Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

1. (a) Using a truth table, show that $p \oplus q \equiv \neg(p \wedge q) \wedge (p \vee q)$.

Solution:

p	q	$p \oplus q$	$\neg(p \wedge q) \wedge (p \vee q)$
1	1	0	0
1	0	1	1
0	1	1	1
0	0	0	0

(b) Using the laws of propositional logic and the result from Part (a), show that $p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$.

Solution:

$$
\begin{aligned}
p \oplus q &\equiv \neg(p \wedge q) \wedge (p \vee q) && \text{(Part (a))} \\
&\equiv (\neg p \vee \neg q) \wedge (p \vee q) && \text{(De Morgan's)} \\
&\equiv (\mathrm{T} \wedge (\neg p \vee \neg q)) \wedge (\mathrm{T} \wedge (p \vee q)) && \text{(identity)} \\
&\equiv ((p \vee \neg p) \wedge (\neg p \vee \neg q)) \wedge ((q \vee \neg q) \wedge (p \vee q)) && \text{(complement)} \\
&\equiv (\neg p \vee (p \wedge \neg q)) \wedge (q \vee (p \wedge \neg q)) && \text{(distributive)} \\
&\equiv (p \wedge \neg q) \vee (\neg p \wedge q) && \text{(distributive)}
\end{aligned}
$$

2. A certain cabal (*cabal: a secret political clique or faction*) within the CS department is plotting to make the final exam *ridiculously hard*. The only way to stop their evil plan is to determine exactly who is in the cabal. The department includes Donald, Grace, Linus, Alan, Ada and Edsger. The cabal is a subset of these six. A membership roster has been found and appears below, but it is deviously encrypted in logic notation. The predicate *cabal* indicates who is in the cabal; that is, $cabal(x)$ is true if and only if x is a member of the cabal. Use the following information to gather who is in the cabal.

   1. $\exists x \exists y \exists z (x \neq y \wedge x \neq z \wedge y \neq z \wedge cabal(x) \wedge cabal(y) \wedge cabal(z))$
   2. $\exists x (\neg cabal(x))$
   3. $cabal(Edsger) \rightarrow \forall x (cabal(x))$
   4. $\neg(cabal(Donald) \wedge cabal(Alan)) \wedge (cabal(Donald) \vee cabal(Alan))$
   5. $cabal(Alan) \rightarrow cabal(Donald)$
   6. $(cabal(Ada) \vee cabal(Linus)) \rightarrow \neg cabal(Grace)$

Solution:

A.	$\exists x(\neg cabal(x))$	(hypothesis 2)
B.	c is a particular element $\wedge \neg cabal(c)$	(existential instantiation A)
C.	c is a particular element	(simplification B)
D.	$\forall x(cabal(Edsger) \rightarrow cabal(x))$	(hypothesis 3)
E.	$\neg cabal(c)$	(simplification B)
F.	$cabal(Edsger) \rightarrow cabal(c)$	(universal instantiation C & D)
G.	$\boldsymbol{\neg cabal(Edsger)}$	(modus tollens E & F)
H.	$\neg(cabal(Donald) \wedge cabal(Alan)) \wedge (cabal(Donald) \vee cabal(Alan))$	(hypothesis 4)
I.	$cabal(Donald) \oplus cabal(Alan)$	(Question 1, part (a) H)
J.	$(cabal(Donald) \wedge \neg cabal(Alan)) \vee (\neg cabal(Donald) \wedge cabal(Alan))$	(Question 1, part (b) I)
K.	$cabal(Alan) \rightarrow cabal(Donald)$	(hypothesis 5)
L.	$\neg cabal(Alan) \vee cabal(Donald)$	(conditional identity K)
M.	$cabal(Donald) \wedge \neg cabal(Alan)$	(disjunctive syllogism J & L)
N.	$\boldsymbol{cabal(Donald)}$	(simplification M)
O.	$\boldsymbol{\neg cabal(Alan)}$	(simplification M)
P.	$(cabal(Ada) \wedge cabal(Linus)) \ \vee \ (cabal(Ada) \wedge cabal(Grace)) \ \vee$ $(cabal(Linus) \wedge cabal(Grace))$	(hypothesis 1 & G & N & O and a lot of hard work)
Q.	$(cabal(Ada) \vee cabal(Linus)) \rightarrow \neg cabal(Grace)$	(hypothesis 6)
R.	$\neg(cabal(Ada) \vee cabal(Linus)) \vee \neg cabal(Grace)$	(conditional identity Q)
S.	$(\neg cabal(Ada) \wedge \neg cabal(Linus)) \vee \neg cabal(Grace)$	(De Morgan's R)
T.	$(\neg cabal(Ada) \vee \neg cabal(Grace)) \wedge (\neg cabal(Linus) \vee \neg cabal(Grace))$	(distributive S)
U.	$\neg cabal(Ada) \vee \neg cabal(Grace)$	(simplification T)
V.	$\neg(cabal(Ada) \wedge cabal(Grace))$	(De Morgan's U)
W.	$(cabal(Ada) \wedge cabal(Linus)) \vee (cabal(Linus) \wedge cabal(Grace))$	(disjunctive syllogism P & V)
X.	$\neg cabal(Linus) \vee \neg cabal(Grace)$	(simplification T)
Y.	$\neg(cabal(Linus) \wedge cabal(Grace))$	(De Morgan's X)
Z.	$cabal(Ada) \wedge cabal(Linus)$	(disjunctive syllogism W & Y)
AA.	$\boldsymbol{cabal(Ada)}$	(simplification Z)
BB.	$\boldsymbol{cabal(Linus)}$	(simplification Z)
CC.	$\boldsymbol{\neg cabal(Grace)}$	(disjunctive syllogism U & AA)