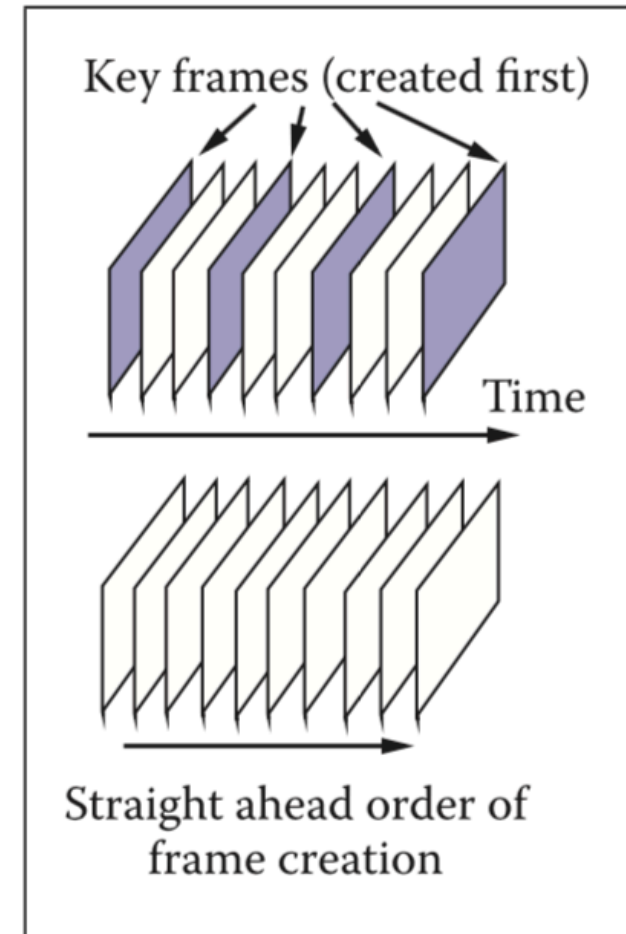


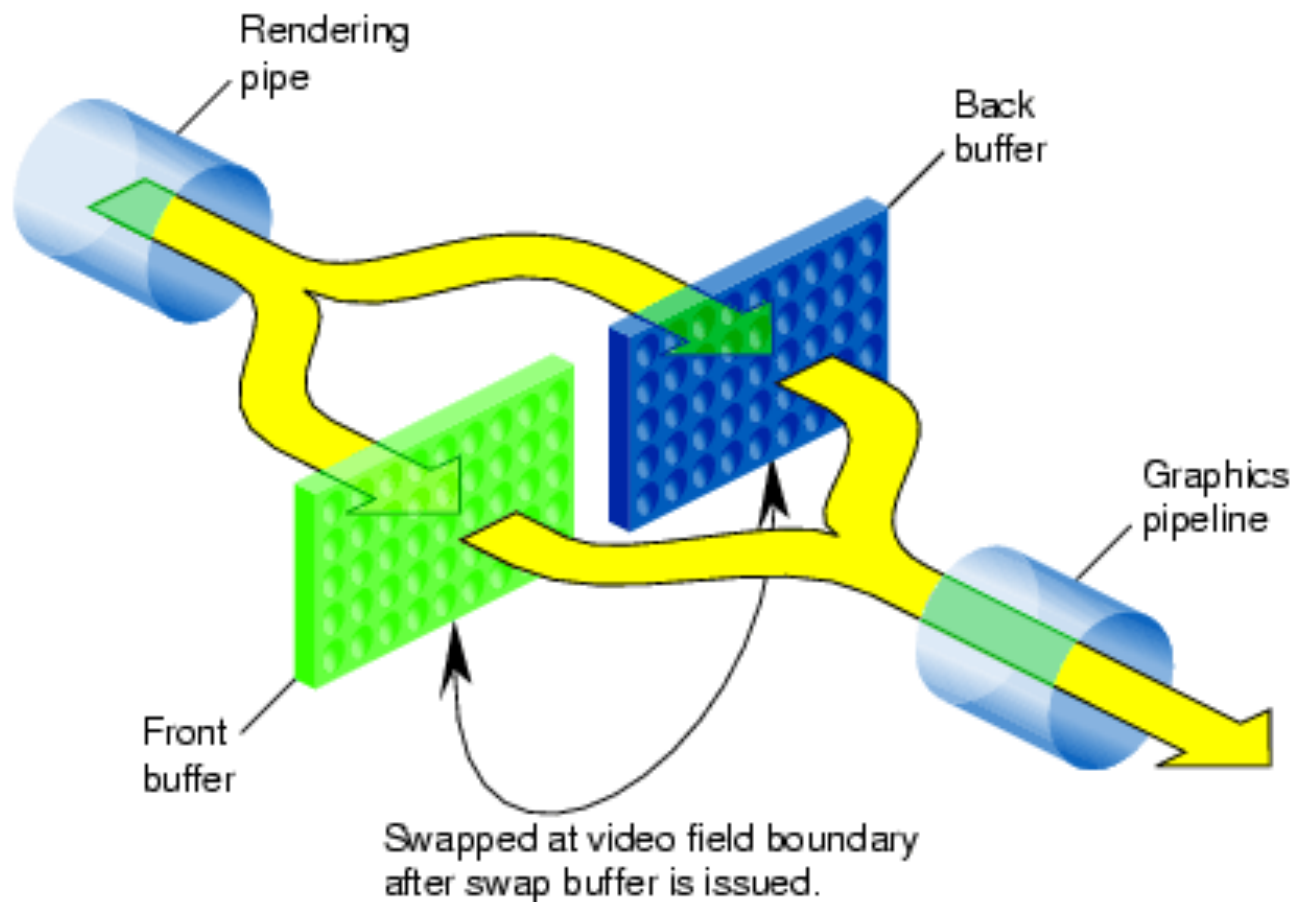
# Keyframe Animation

- Idea: Draw a subset of important frames (called **key frames**) and fill in the rest with *in-betweens*
- In hand-drawn animation, the head animator would draw the poses and the assistants would do the rest
- In computer animation, the artist draws the keys and the computer does the in-betweening
  - Interpolation is used to fill in the rest!



