Answer the questions in the spaces provided on the question sheets. If you run out of
room for an answer, continue on a separate sheet of paper.

1. Define the set $A = \{ r, o, t, p, c \}$ and $B = \{ discrete, math, proof, proposition \}$. Define the relation
$R \subseteq A \times B$ such that (letter, word) is in the relation if that letter occurs somewhere in the word. Draw the
arrow diagram and the matrix representation for each relation.

2. The domain of relation D is $\{ 2, 3, 12, 16, 27, 48 \}$. For x, y in the domain, xDy if y is an integer multiple of
x. Draw the arrow diagram and the matrix representation for the relation.

3. Consider the following relation F. The domain of the relation F is all Facebook users. For x, y in the domain,
xFy if x and y are Facebook friends. Furthermore, $xFy \rightarrow yFx$.

   (a) List all properties of the relation F.

   (b) Read about "*Six degrees of separation*" on Wikipedia. What is the implication of this claim for the relation
   F described above?

   (c) Now read this article `https://research.fb.com/blog/2016/02/three-and-a-half-degrees-of-separation/`.
   What does this mean for you?

4. Define the directed graph G as follows:
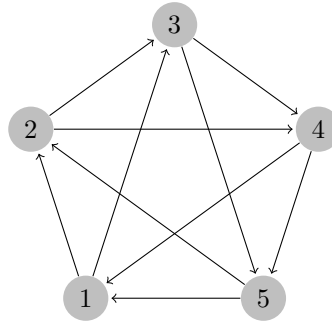
(a) Classify each of the following sequences of vertices as either a *walk* in G or *not a walk* in G. If a sequence, w represents a walk in G, characterize the walk as an *open walk*, *closed walk*, *trail*, *circuit*, *path* or *cycle*, being as specific as possible.

    i. $\langle 1,\ 2,\ 4,\ 5,\ 2 \rangle$ ii. $\langle 1,\ 3,\ 5,\ 2,\ 4,\ 1 \rangle$ iii. $\langle 1,\ 2,\ 3 \rangle$ iv. $\langle 3,\ 5,\ 1,\ 2,\ 4,\ 1,\ 3 \rangle$

(b) For each of the following, find a walk in G that satisfies the requirements specified.

    i. cycle of length 3   ii. trail of length 8   iii. path of length 4 starting at 5 and ending at 3