Performance analysis

Jeremy Iverson

College of Saint Benedict & Saint John's University

plan for the day

- `reduction`
- theoretical analysis framework
- performing analysis
- experimental analysis
- conducting experiments

theoretical analysis framework

seeks to answer the following questions:

- how do we reason about parallel algorithms?
- how can we compare two algorithms and determine which is better?
- how do we measure improvement?

performance metrics

- execution time (T_s and T_p)
- speedup (S)
- efficiency (E)
- cost (C)

execution time

serial (T_s)

- time elapsed between beginning and end of execution

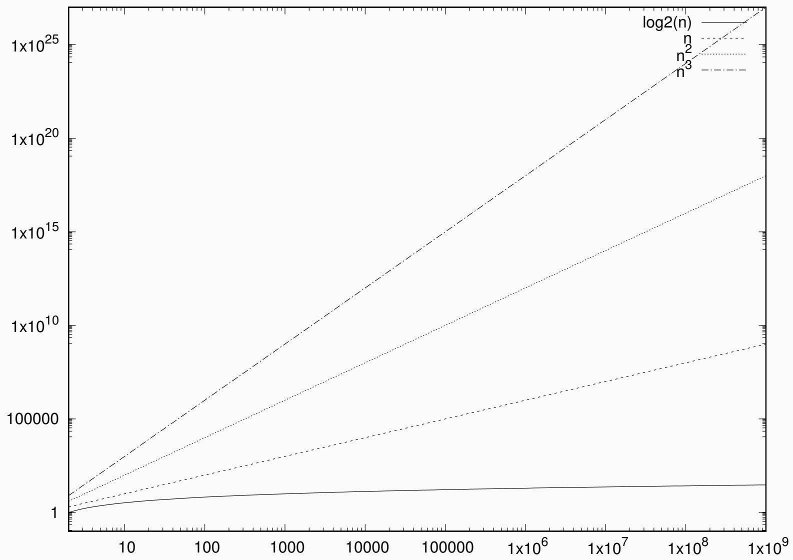
parallel (T_p)

- time elapsed between beginning of execution and the moment the last processing element finishes execution

- Axpy
- Reduction
- Dot-product
- Matrix-vector multiplication
- Matrix-matrix multiplication

1. For now, let's assume that $n = p$
2. Axpy — $T_s = \Theta(n)$ — $T_p = \Theta(1)$
3. Reduction — $T_s = \Theta(n)$ — $T_p = \Theta(\log n)$
4. Matrix-vector — $T_s = \Theta(n^2)$ — $T_p = \Theta(n)$
5. Matrix-matrix — $T_s = \Theta(n^3)$ — $T_p = \Theta(n^2)$
6. So how long to compute mat-mat for a 10000×10000 mat?

execution time



speedup

speedup ($S = T_s/T_p$)

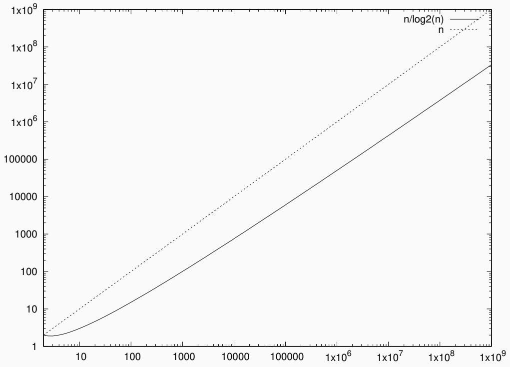
- the ratio of time taken to solve a problem on a single processing element to the time required to solve the same problem on a parallel computer with p processing elements

1. Axpy — $S = \Theta(\frac{n}{n/p}) = \Theta(p)$
2. Reduction — $S = \Theta(n/\log n)$
3. Matrix-vector — $S = \Theta(n^2/n) = \Theta(n)$
4. Matrix-matrix — $S = \Theta(n^3/n^2) = \Theta(n)$
5. What are the limits of speedup?

speedup

speedup ($S = T_s/T_p$)

- the ratio of time taken to solve a problem on a single processing element to the time required to solve the same problem on a parallel computer with p processing elements



1. Axpy — $S = \Theta(\frac{n}{n/p}) = \Theta(p)$
2. Reduction — $S = \Theta(n/\log n)$
3. Matrix-vector — $S = \Theta(n^2/n) = \Theta(n)$
4. Matrix-matrix — $S = \Theta(n^3/n^2) = \Theta(n)$
5. What are the limits of speedup?

