Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

1. Characterize the rate of growth of each function f below by giving a function g such that $f = \Theta(g)$. The function g should be one of the functions in the table of common functions.

   (a) $f(n) = n^8 + 3n - 4$

   Solution: $f = \Theta(n^8)$

   (b) $f(n) = 2 \cdot 3^n$

   Solution: $f = \Theta(3^n)$

   (c) $f(n) = 7(\log\log n) + 3(\log n) + 12n$

   Solution: $f = \Theta(n)$

   (d) $f(n) = 9(n \log n) + 5(\log\log n) + 5$

   Solution: $f = \Theta(n \log n)$

   (e) $f(n) = n \log_{37} n$

   Solution: $f = \Theta(n \log n)$

   (f) $f(n) = n^2 1 + (1.1)^n$

   Solution: $f = \Theta((1.1)^n)$

   (g) $f(n) = 23n + n^3 - 2$

   Solution: $f = \Theta(n^3)$

2. Give complete proofs for the growth rates of the polynomials below. You should provide specific values for c and n_0 and prove algebraically that the functions satisfy the definitions for O, Ω, and Θ.

   (a) $f(n) = (1/2)n^5 - 100n^3 + 3n - 1$. Prove that $f = O(n^5)$.

Solution:

Proof. Let $c = 7/2$ and $n_0 = 1$. For $n \geq 1$, the -1 and the $-100n3$ terms are both negative. Therefore, those terms can be dropped from the expression for $f(n)$ and the resulting expression is larger:
$(1/2)n^5 - 100n^3 + 3n - 1 < (1/2)n^5 + 3^n$
For $1 \leq n$, $n \leq n^5$, so
$(1/2)n^5 + 3n \leq (1/2)n^5 + 3n^5 = (7/2)n^5$.
Therefore for $n \geq 1$, $f(n) \leq (7/2)n^5$.
$f = O(n^5)$. □

   (b) $f(n) = n^3 + 3n^2 + 4$. Prove that $f = \Theta(n^3)$.

Solution:

Proof. Let $c = 8$ and $n_0 = 1$.
For $n \geq 1$, $n^3 \geq n^2$ and $n^3 \geq 1$, so
$n^3 + 3n^2 + 4 \leq n^3 + 3n^3 + 4n^3 = 8n^3$
Therefore for $n \geq 1$, $f(n) \leq 8n^3$.
$f = O(n^3)$. □

Proof. Let $c = 1$ and $n_0 = 1$. Since n is positive, the $3n^2$ term in $f(n)$ is also positive. The positive terms can be dropped from the expression $n^3 + 3n^2 + 4$ and the resulting expression is smaller, so
$n^3 + 3n^2 + 4 \geq n^3$.
Therefore for $n \geq 1$, $f(n) \geq 1 \cdot n^3$.
$f = \Omega(n^3)$. □