in  $\mathcal{O}(n + \log p)$  time
 */
void inverses(long long p) {
    inv[MAXN] = exp(fac[MAXN], p - 2, p);
}
```

```
for (int i = MAXN; i >= 1; i--) { inv[i - 1] = inv[i] * i % p; }

/** @return nCr mod p */
long long choose(long long n, long long r, long long p) {
    return fac[n] * inv[r] % p * inv[n - r] % p;
}

int main() {
    factorial();
    inverses();
    int n;
    cin >> n;
    for (int i = 0; i < n; i++) {
        int a, b;
        cin >> a >> b;
        cout << choose(a, b) << '\n';
    }
}
```

7 Graph Theory

7.1 Shortest Path algorithms

7.1.1 Dijkstra Algorithm

Dijkstra Implementation

```
#define INF (1e18) // for int defined as ll

int n, m;
vector<vector<pair<int, int>>> adj;
vector<int> cost;
vector<int> parent;

void dijkstra(int startNode = 1) {
    priority_queue<pair<ll, int>, vector<pair<ll, int>>, greater<>> pq;

    cost[startNode] = 0;
    pq.emplace(0, startNode);
```

```
while (!pq.empty()) {
    int u = pq.top().second;
    ll d = pq.top().first;
    pq.pop();

    if (d > cost[u]) continue;

    for (auto &p: adj[u]) {
        int v = p.first;
        int w = p.second;
        if (cost[v] > cost[u] + w) {
            cost[v] = cost[u] + w;
            parent[v] = u;
            pq.emplace(cost[v], v);
        }
    }
}

void run_test_case(int testNum) {
    cin >> n >> m;

    adj.assign(n + 1, {});
    cost.assign(n + 1, INF);
    parent.assign(n + 1, -1);

    while (m--) {
        // Read Edges
    }

    dijkstra();

    if (cost[n] == INF) {
        cout << -1 << endl; // not connected {Depends on you use case}
        return;
    }

    stack<int> ans;
    for (int v = n; v != -1; v = parent[v]) ans.push(v);

    while (!ans.empty()) { // printing the path
        cout << ans.top() << ' ';
        ans.pop();
    }
```

```
    cout << e1;
}
```

7.1.2 Floyd Warshal Algorithm

FloydWarshal Implementation

```
int main() {
    int n, m; cin >> n >> m;
    vector <vector <int>> adj(n + 1, vector <int> (n + 1, 2e9));
    for (int i = 0; i < n; i++) adj[i][i] = 0;

    while(m--) {
        int u, v, w;
        cin >> u >> v >> w;
        adj[u][v] = min(adj[u][v], w);
        adj[v][u] = min(adj[v][u], w);
    }

    for (int mid = 1; mid <= n; mid++) {
        for (int start = 1; start <= n; start++) {
            for (int end = 1; end <= n; end++) {
                adj[start][end] = min(adj[start][end], adj[start][mid] +
                    adj[mid][end]);
            }
        }
    }

    return 0;
}
```

7.1.3 Bellman Ford Algorithm

BellmanFord Implementation

```
vector <vector <pair<int, int>>> &adj

vector <long long> BellmanFord(int src) {
    int n = (int)adj.size();
    vector <long long> dist(n, 2e18);
```

```
    dist[src] = 0;
    for (int it = 0; it < n-1; it++) {
        bool in = false;
        for (int i = 0; i < n; i++) { // iterate on the edges
            for (auto &[j, w] : adj[i]) {
                if (dist[j] > dist[i] + w) {
                    in = true;
                    dist[j] = dist[i] + w;
                }
            }
        }
        if (!in) return dist;
    }

    for (int i = 0; i < n; i++) {
        for (auto &[j, w] : adj[i]) {
            if (dist[j] > dist[i] + w) { //negative cycle
                return vector <long long> (n, -1); // or any flag
            }
        }
    }

    return dist;
}
```

7.2 Cycle Detection

7.2.1 DFS Implementation

DFS Implementation

```
// return true with number of nodes in the cycle, either odd cycle or even
bool cycle_detection(unordered_map<int, vector<int>> &graph, int source,
    int par, unordered_map<int, bool> vis, int c){
    if(vis[source]) return true;

    vis[source] = true;

    for(int v: graph[source]){
        if(v != par){
            c++;
```



```
        if(dfs(graph,v, source, vis, c)) return true;
    }
}
return false;
}
```

7.2.2 Another way for undirected graphs

Another way for undirected graphs

```
// this is true only for undirected graphs
bool dfs1(int cur, int par) {
    bool ret = false;
    vis[cur] = true;
    for (auto &i : adj[cur]) {
        if (!vis[i]) ret|=dfs1(i, cur);
        else if (par != i) ret = true;
    }
    return ret;
}
```

7.2.3 General Way

General Way

```
// general algorithm
vector<bool> cyc;
bool dfs(int cur, int par) {
    bool ret = false;
    vis[cur] = cyc[cur] = true;
    for (auto &i : adj[cur]) {
        if (par == i) continue;
        if (!vis[i]) ret|=dfs(i, cur);
        else if (cyc[i]) ret = true;
    }
    cyc[cur] = false;
    return ret;
}
```

7.2.4 DSU Implementation

DSU Implementation

```
#include <iostream>
#include <vector>

class UnionFind {
public:
    UnionFind(int n) {
        parent.resize(n);
        rank.resize(n, 0);
        for (int i = 0; i < n; ++i) {
            parent[i] = i;
        }

        int find(int u) {
            if (parent[u] != u) {
                parent[u] = find(parent[u]);
            }
            return parent[u];
        }

        void unionSets(int u, int v) {
            int rootU = find(u);
            int rootV = find(v);

            if (rootU != rootV) {
                if (rank[rootU] > rank[rootV]) {
                    parent[rootV] = rootU;
                } else if (rank[rootU] < rank[rootV]) {
                    parent[rootU] = rootV;
                } else {
                    parent[rootV] = rootU;
                    ++rank[rootU];
                }
            }
        }

private:
        std::vector<int> parent;
        std::vector<int> rank;
    };
};
```

```
bool detectCycle(const std::vector<std::pair<int, int>>& edges, int n) {
    UnionFind uf(n);

    for (const auto& edge : edges) {
        int u = edge.first;
        int v = edge.second;

        if (uf.find(u) == uf.find(v)) {
            return true;
        }
        uf.unionSets(u, v);
    }

    return false;
}

int main() {
    std::vector<std::pair<int, int>> edges = { {0, 1}, {1, 2}, {2, 3}, {3,
        0} };
    int n = 4; // Number of vertices

    if (detectCycle(edges, n)) {
        std::cout << "Cycle detected" << std::endl;
    } else {
        std::cout << "No cycle detected" << std::endl;
    }

    return 0;
}
```

7.3 Algorithms

7.3.1 Heavy Light Decomposition

Basic HLD Impl

```
struct Node {
    int val;
};

const Node nullNode = {0};
```

```
const int N = 2e5 + 5, S = 1 << 19;
int n, q;
int val[N];
int sz[N], par[N], dep[N], id[N], top[N];
vector<int> adj[N];

Node st[S];