Amdahl's law

Speedup is limited by the fraction of a parallel program that is serial.

If r is the fraction of the code which is serial, then

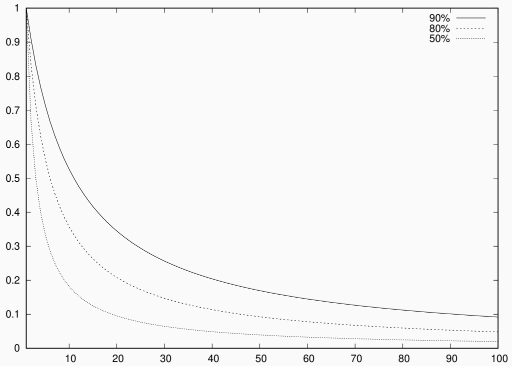
$$S = \frac{T_s}{T_p} = \frac{T_s}{(1 - r)T_s/p + rT_s}$$

Amdahl's law

Speedup is limited by the fraction of a parallel program that is serial.

If r is the fraction of the code which is serial, then

$$S = \frac{T_s}{T_p} = \frac{T_s}{(1-r)T_s/p + rT_s}$$



Amdahl's law

Speedup is limited by the fraction of a parallel program that is serial.

If r is the fraction of the code which is serial, then

$$S = \frac{T_s}{T_p} = \frac{T_s}{(1-r)T_s/p + rT_s}$$

In general, we cannot get a speed up better than

$$\frac{1}{r}$$

efficiency

efficiency ($E = S/p$)

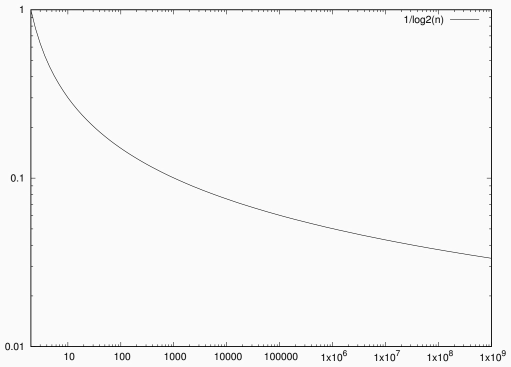
- the ratio of speedup to the number of processing elements — the fraction of time for which a processing element is usefully employed

1. Reduction — $E = \Theta(n/\log n)/n = \Theta(1/\log n)$
2. Matrix-vector — $E = \Theta(n)/n = \Theta(1)$
3. Matrix-matrix — $E = \Theta(n)/n = \Theta(1)$
4. Why do you think that the efficiency of adding numbers is less than matrix-vector or matrix-matrix?
5. What are the limits of efficiency?

efficiency

efficiency ($E = S/p$)

- the ratio of speedup to the number of processing elements — the fraction of time for which a processing element is usefully employed



1. Reduction — $E = \Theta(n/\log n)/n = \Theta(1/\log n)$
2. Matrix-vector — $E = \Theta(n)/n = \Theta(1)$
3. Matrix-matrix — $E = \Theta(n)/n = \Theta(1)$
4. Why do you think that the efficiency of adding numbers is less than matrix-vector or matrix-matrix?
5. What are the limits of efficiency?

cost

cost ($C = pT_p$)

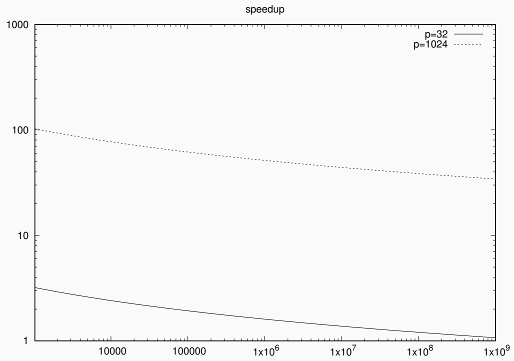
- the sum of the time spent by all processing elements solving the problem
- *cost-optimal* if $C = T_s$

