# CSCI 239—Discrete Structures of Computer Science

## Lab 7—Inductive Proofs

This lab consists of pencil-and-paper exercises constructing inductive proofs.

### Objectives:

* to be able to identify the base case, the inductive hypothesis, and the target of the inductive step for a given statement that can be proved using induction.
* to be able to complete the steps of an inductive proof once the components are identified.

### Instructions

For each of the problems below, where the example problem is to prove *f*(*n*) = *g*(*n*) for all *n* ≥ 1:

1. Identify the base value for the induction and state what is to be proved for the base case.  
   In the example: *n* = 1, show *f*(1) = *g*(1).
2. Identify the inductive hypothesis.  
   In the example: *f*(*k*) = *g*(*k*) for some *k* ≥ 1.
3. Identify the statement to be shown in the inductive step.  
   In the example: *f*(*k*+1) = *g*(*k*+1).
4. Complete the inductive proof using *a*, *b*, and *c*.

### Problems

1. .
2. Find a closed-form formula for by inspecting the values of this expression for small values of *n*. Then follow the steps above to show your formula is correct using induction.
3. , for positive integers *n*.
4. 3 divides, for all positive integers *n*.
5. 2 divides , for all positive integers *n*.  
   Give a second proof by cases that does not use induction.
6. 5 divides , for all non-negative integers *n*.
7. , if for *j* = 1, 2, …, *n*. The proof should be similar to one for a summation. Think in terms of showing that every element in the first union is an element of the second union.
8. If *A* = , then *An* = . Recall that *a* = *a*1.