Node merge(const Node& a, const Node& b) {
    return {a.val + b.val};
}

void update(int idx, Node val) {
    st[idx += n] = val;
    for (idx /= 2; idx; idx /= 2) st[idx] = merge(st[idx * 2], st[idx * 2
        + 1]);
}

Node query(int lo, int hi) {
    Node ra = nullNode, rb = nullNode;

    for (lo += n, hi += n + 1; lo < hi; lo /= 2, hi /= 2) {
        if (lo & 1) ra = merge(ra, st[lo++]);
        if (hi & 1) rb = merge(st[--hi], rb);
    }

    return merge(ra, rb);
}

int dfs_size(const int& node, const int& parent) {
    sz[node] = 1;
    par[node] = parent;
    for (const int& ch : adj[node]) {
        if (ch == parent) continue;
        dep[ch] = dep[node] + 1;
        par[ch] = node;
        sz[node] += dfs_size(ch, node);
    }
    return sz[node];
}

int curId = 0;

void dfs_hld(const int& cur, const int& parent, const int& curTop) {
```

```

id[cur] = curId++;
top[cur] = curTop;
update(id[cur], {val[cur]});
int heavyChild = -1, heavyMax = -1;
for (const int& ch : adj[cur]) {
    if (ch == parent) continue;
    if (sz[ch] > heavyMax) {
        heavyMax = sz[ch];
        heavyChild = ch;
    }
}

if (heavyChild == -1) return;
dfs_hld(heavyChild, cur, curTop);
for (int ch : adj[cur]) {
    if (ch == parent || ch == heavyChild) continue;

    dfs_hld(ch, cur, ch);
}
}

Node path(int u, int v) {
    Node ans = nullNode;

    while (top[u] != top[v]) {
        if (dep[top[u]] < dep[top[v]]) swap(u, v);
        ans = merge(ans, query(id[top[u]], id[u]));
        u = par[top[u]];
    }

    if (dep[u] > dep[v]) swap(u, v);
    ans = merge(ans, query(id[u], id[v]));
    return ans;
}

void init() {
    for (int i = 0; i < S; i++) st[i] = nullNode;
    dfs_size(1, 1);
    dfs_hld(1, 1, 1);
}

int main() {
    cin >> n >> q;
    for (int i = 1; i <= n; i++) cin >> val[i];

```

```

int a, b;
for (int i = 2; i <= n; i++) {
    cin >> a >> b;
    adj[a].pb(b);
    adj[b].pb(a);
}

init(); // <----- DON'T FORGET TO CALL THIS FUNCTION

int type;
while (q--) {
    cin >> type;
    if (type == 1) {
        cin >> a >> b;
        val[a] = b;
        update(id[a], {val[a]});
    }
    else {
        cin >> a;
        cout << path(1, a).val << el;
    }
}
}

```

7.3.2 Heavy Light Decomposition with lazy SegTree

Basic HLD Impl

```

struct Node {
    int val;
};

const Node nullNode = {0};

const int N = 2e5 + 5, S = 1 << 19;
int n, q;
int val[N];
int sz[N], par[N], dep[N], id[N], top[N];
vector<int> adj[N];

Node st[S];
int lazy[S];

```

```
Node merge(const Node& a, const Node& b) {
    return {a.val + b.val};
}

void push(int idx, int l, int r) {
    if (lazy[idx] != 0) {
        st[idx].val += lazy[idx] * (r - l + 1);
        if (l != r) {
            lazy[idx * 2] += lazy[idx];
            lazy[idx * 2 + 1] += lazy[idx];
        }
        lazy[idx] = 0;
    }
}

void update_range(int lo, int hi, int l, int r, int idx, int value) {
    push(idx, l, r);
    if (lo > r || hi < l) return;
    if (lo <= l && r <= hi) {
        lazy[idx] += value;
        push(idx, l, r);
        return;
    }
    int mid = (l + r) / 2;
    update_range(lo, hi, l, mid, idx * 2, value);
    update_range(lo, hi, mid + 1, r, idx * 2 + 1, value);
    st[idx] = merge(st[idx * 2], st[idx * 2 + 1]);
}

void update(int idx, Node val) {
    update_range(idx, idx, 0, n - 1, 1, val.val);
}

void update_range(int lo, int hi, int value) {
    update_range(lo, hi, 0, n - 1, 1, value);
}

Node query(int lo, int hi, int l, int r, int idx) {
    push(idx, l, r);
    if (lo > r || hi < l) return nullNode;
    if (lo <= l && r <= hi) return st[idx];
    int mid = (l + r) / 2;
```

```
    return merge(query(lo, hi, l, mid, idx * 2), query(lo, hi, mid + 1, r,
        idx * 2 + 1));
}

Node query(int lo, int hi) {
    return query(lo, hi, 0, n - 1, 1);
}

int dfs_size(const int& node, const int& parent) {
    sz[node] = 1;
    par[node] = parent;
    for (const int& ch : adj[node]) {
        if (ch == parent) continue;
        dep[ch] = dep[node] + 1;
        par[ch] = node;
        sz[node] += dfs_size(ch, node);
    }
    return sz[node];
}

int curId = 0;

void dfs_hld(const int& cur, const int& parent, const int& curTop) {
    id[cur] = curId++;
    top[cur] = curTop;
    update(id[cur], {val[cur]});
    int heavyChild = -1, heavyMax = -1;
    for (const int& ch : adj[cur]) {
        if (ch == parent) continue;
        if (sz[ch] > heavyMax) {
            heavyMax = sz[ch];
            heavyChild = ch;
        }
    }

    if (heavyChild == -1) return;
    dfs_hld(heavyChild, cur, curTop);
    for (int ch : adj[cur]) {
        if (ch == parent || ch == heavyChild) continue;

        dfs_hld(ch, cur, ch);
    }
}
```

```
int get(int u) {
    return query(id[u], id[u]).val;
}

void path(int u, int v, int val) {
    // Node ans = nullNode;

    while (top[u] != top[v]) {
        if (dep[top[u]] < dep[top[v]]) swap(u, v);
        // ans = merge(ans, query(id[top[u]], id[u]));
        update_range(id[top[u]], id[u], val);
        u = par[top[u]];
    }

    if (dep[u] > dep[v]) swap(u, v);
    // ans = merge(ans, query(id[u], id[v]));
    update_range(id[u], id[v], val);
    // return ans;
}

void init() {
    for (int i = 0; i < S; i++) st[i] = nullNode;
    memset(lazy, 0, sizeof(lazy));
    dfs_size(1, 1);
    dfs_hld(1, 1, 1);
}

int main(void) {
    cin >> n >> q;
    for (int i = 1; i <= n; i++) val[i] = 0;
    int a, b;
    for (int i = 2; i <= n; i++) {
        cin >> a >> b;
        adj[a].push_back(b);
        adj[b].push_back(a);
    }

    init(); // <----- DON'T FORGET TO CALL THIS FUNCTION
    int v;
    while (q--) {
        cin >> a >> b >> v;
        path(a, b, v);
    }
}
```

```
for (int i = 1; i <= n; i++) {
    cout << get(i) << " ";
}
cout << el;
}
```

7.3.3 LCA functions using Binary Lifting

LCA functions using Binary Lifting

