Software Engineering

CSC440/640
Prof. Schweitzer
Week 12

Test Plan Components

- Test Plans Should Cover Every User Story
- For Each User Story
  - Identify the Steps a Tester Should Perform
    - Likely the same as one or more of flows of a User Story, but now more tightly coupled with the implementation
  - Test Data That Will Be Used
    - Create Both Positive and Negative Steps (Not Just the Happy Path)
  - Expected Outcomes
  - Did the Test Pass?
- There are Many Formats – Often a Combination of Word Document (for steps) and Grid/Excel to have the different combinations of data that will be put through the same steps

Final Project Deliverables

- Project Documentation
  - Communication Plan
  - Requirements Definitions – Use Cases
  - Class Diagram
  - Representative Sequence Diagram
  - Test Plan
  - *Delivered in Word or PDF Format*
- Source Code + Compiled Binaries
  - Delivered in Zip File
- Due at Final Exam

Reflection Paper

- Paper is to be created by each individual on the team, not by the team as a whole
- Per the Syllabus:
  - This paper summarizes your software development experience gained during the semester
- Submitted in Word or PDF Format
- Suggested Length
  - 2-3 Pages @ standard font, margins, etc. is acceptable
  - Doesn't have to be a dissertation
- Due at Final Exam

Reflection Paper – Potential Topics

- What Development Experience Did You Gain?
- Team Dynamics
  - What made working in a team difficult
  - What made working in a team beneficial
  - How did work division help or hinder the project
  - Was it difficult to combine code created by different individuals?
- Software Design
  - How did the code at the end of the project compare with the design at the beginning of the project?
  - What lessons did you learn from implementing a design created ahead of time

Final Presentation

- Explain Design
  - What Drove Decisions
- Technology Choice
  - What Drove Decisions
- Demonstrate Working Project
- What Testing Strategies Were Employed

Team Assessments

- Assess Your Team Members
- For the Project:
  - Did Each Team Member Participate Fully?
  - Attended Meetings Regularly
  - Communicate with the Team in a timely fashion?
  - Perform the Tasks assigned to them in a timely manner?
- Would you work with this person again in a future project?
- Provided in Word or PDF Format
- Due at Final Exam

Background

- Several different styles of programming, including:
  - Functional programming
  - Logic programming
  - Object-oriented programming
- Different languages have evolved to support each style of programming
  - Each type of language rests on a distinct model of computation, which is different from the von Neumann model

Background

- Functional programming:
  - Provides a uniform view of programs as functions
  - Treats functions as data
  - Provides prevention of side effects
- Functional programming languages generally have simpler semantics and a simpler model of computation
  - Useful for rapid prototyping, artificial intelligence, mathematical proof systems, and logic applications

Background

- Until recently, most functional languages suffered from inefficient execution
  - Most were originally interpreted instead of compiled
- Today, functional languages are very attractive for general programming
  - They lend themselves very well to parallel execution
    - Due to their heavy usage of immutable values
  - May be more efficient than imperative languages on multicore hardware architectures
  - Have mature application libraries

Background

- Despite these advantages, functional languages have not become mainstream languages for several reasons:
  - Programmers learn imperative or object-oriented languages first
  - OO languages provide a strong organizing principle for structuring code that mirrors the everyday experience of real objects
- Functional methods such as recursion, functional abstraction, and higher-order functions have become part of many programming languages
  - Lambda Expressions in .NET
  - Clojure (Java)

Functional Languages

- Lisp – The God Father. Invented in 1958
  - Scheme
  - Racket
  - Clojure (JVM)
- Haskell
- ML
  - OCaml
  - F# (.NET)
- Scala (JVM)

Caveat

- You're Not Going to Understand a Lot of This
- Functional Programming is a Different Paradigm of Writing Code… a Different Mindset of Thinking
- The idea is to get a sense of a different way of thinking of programming

Programs as Functions

- A program is a description of specific computation
- If we ignore the "how" and focus on the result, or the "what" of the computation, the program becomes a virtual black box that transforms input into output
  - A program is thus essentially equivalent to a mathematical function
- **Function**: a rule that associates to each *x* from set of *X* of values a unique *y* from a set *Y* of values

