Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

Discrete probability

1. If you flip a coin nine times, what is the probability that you will flip six heads and three tails?

   (a) What is the event, E, and what is $|E|$?

   Solution: The event is the number of permutations of {H,H,H,H,H,H,T,T,T}.
   This a counting permutations with repetitions. So $\frac{n!}{n_1!n_2!\cdots n_k!} = \frac{9!}{6!3!} = \frac{9\cdot 8\cdot 7}{3\cdot 2\cdot 1} = 84$

   (b) What is the sample space, S, and what is $|S|$?

   Solution: All possible coin flips. So $|S| = 2^9$.

   (c) What is the probability mentioned above, namely $p(E)$?

   Solution: $\frac{|E|}{|S|} = \frac{84}{512} = .1641$.

2. What is the probability that any pair of people, chosen at random, have the same birthday?

Solution: $\frac{P(366,2)}{366^2} = \frac{1}{366^2} \cdot \frac{366!}{(366-2)!} = \frac{366 \cdot 365}{366 \cdot 366} = \frac{365}{366} = .9973$

3. What is the probability that some pair of people *in our class* have the same birthday?
   (a) Define the event $\overline{E}$ to be the set of outcomes where everyone in the class has a different birthday, i.e., the complement of the event we would like to count. Compute $|\overline{E}|$.

   Solution: $P(366, 23)$

   (b) What is the size of the sample space S for this outcome?

   Solution: 366^{23}

   (c) What is the probability that everyone in the class has a different birthday?

   Solution: $\frac{P(366,23)}{366^{23}} = 0.4937$

   (d) What is the probability that some pair of people in the class have the same birthday?

   Solution: $1 - .4937 = .5063$