

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

1. Write a Haskell function to add the first n odd numbers of a list, using only the following functions: `sum`, `filter`, `even`, `not` and `take`. The function signatures are given below.

```
sum :: Integral a => [a] -> a
filter :: (a -> Bool) -> [a] -> [a]
even :: Integral a => a -> Bool
not :: Bool -> Bool
take :: Integral -> [a] -> [a]
```

2. Consider the following functions:

$$f : \mathbb{R} \rightarrow \mathbb{R}. f(x) = x^2$$

$$g : \mathbb{Z} \rightarrow \mathbb{R}. g(x) = \frac{x}{2}$$

$$h : \mathbb{R} \rightarrow \mathbb{Z}. h(x) = \lceil x \rceil$$

What will be the definition of the following function compositions

- (a) $(f \circ g)$

Solution: $(f \circ g) : \mathbb{Z} \rightarrow \mathbb{R}. (f \circ g)(x) = \left(\frac{x}{2}\right)^2 = \frac{x^2}{4}$

- (b) $(f \circ h)$

Solution: $(f \circ h) : \mathbb{R} \rightarrow \mathbb{R}. (f \circ h)(x) = (\lceil x \rceil)^2$

- (c) $(h \circ g \circ h \circ f)$

Solution: $(h \circ g \circ h \circ f) : \mathbb{R} \rightarrow \mathbb{Z}. (h \circ g \circ h \circ f) = \lceil \frac{\lceil x^2 \rceil}{2} \rceil$

Use your definitions to evaluate the following function compositions:

- (d) $(f \circ g)(1)$

Solution: $(f \circ g)(0) = \frac{1}{4}$

- (e) $(f \circ h)(3.5)$

Solution: $(f \circ h)(3.5) = 16$

- (f) $(h \circ g \circ h \circ f)(\sqrt{3})$

Solution: $(h \circ g \circ h \circ f)(\sqrt{3}) = 2$

For each of the following functions, indicate whether the function has a well-defined inverse. If the inverse is well-defined, give the input/output relationship.

(g) f

Solution: f is not onto, thus is not bijective, thus is not invertible.

(h) g

Solution: $g^{-1}(x) = 2x$

(i) $(g \circ f)$

Solution: $(g \circ f)^{-1}(x) = \sqrt{2x}$