Programs as Functions

- In mathematical terminology, the function can be written as $y=f(x)$ or $f:X \rightarrow Y$
- **Domain** of *f*: the set *X*
- **Range** of *f*: the set *Y*
- **Independent variable**: the *x* in *f(x)*, representing any value from the set *X*
- **Dependent variable**: the *y* from the set *Y*, defined by *y=f(x)*
- **Partial function**: occurs when *f* is not defined for all *x* in *X*

Programs as Functions

- **Total function**: a function that is defined for all x in the set X
- Programs, procedures, and functions can all be represented by the mathematical concept of a function
  - At the program level, *x* represents the input, and *y* represents the output
  - At the procedure or function level, *x* represents the parameters, and *y* represents the returned values

Programs as Functions

- **Functional definition**: describes how a value is to be computed using formal parameters
- **Functional application**: a call to a defined function using actual parameters, or the values that the formal parameters assume for a particular computation
- In math, there is not always a clear distinction between a parameter and a variable
  - The term independent variable is often used for parameters

Programs as Functions

- A major difference between imperative programming and functional programming is the concept of a variable
  - In math, variables always stand for actual values
  - In imperative programming languages, variables refer to memory locations that store values
- Assignment statements allow memory locations to be reset with new values
  - In math, there are no concepts of memory location and assignment

Programs as Functions

- Functional programming takes a mathematical approach to the concept of a variable
  - Variables are bound to values, not memory locations
  - A variable's value cannot change, which eliminates assignment as an available operation
- Most functional programming languages retain some notion of assignment
  - It is possible to create a **pure functional program** that takes a strictly mathematical approach to variables

Programs as Functions

- Lack of assignment makes loops impossible
  - A loop requires a control variable whose value changes as the loop executes
  - Recursion is used instead of loops
- There is no notion of the internal state of a function
  - Its value depends only on the values of its arguments (and possibly nonlocal variables)
- A function's value cannot depend on the order of evaluation of its arguments
  - An advantage for concurrent applications

Programs as Functions

```
void gcd( int u, int v, int* x)
{ int y, t, z;
  z = u ; y = v;
  while (y != 0)
  { t = y;
    y = z % y;
    z = t;
  }
  *x = z;
}
```

(a) Imperative version using a loop

```
int gcd( int u,  int v)
{ if (v == 0) return u;
  else return gcd(v, u % v);
}
```

(b) Functional version with recursion

Programs as Functions

- **Referential transparency**: the property whereby a function's value depends only on the values of its variables (and nonlocal variables)
- Examples:
  - `gcd` function is referentially transparent
  - `rand` function is not because it depends on the state of the machine and previous calls to itself
- A referentially transparent function with no parameters must always return the same value
  - Thus it is no different than a constant

Programs as Functions

- Referential transparency and the lack of assignment make the semantics straightforward
- **Value semantics**: semantics in which names are associated only to values, not memory locations
- Lack of local state in functional programming makes it opposite of OO programming, wherein computation proceeds by changing the local state of objects
- In functional programming, functions must be general language objects, viewed as values themselves

Programs as Functions

- In functional programming, functions are **first-class data values**
  - Functions can be computed by other functions
  - Functions can be parameters to other functions
- **Composition**: essential operation on functions
  - A function takes two functions as parameters and produces another function as its returned value
- In math, the composition operator *o* is defined:

If $f:X \rightarrow Y$ and $g:Y \rightarrow Z$, then $g \circ f:X \rightarrow Z$ is given by

$$(g \circ f)(x) = g(f(x))$$

Programs as Functions

- Qualities of functional program languages and functional programs:
  - All procedures are functions that distinguish incoming values (parameters) from outgoing values (results)
  - In pure functional programming, there are no assignments
  - In pure functional programming, there are no loops
  - Value of a function depends only on its parameters, not on order of evaluation or execution path
  - Functions are first-class data values

