Data representation

College of Saint Benedict & Saint John's University

decimal refresher

58036

decimal refresher

5		8		0		3		6
50000	+	8000	+	0	+	30	+	6

decimal refresher

5		8		0		3		6
50000	+	8000	+	0	+	30	+	6
5×10000	+	8×1000	+	0×100	+	3×10	+	6×1

decimal refresher

5		8		0		3		6
50000	+	8000	+	0	+	30	+	6
5×10000	+	8×1000	+	0×100	+	3×10	+	6×1
5×10^4	+	8×10^3	+	0×10^2	+	3×10^1	+	6×10^0

binary refresher

10110

binary refresher

1		0		1		1		0
1×2^4	$+$	0×2^3	$+$	1×2^2	$+$	1×2^1	$+$	0×2^0

binary refresher

1		0		1		1		0
1×2^4	$+$	0×2^3	$+$	1×2^2	$+$	1×2^1	$+$	0×2^0
1×16	$+$	0×8	$+$	1×4	$+$	1×2	$+$	0×1

binary refresher

1		0		1		1		0
1×2^4	+	0×2^3	+	1×2^2	+	1×2^1	+	0×2^0
1×16	+	0×8	+	1×4	+	1×2	+	0×1
16	+	0	+	4	+	2	+	0

unsigned addition

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10$$

$$0\ 0\ 0\ 1\ 0\ 1 = 5$$

$$\text{ADD}\quad 0\ 0\ 0\ 1\ 0\ 1 = 5$$

---

unsigned addition

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10$$

$$0\ 0\ 0\ 1\ 0\ 1 = 5$$

$$\text{ADD}\quad 0\ 0\ 0\ 1\ 0\ 1 = 5$$

$$\text{C} \leftarrow 0\quad 0\ 0\ 1\ 0\ 1\ 0 = 10$$

more unsigned addition

	1	1		0	1	0	1	$= 53$
ADD	1	1		0	0	0	0	$= 48$

more unsigned addition

	1	1	0	1	0	1	$= 53$
ADD	1	1	0	0	0	0	$= 48$
C $\leftarrow$ 1	1	0	0	1	0	1	$= 37$

signed addition

$$
\begin{array}{lcccccc}
 & 0 & 0 & 0 & 1 & 0 & 1 \quad = +5 \\
\text{ADD} & 1 & 0 & 0 & 1 & 0 & 1 \quad = -5 \\
\hline
\end{array}
$$

signed addition

$$
\begin{array}{rcccccc}
 & 0 & 0 & 0 & 1 & 0 & 1 \quad = +5 \\
\text{ADD} & 1 & 0 & 0 & 1 & 0 & 1 \quad = -5 \\
\hline
C = 0 & 1 & 0 & 1 & 0 & 1 & 0 \quad = -10
\end{array}
$$

method of complements

- technique used to subtract one number from another using only addition of positive numbers
- represent negative numbers as two's complement of their positive counterparts

method of complements

- technique used to subtract one number from another using only addition of positive numbers
- represent negative numbers as two's complement of their positive counterparts

two's complement

- find one's complement
- add one

one’s complement

NOT 0 0 0 1 0 1

one’s complement

NOT	0	0	0	1	0	1
	1	1	1	0	1	0

one’s complement

NOT	0	0	0	1	0	1
	1	1	1	0	1	0
	0	0	0	1	0	1
ADD	1	1	1	0	1	0

one’s complement

NOT	0	0	0	1	0	1
	1	1	1	0	1	0
	0	0	0	1	0	1
ADD	1	1	1	0	1	0
C = 0	1	1	1	1	1	1

two's complement

NOT	0	0	0	1	0	1
	1	1	1	0	1	0
	0	0	0	1	0	1
ADD	1	1	1	0	1	0
C = 0	1	1	1	1	1	1
ADD	0	0	0	0	0	1

two's complement

NOT	0	0	0	1	0	1
	1	1	1	0	1	0
	0	0	0	1	0	1
ADD	1	1	1	0	1	0
C = 0	1	1	1	1	1	1
ADD	0	0	0	0	0	1
C = 1	0	0	0	0	0	0

two’s complement cont’d

	1	1	1	0	1	0
ADD	0	0	0	0	0	1
	1	1	1	0	1	1

cpu bits

	0	1
N	otherwise	result is negative
Z	otherwise	result is all zeros
V	otherwise	signed integer overflow occurred
C	otherwise	unsigned integer overflow occurred

