Data representation

College of Saint Benedict & Saint John's University

decimal refresher

58036

decimal refresher

5		8		0		3		6
50000	+	8000	+	0	+	30	+	6

decimal refresher

5		8		0		3		6
50000	+	8000	+	0	+	30	+	6
5×10000	+	8×1000	+	0×100	+	3×10	+	6×1

decimal refresher

5		8		0		3		6
50000	+	8000	+	0	+	30	+	6
5×10000	+	8×1000	+	0×100	+	3×10	+	6×1
5×10^4	+	8×10^3	+	0×10^2	+	3×10^1	+	6×10^0

binary refresher

10110

binary refresher

1		0		1		1		0
1×2^4	$+$	0×2^3	$+$	1×2^2	$+$	1×2^1	$+$	0×2^0

binary refresher

1		0		1		1		0
1×2^4	$+$	0×2^3	$+$	1×2^2	$+$	1×2^1	$+$	0×2^0
1×16	$+$	0×8	$+$	1×4	$+$	1×2	$+$	0×1

binary refresher

1		0		1		1		0
1×2^4	+	0×2^3	+	1×2^2	+	1×2^1	+	0×2^0
1×16	+	0×8	+	1×4	+	1×2	+	0×1
16	+	0	+	4	+	2	+	0

unsigned addition

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10$$

$$0\ 0\ 0\ 1\ 0\ 1 = 5$$

$$\text{ADD}\quad 0\ 0\ 0\ 1\ 0\ 1 = 5$$

---

unsigned addition

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10$$

$$\begin{array}{rcccccl} & 0 & 0 & 0 & 1 & 0 & 1 = 5 \\ \text{ADD} & 0 & 0 & 0 & 1 & 0 & 1 = 5 \\ \hline \text{C} = 0 & 0 & 0 & 1 & 0 & 1 & 0 = 10 \end{array}$$

signed addition

$$
\begin{array}{lcccccc}
 & 0 & 0 & 0 & 1 & 0 & 1 \quad = +5 \\
\text{ADD} & 1 & 0 & 0 & 1 & 0 & 1 \quad = -5 \\
\hline
\end{array}
$$

signed addition

$$
\begin{array}{rcccccc}
 & 0 & 0 & 0 & 1 & 0 & 1 \quad = +5 \\
\text{ADD} & 1 & 0 & 0 & 1 & 0 & 1 \quad = -5 \\
\hline
C = 0 & 1 & 0 & 1 & 0 & 1 & 0 \quad = -10
\end{array}
$$

one’s complement

NOT 0 0 0 1 0 1

one’s complement

NOT	0	0	0	1	0	1
	1	1	1	0	1	0

one’s complement

NOT	0	0	0	1	0	1
	1	1	1	0	1	0
	0	0	0	1	0	1
ADD	1	1	1	0	1	0

one’s complement

NOT	0	0	0	1	0	1
	1	1	1	0	1	0
	0	0	0	1	0	1
ADD	1	1	1	0	1	0
C = 0	1	1	1	1	1	1

one’s complement

NOT	0	0	0	1	0	1
	1	1	1	0	1	0
	0	0	0	1	0	1
ADD	1	1	1	0	1	0
C = 0	1	1	1	1	1	1
ADD	0	0	0	0	0	1

one's complement

NOT	0	0	0	1	0	1
	1	1	1	0	1	0
	0	0	0	1	0	1
ADD	1	1	1	0	1	0
C = 0	1	1	1	1	1	1
ADD	0	0	0	0	0	1
C = 1	0	0	0	0	0	0

cpu bits

	0	1
N	otherwise	result is negative
Z	otherwise	result is all zeros
V	otherwise	signed integer overflow occurred
C	otherwise	unsigned integer overflow occurred

register transfer language

operation	RTL symbol
AND	$\wedge$
OR	$\vee$
XOR	$\oplus$
NOT	$\neg$
Implies	$\rightarrow$
Transfer	$\leftarrow$
Bit index	$\langle\,\rangle$
Informal description	{ }
Sequential separator	;
Concurrent separator	,

register transfer language

operation	RTL symbol
AND	$\wedge$
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Implies	$\rightarrow$
Transfer	$\leftarrow$
Bit index	$\langle\,\rangle$
Informal description	{ }
Sequential separator	;
Concurrent separator	,

$$c \leftarrow a \oplus b; N \leftarrow c < 0, Z \leftarrow c = 0$$

another example

		0	0	0	1	0	1
	ADD	1	1	1	0	1	1
N ←	0	0	0	0	0	0	0
Z ←	1						
V ←	?						
C ←	1						

another example

		0	0		0	1	0	1
	ADD	1	1		1	0	1	1
N $\leftarrow$	0	0	0		0	0	0	0

$Z \leftarrow 1$

$V \leftarrow \neg(a\langle 0\rangle \oplus b\langle 0\rangle) \wedge (a\langle 0\rangle \oplus N)$

$C \leftarrow 1$

arithmetic shift

arithmetic shift left (asl)

$$C \leftarrow r\langle 0\rangle,\ r\langle 0..4\rangle \leftarrow \langle 1..5\rangle,\ r\langle 5\rangle \leftarrow 0;$$
$$N \leftarrow r < 0,\ Z \leftarrow r = 0,\ V \leftarrow \{\text{overflow}\}$$

arithmetic shift right (asr)

?

arithmetic shift

arithmetic shift left (asl)

$$C \leftarrow r\langle 0\rangle,\ r\langle 0..4\rangle \leftarrow \langle 1..5\rangle,\ r\langle 5\rangle \leftarrow 0;$$
$$N \leftarrow r < 0,\ Z \leftarrow r = 0,\ V \leftarrow \{\text{overflow}\}$$

arithmetic shift right (asr)

$$C \leftarrow r\langle 5\rangle,\ r\langle 1..5\rangle \leftarrow \langle 0..4\rangle;$$
$$Z \leftarrow r = 0$$

hexadecimal

unicode

Hello world.

¡Hola!, Grüß Gott, Hyvää päivää, Tere õhtust, Bonġu Cześć!, Dobrý den

你好, 早晨, こんにちは

unicode

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floating-point

IEEE 754

single precision 1.8.23 — excess 127 / 126

double precision 1.11.52 — excess 1023 / 1022

special values

	exponent	significand
zero	all zeros	all zeros
denormalized	all zeros	non-zero
inifinity	all ones	all zeros
not a number (NaN)	all ones	non-zero

operations that result in NaN

- The divisions $0/0$ and $\pm\infty/\pm\infty$
- The multiplications $0\times\pm\infty$ and $\pm\infty\times 0$
- The additions $\infty + (-\infty)$, $(-\infty) + \infty$ and equivalent subtractions

floating-point

IEEE 754

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