CSCI 239—Discrete Structures of Computer Science
Lab 9—Solving Recurrence Relations

This lab will explore will explore ways to solve recurrence relations using computing tools

Objectives:

- to learn to use linear algebra and MATLAB to solve systems of linear equations
- to get practice proving the correctness of closed forms for recurrence relations

This lab will work with the same recurrence relations as in Lab 8. You'll need to refer to the results from that lab to complete this one.

Part 1: Solving linear non-homogeneous recurrence relations

For each of the recurrence relations *F*, *G*, *H*, *J*, and *Q* from Lab 8, complete the following steps. See the Lab 8 write-up to find the recurrence relations.

a) Determine whether the closed form solution is quadratic, cubic, or quartic (one degree higher than the polynomial in the recurrence relation), and note whether the base value is zero or one.

b) For the quadratic case, we need to find the coefficients for the equation

$$f(n) = a_2x^2 + a_1x + a_0$$

for the cubic case, we need the coefficients for the equation

$$f(n) = a_3x^3 + a_2x^2 + a_1x + a_0$$

c) Open MATLAB from the Windows apps list and use the appropriate matrix and vector indicated below to compute *A* \ *v*, where the elements of *v* are the corresponding values of the recurrence relation computed in Lab 8.

For a quadratic solution with a zero base value, the matrix and vector are:

$$A_{3,0} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{pmatrix} \qquad v = \begin{pmatrix} F(0) \\ F(1) \\ F(2) \end{pmatrix}$$

For a quadratic solution with a one base value, they are:

$$A_{3,1} = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{pmatrix} \qquad v = \begin{pmatrix} F(1) \\ F(2) \\ F(3) \end{pmatrix}$$

For a cubic solution with a zero base value, they are:

$$A_{4,0} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \end{pmatrix} \qquad v = \begin{pmatrix} F(0) \\ F(1) \\ F(2) \\ F(3) \end{pmatrix}$$

For a cubic solution with a one base value, use the last three rows of the matrix and vector above and add a new fourth row that fits the pattern. (That is, remove the top rows pertaining to $F(0)$ and add new rows pertaining to $F(4)$ at the bottom.)

For a quartic solution with a zero base value, you should be able to figure out the values of the 5 × 5 matrix from the cubic matrix above; the vector values just extend the pattern.

The result of the backslash division in MATLAB will be a column vector containing the coefficients of the solution polynomial from highest degree to lowest.

d) Write a Haskell function that computes the value of the closed form polynomial. Use the map function again to compute the first ten values of the function. Compare the values you get for the closed form with the values you got in Lab 8 and verify they are the same. If not, determine what went wrong and go back and correct it. (If your coefficients are not integers, there may be round-off error that will cause the values to be slightly different.)

e) Once your values agree, use pencil and paper to prove by induction that your closed-form solution is correct. You'll need to use exact (fractional) values for the coefficients if they are not integers.

Part 2: A recurrence relation with a poly-log solution

The recurrence relation *L* in Lab 8 has a *poly-log* solution, that is, a solution that is a polynomial with a logarithmic term. The exact solution is difficult to express and even harder to prove because it only works out nicely when *n* is a power of two. Instead, we'll use an upper-bound approximation: $L(n) \leq n[(log_2 n) + 1]$.

Use pencil and paper to prove by strong induction that the inequality holds.

Part 3: A closed form solution to a recurrence relation with a non-polynomial solution

Use your solutions from Lab 8 to make a good guess about the closed-form solution for recurrence relation *T*. Implement this guess as a Haskell function, and use *map* to verify that you get the same values. If not, guess again until you get matching values.

Use pencil and paper to prove your answer is correct by induction.