

Language Modeling and Natural Language Generation

CSCI 1460: Computational Linguistics

Lecture 9

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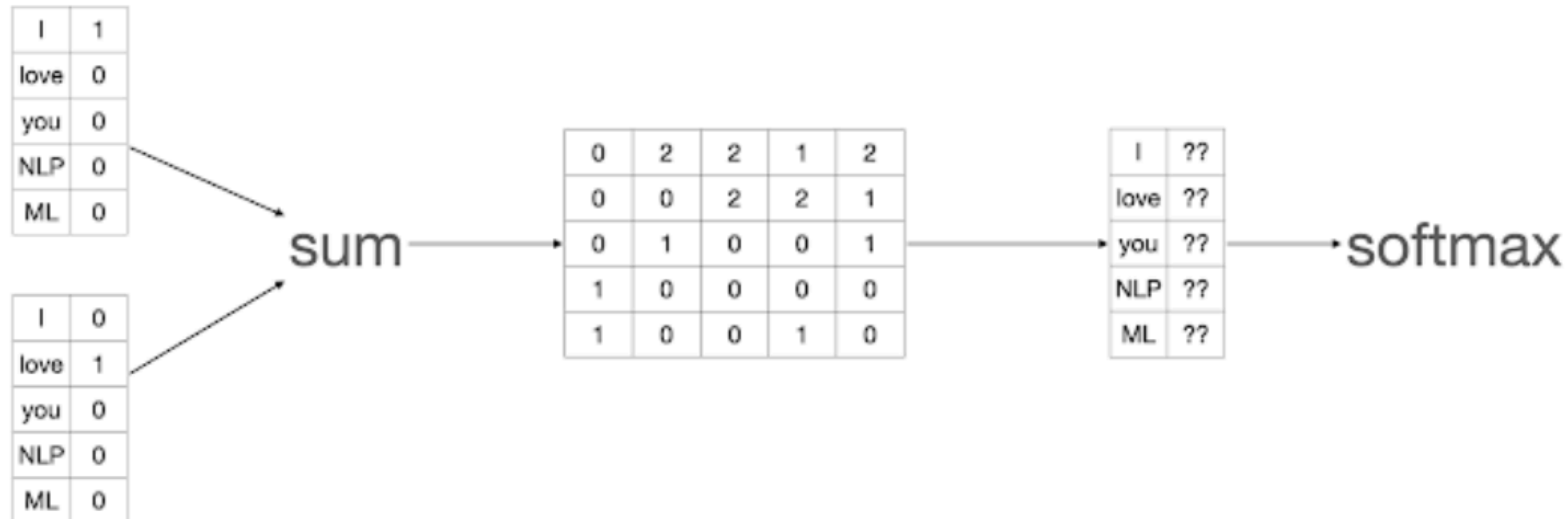
Consider the below network. It is a single-layer perceptron using one-hot encodings. * 1 point

I am using a "bag of vectors" approach, i.e., I summed up the word representations element-wise to obtain a representation for a phrase.

What will be the predicted next word?

Vocab: {I, love, you, NLP, ML}

Input: I love



Consider the
encodings. I a
word represe
What will be t

I	1
love	0
you	0
NLP	0
ML	0

I	0
love	1
you	0
NLP	0
ML	0



```
import numpy as np

v = np.array([1, 1, 0, 0, 0])

w = np.array([[0, 2, 2, 1, 2],
              [0, 0, 2, 2, 1],
              [0, 1, 0, 0, 1],
              [1, 0, 0, 0, 0],
              [1, 0, 0, 1, 0]])

print(v.dot(w))
print(v.T.dot(w))
print(np.matmul(v,w))
print(np.matmul(v.T,w))
```

```
➞ [0 2 4 3 3]
   [0 2 4 3 3]
   [0 2 4 3 3]
   [0 2 4 3 3]
```

one-hot * 1 point
d up the
r a phrase.

→ softmax

I am building an MLP classifier to predict whether a review is positive or negative (the same as in question 1). I train my word embedding layer in the process of training the network, and after I've finished training, I cluster words using the trained embeddings. Which of the following should I expect to see? * 1 point

- ☒ words that occur in positive reviews cluster together, and words that occur in negative reviews cluster together (e.g., good/great/awesome vs. bad/awful/terrible)
- ☐ nouns cluster together, and verbs cluster together (e.g., food/service/ambiance vs. eat/drink/meet)
- ☐ content words cluster together, and stop words cluster together (e.g., food/eat/favorite vs. is/of/and)

Topics

- What is language modeling? When do we use it?
- Ngram language models
- Smoothing
- Perplexity

Topics

- **What is language modeling? When do we use it?**
- Ngram language models
- Smoothing
- Perplexity

