Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

1. zyBooks 7.1.1 (a), (b), (f), and (g).

2. zyBooks 7.2.1

3. zyBooks 7.3.1 (a), (b), (e), (f), and (g).

4. zyBooks 7.3.1 (a), (b), (e), (f), and (g).

5. zyBooks 7.3.2 (a), (b), (c)

6. zyBooks 7.3.3

Consider the following algorithm for counting the triangles in a *symmetric, non-reflexive* graph.

Algorithm 1: Triangle counting

Input: An $n \times n$ *adjacency matrix* called A, of a *symmetric, non-reflexive* graph G
Result: The number of triangles in G

```
/* triu(A) is A with only those edges (x,y) where x ≤ y.                    */
U ← triu(A)

/* U.' is the transpose of U and U ∗ U.' is matrix multiplication.          */
B ← U ∗ U.'

/* U ∘ B is the Hadamard product, which is similar to matrix addition, but multiplies
   rather than adds corresponding elements.                                 */
T ← U ∘ B

/* nnz(T) is the number of ones in T.                                       */
return nnz(T)
```

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7. Analyze the algorithm `Triangle counting` and express the total number of additions, multiplications, and comparisons required for an $n \times n$ matrix as a function of n.

> **Solution:** $f(n) = n^2 + 2n^3 + n^2 + 2n^2 = 2n^3 + 4n^2$
> n^2 comparisons for `triu`.
> n^3 multiplications and additions for matrix multiplication.
> n^2 multiplications for Hadamard product.
> n^2 comparisons and additions for counting ones.

8. Prove that this algorithm is $\Theta(n^3)$.

> **Solution:**
>
> *Proof.* Let $c = 6$ and $n_0 = 1$.
>
> For $n \geq 1$, $n^3 \geq n^2$, so
>
> $2n^3 + 4n^2 \leq 2n^3 + 4n^3 = 6n^3$
>
> Therefore for $n \geq 1$, $f(n) \leq 6n^3$.
>
> $f = O(n^3)$. □
>
> *Proof.* Let $c = 2$ and $n_0 = 1$.
>
> Since n is positive, the $4n^2$ term in $f(n)$ is also positive. The positive terms can be dropped from the expression $2n^3 + 4n^2$ and the resulting expression is smaller, so
>
> $2n^3 + 4n^2 \geq 2n^3$.
>
> Therefore for $n \geq 1$, $f(n) \geq 2n^3$.
>
> $f = \Omega(n^3)$. □