# Double Buffering

- If you draw directly to video buffer, the user will see the drawing happen
- Particularly noticeable artifacts when doing animation



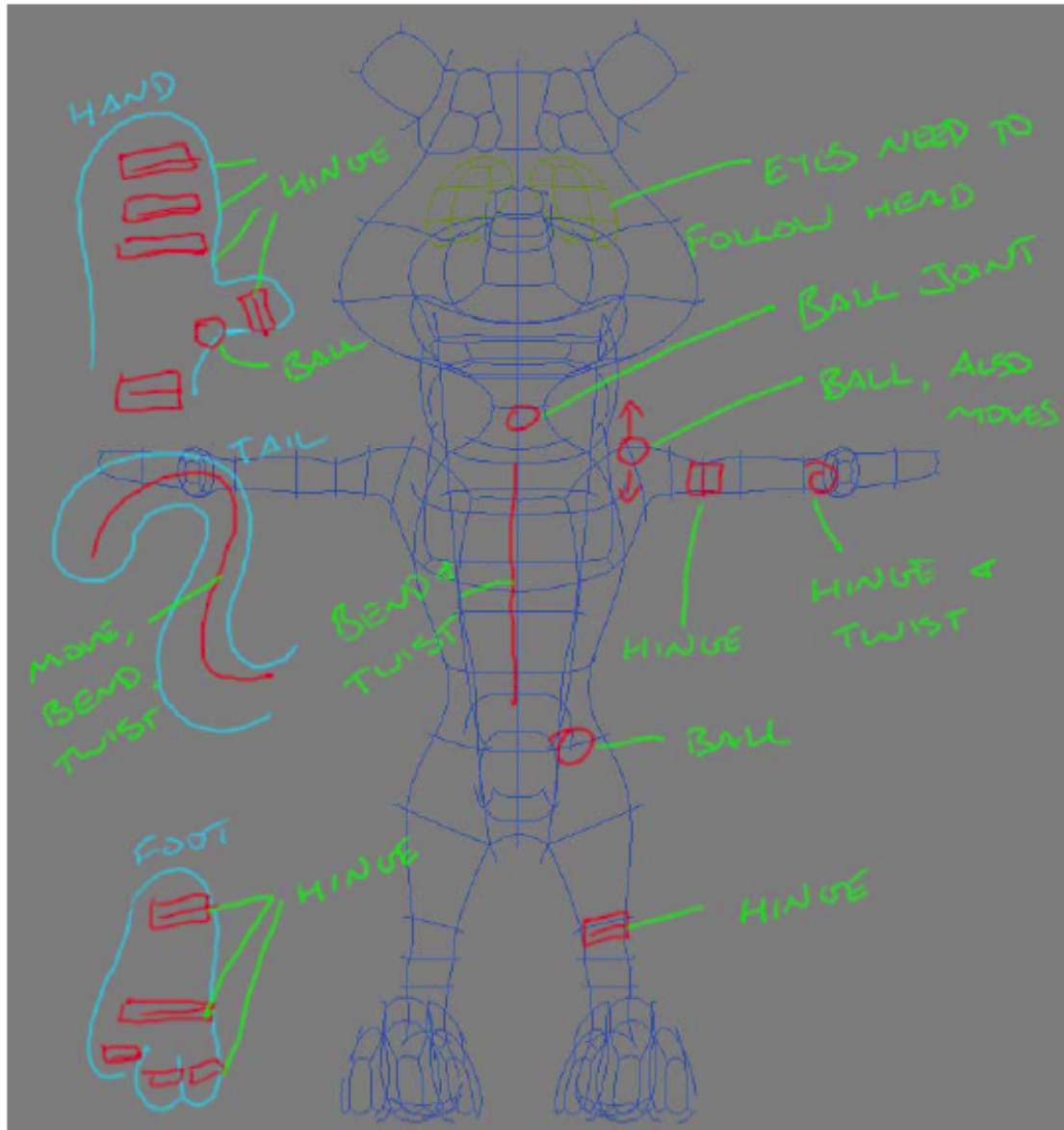
# Controlling geometry conveniently

- Manually place every control point at every keyframe?
  - labor intensive
  - hard to get smooth, consistent motion
- Animate using smaller set of meaningful *degrees of freedom*
  - modeling DOFs are inappropriate for animation  
e.g. “*move one square inch of left forearm*”
  - animation DOFs need to be higher level  
e.g. “*bend the elbow*”

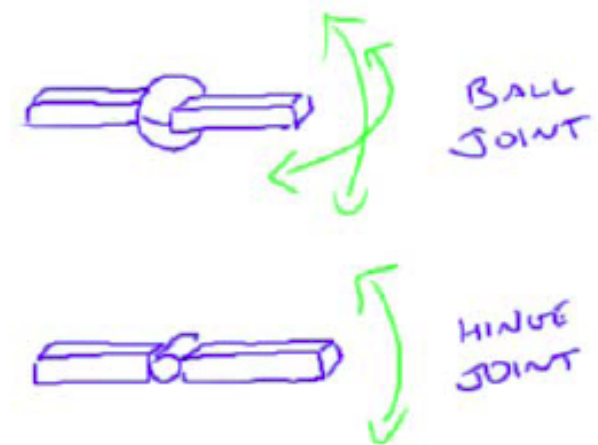
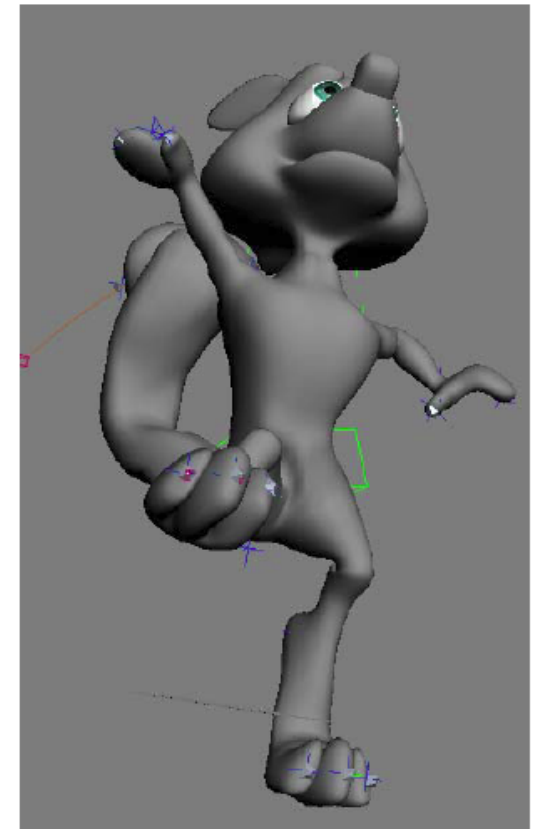
# Controlling shape for animation

