

1. Using set-builder notation, give formal descriptions of the following sets:

(a) The set of positive integers that are even.

**Solution:**  $\{ x \in \mathbb{Z}^+ : x \% 2 = 0 \}$

(b)  $\{ -2, -1, 0, 1, 2 \}$

**Solution:**  $\{ x \in \mathbb{Z} : |x| \leq 2 \}$  or  $\{ x \in \mathbb{Z} : -2 \leq x \leq 2 \}$

(c)  $\{ 3, 6, 9, 12, \dots \}$

**Solution:**  $\{ x \in \mathbb{Z}^+ : x \% 3 = 0 \}$

(d)  $\{ -3, -1, 1, 3, 5, 7, 9 \}$

**Solution:**  $\{ x \in \mathbb{Z} : x \% 2 = 1 \wedge -3 \leq x \leq 9 \}$

(e)  $\{ 0, 10, 20, 30, \dots, 1000 \}$

**Solution:**  $\{ x \in \mathbb{Z} : x \% 10 = 0 \wedge 0 \leq x \leq 1000 \}$

\*(f) The power set of  $X$ , denoted  $P(X)$ .

**Solution:**  $\{ A : A \subseteq X \}$

2. State the cardinality of the following sets:

(a) Question 1, part (a)

**Solution:**  $\infty$

(b) Question 1, part (b)

**Solution:** 5

(c) Question 1, part (d)

**Solution:**  $\infty$

(d) Question 1, part (e)

**Solution:** 101

(e) Question 1, part (f)

**Solution:**  $2^{|X|}$

Consider the following sets:

$$A = \{ 2, 4, 6, 8 \}$$

$$B = \{ x \in \mathbb{Z} : x \% 2 = 0 \wedge 0 < x < 10 \}$$

$$C = \{ x \in \mathbb{Z} : x \% 2 = 0 \wedge 0 < x \leq 10 \}$$

3. Indicate whether each statement about the sets  $A$ ,  $B$  and  $C$  is true or false.

(a)  $A \subseteq B$

**Solution:** True. Every element of  $A$  is also an element of  $B$ .

(b)  $A \subset B$

**Solution:** False.  $A = B$ . There is no element of  $B$  that is not also an element of  $A$ .

(c)  $A \subseteq C$

**Solution:** True. Every element of  $A$  is also an element of  $C$ .

(d)  $A \subset C$

**Solution:** True. Every element of  $A$  is also an element of  $C$ . Also  $10 \in C$  and  $10 \notin A$ .

(e)  $C \subseteq B$

**Solution:** False.  $10 \in C$  and  $10 \notin B$ .

(f)  $A = C$

**Solution:** False.  $10 \in C$  and  $10 \notin A$ .

(g)  $A = B$

**Solution:** True.  $B = \{ 2, 4, 6, 8 \}$ .

4. Using roster notation, give formal descriptions of the following power sets:

(a)  $P(A)$

**Solution:**  $P(A) = \{ \emptyset, \{ 2 \}, \{ 4 \}, \{ 6 \}, \{ 8 \}, \{ 2, 4 \}, \{ 2, 6 \}, \{ 2, 8 \}, \{ 4, 6 \}, \{ 4, 8 \}, \{ 6, 8 \}, \{ 2, 4, 6 \}, \{ 2, 4, 8 \}, \{ 2, 6, 8 \}, \{ 4, 6, 8 \}, \{ 2, 4, 6, 8 \} \}$

(b)  $P(B)$

**Solution:**