1. Axpy — $C = \Theta(n)$
2. Reduction — $C = \Theta(n \log n)$
3. Matrix-vector — $C = n \times \Theta(n) = \Theta(n^2)$
4. Matrix-matrix — $C = n \times \Theta(n^2) = \Theta(n^3)$
5. Reduction is not cost-optimal, the others are

cost

cost ($C = pT_p$)

- the sum of the time spent by all processing elements solving the problem
- *cost-optimal* if $C = T_s$



1. Axpy — $C = \Theta(n)$
2. Reduction — $C = \Theta(n \log n)$
3. Matrix-vector — $C = n \times \Theta(n) = \Theta(n^2)$
4. Matrix-matrix — $C = n \times \Theta(n^2) = \Theta(n^3)$
5. Reduction is not cost-optimal, the others are

exercise — axpy

- $T_p =?$
- $S =?$
- $E =?$
- $C =?$

exercise — axpy

$p = n$ — *cost-optimal*

- $T_p = \Theta(n/p)$
- $S = \Theta(\frac{n}{n/p} = p)$
- $E = \Theta(1)$
- $C = \Theta(n)$

exercise — reduction

$p = n$ — *not cost-optimal*

- $T_p = \Theta(\log n)$
- $S = \Theta(\frac{n}{\log n})$
- $E = \Theta(\frac{1}{\log n})$
- $C = \Theta(n \log n)$

$p > n$ — too many processing elements, use less

$p < n$ — ?

exercise — reduction

$p = n$ — *not cost-optimal*

- $T_p = \Theta(\log n)$
- $S = \Theta(\frac{n}{\log n})$
- $E = \Theta(\frac{1}{\log n})$
- $C = \Theta(n \log n)$

$p > n$ — too many processing elements, use less

$p < n$ — *not cost-optimal?*

- $T_p = \Theta(\frac{n}{p} + \log p)$
- $S = \Theta(\frac{n}{\frac{n}{p} + \log p})$
- $E = \Theta(\frac{n}{n + \log p})$
- $C = \Theta(n + p \log p)$

exercise — reduction

$p = n$ — *not cost-optimal*

- $T_p = \Theta(\log n)$
- $S = \Theta(\frac{n}{\log n})$
- $E = \Theta(\frac{1}{\log n})$
- $C = \Theta(n \log n)$

$p > n$ — too many processing elements, use less

$p < n$ — *cost-optimal iff* $n = \Omega(p \log p)$

- $T_p = \Theta(\frac{n}{p} + \log p)$
- $S = \Theta(\frac{n}{\frac{n}{p} + \log p})$
- $E = \Theta(\frac{n}{n + \log p})$
- $C = \Theta(n + p \log p)$

1. if you are given a problem size, what is the maximum number of processing elements that can be used in a cost optimal way?
2. what are the limits of efficiency...how about speedup?

algorithm analysis

case analysis
best, worst, average

asymptotic analysis
Ω, O, Θ

empirical analysis

- use wall clock time
- time only the code that directly corresponds to your algorithm
- repeat experiments multiple times to account for discrepencies in timings
    - take minimum execution time among several trials

1. your code will not magically run faster than its best possible exeuction time, but it may run slower, so take minimum

scalability

How the execution time varies with the number of processors.

- *strong* scaling — how the solution time varies with the number of processors for a fixed *total* problem size.
- *weak* scaling — how the solution time varies with the number of processors for a fixed problem size *per processor*.

1. Reduction — $E = \Theta(n/\log n)/n = \Theta(1/\log n)$
2. Matrix-vector — $E = \Theta(n)/n = \Theta(1)$
3. Matrix-matrix — $E = \Theta(n)/n = \Theta(1)$
4. Why do you think that the efficiency of adding numbers is less than matrix-vector or matrix-matrix?
5. What are the limits of efficiency?

scalability

How the execution time varies with the number of processors.

- *strong* scaling — how the solution time varies with the number of processors for a fixed *total* problem size.
- *weak* scaling — how the solution time varies with the number of processors for a fixed problem size *per processor*.

Algorithms that require the problem size to grow at a lower rate are more scalable.

1. Reduction — $E = \Theta(n/\log n)/n = \Theta(1/\log n)$
2. Matrix-vector — $E = \Theta(n)/n = \Theta(1)$
3. Matrix-matrix — $E = \Theta(n)/n = \Theta(1)$
4. Why do you think that the efficiency of adding numbers is less than matrix-vector or matrix-matrix?
5. What are the limits of efficiency?