

Binary Space Partitions (BSP)

BSP and the painter algorithm

Quad trees and R-trees



BSP tree

Given a set of triangles $S = \{t_1 \dots t_n\}$ in 3D, a BSP T for S is a tree where

1. Each leaf stores a triangle t_i
1. Each internal (non-leaf) node v stores a plane h_v and pointers to two children $v.right, v.left$
3. All triangles in the subtree $v.left$ are fully **below** h_v , and all triangles in $v.right$ are fully on or above h_v



See further example on the board.

Sometimes we need to split triangles to construct the BSP

If a (perfect) BSP exist, then for any location of a viewer, we can use the painter algorithm.

Numerous other applications in graphics. (e.g. combine with imposters/billboards)

If the number of triangles above and below h_v are roughly the same, then the height is $O(\log n)$

Quad Trees

A data simple data structure for geometric objects(e.g. points, houses, an image, 3D scene)

Support efficiently a very wide variety of queries.

Hierarchical Partition of the scene

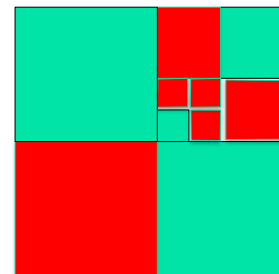


QuadTrees

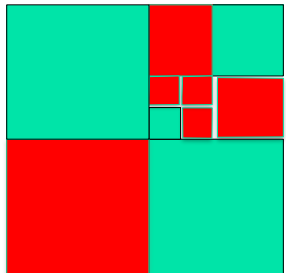
Assume we are given a red/green picture defined a $2^h \times 2^h$ grid. E.g. pixels. Each pixel is either **green** or **red**.

(more general and interesting examples – soon)

Need to represent the shape “compactly”



QuadTrees



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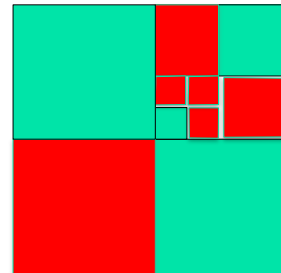
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Need a data structure that could answers multiple types of queries. For example:

4

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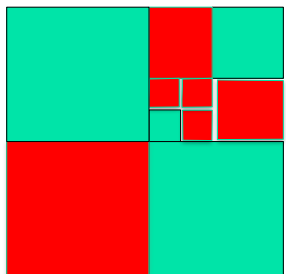
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1. For a given point q , is q **red** or **green** ?

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QuadTrees



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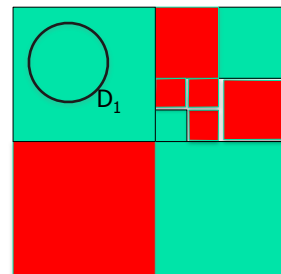
Need a data structure that could answers multiple types of queries. For example:

1. For a given point q , is q **red** or **green** ?

2. For a given query disk D , are there any green points in D ?

4

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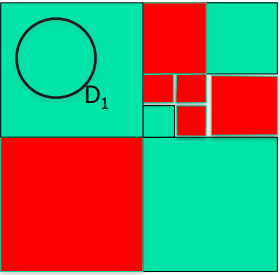
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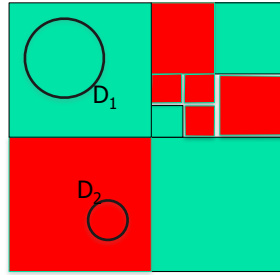
Need to represent the shape “compactly”

Need a data structure that could answers multiple types of queries. For example:

1. For a given point q , is q **red** or **green** ?
2. For a given query disk D , are there any green points in D ?
3. How many green points are there in D ?
4. Etc etc

4

QuadTrees



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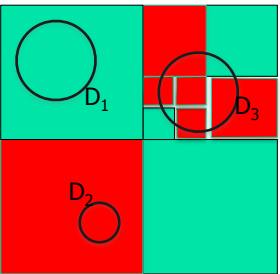
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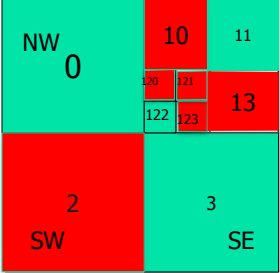
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QuadTrees



Assume we are given a red/green picture defined on a $2^h \times 2^h$ grid of pixels.