```
const int N = 2e5 + 15, M = 23;
int ancestors[N][M], depth[N], parent[N], val[N];
vector<vector<int>> adj;
//int tin[N], tout[N], timer;

void dfs_LCA(const int &node, const int &par) {
    // tin[node] = timer++;
    parent[node] = par;
    ancestors[node][0] = par;
    depth[node] = depth[par] + 1;

    for (int i = 1; i < M; i++) {
        int p = ancestors[node][i - 1];
        ancestors[node][i] = ancestors[p][i - 1];
    }

    for (const int &v: adj[node]) {
        if (v == par) continue;
        dfs_LCA(v, node);
    }
    // tout[node] = timer++;
}

//bool is_ancestor(int u, int v) {
//    return tin[u] <= tin[v] && tout[u] >= tout[v];
//}

int findKth(int u, int k) {
    if (depth[u] <= k) return -1;
    for (int i = M - 1; i >= 0; i--) {
        if (k & (1 << i)) {
            u = ancestors[u][i];
        }
    }
}
```

```

    }
    return u;
}

int getLCA(int u, int v) {
    if (depth[u] < depth[v])
        swap(u, v);

    u = findKth(u, depth[u] - depth[v]);
    if (u == v) return u;

    for (int i = M - 1; i >= 0; i--) {
        if (ancestors[u][i] == ancestors[v][i]) continue;
        u = ancestors[u][i];
        v = ancestors[v][i];
    }
    return ancestors[u][0];
}

int getDistance(int u, int v) {
    int lca = getLCA(u, v);
    return (depth[u] + depth[v]) - (2 * depth[lca]);
}

int dfs_accumulate(const int &node, const int &par) {

    for (const int& ch: adj[node]) {
        if (ch == par) continue;
        val[node] += dfs_accumulate(ch, node);
    }
    return val[node];
}

void applyOpOnPath(const int a, const int b, const int w) {
    // adding w to each node on the path a to b
    val[a] += w;
    val[b] += w;
    int lca = getLCA(a, b);
    val[lca] -= w;
    val[parent[lca]] -= w;
}

int main(void) {
    int n, q;

```

```

    cin >> n >> q;
    adj.resize(n + 1);

    int u, v;
    for (int i = 2; i <= n; ++i) {
        cin >> u >> v;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }

    dfs_LCA(1, 1);
    parent[1] = -1;

    int w;
    for (int i = 0; i < q; ++i) {
        cin >> u >> v;
        cout << getDistance(u, v) << el;
    }

    // dfs_accumulate(1, 0);
    //
    // for (int i = 1; i <= n; i++) {
    //     cout << val[i] << " ";
    // }
    // cout << el;
}

```

7.3.4 Topological Sort

Topological Sort Using DFS

```

//REF: USACO Guide
vector<int> top_sort;
vector<vector<int>> graph;
vector<bool> visited;

void dfs(int node) {
    for (int next : graph[node]) {
        if (!visited[next]) {
            visited[next] = true;
            dfs(next);
        }
    }
}

```

```
top_sort.push_back(node);
}

int main() {
    int n, m; // The number of nodes and edges respectively
    std::cin >> n >> m;

    graph = vector<vector<int>>>(n);
    for (int i = 0; i < m; i++) {
        int a, b;
        std::cin >> a >> b;
        graph[a - 1].push_back(b - 1);
    }

    visited = vector<bool>(n);
    for (int i = 0; i < n; i++) {
        if (!visited[i]) {
            visited[i] = true;
            dfs(i);
        }
    }
    std::reverse(top_sort.begin(), top_sort.end());

    vector<int> ind(n);
    for (int i = 0; i < n; i++) { ind[top_sort[i]] = i; }

    // Check if the topological sort is valid
    bool valid = true;
    for (int i = 0; i < n; i++) {
        for (int j : graph[i]) {
            if (ind[j] <= ind[i]) {
                valid = false;
                goto answer;
            }
        }
    }
    answer::;

    if (valid) {
        for (int i = 0; i < n - 1; i++) { cout << top_sort[i] + 1 << ' '; }
        cout << top_sort.back() + 1 << endl;
    } else {
        cout << "IMPOSSIBLE" << endl;
    }
}
```

```
}
```

8 Techniques

8.1 Coordinate Compression

```
void coordinate_compress(vector<int> &x, int start=0, int
    step=1) {
    set unique(x.begin(), x.end());
    map<int, int> valPos;

    int idx=0;
    for (auto i: unique) {
        valPos[i] = start + idx * step;
        ++idx;
    }
    for(auto &i: x) i = valPos[i];
}
```

Coordinate Compression

8.2 Binary to decimal

Binary to decimal

```
// Function to convert binary to decimal
// 0(32)
int binaryToDecimal(string str)
{
    int dec_num = 0;
    int power = 0 ;
    int n = str.length() ;

    for(int i = n-1 ; i>=0 ; i--){
        if(str[i] == '1'){
            dec_num += (1<<power) ;
        }
        power++ ;
    }
}
```

```

    }

    return dec_num;
}

```

8.3 Decimal to binary

Decimal to binary

```

// Function that convert Decimal to binary
// O(32)
void decToBinary(int n)
{
    // Size of an integer is assumed to be 32 bits
    for (int i = 31; i >= 0; i--) {
        int k = n >> i;
        if (k & 1)
            cout << "1";
        else
            cout << "0";
    }
}

// O(logn)
string DecimalToBinary(int num)
{
    string str;
    while(num){
        if(num & 1) // 1
            str+='1';
        else // 0
            str+='0';
        num>>=1; // Right Shift by 1
    }
    return str;
}

```

9 Number Theory

9.1 Divisors

9.1.1 formulas

number of divisors

```

int d(int n){
    unordered_map<int, int> factors = pf(n);
    int c = 1;
    for(const auto& factor: factors){
        c *= (factor.second+1);
    }
    return c;
}

// range Count Divisors backward thinking MAXN = 2e6
for(int i=1; i <= n; ++i) {
    for(int j = i; j <= n; j += i) {
        numFactors[j]++;
    }
}

int countDivisors(int n) {
    int count = 0;
    for (int i = 1; i * i <= n; ++i) {
        if (n % i == 0) {
            if (i == n / i) {
                count++; // Perfect square
            } else {
                count += 2; // Pair of divisors
            }
        }
    }
    return count;
}

```

sum of divisors

```

int s(int n){

```



```
unordered_map<int,int> factors = pf(n);
int sum = 1;
for(const auto& factor: factors){
    int p = factor.first;
    int exp = factor.second;
    sum *= (pow(p,exp+1)-1)/p-1;
}
return sum;
}
```

9.2 Primes

prime factorization

```
void factorize(int x, unordered_map<int, int>& factors) {
    while (x % 2 == 0) {
        factors[2]++;
        x /= 2;
    }
    for (int i = 3; i * i <= x; i += 2) {
        while (x % i == 0) {
            factors[i]++;
            x /= i;
        }
    }
    if (x > 2) factors[x]++;
}
```

number of co-primes with n

```
int eulerTotient(int n){
    int result = n;

    for(int i = 2; SQ(i) <= n; i++){
        if(n%i == 0){
            while(n%i == 0){
                n/=i;
            }
            result -= result/i;
        }
    }
}
```

```
}

if(n > 1) result -= result/n;
return result;
}

//Phi(n) = n * (1 - 1/P1) * (1 - 1/P2) * ...