Scheme: A Dialect of Lisp

- **Lisp** (**LISt Processing**): first language that contained many of the features of modern functional languages
  - Based on the lambda calculus
- Features included:
  - Uniform representation of programs and data using a single general structure: the list
  - Definition of the language using an interpreter written in the same language (**metacircular interpreter**)
  - Automatic memory management by the runtime system

Scheme: A Dialect of Lisp

- No single standard evolved for Lisp, and there are many variations
- Two dialects that use static scoping and a more uniform treatment of functions have become standard:
  - Common Lisp
  - Scheme

The Elements of Scheme

- All programs and data in Scheme are considered expressions
- Two types of expressions:
  - **Atoms**: like literal constants and identifiers of an imperative language
  - **Parenthesized expression**: a sequence of zero or more expressions separated by spaces and surrounded by parentheses
- Syntax is expressed in **extended Backus-Naur form** notation
  - Analogous to the RPN Notation in math

The Elements of Scheme

Table 3.1 Symbols used in an extended Backus-Naur form grammar

Symbol	Use
→	Means "is defined as"
|	Indicates an alternative
{ }	Enclose an item that may be seen zero or more times
' '	Enclose a literal item

The Elements of Scheme

- Syntax of Scheme:

```
expression → atom | '(' {expression} ')'
atom → number | string | symbol |
character | boolean
```

- When parenthesized expressions are viewed as data, they are called lists
- **Evaluation rule**: the meaning of a Scheme expression
- An **environment** in Scheme is a symbol table that associates identifiers with values

The Elements of Scheme

- Standard evaluation rule for Scheme expressions:
  - Atomic literals evaluate to themselves
  - Symbols other than keywords are treated as identifiers or variables that are looked up in the current environment and replaced by values found there
  - A parenthesized expression or list is evaluated in one of two ways:
    - If the first item is a keyword, a special rule is applied to evaluate the rest of the expression
    - An expression starting with a keyword is called a **special form**

The Elements of Scheme

    - Otherwise, the parenthesized expression is a function application
    - Each expression within the parentheses is evaluated recursively
    - The first expression must evaluate to a function, which is then applied to remaining values (its arguments)
- The Scheme evaluation rule implies that all expressions must be written in prefix form
  - Example: `(+ 2 3)`
    - `+` is a function, and it is applied to the values 2 and 3, to return the value 5

The Elements of Scheme (cont'd.)

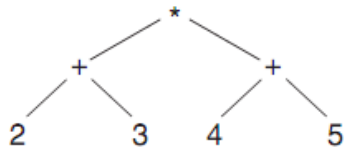
- Evaluation rule also implies that the value of a function (as an object) is clearly distinguished from a call to the function
  - Function is represented by the first expression in an application
  - Function call is surrounded by parentheses
- Evaluation rule represents applicative order evaluation:
  - All subexpressions are evaluated first
  - A corresponding expression tree is evaluated from leaves to root

The Elements of Scheme

C	Scheme
`3 + 4 * 5`	`(+ 3 (* 4 5))`
`(a == b) && (a != 0)`	`(and (= a b) (not (= a 0)))`
`gcd(10,35)`	`(gcd 10 35)`
`gcd`	`gcd`
`getchar()`	`(read-char)`

The Elements of Scheme

- Example: `(* (+ 2 3) (+ 4 5 ))`
  - Two additions are evaluated first, then the multiplication



The Elements of Scheme

- A problem arises when data are represented directly in a program, such as a list of numbers
- Example: `(2.1 2.2 3.1)`
  - Scheme will try to evaluate it as a function call
  - Must prevent this and consider it to be a list literal, using a special form with the keyword `quote`
- Example: `(quote (2.1 2.2 3.1))`
- Rule for evaluating a `quote` special form is to simply return the expression following quote without evaluating it

The Elements of Scheme

- Loops are provided by recursive call
- Selection is provided by special forms:
  - `if` form: like an `if-else` construct
  - `cond` form: like an `if-elseif` construct; `cond` stands for conditional expression

```
(if (= a 0) 0          ; if a = 0 then return 0
    (/ 1 a))           ; else return 1/a

(cond((= a 0) 0)       ; if a=0 then return 0
     ((= a 1) 1)       ; elsif a=1 then return 1
     (else (/ 1 a)))   ; else return 1/a
```