Language Modeling

Definition

- Assign a probability to a sequence of words

OR

- Given a sequence of words, predict the most likely next word

OR

- Generate likely sequences of words

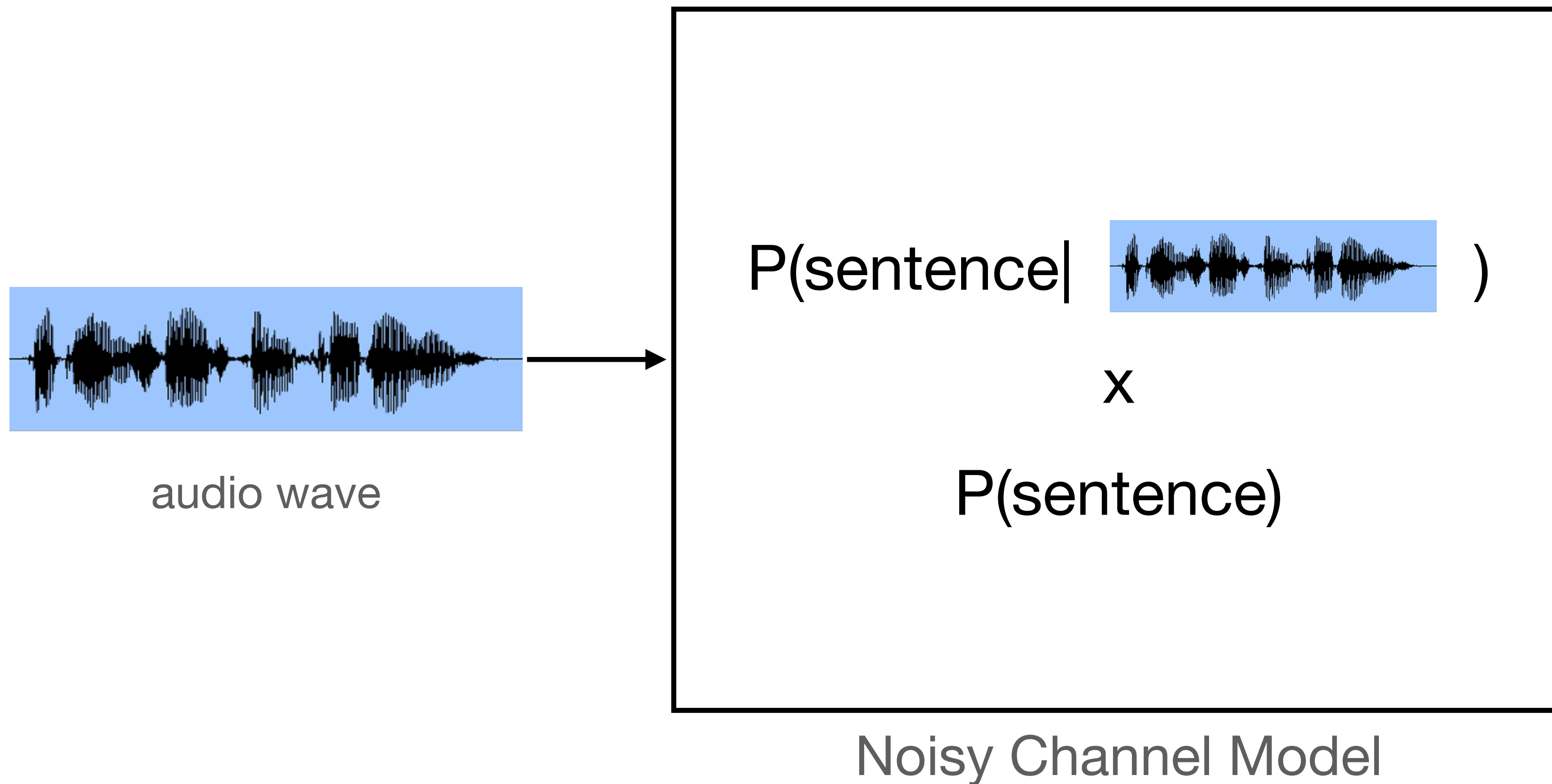
Language Modeling

Applications

- Unconstrained text generation (fun, but not super practical)
- Conditional text generation, e.g.,:
 - Machine translation: e.g., find most likely English sentence given Mandarin sentence
 - Speech recognition: e.g., find most likely English sentence given acoustic input
 - Summarization: e.g., find most likely 50 word English document given a 1000 word English document
 - ...
- Representation learning

