

# Competitive Programming Library

Too bad to be Accepted 2023/2024

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# 1 Templates

## 1.1 Setup

### 1.1.1 IO Manipulation

#### *Input/Output*

```
#include <bits/stdc++.h>

freopen("input.txt", "r", stdin);
freopen("output.txt", "w", stdout);

#define fastIO \
    ios_base::sync_with_stdio(false), cin.tie(nullptr), cout.tie(nullptr);
```

### 1.1.2 GCC Compiler Optimization (Vectorization)

#### *GCC Opt*

```
// Ref: USACO guide
// will make GCC auto-vectorize for loops and optimizes floating points
// better (assumes associativity and turns off denormals).
#pragma GCC optimize ("Ofast")
// can double performance of vectorized code, but causes crashes on old
// machines.
#pragma GCC target ("avx,avx2,fma")

// slows down run time but throws a Runtime Error if an overflow occurred
#pragma GCC optimize("trapv")
```

## 1.2 MOD Template

```
constexpr int MOD = 1e9+7; // must be a prime number

int add(int a, int b) {
    int res = a+b;
    if(res >= MOD) return res -= MOD;
}
```

```
int sub(int a, int b) {
    int res = a-b;
    if(res < 0) return res += MOD;
}

int power(int a, int e) {
    int res = 1;
    while(e) {if(e & 1) res = res * a % MOD; a = a * a % MOD;
    e >>= 1;}
    return res;
}

int inverse(int a) {
    return power(a, MOD-2);
}

int div(int a, int b) {
    return a * inverse(b) % MOD;
}
```

### MOD Template

## 1.3 Macros

### Macros

```
#define getBit(n, k) (n >> k)
#define ON(n, idx) (n | (1ll << idx))
#define OFF(n, idx) (n & ~(1ll << idx))
#define toggle(n, idx) ((n) ^ (1ll<<(idx)))
#define gray(n) (n ^ (n >> 1))
#define bitCount(x) (__builtin_popcountll(x))
#define uniq(x) x.resize(unique(x.begin(), x.end())-x.begin());

#define angle(a) (atan2((a).imag(), (a).real()))
//#define vec(a, b) ((b)-(a))
#define same(v1, v2) (dp(vec(v1,v2),vec(v1,v2)) < EPS)
#define dotProduct(a, b) ((conj(a)*(b)).real()) // a*b cos(T), if zero ->
    prep
#define crossProduct(a, b) ((conj(a)*(b)).imag()) // a*b sin(T), if zero
    -> parallel
//#define length(a) (hypot((a).imag(), (a).real()))
```

```
#define normalize(a) ((a)/length(a))
#define rotate0(v, ang) ((v)*exp(point(0,ang)))
#define rotateA(p, ang, about) (rotate0(vec(about,p),ang)+about)
#define reflect0(v, m) (conj((v)/(m))*(m))
#define ceil_i(a, b) (((1ll)(a)+(1ll)(b-1))/(1ll)(b))
#define floor_i(a, b) (a/b)
#define round_i(a, b) ((a+(b/2))/b) // if a>0
#define round_m(a, b) ((a-(b/2))/b) // if a<0
#define round_multiple(n, m) round_i(n,m)*m // round to multiple if
    specified element

const double PI = acos(-1.0);
```

## 1.4 Grid Navigation

### Grid Nav

```
// knight moves on a chess board
int dx[] = { -2, -1, 1, 2, -2, -1, 1, 2 };
int dy[] = { -1, -2, -2, -1, 1, 2, 2, 1 };

// Grid up, down, right, left (Moves for Chess Rook)
int dx[4] = {1, -1, 0, 0};
int dy[4] = {0, 0, 1, -1};

// Grid cell all neighbours
const int dx[8] = {1, 0, -1, 0, 1, 1, -1, -1}
const int dy[8] = {0, 1, 0, -1, -1, 1, -1, 1};

// Grid Diagonal (Moves for Chess Bishop)
int dx[] = {1, 1, -1, -1};
int dy[] = {1, -1, 1, -1};
```

## 1.5 Integer 128

### i128

```
typedef __int128 i128;
```

```
__int128 read() {
    __int128 x = 0, f = 1;
    char ch = getchar();
    while (ch < '0' || ch > '9') {
        if (ch == '-') f = -1;
        ch = getchar();
    }
    while (ch >= '0' && ch <= '9') {
        x = x * 10 + ch - '0';
        ch = getchar();
    }
    return x * f;
}

void print(__int128 x) {
    if (x < 0) {
        putchar('-');
        x = -x;
    }
    if (x > 9) print(x / 10);
    putchar(x % 10 + '0');
}

bool cmp(__int128 x, __int128 y) { return x > y; }
```

---

## 2 Dynamic Programming

### 2.1 Some dp patterns

#### *Maximumu/Minimum path cost*

```
const int MAX = 21;
int grid[MAX][MAX];
int mem[MAX][MAX];
int n = 20;
bool valid(int r, int c){
    return r >= 0 && r < n && c >= 0 && c < n;
}

int maxPathSum(int r, int c){
```

```
    if(!valid(r,c)){
        return 0;
    }

    if(r == n-1 && c == n-1){
        return mem[r][c] = grid[r][c];
    }
    // available moves
    int path1 = maxPathSum(r+1,c);
    int path2 = maxPathSum(r,c+1);

    return grid[r][c] + max(path1,path2);
}
```

---

#### *add operators between numbers to get max prod/sum*

```
// put +, -, between sequence of numbers such that the sum is divisible by
// k, and maximum as possible
const int MAX = 21;
long long mem[MAX][MAX];
const int n = 20;
int k = 4; // example
int v[20];
int fix(int a){
    return (a % k + k) % k;
}
long long tryAll(int pos, int mod){
    long long &ret = mem[pos][mod];
    if(ret != -1){
        return ret;
    }
    if(pos == n){
        return ret = mod == 0;
    }
    if(tryAll(pos+1,fix(mod + v[pos])) || tryAll(pos+1,fix(mod-v[pos]))){
        return ret = 1;
    }
    return ret = 0;
}
```

---

#### *pick choices with no two similar consecutive choices*

```
// pick minimum of choinces costs with no two similar consecutive choices
const int choices = 4;
const int n = 20;
int MAX = n;
int mem[MAX][choices];
const int OO = 1e6+1;
int minCost(int pos, int lastChoice){
    if(pos == n){
        return 0; // invalid move
    }
    int &ret = mem[pos][lastChoice];

    if(ret != -1){
        return ret;
    }

    ret = OO; // want to minimize
    // let choices are 0, 1, 2
    if(lastChoice != 0){
        ret = min(ret, minCost(pos+1,0));
    }
    if(lastChoice != 1){
        ret = min(ret, minCost(pos+1,1));
    }
    if(lastChoice != 2){
        ret = min(ret, minCost(pos+1,2));
    }
    return ret;
}
```