//NOTE: summation of Euler function over divisors of n is equal to n

// using seive
void phi_generator() {
    const int MAX = 1000000;
    char primes[MAX];
    int phi[MAX];

    memset(primes, 1, sizeof(primes));

    for (int k = 0; k < MAX; ++k)
        phi[k] = 1;

    for (int i = 2; i <= MAX; ++i) {
        if (primes[i]) {
            phi[i] = i - 1; // phi(prime) = p-1

            for (int j = i * 2; j <= MAX; j += i) {
                primes[j] = 0;
                int n = j, pow = 1;
                while (n % i == 0) {
                    pow *= i;
                    n /= i;
                }
                phi[j] *= (pow / i) * (i - 1);
            }
        }
    }
}

// phi(N!) = (N is prime ? N-1 : N) * phi((N-1)!)
ll phi_factn(int n) {
    ll ret = 1;
    for (int i = 2; i <= n; ++i)
        ret = ret * (isprime(i) ? i - 1 : i);
}
```

```
    return ret;
}
```

Prime Check

```
vector<bool> isPrime(MAXN, true);

void sieve() {
    isPrime[0] = isPrime[1] = false;

    for (int i=2; i * i <= isPrime.size(); ++i) {
        if(isPrime[i]) {
            for (int j = 2 * i; i <= isPrime.size(); j += i)
                prime[j] = false;
        }
    }
}

bool Prime(int n) {
    if(n == 2) return true;
    if(n < 2 || n % 2 == 0) return false;

    for(int i=3; i * i <= n; i += 2) {
        if(n % i == 0) return false;
    }
    return true;
}

// Generate Primes
const int sz = sqrt(MAXN);
vector<int> prime;
vector<bool> vis(sz);

void pre() {
    prime.push_back(2);
    for (int j = 4; j < sz; j += 2) vis[j] = true;
    for (int i = 3; i < sz; i += 2) {
        if (vis[i]) continue;
        prime.push_back(i);
        for (int j = i * i; j < sz; j += i) vis[j] = true;
    }
}
```

```
// Preprocessing Prime Factorization of range numbers
constexpr int N = 5e6+1;
int a[N];

