Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

# Product and sum rule

Consider the following definitions for sets of characters:

- Digits = { 0 .. 9 }
- Letters = { $a$ .. $z$ }
- Special characters = { $*$, &, \$, # }

1. Compute the number of passwords that satisfy the given constraints.
   (a) Strings of length 6. Characters can be special characters, digits, or letters.

   (b) Strings of length 7, 8, or 9. Characters can be special characters, digits, or letters.

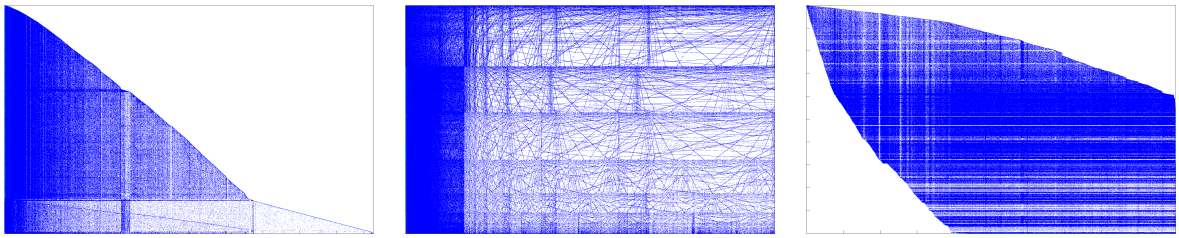
   (c) Strings of length 7, 8, or 9. Characters can be special characters, digits, or letters. The first character cannot be a letter.

2. Consider the numbers in the range 1 to $10^4$ (inclusive).
   (a) How many of the numbers in the range $10^3$ to $10^4$ (inclusive) are composed of all distinct digits?

   (b) How many of the numbers in the range 1 to $10^4$ (inclusive) are composed of all distinct digits?

## Permuations

Consider the following permutations of a single adjacency matrix:



(a) Original (b) Clean (c) RCM

Permuting the rows / columns of a matrix can be beneficial for a couple reasons:

1. Improve computational efficiency by improving structure of adjacency matrix
2. Expose hidden structure in the graph

Unfortunately, knowing *apriori* which permutation is best is a difficult problem. The problem is compounded by the huge number of permutations possible.

As a result, *heuristic* methods are usually employed. These methods, like (b) and (c) above, are efficient to find and lead to good, but not necessarily optimal, permutations of the matrix.

3. How many ways are there to permute the rows / columns of a
   (a) $10 \times 10$ matrix?

   (b) $n \times n$ matrix?

## Combinations

4. Suppose a network has 40 computers of which 5 fail.
   (a) How many possibilities are there for the five that fail?

   (b) Suppose that 3 of the computers in the network have a copy of a particular file. How many sets of failures wipe out all the copies of the file? That is, how many 5-subsets contain the three computers that have the file?