- Each pixel has as a unique color (**Green** or **Red**)
- Every node $v \in T$ is associated with a geometric region $R(v)$.
- This is the region that v is “in charge of”.

Alg **ConstructQT** for a shape S .

- **input** – a node $v \in T$, and a shape S .
- **Output** – a Quadtree T_v representing the shape of S within $R(v)$.

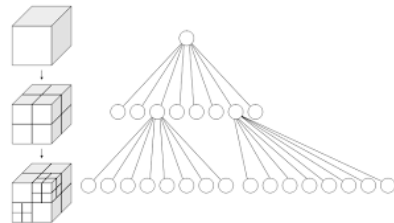
- If S is fully **green** in $R(v)$, or S is fully **red** in $R(v)$ – then
 - v is a leaf, labeled **Green** or **Red**. Return ;
- Otherwise, divide $R(v)$ into 4 equal-sized quadrants, corresponding to nodes $v.NW$, $v.NE$, $v.SW$, $v.SE$.
- Call **ConstructQT** recursively for each quadrant.

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3D-Quadtree

There are no 3D Quadtrees. We call them Oct-trees

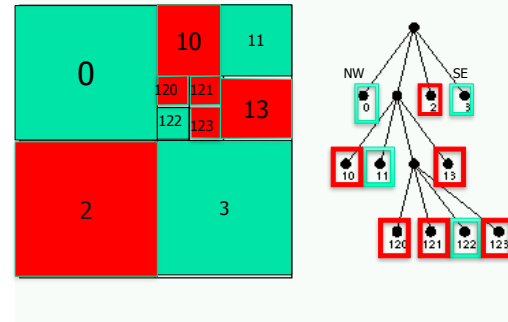
Each node is either a leaf, or is split into 8 equal-volume octants.



credit for image: wikipedia.

6

QuadTrees



Consider a picture stored on an $2^h \times 2^h$ grid. Each pixel is either **red** or **green**.

We can represent the shape “compactly” using a QT.

Height – at most h .

Point location operation – given a point q , is it black or white

– takes time $O(h)$

- could it be much smaller ?

Many other operations are very simple to implement.

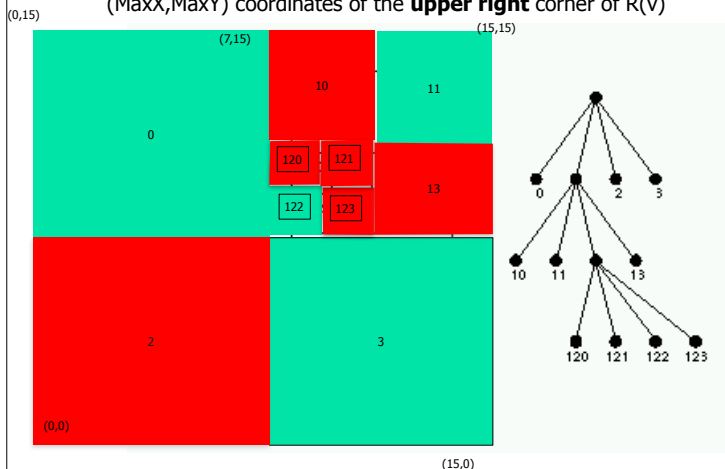
7

Storing the **range** $R(v)$ of a node

Each node v is associated with a range $R(v)$ – a square. The node v stores (in addition to other info) 4 values

(MinX,MinY) – coordinates of the **lower left** corner of $R(v)$

(MaxX,MaxY) coordinates of the **upper right** corner of $R(v)$

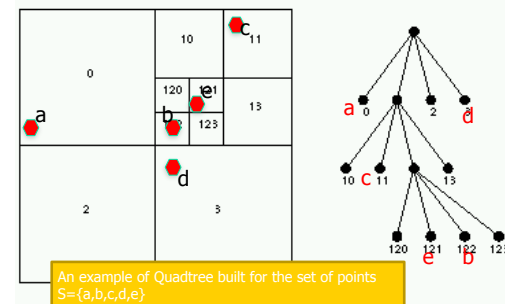


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QuadTree for a set of points

Now consider a set of points (red) but on a $2^h \times 2^h$ grid.

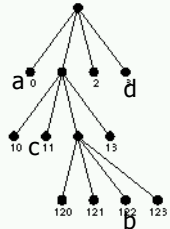
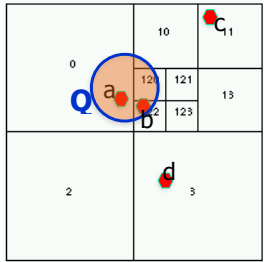
Splitting policy: Split until each quadrant contains ≤ 1 point.



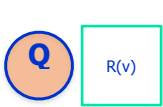
- Build a similar QT, but we stop splitting a quadrant when it contains ≤ 1 point (or some other small constant).
- Could be easily built by inserting the points one after the other. A leaf is split if contains 2 points.
- Point location operation – given a point q , is it black or white
- – takes time $O(h)$ (and less in practice)
- Many other splitting policies are very simple to implement. (eg. a leaf could contain **contains ≤ 17** points)

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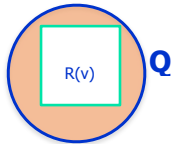
QuadTrees for a set of points



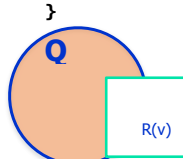
```
Report(Q,v) {
  // Q – a query disk. v- node of a quad tree that stores
  // a set S of points (e.g. S={a,b,c,s}).
  // Prints all data point the points in stored at the subtree
  // rooted at v, which are also inside Q. */
  1. If R(v) is disjoint from Q –return //no point to report
  // at v's subtree
  2. If R(v) is fully contained in Q – print all of S stores at
  // the subtree rooted at v.
  3. Else // R(v) partially overlaps Q. {
    • If v is a leaf – check each point in R(v) if inside Q
    // and print if yes.
    • Else // v internal node
      • Report(Q, NW(v)) and ..
      • Report(Q, NE(v)) and ..
      • Report(Q, SW(v)) and ..
      • Report(Q, SE(v)) }
```



$R(v)$ is disjoint from Q



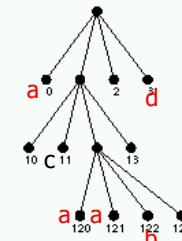
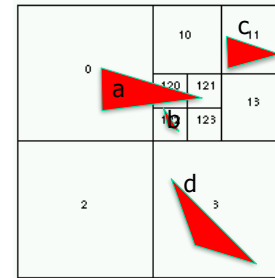
$R(v)$ is contained in Q



$R(v)$ partially overlap Q

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QuadTrees for shape



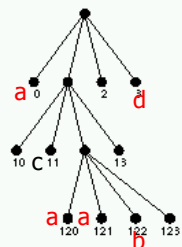
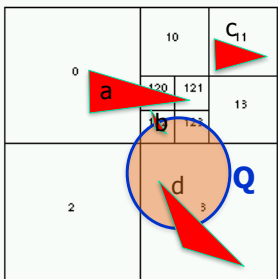
- Input: A set S of triangles $S=\{t_1, \dots, t_n\}$.
- Each leaf v stores a list $v.TriangleList$ of all triangles intersecting $R(v)$.
- Splitting policy: Split a quadrant if it intersects more than 5 (say) triangle of S .

Note – a triangle might be stored in multiple leaves. Some leaves might store no triangles.

Finding all triangles inside a query region Q . We essentially use the function $Report(Q,v)$ from the previous slide (with minor modifications)

11

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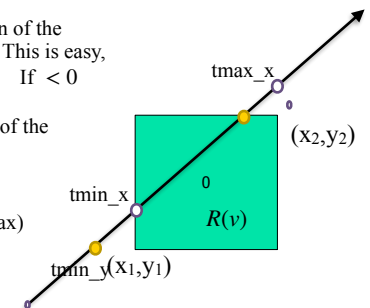
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Ray tracing and QuadTrees

- Consider a quadrant $R(v)$ with corners $LL=(x_1,y_1)$ and $UR=(x_2,y_2)$.
- To find if a ray $r = p_0 + t \cdot \vec{v}$ intersects this quadrant
 - Find t_{min_x}, t_{max_x} , where the ray is in the x-span of the quadrant (the vertical slab containing the quadrant). This is easy, since we only need to check the x-component of \vec{v} . If < 0 then this ray does not intersect $R(v)$
 - Find t_{min_y}, t_{max_y} , where the ray is in the y-span of the quadrant
 - Set $t_{min} = \max(t_{min_x}, t_{min_y})$
 - Set $t_{max} = \min(t_{max_x}, t_{max_y})$
 - The ray is inside the quadrant only for $t \in (t_{min}, t_{max})$
- In 3D, we also check t_{min_z}, t_{max_z}