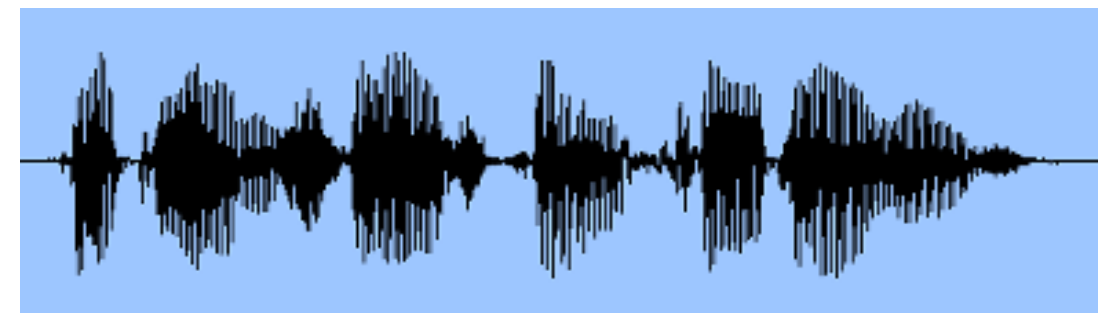
Language Modeling

Application: Noisy Channel Speech Recognition Model



Language Modeling

Application: Noisy Channel Speech Recognition Model



audio wave

$$P(\text{Its easy to recognize speech} \mid \text{audio wave}) = 0.3$$

$$P(\text{Its easy to wreck a nice beach} \mid \text{audio wave}) = 0.5$$

$$P(\text{Its easy to recognize speech}) = 0.5$$

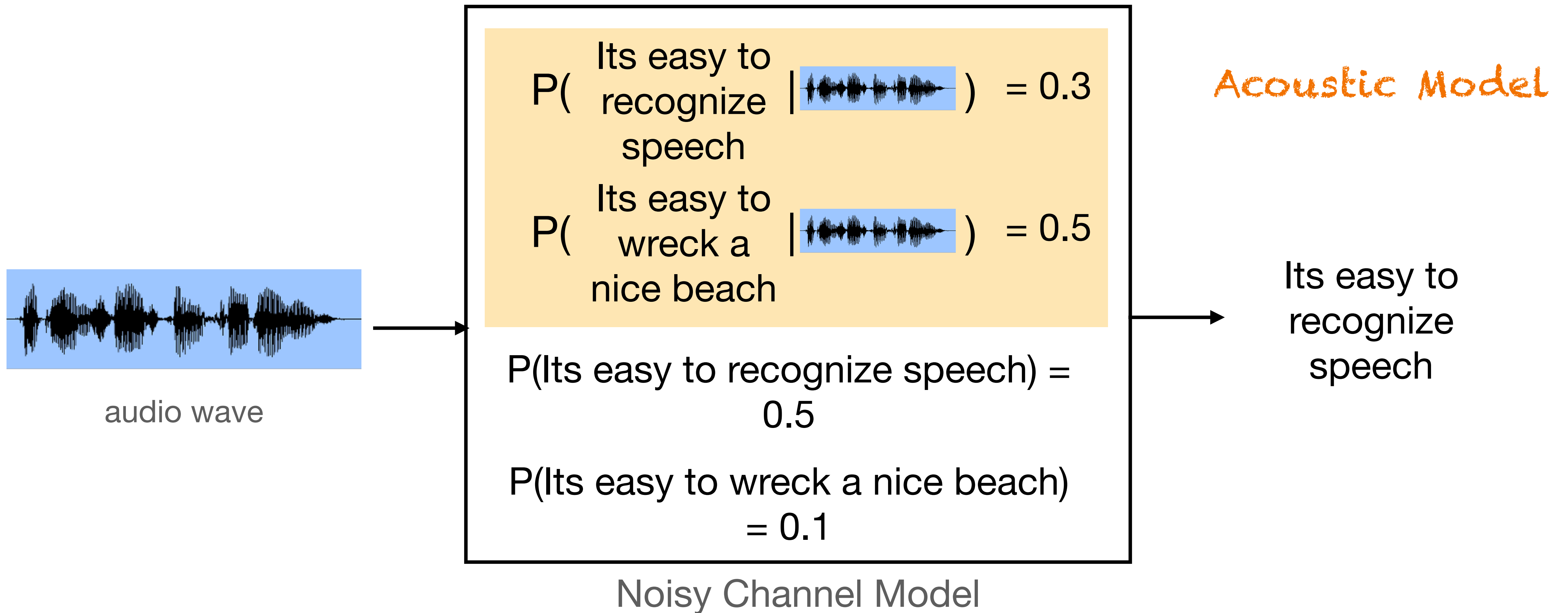
$$P(\text{Its easy to wreck a nice beach}) = 0.1$$

