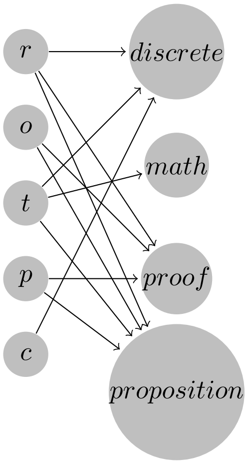
Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on a separate sheet of paper.

1. Define the set $A = \{ \ r, \ o, \ t, \ p, \ c \ \}$ and $B = \{ \ \textit{discrete}, \ \textit{math}, \ \textit{proof}, \ \textit{proposition} \ \}$. Define the relation $R \subseteq A \times B$ such that (letter, word) is in the relation if that letter occurs somewhere in the word. Draw the arrow diagram and the matrix representation for each relation.

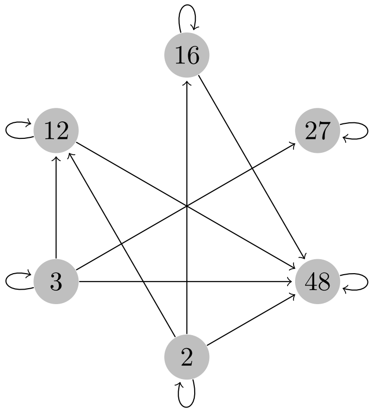
Solution:



$$A = \begin{array}{cc} \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} & \begin{array}{c} r \\ o \\ t \\ p \\ c \end{array} \end{array}$$

2. The domain of relation D is $\{\ 2,\ 3,\ 12,\ 16,\ 27,\ 48\ \}$. For x, y in the domain, xDy if y is an integer multiple of x. Draw the arrow diagram and the matrix representation for the relation.

Solution:



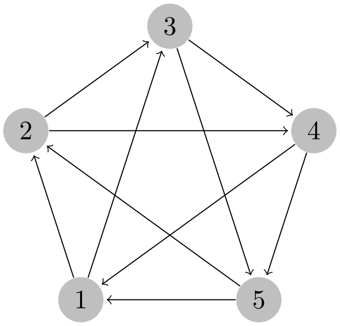
$$A = \begin{array}{cc} \begin{matrix} 2 & 3 & 12 & 16 & 27 & 48 \end{matrix} & \\ \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} & \begin{matrix} 2 \\ 3 \\ 12 \\ 16 \\ 27 \\ 48 \end{matrix} \end{array}$$

3. Consider the following relation F. The domain of the relation F is all Facebook users. For x, y in the domain, xFy if x and y are Facebook friends. Furthermore, $xFy \rightarrow yFx$.
   (a) List all properties of the relation F.

   (b) Read about "*Six degrees of separation*" on Wikipedia. What is the implication of this claim for the relation F described above?

   (c) Now read this article `https://research.fb.com/blog/2016/02/three-and-a-half-degrees-of-separation/`. What does this mean for you?

4. Define the directed graph G as follows:



(a) Classify each of the following sequences of vertices as either a *walk* in G or *not a walk* in G. If a sequence, w represents a walk in G, characterize the walk as an *open walk*, *closed walk*, *trail*, *circuit*, *path* or *cycle*, being as specific as possible.

i. $\langle 1,\ 2,\ 4,\ 5,\ 2\rangle$

Solution: trail

ii. $\langle 1,\ 3,\ 5,\ 2,\ 4,\ 1\rangle$

Solution: cycle

iii. $\langle 1,\ 2,\ 3\rangle$

Solution: path

iv. $\langle 3,\ 5,\ 1,\ 2,\ 4,\ 1,\ 3\rangle$

Solution: circuit

(b) For each of the following, find a walk in G that satisfies the requirements specified.

i. cycle of length 3

Solution:
$\langle 1,\ 3,\ 5,\ 1\rangle$

ii. trail of length 8

Solution:
$\langle 1,\ 2,\ 3,\ 4,\ 5,\ 1,\ 3,\ 5,\ 2\rangle$

iii. path of length 4 starting at 5 and ending at 3

Solution:
$\langle 5,\ 2,\ 4,\ 1,\ 3\rangle$