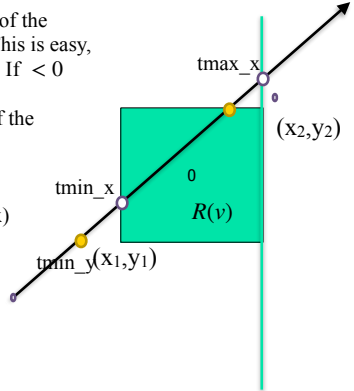


12

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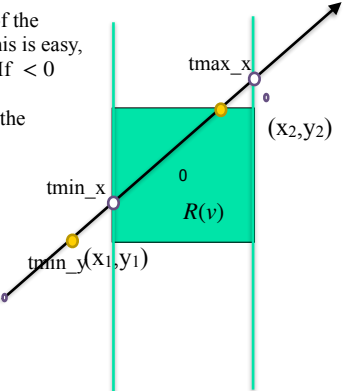


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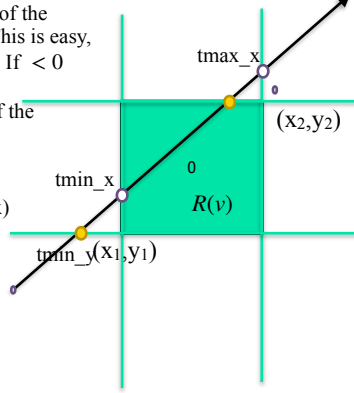


12

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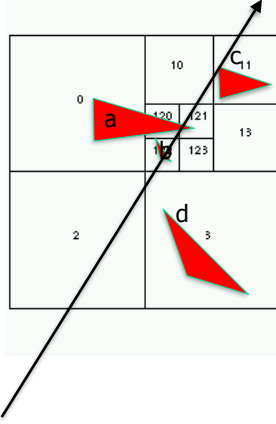
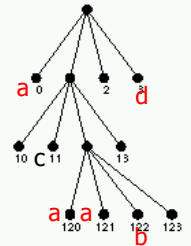
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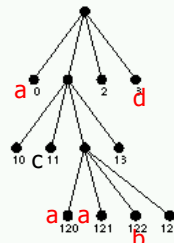
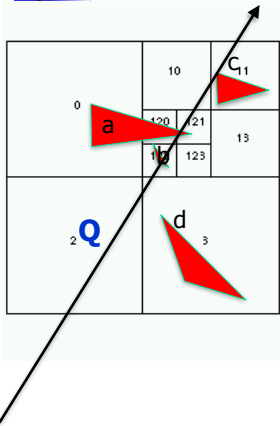
Ray tracing and QuadTrees

- Now, it is easy to find the first triangle hit by a ray r :
- Start from $v = \text{root}$. If empty, then continue tracing the ray from the point it leaves the quadrant.
- If v is internal node, check which of its quadrants is first hit by r , and continue recursively.
- If $v = \text{leaf}$, check each triangle in v

