Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

1. For each of the following functions $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, determine if the function is one-to-one, onto, both, or none.

   (a) $f(x, y) = 2x - 4y$

   Solution:

   (b) $f(x, y) = (x + 1, 2y)$

   Solution: One-to-one, but not onto. There is no pair (x, y) such that $f(x, y) = (1, 1)$.

   (c) $f(x, y) = (1 - y, 1 - x)$

   Solution: One-to-one and onto.

2. For a function $f : A \to X$, what can we say about the relationship between the cardinality of the domain and the target if f has the following properties.

   (a) Onto

   Solution: $|A| \geq |X|$

   (b) One-to-one

   Solution: $|A| \leq |X|$

   (c) One-to-one correspondence

   Solution: $|A| = |X|$

3. For each of the properties, determine a function $f : \mathbb{Z} \to \mathbb{Z}$, that satisfies the properties.

   (a) Neither one-to-one nor onto.

   Solution: $f(x) = x^2$

   (b) Onto, but not one-to-one.

   Solution: $f(x) = |x|$.

   *(c) One-to-one, but not onto.

   Solution: