Adapting the Chumbley Score to Bullet Striations

Ganesh Krishnan, Heike Hofmann

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Objective and Motivation

- Same Source Matching of Bullet lands
- Evaluate performance of Chumbley Score method when used for Bullet Striations
- Bullet striations have curvature, not present in toolmarks
- Identify Error rates and effect of different parameters on them (In short finding the best error rates possible)

Structure

- Data being used
- What is the Chumbley Score Method?
- Identifying Best parameter Settings for Bullets
- Modifications to the Algorithm
- Results using Hadler and Morris [2017] method and results using Modified method

Data

- Ruger P85s Bullet Lands, or Hamby scans (Hamby et al. [2009]) provided by NIST (85,491 comparisons)
- Bullet striation marks ≈ 2mm | Screwdriver marks ≈ 7mm (all chumbley score papers)

Getting from Profiles to Signatures

- Lowess smoothing or Loess fit residuals = Signatures
- Removes topographic structure (curvature)
- Improve the signal to noise Ratio

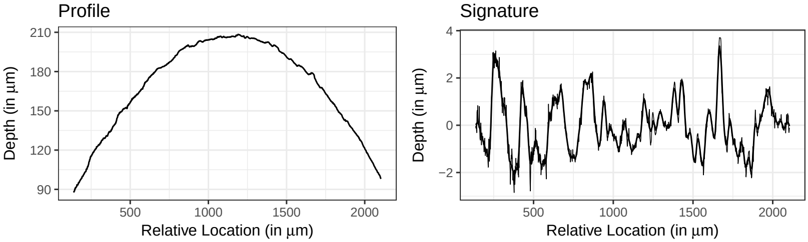


Figure 1: Bullet land profile (left) and the corresponding signature (right).

Error Rates for toolmarks

Research paper	Method	Data Source	False Positives	False Negatives
Faden et al. [2007]	Maximized Correlation	Screwdrivers	-	-
Chumbley et al. [2010] (Same-Surface Same-Angle)	Randomized Chumbley Score	Screwdrivers	2.3%	8.9%
Grieve et al. [2014]	Randomized Chumbley Score	Slip-joint	-	-
Hadler and Morris [2017] (Same-Surface Same-Angle)	Deterministic Chumbley Score	Screwdrivers	0%	6%
Bachrach et al. [2010]	Similarity Measure	Screwdrivers		
(Different Surfaces-same angle)	Relative Distance Metric		5.9%	9.4%
(Same Surfaces-same angle)			0.22%	0%

Table 1: Error Rates for Toolmarks

Digitized Striation Marks

- Let $x(t_1)$, $t_1 = 1, 2, \ldots T_1$ and $y(t_2)$, $t_2 = 1, 2 \ldots T_2$ be two digitized marks (where T_1 and T_2 are not necessarily equal).
- T_1 and T_2 are the final pixel indexes of each marking. Therefore give the respective lengths of the markings.
- Signatures/ Profiles (NIST- Hamby) $\approx$ 1200 pixels (2 mm) Screwdriver toolmarks (Chumbley Papers) $\approx$ 9000 pixels (7 mm)
- Markings are smoothed as a preprocessing step
  - Used to remove drift and (sub)class characteristics from individual markings

Overview

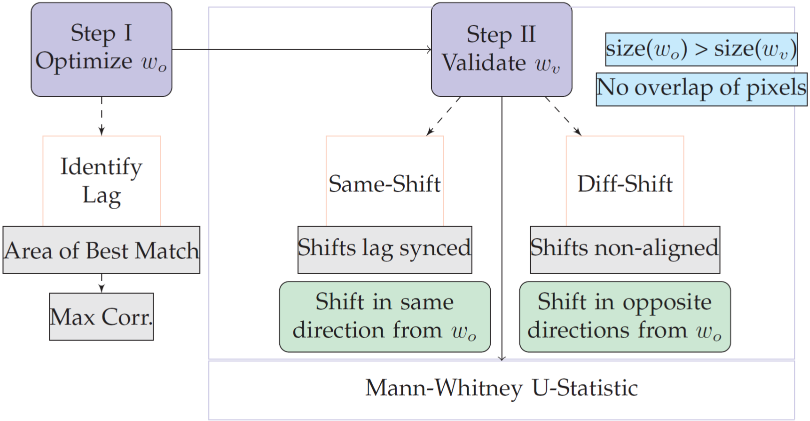


Figure 2: An overview of the chumbley score method

Chumbley Score

- Two steps: Optimization (1^{st}) and Validation (2^{nd}).
- Windows $\implies$ short segments of the markings
  - Have *predefined sizes*. (T_1 or $T_2 >>> w_o >> w_v$)
    1. w_o used in the Optimization step
    2. w_v used in the Validation step

Optimization step

- Aligns markings horizontally
- Correlation of all possible windows of size w_o between $x(t_1)$ and $y(t_2)$ computed
- *Possible lag in the 2 markings*
  Window Pair with maximized correlation $\implies$
  Optimal vertical (in-phase) shift of $t_1^o - t_2^o$
  - For aligning the two markings.