---

### *sum S and max/min Product*

```
int maxK;

ll mem[21][101]; // k, and s

// You are given an integer s and an integer k. Find k positive integers
// a1, a2, ..., ak
// such that their sum is equal to s and their product is the maximal
// possible. Return their product.

ll maxProd(int k, int rem)
{
```

```
if(k == maxK){
    // base case
    if(rem == 0)
        return 1;
    return 0;
}

if(rem == 0) // invalid case
    return 0;

ll &ret = mem[k][rem];

if(ret != -1)
    return ret;

ret = 0;

for (int i = 1; i <= rem; ++i) {
    ll sol = maxProd(k+1, rem - i) * i;
    ret = max(ret, sol);
}

return ret;
}
```

---

## 2.2 DP solutions

### 2.2.1 Max Subarray sum (Kadane's Algorithm)

#### *Max Subarray sum*

```
int maxSubarraySum(vector<int>& arr, int len) {
    int ans = INT_MIN, cur = 0;

    for (int i = 0; i < len; i++) {
        cur = cur + arr[i];
        if (ans < cur)
            ans = cur;

        if (cur < 0)
            cur = 0;
    }
}
```

```
    return ans;
}
```

---

## 2.2.2 Maximum Subarray Alternating Sum

### *Maximum Subarray Alternating Sum*

```
/* REF: GeeksForGeeks
Input: arr[] = {-4, -10, 3, 5}
Output: 9
Explanation: Subarray {arr[0], arr[2]} = {-4, -10, 3}. Therefore, the sum
of this subarray is 9.
*/
int maxSubarraySumALT(vector<int>& a, int len) {
    int ans = INT_MIN, cur = 0;
    for (int i = 0; i < len; i++) {
        if (i % 2 == 0)
            cur = max(cur + a[i], a[i]);
        else
            cur = max(cur - a[i], -a[i]);

        ans = max(ans, cur);
    }

    cur = 0;

    for (int i = 0; i < len; i++) {
        if (i % 2 == 1)
            cur = max(cur + a[i], a[i]);
        else
            cur = max(cur - a[i], -a[i]);

        ans = max(ans, cur);
    }

    return ans;
}
```

---

## 2.2.3 Count number of DISTINCT ordered ways to produce coins sums to x

### *Count distinct*

```
/*
For example, if the coins are {2,3,5} and the desired sum is 9, there
are 3 ways:

2+2+5
3+3+3
2+2+2+3
*/
int n, x;
cin >> n >> x;
vector<int> coins(n);
read(coins);

vector dp(x + 1, 0);

dp[0] = 1;
for (int i = 0; i < n; ++i) {
    for (int j = coins[i]; j <= x; ++j) {
        dp[j] = add(dp[j], dp[j - coins[i]]);
    }
}

cout << dp[x] << endl;
```

---

## 2.2.4 Min absolute difference between 2 elements from (L, R) (DP Ranges)

### *Min absolute difference*

```
const int N = 1e4 + 1;

int dp[N][N];

int n;
cin >> n;
vector<int> a(n);
read(a);
```

```
for (int i = 0; i < n; ++i) dp[i][i] = 1e6; // INF, you can't take the
    element with it self
for (int i = 1; i < n; ++i) dp[i - 1][i] = abs(a[i] - a[i - 1]);

for (int len = 3; len <= n; ++len) {
    for (int l = 0, r = len - 1; r < n; ++l, ++r) {
        dp[l][r] = min(dp[l][r - 1], dp[l + 1][r]);
        dp[l][r] = min(dp[l][r], abs(a[l] - a[r]));
    }
}

int q;
cin >> q;
while (q--) {
    int l, r;
    cin >> l >> r;
    --l, --r;

    cout << dp[l][r] << el;
}
```

### 2.2.5 Longest common subsequence between 2 Strings

$$dp[i][j] = \begin{cases} \max(dp[i-1][j], dp[i][j-1]) & \text{if } A_i \neq B_j \\ dp[i-1][j-1] + 1 & \text{if } A_i = B_j \end{cases}$$

#### LIS 2 Strings

```
// REF: USACO guide
int longestCommonSubsequence(string a, string b) {
    int dp[a.size()][b.size()];
    for (int i = 0; i < a.size(); i++) { fill(dp[i], dp[i] + b.size(), 0); }
    for (int i = 0; i < a.size(); i++) {
        if (a[i] == b[0]) dp[i][0] = 1;
        if (i != 0) dp[i][0] = max(dp[i][0], dp[i - 1][0]);
    }
    for (int i = 0; i < b.size(); i++) {
        if (a[0] == b[i]) dp[0][i] = 1;
        if (i != 0) dp[0][i] = max(dp[0][i], dp[0][i - 1]);
    }
    for (int i = 1; i < a.size(); i++) {
        for (int j = 1; j < b.size(); j++) {
```

```
if (a[i] == b[j]) {
    dp[i][j] = dp[i - 1][j - 1] + 1;
} else {
    dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
}
}
}
return dp[a.size() - 1][b.size() - 1];
}
```

## 3 Bit Manipulation

### 3.1 Subset Operations

#### *count subsets with give sum*

```
int countDistinctSubsetsWithSum(vector<int>& arr, int n, int k) {
    // Count distinct subsets of array arr that sum up to k
    vector<int> dp(k + 1, 0);
    dp[0] = 1;
    for (int i = 0; i < n; ++i) {
        for (int j = k; j >= arr[i]; --j) {
            dp[j] += dp[j - arr[i]];
        }
    }
    return dp[k]; // Number of distinct subsets with sum k
}
```

#### *max xor of any subset of elements in the array*

```
int maximalSubsetXOR(vector<int>& arr, int n) {
    // Find the maximum XOR of any subset of elements in array arr
    int maxXor = 0;
    for (int mask = 0; mask < (1 << n); ++mask) {
        int xorSum = 0;
        for (int i = 0; i < n; ++i) {
            if (mask & (1 << i)) {
                xorSum ^= arr[i];
            }
        }
    }
}
```

```
        maxXor = max(maxXor, xorSum);
    }
    return maxXor;
}
```