for(int i=2; i < N; ++i) {
    if(!a[i]) {
        for(int j=1; i*j < N; ++j) {
            for(int k=i*j; k%i==0; k/=i) a[i*j]++;
        }
        a[i] += a[i-1];
    }
}
```

9.3 Math

9.3.1 Vieta's Formula for a Polynomial of Degree n

Problem: Given a polynomial of degree n :

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

with roots r_1, r_2, \dots, r_n , express the sums and products of its roots using Vieta's formulas.

Solution: Using Vieta's formulas, we can relate the coefficients of the polynomial to sums and products of its roots:

- Sum of the roots taken one at a time:

$$r_1 + r_2 + \cdots + r_n = -\frac{a_{n-1}}{a_n}$$

- Sum of the products of the roots taken two at a time:

$$r_1 r_2 + r_1 r_3 + \cdots + r_{n-1} r_n = \frac{a_{n-2}}{a_n}$$

- Sum of the products of the roots taken three at a time:

$$r_1 r_2 r_3 + r_1 r_2 r_4 + \cdots + r_{n-2} r_{n-1} r_n = -\frac{a_{n-3}}{a_n}$$

- Continue this pattern until:
- Product of the roots (for even n):

$$r_1 r_2 \cdots r_n = (-1)^n \frac{a_0}{a_n}$$

Example Problem Using Vieta's Formula

Problem: Given a quadratic equation $x^2 + bx + c = 0$ with roots r_1 and r_2 , find $r_1 + r_2$ and $r_1 r_2$.

Solution: Using Vieta's formulas for a quadratic equation $ax^2 + bx + c = 0$:

- Sum of the roots:

$$r_1 + r_2 = -\frac{b}{a}$$

- Product of the roots:

$$r_1 r_2 = \frac{c}{a}$$

For the given quadratic $x^2 + bx + c = 0$ (where $a = 1$):

Also: To find the roots of the quadratic equation, we can use the discriminant formula, $D = b^2 - 4ac$. The roots will then be $x_1 = \frac{b - \sqrt{D}}{2}$ and $x_2 = \frac{b + \sqrt{D}}{2}$.

- $r_1 + r_2 = -b$
- $r_1 r_2 = c$

Example Vieta's Formula for Cubic Equation

When considering a cubic equation in the form of $f(x) = ax^3 + bx^2 + cx + d$, Vieta's formula states that if the equation $f(x) = 0$ has roots r_1, r_2 , and r_3 , then:

- $r_1 + r_2 + r_3 = -\frac{b}{a}$
- $r_1 r_2 + r_2 r_3 + r_3 r_1 = \frac{c}{a}$
- $r_1 r_2 r_3 = -\frac{d}{a}$

9.4 Phi Function

- Count integers $i < n$ such that $\gcd(i, n) = 1$
- $\gcd(a, b) = 1 \Rightarrow$ then coprimes: $\gcd(5, 7), \gcd(4, 9)$
- $\gcd(\text{prime}, i) = 1$ for $i < \text{prime}$
- $\varphi(10) = 4 \Rightarrow 1, 3, 7, 9$
- $\varphi(5) = 4 \Rightarrow 1, 2, 3, 4 \dots \varphi(\text{prime}) = \text{prime} - 1$
- If a, b, c are pairwise coprimes, then

$$\varphi(a \cdot b \cdot c) = \varphi(a) \cdot \varphi(b) \cdot \varphi(c)$$

- If $k \geq 1$

$$\varphi(p^k) = p^k - p^{k-1} = p^{k-1}(p - 1) = p^k \left(1 - \frac{1}{p}\right)$$

9.5 Euler's Totient Numbers

Online Sequence

$\varphi(n) = 1, 1, 2, 2, 4, 2, 6, 4, 6, 4, 10, 4, 12, 6, 8, 8, 16, 6, 18, 8, 12, 10, 22, 8, 20, 12, 18, 12, 24, 16, 28, 12, 30, 16, 32, 20, 36, 24, 40, 24, 42, 28, 48, 24, 50, 28, 54, 36, 60, 36, 64, 40, 72, 48, 80, 48, 84, 48, 90, 60, 96, 64, 100, 64, 108, 72, 112, 72, 120, 72, 128, 80, 144, 80, 150, 80, 160, 80, 180, 90, 192, 96, 200, 96, 210, 100, 224, 100, 240, 100, 252, 108, 270, 108, 280, 112, 300, 120, 320, 120, 336, 120, 360, 120, 384, 128, 400, 128, 420, 128, 440, 128, 450, 144, 480, 144, 504, 144, 540, 144, 560, 144, 600, 144, 640, 144, 672, 144, 700, 144, 720, 144, 756, 144, 768, 144, 800, 144, 840, 144, 864, 144, 900, 144, 960, 144, 1000, 160, 1056, 160, 1120, 160, 1152, 160, 1200, 160, 1260, 160, 1280, 160, 1344, 160, 1400, 160, 1440, 160, 1500, 160, 1560, 160, 1600, 160, 1680, 160, 1764, 168, 1800, 168, 1920, 168, 2000, 176, 2100, 180, 2160, 180, 2240, 180, 2310, 180, 2400, 180, 2520, 180, 2640, 180, 2700, 180, 2800, 180, 2880, 180, 3000, 180, 3120, 180, 3200, 180, 3360, 180, 3456, 180, 3600, 180, 3744, 180, 3840, 180, 4000, 180, 4200, 180, 4320, 180, 4400, 180, 4500, 180, 4608, 180, 4800, 180, 5040, 180, 5280, 180, 5400, 180, 5600, 180, 5760, 180, 6000, 180, 6240, 180, 6400, 180, 6720, 180, 7000, 180, 7200, 180, 7560, 180, 7680, 180, 8000, 180, 8400, 180, 8640, 180, 9000, 180, 9600, 180, 10000, 180, 10560, 180, 11200, 180, 11520, 180, 12000, 180, 12600, 180, 12800, 180, 13440, 180, 14000, 180, 14400, 180, 15000, 180, 15600, 180, 16000, 180, 16800, 180, 17640, 180, 18000, 180, 19200, 180, 20000, 180, 21000, 180, 22400, 180, 23100, 180, 24000, 180, 25200, 180, 26400, 180, 27000, 180, 28000, 180, 28800, 180, 30000, 180, 31200, 180, 32000, 180, 33600, 180, 34560, 180, 36000, 180, 37440, 180, 38400, 180, 40000, 180, 42000, 180, 43200, 180, 44000, 180, 45000, 180, 46080, 180, 48000, 180, 50400, 180, 52800, 180, 54000, 180, 56000, 180, 57600, 180, 60000, 180, 62400, 180, 64000, 180, 67200, 180, 70000, 180, 72000, 180, 75600, 180, 76800, 180, 80000, 180, 84000, 180, 86400, 180, 90000, 180, 96000, 180, 100000, 180, 105600, 180, 112000, 180, 115200, 180, 120000, 180, 126000, 180, 128000, 180, 134400, 180, 140000, 180, 144000, 180, 150000, 180, 156000, 180, 160000, 180, 168000, 180, 176400, 180, 180000, 180, 192000, 180, 200000, 180, 210000, 180, 224000, 180, 231000, 180, 240000, 180, 252000, 180, 264000, 180, 270000, 180, 280000, 180, 288000, 180, 300000, 180, 312000, 180, 320000, 180, 336000, 180, 345600, 180, 360000, 180, 374400, 180, 384000, 180, 400000, 180, 420000, 180, 432000, 180, 440000, 180, 450000, 180, 460800, 180, 480000, 180, 504000, 180, 528000, 180, 540000, 180, 560000, 180, 576000, 180, 600000, 180, 624000, 180, 640000, 180, 672000, 180, 700000, 180, 720000, 180, 756000, 180, 768000, 180, 800000, 180, 840000, 180, 864000, 180, 900000, 180, 960000, 180, 1000000, 180, 1056000, 180, 1120000, 180, 1152000, 180, 1200000, 180, 1260000, 180, 1280000, 180, 1344000, 180, 1400000, 180, 1440000, 180, 1500000, 180, 1560000, 180, 1600000, 180, 1680000, 180, 1764000, 180, 1800000, 180, 1920000, 180, 2000000, 180, 2100000, 180, 2240000, 180, 2310000, 180, 2400000, 180, 2520000, 180, 2640000, 180, 2700000, 180, 2800000, 180, 2880000, 180, 3000000, 180, 3120000, 180, 3200000, 180, 3360000, 180, 3456000, 180, 3600000, 180, 3744000, 180, 3840000, 180, 4000000, 180, 4200000, 180, 4320000, 180, 4400000, 180, 4500000, 180, 4608000, 180, 4800000, 180, 5040000, 180, 5280000, 180, 5400000, 180, 5600000, 180, 5760000, 180, 6000000, 180, 6240000, 180, 6400000, 180, 6720000, 180, 7000000, 180, 7200000, 180, 7560000, 180, 7680000, 180, 8000000, 180, 8400000, 180, 8640000, 180, 9000000, 180, 9600000, 180, 10000000, 180, 10560000, 180, 11200000, 180, 11520000, 180, 12000000, 180, 12600000, 180, 12800000, 180, 13440000, 180, 14000000, 180, 14400000, 180, 15000000, 180, 15600000, 180, 16000000, 180, 16800000, 180, 17640000, 180, 18000000, 180, 19200000, 180, 20000000, 180, 21000000, 180, 22400000, 180, 23100000, 180, 24000000, 180, 25200000, 