# Adapting the Chumbley Score to Bullet Striations

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April 21, 2018

# Objective and Motivation

- Same Source Matching of Bullet lands
- Evaluate performance of Chumbley Score method when used for Bullet Striations
- Bullet striations have curvature, not present in toolmarks
- Identify Error rates and effect of different parameters on them (In short finding the best error rates possible )

## Structure

- Error rates in toolmarks
- Data being used
- What is the Chumbley Score Method?
- Identifying Best parameter Settings for Bullets
- Modifications to the Algorithm
- Results

# Variations of Chumbley score method and Error Rates for toolmarks

| Research paper | Method | Data Source | False Positives | False Negatives |
|---|---|---|---|---|
| **Faden et al. [2007]** | Maximized Correlation | Screwdrivers | - | - |
| **Chumbley et al. [2010]** (Same-Surface Same-Angle) | Randomized Chumbley Score | Screwdrivers | 2.3% | 8.9% |
| **Grieve et al. [2014]** | Randomized Chumbley Score | Slip-joint | - | - |
| **Hadler and Morris [2017]** (Same-Surface Same-Angle) | Deterministic Chumbley Score | Screwdrivers | 0% | 6% |

Table 1: Error Rates for Toolmarks using variations of the chumbley score method

# Digitized Striation Marks

- Data
  - Ruger P85s Bullet Lands, or Hamby scans (Hamby et al. [2009]) provided by NIST (85,491 comparisons)
  - Bullet striation marks $\approx$ 2mm
  - Screwdriver marks $\approx$ 7mm (all chumbley score papers)
- Let $x(t_1)$, $t_1 = 1, 2, ... T_1$ and $y(t_2)$, $t_2 = 1, 2 ... T_2$ be two digitized marks (where $T_1$ and $T_2$ are not necessarily equal).
- $T_1$ and $T_2$ are the final pixel indexes of each marking. Therefore give the respective lengths of the markings.
- Signatures/ Profiles (NIST- Hamby) $\approx$ 1200 pixels (2 mm) Screwdriver toolmarks (Chumbley Papers) $\approx$ 9000 pixels (7 mm)

# Chumbley Score

## Step 0 : Defining a coarseness parameter

- Used to remove drift and (sub)class characteristics from individual markings
- Lowess or Loess fit residuals = Signatures
- Removes topographic structure (curvature)
- Improve the signal to noise Ratio

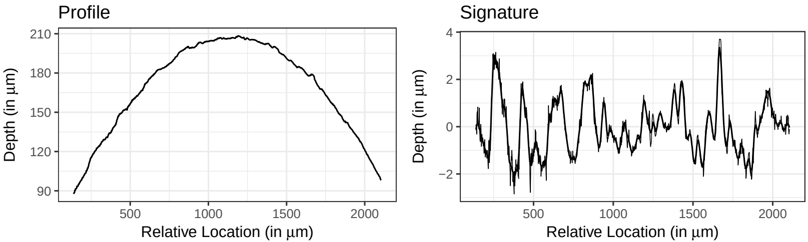



Figure 1: Bullet land profile (left) and the corresponding signature (right).

# Algorithm

- Two steps: Optimization ($1^{st}$) and Validation ($2^{nd}$).
- Windows $\implies$ short segments of the markings
  - Have *predefined sizes*. ($T_1$ or $T_2 >>> w_o$ & w_v$)
    1. $w_o$ used in the Optimization step
    2. $w_v$ used in the Validation step

## Optimization step

- **Goal :**Align markings horizontally as best as possible
- Correlation Matrix of all possible windows of size $w_o$ between $x(t_1)$ and $y(t_2)$ computed
- *Identify lag for horizontal alignment*
  Window Pair with maximized correlation $\implies$
  Optimal vertical (in-phase) shift of $t_1^o - t_2^o$
  - For aligning the two markings.

$$(t_1^o, t_2^o) = \underset{1 \leq t_1 \leq T_1, 1 \leq t_2 \leq T_2}{\arg\max} \operatorname{cor}\left(x^{w_o}(t_1), y^{w_o}(t_2)\right)$$

where $t_1^o, t_2^0$ are the respective starting points of $w_o$ in $x(t_1)$ and $y(t_2)$

