

Adaption of the Chumbley Score to matching of bullet striation marks

Ganesh Krishnan *

Department of Statistics, Iowa State University
and

Heike Hofmann

Department of Statistics and CSAFE, Iowa State University

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Abstract

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1 Introduction and Background

1.1 Motivation

Same source analyses are a major part of an Forensic Toolmark Examiner’s job. In current practice examiners make these comparisons by visual inspection under a comparison microscope and come to one of the following four conclusions: identification, inconclusive, elimination or unsuitable for examination~(AFTE Glossary 1998). These conclusions are made on the basis of “unique surface contours” of the two toolmarks being in “sufficient agreement” (AFTE Glossary 1998). AFTE describes the term “sufficient agreement” as the possibility of another tool producing the markings under comparison, as practically impossible (AFTE Glossary 1998). Potential subject bias in the assessment as well as the lack of specified error rates are the main points of criticisms first raised by the National Research Council in 2009 (National Research Council 2009) and later emphasized further by the President’s Council of Advisors on Science and Technology (President’s Council of Advisors on Science and Technology 2016).

Technological advances, such as profilometers and confocal microscopy allow to measure 3D surfaces in a high-resolution digitized form. This technology has become more accessible over the last decade, and has made its way into topological images of ballistics evidence, such as bullet lands and breech faces (De Kinder et al. 1998, De Kinder & Bonifanti 1999, Bachrach 2002, Vorburger et al. 2016). Digitized images of 3D surfaces of form the data basis of statistical analysis of toolmarks. A statistical approach based on data removes both subjectivity from the assessment and allows a quantification of error rates for both false positive and false negative identifications.

In the next page and a half it is easy to lose the red line. It might help to include a table with an overview. The table should include the reference to the paper, the data used, the statistical method and the associated error rates. Various toolmarks have been studied in the literature: Faden et al. (2007) and Chumbley et al. (2010) have been analyzing screwdriver marks digitized using a profilometer; Bachrach et al. (2010) have investigated 3D marks from screwdriver, tongue and groove pliers captured using a confocal microscope; Grieve et al. (2014) have been investigated digitized marks from slip-joint pliers generated

by a surface profilometer.

We need an additional sentence here to get from the data to the statistical methods ...

Bachrach et al. (2010) define a relative distance metric and use it as similarity measure between two toolmarks. Faden et al. (2007) extract many small segments in the markings of two toolmarks and compare similarity using a maximum pearson correlation coefficient. The Chumbley scoring method, first introduced by Chumbley et al. (2010), uses a similar but more extensive framework based on a Mann-Whitney U test of the resulting correlation coefficients. This approach is non-deterministic, because segments are chosen randomly. (Hadler & Morris 2017) make the score deterministic for each pair of toolmarks by choosing segments for comparison systematically. This approach also ensures independence between segments of striae. In this paper, we are investigating the applicability of the Chumbley scoring method by Hadler & Morris (2017) to assess striation marks on bullet lands for same-source identification.

Striation marks on bullets are made by impurities in the barrel. As the bullet travels through the barrel, these imperfections leave “scratches” on the bullet surface. Typically, only striation marks in the land engraved areas (LEAs) are considered AFTE Criteria for Identification Committee (1992). Bullet lands are depressed areas between the grooves made by the rifling action of the barrel. Compared to toolmarks made by screwdrivers striation marks on bullets are typically much smaller, both in length and in width. Bullets also have a curved cross-sectional topography. Figure 1 shows us how the signature from a bullet land (bottom) lines up with the image of the land (top) from which it was extracted. We can also see in the figure how the depth and relative position of the striation markings seen in the image are interpreted as the signature.

Bullet matching methods are usually based on these associated signatures. Chu et al. (2013) use an automatic method for counting consecutive matching striae (CMS). The authors report an error rate of 52% of the known same source lands comparisons as misidentified (false negative) and zero false positives for known different source lands. Ma et al. (2004) and Vorburger et al. (2011) discuss CCF (cross-correlation function) and its discriminating power and applicability for same-source analyses of bullets, but do not provide any error rates in their discussion. Hare et al. (2016) use multiple features like CCF, CMS, D

(distance measure) etc in a random forest based method and compare every land against every other land of digitised versions of Hamby 252 and Hamby 44 (Hamby et al. 2009) published on the NIST Ballistics Database (Zheng 2016). The authors report an out-of-bag overall error rate of 0.46%, comprised of an error rate of 30.05% of same-source pairs that were not identified and an error rate of 0.026% of different-source pairs that were incorrectly identified as same-source.

