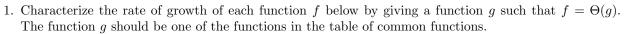
Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.



(a)
$$f(n) = n^8 + 3n - 4$$

Solution: $f = \Theta(n^8)$

(b)
$$f(n) = 2 \cdot 3^n$$

Solution: $f = \Theta(3^n)$

(c)
$$f(n) = 7(\log \log n) + 3(\log n) + 12n$$

Solution: $f = \Theta(n)$

(d)
$$f(n) = 9(n \log n) + 5(\log \log n) + 5$$

Solution: $f = \Theta(n \log n)$

(e) $f(n) = n \log_{37} n$

Solution: $f = \Theta(n \log n)$

(f)
$$f(n) = n^2 1 + (1.1)^n$$

Solution: $f = \Theta((1.1)^n)$

(g)
$$f(n) = 23n + n^3 - 2$$

Solution: $f = \Theta(n^3)$

- 2. Give complete proofs for the growth rates of the polynomials below. You should provide specific values for c and n_0 and prove algebraically that the functions satisfy the definitions for O, Ω , and Θ .
 - (a) $f(n) = (1/2)n^5 100n^3 + 3n 1$. Prove that $f = O(n^5)$.

Solution:

Proof. Let c = 7/2 and $n_0 = 1$. For $n \ge 1$, the -1 and the -100n3 terms are both negative. Therefore, those terms can be dropped from the expression for f(n) and the resulting expression is larger:

$$(1/2)n^5 - 100n^3 + 3n - 1 < (1/2)n^5 + 3^n$$

For $1 \le n, n \le n^5$, so

$$(1/2)n^5 + 3n \le (1/2)n^5 + 3n^5 = (7/2)n^5.$$

Therefore for $n \ge 1$, $f(n) \le (7/2)n^5$.

$$f = O(n^5).$$

(b) $f(n) = n^3 + 3n^2 + 4$. Prove that $f = \Theta(n^3)$.

Solution:

Proof. Let c = 8 and $n_0 = 1$.

For $n \ge 1$, $n^3 \ge n^2$ and $n^3 \ge 1$, so

$$n^3 + 3n^2 + 4 \le n^3 + 3n^3 + 4n^3 = 8n^3$$

Therefore for $n \ge 1$, $f(n) \le 8n^3$.

$$f = O(n^3)$$
.

Proof. Let c = 1 and $n_0 = 1$. Since n is positive, the $3n^2$ term in f(n) is also positive. The positive terms can be dropped from the expression $n^3 + 3n^2 + 4$ and the resulting expression is smaller, so

$$n^3 + 3n^2 + 4 > n^3$$
.

Therefore for $n \ge 1$, $f(n) \ge 1 \cdot n^3$.

$$f = \Omega(n^3)$$
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