- Let $t_1^*$ and $t_2^*$ be relative optimal locations, where $t_i^* = t_i^o/(T_i - w_o)$ for $i = 1, 2$, such that $t_1^*, t_2^* \in [0, 1]$.
- Once (sub-)class characteristics are removed, these locations have uniform distribution in $[0, 1]$

## Validation Step

- Two sets of windows of size $w_v$ chosen from both markings (see Figure 2)
- First set or **Same Shift**
  - pairs of windows are extracted from the two markings using the optimal vertical shift. $t_1^o - t_2^o$
- Second set or **Different Shift**
  - the windows are extracted using a different (out-of-phase) shift.

# In-phase and Out-of-phase

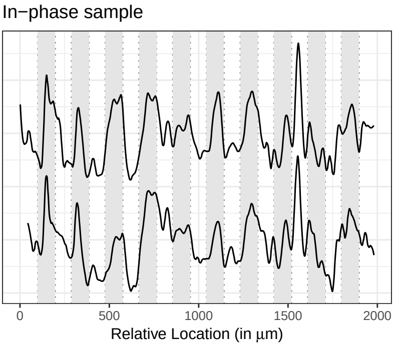



Figure 2: Two markings made by the same source. For convenience, the markings are moved into phase on the left and out-of phase on the right. In-phase (left) and out-of-phase (right) samples are shown by the light grey background. The Chumbley-score is based on a Mann-Whitney U test of the correlations derived from these two sets of samples.

- Both same- and different-shift pairs correlations between the markings are calculated.
- For Same-Source markings, correlations
  - for the in-phase shift should be high
  - for out-of-phase shift should be low.
    - Provide a measure for the base-level correlation to which in-phase shift correlations can be compared.
- The Chumbley score is the Mann Whitney U statistic computed by comparing between in-phase sample and out-of-phase sample.

# Block Diagram

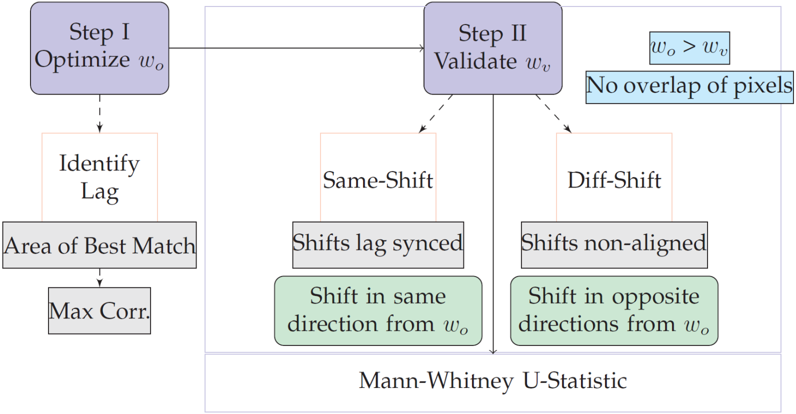



Figure 3: An overview of the adjusted chumbley score method as given by Hadler and Morris [2017]

# Starting Points

More precisely, let us define starting points of the windows of validation $s_i^{(k)}$ for each marking $k = 1, 2$ as

$$s_i^{(k)} = \begin{cases} t_k^o + iw_v & \text{for } i < 0 \\ t_k^o + w_o + iw_v & \text{for } i \geq 0, \end{cases} \tag{1}$$

for integer values of $i$ with $0 < s_i^{(k)} \leq T_k - w_v$ where $s \in \mathbb{Z}$

# The Hadler and Morris [2017] method (CS1)

- Same-shift pairs of length $w_v$ are all pairs that start in:

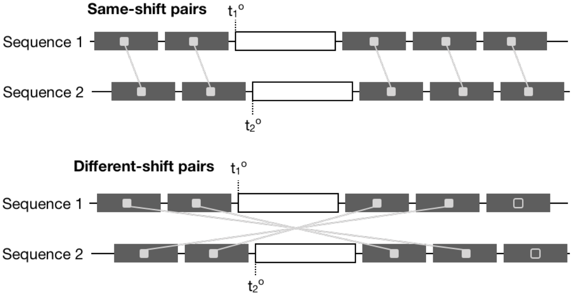
$$(s_i^{(1)}, s_i^{(2)}) \quad \forall\, i \in \mathbb{Z}$$

for which both $s_i^{(1)}$ and $s_i^{(2)}$ are defined.