Noisy Channel Model

Its easy to recognize speech

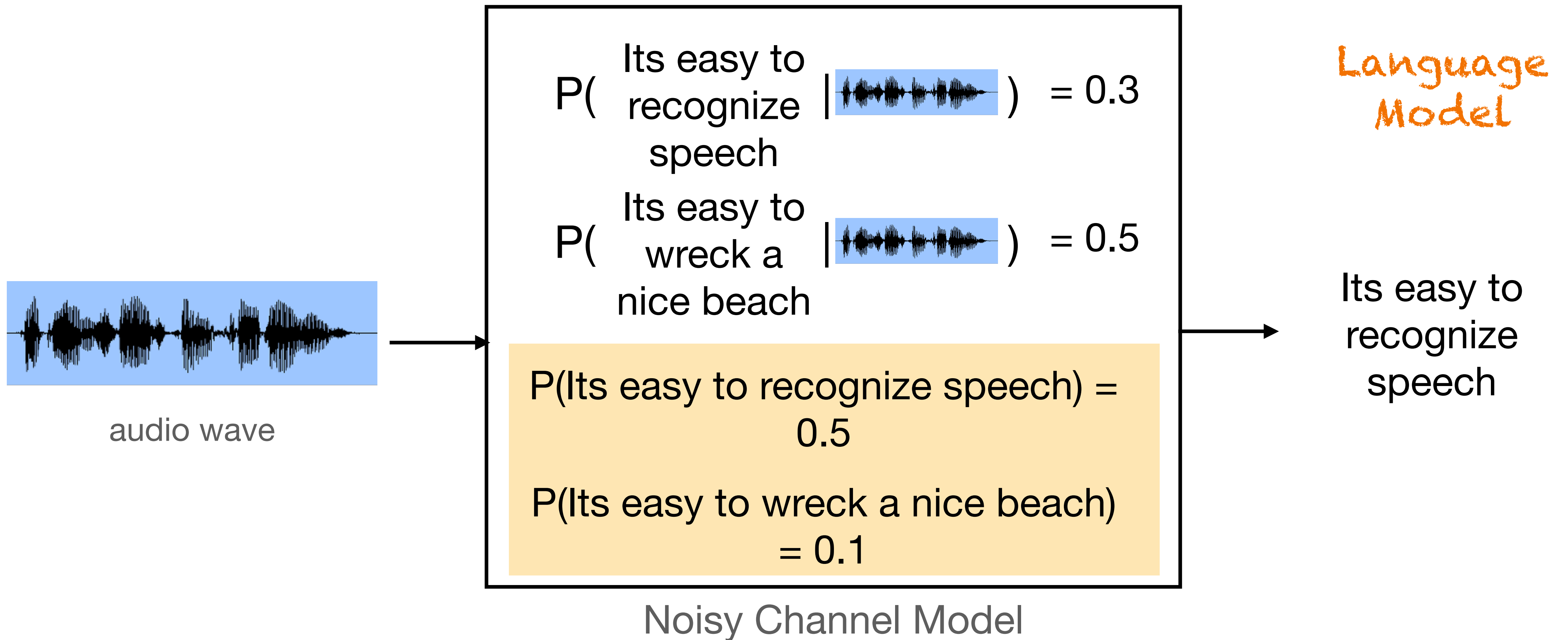
Language Modeling

Application: Noisy Channel Speech Recognition Model



Language Modeling

Application: Noisy Channel Speech Recognition Model



Topics

- What is language modeling? When do we use it?
- **Ngram language models**
- Smoothing
- Perplexity

Ngram Language Models

Directly computing corpus stats

- Simple idea: Just compute the probability $P(w_0, \dots w_n)$ directly from a corpus!
- I.e.:

occurrences of $w_0, \dots w_n$

sequences of length n

Ngram Language Models

Directly computing corpus stats

can you tell me about any good cantonese restaurants close by
mid priced thai food is what i'm looking for
tell me about chez panisse
can you give me a listing of the kinds of food that are available
i'm looking for a good place to eat breakfast
when is caffe venezia open during the day

Ngram Language Models

Directly computing corpus stats

can you tell me about any good cantonese restaurants close by
mid priced thai food is what i'm looking for
tell me about chez panisse
can you give me a listing of the kinds of food that are available
i'm looking for a good place to eat breakfast
when is caffe venezia open during the day

$P(\text{tell me about chez panisse}) =$

Ngram Language Models

Directly computing corpus stats

can you tell me about any good cantonese restaurants close by
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$P(\text{tell me about chez panisse}) =$

1

Ngram Language Models

Directly computing corpus stats

can you tell me about any good cantonese restaurants close by
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tell me about chez panisse
can you give me a listing of the kinds of food that are available
i'm looking for a good place to eat breakfast
when is caffe venezia open during the day

$$P(\text{tell me about chez panisse}) = \frac{1}{\quad}$$

Ngram Language Models

Directly computing corpus stats

can you tell me about any good cantonese restaurants close by
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Ngram Language Models

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Ngram Language Models

Directly computing corpus stats

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when is caffe venezia open during the day

$$P(\text{tell me about chez panisse}) = \frac{1}{51}$$

Ngram Language Models

Directly computing corpus stats

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$$P(\text{tell me about chez panisse}) = \frac{1}{51}$$

Problems?