---

### *min xor of any subset*

```
int minimumSubsetXOR(vector<int>& arr, int n) {
    // Find the minimum XOR of any pair of elements in array arr
    int minSubsetXor = INT_MAX;
    for (int mask = 0; mask < (1 << n); ++mask) {
        int xorSum = 0;
        for (int i = 0; i < n; ++i) {
            if (mask & (1 << i)) {
                xorSum ^= arr[i];
            }
        }
        minSubsetXor = min(minSubsetXor, xorSum);
    }
    return minSubsetXor;
}
```

---

### *subset generation*

```
void subsetGeneration(int x, int n) {
    // Generate all non-empty subsets of a set represented by an integer x
    for (int subset = x; subset > 0; subset = (subset - 1) & x) {
        // Process subset
        cout << subset << endl;
    }
}
```

---

### *check if subset of elements in the array sum up to k*

```
void subsetSumCheck(vector<int>& arr, int n, int k) {
    // Check if a subset of elements in array arr sums up to k
    for (int subset = 0; subset < (1 << n); ++subset) {
        int sum = 0;
        for (int i = 0; i < n; ++i) {
            if (subset & (1 << i)) {
```

```
                sum += arr[i];
            }
        }
        if (sum == k) {
            // Found subset with sum k
            cout << "Subset with sum " << k << ": " << subset << endl;
        }
    }
}
```

---

### *max subset sum mod m*

```
int subsetWithMaxSumModuloM(vector<int>& arr, int n, int m) {
    // Find the maximum subset sum modulo m
    vector<int> dp(m, -1);
    dp[0] = 0;
    int currentMod = 0;
    for (int i = 0; i < n; ++i) {
        currentMod = (currentMod + arr[i]) % m;
        for (int j = 0; j < m; ++j) {
            if (dp[j] != -1) {
                dp[(j + currentMod) % m] = max(dp[(j + currentMod) % m], dp[j] + arr[i]);
            }
        }
        dp[currentMod] = max(dp[currentMod], arr[i]);
    }
    return dp[0]; // Maximum subset sum modulo m
}
```

---

### *iterate over all supersets represented by x*

```
void iterateOverSupersets(int x, int n) {
    // Iterate over all supersets of a set represented by x
    int subset = x;
    do {
        // Process subset
        cout << subset << endl;
        subset = (subset + 1) | x;
    } while (subset <= (1 << n) - 1);
}
```

---



## 4 Algorithms

### 4.1 MO

#### *MO Algorithm*

```
// MO      -> O(N+Q SQRT(N)) <= 10^5

const int N = 1e5+5, M = 1e5+5;
int n, m;
int nums[N], q_ans[M];

struct query {
    int idx, block_idx, l, r;

    query() = default;
    query(int _l, int _r, int _idx) {
        idx = _idx;
        r = _r - 1;
        l = _l - 1;
        block_idx = _l / sqrt(n);
    }

    bool operator <(const query & y) const {
        if(y.block_idx == block_idx) return r < y.r;
        return block_idx < y.block_idx;
    }
};

int freq[N], ans;

void add(int idx) {
    freq[nums[idx]]++;
    if (freq[nums[idx]] == 2) ans++;
}

void remove(int idx) {
    freq[nums[idx]]--;
    if (freq[nums[idx]] == 1) ans--;
}

cin >> n >> m;
for (int i = 0; i < n; ++i) cin >> nums[i];
```

```
vector<query> Query(m);
for (int i = 0; i < m; ++i) {
    int l, r; cin >> l >> r;
    Query[i] = query(l, r, i);
}

sort(Query.begin(), Query.end());
int l0 = 1, r0 = 0;
for (int i = 0; i < m; ++i) {
    while (l0 < Query[i].l) remove(l0++);
    while (l0 > Query[i].l) add(--l0);
    while (r0 < Query[i].r) add(++r0);
    while (r0 > Query[i].r) remove(r0--);
    q_ans[Query[i].idx] = ans;
}
for (int i = 0; i < m; ++i) {
    cout << q_ans[i] << '\n';
}
```

---

### 4.2 Intervals

#### 4.2.1 Prefix Sum (L, R) intervals

##### *Prefix Sum (L, R) intervals*

// NOTE: works fine with small n or with large memory

```
int main() {
    int n, k;
    cin >> n >> k;

    vector<int> a(n + 1);
    vector<vector<int>>> rangesPrefix(n + 1, vector<int>(n + 1, 0));
    for (int i = 1; i <= n; ++i)
        cin >> a[i];

    int l = 1, r = 1, sum = 0;
    // validate your intervals
    // here the intervals are the ones that have a sum of k
    while (r <= n) {
        sum += a[r];
```

```

while (sum > k) {
    sum -= a[l];
    ++l;
}

while (l <= r && a[l] == 0) {
    if (sum != k)
        break;

    rangesPrefix[r][l]++;

    ++l;
}

if (sum == k) {
    rangesPrefix[r][l]++;
}

++r;
}

// prefix sum the columns
for (int i = 1; i <= n; ++i) {
    for (int j = n - 1; j >= 0; --j) {
        rangesPrefix[i][j] += rangesPrefix[i][j + 1];
    }
}

// prefix sum the rows
for (int i = 0; i <= n; ++i) {
    for (int j = 1; j <= n; ++j) {
        rangesPrefix[j][i] += rangesPrefix[j - 1][i];
    }
}

int q; cin >> q;

while (q--) {
    cin >> l >> r;
    // answer the number of intervals (X, Y) X <= Y that are included
    // between L, R
    cout << rangesPrefix[r][l] - rangesPrefix[l - 1][l] << endl;
}

```

```

}
}

```

## 4.2.2 Find subarrays intervals that sum to K Using Map

### *Find subarray intervals that sum to K Using Map*

```

int n, k;
cin >> n >> k;

vector<int> a(n + 1);
vector<pair<int, int>> rng;
for (int i = 1; i <= n; ++i)
    cin >> a[i];

map<int, set<int>> prev;
int currSum = 0;

for (int i = 1; i <= n; ++i) {
    currSum += a[i];
    if (currSum == k) {
        rng.push_back({1, i});
    }
    if (prev.find(currSum - k) != prev.end()) {
        for (auto &j : prev[currSum - k]) {
            rng.push_back({j + 1, i});
        }
    }
    prev[currSum].insert(i);
}

```

## 4.3 Ad-hoc

### 4.3.1 Find duplicates

#### *Find duplicates using XOR*

```

int findDuplicate(int arr[] , int n)
{
    int answer=0;
    //XOR all the elements with 0
}

```

```
for(int i=0; i<n; i++){
    answer=answer^arr[i];
}
//XOR all the elements with no from 1 to n
// i.e answer^0 = answer
for(int i=1; i<n; i++){
    answer=answer^i;
}
return answer;
}
```