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Ray tracing and QuadTrees



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Inserting a new triangle

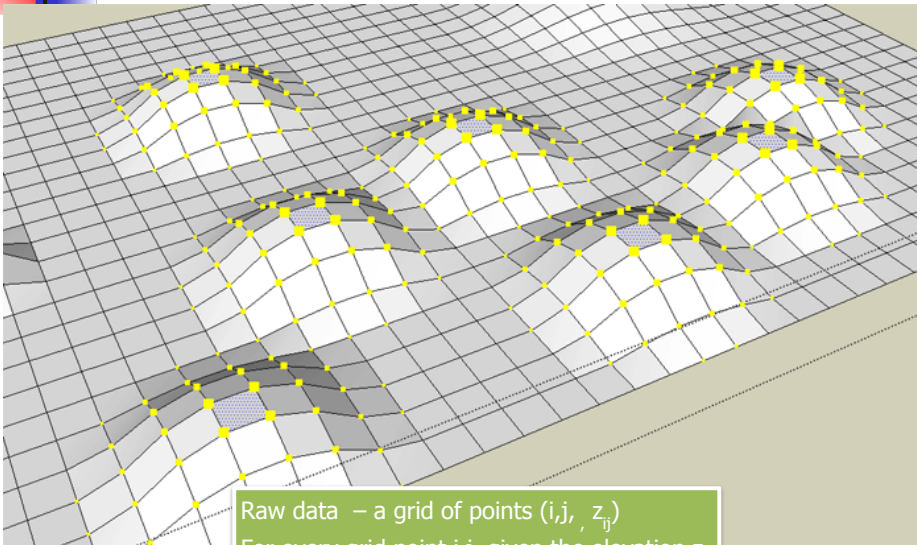
```

insert(triangle  $t_i$ , node  $v$ ) {
    // Inserting a new triangle  $t_i$  into an existing node  $v$  of the Quadtree.
    //  $v$  is not necessarily a leaf.
    If  $v$  is NULL - Error
    If  $R(v)$  is disjoint from  $t_i$  (share no points)– Return. Nothing to do.
    If  $v$  is not a leaf, then for each child  $u$  of  $v$ , call insert( $t_i, u$ );
    Else //  $v$  is a leaf
        Add  $t_i$  to  $v.\text{TrianglesList}$ 
        If number of triangles in  $v.\text{SegmentsList}$  is too long (e.g.  $>5$ ) Call Split( $v$ )
}
    
```

```

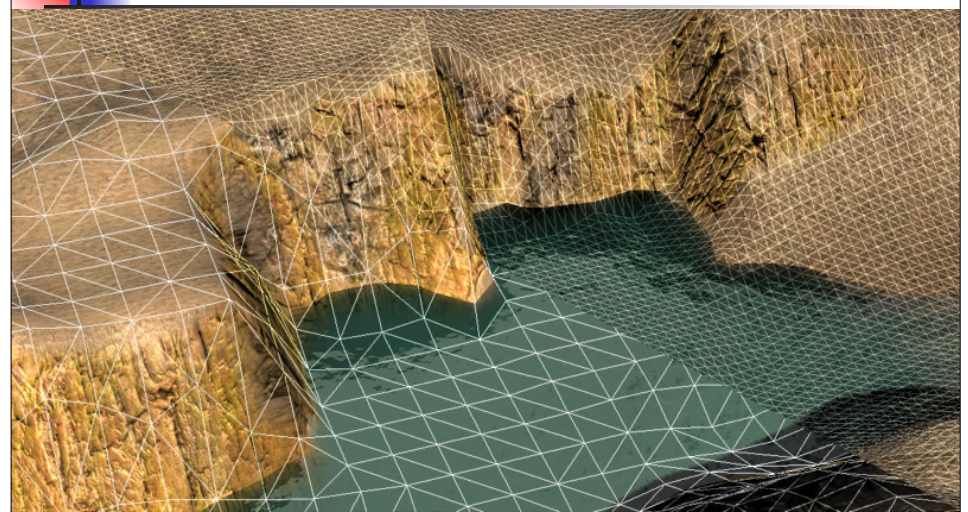
Split( $v$ ){
    // Assumption –  $v$  is a leaf, but has too many triangles in its list.
    // Create 4 children for  $v$  (make sure they know which regions they cover.)
    For each child  $u$  of  $v$ 
        For each segment  $s$  in  $v.\text{TrianglesList}$  Call insert( $s, u$ )
    Empty  $v.\text{TrianglesList}$ 
}
    
```

Terrain representations and levels-of-details



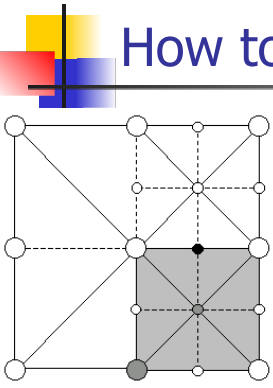
Raw data – a grid of points (i,j, z_{ij})
For every grid point i,j , given the elevation z_{ij}

(TIN – Triangulated Irregular Network)



Each triangle approximately fits the surface below it

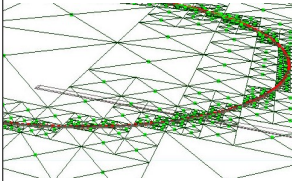
How to find good triangulation ?



- Input – a very large set of points $S = \{(i, j, z_{ij})\}$.
- z_{ij} is the elevation at point (i, j) (*latitude and longitude*)
- Want to create a surface, consists of triangles, where each triangle interpolates the data points underneath it.
- Idea: Build a QT T for the 2D points.
- (If want triangles: Each quadrant is split into 2 right-hand triangles)
- Assign to each vertex the height of the terrain above it.
- The approximated elevation of the terrain at any point (x, y) is the linear interpolation of its elevated vertices.