$$(t_1^o, t_2^o) = \underset{1 \le t_1 \le T_1, 1 \le t_2 \le T_2}{\arg\max} \operatorname{cor}\left(x^{w_o}(t_1), y^{w_o}(t_2)\right)$$

where t_1^o, t_2^0 are the respective starting points of w_o in $x(t_1)$ and $y(t_2)$

- Let t_1^* and t_2^* be relative optimal locations, where $t_i^* = t_i^o/(T_i - w_o)$ for $i = 1, 2$, such that $t_1^*, t_2^* \in [0, 1]$.
- Once (sub-)class characteristics are removed, these locations have uniform distribution in $[0, 1]$

Validations Step

- Two sets of windows of size w_v chosen from both markings (see Figure 3)
- First set or **Same Shift**
  - pairs of windows are extracted from the two markings using the optimal vertical shift. $t_1^o - t_2^o$
- Second set or **Different Shift**
  - the windows are extracted using a different (out-of-phase) shift.

In-phase and Out-of-phase

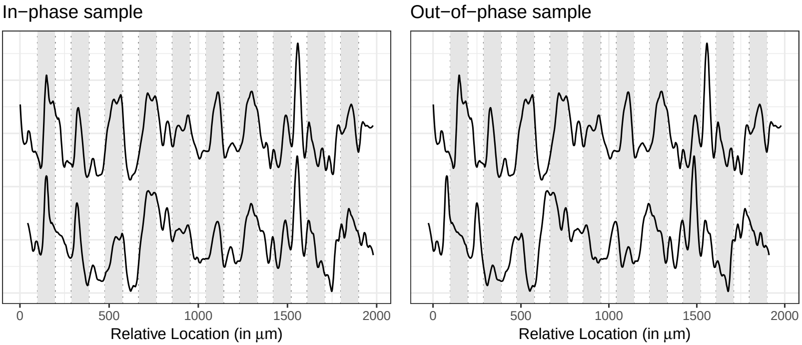


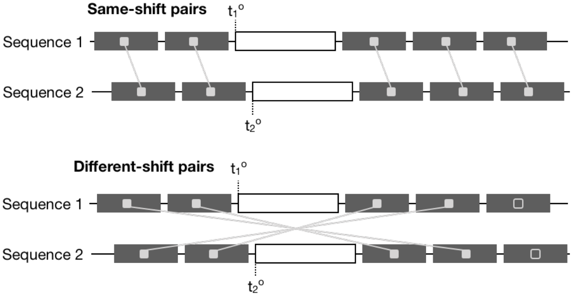
Figure 3: Two markings made by the same source. For convenience, the markings are moved into phase on the left and out-of phase on the right. In-phase (left) and out-of-phase (right) samples are shown by the light grey background. The Chumbley-score is based on a Mann-Whitney U test of the correlations derived from these two sets of samples.

More precisely, let us define starting points of the windows of validation $s_i^{(k)}$ for each signature $k = 1, 2$ as

$$s_i^{(k)} = \begin{cases} t_k^* + iw_v^* & \text{for } i < 0 \\ t_k^* + w_o^* + iw_v^* & \text{for } i \geq 0, \end{cases} \tag{1}$$

for integer values of i with $0 < s_i^{(k)} \leq 1 - w_v^*$

- Same-shift pairs of length w_v are all pairs
$$(s_i^{(1)}, s_i^{(2)}) \quad \forall\, i \in \mathbb{Z}$$
for which both $s_i^{(1)}$ and $s_i^{(2)}$ are defined.
- Different-shift pairs are defined as
$$(s_i^{(1)}, s_{-i-1}^{(2)}) \quad \forall\, i \in \mathbb{Z}$$
where both $s_i^{(1)}$ and $s_{-i-1}^{(2)}$ are defined (see fig. 4).



- No overlaps of pairs within selected marks $\Longrightarrow$ independence.
- Both same- and different-shift pairs correlations between the markings are calculated.
- For Same-Source markings correlations
  - for the in-phase shift should be high
  - for out-of-phase shift should be low.
    - Provide a measure for the base-level correlation to which in-phase shift correlations can be compared.
- The Chumbley score is the Mann Whitney U statistic computed by comparing between in-phase sample and out-of-phase sample.

Failed Tests

- By definition (equation 1), some number of tests fail to produce a result
- Either because the number of eligible same-shift pairs is 0, or the number of different-shift pairs is 0.

No Same-shift pairs if

- the optimal locations t_1^o and t_2^o are in the extremes
  - no segments of size w_v are left on the same sides of the optimal locations,
  - For all $t_1^o < w_v$ and $t_2^o > T_2 - w_o - w_v$ or $t_1^o < T_1 - w_o - w_v$ and $t_2^o < w_v$

No Different Shift Pairs if

- the optimal locations are close to the boundaries
  - $\#$ of different-shift pairs also depends on the placement of the optimal locations t_i^o.
  - For all $t_i^{(o)} < w_v$ or $t_i^{(o)} > T_i - w_o - w_v$.

