Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

Random variables and expectation

1. Consider a game in which a fair die is rolled. If the die comes up 1, the player wins \$2. If the die comes up 2, the player wins \$1. For all other outcomes, the player loses \$1. Let X denote the amount of money won by the player for a single role of the die.

   (a) What is the range of X?

   Solution: { -1, 1, 2 }

   (b) What is the distribution over the random variable X?

   Solution: $(-1, \frac{4}{6}),\ (1, \frac{1}{6}), (2, \frac{1}{6})$

   (c) What is the expected amount that the player wins or loses, i.e., what is $E[X]$? Round to the nearest cent.

   Solution:
   $E[X] = \sum_{s \in S} X(s)p(s) = 2 \cdot \frac{1}{6} + 1 \cdot \frac{1}{6} + (-1) \cdot \frac{1}{6} + (-1) \cdot \frac{1}{6} + (-1) \cdot \frac{1}{6} + (-1) \cdot \frac{1}{6} = -.16\overline{7}$
   $E[X] = \sum_{r \in X(S)} r \cdot p(X = r) = 2 \cdot \frac{1}{6} + 1 \cdot \frac{1}{6} + (-1) \cdot \frac{4}{6} = -.16\overline{7}$
   The player loses 17 cents.

2. In a network of 40 computers, 5 hold a copy of a particular file. Suppose that 7 computers at random fail. Let F denote the number of computers that fail and have a copy of the file.

(a) What is the range of F?

Solution: $\{ 0, 1, 2, 3, 4, 5 \}$

(b) What is $p(F = 2)$?

Solution: $\frac{\binom{5}{2}\binom{35}{5}}{\binom{40}{7}}$

(c) What is the distribution over the random variable F?

Solution: $(0, \frac{\binom{35}{7}}{\binom{40}{7}}),\ (1, \frac{\binom{5}{1}\binom{35}{6}}{\binom{40}{7}}),\ (2, \frac{\binom{5}{2}\binom{35}{5}}{\binom{40}{7}}),\ (3, \frac{\binom{5}{3}\binom{35}{4}}{\binom{40}{7}}),\ (4, \frac{\binom{5}{4}\binom{35}{3}}{\binom{40}{7}}),\ (5, \frac{\binom{5}{5}\binom{35}{2}}{\binom{40}{7}})$
$\approx (0, .3606),\ (1, .4353),\ (2, .1741),\ (3, .0281),\ (4, .0018),\ (5, .0001)$

(d) What is the expected number of computers that will fail and have a copy of the file, i.e., what is $E[F]$? Round to the nearest computer.

Solution: $E[F] = \sum_{s \in S} F(s)p(s) = \sum_{r \in F(S)} r \cdot p(F = r)$
$= 0 \cdot .3606 + 1 \cdot .4353 + 2 \cdot .1741 + 3 \cdot .0281 + 4 \cdot .0018 + 5 \cdot .0001 = .8755$
1 computer with the file will fail.