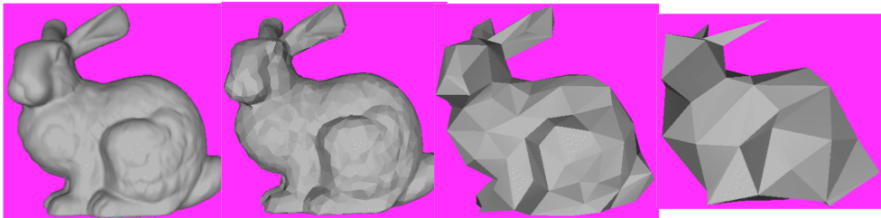
QT Split Policy: Splitting a quadrant into 4 sub-quadrants:

- split a node v if for some data point $(x_i, y_i) \in R(v)$, the elevation of z_{ij} is too far from the the corresponding triangle. If not, leave v as a leaf.
- That is, for any point (i, j) on the plane, the elevation (i, j, z_{ij}) it is too far from the interpolated elevation.
- Note: A quadrant might contain a huge number of points, but they behave smoothly. E.g. all a the sloop of a mountain, but this slope is more or less linear.



Level Of Details

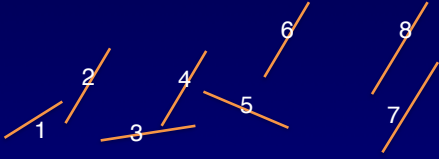
- Idea – the same object is stored several times, but with a different level of details
- Coarser representations for distant objects
- Decision which level to use is accepted 'on the fly'
(eg in graphics applications, if we are far away from a terrain, we could tolerate usually large error. E.g., sub pixels error are not noticeable.)



69,451 polys 2,502 polys 251 polys 76 polys

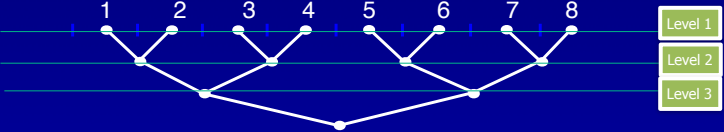
R-trees

- Input: A set S of shapes (segments in this example. Triangles in graphics apps)
- Build a tree that could expedite
 - (i) finding the segments intersecting a query region,
 - (ii) answering ray tracing
 - (iii) Emptiness queries. etc



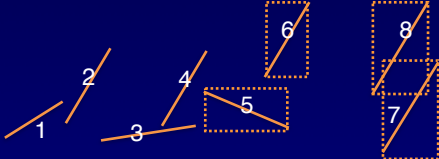
- We compute for each segment its bounding box (rectangle).
- These are the leaves of T . Call them "Level 1".
- Find the nearest pair of segments (say 7,8). Remove them from level 1, and replace them by a single BB encapsulate both. It corresponds to a node of level 2.
- Repeat until no vertex is left in level 1.
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- Repeat until we are left with one bounding box.



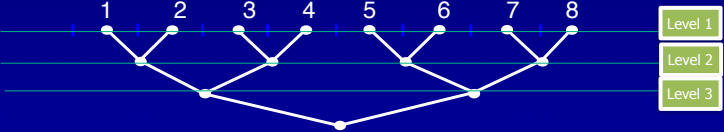
R-trees

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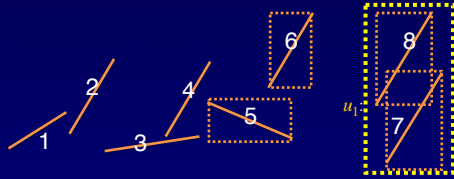
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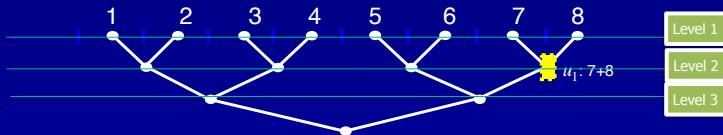


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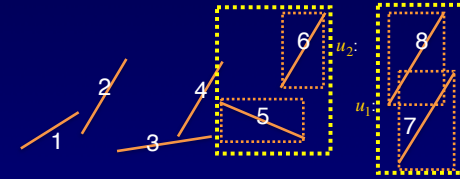


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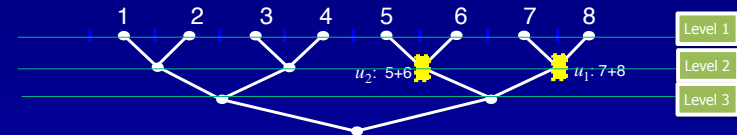