The Chumbley score provides us with another approach in the same-source assessment of bullet striation marks. Chumbley et al. (2010) compare two toolmarks for same-source. The data for this study was obtained from 50 sequentially manufactured screwdriver tips. Chumbley et al. (2010) report error rates for markings made by the tips at different angles. For markings made at 30 degree the authors report an average false negative error rate of 0.089 and an average false positive error rate of 0.023. For other angles of 60 and 85 degrees the false negatives error rate is 0.09 while the rate of false positives decreases to 0.01. The paper by Hadler & Morris (2017) is based on the same data but the authors focus on markings made under the same angle. The error rates associated with the deterministic version of the score are 0.06 for false negatives and a false positive error rate of 0.

1.2 Scans for land engraved areas

Comparisons of striae from bullets are usually based on comparisons of striae in land engraved areas, which are extracted in form of cross sections, called *profiles* (Hare et al. 2016, Ma et al. 2004). From profiles bullet *signatures* (Chu et al. 2013, Hare et al. 2016) are extracted as residuals of a loess fit or Gaussian filter. This effectively removes topographic structure from the data in the attempt to increase the signal to noise ratio. The span of the loess fit was found using cross-validation, as described by Hare et al. (2016).

don't split the discussion on the size. between the next paragraph and There are two sources of scans for sets from the Hamby study available to us: scans of Hamby 44 and Hamby 252 are available from the NIST database (Zheng 2016). Hamby 44 has also been made available to us and has been scanned locally for CSAFE at the Roy J. Carver High Resolution Microscopy Facility using a Sensofar confocal light microscope. Scans in the NIST database are made with a NanoFocus at 20x magnification. The resolutions of the



Figure 1: Image of a bullet land from a confocal light microscope at 20 fold magnification (top) and a chart of the corresponding signature of the same land (bottom). The dotted lines connect some peaks visible in both visualizations.

two instruments are different: the NIST scans are taken at a resolution of $1.5625 \mu m$ per pixel, while the CSAFE scans are available at a resolution of $0.645 \mu m$ per pixel. The length of an average bullet land from Hamby (9 mm Ruger P85) is about 2 millimeter, resulting in signatures of about 1200 pixels for NIST scans, and about 3000 pixels for CSAFE scans. In comparison, scans from the profilometer used by Chumbley et al. (2010), Hadler & Morris (2017) were taken at a resolution of about $0.73 \mu m$ per pixel. The screw driver toolmarks are about 7 mm in length (Faden et al. 2007), for a total of over 9000 pixels for the width of these scans.

This severe limitation in the amount of available data poses the main challenge in adapting the Chumbley score to matching bullet lands, because of the resulting loss in power.

1.3 The Chumbley Score Test

The Chumbley score algorithm takes input in form of two digitized toolmarks. The toolmark is in form of $z(t)$ which is a spatial process for location indexed by t . t here denotes equally spaced pixel locations for the striation marks under consideration, $t = 1, \dots, T$. Let further $z^s(t)$ denote a vector of markings of length s starting in location t .

Let $x(t_1)$, $t_1 = 1, 2, \dots, T_1$ and $y(t_2)$, $t_2 = 1, 2, \dots, T_2$ be two digitized toolmarks (where T_1 and T_2 are not necessarily equal). The toolmarks under consideration are potentially from two different sources or the same source. T_1 and T_2 , as represented above, are the final pixel indexes of each marking and therefore give the respective lengths of the markings.

In a pre-processing step the two markings are smoothed using a lowess (Cleveland 1979) with coarseness parameter c . Originally, this smoothing is intended to remove drift and (sub)class characteristics from individual markings, however, in the setting of matching bullet striae, we can also make use of this mechanism to separate bullet curvature in profiles from signatures before matching signatures.

After removing sub-class structure, the Chumbley scores is calculated in two steps: an optimization step and a validation step. In the optimization step, the two markings are aligned horizontally such that within a pre-defined window of length w_o the correlation between $x(t_1)$ and $y(t_2)$ is maximized:

$$(t_1^o, t_2^o) = \arg \max_{1 \leq t_1 \leq T_1, 1 \leq t_2 \leq T_2} \text{cor}(x^{w_o}(t_1), y^{w_o}(t_2))$$

This results in an optimal vertical (in-phase) shift of $t_1^o - t_2^o$ for aligning the two markings. We will denote the relative optimal locations as t_1^* and t_2^* , where $t_i^* = t_i^o / (T_i - w_o)$ for $i = 1, 2$, such that $t_1^*, t_2^* \in [0, 1]$. Once (sub-)class characteristics are removed, the relative optimal locations should be distributed according to a uniform distribution in $[0, 1]$.

In the validation step, two sets of windows of size w_v are chosen from both markings (see Figure 3). In the first set, pairs of windows are extracted from the two markings using the optimal vertical shift as determined in the first step, whereas for the second set the windows are extracted using a different (out-of-phase) shift.