---

## 5 Data Structures

### 5.1 Strings

#### 5.1.1 Trie (Prefix Tree)

##### *Basic Implementation*

```
#define MAX_CHAR 26

struct TrieNode {
    TrieNode *pTrieNode[MAX_CHAR]{};
    bool isWord;

    TrieNode() {
        isWord = false;
        fill(pTrieNode, pTrieNode + 26, (TrieNode *) NULL);
    }

    virtual ~TrieNode() = default;
};

class Trie {
private:
    TrieNode *root;
public:
    Trie() {
        root = new TrieNode();
    }
};
```

```
virtual ~Trie() = default;

TrieNode *getTrieNode() {
    return this->root;
}

void insert(const string &word) {
    TrieNode *current = root;
    for (char c: word) {
        int i = c - 'a';
        if (current->pTrieNode[i] == nullptr)
            current->pTrieNode[i] = new TrieNode();
        current = current->pTrieNode[i];
    }
    current->isWord = true;
}

bool search(const string &word) {
    TrieNode *current = root;
    int ch = 0;
    for (char c: word) {
        ch = c - 'a';
        if (current->pTrieNode[ch] == nullptr)
            return false;
        current = current->pTrieNode[ch];
    }
    return current->isWord;
}

bool startsWith(const string &prefix) {
    TrieNode *current = root;
    int ch = 0;
    for (char c: prefix) {
        ch = c - 'a';
        if (current->pTrieNode[ch] == nullptr)
            return false;
        current = current->pTrieNode[ch];
    }
    return true;
};
```

## 5.2 Range Queries

### 5.2.1 Segment Tree

#### *Basic Implementation*

```
struct Node {
    long long val;
};

struct SegTree {
private:
    const Node NEUTRAL = {INT_MIN};

    static Node merge(const Node& x1, const Node& x2) {
        return {x1.val + x2.val};
    }

    void set(const int& idx, const int& val, int x, int lx, int rx) {
        if (rx - lx == 1) return void(values[x].val = val);

        int mid = (rx + lx) / 2;

        if (idx < mid)
            set(idx, val, 2 * x + 1, lx, mid);
        else
            set(idx, val, 2 * x + 2, mid, rx);

        values[x] = merge(values[2 * x + 1], values[2 * x + 2]);
    }

    Node query(const int& l, const int& r, int x, int lx, int rx) {
        if (lx >= r || l >= rx) return NEUTRAL;
        if (lx >= l && rx <= r) return values[x];

        int mid = (rx + lx) / 2;

        return merge(query(l, r, 2 * x + 1, lx, mid), query(l, r, 2 * x + 2, mid, rx));
    }

    void build(vector<int> &a, int x, int lx, int rx) {
        if (rx - lx == 1) {
            if (lx < a.size()) {
                values[x].val = a[lx];
            }
        }
    }
};
```

```
        return;
    }
    int m = (lx + rx) / 2;
    build(a, 2 * x + 1, lx, m);
    build(a, 2 * x + 2, m, rx);
    values[x] = merge(values[2 * x + 1], values[2 * x + 2]);
}

void assign_range(int l, int r, int node, int lx, int rx, int time,
    int val) {
    if (lx > r || l > rx) return;
    if (lx >= l && rx <= r) {
        lazy[node] = {time, val};
        return;
    }
    int mid = (lx+rx) / 2;

    assign_range(l, r, 2*node+1, lx, mid, time, val);
    assign_range(l, r, 2*node+2, mid+1, rx, time, val);
}

pair<int, int> point_query(int lx, int rx, int node, int idx) {
    if(rx == lx) return lazy[node];
    int mid = (lx+rx) / 2;

    if(idx <= mid) {
        auto x = point_query(lx, mid, 2*node+1, idx);
        if(x.first > lazy[node].first) return x;
        return lazy[node];
    }
    auto x = point_query(mid+1, rx, 2*node+2, idx);
    if(x.first > lazy[node].first) return x;
    return lazy[node];
}

public:
    int size{};
    vector<Node> values;

    void build(vector<int> &a) {
        build(a, 0, 0, size);
    }

    void init(int _size) {
```

```

    size = 1;
    while (size < _size) size *= 2;
    values.assign(2 * size, NEUTRAL);
}

void set(int idx, int val) {
    set(idx, val, 0, 0, size);
}

Node query(const int& l, const int& r) {
    return query(l, r, 0, 0, size);
}
};

```

---

## 5.2.2 Lazy Propagation

### *Lazy Propagation*

```

struct SegTree {
private:
    void propagate(int lx, int rx, int node) {
        if(!lazy[node]) return;

        if(lx != rx) {
            lazy[2*node+1] = lazy[node];
            lazy[2*node+2] = lazy[node];
        }
        values[node] = lazy[node] * (rx - lx + 1);
        lazy[node] = 0;
    }

    // assign val in range [l, r]
    void update_range(int l, int r, int node, int lx, int rx, int val,
        bool f) {
        propagate(lx, rx, node);
        if (lx > r || l > rx) return;
        if (lx >= l && rx <= r) {
            lazy[node] = val;
            propagate(lx, rx, node);
            return;
        }
        int mid = (lx+rx) / 2;

```

```

        update_range(l, r, 2*node+1, lx, mid, val, f);
        update_range(l, r, 2*node+2, mid+1, rx, val, f);
        values[node] = values[2*node+1] + values[2*node+2];
    }

    // get sum in range [l, r]
    int range_query(int l, int r, int lx, int rx, int node) {
        propagate(lx, rx, node);
        if (lx > r || l > rx) return 0;
        if (lx >= l && rx <= r) return values[node];

        int mid = (lx+rx) / 2;
        return range_query(l, r, lx, mid, 2*node+1) + range_query(l, r, mid
            +1, rx, 2*node+2);
    }
}

```

```

public:
    int size{};
    vector<int> values, lazy;

    void init(int _size) {
        size = 1;
        while (size < _size) size *= 2;
        values.assign(2 * size, 0);
        lazy.assign(2 * size, 0);
    }

    void update_range(int l, int r, int v, bool f) {
        update_range(l, r, 0, 0, size-1, v, f);
    }

    int range_query(int l, int r) {
        return range_query(l, r, 0, size-1, 0);
    }
};