- Start with *modeling DOFs* (control points)
- *Deformations* control those DOFs at a higher level
  - Example: move first joint of second finger on left hand
- *Animation controls* control *those* DOFs at a higher level
  - Example: open/close left hand
- Both cases can be handled by the same kinds of deformer

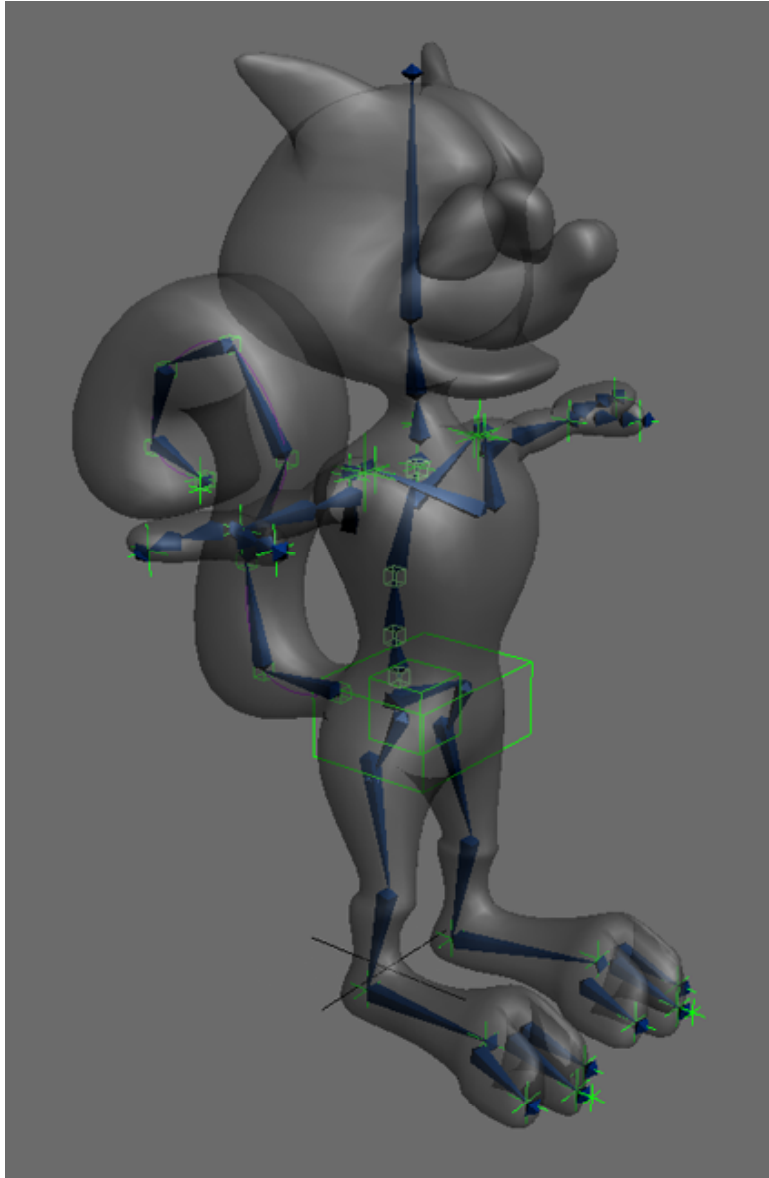
# Character with DOFs



*A visual description of the possible movements for the squirrel*



# Rigged character



- Surface is deformed by a set of *bones*
- Bones are in turn controlled by a smaller set of *controls*
- The controls are useful, intuitive DOFs for an animator to use

[CIS 565 staff]

# Interpolating Rotations

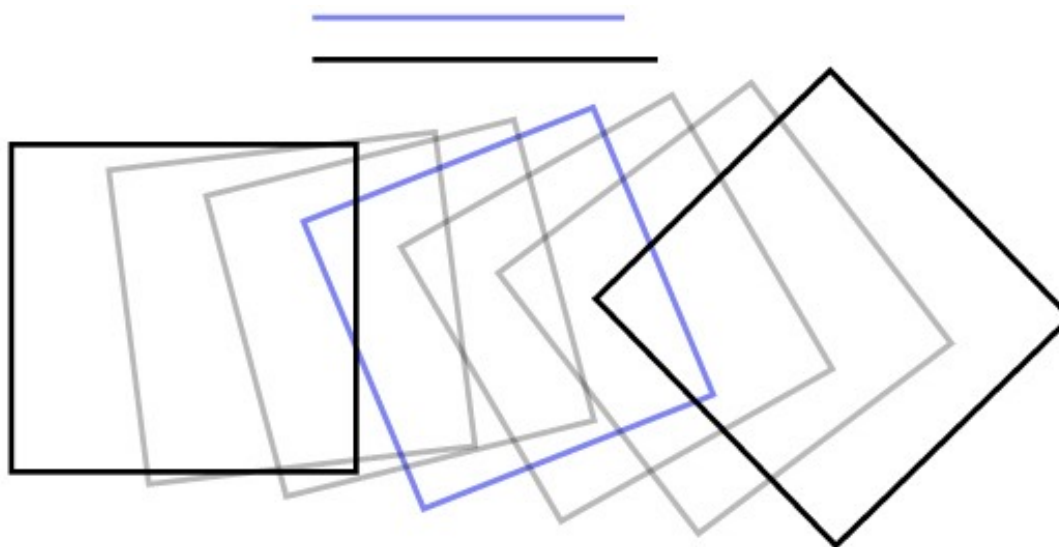
# The most basic animation control

- Affine transformations position things in modeling
- Time-varying affine transformations move things around in animation
- A hierarchy of time-varying transformations is the main workhorse of animation
  - and the basic framework within which all the more sophisticated techniques are built



# Interpolating transformations

- Move a set of points by applying an affine transformation
- How to animate the transformation over time?
  - interpolate the matrix entries from keyframe to keyframe?  
*this is fine for translations but bad for rotations*