Ngram Language Models

Directly computing corpus stats

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$$P(\text{tell me about caffe venezia}) = \frac{0}{51}$$

Ngram Language Models

Directly computing corpus stats

can you tell me about any good cantonese restaurants close by
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can you give me a listing of the kinds of food that are available
i'm looking for a good place to eat breakfast
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$$P(\text{tell me about caffe venezia}) = \frac{0}{51}$$

Ngram Language Models

Unigram Language Model

Chain Rule of Probability

$$P(w_0 \dots w_n) = P(w_0) \times P(w_1 | w_0) \times P(w_2 | w_0, w_1) \times \dots \times P(w_n | w_0 \dots w_{n-1})$$

Ngram Language Models

Unigram Language Model

Chain Rule of Probability

$$P(w_0 \dots w_n) = P(w_0) \times P(w_1 | w_0) \times P(w_2 | w_0, w_1) \times \dots \times P(w_n | w_0 \dots w_{n-1})$$

Not helpful (yet). Still requires observing $w_0 \dots w_n$

Ngram Language Models

Unigram Language Model

$$P(w_0 \dots w_n) = P(w_0) \times P(w_1 | w_0) \times P(w_2 | w_0, w_1) \times \dots \times P(w_n | w_0 \dots w_{n-1})$$

$$P(w_i | w_0 \dots w_{i-1}) \approx P(w_i)$$

Independence Assumption
(Just Like Naive Bayes)

Ngram Language Models

Unigram Language Model

$$P(w_0 \dots w_n) = P(w_0) \times P(w_1 | w_0) \times P(w_2 | w_0, w_1) \times \dots \times P(w_n | w_0 \dots w_{n-1})$$

$$P(w_i | w_0 \dots w_{i-1}) \approx P(w_i)$$

$$P(w_0 \dots w_n) \approx P(w_0) \times P(w_1) \times P(w_2) \times \dots \times P(w_n)$$

Unigram Model

Ngram Language Models

Unigram Language Model

can you tell me about any good cantonese restaurants close by
mid priced thai food is what i'm looking for
tell me about chez panisse
can you give me a listing of the kinds of food that are available
i'm looking for a good place to eat breakfast
when is caffe venezia open during the day

$$P(w_0 \dots w_n) \approx P(w_0) \times P(w_1) \times P(w_2) \times \dots \times P(w_n)$$

$$P(\text{tell me about caffe venezia}) = P(\text{tell}) \times P(\text{me}) \times P(\text{about}) \times P(\text{caffe}) \times P(\text{venezia})$$

Ngram Language Models

Unigram Language Model

can you tell me about any good cantonese restaurants close by
mid priced thai food is what i'm looking for
tell me about chez panisse
can you give me a listing of the kinds of food that are available
i'm looking for a good place to eat breakfast
when is caffe venezia open during the day

$$P(w_0 \dots w_n) \approx P(w_0) \times P(w_1) \times P(w_2) \times \dots \times P(w_n)$$

$$P(\text{tell me about caffe venezia}) = \frac{2}{56} \times \frac{3}{56} \times \frac{3}{56} \times \frac{1}{56} \times \frac{1}{56}$$

Ngram Language Models

Unigram Language Model

can you tell me about any good cantonese restaurants close by
mid priced thai food is what i'm looking for
tell me about chez panisse
can you give me a listing of the kinds of food that are available
i'm looking for a good place to eat breakfast
when is caffe venezia open during the day

$$P(w_0 \dots w_n) \approx P(w_0) \times P(w_1) \times P(w_2) \times \dots \times P(w_n)$$

$$\begin{aligned} P(\text{tell me about caffe venezia}) &= \frac{2}{56} \times \frac{3}{56} \times \frac{3}{56} \times \frac{1}{56} \times \frac{1}{56} \\ &= 3.26 \times 10^{-8} \end{aligned}$$

Ngram Language Models

Unigram Language Model

can you tell me about any good cantonese restaurants close by
mid priced thai food is what i'm looking for
tell me about chez panisse
can you give me a listing of the kinds of food that are available
i'm looking for a good place to eat breakfast
when is caffe venezia open during the day

$$P(w_0 \dots w_n) \approx P(w_0) \times P(w_1) \times P(w_2) \times \dots \times P(w_n)$$

P(tell me about caffe venezia)

$$= \log(2/56) + \log(3/56) + \log(3/56) + \log(1/56) + \log(1/56)$$

$$= -13.9$$

Ngram Language Models

Unigram Language Model

can you tell me about any good cantonese restaurants close by
mid priced thai food is what i'm looking for
tell me about chez panisse
can you give me a listing of the kinds of food that are available
i'm looking for a good place to eat breakfast
when is caffe venezia open during the day

$P(\text{tell me about caffe venezia})$

$P(\text{caffe about tell venezia me})$

Which is more probable?

Ngram Language Models

Bigram Language Model

$$P(w_0 \dots w_n) = P(w_0) \times P(w_1 | w_0) \times P(w_2 | w_0, w_1) \times \dots \times P(w_n | w_0 \dots w_{n-1})$$

$$P(w_i | w_0 \dots w_{i-1}) \approx P(w_i | w_{i-1})$$

Markov Assumption!

Ngram Language Models

Bigram Language Model

$$P(w_0 \dots w_n) = P(w_0) \times P(w_1 | w_0) \times P(w_2 | w_0, w_1) \times \dots \times P(w_n | w_0 \dots w_{n-1})$$

$$P(w_i | w_0 \dots w_{i-1}) \approx P(w_i | w_{i-1})$$

$$P(w_0 \dots w_n) \approx P(w_0 | \langle \mathbf{s} \rangle) \times P(w_1 | w_0) \times P(w_2 | w_1) \times \dots \times P(w_n | w_{n-1})$$

Bigram Language Model

Ngram Language Models

Unigram Language Model

<s> can you tell me about any good cantonese restaurants close by </s>
 <s> mid priced thai food is what i'm looking for </s>
 <s> tell me about chez panisse </s>
<s> can you give me a listing of the kinds of food that are available </s>
 <s> i'm looking for a good place to eat breakfast </s>
 <s> when is caffe venezia open during the day </s>

$$P(w_0 \dots w_n) \approx P(w_0 | \text{<S>}) \times P(w_1 | w_0) \times P(w_2 | w_1) \times \dots \times P(w_n | w_{n-1})$$

$$P(\text{tell me about caffe venezia}) = P(\text{tell} | \text{<s>}) \times P(\text{me} | \text{tell}) \times P(\text{about} | \text{me}) \times \\ P(\text{caffe} | \text{about}) \times P(\text{venezia} | \text{caffe}) \times P(\text{</s>} | \text{venezia})$$

Ngram Language Models

Ngram Language Models

Unigram Language Model

$$P(w_0 \dots w_n) \approx P(w_0) \times P(w_1) \times P(w_2) \times \dots \times P(w_n)$$

Bigram Language Model

$$P(w_0 \dots w_n) \approx P(w_0 | \langle s \rangle) \times P(w_1 | w_0) \times P(w_2 | w_1) \times \dots \times P(w_n | w_{n-1})$$

n-gram Language Model

$$P(w_0 \dots w_n) \approx \prod_{i=0}^n P(w_i | w_{i-(n-1)} \dots w_{i-1})$$

Ngram Language Models

Ngram Language Models

To him swallowed confess hear
both. Which. Of save on trail for
are ay device and rote life have

Unigram

Ngram Language Models

Ngram Language Models

To him swallowe
both. Which. Of
are ay device a

Unigram

Why dost stand forth thy canopy,
forsooth; he is this palpable hit
the King Henry. Live king. Follow

Bigram

Ngram Language Models

Ngram Language Models

Trigram

To him swallowe
both. Which. Of
are ay device a

Why dost st
forsooth; h
the King He

Fly, and will rid me these news of
price. Therefore the sadness of
parting, as they say, 'tis done

Unigram

Bigram

Ngram Language Models

Ngram Language Models

Trigram

To him swallowe
both. Which. Of
are ay device a

Why dost st
forsooth; h
the King He

Fly, and will rid me these evils of
price. Th
parting.

King Henry. What! I will go seek
the traitor Gloucester. Exeunt
some of the watch. A great
banquet serv'd in;

Unigram

Bigram

4-gram

Topics

- What is language modeling? When do we use it?
- Ngram language models
- **Smoothing**
- Perplexity

Ngram Language Models

<s> can you tell me about any good cantonese restaurants close by </s>
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<s> can you give me a listing of the kinds of food that are available </s>
 <s> i'm looking for a good place to eat breakfast </s>
 <s> when is caffe venezia open during the day </s>

$$P(w_0 \dots w_n) \approx P(w_0 | \text{<S>}) \times P(w_1 | w_0) \times P(w_2 | w_1) \times \dots \times P(w_n | w_{n-1})$$

$$P(\text{tell me about caffe venezia}) = P(\text{tell} | \text{<s>}) \times P(\text{me} | \text{tell}) \times P(\text{about} | \text{me}) \times \\ P(\text{caffe} | \text{about}) \times P(\text{venezia} | \text{caffe}) \times P(\text{</s>} | \text{venezia})$$

Ngram Language Models

<s> can you tell me about any good cantonese restaurants close by </s>
<s> mid priced thai food is what i'm looking for </s>
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$$P(w_0 \dots w_n) \approx P(w_0 | \text{<s>}) \times P(w_1 | w_0) \times P(w_2 | w_1) \times \dots \times P(w_n | w_{n-1})$$

$$P(\text{tell me about caffe venezia}) = P(\text{tell} | \text{<s>}) \times P(\text{me} | \text{tell}) \times P(\text{about} | \text{me}) \times \\ P(\text{caffe} | \text{about}) \times P(\text{venezia} | \text{caffe}) \times P(\text{</s>} | \text{venezia})$$

Ngram Language Models

<s> can you tell me about any good cantonese restaurants close by </s>
<s> mid priced thai food is what i'm looking for </s>
<s> tell me about chez panisse </s>
<s> can you give me a listing of the kinds of food that are available </s>
<s> i'm looking for a good place to eat breakfast </s>
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$$P(w_0 \dots w_n) \approx P(w_0 | \text{<s>}) \times P(w_1 | w_0) \times P(w_2 | w_1) \times \dots \times P(w_n | w_{n-1})$$

$$P(\text{tell me about caffe venezia}) = P(\text{tell} | \text{<s>}) \times P(\text{me} | \text{tell}) \times P(\text{about} | \text{me}) \times \\ P(\text{caffe} | \text{about}) \times P(\text{venezia} | \text{caffe}) \times P(\text{</s>} | \text{venezia})$$

Zero counts!