```

---

## 5.2.3 Fenwick Tree

### *Fenwick Tree*

```

struct Fenwick {
    // One Based
    vector<int> tree;

```

```

explicit Fenwick(int n) {tree.assign(n + 5, {});}

// Computes the prefix sum from [1, i], O(log(n))
int query(int i) {
    int res = 0;
    while (i > 0) {
        res += tree[i];
        i &= ~(i & -i);
    }
    return res;
}

int query(int l, int r) {
    return query(r) - query(l-1);
}

// Get the value at index i
int get(int i) {
    return query(i, i);
}

// Add 'v' to index 'i', O(log(n))
void update(int i, int v) {
    while (i < tree.size()) {
        tree[i] += v;
        i += (i & -i);
    }
}

// Update range, Point query
// To get(k) do prefix sum [1, k] and in insert update_range(i, i, a[i])
void update_range(int l, int r, int v) {
    update(l, v);
    update(r+1, -v);
}
};

```

#### 5.2.4 Fenwick UpdateRange

#### BIT UpdateRange

```

struct BITUpdateRange {
private:
    int n;
    vector<int> B1, B2;

    void add(vector<int> &b, int idx, int x) {
        while (idx <= n) {
            b[idx] += x;
            idx += idx & -idx;
        }
    }

    int sum(vector<int> &b, int idx) {
        int total = 0;
        while (idx > 0) {
            total += b[idx];
            idx &= ~(idx & -idx);
        }
        return total;
    }

    int prefix(int idx) {
        return sum(B1, idx) * idx - sum(B2, idx);
    }

public:
    explicit BITUpdateRange(int n) : n(n) {
        B1.assign(n + 1, {});
        B2.assign(n + 1, {});
    }

    void update(int l, int r, int x) {
        add(B1, l, x);
        add(B1, r + 1, -x);
        add(B2, l, x * (l - 1));
        add(B2, r + 1, -x * r);
    }

    int query(int i) {
        return prefix(i) - prefix(i - 1);
    }

    int query(int l, int r) {

```

```

        return prefix(r) - prefix(l - 1);
    }
};

```

## 5.2.5 2D BIT

### 2D BIT

```

struct BIT2D {
    int n, m;
    vector<vector<int>> bit;

    BIT2D(int n, int m) : n(n), m(m) {
        bit.assign(n + 2, vector<int>(m + 2));
    }

    void update(int x, int y, int val) {
        for (; x <= n; x += x & -x) {
            for (int i = y; i <= m; i += i & -i) {
                bit[x][i] += val;
            }
        }
    }

    int prefix(int x, int y) {
        int res = 0;
        for (; x > 0; x &= ~(x & -x)) {
            for (int i = y; i > 0; i &= ~(i & -i)) {
                res += bit[x][i];
            }
        }
        return res;
    }

    int query(int sx, int sy, int ex, int ey) {
        int ans = 0;
        ans += prefix(ex, ey);
        ans -= prefix(ex, sy - 1);
        ans -= prefix(sx - 1, ey);
        ans += prefix(sx - 1, sy - 1);
        return ans;
    }
};

```

## 5.2.6 Sparse Table

### Impl with the index

```

// storing the index also
struct SNode {
    int val;
    int index;
};

class SparseTable {
private:
    vector<vector<SNode>> table;

    function<SNode(const SNode&, const SNode&)> merge;

    static SNode StaticMerge(const SNode& a, const SNode& b) {
        return a.val < b.val ? a : b;
    }

public:
    explicit SparseTable(const vector<int>& arr, const function<SNode(
        const SNode&, const SNode&)>& mergeFunc = StaticMerge) {
        int n = static_cast<int>(arr.size());
        int log_n = static_cast<int>(log2(n)) + 1;
        this->merge = mergeFunc;

        table.resize(n, vector<SNode>(log_n));

        for (int i = 0; i < n; i++) {
            table[i][0] = {arr[i], i};
        }

        for (int j = 1; (1 << j) <= n; j++) {
            for (int i = 0; i + (1 << j) <= n; i++) {
                table[i][j] = mergeFunc(table[i][j - 1], table[i + (1 << (j
                    - 1))] [j - 1]);
            }
        }
    }

    SNode query(int left, int right) {

```

```

        int j = static_cast<int>(log2(right - left + 1));
        return merge(table[left][j], table[right - (1 << j) + 1][j]);
    }
};

int main(void) {
    int n;
    cin >> n;
    vector<int> arr(n);
    for (auto& element : arr) cin >> element;

    SparseTable minSt(arr, [](const SNode& a, const SNode& b) -> SNode {
        return a.val < b.val ? a : b;
    });

    SparseTable maxSt(arr, [](const SNode& a, const SNode& b) -> SNode {
        return a.val > b.val ? a : b;
    });
}

```

## 5.3 Ordered Set

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;

template<typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

void erase_set(ordered_set &os, int v) {
    // Number of elements less than v
    int rank = os.order_of_key(v);

    auto it = os.find_by_order(rank);
    os.erase(it);
}

```

*Ordered Set*

## 6 Counting Principles

### 6.1 nCr

$$C(n, k) = \frac{n!}{(n-k)!k!} = \frac{n * (n-1) * (n-2) * \dots * (n-k+1)}{k!}$$

#### 6.1.1 Fast nCr

$$C(n, k) = \frac{n * (n-1) * (n-2) * \dots * (n-k+1)}{1 * 2 * 3 * \dots * k} = \prod_{i=0}^{k-1} \frac{n-i}{i+1} = \prod_{i=0}^{k-1} (n-i)(i+1)^{-1}$$

*Fast nCr*

```

int nCr(const int& n, const int& r) {
    double res = 1;
    for (int i = 1; i <= r; ++i)
        res = res * (n - r + i) / i;
    return (int)(res + 0.01);
}

```

#### 6.1.2 Method 1: Pascal's Triangle (Dynamic Programming) - $\mathcal{O}(n^2)$

*nCk using dp*

```

// REF: USACO guide

/** @return nCk mod p using dynamic programming */
int binomial(int n, int k, int p) {
    // dp[i][j] stores iCj
    vector<vector<int>> dp(n + 1, vector<int>(k + 1, 0));

    // base cases described above
    for (int i = 0; i <= n; i++) {
        /*
         * i choose 0 is always 1 since there is exactly one way

```



```

    * to choose 0 elements from a set of i elements
    * (don't choose anything)
    */
dp[i][0] = 1;
/*
    * i choose i is always 1 since there is exactly one way
    * to choose i elements from a set of i elements
    * (choose every element in the set)
    */
if (i <= k) { dp[i][i] = 1; }
}

for (int i = 0; i <= n; i++) {
    for (int j = 1; j <= min(i, k); j++) {
        if (i != j) { // skips over the base cases
            // uses the recurrence relation above
            dp[i][j] = (dp[i - 1][j - 1] + dp[i - 1][j]) % p;
        }
    }
}

return dp[n][k]; // returns nCk modulo p
}

```

### 6.1.3 Method 2: Factorial Definition (Modular Inverses) - $\mathcal{O}(n + \log MOD)$

#### *nCk using Modular Inverses*

// REF: USACO guide

