Use the following spaces to record any information about key topics that you find useful.

⇒ Prove the following propositional statements using strong induction.

Proposition. *Every positive integer can be written as a sum of distinct powers of 2.*

Proof. By strong mathematical induction on n.

(1) *Base case.* $n = 1$.

Solution: $2^0 = 1$.

(2) *Inductive step.*

Solution:
Assume for $k \geq 1$, k can be written as the sum of distinct powers of 2.

Show that $k + 1$ can be written as the sum of distinct powers of 2.

$\exists_j \; 2^j \leq k + 1 < 2^{j+1}$.

If $2^j = k+1$, then $k+1$ can be written as sum of distinct powers of 2, namely 2^j, and we are done.

If $2^j \neq k + 1$, then $2^j < k + 1 < 2^{j+1}$, so

$0 < k + 1 - 2^j < 2^{j+1} - 2^j = 2^j \leq k$.

Since $0 < k + 1 - 2^j < k$, the inductive hypothesis guarantees that $k + 1 - 2^j$ can be written as a sum of distinct powers of 2 and the powers are less than j. Thus, $k + 1 = 2^j +$ a sum of distinct powers of 2 where the powers of 2 are less than j.

□