Failure Rate

$$P\left(t_1^o < w_v \bigcap t_2^o > T_2 - w_o - w_v\right) +$$
$$P\left(t_1^o < T_1 - w_o - w_v \bigcap t_2^o < w_v\right).$$

Same-shift failure

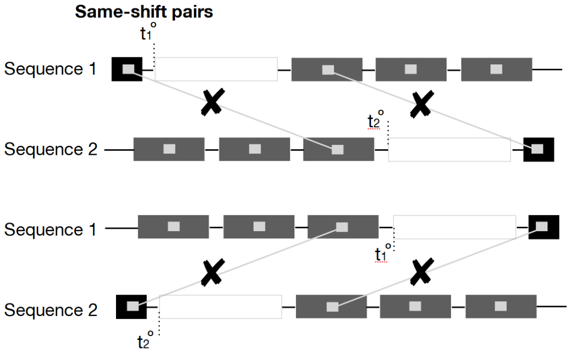


Figure 5: Sketch of same-shift pairings (top) when the lag is too large to accomodate a a vaildation window in either of the two signatures

- For different-source matches
  - we can assume t_1^o and t_2^o are independent.
  - Failure rate $= \frac{2w_v^2}{(T_1 - w_o)(T_2 - w_o)}$
  - For an average length of T_i of 1200 pixels, $w_o = 120$ pixels and $w_v = 30$ pixels this probability is about 0.0015.
- For same-source matches,
  - we expect a strong dependency between optimal locations t_i^o
  - t_1^o and t_2^o are not independent of each other
  - A large difference between locations is unlikely
  - Failure Rate $= 2w_v/(T_i - w_o)$, or about 5.6% for an average length of T_i of 1200 pixels, $w_o = 120$ pixels and $w_v = 30$ pixels.

Failure rate for same-source matches

The failure rate for same-source matches can also be written as follows

$$P\left(t_1^o < w_v \bigcup t_2^o > T_2 - w_o - w_v\right) +$$
$$P\left(t_1^o < T_1 - w_o - w_v \bigcup t_2^o < w_v\right)$$

Taking first part of the above equation, let $A = t_1^o < w_v$ and $B = t_2^o > T_2 - w_o - w_v$

- Again we assume high correlation between optimal locations
- In order to have at least one same-shift pair or different shift pair
  - **Either**, if $x \in A$, then B is empty and $P(A \cup B) = P(A)$
  - **or**, if $x \in B$, then A is empty and $P(A \cup B) = P(B)$

This would hold for the second part too, and would therefore give an expected rate of failure as $2w_v/(T_i - w_o)$

Proposed Modification

- Failures due to missing Same-shift pairs unavoidable
- Failures due to missing different-shift pairs preventable

Define same-shift pairs identical to Hadler and Morris [2017] as pairs

$$(s_i^{(1)}, s_i^{(2)}) \quad \forall\, i \in \mathbb{Z}$$

where the boundary conditions of both sequences are met simultaneously.

- Let us assume that this results in I pairs.
- Let $s_{(j)}^{(k)}$ to be the jth starting location in sequence $k = 1, 2$, i.e. $s_{(1)}^{(k)} < s_{(2)}^{(k)} < ... < s_{(I)}^{(k)}$.

We then define the pairs for different-shifts as

$$\left(s_{(j)}^{(1)}, s_{(I-j+1)}^{(1)}\right) \text{ for } j = \begin{cases} 1, ..., I & \text{for even } I \\ 1, ..., (I-1)/2, (I-1)/2+2, ..., I & \text{for odd } I \end{cases} \tag{2}$$

- For an odd number of same-shift correlations
  - We skip the middle pair for the different-shift correlations (see fig. 6).
- This pairing ensures that the number of different-shift pairings is the same or at most one less than the number of same-shift pairings in all tests.

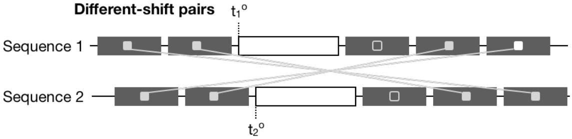


Figure 6: Sketch of adjusted different-shift pairings. At most one of the same-shift pairings can not be matched with a different-shift pair.

Case where CS1 fails but CS2 does not fail

CS1 Hadler and Morris [2017] algorithm
CS2 the suggested modified algorithm

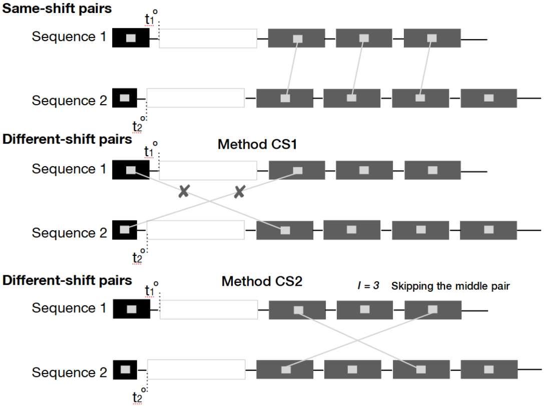


Figure 7: Sketch of a case where CS1 fails but CS2 does not fail

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