

CSC309 Programming on the Web

week 11: cryptography

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some contents are from:

- Computer networking: a top-down approach, Ross

review

- ❖ security requirements



- ❖ attack vectors

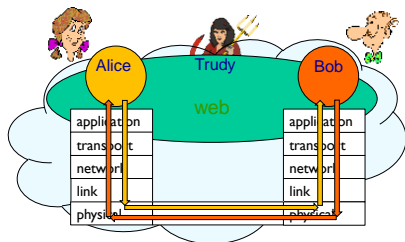
- ❖ this week

- **cryptography**
 - to preserve confidentiality and integrity

cryptography 11-2

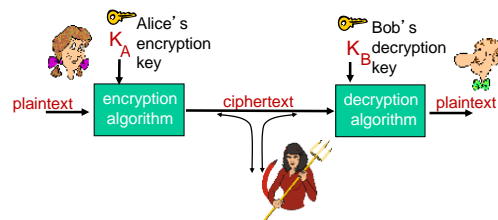
friends and enemies: Alice, Bob, Trudy

- ❖ well-known in network security world
- ❖ Bob, Alice (lovers!) want to communicate “securely”
- ❖ Trudy (intruder) may intercept, delete, add messages



cryptography 11-3

the language of cryptography



m plaintext message

$K_A(m)$ ciphertext, encrypted with key K_A

$m = K_B(K_A(m))$

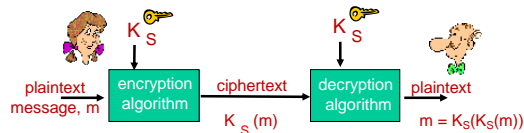
cryptography 11-4

breaking an encryption scheme

- ❖ **cipher-text only attack:** Trudy has ciphertext she can analyze
- ❖ **two approaches:**
 - brute force: search through all keys
 - statistical analysis
- ❖ **known-plaintext attack:** Trudy has plaintext corresponding to ciphertext
 - e.g., in monoalphabetic cipher, Trudy determines pairings for a,l,i,c,e,b,o,
- ❖ **chosen-plaintext attack:** Trudy can get ciphertext for chosen plaintext

cryptography 11-5

symmetric key cryptography



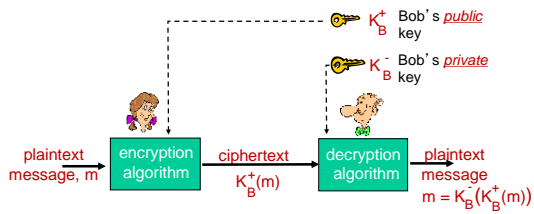
symmetric key crypto: Bob and Alice share same (symmetric) key: K_S

- ❖ e.g., key is knowing substitution pattern in mono alphabetic substitution cipher

Q: how do Bob and Alice agree on key value?

cryptography 11-6

public key crypto



cryptography 11-13

public key encryption algorithms

requirements:

- ① need $K_B^+(\cdot)$ and $K_B^-(\cdot)$ such that

$$K_B^-(K_B^+(m)) = m$$
- ② given public key K_B^+ , it should be impossible to compute private key K_B^-

RSA: Rivest, Shamir, Adelson algorithm

cryptography 11-14

prerequisite: modular arithmetic

- ❖ $x \bmod n$ = remainder of x when divide by n
- ❖ facts:
 - $(a+b) \bmod n = [(a \bmod n) + (b \bmod n)] \bmod n$
 - $(a-b) \bmod n = [(a \bmod n) - (b \bmod n)] \bmod n$
 - $(a*b) \bmod n = [(a \bmod n) * (b \bmod n)] \bmod n$
- ❖ thus

$$a^d \bmod n = (a \bmod n)^d \bmod n$$
- ❖ example: $x=14, n=10, d=2$:

$$(x \bmod n)^d \bmod n = 4^2 \bmod 10 = 6$$

$$x^d = 14^2 = 196 \quad x^d \bmod 10 = 6$$

cryptography 11-15

rsa: getting ready

- ❖ message: just a bit pattern
- ❖ bit pattern can be represented by an integer number
- ❖ thus, encrypting a message is equivalent to encrypting a number

example:

- ❖ $m = 10010001$. This message is uniquely represented by the decimal number 145.
- ❖ to encrypt m , we encrypt the corresponding number, which gives a new number (the ciphertext).

cryptography 11-16

rsa: creating public/private key pair

1. choose two large prime numbers p, q (e.g., 1024 bits each)
 2. compute $n = pq, z = (p-1)(q-1)$
 3. choose e (with $e < n$) that has no common factors with z (e, z are "relatively prime").
 4. choose d such that $ed-1$ is exactly divisible by z (in other words: $ed \bmod z = 1$).
 5. public key is (n, e) . private key is (n, d) .
- $\underbrace{(n, e)}_{K_B^+} \quad \underbrace{(n, d)}_{K_B^-}$

cryptography 11-17

rsa: encryption, decryption

0. given (n, e) and (n, d) as computed above
1. to encrypt message $m (< n)$, compute

$$c = m^e \bmod n$$
2. to decrypt received bit pattern, c , compute

$$m = c^d \bmod n$$

magic happens!

$$m = \underbrace{(m^e \bmod n)^d}_{c} \bmod n$$

cryptography 11-18

rsa example:

Bob chooses $p=5$, $q=7$. Then $n=35$, $z=24$.
 $e=5$ (so e, z relatively prime).
 $d=29$ (so $ed-1$ exactly divisible by z).

encrypting 8-bit messages.

encrypt: $\underbrace{\text{bit pattern}}_{00001100} \quad \underbrace{m}_{12} \quad \underbrace{m^e}_{24832} \quad \underbrace{c = m^e \bmod n}_{17}$

decrypt: $\underbrace{c}_{17} \quad \underbrace{c^d}_{481968572106750915091411825223071697} \quad \underbrace{m = c^d \bmod n}_{12}$

cryptography 11-19

rsa: another important property

The following property will be *very* useful later:

$$K_B^-(K_B^+(m)) = m = K_B^+(K_B^-(m))$$

use public key first,
followed by
private key

use private key
first, followed by
public key

result is the same!

cryptography 11-20

why $K_B^-(K_B^+(m)) = m = K_B^+(K_B^-(m))$?

follows directly from modular arithmetic:

$$\begin{aligned} (m^e \bmod n)^d \bmod n &= m^{ed} \bmod n \\ &= m^{de} \bmod n \\ &= (m^d \bmod n)^e \bmod n \end{aligned}$$

cryptography 11-21

why is rsa secure?

- ❖ suppose you know Bob's public key (n, e) . How hard is it to determine d ?
- ❖ essentially need to find factors of n without knowing the two factors p and q
 - fact: factoring a big number is hard
 - e.g., 2 years to factor a 232-digit number

cryptography 11-22

pk crypto in practice: digital signatures

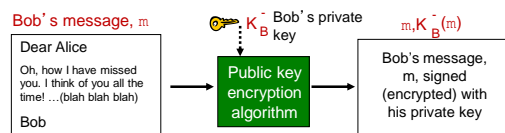
- ❖ cryptographic technique analogous to hand-written signatures:
- ❖ sender (Bob) digitally signs document, establishing he is document owner/creator
- ❖ *verifiable, nonforgeable*: recipient (Alice) can prove to someone that Bob, and no one else (including Alice), must have signed document

cryptography 2-23

digital signatures

simple digital signature for message m :

- ❖ Bob signs m by encrypting with his private key K_B^- , creating "signed" message, $K_B^-(m)$



cryptography 2-24

digital signatures

- ❖ suppose Alice receives msg m , with signature: $m, K_B^-(m)$
- ❖ Alice verifies m signed by Bob by applying Bob's public key K_B^+ to $K_B^-(m)$ then checks $K_B^+(K_B^-(m)) = m$
- ❖ If $K_B^+(K_B^-(m)) = m$, whoever signed m must have used Bob's private key