# Interpolating Rotations

$$\frac{1}{2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$90^\circ \text{ CW}$                        $90^\circ \text{ CCW}$



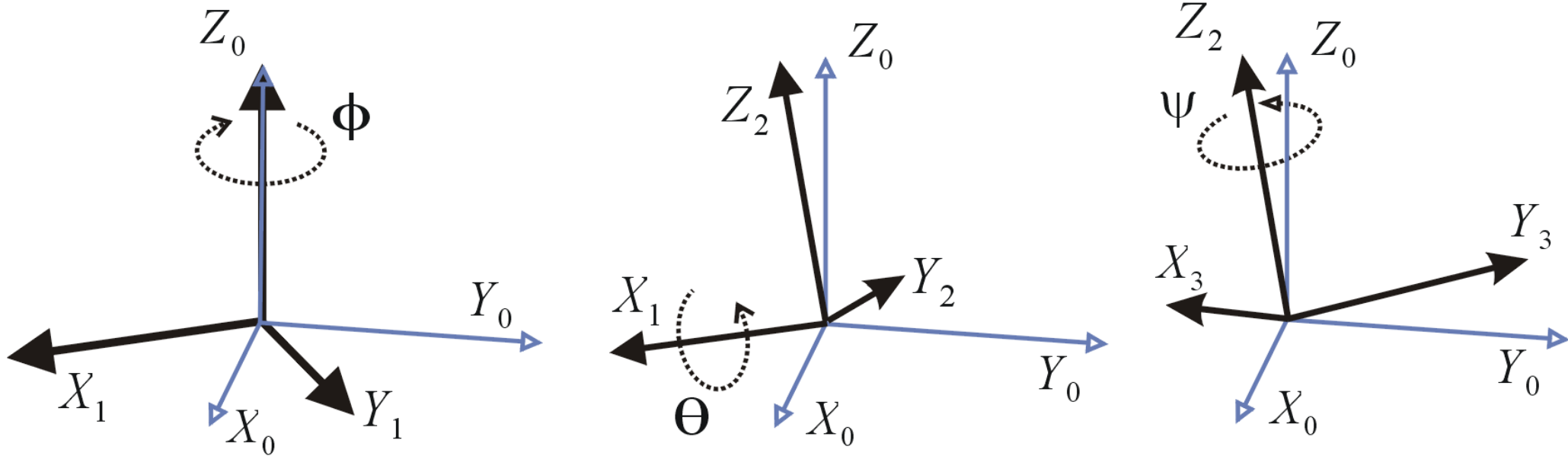
**Not a rotation matrix!**

# Interpolating transformations

- Linear interpolation of matrices is not effective
  - leads to shrinkage when interpolating rotations
- One approach: always keep transformations in a canonical form (e.g. translate-rotate-scale)
  - then the pieces can be interpolated separately
  - rotations stay rotations, scales stay scales, all is good

**Issues occurs when the source and target angles are not close to each other**

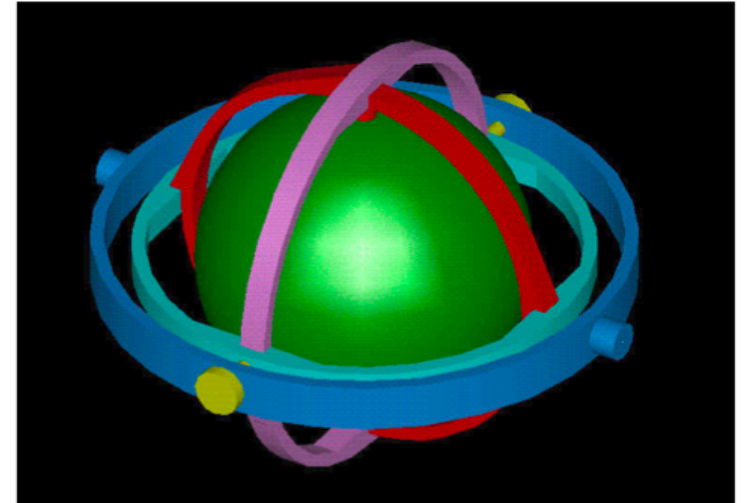
# Could Instead Decompose Rotation by Euler Angles



# Parameterizing rotations

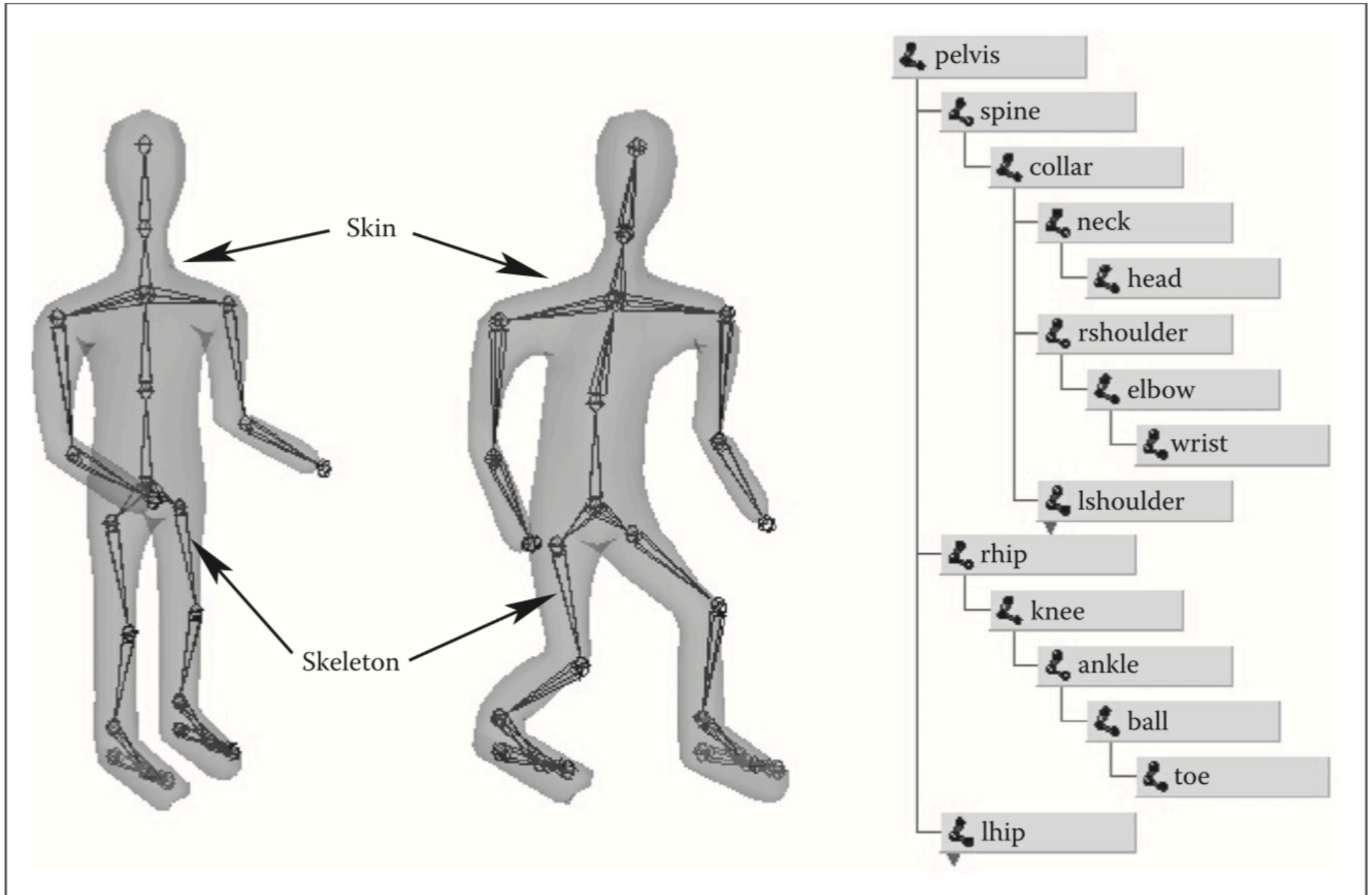
- Euler angles
  - rotate around x, then y, then z
  - nice and simple