More precisely, let us define starting points $s_i^{(k)}$ for each signature $k = 1, 2$ as

$$s_i^{(k)} = \begin{cases} t_k^* + iw_v & \text{for } i < 0 \\ t_k^* + w_o + iw_v & \text{for } i \geq 0, \end{cases} \quad (1)$$

for integer values of i with $0 < s_i^{(k)} \leq T_k - w_v$.

Same-shift pairs of length w_v are defined in Hadler & Morris (2017) as all pairs $(s_i^{(1)}, s_i^{(2)})$ for all integer values i for which both $s_i^{(1)}$ and $s_i^{(2)}$ are defined. Similarly, different-shift pairs are defined as $(s_i^{(1)}, s_{-i-1}^{(2)})$ for all i where both $s_i^{(1)}$ and $s_{-i-1}^{(2)}$ are defined (see Figure 2).



Figure 2: Sketch of same-shift pairings (top) and different-shift pairings (bottom). Filled in rectangles show pairings resulting in correlations, unfilled rectangles are segments without a match.

For both same- and different-shift pairs correlations between the markings are calculated. The intuition here is that for two markings from the same source the correlation for the



Figure 3: Two markings made by the same source. For convenience, the markings are moved into phase on the left and out-of phase on the right. In-phase (left) and out-of-phase (right) samples are shown by the light grey background. The Chumbley-score is based on a Mann-Whitney U test of the correlations derived from these two sets of samples.

in-phase sample should be high, while the correlations of the out-of-phase sample provide a measure for the base-level correlation for non-matching marks of a given length w_v . The Chumbley score is then computed as a Mann Whitney U statistic to compare between in-phase sample and out-of-phase sample. In the original method proposed in Chumbley et al. (2010) both in-phase and out-of-phase sample are extracted randomly, whereas Hadler & Morris (2017) proposed the above specified deterministic rules for both samples to make the resulting score deterministic while simultaneously avoiding overlaps within selected marks to ensure independence.

1.4 A problem with failures and modification

Looking closer at Equation 1, we see that by definition, some number of tests will fail to produce a result, either because the number of eligible same-shift pairs is zero, or the number of different-shift opairs is zero.

The number of same-shift pairs will be zero, if the optimal locations t_1^o and t_2^o are so far apart, that no segments of size w_v are left on the same sides of the optimal locations, i.e.

$t_1^o < w_v$ and $t_2^o > T_2 - w_o - w_v$ or $t_1^o < T_1 - w_o - w_v$ and $t_2^o < w_v$. While we can assume that once (sub-)class characteristics are removed, optimal locations t_i^o are uniformly distributed across the length of the profile, we cannot assume that t_1^o and t_2^o are independent of each other. In particular, for same-source profiles, we would expect a strong dependency between these locations. However, for different source matches, an argument of independence in the locations could be made. In that case, we expect a test to fail with a probability of $\frac{2w_v^2}{(T_1 - w_o)(T_2 - w_o)}$. For an average length of T_i of 1200 pixels, $w_o = 120$ pixels and $w_v = 30$ pixels this probability is about 0.0015.

The number of possible different-shift pairs also depends on the location of the optimal locations t_1^o and t_2^o . Whenever the optimal locations are close to the boundaries, the number of possible pairings decreases and reaches zero, if $t_i^{(o)} < w_v$ or $t_i^{(o)} > T_i - w_o - w_v$. Assuming a correlation between optimal locations t_1^o and t_2^o of close to one for same-source profiles, this results in an expected rate of failure of $2w_v/(T_i - w_o)$, or about 5.6% for an average length of T_i of 1200 pixels, $w_o = 120$ pixels and $w_v = 30$ pixels. Assuming independence in the optimal locations for different source profiles the expected probability for a failed test is, again, $\frac{2w_v^2}{(T_1 - w_o)(T_2 - w_o)}$.

While failures due to missing correlations from same-shift pairs are unavoidable by definition of the Chumbley score, failures due to missing correlations from different-shift pairs can be prevented by using a different strategy in assigning pairs.

Using the same notation as in Equation 1, we define same-shift pairs identical to Hadler & Morris (2017) as pairs $(s_i^{(1)}, s_i^{(2)})$ for all i where the boundary conditions of both sequences are met simultaneously. Let us assume that this results in I pairs. Define $s(j)^{(k)}$ to be the j th starting location in sequence $k = 1, 2$, i.e. $s(1)^{(k)} < s(2)^{(k)} < \dots < s(I)^{(k)}$.