- Different-shift pairs are defined as

$$(s_i^{(1)}, s_{-i-1}^{(2)}) \quad \forall\, i \in \mathbb{Z}$$

where both $s_i^{(1)}$ and $s_{-i-1}^{(2)}$ are defined (see fig. 4).



# Failed Tests

- By definition (equation 1), some number of tests fail to produce a result
- Either because the number of eligible same-shift pairs is 0, or the number of different-shift pairs is 0.
- $t_1^o, t_2^o$ not necessarily independent
  - **same-source:** Assume high dependence, $\text{corr}(t_1^o, t_2^o) \approx 1$
    - Example: $w_o = 120$, coarseness (c) $= 0.3$, $\text{corr}(t_1^o, t_2^o) = 0.85$
  - **diff-source:** Assume independence of $t_1^o, t_2^o$
    - Example: $w_o = 120$, coarseness (c) $= 0.3$, $\text{corr}(t_1^o, t_2^o) = 0.12$

## Failure Rate

$$P\left(t_1^o < w_v \bigcap t_2^o > T_2 - w_o - w_v\right) +$$
$$P\left(t_1^o < T_1 - w_o - w_v \bigcap t_2^o < w_v\right).$$

# Same-shift failure

- Same-source $\approx 0$
- Different-source $\approx 2\ P(t_i < w_o)^2 = \frac{2w_v^2}{(T_1 - w_o)(T_2 - w_o)}$

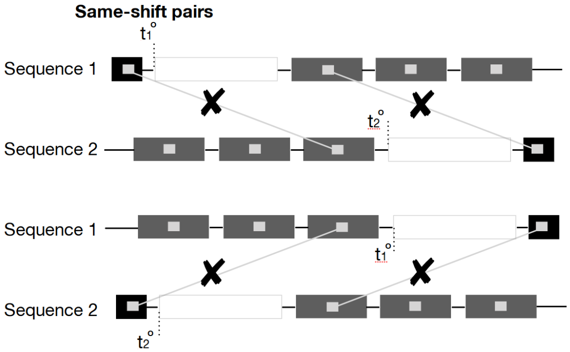



Figure 5: Sketch of same-shift pairings (top) when the lag is too large to accomodate a a vaildation window in either of the two signatures

# Different-Shift Failure

- Same-source (Assuming $t_1^o \approx t_2^o$) $\approx 2w_v/(T_i - w_o)$

$$P(t_1^o < w_v \cap t_2^o < w_v) + P(t_1^o < w_v \cap t_2^o < w_v) = 2P(t_o^1 < w_v)$$

- Different-source $\approx 2P(t_i < w_o)^2 = \frac{2w_v^2}{(T_1 - w_o)(T_2 - w_o)}$

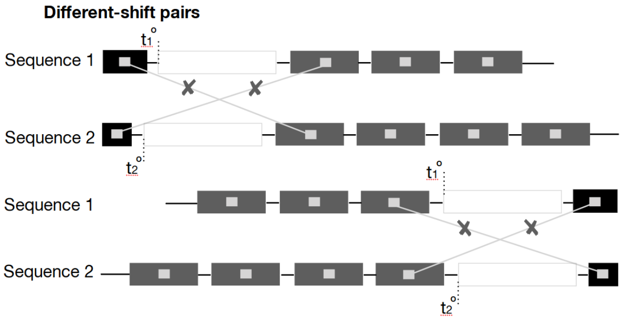



Figure 6: Sketch of diff-shift pairings (top) when the number of diff-shift computations is likely to 0

# Proposed Modification

- Failures due to missing Same-shift pairs unavoidable
- Failures due to missing different-shift pairs preventable

Define same-shift pairs identical to Hadler and Morris [2017] as pairs

$$(s_i^{(1)}, s_i^{(2)}) \quad \forall\, i \in \mathbb{Z}$$

where the boundary conditions of both sequences are met simultaneously.