```

const int MAXN = 1e6;

long long fac[MAXN + 1];
long long inv[MAXN + 1];

/** @return x^n modulo m in O(log p) time. */
long long exp(long long x, long long n, long long m) {
    x %= m; // note: m * m must be less than 2^63 to avoid ll overflow
    long long res = 1;
    while (n > 0) {
        if (n % 2 == 1) { res = res * x % m; }

```

```

        x = x * x % m;
        n /= 2;
    }
    return res;
}

/** Precomputes n! from 0 to MAXN. */
void factorial(long long p) {
    fac[0] = 1;
    for (int i = 1; i <= MAXN; i++) { fac[i] = fac[i - 1] * i % p; }
}

/**
    * Precomputes all modular inverse factorials
    * from 0 to MAXN in O(n + log p) time
    */
void inverses(long long p) {
    inv[MAXN] = exp(fac[MAXN], p - 2, p);
    for (int i = MAXN; i >= 1; i--) { inv[i - 1] = inv[i] * i % p; }
}

/** @return nCr mod p */
long long choose(long long n, long long r, long long p) {
    return fac[n] * inv[r] % p * inv[n - r] % p;
}

int main() {
    factorial();
    inverses();
    int n;
    cin >> n;
    for (int i = 0; i < n; i++) {
        int a, b;
        cin >> a >> b;
        cout << choose(a, b) << '\n';
    }
}

```

## 7 Graph Theory

### 7.1 Shortest Path algorithms

#### 7.1.1 Dijkstra Algorithm

##### *Dijkstra Implementation*

```
#define INF (1e18) // for int defined as ll

int n, m;
vector<vector<pair<int, int>>> adj;
vector<int> cost;
vector<int> parent;

void dijkstra(int startNode = 1) {
    priority_queue<pair<ll, int>, vector<pair<ll, int>>, greater<>> pq;

    cost[startNode] = 0;
    pq.emplace(0, startNode);

    while (!pq.empty()) {
        int u = pq.top().second;
        ll d = pq.top().first;
        pq.pop();

        if (d > cost[u]) continue;

        for (auto &p: adj[u]) {
            int v = p.first;
            int w = p.second;
            if (cost[v] > cost[u] + w) {
                cost[v] = cost[u] + w;
                parent[v] = u;
                pq.emplace(cost[v], v);
            }
        }
    }
}

void run_test_case(int testNum) {
    cin >> n >> m;

    adj.assign(n + 1, {});
```

```
cost.assign(n + 1, INF);
parent.assign(n + 1, -1);

while (m--) {
    // Read Edges
}

dijkstra();

if (cost[n] == INF) {
    cout << -1 << endl; // not connected {Depends on you use case}
    return;
}

stack<int> ans;
for (int v = n; v != -1; v = parent[v]) ans.push(v);

while (!ans.empty()) { // printing the path
    cout << ans.top() << ' ';
    ans.pop();
}
cout << endl;
}
```

#### 7.1.2 Floyd Warshal Algorithm

##### *Floyd Warshal Implementation*

```
int main() {
    int n, m; cin >> n >> m;
    vector <vector <int>> adj(n + 1, vector <int> (n + 1, 2e9));
    for (int i = 0; i < n; i++) adj[i][i] = 0;

    while(m--) {
        int u, v, w;
        cin >> u >> v >> w;
        adj[u][v] = min(adj[u][v], w);
        adj[v][u] = min(adj[v][u], w);
    }

    for (int mid = 1; mid <= n; mid++) {
        for (int start = 1; start <= n; start++) {
            for (int end = 1; end <= n; end++) {
```

```
        adj[start][end] = min(adj[start][end], adj[start][mid] +
                               adj[mid][end]);
    }
}

return 0;
}
```

---

### 7.1.3 Bellman Ford Algorithm

#### *BellmanFord Implementation*

```
vector <vector <pair<int, int>>> &adj

vector <long long> BellmanFord(int src) {
    int n = (int)adj.size();
    vector <long long> dist(n, 2e18);

    dist[src] = 0;
    for (int it = 0; it < n-1; it++) {
        bool in = false;
        for (int i = 0; i < n; i++) { // iterate on the edges
            for (auto &[j, w] : adj[i]) {
                if (dist[j] > dist[i] + w) {
                    in = true;
                    dist[j] = dist[i] + w;
                }
            }
        }
        if (!in) return dist;
    }

    for (int i = 0; i < n; i++) {
        for (auto &[j, w] : adj[i]) {
            if (dist[j] > dist[i] + w) { //negative cycle
                return vector <long long> (n, -1); // or any flag
            }
        }
    }

    return dist;
}
```

---

## 7.2 Cycle Detection

### 7.2.1 DFS Implementation

#### *DFS Implementation*

```
// return true with number of nodes in the cycle, either odd cycle or even
bool cycle_detection(unordered_map<int, vector<int>> &graph, int source,
                    int par, unordered_map<int, bool> vis, int c){
    if(vis[source]) return true;

    vis[source] = true;

    for(int v: graph[source]){
        if(v != par){
            c++;
            if(dfs(graph,v, source, vis, c)) return true;
        }
    }
    return false;
}
```

---

### 7.2.2 Another way for undirected graphs

#### *Another way for undirected graphs*

```
// this is true only for undirected graphs
bool dfs1(int cur, int par) {
    bool ret = false;
    vis[cur] = true;
    for (auto &i : adj[cur]) {
        if (!vis[i]) ret|=dfs1(i, cur);
        else if (par != i) ret = true;
    }
    return ret;
}
```

---

### 7.2.3 General Way

#### *General Way*

```
// general algorithm
vector<bool> cyc;
bool dfs(int cur, int par) {
    bool ret = false;
    vis[cur] = cyc[cur] = true;
    for (auto &i : adj[cur]) {
        if (par == i) continue;
        if (!vis[i]) ret |= dfs(i, cur);
        else if (cyc[i]) ret = true;
    }
    cyc[cur] = false;
    return ret;
}
```

---

### 7.2.4 DSU Implementation

#### *DSU Implementation*