Generalization in LMs

Smoothing Strategies

- Laplace Smoothing (i.e., “Add-One” smoothing)
- Backoff
- Kneser-Ney Smoothing

Generalization in LMs

Smoothing Strategies

- **Laplace Smoothing (i.e., “Add-One” smoothing)**
- Backoff
- Kneser-Ney Smoothing

Smoothing

Laplace (“Add One”)

Bigram Probabilities

	i	want	to	eat	chinese	food	lunch	spend
i	5	827		9				2
want	2		608	1	6	6	5	1
to	2		4	686	2		6	211
eat			2		16	2	42	
chinese	1					82	1	
food	15		15		1	4		
lunch	2					1		
spend	1		1					

Smoothing

Laplace (“Add One”)

$P(\text{want} | i)$

	i	want	to	eat	chinese	food	lunch	spend
i	5	827		9				2
want	2		608	1	6	6	5	1
to	2		4	686	2		6	211
eat			2		16	2	42	
chinese	1					82	1	
food	15		15		1	4		
lunch	2					1		
spend	1		1					

Smoothing

Laplace (“Add One”)

$$P(w_n \mid w_{n-1}) = \frac{\#(w_{n-1}w_n)}{\#w_{n-1}}$$

	i	want	to	eat	chinese	food	lunch	spend
i	5	827		9				2
want	2		608	1	6	6	5	1
to	2		4	686	2		6	211
eat			2		16	2	42	
chinese	1					82	1	
food	15		15		1	4		
lunch	2					1		
spend	1		1					

Smoothing

Laplace (“Add One”)

Simple Idea: Just add 1
to everything!

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Smoothing

Laplace (“Add One”)

Need to renormalize to
keep it a probability
distribution

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Smoothing

Laplace (“Add One”)

$$P(w_n \mid w_{n-1}) = \frac{\#(w_{n-1}w_n) + 1}{\sum_w (\#(w_{n-1}w) + 1)}$$

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Smoothing

Laplace (“Add One”)

$$P(w_n | w_{n-1}) = \frac{\#(w_{n-1}w_n) + 1}{\#w_{n-1} + V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Smoothing

Laplace (“Add One”)

$$P(w_n | w_{n-1}) = \frac{\#(w_{n-1}w_n) + 1}{\#w_{n-1} + V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	4	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Often interpreted as “discounting”.

I.e., we borrow probability mass from high-count words in order to make room for unseen words.

Generalization in LMs

Smoothing Strategies

- Laplace Smoothing (i.e., “Add-One” smoothing)
- **Backoff**
- Kneser-Ney Smoothing

Generalization in LMs

Smoothing Strategies

- Laplace Smoothing (i.e., “Add-One” smoothing)
- **Backoff/Interpolation**
- Kneser-Ney Smoothing

Smoothing

Backoff

- Intuition: We can estimate the probability of a longer sequence from the probabilities of its subsequences
- If an ngram of length n is not observed, use the corresponding length $n-1$ ngram instead
- $P(\text{"tell me about caffe venezia"}) \cong P(\text{"me about caffe venezia"})$
 $\cong P(\text{"about caffe venezia"}) \cong P(\text{"caffe venezia"}) \cong P(\text{"venezia"})$

Smoothing

Interpolation

- All counts are estimated using a weighted combination of smaller ngrams
- $P(\text{"tell me about caffe venezia"}) = \lambda_1 P(\text{"tell me about caffe venezia"}) \times \lambda_2 P(\text{"me about caffe venezia"}) \times \lambda_3 P(\text{"about caffe venezia"}) \times \lambda_4 P(\text{"caffe venezia"}) \times \lambda_5 P(\text{"venezia"})$
- Requires some renormalization (like in Laplace Smoothing)

Generalization in LMs

Smoothing Strategies

- Laplace Smoothing (i.e., “Add-One” smoothing)
- Backoff
- **Kneser-Ney Smoothing**

Smoothing

Kneser-Ney

- State-of-the-art smoothing algorithm that combines several ideas:
 1. Absolute discounting (estimated from data)
 2. Replace ngram probabilities with *continuation* probabilities

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$$P_{\text{AbsoluteDiscounting}}(w_i | w_{i-1}) = \frac{C(w_{i-1} w_i) - d}{\sum_v C(w_{i-1} v)} + \lambda(w_{i-1}) P(w_i)$$

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Similar to Laplace, except we discount factor doesn't necessarily correspond to adding 1...

