

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

Counting by complement

1. Give numerical answers for the questions below.

(a) There are 5 kids on the math team. Two kids will be selected from the team to compete in the state competition. How many ways are there to select the 2 competitors?

(b) The math team has 3 girls and 2 boys. How many ways are there to select the two competitors if they are both girls?

(c) The math team has 3 girls and 2 boys. How many ways are there to select the two competitors so that at least one boy is chosen?

Inclusion-exclusion principle

2. You are contracted by the FBI to unlock the phone of a suspected criminal. Your forensics team informs you that the material on the glass where the 8-key would appear during passcode entry has more oil-residue, left behind by fingers, than other places on the screen. This is understood to be a strong indication that the passcode contains at least one 8. How much does knowing that the passcode includes the digit 8 narrow the search space, considering that the passcode is 4 digits?
 - (a) Use the inclusion-exclusion principle to count the number of 2 digit passcode that include an 8.
 - (b) Use the inclusion-exclusion principle to count the number of 3 digit passcode that include an 8.
 - (c) Use the inclusion-exclusion principle to count the number of 4 digit passcode that include an 8.
3. Solve the previous three sub-problems using counting by complement rather than the inclusion-exclusion principle, to verify that you have the correct answers, i.e., the total number of passcodes should be the same.
 - (a)
 - (b)
 - (c)