- Let us assume that this results in $I$ pairs.
- Let $s_{(j)}^{(k)}$ to be the $j$th starting location in sequence $k = 1, 2$, i.e. $s_{(1)}^{(k)} < s_{(2)}^{(k)} < \ldots < s_{(I)}^{(k)}$.

We then define the pairs for different-shifts as

$$\left(s^{(1)}_{(j)}, s^{(1)}_{(I-j+1)}\right) \text{ for } j = \begin{cases} 1, ..., I & \text{for even } I \\ 1, ..., (I-1)/2, (I-1)/2+2, ..., I & \text{for odd } I \end{cases} \tag{2}$$

- For an odd number of same-shift correlations
  - We skip the middle pair for the different-shift correlations (see fig. 7).
- This pairing ensures that the number of different-shift pairings is the same or at most one less than the number of same-shift pairings in all tests.

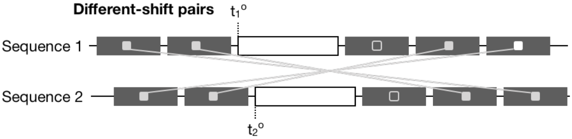



Figure 7: Sketch of adjusted different-shift pairings. At most one of the same-shift pairings can not be matched with a different-shift pair.

# Case where CS1 fails but CS2 does not fail

**CS1** Hadler and Morris [2017] algorithm
**CS2** the suggested modified algorithm

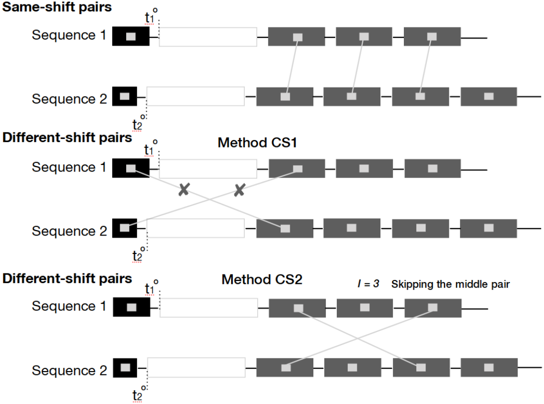



Figure 8: Sketch of a case where CS1 fails but CS2 does not fail

# Results Failed Tests

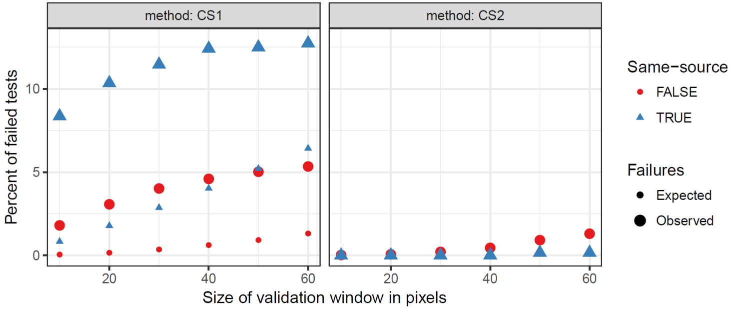



Figure 9: Percent of failed land-to-land comparisons for $w_o = 120$ and coarseness $c = 0.25$

# Conclusions Failed Tests

- With an increase in the $w_v$ higher percent of tests fail under both CS1 and CS2
- Number is highly dependent on the comparison window sizes
- Correlated to the ground truth,
- **Higher for known same-sourced lands (CS1)** than for known different sourced lands.
- **CS1** fails to conduct a test about 8 to 13 % of the time for **known same-source lands**, and 2 to 6% of the time for **known different source lands**.
- Number for CS1 always higher than the corresponding theoretical number of failed tests.
- Using CS2, the case with **largest** # of failed tests is still **lower** than the case where CS1 gives the **lowest** # of failed tests
- Even for high coarseness, CS2 will have lower number of failed tests than CS1, Making it more robust.
- CS2 performs better for **both** same and different-sources
- Solves a **critical issue** of CS1 known same-source matching, by having a **negligible** number of Known same-source failed tests