The Elements of Scheme

- Neither the `if` nor the `cond` special form obey the standard evaluation rule
  - If they did, all arguments would be evaluated each time, rendering them useless as control mechanisms
  - Arguments to special forms are **delayed** until the appropriate moment
- Scheme function applications use pass by value, while special forms in Scheme and Lisp use delayed evaluation

The Elements of Scheme

- Special form `let`: binds a variable to a value within an expression
  - Example: `(let ((a 2) (b 3)) (+ 1 b))`
    - First expression in a `let` is a **binding list**
- `let` provides a local environment and scope for a set of variable names
  - Similar to temporary variable declarations in block-structured languages
  - Values of the variables can be accessed only within the `let` form, not outside it

The Elements of Scheme

- `lambda` special form: creates a function with the specified formal parameters and a body of code to be evaluated when the function is applied
  - Example:

```
(lambda (radius) (* 3.14 (* radius
radius)))
```

  - Can apply the function to an argument by wrapping it and the argument in another set of parentheses:

```
((lambda (radius) (* 3.14 (* radius
radius))) 10)
```

The Elements of Scheme

- Can bind a name to a `lambda` within a `let`:

```
(let ((circlearea (lambda (radius) (* 3.14
(* radius radius))))) (circlearea 10))
```

- `let` cannot be used to define recursive functions since `let` bindings cannot refer to themselves or each other
- `letrec` special form: works like a `let` but allows arbitrary recursive references within the binding list

```
(letrec ((factorial (lambda (n) (if (= n 0)
1 (* n (factorial (- n 1)))))))) (factorial
10)
```

The Elements of Scheme

- `let` and `letrec` forms create variables visible within the scope and lifetime of the `let` or `letrec`
- `define` special form: creates a global binding of a variable visible in the top-level environment

Dynamic Type Checking

- Scheme's semantics include dynamic or latent type checking
  - Only values, not variables, have data types
  - Types of values are not checked until necessary at runtime
- Automatic type checking happens right before a primitive function, such as +
- Arguments to programmer-defined functions are not automatically checked
- If wrong type, Scheme halts with an error message

Dynamic Type Checking

- Can use built-in type recognition functions such as `number?` and `procedure?` to check a value's type
  - This slows down programmer productivity and the code's execution speed

Tail and Non-Tail Recursion

- Because of runtime overhead for procedure calls, loops are always preferable to recursion in imperative languages
- **Tail recursive**: when the recursive steps are the last steps in any function
  - Scheme compiler translates this to code that executes as a loop with no additional overhead for function calls other than the top-level call
  - Eliminates the performance hit of recursion

Tail and Non-Tail Recursion

Non-Tail Recursive factorial

```
> (define factorial
    (lambda (n)
      (if (= n 1)
          1
          (* n (factorial (- n 1))))))

> (factorial 6)
720
```

Tail Recursive factorial

```
> (define factorial
    (lambda (n result)
      (if (= n 1)
          result
          (factorial (- n 1) (* n
                                result)))))
> (factorial 6 1)
720
```

Tail and Non-Tail Recursion

- Non-tail recursive function example:
  - *After* each recursive call, the value returned by the call must be multiplied by n (the argument to the previous call)
  - Requires a runtime stack to track the value of this argument for each call as the recursion unwinds
  - Entails a linear growth of memory and a substantial performance hit

Tail and Non-Tail Recursion

- Tail recursive function example:
  - All the work of computing values is done when the arguments are evaluated *before* each recursive call
  - Argument result is used to accumulate intermediate products on the way down through the recursive calls
  - No work remains to be done after each recursive call, so no runtime stack is necessary to remember arguments of previous calls

Data Structures in Scheme

- Basic data structure in Scheme is the list
  - Can represent a sequence, a record, or any other structure
- Scheme also supports structured types for vectors (one-dimensional arrays) and strings
- List functions:
  - `car`: accesses the head of the list
  - `cdr`: returns the tail of the list (minus the head)
  - `cons`: adds a new head to an existing list

