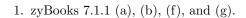
Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.



3. zyBooks 
$$7.3.1$$
 (a), (b), (e), (f), and (g).

## 6. zyBooks 7.3.3

Consider the following algorithm for counting the triangles in a symmetric, non-reflexive graph.

## Algorithm 1: Triangle counting

7. Analyze the algorithm Triangle counting and express the total number of additions, multiplications, and comparisons required for an  $n \times n$  matrix as a function of n.

```
Solution: f(n) = n^2 + 2n^3 + n^2 + 2n^2 = 2n^3 + 4n^2

n^2 comparisons for triu.

n^3 multiplications and additions for matrix multiplication.

n^2 multiplications for Hadamard product.

n^2 comparisons and additions for counting ones.
```

8. Prove that this algorithm is  $\Theta(n^3)$ .

## Solution:

*Proof.* Let c = 6 and  $n_0 = 1$ .

For  $n \ge 1$ ,  $n^3 \ge n^2$ , so

$$2n^3 + 4n^2 \le 2n^3 + 4n^3 = 6n^3$$

Therefore for n > 1,  $f(n) < 6n^3$ .

$$f = O(n^3).$$

*Proof.* Let c=2 and  $n_0=1$ .

Since n is positive, the  $4n^2$  term in f(n) is also positive. The positive terms can be dropped from the expression  $2n^3 + 4n^2$  and the resulting expression is smaller, so

$$2n^3 + 4n^2 \ge 2n^3$$
.

Therefore for n > 1,  $f(n) > 2n^3$ .

$$f = \Omega(n^3).$$