# Coarseness

- Remove (sub-)class characteristics from profiles before comparisons for matching.
- Hadler and Morris [2017] suggest a coarseness parameter of 0.25 for toolmark comparisons.
- For bullet lands, coarseness might need to be adjusted because of the strong effect bullet curvature has on profiles.
- Optimal locations are distributed uniformly once (sub-)class characteristics are removed.
- Distinct boundary effects: c $> 0.20$ optimal locations $t^*$ are found at the very extreme ends of a profile more often than one would expect based on a uniform distribution. -smaller coarseness value of $c = 0.15$ to be suitable

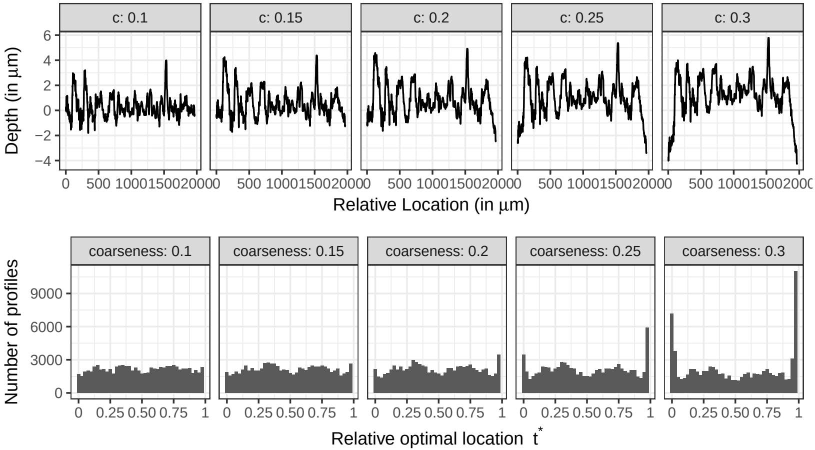



Figure 10: Overview of the effect of different coarseness parameters $c$ on the profile shown in Figure 1 (top). The bottom row shows histograms of the (relative) optimal locations $t^o$ identified in the optimization step for different values of the coarseness parameter $c$.

# Type II error rates

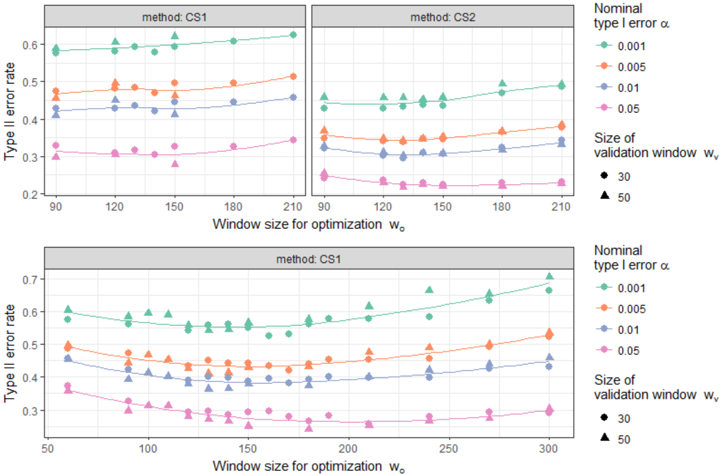



Figure 11: Type 2 error of methods CS1 and CS2 across a range of different optimization windows $w_o$. Top two figures are for a **coarseness** $\approx$ *0.15* and the bottom one is for *0.3*

# Conclusions for Type II Error

## CS1

- Best works best for $w_o$ of $\approx 130$ to 160 and $w_v$ 50 when the smoothing is $c \approx 0.3$.
- The Type II rate is lowest for a nominal $\alpha$ of 5%, with type I error rate of 6.2% and the Type II error rate of 24%.
- For lower nominal alpha levels of 1%, 0.5% and 0.1% the lowest type II error rate increases to about 36.4%, 41% and 52.5% respectively.
- Gets worse for coarseness 0.15

## CS2

- Significantly reduced over CS1
- For a window size of $w_o = 130$ we see a minimum in type II error rate across all type I rates considered. - Smaller validation sizes $w_v$ are typically associated with a smaller type II error.
- CS2 shows an increase in the power of the test.
- Type II CS2, still much higher for bullet lands than for toolmarks.
- Fix in CS2 will also improve power for matching toolmarks thans CS1
- Bullet-to-bullet comparison using CS2 $\approx$ more power out of the test.

