Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

Product and sum rule

Consider the following definitions for sets of characters:

- Digits = { 0 .. 9 }
- Letters = { a .. z }
- Special characters = { $*$, &, \$, # }

1. Compute the number of passwords that satisfy the given constraints.

   (a) Strings of length 6. Characters can be special characters, digits, or letters.

   Solution: Let D be the set of digits, L the set of letters, and S the set of special characters. The three sets are mutually disjoint, so the total number of characters is

   $|D \cup L \cup S| = |D| + |L| + |S| = 10 + 26 + 4 = 40$

   Each of the six characters in the string can be any of the 40 characters, so there are a total of 40^6 strings of length 6.

   (b) Strings of length 7, 8, or 9. Characters can be special characters, digits, or letters.

   Solution: The sets of strings of length 7, 8, and 9 are mutually disjoint. The number of strings of length j with no other restrictions on the characters is 40^j. Therefore, by the sum rule, the total number of strings of length 7, 8, or 9 is:

   $40^7 + 40^8 + 40^9$

   (c) Strings of length 7, 8, or 9. Characters can be special characters, digits, or letters. The first character cannot be a letter.

   Solution: In selecting a string of length j in which the first character is not a letter, there are 14 choices for the first character because there are 4 + 10 digits and special characters. There are 40 choices for each of the remaining characters. Putting the choices together by the product rule, the total number of strings of length j in which the first character is not a letter is $14 \cdot 40^{j-1}$. Sets of strings of length 7, 8, and 9 are mutually disjoint. Therefore, by the sum rule, the number of strings of length 7, 8, or 9 which do not start with a letter is:

   $14 \cdot 40^6 + 14 \cdot 40^7 + 14 \cdot 40^8 = 14(40^6 + 40^7 + 40^8)$

2. Consider the numbers in the range 1 to 10^4 (inclusive).

   (a) How many of the numbers in the range 10^3 to 10^4 (inclusive) are composed of all distinct digits?

   Solution: By the generalized product rule, $9 \cdot 9 \cdot 8 \cdot 7 = 4536$ numbers are composed of distinct digits. *Note: the first digit cannot be zero, thus this initial 9 factor.* 10^4 has repeated digits, so we don't count it.

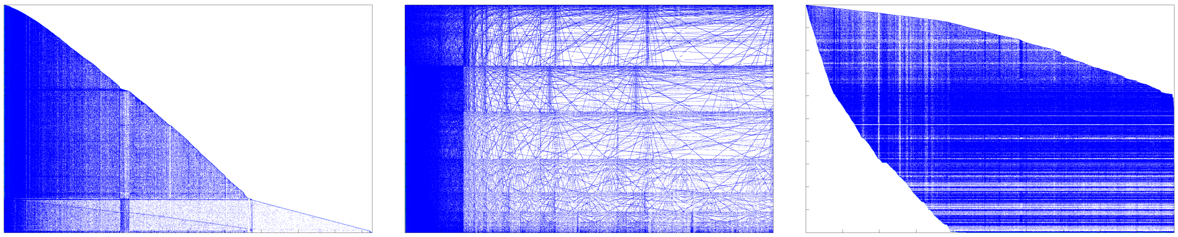
(b) How many of the numbers in the range 1 to 10^4 (inclusive) are composed of all distinct digits?

Solution: 9 single digit numbers, $9 \cdot 9 = 81$ two digit numbers, $9 \cdot 9 \cdot 8 = 648$ three digits numbers, and 4536 four digit numbers.

$9 + 81 + 648 + 4536 = 5274$

Permuations

Consider the following permutations of a single adjacency matrix:



(a) Original (b) Clean (c) RCM

Permuting the rows / columns of a matrix can be beneficial for a couple reasons:

1. Improve computational efficiency by improving structure of adjacency matrix
2. Expose hidden structure in the graph

Unfortunately, knowing *apriori* which permutation is best is a difficult problem. The problem is compounded by the huge number of permutations possible.

As a result, *heuristic* methods are usually employed. These methods, like (b) and (c) above, are efficient to find and lead to good, but not necessarily optimal, permutations of the matrix.

3. How many ways are there to permute the rows / columns of a
   (a) 10×10 matrix?

   Solution: $10! = 3628800$

   (b) $n \times n$ matrix?

   Solution: $n!$

Combinations

4. Suppose a network has 40 computers of which 5 fail.
   (a) How many possibilities are there for the five that fail?

   Solution: $\binom{40}{5} = \frac{40!}{5!(40-5)!} = \frac{40\cdot39\cdot38\cdot37\cdot36}{5\cdot4\cdot3\cdot2\cdot1} = 658008$

   (b) Suppose that 3 of the computers in the network have a copy of a particular file. How many sets of failures wipe out all the copies of the file? That is, how many 5-subsets contain the three computers that have the file?

Solution: Any such 5-subset of computers that fail must contain the 3 computers that have the file. The two remaining computers in the subset must be selected from the 37 computers that did not contain the file. Thus, the number of such 5-subsets in which the 3 computers with the file fail is the number of ways to select 2 computers from the 37 computers without the file, or $\binom{37}{2}$.