Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

1. Write a Haskell function to add the first `n` odd numbers of a list, using only the following functions: `sum`, `filter`, `even`, `not` and `take`. The function signatures are given below.

```
sum ::  Integral a => [a] -> a
filter ::  (a -> Bool) -> [a] -> [a]
even ::  Integral a => a -> Bool
not ::  Bool -> Bool
take ::  Integral -> [a] -> [a]
```

2. Consider the following functions:

   $f : \mathbb{R} \to \mathbb{R}.\ f(x) = x^2$
   $g : \mathbb{Z} \to \mathbb{R}.\ g(x) = \frac{x}{2}$
   $h : \mathbb{R} \to \mathbb{Z}.\ h(x) = \lceil x \rceil$

   What will be the definition of the following function compositions

   (a) $(f \circ g)$

   **Solution:** $(f \circ g) : \mathbb{Z} \to \mathbb{R}.\ (f \circ g)(x) = (\frac{x}{2})^2 = \frac{x^2}{4}$

   (b) $(f \circ h)$

   **Solution:** $(f \circ h) : \mathbb{R} \to \mathbb{R}.\ (f \circ g)(x) = (\lceil x \rceil)^2$

   (c) $(h \circ g \circ h \circ f)$

   **Solution:** $(h \circ g \circ h \circ f) : \mathbb{R} \to \mathbb{Z}.\ (h \circ g \circ h \circ f) = \lceil \frac{\lceil x^2 \rceil}{2} \rceil$

   Use your definitions to evaluate the following function compositions:

   (d) $(f \circ g)(1)$

   **Solution:** $(f \circ g)(0) = \frac{1}{4}$

   (e) $(f \circ h)(3.5)$

   **Solution:** $(f \circ h)(3.5) = 16$

   (f) $(h \circ g \circ h \circ f)(\sqrt{3})$

   **Solution:** $(h \circ g \circ h \circ f)(\sqrt{3}) = 2$

For each of the following functions, indicate whether the function has a well-defined inverse. If the inverse is well-defined, give the input/output relationship.

(g) $f$

> **Solution:** $f$ is not onto, thus is not bijective, thus is not invertible.

(h) $g$

> **Solution:** $g$ is not onto, thus is not bijective, thus is not invertible.

(i) $(g \circ f)$

> **Solution:** $f$ and $g$ cannot be composed, since the domain of $g$ is not a subset of the target of $f$, thus is not invertible.