CSCI 239—Discrete Structures of Computer Science

Lab 7—Inductive Proofs

This lab consists of pencil-and-paper exercises constructing inductive proofs.

Objectives:

- to be able to identify the base case, the inductive hypothesis, and the target of the inductive step for a given statement that can be proved using induction.
- to be able to complete the steps of an inductive proof once the components are identified.

Instructions

For each of the problems below, where the example problem is to prove *f*(*n*) = *g*(*n*) for all *n* ≥ 1:

a. Identify the base value for the induction and state what is to be proved for the base case.
   In the example: *n* = 1, show *f*(1) = *g*(1).
b. Identify the inductive hypothesis.
   In the example: *f*(*k*) = *g*(*k*) for some *k* ≥ 1.
c. Identify the statement to be shown in the inductive step.
   In the example: *f*(*k*+1) = *g*(*k*+1).
d. Complete the inductive proof using *a*, *b*, and *c*.

Problems

1. $\sum_{j=0}^{n} (2j+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$.
2. Find a closed-form formula for $\sum_{j=1}^{n} \frac{1}{2^j}$ by inspecting the values of this expression for small values of *n*. Then follow the steps above to show your formula is correct using induction.
3. $n < 2^n$, for positive integers *n*.
4. 3 divides $n^3 - n$, for all positive integers *n*.
5. 2 divides $n^2 + n$, for all positive integers *n*.
   Give a second proof by cases that does not use induction.
6. 5 divides $n^5 - n$, for all non-negative integers *n*.
7. $\bigcup_{j=1}^{n} A_j \subseteq \bigcup_{j=1}^{n} B_j$, if $A_j \subseteq B_j$ for *j* = 1, 2, …, *n*. The proof should be similar to one for a summation. Think in terms of showing that every element in the first union is an element of the second union.
8. If $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, then $A^n = \begin{pmatrix} a^n & 0 \\ 0 & b^n \end{pmatrix}$. Recall that $a = a^1$.