We then define the pairs for different-shifts as

$$\left(s_{(j)}^{(1)}, s_{(I-j+1)}^{(1)}\right) \text{ for } j = \begin{cases} 1, \dots, I & \text{for even } I \\ 1, \dots, (I-1)/2, (I-1)/2 + 2, \dots, I & \text{for odd } I \end{cases}, \quad (2)$$

i.e. for an odd number of same-shift correlations, we skip the middle pair for the different-shift correlations (see Figure 4). This pairing ensures that the number of different-shift pairings is the same or at most one less than the number of same-shift pairings in all tests.

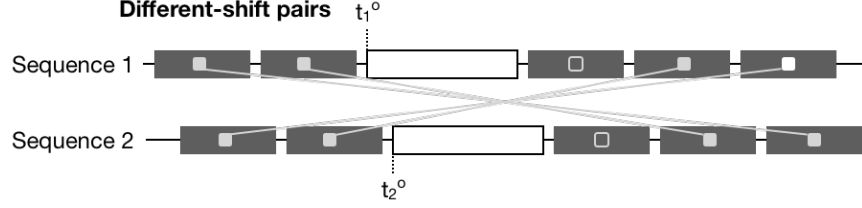


Figure 4: Sketch of adjusted different-shift pairings. At most one of the same-shift pairings can not be matched with a different-shift pair.

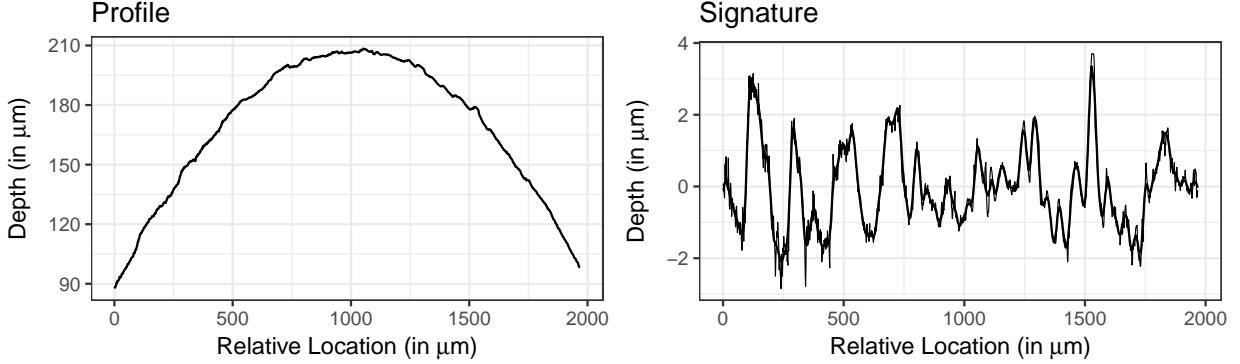


Figure 5: Bullet land profile (left) and the corresponding signature (right) for one of the lands of Hamby-44.

2 Testing setup

2.1 The Data

Introduce profiles and signatures, as shown in figure 5.

Lands for all Hamby-44 and Hamby-252 scans are made available through the NIST ballistics database (Zheng 2016) and are considered, here. Both of these sets of scans are part of the larger Hamby study (Hamby et al. 2009). Each set consists of twenty known bullets (two each from ten consecutively rifled Ruger P85 barrels) and fifteen questioned bullets (each matching one of the ten barrels). Ground truth for both of these Hamby sets is known and was used to assess correctness of the tests results.

Discuss CSAFE scans

Profiles and signatures were extracted from all scans as described in Hare et al. (2016).

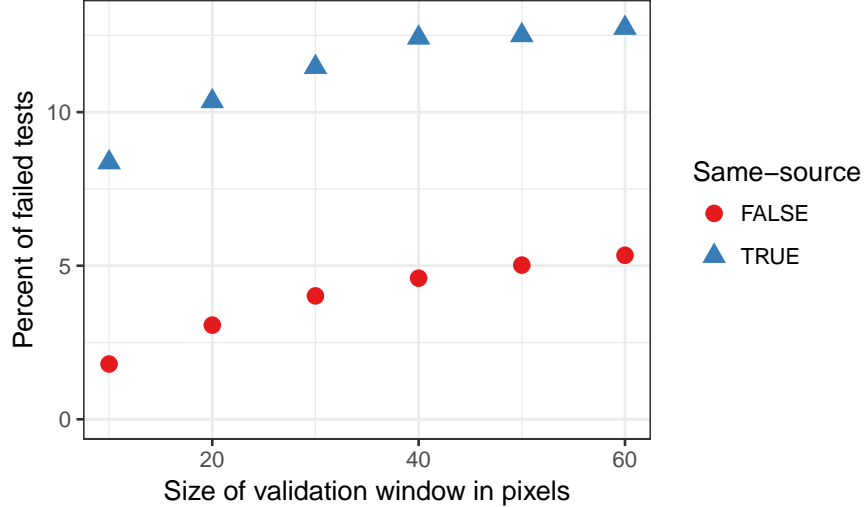


Figure 6: Percent of failed land-to-land comparisons using an optimization window $w_o = 120$ and a coarseness of $c = 0.25$. With an increase in the size of the validation window a higher percentage of tests fails. The percentage of failures is higher for same-source lands than for different source lands.