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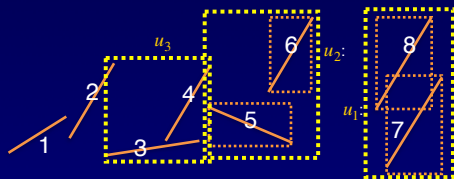


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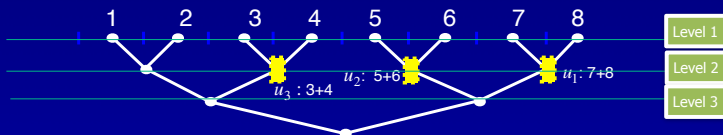


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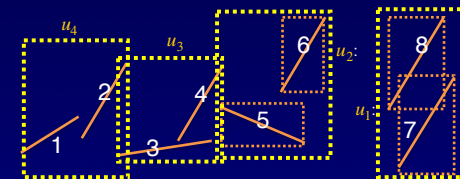


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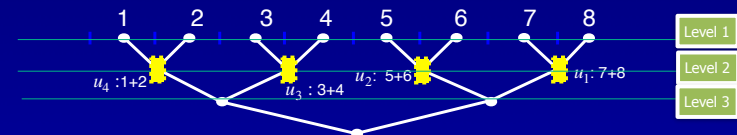


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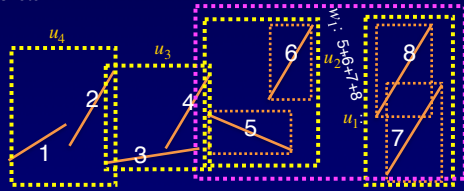


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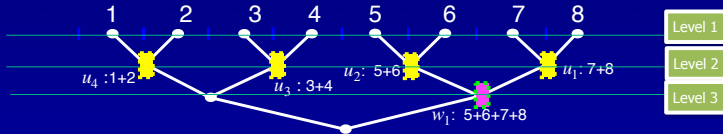


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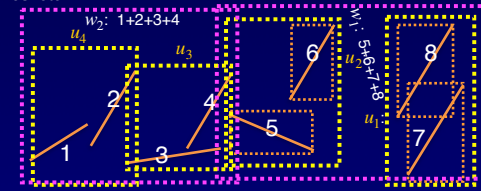


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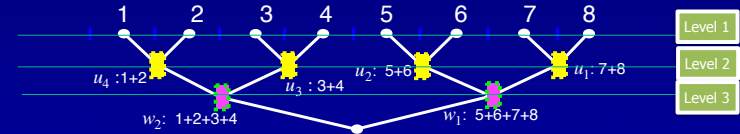


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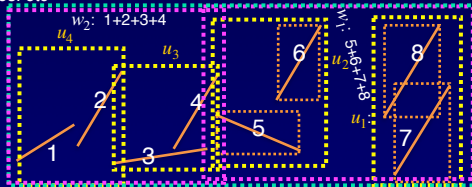


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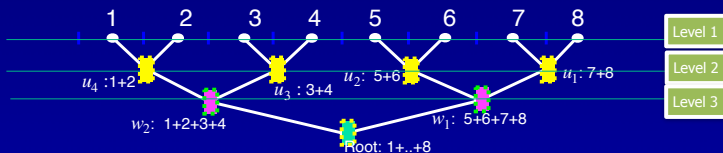


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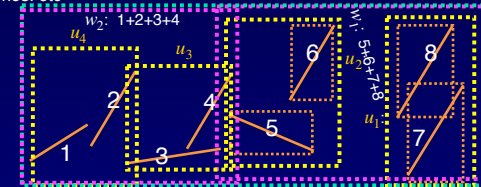


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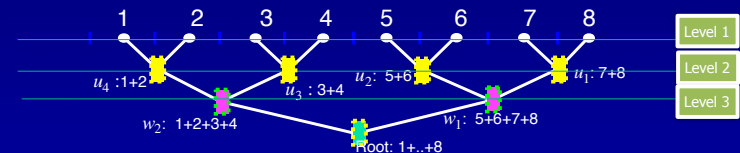


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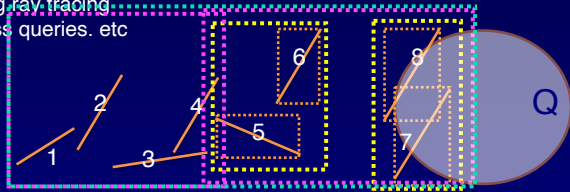