```
#include <iostream>
#include <vector>

class UnionFind {
public:
    UnionFind(int n) {
        parent.resize(n);
        rank.resize(n, 0);
        for (int i = 0; i < n; ++i) {
            parent[i] = i;
        }
    }

    int find(int u) {
        if (parent[u] != u) {
            parent[u] = find(parent[u]);
        }
        return parent[u];
    }

    void unionSets(int u, int v) {
```

```
int rootU = find(u);
int rootV = find(v);

if (rootU != rootV) {
    if (rank[rootU] > rank[rootV]) {
        parent[rootV] = rootU;
    } else if (rank[rootU] < rank[rootV]) {
        parent[rootU] = rootV;
    } else {
        parent[rootV] = rootU;
        ++rank[rootU];
    }
}
```

```
private:
    std::vector<int> parent;
    std::vector<int> rank;
};
```

```
bool detectCycle(const std::vector<std::pair<int, int>>& edges, int n) {
    UnionFind uf(n);

    for (const auto& edge : edges) {
        int u = edge.first;
        int v = edge.second;

        if (uf.find(u) == uf.find(v)) {
            return true;
        }
        uf.unionSets(u, v);
    }

    return false;
}

int main() {
    std::vector<std::pair<int, int>> edges = { {0, 1}, {1, 2}, {2, 3}, {3, 0} };
    int n = 4; // Number of vertices

    if (detectCycle(edges, n)) {
        std::cout << "Cycle detected" << std::endl;
    } else {
```

```

        std::cout << "No cycle detected" << std::endl;
    }

    return 0;
}

```

---

## 7.3 Algorithms

### 7.3.1 Heavy Light Decomposition

#### *Basic HLD Impl*

```

struct Node {
    int val;
};

const Node nullNode = {0};

const int N = 2e5 + 5, S = 1 << 19;
int n, q;
int val[N];
int sz[N], par[N], dep[N], id[N], top[N];
vector<int> adj[N];

Node st[S];

Node merge(const Node& a, const Node& b) {
    return {a.val + b.val};
}

void update(int idx, Node val) {
    st[idx += n] = val;
    for (idx /= 2; idx; idx /= 2) st[idx] = merge(st[idx * 2], st[idx * 2
+ 1]);
}

Node query(int lo, int hi) {
    Node ra = nullNode, rb = nullNode;

    for (lo += n, hi += n + 1; lo < hi; lo /= 2, hi /= 2) {
        if (lo & 1) ra = merge(ra, st[lo++]);
        if (hi & 1) rb = merge(st[--hi], rb);
    }
}

```

```

    }

    return merge(ra, rb);
}

int dfs_size(const int& node, const int& parent) {
    sz[node] = 1;
    par[node] = parent;
    for (const int& ch : adj[node]) {
        if (ch == parent) continue;
        dep[ch] = dep[node] + 1;
        par[ch] = node;
        sz[node] += dfs_size(ch, node);
    }
    return sz[node];
}

int curId = 0;

void dfs_hld(const int& cur, const int& parent, const int& curTop) {
    id[cur] = curId++;
    top[cur] = curTop;
    update(id[cur], {val[cur]});
    int heavyChild = -1, heavyMax = -1;
    for (const int& ch : adj[cur]) {
        if (ch == parent) continue;
        if (sz[ch] > heavyMax) {
            heavyMax = sz[ch];
            heavyChild = ch;
        }
    }

    if (heavyChild == -1) return;
    dfs_hld(heavyChild, cur, curTop);
    for (int ch : adj[cur]) {
        if (ch == parent || ch == heavyChild) continue;

        dfs_hld(ch, cur, ch);
    }
}

Node path(int u, int v) {
    Node ans = nullNode;
}

```

```

while (top[u] != top[v]) {
    if (dep[top[u]] < dep[top[v]]) swap(u, v);
    ans = merge(ans, query(id[top[u]], id[u]));
    u = par[top[u]];
}

if (dep[u] > dep[v]) swap(u, v);
ans = merge(ans, query(id[u], id[v]));
return ans;
}

void init() {
    for (int i = 0; i < S; i++) st[i] = nullNode;
    dfs_size(1, 1);
    dfs_hld(1, 1, 1);
}

int main() {
    cin >> n >> q;
    for (int i = 1; i <= n; i++) cin >> val[i];

    int a, b;
    for (int i = 2; i <= n; i++) {
        cin >> a >> b;
        adj[a].pb(b);
        adj[b].pb(a);
    }

    init(); // <----- DON'T FORGET TO CALL THIS FUNCTION

    int type;
    while (q--) {
        cin >> type;
        if (type == 1) {
            cin >> a >> b;
            val[a] = b;
            update(id[a], {val[a]});
        }
        else {
            cin >> a;
            cout << path(1, a).val << endl;
        }
    }
}

```

## 7.3.2 Topological Sort

### *Topological Sort Using DFS*

```

//REF: USACO Guide
vector<int> top_sort;
vector<vector<int>> graph;
vector<bool> visited;

void dfs(int node) {
    for (int next : graph[node]) {
        if (!visited[next]) {
            visited[next] = true;
            dfs(next);
        }
    }
    top_sort.push_back(node);
}

int main() {
    int n, m; // The number of nodes and edges respectively
    std::cin >> n >> m;

    graph = vector<vector<int>>(n);
    for (int i = 0; i < m; i++) {
        int a, b;
        std::cin >> a >> b;
        graph[a - 1].push_back(b - 1);
    }

    visited = vector<bool>(n);
    for (int i = 0; i < n; i++) {
        if (!visited[i]) {
            visited[i] = true;
            dfs(i);
        }
    }

    std::reverse(top_sort.begin(), top_sort.end());

    vector<int> ind(n);
    for (int i = 0; i < n; i++) { ind[top_sort[i]] = i; }
}

```

```
// Check if the topological sort is valid
bool valid = true;
for (int i = 0; i < n; i++) {
    for (int j : graph[i]) {
        if (ind[j] <= ind[i]) {
            valid = false;
            goto answer;
        }
    }
}
answer;;

if (valid) {
    for (int i = 0; i < n - 1; i++) { cout << top_sort[i] + 1 << ' '; }
    cout << top_sort.back() + 1 << endl;
} else {
    cout << "IMPOSSIBLE" << endl;
}
}
```

---

## 8 Techniques

### 8.1 Coordinate Compression

```
void coordinate_compress(vector<int> &x, int start=0, int
    step=1) {
    set unique(x.begin(), x.end());
    map<int, int> valPos;

    int idx=0;
    for (auto i: unique) {
        valPos[i] = start + idx * step;
        ++idx;
    }
    for(auto &i: x) i = valPos[i];
}
```

*Coordinate Compression*

---

### 8.2 Binary to decimal

*Binary to decimal*

