

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

1. Using the laws of propositional logic, determine which of the following are equivalent to $(p \wedge q) \rightarrow r$ and which are equivalent to $(p \vee q) \rightarrow r$. Confirm your answer using truth-tables.
 - (a) $p \rightarrow (q \rightarrow r)$
 - (b) $q \rightarrow (p \rightarrow r)$
 - (c) $(p \rightarrow r) \wedge (q \rightarrow r)$
 - (d) $(p \rightarrow r) \vee (q \rightarrow r)$

2. Convert the following English sentences into logical formulas. You may use expressions like $x = y$ or $x \neq y$ to indicate whether or not the variables x and y denote different people. The domain of discourse is all people. Let the predicate $H(x)$ mean that “ x is happy,” and let the predicate $L(x, y)$ mean that “ x loves y .”
 - (a) At least one person is happy.
 - (b) No one is happy.
 - (c) At least one person is unhappy.
 - (d) Exactly one person is happy.
 - (e) Not everyone loves someone else.
 - (f) Everyone loves someone else.

3. Let the domain of discourse be all members of the class and let $L(x, y)$ be the predicate “ x likes y .” Translate the following into plain language:

- (a) $\forall x \exists y (L(x, y) \wedge x \neq y)$
- (b) $\exists x \neg \exists y (L(x, y) \vee L(y, x))$
- (c) $\exists x \exists y \exists z \exists w (L(x, w) \wedge L(y, w) \wedge L(z, w) \wedge x \neq y \neq z)$

4. A certain cabal (*cabal*: a secret political clique or faction) within the CS department is plotting to make the final exam *ridiculously hard*. The only way to stop their evil plan is to determine exactly who is in the cabal. The department includes Donald, Grace, Linus, Alan, Ada and Edsger. The cabal is a subset of these six. A membership roster has been found and appears below, but it is deviously encrypted in logic notation. The predicate *cabal* indicates who is in the cabal; that is, *cabal*(x) is true if and only if x is a member of the cabal. Use the following information to gather who is in the cabal.

- 1. $\exists x \exists y \exists z (x \neq y \wedge x \neq z \wedge y \neq z \wedge \text{cabal}(x) \wedge \text{cabal}(y) \wedge \text{cabal}(z))$
- 2. $\exists x (\neg \text{cabal}(x))$
- 3. $\text{cabal}(\text{Edsger}) \rightarrow \forall x (\text{cabal}(x))$
- 4. $\neg(\text{cabal}(\text{Donald}) \wedge \text{cabal}(\text{Alan})) \wedge (\text{cabal}(\text{Donald}) \vee \text{cabal}(\text{Alan}))$
- 5. $\text{cabal}(\text{Alan}) \rightarrow \text{cabal}(\text{Donald})$
- 6. $(\text{cabal}(\text{Ada}) \vee \text{cabal}(\text{Linus})) \rightarrow \neg(\text{cabal}(\text{Grace}))$