$$R(\theta_x, \theta_y, \theta_z) = R_z(\theta_z)R_y(\theta_y)R_x(\theta_x)$$

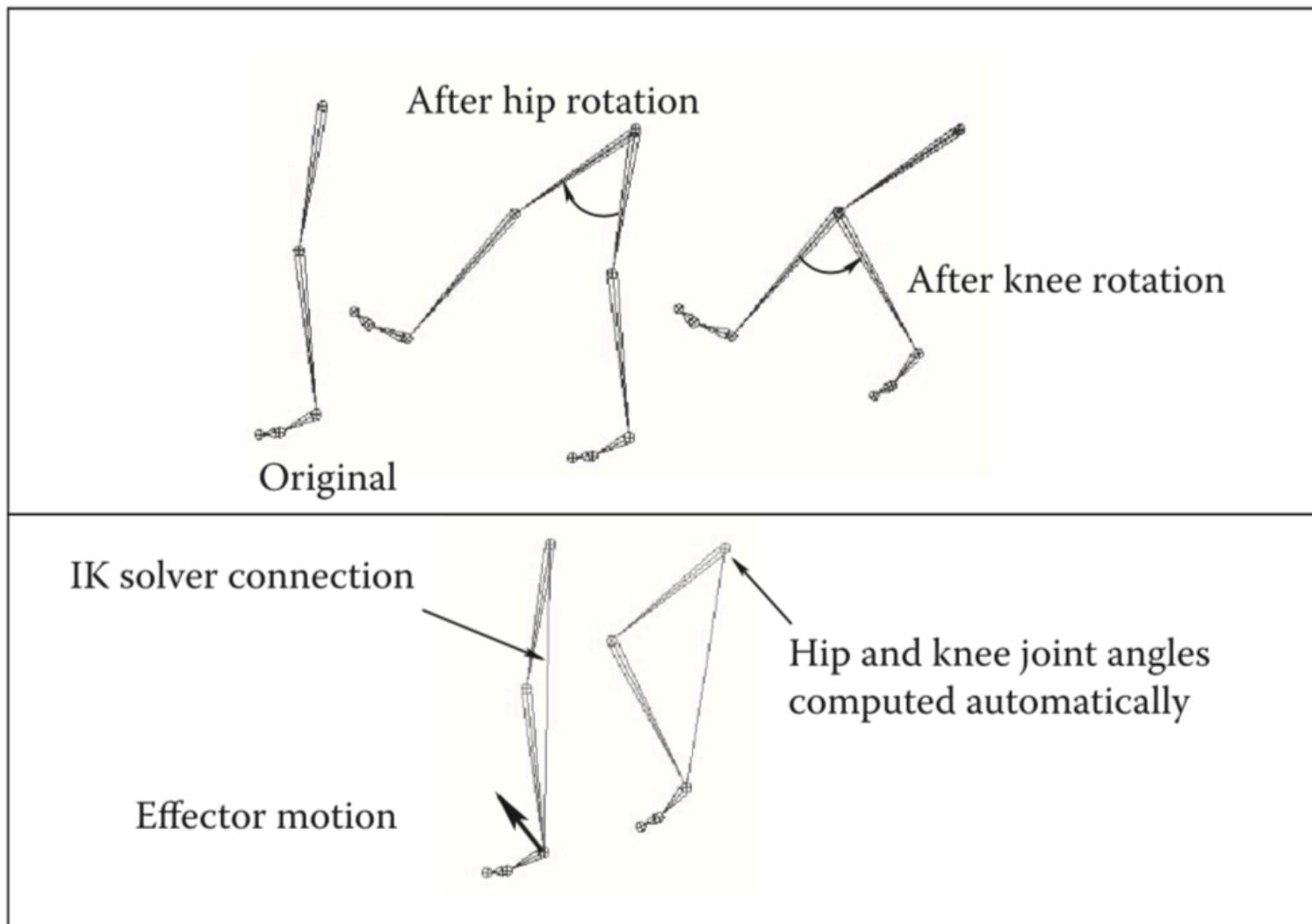


# Character Animation

# Animating w/ Skeletal Hierarchies



# Forward vs. Inverse Kinematics



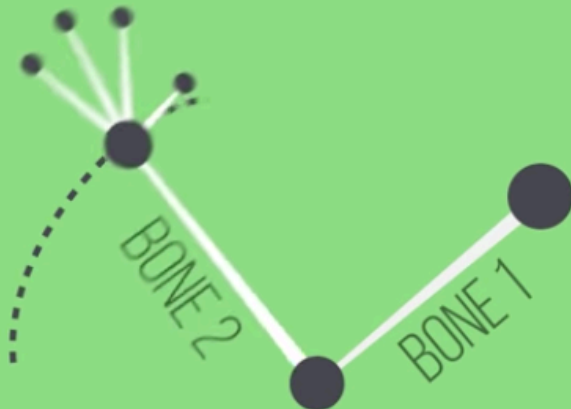


# Inverse Kinematics Solves for all Intermediate Constraints

← → ↻ https://www.youtube.com/watch?v=0a9qlj7kwiA ☆ 2

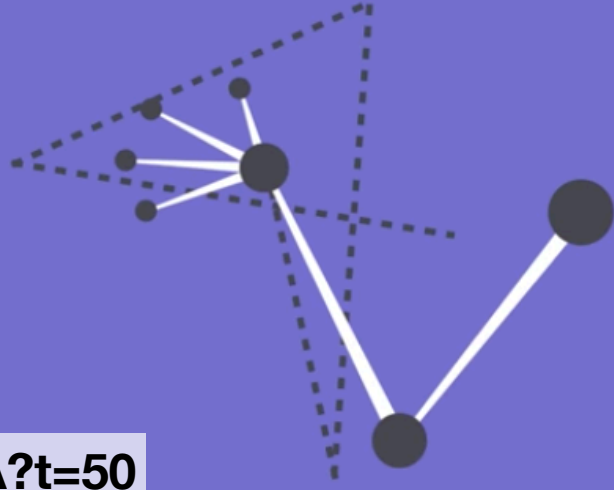
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## FORWARD KINEMATICS



A diagram illustrating forward kinematics on a green background. It shows a skeletal structure with two bones, labeled 'BONE 1' and 'BONE 2', connected at a joint. Bone 1 is a solid line, and Bone 2 is a dashed line. A cluster of points is shown at the end of Bone 2, indicating the range of possible positions for the end effector.

## INVERSE KINEMATICS



A diagram illustrating inverse kinematics on a purple background. It shows the same skeletal structure as the forward kinematics diagram, but with a target point (a black dot) and a dashed line representing the path of the end effector to reach that target. The bones are labeled 'BONE 1' and 'BONE 2'.

<https://youtu.be/0a9qlj7kwiA?t=50>

# Physics-Based Animation

# Animation vs. Simulation

- Animation methods use scripted actions to make objects change
- Simulation: simulate physical laws by associating physical properties to objects