Data Structures in Scheme

- Example: a list representation of a binary search tree

```
("horse" ("cow" () ("dog" () ()))
("zebra" ("yak" () ()) () ))
```

- A tree node is a list of three items (name left right)

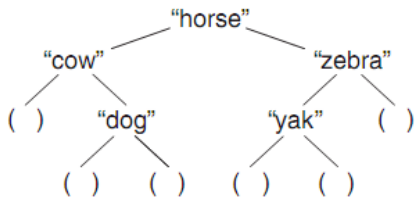


Figure 3.5 A binary search tree containing string data

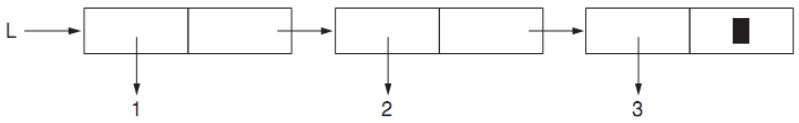
Data Structures in Scheme

- List can be visualized as a pair of values: the `car` and the `cdr`
  - List `L` is a pointer to a box of two pointers, one to its `car` and the other to its `cdr`

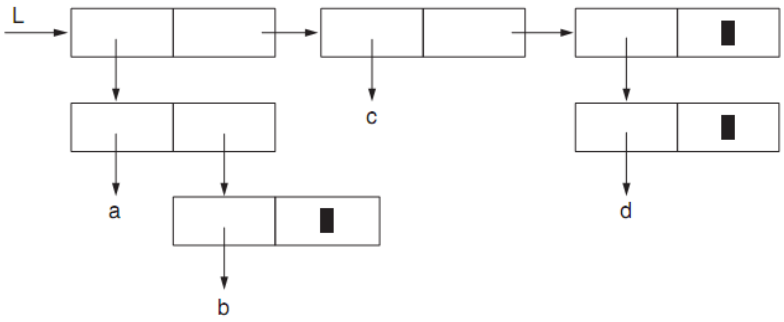
Figure 3.6 Visualizing a list with box and pointer notation

Data Structures in Scheme

- **Box and pointer notation** for a simple list `(1 2 3)`
  - Black rectangle in the end box stands for the empty list ( )



Data Structures in Scheme



Data Structures in Scheme

- All the basic list manipulation operations can be written as functions using the primitives `car`, `cdr`, `cons`, and `null?`
  - `null?` returns true if the list is empty or false otherwise

Programming Techniques in Scheme

- Scheme relies on recursion to perform loops and other repetitive operations
  - To apply repeated operations to a list, "cdr down and cons up": apply the operation recursively to the tail of a list and then use the cons operator to construct a new list with the current result
- Example:

```
(define square-list (lambda (L)
   (if (null? L) '()
      (cons (* (car L) (car L)) (square-
list
          (cdr L)))))
```

Higher-Order Functions

- **Higher-order functions**: functions that take other functions as parameters and functions that return functions as values
- Example: function with a function parameter that returns a function value

```
(define make-double (lambda (f)
     (lambda (x) (f x x)))
```

- Can now create functions using this:

```
(define square (make-double *))
(define double (make-double +))
```

Higher-Order Functions

- Runtime environment of functional languages is more complicated than the stack-based environment of a standard block-structured imperative language
- **Garbage collection**: automatic memory management technique to return memory used by functions

Static (Lexical) Scoping

- Early dialects of Lisp were dynamically scoped
- Modern dialects, including Scheme and Common Lisp, are statically scoped
- **Static scope** (or **lexical scope**): the area of a program in which a variable declaration is visible
  - For static scoping, the meaning or value of a variable can be determined by reading the source code
  - For dynamic scoping, the meaning depends on the runtime context

Static (Lexical) Scoping

- Declaration of variables can be nested in block-structured languages
- Scope of a variable extends to the end of the block in which it is declared, including any nested blocks (unless it is redeclared within a nesting block)

```
> (let ((a 2) (b 3))
    (let ((a (+ a b)))
      (+ a b)))
8
```