Smoothing

Kneser-Ney

- State-of-the-art smoothing algorithm for n-grams
 - 1. Absolute discounting (estimated from heldout data)
 - 2. Replace ngram probabilities with c

Bigram count in training set	Bigram count in heldout set
0	0.0000270
1	0.448
2	1.25
3	2.24
4	3.23
5	4.21
6	5.23
7	6.21
8	7.21
9	8.26

$$P_{\text{AbsoluteDiscounting}}(w_i | w_{i-1}) = \frac{C(w_{i-1} w_i) - d}{\sum_v C(w_{i-1} v)} + \lambda(w_{i-1}) P(w_i)$$

Instead, we estimate it from data!

Smoothing

Kneser-Ney

- State-of-the-art smoothing algorithm for n-grams
 - 1. Absolute discounting (estimated from heldout data)
 - 2. Replace ngram probabilities with discounted counts

Bigram count in training set	Bigram count in heldout set
0	0.0000270
1	0.448
2	1.25
3	2.24
4	3.23
5	4.21
6	5.23
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$$P_{\text{AbsoluteDiscounting}}(w_i | w_{i-1}) = \frac{C(w_{i-1} w_i) - d}{\sum_v C(w_{i-1} v)} + \lambda(w_{i-1}) P(w_i)$$

Can be fixed (e.g., 0.75) or a function of n

Smoothing

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Interpolated with observed unigram probability

Smoothing

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 2. Replace ngram probabilities with *continuation* probabilities

Consider: I can't see without my reading ____ .

In corpus:

- $P(\text{reading glasses}) = P(\text{reading Kong}) = 0$
- $P(\text{Kong}) > P(\text{glasses})!$
- What to do?

Smoothing

Kneser-Ney

- State-of-the-art smoothing algorithm that combines several ideas:
 1. Absolute discounting (estimated from data)
 2. Replace ngram probabilities with *continuation* probabilities

unique contexts for “glasses” > # unique contexts for “Kong”

so we assume:

$P(\text{glasses}|\text{as-yet-unseen-ctx}) > P(\text{Kong}|\text{as-yet-unseen-ctx})$

Smoothing

Kneser-Ney

- State-of-the-art smoothing algorithm that combines several ideas:
 1. Absolute discounting (estimated from data)
 2. Replace ngram probabilities with *continuation* probabilities

$$P_{\text{KN}}(w_i | w_{i-1}) = \frac{\max(C(w_{i-1}w_i) - d, 0)}{C(w_{i-1})} + \lambda(w_{i-1})P_{\text{CONTINUATION}}(w_i)$$

Topics

- What is language modeling? When do we use it?
- Ngram language models
- Smoothing
- **Perplexity**

Perplexity

- How do we decide if a language model is “good”?
- A good language model should assign high probability to sentences that actually appear
- Instead of using probability directly, we use a metric called “perplexity”
 - Inverse probability of test set, normalized by # of words

Perplexity

$$ppl(W) = \sqrt[n]{\prod_{i=1}^n \frac{1}{P(w_1 \dots w_n)}}$$

Perplexity

$$ppl(W) = \sqrt[n]{\prod_{i=1}^n \frac{1}{P(w_1 \dots w_n)}} = \sqrt[n]{\prod_{i=1}^n \frac{1}{P(w_i | w_{i-1})}}$$

(for bigram model)

Perplexity

Intuition

$$ppl(W) = \sqrt[n]{\prod_{i=1}^n \frac{1}{P(w_1 \dots w_n)}} = \sqrt[n]{\prod_{i=1}^n \frac{1}{P(w_i | w_{i-1})}}$$

Language with
digits 0-9, all
equally probable

$W = 1\ 3\ 5\ 6\ 7\ 2\ 3\ 5\ 4$

Perplexity

Intuition

$$ppl(W) = \sqrt[n]{\prod_{i=1}^n \frac{1}{P(w_1 \dots w_n)}} = \sqrt[n]{\prod_{i=1}^n \frac{1}{P(w_i | w_{i-1})}}$$

Language with
digits 0-9, all
equally probable

$W = 1\ 3\ 5\ 6\ 7\ 2\ 3\ 5\ 4$

$$ppl(W) = \left(\frac{1}{10}\right)^n)^{-\frac{1}{n}} = \frac{1}{10}^{-1} = 10$$

Perplexity

Intuition

- “Weighted average branching factor”, i.e., how many next words can follow any given word?
- In PPL, **lower is better!** A model with lower PPL is less “surprised” by new data
- I.e., a model with lower PPL has more certainty about true sequences. It considers branching factors to be lower, because it has a good sense of what should come next

Perplexity

Intuition

- In natural language, distributions are highly non-uniform, so branching factors are (relatively) low
- PPL will never be zero! Natural language has inherent uncertainty
- PPL is not comparable across different datasets! Some datasets/languages/corpora are “easier”/lower uncertainty than others

Perplexity

Intuition

- Higher-order n-grams lead to lower ppl in general, but:
 - More likely to overfit to training data
 - Require more memory
 - Result in many more zero-counts