```
// Function to convert binary to decimal
// O(32)
int binaryToDecimal(string str)
{
    int dec_num = 0;
    int power = 0 ;
    int n = str.length() ;

    for(int i = n-1 ; i>=0 ; i--){
        if(str[i] == '1'){
            dec_num += (1<<power) ;
        }
        power++ ;
    }

    return dec_num;
}
```

---

### 8.3 Decimal to binary

*Decimal to binary*

```
// Function that convert Decimal to binary
// O(32)
void decToBinary(int n)
{
    // Size of an integer is assumed to be 32 bits
    for (int i = 31; i >= 0; i--) {
        int k = n >> i;
        if (k & 1)
            cout << "1";
        else
            cout << "0";
    }
}

// O(logn)
string DecimalToBinary(int num)
{
}
```

```
string str;
while(num){
    if(num & 1) // 1
        str+='1';
    else // 0
        str+='0';
    num>>=1; // Right Shift by 1
}
return str;
}
```

---

## 9 Number Theory

### 9.1 Divisors

#### 9.1.1 formulas

##### *number of divisors*

```
int d(int n){
    unordered_map<int, int> factors = pf(n);
    int c = 1;
    for(const auto& factor: factors){
        c *= (factor.second+1);
    }
    return c;
}
```

```
// range Count Divisors backward thinking MAXN = 2e6
for(int i=1; i <= n; ++i) {
    for(int j = i; j <= n; j += i) {
        numFactors[j]++;
    }
}
```

```
int countDivisors(int n) {
    int count = 0;
    for (int i = 1; i * i <= n; ++i) {
        if (n % i == 0) {
            if (i == n / i){
```

```
                count++; // Perfect square
            } else {
                count += 2; // Pair of divisors
            }
        }
    }
    return count;
}
```

---

##### *sum of divisors*

```
int s(int n){
    unordered_map<int,int> factors = pf(n);
    int sum = 1;
    for(const auto& factor: factors){
        int p = factor.first;
        int exp = factor.second;
        sum *= (pow(p,exp+1)-1)/p-1;
    }
    return sum;
}
```

---

### 9.2 Primes

##### *prime factorization*

```
void factorize(int x, unordered_map<int, int>& factors) {
    while (x % 2 == 0) {
        factors[2]++;
        x /= 2;
    }
    for (int i = 3; i * i <= x; i += 2) {
        while (x % i == 0) {
            factors[i]++;
            x /= i;
        }
    }
    if (x > 2) factors[x]++;
}
```

---



### *number of co-primes with n*

```
int eulerTotient(int n){
    int result = n;

    for(int i = 2; SQ(i) <= n; i++){
        if(n%i == 0){
            while(n%i == 0){
                n/=i;
            }
            result -= result/i;
        }
    }

    if(n > 1) result -= result/n;
    return result;
}

//Phi(n) = n * (1 - 1/P1) * (1 - 1/P2) * ...

//NOTE: summation of Euler function over divisors of n is equal to n
```

---

### *Prime Check*

```
vector<bool> isPrime(MAXN, true);

void sieve() {
    isPrime[0] = isPrime[1] = false;

    for (int i=2; i * i <= isPrime.size(); ++i) {
        if(isPrime[i]) {
            for (int j = 2 * i; i <= isPrime.size(); j += i)
                prime[j] = false;
        }
    }
}

bool Prime(int n) {
    if(n == 2) return true;
    if(n < 2 || n % 2 == 0) return false;

    for(int i=3; i * i <= n; i += 2) {
        if(n % i == 0) return false;
    }
}
```

```
    }
    return true;
}

// Generate Primes
const int sz = sqrt(MAXN);
vector<int> prime;
vector<bool> vis(sz);

void pre() {
    prime.push_back(2);
    for (int j = 4; j < sz; j += 2) vis[j] = true;
    for (int i = 3; i < sz; i += 2) {
        if (vis[i]) continue;
        prime.push_back(i);
        for (int j = i * i; j < sz; j += i) vis[j] = true;
    }
}

// Preprocessing Prime Factorization of range numbers
constexpr int N = 5e6+1;
int a[N];

for(int i=2; i < N; ++i) {
    if(!a[i]) {
        for(int j=1; i*j < N; ++j) {
            for(int k=i*j; k%i==0; k/=i) a[i*j]++;
        }
        a[i] += a[i-1];
    }
}
```

---

## 10 Geometry

### 10.1 Linearity

#### 10.1.1 co-linear points

*check if two points are co-linear*

```
bool co_linear(int x1, int y1, int x2, int y2, int x3, int y3){
    int area = x1*(y2-y3) + x2*(y3-y1) + x3*(y1-y2);
    return area == 0;
}
```

---

## 10.2 polygons

### 10.2.1 Polygon formation

*check if can form polygon with given angle*

```
bool possible(double angle){
    if(angle <= 0 || angle >= 180) return false;

    double sides = 360.0/(180.0-angle);

    return (sides == static_cast<int>(sides) && sides >= 3);
}
```

---

### 10.2.2 Rectangle Intersection

*intersection area between 2 rectangles*

```
struct Rectangle {
    int x1, y1; // Bottom-left corner
    int x2, y2; // Top-right corner
};

int intersectionArea(const Rectangle& rect1, const Rectangle& rect2){

    int x_left = max(rect1.x1, rect2.x1);
    int y_bottom = max(rect1.y1, rect2.y1);
    int x_right = min(rect1.x2, rect2.x2);
    int y_top = min(rect1.y2, rect2.y2);

    int intersection_width = x_right - x_left;
    int intersection_height = y_top - y_bottom;

    if (intersection_width > 0 && intersection_height > 0) {
```

```
        return intersection_width * intersection_height;
    }

    return 0;
}
```

---

## 11 Miscellaneous

### 11.1 Faster implementations

#### 11.1.1 hashes

*custom hash*

```
#define safe hash unordered_map<type, type, custom_hash> // same for
gp_hash_table
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM = chrono::steady_clock::now().
            time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
```

---

*gb hash table*

```
//policy based ds (faster hash table)
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
gp_hash_table<int, int> table;
```

---

### 11.1.2 Binary Search the value

*nearest sqrt*

```
long long my_sqrt(long long a)
{
    long long l=0,r=5000000001;
    while(r-l>1)
    {
        long long mid=(l+r)/2;
        if(1ll*mid*mid<=a)l=mid;
        else r=mid;
    }
    return l;
}
```

---