Static (Lexical) Scoping

- **Free variable**: a variable referenced within a function that is not also a formal parameter to that function and is not bound within a nested function
- **Bound variable**: a variable within a function that is also a formal parameter to that function
- Lexical scoping fixes the meaning of free variables in one place in the code, making a program easier to read and verify than dynamic scoping

Symbolic Information Processing and Metalinguistic Power

- **Metalinguistic power**: the capacity to build, manipulate, and transform lists of symbols that are then evaluated as programs
- Example: `let` form is actually syntactic sugar for the application of a `lambda` form to its arguments

```
(let ((a 3) (b 4))
  (* a b))
```

```
((lambda (a b) (* a b)) 3 4)
```

Delayed Evaluation

- In a language with an applicative order evaluation rule, all parameters to user-defined functions are evaluated at the time of a call
- Examples that do not use applicative order evaluation:
  - Boolean special forms `and` and `or`
  - `if` special form
- Short-circuit evaluation of Boolean expressions allows a result without evaluating the second parameter

Delayed Evaluation

- Delayed evaluation is necessary for `if` special form
- Example: `(if a b c)`
  - Evaluation of b and c must be delayed until the result of a is known; then either b or c is evaluated, but not both
- Must distinguish between forms that use standard evaluation (function applications) and those that do not (special forms)
- Using applicative order evaluation for functions makes semantics and implementation easier

Delayed Evaluation

- **Nonstrict**: a property of a function in which delayed evaluation leads to a well-defined result, even though subexpressions or parameters may be undefined
- Languages with the property that functions are strict are easier to implement, although nonstrictness can be a desirable property
- Algol60 included delayed execution in its pass by name parameter passing convention
  - A parameter is evaluated only when it is actually used in the code of a called procedure

Delayed Evaluation

- Example: Algol60 delayed execution

```
function p(x: boolean; y: integer): integer;
begin
  if x then p := 1
  else p := y;
end;
```

- When called as `p(true, 1 div 0)`, it returns 1 since `y` is never reached in the code of `p`
  - The undefined expression `1 div 0` is never computed

Delayed Evaluation

- In a language with function values, it is possible to delay evaluation of a parameter by enclosing it in a function "shell" (a function with no parameters)
- Example: C pass by name equivalent

```
typedef int (*IntProc) ();
int divByZero ()
{  return 1 / 0;
}
int p(int x, IntProc y)
{ if (x) return 1;
  else return y();
}
```

Delayed Evaluation

- Such "shell" procedures are sometimes referred to as **pass by name thunks**, or just **thunks**
- In Scheme and ML, the `lambda` and `fn` function value constructors can be used to surround parameters with function shells
- Example:

```
(define (p x y) (if x 1 (y)))
```

which can be called as follows:

```
(p #T (lambda () (/ 1 0)))
```

Delayed Evaluation

- `delay` special form: delays evaluation of its arguments and returns an object like a `lambda` "shell" or **promise** to evaluate its arguments
- `force` special form: causes its parameter, a delayed object, to be evaluated
- Previous function can now be written as:

```
(define ( p x y) (if x 1 (force y)))
```

and called as:

```
(p #T (delay (/ 1 0)))
```

Delayed Evaluation

- Delayed evaluation can introduce inefficiency when the same delayed expression is repeatedly evaluated
- Scheme uses a **memoization** process to store the value of the delayed object the first time it is forced and then return this value for each subsequent call to force
  - This is sometimes referred to as **pass by need**

Delayed Evaluation

- **Lazy evaluation**: only evaluate an expression once it is actually needed
- This can be achieved in a functional language without explicit calls to `delay` and `force`
- Required runtime rules for lazy evaluation:
  - All arguments to user-defined functions are delayed
  - All bindings of local names in `let` and `letrec` expressions are delayed
  - All arguments to constructor functions are delayed

Delayed Evaluation

- Required runtime rules for lazy evaluation (cont'd.):
  - All arguments to other predefined functions are forced
  - All function-valued arguments are forced
  - All conditions in selection forms are forced
- Lists that obey lazy evaluation may be called **streams**
- Primary example of a functional language with lazy evaluation is Haskell

