Bitmask Tricks

Operations

$$a + b = a \oplus b + 2(a\&b)$$
$$a + b = a|b + a\&b$$
$$a \oplus b = a|b - a\&b$$

Notes

1. Kth bit is set in x iff $x \mod 2^{k-1} \geq 2^k$. It comes handy when you need to look at the bits of the numbers which are pair sums or subset sums, etc.
2. Kth bit is set in x iff $x \mod 2^{k-1} - x \mod 2^k \neq 0$ ($= 2^k$ to be exact).
3. $n \mod 2^i = n\&(2^i - 1)$
4. $1 \oplus 2 \oplus 3 \oplus \cdots \oplus (4k-1) = 0$ for any $k \geq 0$

Shortcuts

1. Counting Set Bits (Hamming Weight):

```
__builtin_popcount(x)
```

2. Lowest Set Bit:

```
x & -x
```

3. Turn Off the Lowest Set Bit:

```
x & (x - 1)
```

4. Check if x is a Power of Two:

```
(x & (x - 1)) == 0
```

5. Generate All Subsets of a Set:

```
for (int mask = 0; mask < (1 << n); ++mask)
```

6. Set/Unset k-th Bit:

```
x | (1 << k) // set
```

```
x & ~ (1 << k) // unset
```

7. Flip All Bits:

```
~x
```

8. Modulo with Power of Two:

```
x % (1 << k) // x & (2^k - 1)
```

9. Gray Code:

```
gray(n) = n ^ (n >> 1)
```

10. Binary Representation of a Number:

```
std::bitset<32>(x).to_string()
```