2.2 Setup

XXX the description below needs to be updated.

using (a) signatures and (b) profiles, run chumbly score across scans from NIST and CSAFE for various settings of w_o and w_v (and coarseness c for profiles).

We used the adjusted Chumbly method as proposed in Hadler & Morris (2017) and implemented in the R package `toolmaRk` (Hadler 2017) on all pairwise land-to-land comparisons of the Hamby scans provided by NIST for a total of 85,491 land-to-land comparisons.

2.3 Failed Tests

Initially, the default settings suggested in Hadler & Morris (2017) were used: $w_o = 120$ pixels or about $190 \mu m$ (ten percent of the average length of profiles) and coarseness $c = 0.25$. Figure 6 shows the percentage of failed tests among the 85,491 land-to-land comparisons of the NIST data for different values of the validation window size w_v . For same-source lands up to 12.5 percent of the tests fail.

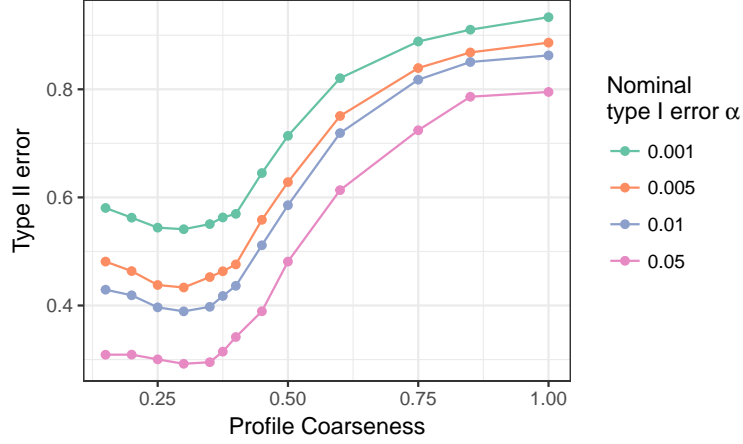


Figure 7: Type II error with respect to coarseness parameter over profiles, $w_o = 120$, $w_v = 30$. Optimal values for coarseness are around $c \approx 0.30$

3 A modified test

fix the algorithm - all of the other properties in Hadler and morris still hold.
number of failed tests down.

4 Comparison of Results

4.1 Profiles

When dealing with profiles, coarseness is an additional parameter that has to be considered in the matching.

Figure 7 shows the type II error rates for profiles using an optimization window $w_o = 120$ and a validation window $w_v = 30$ for varying level of coarseness. The type II error for all nominal levels of α are the lowest for a coarseness range of 0.20 to 0.30, with a minim in type II reached at a coarseness $c = 0.25$. However, for an optimal value of coarseness other aspects have to be considered.

Figure 8 gives an overview of the effect of different coarseness parameters: from left to right, coarseness levels c are 0.2, 0.25, and 0.3, respectively. The top row shows the resulting signature after smoothing the profile shown in Figure 5 with the coarseness specified. The histograms in the bottom row show the relative optimal location t^o (optimal location t^o

