Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

1. Using the laws of propositional logic, determine which of the following are equivalent to $(p \wedge q) \to r$ and which are equivalent to $(p \vee q) \to r$. Confirm your answer using truth-tables.
   (a) $p \to (q \to r)$
   (b) $q \to (p \to r)$
   (c) $(p \to r) \wedge (q \to r)$
   (d) $(p \to r) \vee (q \to r)$

Solution:

1. $p \to (q \to r)$
   $\neg p \vee (q \to r)$ $(conditional\ identity)$
   $\neg p \vee \neg q \vee r$ $(conditional\ identity)$
   $\neg(p \wedge q) \vee r$ $(deMorgan's)$
   $\equiv (p \wedge q) \to r$ $(conditional\ identity)$
2. $q \to (p \to r)$
   $\neg q \vee (p \to r)$ $(conditional\ identity)$
   $\neg q \vee \neg p \vee r$ $(conditional\ identity)$
   $\neg(p \wedge q) \vee r$ $(deMorgan's)$
   $\equiv (p \wedge q) \to r$ $(conditional\ identity)$
3. $(p \to r) \wedge (q \to r)$
   $(\neg p \vee r) \wedge (\neg q \vee r)$ $(cond.\ identity)$
   $r \vee (\neg p \wedge \neg q)$ $(distributive)$
   $r \vee \neg(p \vee q)$ $(deMorgan's)$
   $\equiv (p \vee q) \to r$ $(cond.\ identity)$
4. $(p \to r) \vee (q \to r)$
   $(\neg p \vee r) \vee (\neg q \vee r)$ $(cond.\ identity)$
   $r \vee r \vee \neg p \vee \neg q$ $(commutative)$
   $r \vee \neg p \vee \neg q$ $(idempotent)$
   $r \vee \neg(p \wedge q)$ $(deMorgan's)$
   $\equiv (p \wedge q) \to r$ $(cond.\ identity)$

2. Convert the following English sentences into logical formulas. You may use expressions like $x = y$ or $x \neq y$ to indicate whether or not the variables x and y denote different people. The domain of discourse is all people. Let the predicate $H(x)$ mean that "x is happy," and let the predicate $L(x, y)$ mean that "x loves y."
   (a) At least one person is happy.
   (b) No one is happy.
   (c) At least one person is unhappy.
   (d) Exactly one person is happy.
   (e) Not everyone loves someone else.
   (f) Everyone loves someone else.

Solution:

1. $\exists x H(x)$
2. $\neg \exists x H(x) \equiv \forall x(\neg H(x))$
3. $\exists x(\neg H(x))$
4. $\exists x(H(x) \wedge \forall y(H(y) \to x = y))$
5. $\exists x \forall y(\neg L(x, y))$
   $\neg(\forall x \exists y L(x, y))$
   $\exists x \neg(\exists y L(x, y))$
6. $\neg(\exists x \forall y(\neg L(x, y))) \equiv \forall x \exists y L(x, y)$
   $\neg(\neg(\forall x \exists y L(x, y))) \equiv \forall x \exists y L(x, y)$
   $\neg(\exists x \neg(\exists y L(x, y))) \equiv \forall x \exists y L(x, y)$

3. Let the domain of discourse be all members of the class and let $L(x, y)$ be the predicate "x likes y." Translate the following into plain language:
   (a) $\forall x \exists y (L(x, y) \wedge x \neq y)$
   (b) $\exists x \neg \exists y (L(x, y) \vee L(y, x))$
   (c) $\exists x \exists y \exists z \exists w (L(x, w) \wedge L(y, w) \wedge L(z, w) \wedge x \neq y \neq z)$

Solution:

1. Everyone in the class likes some other member of the class.
2. There is a person who doesn't like anyone and who nobody likes.
3. At least three different people like the same person.

4. A certain cabal (*cabal: a secret political clique or faction*) within the CS department is plotting to make the final exam *ridiculously hard*. The only way to stop their evil plan is to determine exactly who is in the cabal. The department includes Donald, Grace, Linus, Alan, Ada and Edsger. The cabal is a subset of these six. A membership roster has been found and appears below, but it is deviously encrypted in logic notation. The predicate *cabal* indicates who is in the cabal; that is, $cabal(x)$ is true if and only if x is a member of the cabal. Use the following information to gather who is in the cabal.

   1. $\exists x \exists y \exists z (x \neq y \wedge x \neq z \wedge y \neq z \wedge cabal(x) \wedge cabal(y) \wedge cabal(z))$
   2. $\exists x (\neg cabal(x))$
   3. $cabal(Edsger) \rightarrow \forall x (cabal(x))$
   4. $\neg(cabal(Donald) \wedge cabal(Alan)) \wedge (cabal(Donald) \vee cabal(Alan))$
   5. $cabal(Alan) \rightarrow cabal(Donald)$
   6. $(cabal(Ada) \vee cabal(Linus)) \rightarrow \neg(cabal(Grace))$

Solution: Donald, Ada, and Linus are in the cabal