180, 26400000, 180, 27000000, 180, 28000000, 180, 28800000, 180, 30000000, 180, 31200000, 180, 32000000, 180, 33600000, 180, 34560000, 180, 36000000, 180, 37440000, 180, 38400000, 180, 40000000, 180, 42000000, 180, 43200000, 180, 44000000, 180, 45000000, 180, 46080000, 180, 48000000, 180, 50400000, 180, 52800000, 180, 54000000, 180, 56000000, 180, 57600000, 180, 60000000, 180, 62400000, 180, 64000000, 180, 67200000, 180, 70000000, 180, 72000000, 180, 75600000, 180, 76800000, 180, 80000000, 180, 84000000, 180, 86400000, 180, 90000000, 180, 96000000, 180, 100000000, 180, 105600000, 180, 112000000, 180, 115200000, 180, 120000000, 180, 126000000, 180, 128000000, 180, 134400000, 180, 140000000, 180, 144000000, 180, 150000000, 180, 156000000, 180, 160000000, 180, 168000000, 180, 176400000, 180, 180000000, 180, 192000000, 180, 200000000, 180, 210000000, 180, 224000000, 180, 231000000, 180, 240000000, 180, 252000000, 180, 264000000, 180, 270000000, 180, 280000000, 180, 288000000, 180, 300000000, 180, 312000000, 180, 320000000, 180, 336000000, 180, 345600000, 180, 360000000, 180, 374400000, 180, 384000000, 180, 400000000, 180, 420000000, 180, 432000000, 180, 440000000, 180, 450000000, 180, 460800000, 180, 480000000, 180, 504000000, 180, 528000000, 180, 540000000, 180, 560000000, 180, 576000000, 180, 600000000, 180, 624000000, 180, 640000000, 180, 672000000, 180, 700000000, 180, 720000000, 180, 756000000, 180, 768000000, 180, 800000000, 180, 840000000, 180, 864000000, 180, 900000000, 180, 960000000, 180, 1000000000, 180, 1056000000, 180, 1120000000, 180, 1152000000, 180, 1200000000, 180, 1260000000, 180, 1280000000, 180, 1344000000, 180, 1400000000, 180, 1440000000, 180, 1500000000, 180, 1560000000, 180, 1600000000, 180, 1680000000, 180, 1764000000, 180, 1800000000, 180, 1920000000, 180, 2000000000, 180, 2100000000, 180, 2240000000, 180, 2310000000, 180, 2400000000, 180, 2520000000, 180, 2640000000, 180, 2700000000, 180, 2800000000, 180, 2880000000, 180, 3000000000, 180, 3120000000, 180, 3200000000, 180, 3360000000, 180, 3456000000, 180, 3600000000, 180, 3744000000, 180, 3840000000, 180, 4000000000, 180, 4200000000, 180, 4320000000, 180, 4400000000, 180, 4500000000, 180, 4608000000, 180, 4800000000, 180, 5040000000, 180, 5280000000, 180, 5400000000, 180, 5600000000, 180, 5760000000, 180, 6000000000, 180, 6240000000, 180, 6400000000, 180, 6720000000, 180, 7000000000, 180, 7200000000, 180, 7560000000, 180, 7680000000, 180, 8000000000, 180, 8400000000, 180, 8640000000, 180, 9000000000, 180, 9600000000, 180, 10000000000, 180, 10560000000, 180, 11200000000, 180, 11520000000, 180, 12000000000, 180, 12600000000, 180, 12800000000, 180, 13440000000, 180, 14000000000, 180, 14400000000, 180, 15000000000, 180, 15600000000, 180, 16000000000, 180, 16800000000, 180, 17640000000, 180, 18000000000, 180, 19200000000, 180, 20000000000, 180, 21000000000, 180, 22400000000, 180, 23100000000, 180, 24000000000, 180, 25200000000, 180, 26400000000, 180, 27000000000, 180, 28000000000, 180, 28800000000, 180, 30000000000, 180, 31200000000, 180, 32000000000, 180, 33600000000, 180, 34560000000, 180, 36000000000, 180, 37440000000, 180, 38400000000, 180, 40000000000, 180, 42000000000, 180, 43200000000, 180, 44000000000, 180, 45000000000, 180, 46080000000, 180, 48000000000, 180, 50400000000, 180, 52800000000, 180, 54000000000, 180, 56000000000, 180, 57600000000, 180, 60000000000, 180, 62400000000, 180, 64000000000, 180, 67200000000, 180, 70000000000, 180, 72000000000, 180, 75600000000, 180, 76800000000, 180, 80000000000, 180, 84000000000, 180, 86400000000, 180, 90000000000, 180, 96000000000, 180, 100000000000, 180, 105600000000, 180, 112000000000, 180, 115200000000, 180, 120000000000, 180, 126000000000, 180, 128000000000, 180, 134400000000, 180, 140000000000, 180, 144000000000, 180, 150000000000, 180, 156000000000, 180, 160000000000, 180, 168000000000, 180, 176400000000, 180, 180000000000, 180, 192000000000, 180, 200000000000, 180, 210000000000, 180, 224000000000, 180, 231000000000, 180, 240000000000, 180, 252000000000, 180, 264000000000, 180, 270000000000, 180, 280000000000, 180, 288000000000, 180, 300000000000, 180, 312000000000, 180, 320000000000, 180, 336000000000, 180, 345600000000, 180, 360000000000, 180, 374400000000, 180, 384000000000, 180, 400000000000, 180, 420000000000, 180, 432000000000, 180, 440000000000, 180, 450000000000, 180, 460800000000, 180, 480000000000, 180, 504000000000, 180, 528000000000, 180, 540000000000, 180, 560000000000, 180, 576000000000, 180, 600000000000, 180, 624000000000, 180, 640000000000, 180, 672000000000, 180, 700000000000, 180, 720000000000, 180, 756000000000, 180, 768000000000, 180, 800000000000, 180, 840000000000, 180, 864000000000, 180, 900000000000, 180, 960000000000, 180, 1000000000000, 180, 1056000000000, 180, 1120000000000, 180, 1152000000000, 180, 1200000000000, 180, 1260000000000, 180, 1280000000000, 180, 1344000000000, 180, 1400000000000, 180, 1440000000000, 180, 1500000000000, 180, 1560000000000, 180, 1600000000000, 180, 1680000000000, 180, 1764000000000, 180, 1800000000000, 180, 1920000000000, 180, 2000000000000, 180, 2100000000000, 180, 2240000000000, 180, 2310000000000, 180, 2400000000000, 180, 2520000000000, 180, 2640000000000, 180, 2700000000000, 180, 2800000000000, 180, 2880000000000, 180, 3000000000000, 180, 3120000000000, 180, 3200000000000, 180, 3360000000000, 180, 3456000000000, 180, 3600000000000, 180, 3744000000000, 180, 3840000000000, 180, 4000000000000, 180, 4200000000000, 180, 4320000000000, 180, 4400000000000, 180, 4500000000000, 180, 4608000000000, 180, 4800000000000, 180, 5040000000000, 180, 5280000000000, 180, 5400000000000, 180, 5600000000000, 180, 5760000000000, 180, 6000000000000, 180, 6240000000000, 180, 6400000000000, 180, 6720000000000, 180, 7000000000000, 180, 7200000000000, 180, 7560000000000, 180, 7680000000000, 180, 8000000000000, 180, 8400000000000, 180, 8640000000000, 180, 9000000000000, 180, 9600000000000, 180, 10000000000000, 180, 10560000000000, 180, 11200000000000, 180, 11520000000000, 180, 12000000000000, 180, 12600000000000, 180, 12800000000000, 180, 13440000000000, 180, 14000000000000, 180, 14400000000000, 180, 15000000000000, 180, 15600000000000, 180, 16000000000000, 180, 16800000000000, 180, 17640000000000, 180, 18000000000000, 180, 19200000000000, 180, 20000000000000, 180, 21000000000000, 180, 22400000000000, 180, 23100000000000, 180, 24000000000000, 180, 25200000000000, 180, 26400000000000, 180, 27000000000000, 180, 28000000000000, 180, 28800000000000, 180, 30000000000000, 180, 3120000$

9.6 Möbius Function

Möbius Function