- Solve for physics to achieve (predict) realistic effects

# Using Particle Systems

- Idea: Represent the physics on the simplest possible entity: particles
- Used for effects like smoke, fire, water, sparks, and more
- Plenty of other approaches, this is just one family

# Integration Algorithm 1

## Calculating Particle State from Forces: First attempt

- Use forces to update velocity:  $\vec{v}(t+h) = \vec{v}(t) + \frac{h}{m} \vec{f}(t)$
- Use old velocity to update position:  $\vec{x}(t+h) = \vec{x}(t) + h\vec{v}(t)$

# Physically-based Motion

## Acceleration based on Newton's laws

- $\vec{f}(t) = m\vec{a}(t)$  ...or, equivalently...  $\vec{a}(t) = \vec{f}(t)/m$
- i.e., force is mass times acceleration

## Forces are known beforehand

- e.g., gravity, springs, others....
- Multiple forces sum together
- These often depend on the position, i.e.,  $\vec{f}(t) \equiv \vec{f}(\vec{x}(t))$
- Sometimes velocity, too

**If we know the values of the forces, we can solve for particle's state**



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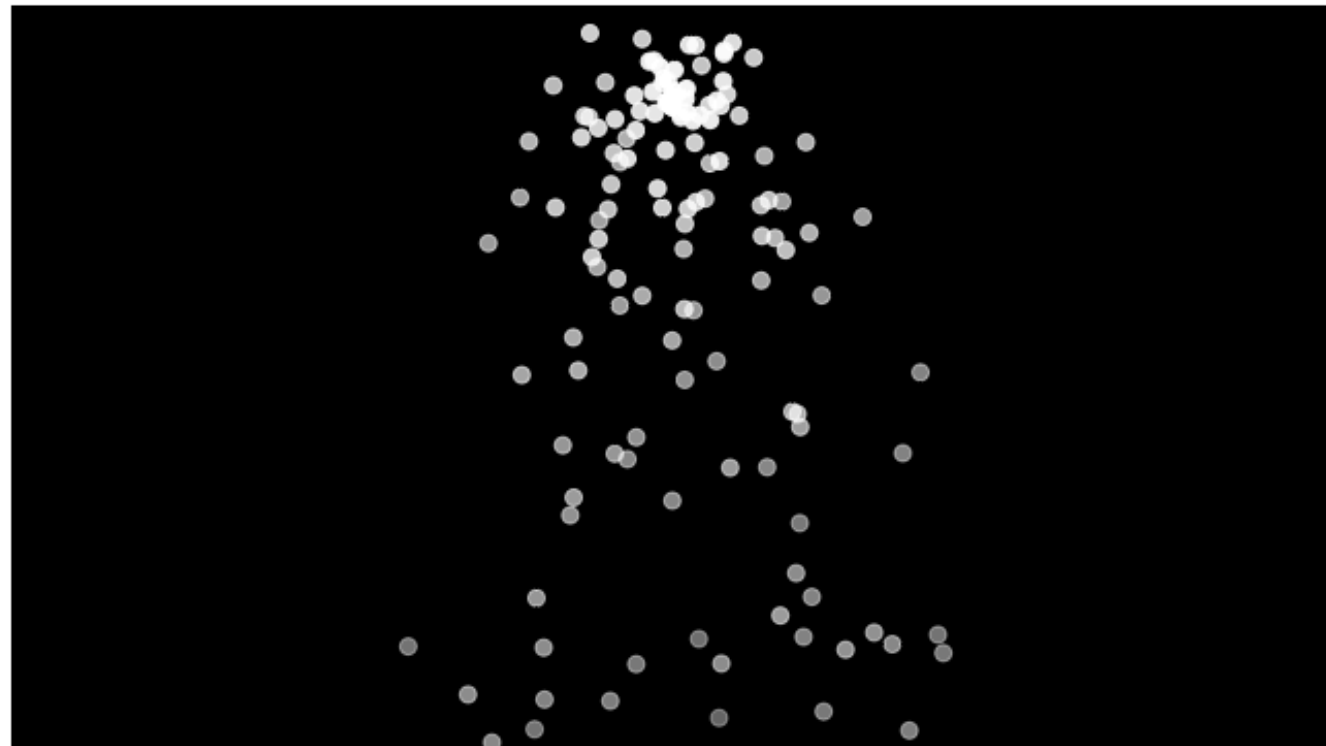
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This example is for Processing 3+. If you have a previous version, use the examples included with your software. If you see any errors or have suggestions, please let us know.

