Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

You may find the following definitions helpful:

$A \cap B = \{ \ x \ : \ x \in A \wedge x \in B \ \}$
$A \cup B = \{ \ x \ : \ x \in A \vee x \in B \ \}$
$A - B = \{ \ x \ : \ x \in A \wedge x \notin B \ \}$
$A \oplus B = \{ \ x \ : \ x \in A \oplus x \in B \ \}$
$A \times B = \{ \ (a, \ b) \ : \ a \in A \wedge b \in B \ \}$
$\overline{A} = \{ \ x \ : \ x \notin A \ \}$

Consider the following sets:

$A = \{ \ 1, \ 2, \ 3, \ 4, \ 5 \ \}$
$B = \{ \ a, \ b, \ c, \ d, \ 4, \ 5 \ \}$
$C = \{ \ a, \ b \ \}$

1. Using roster notation, give formal descriptions of the following sets:

(a) $A \cap B$

Solution: $\{ \ 4, \ 5 \ \}$

(b) $A \cup B$

Solution: $\{ \ 1, \ 2, \ 3, \ 4, \ 5, \ a, \ b, \ c, \ d \ \}$

(c) $B - C$

Solution: $\{ \ c, \ d, \ 4, \ 5 \ \}$

(d) $C - B$

Solution: $\varnothing$

(e) $(A \cap B) \times C$

Solution: $\{ \ (4, \ a), \ (4, \ b), \ (5, \ a), \ (5, \ b) \ \}$

*(f) $\overline{A}$

Solution: $\mathbb{Z} - A$

2. For each of the following sets, draw the corresponding Venn diagram:

   (a) $A \cap B$

   (b) $A \cup B$

   (c) $B - C$

   (d) $C - B$

   (e) $\overline{A}$

3. For each of the following statements, let $|A| = n$ and $|B| = m$. If $A \subseteq B$, then what is the cardinality of each of the following sets:

   (a) $|A \cap B|$

   Solution: n

   (b) $|A - B|$

   Solution: 0

   (c) $|B \oplus A|$

   Solution: $m - n$

   (d) $|B \times A|$

   Solution: nm

Hint: for the following parts, you might find it useful to restate each set as an equivalent set, using applications of the *set identities* before trying to determine cardinality

   *(e) $|A \cup (A \cap B)|$

   Solution: $A \cup (A \cap B) = A$, by *absorption law*, so $|A \cup (A \cap B)| = |A| = n$.

   *(f) $|(\overline{A} \cap B) \cup (A \cap B)|$

   Solution: $(\overline{A} \cap B) \cup (A \cap B) = B \cap (A \cup \overline{A}) = B \cap U = B$, by *distributive law*, *complement law*, *identity law*, respectively, so $|(\overline{A} \cap B) \cup (A \cap B)| = |B| = m$.

   *(g) $|A \cap (B \cap \overline{B})|$

   Solution: $A \cap (B \cap \overline{B}) = A \cap \varnothing = \varnothing$, by *complement law* and *domination law*, respectively, so $|A \cap (B \cap \overline{B})| = |\varnothing| = 0$.