```
int mobius(int n) {
    int p = 0;
    // Handling 2 separately
    if (n%2 == 0){
        n = n/2;
        p++;

        // If 2^2 also divides N
        if (n % 2 == 0)
            return 0;
    }

    // Check for all other prime factors
    for (int i = 3; i <= sqrt(n); i = i+2) {
        if (n%i == 0){
            n = n/i;
            p++;

            if (n % i == 0) return 0;
        }
    }

    return (p % 2 == 0)? -1 : 1;
}

void mobius_generator() const {
    const int MAX = 1000000;
    int moebius[MAX + 1];
    char prime[MAX + 1];

    for (ll i = 2; i <= MAX; i++)
        moebius[i] = -1, prime[i] = 1;

    for (ll i = 2; i <= MAX; ++i)
        if (prime[i]) {
            moebius[i] = 1;

            for (ll j = 2 * i; j <= MAX; j += i)
                prime[j] = 0, moebius[j] = j % (i * i) == 0 ? -moebius[j] : 1;
        }
}
```

```
}

// Mobius Inclusion Exclusion
// Count triples gcd(a, b, c) = 1
int n = 4;
ll sum = n * n * n;
for (ll i = 2; i <= n; ++i)
    sum -= moebius[i] * (n / i) * (n / i) * (n / i);
```

9.7 Möbius and Inclusion Exclusion

Count the triples (a, b, c) such that $a, b, c \leq n$, and $\gcd(a, b, c) = 1$

- Reverse thinking, total - (# triples $\gcd > 1$)
- How many triples with \gcd multiple of 2: $(n/2)^3$
- How many triples with \gcd multiple of 3: $(n/3)^3$
- and 4? Ignore any numbers of internal duplicate primes
- and 6? already computed in 2, 3. Remove it: $-(n/6)^3$

9.8 Totient and Möbius Connection

Sum over divisors d of n

$$\sum_d d\mu\left(\frac{n}{d}\right) = \varphi(n)$$

9.9 Lagrange's four-square theorem

Lagrange's four-square theorem states that, every positive integer can be expressed as the sum of the squares of four integers.

Lagrange's four-square theorem

```
#include <cmath>

bool isPerfectSquare(int n) {
    int rt = sqrt(n);
```

```
    return rt * rt == n;
}

// Function to compute the minimum number of perfect squares
int numSquares(int n) {
    // Case 1:
    if (isPerfectSquare(n)) return 1;

    // Case 2: Check if n can be expressed as the sum of two perfect
    squares
    for (int i = 1; i * i <= n; i++) {
        int d = n - i * i;
        if (isPerfectSquare(d)) return 2;
    }

    // Case 3:
    // If n can be reduced to the form  $4^a * (8b + 7)$ , then it requires 4
    squares
    int m = n;
    while (m % 4 == 0) m /= 4;
    if (m % 8 == 7) return 4;

    // Case 4:
    return 3;
}
```

10 Geometry

10.1 Linearity

10.1.1 co-linear points

check if two points are co-linear

```
bool co_linear(int x1, int y1, int x2, int y2, int x3, int y3){
    int area = x1*(y2-y3) + x2*(y3-y1) + x3*(y1-y2);
    return area == 0;
}
```

10.2 Polygons

10.2.1 Polygon formation

check if can form polygon with given angle

```
bool possible(double angle){
    if(angle <= 0 || angle >= 180) return false;

    double sides = 360.0/(180.0-angle);

    return (sides == static_cast<int>(sides) && sides >= 3);
}
```

10.2.2 Polygon Area

area of any polygon with x vertices

```
double shoelace(vector<pair<double, double>> points) {
    double leftSum = 0.0;
    double rightSum = 0.0;

    for (int i = 0; i < points.size(); ++i) {
        int j = (i + 1) % points.size();
        leftSum += points[i].first * points[j].second;
        rightSum += points[j].first * points[i].second;
    }

    return 0.5 * abs(leftSum - rightSum);
}
```

10.3 Intersections

10.3.1 Rectangle

intersection area between 2 rectangles

```
struct Rectangle {
    int x1, y1; // Bottom-left corner
    int x2, y2; // Top-right corner
};
```

```
int intersectionArea(const Rectangle& rect1, const Rectangle& rect2){

    int x_left = max(rect1.x1, rect2.x1);
    int y_bottom = max(rect1.y1, rect2.y1);
    int x_right = min(rect1.x2, rect2.x2);
    int y_top = min(rect1.y2, rect2.y2);

    int intersection_width = x_right - x_left;
    int intersection_height = y_top - y_bottom;

    if (intersection_width > 0 && intersection_height > 0) {
        return intersection_width * intersection_height;
    }

    return 0;
}
```

10.3.2 Circle

intersection area between 2 circles

```
double area(int x0, int y0, int r0, int x1, int y1, int r1){
    const double PI = 3.14159265358979323846;
    double rr0 = r0 * r0;
    double rr1 = r1 * r1;
    double d = sqrt((x1 - x0) * (x1 - x0) + (y1 - y0) * (y1 - y0));
    if(d >= r0 + r1){
        return 0;
    }
    double phiAngle = (rr0 + (d * d) - rr1) / (2 * r0 * d);
    double phi = acos(phiAngle) * 2;
    double thetaAngle = (rr1 + (d * d) - rr0) / (2 * r1 * d);
    double theta = acos(thetaAngle) * 2;
    double area1 = 0.5 * theta * rr1 - 0.5 * rr1 * sin(theta);
    double area2 = 0.5 * phi * rr0 - 0.5 * rr0 * sin(phi);
    return area1+area2;
}
```

10.3.3 Triangle

intersection area between 2 triangles

```
struct Point {
    double x, y;
};

typedef vector<Point> Polygon;

bool inside(const Point &p, const pair<Point, Point> &edge) {
    Point a = edge.first, b = edge.second;
    return (b.x - a.x) * (p.y - a.y) - (b.y - a.y) * (p.x - a.x) >= 0;
}

Point compute_intersection(const Point &p1, const Point &p2, const pair<
    Point, Point> &edge) {
    Point a = edge.first, b = edge.second;
    double A1 = b.y - a.y, B1 = a.x - b.x, C1 = b.x * a.y - a.x * b.y;
    double A2 = p2.y - p1.y, B2 = p1.x - p2.x, C2 = p2.x * p1.y - p1.x *
        p2.y;
    double det = A1 * B2 - A2 * B1;
    if (det == 0) return {numeric_limits<double>::quiet_NaN(),
        numeric_limits<double>::quiet_NaN()}; // parallel
    double x = (B1 * C2 - B2 * C1) / det;
    double y = (A2 * C1 - A1 * C2) / det;
    return {x, y};
}

Polygon HodgmanClip(const Polygon &subjectPolygon, const Polygon &
    clipPolygon) {
    Polygon outputList = subjectPolygon;
    for (size_t i = 0; i < clipPolygon.size(); ++i) {
        Point a = clipPolygon[i];
        Point b = clipPolygon[(i + 1) % clipPolygon.size()];
        Polygon inputList = outputList;
        outputList.clear();
        if (inputList.empty()) break;
        Point s = inputList.back();
        for (const auto &e : inputList) {
            if (inside(e, {a, b})) {
                if (!inside(s, {a, b})) {
                    Point intersection = compute_intersection(s, e, {a, b});
                    if (!std::isnan(intersection.x)) {
                        outputList.push_back(intersection);
                    }
                }
            }
        }
        outputList.push_back(e);
    }
}
```

```

    } else if (inside(s, {a, b})) {
        Point intersection = compute_intersection(s, e, {a, b});
        if (!std::isnan(intersection.x)) {
            outputList.push_back(intersection);
        }
    }
    s = e;
}
}
return outputList;
}

// area of polygon
double shoelace(const Polygon &vertices) {
    double area = 0;
    for (size_t i = 0; i < vertices.size(); ++i) {
        Point p1 = vertices[i];
        Point p2 = vertices[(i + 1) % vertices.size()];
        area += p1.x * p2.y - p2.x * p1.y;
    }
    return fabs(area) / 2.0;
}

```

10.3.4 Rectangle & Circle

intersection area between Rectangle and Circle

