Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

1. Construct a truth table for the following compound propositions:

(a) $\neg(p \vee q)$

Solution:

p	q	$\neg(p \vee q)$
0	0	1
0	1	0
1	0	0
1	1	0

(b) $\neg(p \wedge \neg(q \wedge s))$

Solution:

p	q	s	$\neg(p \wedge \neg(q \wedge s))$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

2. (a) Using the propositions p="I study", q="I will pass the course", r="The professor accepts bribes", translate the following into statements of propositional logic:
   1. If I do not study, then I will not pass the course unless the professor accepts bribes.
   2. If the professor accepts bribes, then I will pass the course regardless of whether or not I study.
   3. The professor does not accept bribes, but I study and will pass the course.

Solution:

1. $(\neg p \wedge q) \rightarrow r \equiv (\neg p \wedge \neg r) \rightarrow \neg q$
2. $r \rightarrow q$
3. $\neg r \wedge p \wedge q$

(b) Using the propositions p="The night hunting is successful", q="The moon is full", r="The sky is cloudless", translate the following into plain language:
   1. $p \rightarrow (q \wedge r)$
   2. $\neg r \leftrightarrow q$
   3. $(\neg r \wedge p) \rightarrow q$

Solution:

1. For successful night hunting it is necessary that the moon is full and the sky is cloudless.
2. The sky being cloudy is both necessary and sufficient for the night hunting to be successful.
3. If the sky is cloudy, then the night hunting will not be successful unless the moon is full.

3. Using a truth table, find which of the following compound propositions are always true (a tautology), regardless of the values of p and q:

   1. $p \to (p \vee q)$

   2. $p \to (p \to q)$

   3. $\neg(p \to (p \vee q))$

Solution:

1. Always true
2. Not true for $p = 1, q = 0$
3. Always false

4. Find the inverse, converse and contrapositive for the following compound propositions, then evaluate each when p is true and q is false:

   1. $p \to \neg q$

Solution: inverse: $\neg p \to q = T$
converse: $\neg q \to p = T$
contrapositive: $q \to \neg p = T$

   2. $\neg p \to (p \vee q)$

Solution: inverse: $p \to \neg(p \vee q) = F$
converse: $(p \vee q) \to \neg p = F$
contrapositive: $\neg(p \vee q) \to p = T$

   3. $p \to (p \to q)$

Solution: inverse: $\neg p \to \neg(p \to q) = T$
converse: $(p \to q) \to p = T$
contrapositive: $\neg(p \to q) \to \neg p = F$