

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

1. For each of the following functions, determine if the function is one-to-one, onto, both, or none.
 - (a) $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}, f(x, y) = 2x - 4y$

 - (b) $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}, f(x, y) = (x + 1, 2y)$

 - (c) $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}, f(x, y) = (1 - y, 1 - x)$

2. For a function $f : A \rightarrow X$, what can we say about the relationship between the cardinality of the domain and the target if f has the following properties.
 - (a) Onto

 - (b) One-to-one

 - (c) One-to-one correspondence

3. For each of the properties, determine a function $f : \mathbb{Z} \rightarrow \mathbb{Z}$, that satisfies the properties.
 - (a) Neither one-to-one nor onto.

 - (b) Onto, but not one-to-one.

 - *(c) One-to-one, but not onto.