Delayed Evaluation

- **Generator-filter programming**: a style of functional programming in which computation is separated into procedures that generate streams and other procedures that take streams as arguments
- **Generators**: procedures that generate streams
- **Filters**: procedures that modify streams
- **Same-fringe** problem for lists: two lists have the same fringe if they contain the same non-null atoms in the same order

Delayed Evaluation

- Example: these lists have the same fringe:
  `((2 (3)) 4)` and `(2 (34 ()))`
- To determine if two lists have the same fringe, must **flatten** them to just lists of their atoms
- `flatten` function: can be viewed as a filter; reduces a list to a list of its atoms
- Lazy evaluation will compute only enough of the flattened lists as necessary before their elements disagree

Delayed Evaluation

- Delayed evaluation complicates the semantics and increases complexity in the runtime environment
  - Delayed evaluation has been described as a form of parallelism, with `delay` as a form of process suspension and `force` as a kind of process continuation
- Side effects, in particular assignment, do not mix well with lazy evaluation

The Mathematics of Functional Programming: Lambda Calculus

- **Lambda calculus**: invented by Alonzo Church in the 1930s
  - A mathematical formalism for expressing computation by functions
  - Can be used as a model for purely functional programming languages
- Many functional languages, including Lisp, ML and Haskell, were based on lambda calculus

Lambda Calculus Reference

- The following slides are for those interested in the Computer Science details of Lambda Calculus

Lambda Calculus

- **Lambda abstraction**: the essential construct of lambda calculus: $(\lambda\, x\, .\, 1\ 1\ x)$
- Can be interpreted exactly as this Scheme lambda expression:
  `(lambda (x) (+ 1 x))`
  - An unnamed function of parameter x that adds 1 to x
- Basic operation of lambda calculus is the **application** of expressions such as the lambda abstraction

Lambda Calculus

- This expression: $(\lambda\, x\, .\, +\ 1\ x)\ 2$
  - Represents the application of the function that adds 1 to x to the constant 2
- A **reduction rule** permits 2 to be substituted for x in the lambda, yielding this:

$$(\lambda\, x\, .\, +\ 1\ x)\ 2 \Rightarrow (+\ 1\ 2) \Rightarrow 3$$

Lambda Calculus

- Syntax for lambda calculus:

$$exp \rightarrow constant$$
$$\mid variable$$
$$\mid (exp\ exp)$$
$$\mid (\lambda\ variable\ .\ exp)$$

- Third rule represents function application
- Fourth rule gives lambda abstractions
- Lambda calculus as defined here is fully curried

Currying

- Lambdas can only describe functions of a single variable
  - (λn.(λm. …))
  - A wrong example:
    (λn m. if m = 0 then n else …))
- Currying: the process of splitting multiple parameters into single parameters to higher-order functions
  - The general principle
    fname :: (type1, type2) -> type3 =>
    fname :: type1 -> type2 -> type3
  - e.g.,
    (+ 2 3) => ((+ 2) 3)

Lambda Calculus

- Lambda calculus variables do not occupy memory
- The set of constants and the set of variables are not specified by the grammar
  - It is more correct to speak of many **lambda calculi**
- In the expression
  - x is **bound** by the lamb $(\lambda x.E)$
  - The expression E is the scope of the binding
  - **Free occurrence**: any variable occurrence outside the scope
  - **Bound occurrence**: an occurrence that is not free

Lambda Calculus

- Different occurrences of a variable can be bound by different lambdas
- Some occurrences of a variable may be bound, while others are free
- Can view lambda calculus as modeling functional programming:
  - A lambda abstraction as a function definition
  - Juxtaposition of two expressions as function application

Bound and free variables

- (λx.E)
  - The scope of the binding of the lambda is expression E
  - All occurrences of x in E are bound
  - All occurrences of x outside E are not bound by this lambda
- e.g., in (λx. + y x)
  - x is bound, like a local variable
  - y is free, like a nonlocal reference in the function