# ROC Curves

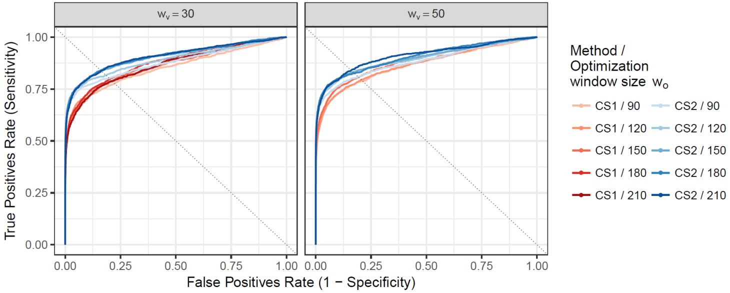



Figure 12: ROC curves of methods CS1 and CS2 for different sizes of optimization window $w_o$.

- Superior performance of CS2 over CS1
- Best performances wrt ROC curves are reached for $w_0$ 150 and higher.
- Points of equal error rates (EERs): intersection of the dotted line and the ROC curves.

**THANK YOU. Questions?**

# Appendix

## U Statistic:

This is computed from the joint rank of all correlations of both the same and different shift samples. As given by Hadler and Morris [2017]

Null Hypothesis: If the toolmarks were not match i.e not made by the same tool.

Let $n_s$ and $n_d$ be the number of same shift and different shift windows

$$N = n_s + n_d$$

Additionally, let $R_s(i)$ and $R_d(j)$ be the ranks associated with the combined vector of correlations for the same-shift and different-shift correlations, for i = 1,2,. . . ,$n_s$ and j = 1,2,. . . ,$n_d$. Then the Mann–Whitney U-statistic is given by

The mann whitney U statistic is given by

$$U = \sum_{i=1}^{n_s} R_s\left(i\right)$$

with the standardized version which includes provision for rank ties

$$\overline{U} = \frac{U - M}{\sqrt{V}}$$

where prior to normalization the U-statistic has the mean as

$$M = n_s \left( \frac{N+1}{2} \right)$$

and variance

$$V = \frac{n_s n_d}{N\,(N-1)} \left[ \Sigma^{n_s} R_s\,(i)^2 + \Sigma^{n_d} R_d\,(j)^2 \right] - \frac{n_s n_d\,(N+1)^2}{4\,(N-1)}$$

L. Scott Chumbley, Max D. Morris, M. James Kreiser, Charles Fisher, Jeremy Craft, Lawrence J. Genalo, Stephen Davis, David Faden, and Julie Kidd. Validation of tool mark comparisons obtained using a quantitative, comparative, statistical algorithm. *Journal of Forensic Sciences*, 55(4):953–961, 2010. ISSN 1556-4029. doi: 10.1111/j.1556-4029.2010.01424.x. URL http://dx.doi.org/10.1111/j.1556-4029.2010.01424.x.

David Faden, Julie Kidd, Jeremy Craft, L. Scott Chumbley, Max D. Morris, M. James Genalo, Lawrence J.and Kreiser, and Stephen Davis. Statistical confirmation of empirical observations concerning toolmark striae. *AFTE Journal*, 39(2):205–214, 2007.

Taylor Grieve, L. Scott Chumbley, Jim Kreiser, Laura Ekstrand, Max Morris, and Song Zhang. Objective comparison of toolmarks from the cutting surfaces of slip-joint pliers. *AFTE Journal*, 46(2): 176–185, 2014.

Jeremy R. Hadler and Max D. Morris. An improved version of a tool mark comparison algorithm. *Journal of Forensic Sciences*, pages n/a–n/a, 2017. ISSN 1556-4029. doi: 10.1111/1556-4029.13640. URL http://dx.doi.org/10.1111/1556-4029.13640.

James E. Hamby, David J. Brundage, and James W. Thorpe. The Identification of Bullets Fired from 10 Consecutively Rifled 9mm Ruger Pistol Barrels: A Research Project Involving 507 Participants from 20 Countries. *AFTE Journal*, 41(2):99–110, 2009.