```

struct Point {
    double x, y;
};

struct Circle {
    Point center;
    double radius;
};

struct Rectangle {
    Point bottomLeft, topRight;
};

const double PI = acos(-1.0);

bool point_inside_circle(const Point &p, const Circle &c) {

```

```

    double dx = p.x - c.center.x;
    double dy = p.y - c.center.y;
    return (dx * dx + dy * dy) <= (c.radius * c.radius);
}

double rect_area(const Rectangle &rect) {
    return (rect.topRight.x - rect.bottomLeft.x) * (rect.topRight.y - rect
        .bottomLeft.y);
}

bool rect_in_circle(const Circle &circle, const Rectangle &rect) {
    vector<Point> corners = {
        rect.bottomLeft,
        {rect.topRight.x, rect.bottomLeft.y},
        rect.topRight,
        {rect.bottomLeft.x, rect.topRight.y}
    };
    for (const auto &corner : corners) {
        if (!isPointInsideCircle(corner, circle)) {
            return false;
        }
    }
    return true;
}

bool circle_in_rect(const Circle &circle, const Rectangle &rect) {
    return circle.center.x - circle.radius >= rect.bottomLeft.x &&
        circle.center.x + circle.radius <= rect.topRight.x &&
        circle.center.y - circle.radius >= rect.bottomLeft.y &&
        circle.center.y + circle.radius <= rect.topRight.y;
}

double intersection(const Circle &circle, const Rectangle &rect) {
    if (rect_in_circle(circle, rect)) {
        return rect_area(rect);
    }

    if (circle_in_rect(circle, rect)) {
        return PI * circle.radius * circle.radius;
    }
}

```

```
double intersectionArea = 0.0;
double dx1 = max(rect.bottomLeft.x, circle.center.x - circle.radius);
double dx2 = min(rect.topRight.x, circle.center.x + circle.radius);
double dy1 = max(rect.bottomLeft.y, circle.center.y - circle.radius);
double dy2 = min(rect.topRight.y, circle.center.y + circle.radius);

for (double x = dx1; x < dx2; x += 0.001) {
    for (double y = dy1; y < dy2; y += 0.001) {
        Point p = {x, y};
        if (point_inside_circle(p, circle)) {
            intersectionArea += 0.001 * 0.001;
        }
    }
}

return intersectionArea;
}
```

10.3.5 Line & Circle

intersection points between Line and Circle

```
struct Point {
    double x, y;
};

struct Circle {
    Point center;
    double radius;
};

struct Line {
    double slope;
    double intercept;
};

vector<Point> intersect_points(const Circle &circle, const Line &line) {
    vector<Point> intersections;

    // solve  $y = mx + b$  with  $(x - h)^2 + (y - k)^2 = r^2$ 
    double h = circle.center.x;
    double k = circle.center.y;
    double r = circle.radius;
```

```
double m = line.slope;
double b = line.intercept;

// Quadratic coefficients
double A = 1 + m * m;
double B = 2 * (m * b - m * k - h);
double C = k * k - r * r + h * h - 2 * b * k + b * b;

// Discriminant
double discriminant = B * B - 4 * A * C;

if (discriminant < 0) {
    // No intersection
    return intersections;
} else if (discriminant == 0) {
    // One intersection (tangent line)
    double x = -B / (2 * A);
    double y = m * x + b;
    intersections.push_back({x, y});
} else {
    // Two intersections
    double sqrtDiscriminant = sqrt(discriminant);
    double x1 = (-B + sqrtDiscriminant) / (2 * A);
    double y1 = m * x1 + b;
    double x2 = (-B - sqrtDiscriminant) / (2 * A);
    double y2 = m * x2 + b;
    intersections.push_back({x1, y1});
    intersections.push_back({x2, y2});
}

return intersections;
}
```

10.4 Linear Operations

10.5 2D Points

Basic Operations

```
#include<bits/stdc++.h>

using namespace std;
```



```
#define ftype double

struct Point2D {
    ftype x, y;
    Point2D() {}
    Point2D(ftype x, ftype y): x(x), y(y) {}
    Point2D& operator+=(const Point2D &p2) {
        x += p2.x;
        y += p2.y;
        return *this;
    }
    Point2D& operator-=(const Point2D &p2) {
        x -= p2.x;
        y -= p2.y;
        return *this;
    }

    Point2D& operator*=(const Point2D p2) {
        x *= p2.x;
        y *= p2.y;
        return *this;
    }
    Point2D& operator/=(const Point2D p2) {
        x /= p2.x;
        y /= p2.y;
        return *this;
    }

    Point2D& operator+(const Point2D p2) {
        return Point2D(*this) += p2;
    }
    Point2D& operator-(const Point2D p2) {
        return Point2D(*this) -= p2;
    }
    Point2D& operator/(const Point2D p2) {
        return Point2D(*this) /= p2;
    }
    Point2D& operator*(const Point2D p2) {
        return Point2D(*this) *= p2;
    }
};
```

```
Point2D operator*(ftype a, Point2D b) {
    return a * b;
}

int main() {
    Point2D p1(1, 2);
    Point2D p2(3, 4);

    Point2D p3 = p1 * p2;
}
```

10.6 3D Points

Basic Operations

```
#include<bits/stdc++.h>

#define ftype double

struct Point3D {
    ftype x, y, z;
    Point3D() {}
    Point3D(ftype x, ftype y, ftype z): x(x), y(y), z(z) {}
    Point3D& operator+=(const Point3D &p2) {
        x += p2.x;
        y += p2.y;
        z += p2.z;
        return *this;
    }
    Point3D& operator-=(const Point3D &p2) {
        x -= p2.x;
        y -= p2.y;
        z -= p2.z;
        return *this;
    }

    Point3D& operator*=(const Point3D p2) {
        x *= p2.x;
        y *= p2.y;
```

```

        z *= p2.z;
        return *this;
    }
    Point3D& operator/=(const Point3D p2) {
        x /= p2.x;
        y /= p2.y;
        z /= p2.z;
        return *this;
    }

    Point3D& operator+(const Point3D p2) {
        return Point3D(*this) += p2;
    }
    Point3D& operator-(const Point3D p2) {
        return Point3D(*this) -= p2;
    }
    Point3D& operator/(const Point3D p2) {
        return Point3D(*this) /= p2;
    }
    Point3D& operator*(const Point3D p2) {
        return Point3D(*this) *= p2;
    }
};

Point3D operator*(ftype a, Point3D b) {
    return a * b;
}

int main() {
    Point3D p1(1, 2);
    Point3D p2(3, 4);

    Point3D p3 = p1 * p2;
}

```

11 Miscellaneous

11.1 Faster implementations

11.1.1 hashes

custom hash

```

#define safe hash unordered_map<type, type, custom_hash> // same for
gp_hash_table
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM = chrono::steady_clock::now().
            time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};

```

gb hash table

```

//policy based ds (faster hash table)
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
gp_hash_table<int, int> table;

```

11.1.2 Binary Search the value

nearest sqrt

```

long long my_sqrt(long long a)
{
    long long l=0,r=5000000001;
    while(r-l>1)

```

Too bad to be Accepted (Alexandria University)

```
{  
    long long mid=(l+r)/2;  
    if(1ll*mid*mid<=a)l=mid;  
    else r=mid;  
}  
return l;  
}
```