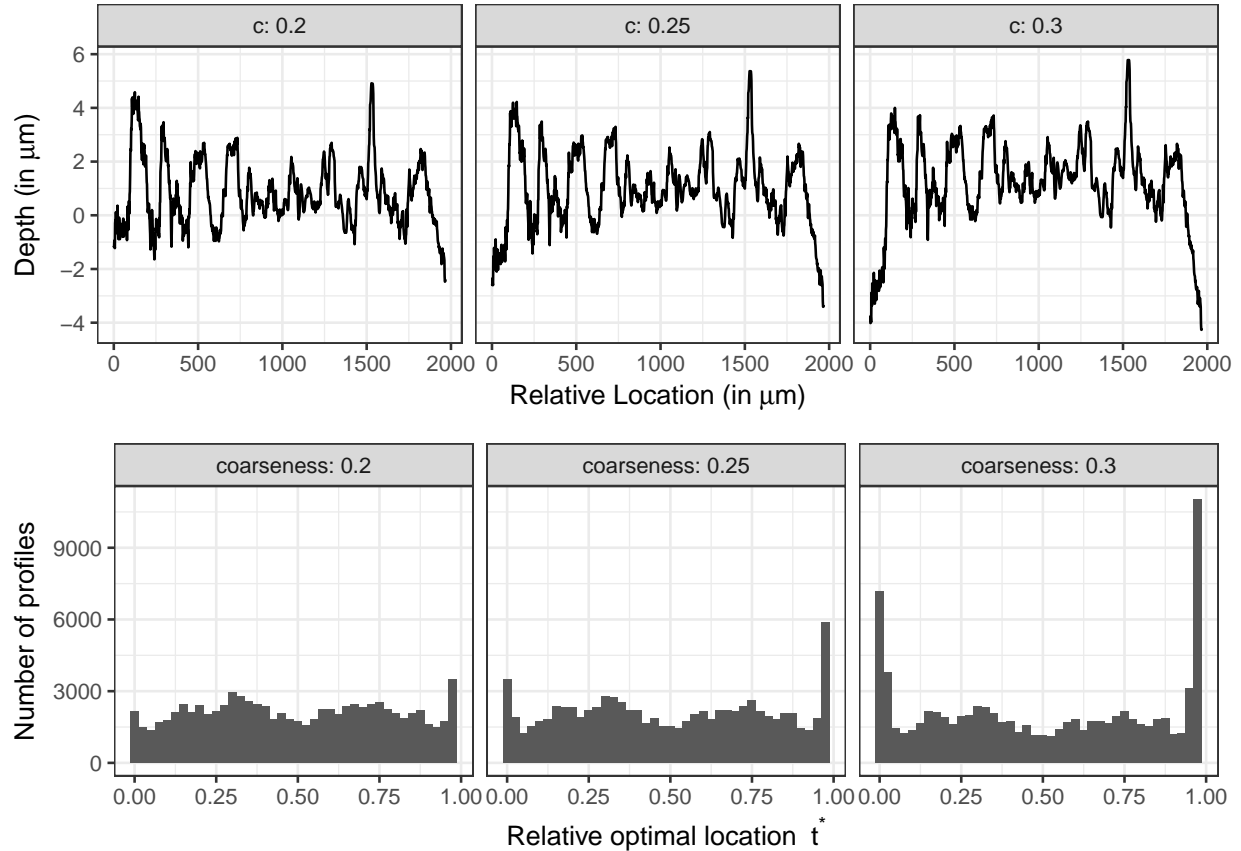


Figure 8: Overview of the effect of different coarseness parameters c on the profile shown in Figure 5 (top). The bottom row shows histograms of the (relative) optimal locations t^o identified in the optimization step for different values of the coarseness parameter c .

divided by the length of the corresponding profile). Optimal locations should be distributed uniformly. However, for coarseness values of $c = 0.25$ and $c = 0.30$ we see very distinct boundary effects: optimal locations t^o are found at the very extreme ends of a profile more often than one would expect based on a uniform location.

4.2 Results

For signatures from NIST scans we see three problems:

1. type-2 error rate is at best 30% for a type-1 error rate of 5%, which is well above the error rates we see for tool marks from screw drivers, see figure 9;
2. the observed type-1 error, which generally close to the nominal type-1 error rate,

depends on the size of the optimization window: as the window size increases, the observed type-1 error decreases, see figure 10;

3. the Chumbley-score fails to provide a result for up to 3% of the cases. The number of failed tests increases linearly in the size used for the window in the optimization step. The rate of failed tests is considerably higher when the two lands are from same source than when the lands are from different sources, see figure 11.

Figure 11 gives an overview of the number of failed tests, i.e. tests in which a particular parameter setting did not return a valid result. This happens e.g. when the shift to align two markings is so large, that the remaining overlap is too small to accommodate windows for validation. The problem is therefore exacerbated by a larger validation window. Figure 11(left) also shows that the number of failed tests is approximately linear in the size of the optimization window. Tests also fail at a higher rate than expected when the markings are from the same source (right). This difference is the least pronounced around an optimized window size w_o of around 120. However, even in this scenario, the number of failed tests for markings from the same source is about twice as high as expected given the number of same source and different source pairings in the data set.

Table 1: Type II error rates for profiles and signatures of bullet lands. For profiles, a coarseness value of $c = 0.25$ is used to remove bullet curvature.

validation		Nominal type I error rate α			
window w_v	source	0.001	0.005	0.01	0.05
30	profiles	54.40	43.80	39.70	30.00
30	signatures	55.00	45.40	41.40	31.10
50	profiles	58.50	44.40	40.70	28.70
50	signatures	62.60	49.60	44.20	30.50

5 Conclusion

The results suggest that the Nominal type I error α value shows dependence on the size of the window of optimization. For a given window of optimization the actual Type I error

