

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

1. For each of the following functions  $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ , determine if the function is one-to-one, onto, both, or none.
  - (a)  $f(x, y) = 2x - 4y$
  
  - (b)  $f(x, y) = (x + 1, 2y)$
  
  - (c)  $f(x, y) = (1 - y, 1 - x)$
  
2. For a function  $f : A \rightarrow X$ , what can we say about the relationship between the cardinality of the domain and the target if  $f$  has the following properties.
  - (a) Onto
  
  - (b) One-to-one
  
  - (c) One-to-one correspondence
  
3. For each of the properties, determine a function  $f : \mathbb{Z} \rightarrow \mathbb{Z}$ , that satisfies the properties.
  - (a) Neither one-to-one nor onto.
  
  - (b) Onto, but not one-to-one.
  
  - \*(c) One-to-one, but not onto.