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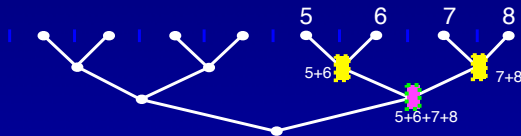


R-trees

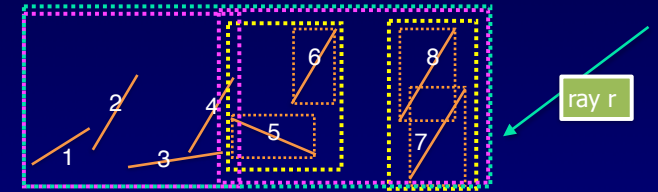
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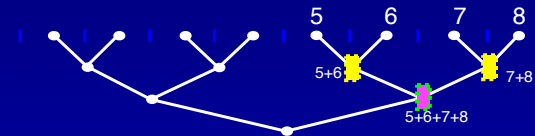
Once a query region Q is given, we need to report the segments intersecting Q .
 Check if Q intersects $BB(\text{root})$
 If not, we are done. If yes, check recursively if Q intersects $BB(v.\text{left})$ and $BB(v.\text{right})$



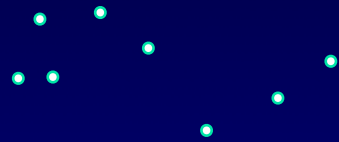
R-trees



Analogously for a query ray r

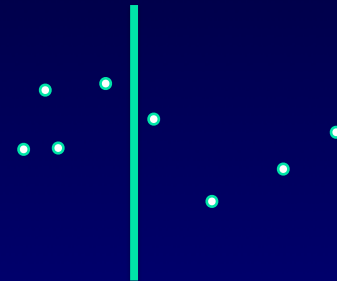


kD-tree



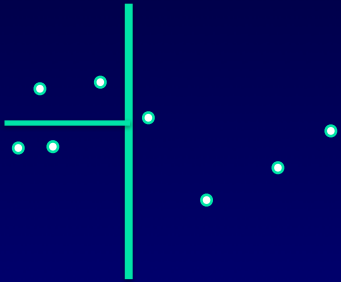
- Another example of hierarchical BSP.
- Input - set of points in d -dimension
- Builds the tree by splitting the data into two equal-cardinality parts
- Always split along a horizontal or a vertical lines.
- First split horizontally, then vertically, next get back to split horizontally etc.

kD-tree



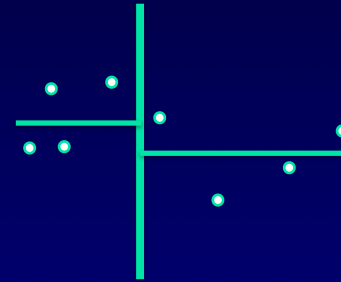
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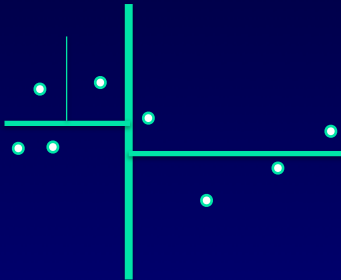
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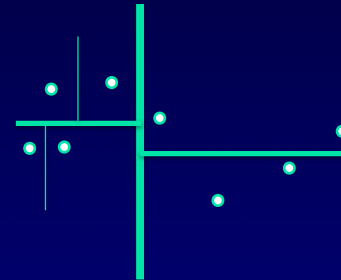
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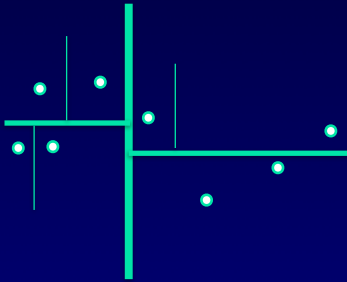
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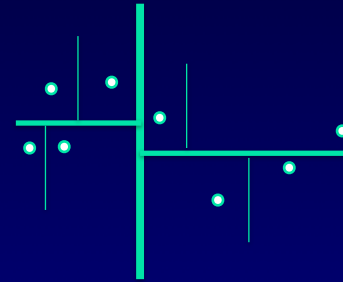
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