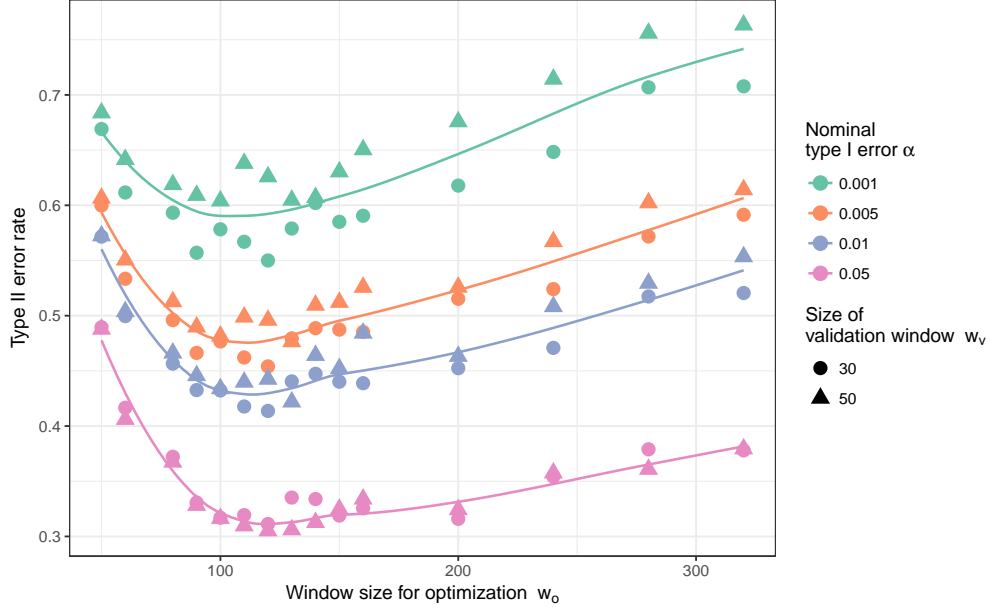


Figure 9: Type II error rates observed across a range of window sizes for optimization w_o . For a window size of $w_o = 120$ we see a drop in type II error rate across all type I rates considered. Smaller validation sizes w_v are typically associated with a smaller type II error.

is comparable to the nominal level for only a select few validation window sizes and for comparable validation window sizes of 30 and 50 as done here, the actual type I error does not seem to vary as much as it varies with the optimization window sizes. A Test Fail, i.e. tests in which a particular parameter setting did not return a valid result, happens, when the shift to align two signatures is so large, that the remaining overlap is too small to accommodate windows for validation, depends on whether known-match or known non-matches has predictive value, with test results from different sources having a much higher chance to fail. On conducting an analysis of all known bullet lands using the adjusted chumbley algorithm, Type II error was identified to be least bad for window of validation 30 and window of optimization 120. In case of unsmoothed raw marks (profiles), Type II error increases with the amount of smoothing and least for LOWESS smoothing coarseness value about 0.25 or 0.3. In an effort to identify the level of adaptiveness of the algorithm, comparisons were made between signatures and profiles. Their comparison with respect to validation window size for a fixed optimization window size suggested that, profiles have a total error (i.e all incorrect classification of known-matches and known non-matches) greater