Lambda Calculus

- **Typed lambda calculus**: more restrictive form that includes the notion of data type, thus reducing the set of expressions that are allowed
- Precise rules must be given for transforming expressions
- **Substitution** (or **function application**): called **beta-reduction** in lambda calculus
- **Beta-abstraction**: reversing the process of substitution
- **Beta-conversion**: either beta-reduction or beta-abstraction

Lambda Calculus

- **Name capture** problem: when doing beta-conversion and replacing variables that occur in nested scopes, an incorrect reduction may occur
  - Must change the name of the variable in the inner lambda abstraction (**alpha-conversion**)
- **Eta-conversion**: allows for the elimination of "redundant" lambda abstractions
  - Helpful in simplifying curried definitions in functional languages

Beta-abstraction and beta-reduction

- beta-reduction

  ((λx. + x 1) 2) => (+ 2 1)
- beta-abstraction

  (+ 2 1) => ((λx. + x 1) 2)
- beta-conversion (β-conversion) - beta-abstraction or beta reduction

  ((λx. E) F) <=> E[F/x]

  where E[F/x] is E with all free occurrences of x replaced by F

Alpha-conversion (α-conversion)

- The name capture problem: if you substitute an expression with free variables into a lambda, one or more of those free variables may conflict with names bound by lambdas (and be *captured* by them), giving incorrect results (since variable bound/free status should not change)

  ((λx.(λy. + x y)) y) => (λy. + y y)
- alpha-conversion
  - General form: (λx. E) <=> (λy. E[y/x]) and y does not occur in E
  - ((λx.(λy. + x y)) y) =>
    ((λx.(λz. + x z)) y) =>
    (λz. + y z)

Eta-conversion (η-conversion)

- The process of eliminating redundant lambda abstractions

  (λx. (E x)) <=> E if E contains no free occurrences of x
- e.g.,
  - (λx. (+ 1 x)) => (+ 1)
  - (λx.(λy.(+ x y))) => (λx. (+ x)) => +

Lambda Calculus

- Applicative order evaluation (pass by value) vs. normal order evaluation (pass by name)
- Example: evaluate this expression: $((\lambda x.\ *\ x\ x)\ (+\ 2\ 3))$
  - Use applicative order; replacing (1 2 3) by its value and then applying beta-reduction gives:
    $((\lambda x.\ *\ x\ x)\ (+\ 2\ 3)) \Rightarrow ((\lambda x.\ *\ x\ x)\ 5) \Rightarrow (*\ 5\ 5) \Rightarrow 25$
  - Use normal order; applying beta-reduction first and then evaluating gives:
    $((\lambda x.\ *\ x\ x)\ (\ +\ 2\ 3)) \Rightarrow (*\ (+\ 2\ 3)\ (+\ 2\ 3)) \Rightarrow (*\ 5\ 5) \Rightarrow 25$
- Normal order evaluation is a kind of **delayed** evaluation

Lambda Calculus

- Different results can occur, such as when parameter evaluation gives an undefined result
  - Normal order will still compute the correct value
  - Applicative order will give an undefined result
- Functions that can return a value even when parameters are undefined are said to be **nonstrict**
- Functions that are undefined when parameters are undefined are said to be **strict**
- **Church-Rosser theorem**: reduction sequences are essentially independent of the order in which they are performed

Semantics

- An expression is a normal form if there are no β-reductions that can be performed on it.
- The semantics of an expression in lambda calculus is expressed by converting it to a normal form, if possible. Such a normal form, if it exists, is unique, by the famous Church-Rosser Theorem.
- Not all reduction sequences lead to a normal form: $(\lambda\, y.\, 2)\ ((\lambda\, x.\, x\ x)(\lambda\, x.\, x\ x))$

Lambda Calculus

- **Fixed point**: a function that when passed to another function as an argument returns a function
- To define a recursive function in lambda calculus, we need a function Y for constructing a fixed point of the lambda expression for the function
  - Y is called a **fixed-point combinator**
- Because by its nature, Y will actually construct a solution that is in some sense the "smallest"; one can refer to the **least-fixed-point semantics** of recursive functions in lambda calculus