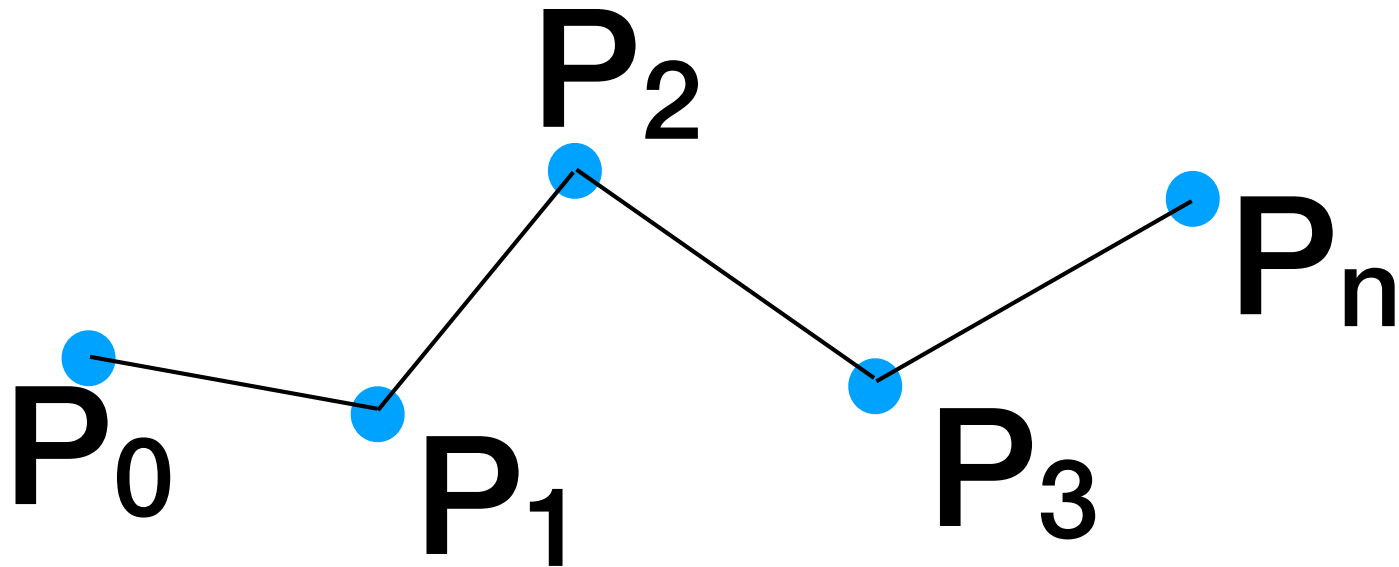


Simple Particle System by Daniel Shiffman.

<https://processing.org/examples/simpleparticlesystem.html>

Particles are generated each cycle through draw(), fall with gravity and fade out over time A ParticleSystem

# How to create a curves that represents the points $P_0, P_1, P_2 \dots P_n$



**Option 1: Linear interpolation. Not great for animation**  
**Need something more smooth.**  
**Solutions - cubic splines or B-splines**



# Hermite Cubic Basis

Lets look at cubic polynomial (max degree =3)

$$h(t) = a t^3 + b t^2 + c t + d$$

*Let prepare actually 4 such polynomials*

Curve	$h(0)$	$h(1)$	$h'(0)$	$h'(1)$
$h_{00}(t)$	1	0	0	0
$h_{01}(t)$	0	1	0	0
$h_{10}(t)$	0	0	1	0
$h_{11}(t)$	0	0	0	1

$$h_{00}(0) = 1 = d,$$

$$h_{00}(1) = 0 = a + b + c + d,$$

$$h_{00}'(0) = 0 = c,$$

$$h_{00}'(1) = 0 = 3a + 2b + c.$$

# Hermite Cubic Basis

Lets look at cubic polynomial (max degree =3)

$$h(t) = a t^3 + b t^2 + c t + d$$

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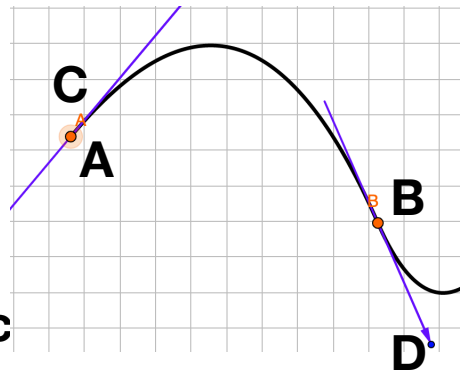
$$h_{00}(0) = 1 = d,$$