register transfer language

operation	RTL symbol
AND	$\wedge$
OR	$\vee$
XOR	$\oplus$
NOT	$\neg$
Implies	$\rightarrow$
Transfer	$\leftarrow$
Bit index	$\langle\,\rangle$
Informal description	{ }
Sequential separator	;
Concurrent separator	,

register transfer language

operation	RTL symbol
AND	$\wedge$
OR	$\vee$
XOR	$\oplus$
NOT	$\neg$
Implies	$\rightarrow$
Transfer	$\leftarrow$
Bit index	$\langle\,\rangle$
Informal description	{ }
Sequential separator	;
Concurrent separator	,

$$c \leftarrow a \oplus b; N \leftarrow c < 0, Z \leftarrow c = 0$$

another example

		0	0	0	1	0	1
	ADD	1	1	1	0	1	1
N ←	0	0	0	0	0	0	0
Z ←	1						
V ←	?						
C ←	1						

another example

		0	0		0	1	0	1
	ADD	1	1		1	0	1	1
N $\leftarrow$	0	0	0		0	0	0	0
Z $\leftarrow$	1							
V $\leftarrow$	$\neg(a\langle 0\rangle \oplus b\langle 0\rangle) \wedge (a\langle 0\rangle \oplus N)$							
C $\leftarrow$	1							

arithmetic shift

arithmetic shift left (asl)

$$C \leftarrow r\langle 0\rangle,\ r\langle 0..4\rangle \leftarrow \langle 1..5\rangle,\ r\langle 5\rangle \leftarrow 0;$$
$$N \leftarrow r < 0,\ Z \leftarrow r = 0,\ V \leftarrow \{\text{overflow}\}$$

arithmetic shift right (asr)

?

arithmetic shift

arithmetic shift left (asl)

$$C \leftarrow r\langle 0\rangle,\ r\langle 0..4\rangle \leftarrow \langle 1..5\rangle,\ r\langle 5\rangle \leftarrow 0;$$
$$N \leftarrow r < 0,\ Z \leftarrow r = 0,\ V \leftarrow \{\text{overflow}\}$$

arithmetic shift right (asr)

$$C \leftarrow r\langle 5\rangle,\ r\langle 1..5\rangle \leftarrow \langle 0..4\rangle;$$
$$Z \leftarrow r = 0$$

hexadecimal

unicode

Hello world.

¡Hola!, Grüß Gott, Hyvää päivää, Tere õhtust, Bonġu Cześć!, Dobrý den

你好, 早晨, こんにちは

unicode

Hello world.

¡Hola!, Grüß Gott, Hyvää päivää, Tere õhtust, Bonġu Cześć!, Dobrý den 你好, 早晨, こんにちは

https://www.paypal.com

unicode

Hello world.

¡Hola!, Grüß Gott, Hyvää päivää, Tere õhtust, Bonġu Cześć!, Dobrý den

你好, 早晨, こんにちは

https://www.paypal.com

floating-point

IEEE 754

single precision 1.8.23 — excess 127 / 126

double precision 1.11.52 — excess 1023 / 1022

special values

	exponent	significand
zero	all zeros	all zeros
denormalized	all zeros	non-zero
inifinity	all ones	all zeros
not a number (NaN)	all ones	non-zero

operations that result in NaN

- The divisions $0/0$ and $\pm\infty/\pm\infty$
- The multiplications $0\times\pm\infty$ and $\pm\infty\times 0$
- The additions $\infty + (-\infty)$, $(-\infty) + \infty$ and equivalent subtractions

floating-point

IEEE 754

single precision 1.8.23 — excess 127 / 126

double precision 1.11.52 — excess 1023 / 1022

special values

	exponent	significand
zero	all zeros	all zeros
denormalized	all zeros	non-zero
inifinity	all ones	all zeros
not a number (NaN)	all ones	non-zero

operations that result in NaN

- The divisions $0/0$ and $\pm\infty/\pm\infty$
- The multiplications $0\times\pm\infty$ and $\pm\infty\times 0$
- The additions $\infty + (-\infty)$, $(-\infty) + \infty$ and equivalent subtractions