Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

Bayes' theorem

1. Suppose we have two coins. The first coin is a fair coin and the second coin is biased. The biased coin comes up heads with probability .75 and tails with probability .25. We select a coin at random and flip the coin ten times. The results of the coin flips are mutually independent. The result of the 10 flips is: T,T,H,T,H,T,T,T,H,T. What is the probability that we selected the biased coin?

Solution: Let B be the event that we selected the biased coin. $p(B) = p(\overline{B}) = 1/2$.

Let Y be the event that the outcomes is T,T,H,T,H,T,T,T,H,T. Because the results of the flips are independent, $p(Y|\overline{B}) = (1/2)^{10}$. Also $p(Y|B) = (1/4)^7(3/4)^3$. Using Bayes' Theorem:

$$p(B|Y) = \frac{p(Y|B)p(B)}{p(Y|B)p(B) + p(Y|\overline{B})p(\overline{B})} = \frac{(1/4)^7(3/4)^3(1/2)}{(1/4)^7(3/4)^3(1/2) + (1/2)^{10}(1/2)} = \frac{27}{27 + 2^{10}} \approx 0.0257.$$

2. Naïve Bayes classifier

   (a) Using Bayes' theorem, compute the probability that a traveler survived (*column D*) given their ticket class (*column A*).

   Solution: $p(0|ticket) = \frac{p(ticket|0) * p(0)}{p(ticket)}$.
   $p(1|ticket) = \frac{p(ticket|1) * p(1)}{p(ticket)}$.

   (b) Given an unknown traveler, make a prediction, using results from Part (a) about whether or not they will survive based on their ticket class.

      i. What do you notice about this predictive model? How can we improve it?

   (c) Improve model.