$$h_{00}(1) = 0 = a + b + c + d,$$

$$h_{00}'(0) = 0 = c,$$

$$h_{00}'(1) = 0 = 3a + 2b + c.$$

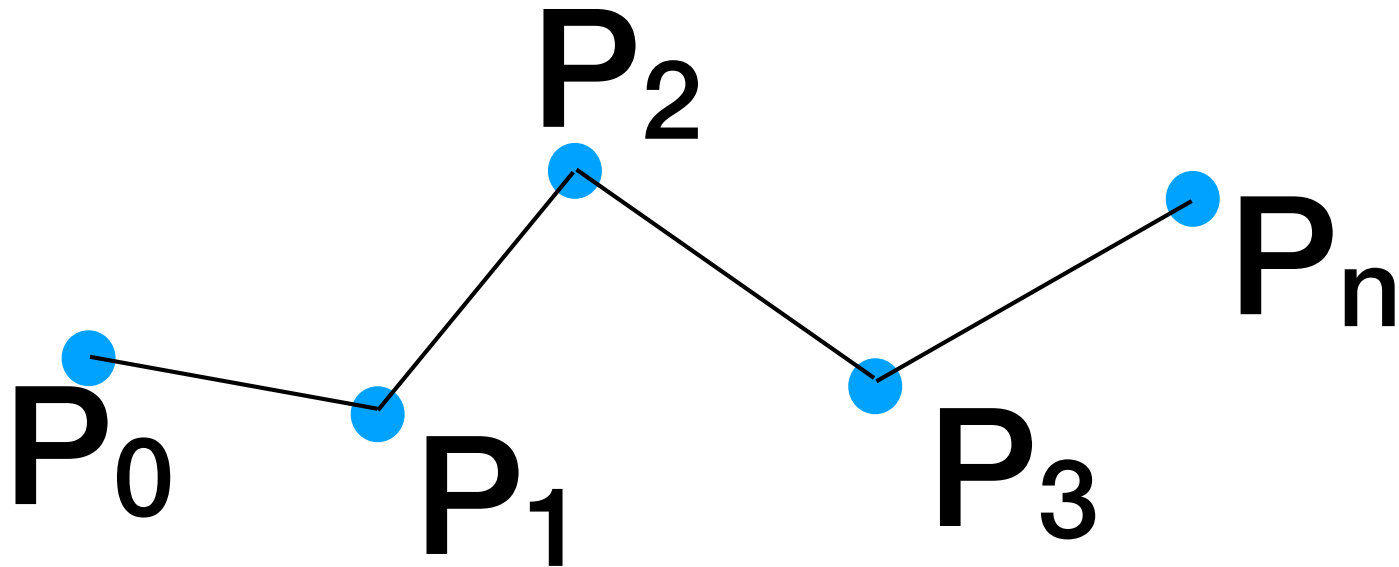
Curve	$h(0)$	$h(1)$	$h'(0)$	$h'(1)$
$h_{00}(t)$	1	0	0	0
$h_{01}(t)$	0	1	0	0
$h_{10}(t)$	0	0	1	0
$h_{11}(t)$	0	0	0	1



Now the animator could decide  
about the location of A,B, and the direc  
(all vectors)

$$h_{00}(t)\mathbf{A} + h_{11}(t)\mathbf{B} + h_{01}(t)\mathbf{C} + h_{10}(t)\mathbf{D}$$

Now we could concatenate several  
terminals, to form a curve to connect all  
the points



## Hermite Cubic Basis (cont'd)

□ Lets solve for  $h_{00}(t)$  as an example.

□  $h_{00}(t) = a t^3 + b t^2 + c t + d$

must satisfy the following four constraints:

$$\begin{aligned} h_{00}(0) &= 1 = d, \\ h_{00}(1) &= 0 = a + b + c + d, \\ h_{00}'(0) &= 0 = c, \\ h_{00}'(1) &= 0 = 3a + 2b + c. \end{aligned}$$

Curve	$h(0)$	$h(1)$	$h'(0)$	$h'(1)$
$h_{00}(t)$	1	0	0	0
$h_{01}(t)$	0	1	0	0
$h_{10}(t)$	0	0	1	0
$h_{11}(t)$	0	0	0	1

□ Four linear equations in four unknowns.

# Hermite Cubic Basis

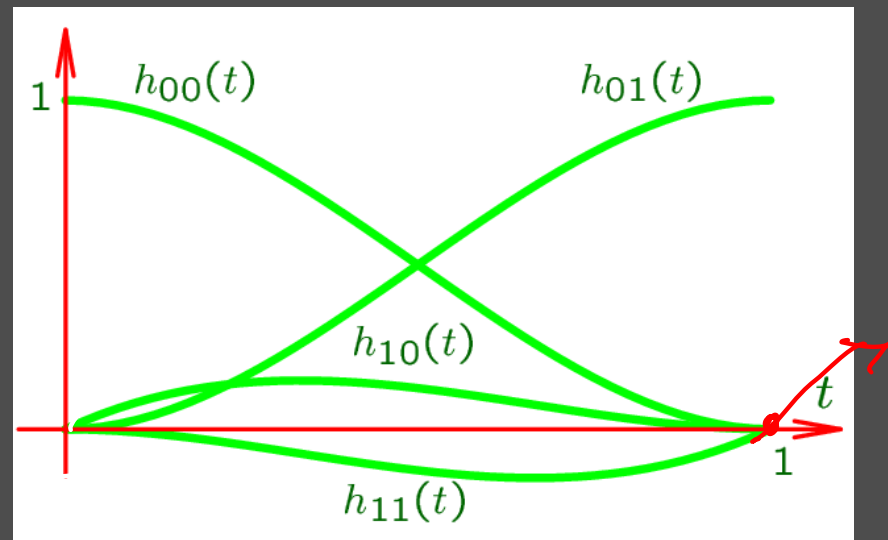
Curve	$h(0)$	$h(1)$	$h'(0)$	$h'(1)$
$h_{00}(t)$	1	0	0	0
$h_{01}(t)$	0	1	0	0
$h_{10}(t)$	0	0	1	0
$h_{11}(t)$	0	0	0	1

- The four cubics which satisfy these conditions are

$$\begin{aligned}h_{00}(t) &= t^2(2t - 3) + 1 & h_{01}(t) &= -t^2(2t - 3) \\h_{10}(t) &= t(t - 1)^2 & h_{11}(t) &= t^2(t - 1)\end{aligned}$$

- Obtained by solving four linear equations in four unknowns for each basis function

- **Prove:** Hermite cubic polynomials are linearly independent and form a basis for cubics



$$C(t) = P_0 h_{00}(t) + P_1 h_{01}(t) + T_0 h_{10}(t) + T_1 h_{11}(t)$$



## Hermite Cubic Basis (cont'd)

- Let  $C(t)$  be a cubic polynomial defined as the linear combination:



- Then  $C(0) = P_0$ ,  $C(1) = P_1$ ,  $C'(0) = T_0$ ,  $C'(1) = T_1$
- To generate a curve through  $P_0$  &  $P_1$  with slopes  $T_0$  &  $T_1$ , use

$$C(t) = P_0 h_{00}(t) + P_1 h_{01}(t) + T_0 h_{10}(t) + T_1 h_{11}(t)$$