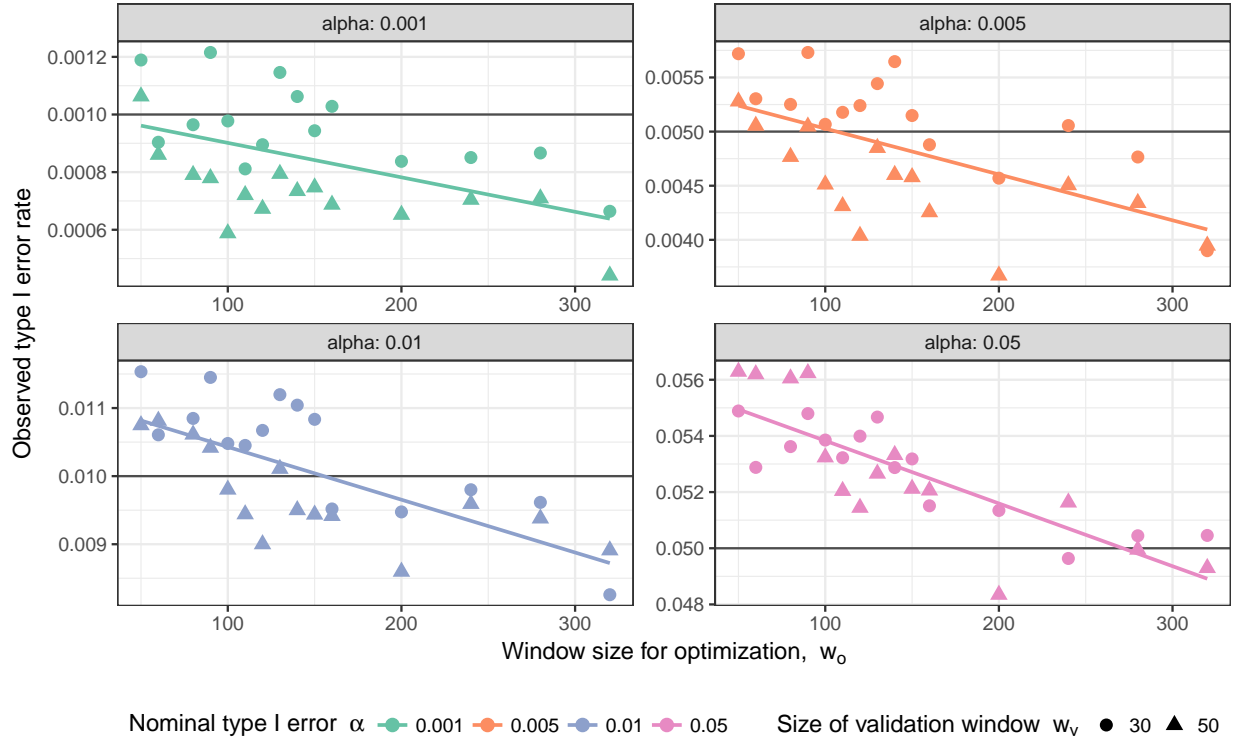


Figure 10: Comparison of observed and nominal type I error rates across a range of window sizes for optimization w_o . The horizontal line in each facet indicates the nominal type I error rate.

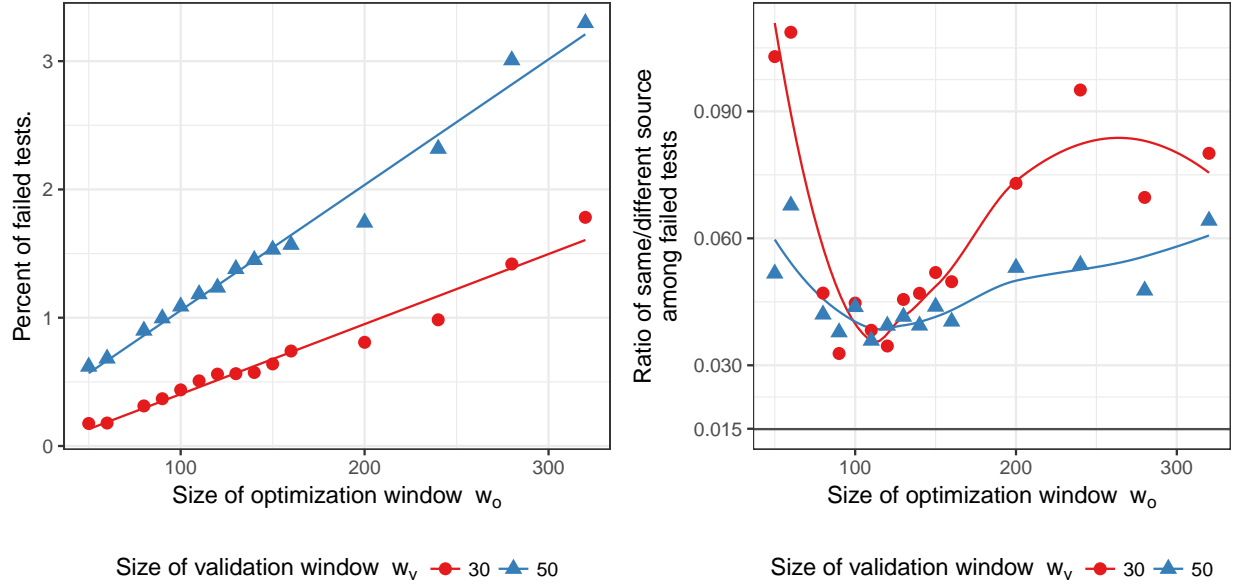


Figure 11: The number of failed tests increases with an increase in the size of the optimization window (left). Unfortunately there is also a dependency between failed tests and ground truth. The plot on the right shows the ratio of the number of land pairs from same sources and different sources for failed tests. For small optimization windows and large windows the number of failed tests for same-source land-to-land comparisons is increasing. Even in the minimum, same-source land-to-land comparisons fail at twice the rate that they are expected to based on the ratio of the number of known matches and known non-matches (horizontal line).

than or equal to the total error of signatures for all sizes of validation window. Profiles also fail more number of times than signatures in a test fail (for different coarseness keeping windows fixed and also for different validation windows keeping coarseness fixed) which lets us conclude that the behaviour of the algorithm for the profiles instead of pre-processed signatures is not better. Finally it should be noted that the current version of the adjusted chumbley algorithm seems to fall short when compared to other machine-learning based methods Hare et al. (2016), and some level of modification to the deterministic algorithm needs to be identified and tested that would reduce the number of incorrect classifications.

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