Alice thus verifies that:

- ✓ Bob signed m
- ✓ no one else signed m
- ✓ Bob signed m and not m'

non-repudiation:

- ✓ Alice can take m , and signature $K_B^-(m)$ to court and prove that Bob signed m

cryptography 2-25

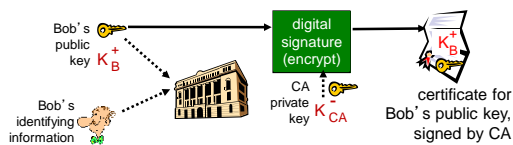
pk crypto in practice: pk certification

- ❖ motivation: Trudy plays pizza prank on Bob
 - Trudy creates e-mail order: *Dear Pizza Store, Please deliver to me four pepperoni pizzas. Thank you, Bob*
 - Trudy signs order with her private key
 - Trudy sends order to Pizza Store
 - Trudy sends to Pizza Store her public key, but says it's Bob's public key
 - Pizza Store verifies signature; then delivers four pepperoni pizzas to Bob
 - Bob doesn't even like pepperoni

cryptography 2-26

public-key certification

- ❖ **certification authority (CA)**: binds public key to particular entity, E.
- ❖ E (person, router) registers its public key with CA.
 - E provides "proof of identity" to CA.
 - CA creates certificate binding E to its public key.
 - certificate containing E's public key digitally signed by CA – CA says "this is E's public key"



cryptography 11-27

pk crypto in practice: session keys

- ❖ exponentiation in **rsa** is computationally intensive
- ❖ **des** is at least 100 times faster than **rsa**
- ❖ idea:
 - public key crypto is used to establish secure connection, then second key (e.g. symmetric session key) is exchanged for encrypting data

session key, K_S

- ❖ Bob and Alice use **rsa** to exchange a symmetric key K_S
- ❖ once both have K_S , they use symmetric key cryptography

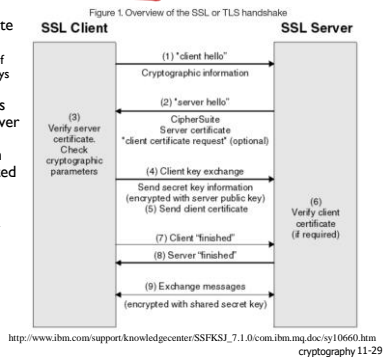
concept underlying SSL/TLS

- ❖ Secure Socket Layer/Transport Layer Security

cryptography 11-28

pk crypto in practice:

- ❖ you get a ssl certificate for your server
 - this includes, a pair of public and private keys
- ❖ once the certificate is installed on your server
- ❖ clients' browsers can verify it, via the trusted CA's they know
- ❖ before sending you a secret key and exchanging messages with your server



final exam: cover page

CSC309H1S
Duration – 3 hours
No Aids Allowed

Student Number: _____

Last (Family) Name(s): _____

First (Given) Name(s): _____

Team Number: _____

Do not turn this page until you have received the signal to start.
In the meantime, please fill out the identification section above,
and read the instructions below carefully.

This exam consists of 7 questions on 32 pages (including this one). Pages 10 to 31 consist of code snippets from projects.

Please answer questions in the space provided. You will earn 20% for any question you leave blank or write "I cannot answer this question," etc. We think we have provided a lot of space for your work, but please do not feel you need to fill all available space.

You must achieve at least 40% of this exam to pass the course.

Write neatly and concisely. If we cannot read it, we cannot grade it.

GOOD LUCK!

Question	1	2	3	4	5	6	7	Total
Initial								
Mark	/16	/15	/12	/12	/10	/15	/20	/100

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