# Recursion

# Lecture Task

## - Genetic Algorithm: Lecture Task 3 -

**Functionality**: In the statistics.Statistics object, implement the following method:

- A method named “bayesianAverage” with:
  - A type Parameter T
  - Has parameters of type List of T, a function of type T to Double, an Int representing the number of extra “fake” ratings, and an Int representing the value of the extra ratings
  - Returns the bayesian average of the elements in the list after the input function is applied to them

**Testing**: In the tests package, complete the test suite named LectureTask3 that tests this functionality.

# Recursion Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Computes the geometric sum of the input
  - ex: if n == 3, geometric sum is 3+2+1 == 6

# Recursion Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Base Case:
  - An input with a trivial output
  - Geometric sum of 0 is defined as 0
  - We could also add 1 -> 1 as a base case

# Recursion Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Recursive Step:
  - Any input that is not a base case will put another recursive call on the stack
  - Write the recursive step with the assumption that the recursive call will return the correct value

# Recursion Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Recursive calls must get closer to the base case
  - All calls must eventually reach a base case or we'll go infinite
  - n-1 is closer to n<=0 than n
  - Regardless of the original value of n, it will eventually be decremented until the base case condition is true

# Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

| Program Stack | |
|---|---|
| Main Frame | args |
| | pointer -> line 11 |
| Method Frame | name:n, value:3 |
| | pointer -> line 5 |
| Method Frame | name:n, value:2 |
| | pointer -> line 5 |
| Method Frame | name:n, value:1 |
| | pointer -> line 5 |
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- Each recursive call creates a new stack frame
- Each frame remembers where it will resume running when it's on the top of the stack

# Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- New frames start at the first line of the method
- Top frame on the stack executes the method one line at a time

| Program Stack | |
|---|---|
| Main Frame | args |
| | pointer -> line 11 |
| Method Frame | name:n, value:3 |
| | pointer -> line 5 |
| Method Frame | name:n, value:2 |
| | pointer -> line 5 |
| Method Frame | name:n, value:1 |
| | pointer -> line 5 |
| Method Frame | name:n, value:0 |
| | pointer -> line 1 |
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# Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Recursive calls are added to the stack until a base case is reached

| Program Stack | |
|---|---|
| Main Frame | args |
| | pointer -> line 11 |
| Method Frame | name:n, value:3 |
| | pointer -> line 5 |
| Method Frame | name:n, value:2 |
| | pointer -> line 5 |
| Method Frame | name:n, value:1 |
| | pointer -> line 5 |
| Method Frame | name:n, value:0 |
| | pointer -> line 3 |
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# Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

| Program Stack | |
|---|---|
| Main Frame | args |
| | pointer -> line 11 |
| Method Frame | name:n, value:3 |
| | pointer -> line 5 |
| Method Frame | name:n, value:2 |
| | pointer -> line 5 |
| Method Frame | name:n, value:1 |
| | pointer -> line 5 |
| Method Frame | returning 0 |
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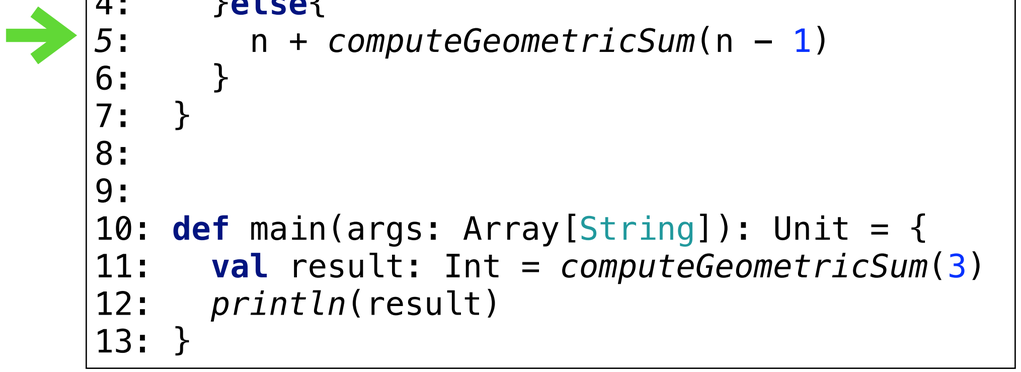
- When a method call returns, its frame is destroyed
- The calling frame resumes and uses the returned value

# Recursive Example



```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

| Program Stack | |
|---|---|
| Main Frame | args |
| | pointer -> line 11 |
| Method Frame | name:n, value:3 |
| | pointer -> line 5 |
| Method Frame | name:n, value:2 |
| | pointer -> line 5 |
| Method Frame | name:n, value:1 |
| | pointer -> line 5 |
| | gets return value of 0 |
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- Method continues after the recursive call
- Sums n + return value and returns this value

# Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

| Program Stack | |
|---|---|
| Main Frame | args |
| | pointer -> line 11 |
| Method Frame | name:n, value:3 |
| | pointer -> line 5 |
| Method Frame | name:n, value:2 |
| | pointer -> line 5 |
| Method Frame | returning 1 |
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- This frame reaches the end of the method
- Repeat the process

# Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

| Program Stack | |
|---|---|
| Main Frame | args |
| | pointer -> line 11 |
| Method Frame | name:n, value:3 |
| | pointer -> line 5 |
| Method Frame | name:n, value:2 |
| | pointer -> line 5 |
| | gets return value of 1 |
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- Return value
- Pop off the stack
- Resume execution of the top frame

# Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

| Program Stack | |
|---|---|
| Main Frame | args |
| | pointer -> line 11 |
| Method Frame | returning 6 |
| | |

- Process continues until all recursive calls resolve
- Last frame returns to main

# Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Main continues with the result from the recursive calls

| Program Stack | |
|---|---|
| Main Frame | args |
| | pointer -> line 11 |
| | gets return value of 6 |
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# Recursive Example

```
1:  def computeGeometricSum(n: Int): Int ={
2:    if(n <= 0){
3:      0
4:    }else{
5:      n + computeGeometricSum(n - 1)
6:    }
7:  }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Main continues with the result from the recursive calls

| Program Stack | |
|---|---|
| Main Frame | args |
| | name:result, value:6 |
| | pointer -> line 12 |
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# Anagrams Revisited

# Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Recall anagrams
  - Rewritten to use functional programming and no vars
  - The syntax may not fully make sense until the next Functional Programming lecture

# Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Base Case
  - A String of length 1 is itself its only anagram
  - If the length is 1, return a new list containing only that String

# Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Base Case Note
  - We will eventually return a list containing all anagrams from the top level call
  - The base case is the only time we create a new List

# Anagrams Example

```scala
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Recursive Step
  - For each character in the input String
    - Remove that character and make a recursive call with the remaining characters
    - Append the removed character to all the returned anagrams

# Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Recursive Step
  - We write this code with the assumption that our recursive calls will return all the anagrams of the new Strings
  - If our logic is sound, this assumption will be true through the power of recursion!

# Anagrams Example

```scala
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Always reach a base case
  - We always make recursive calls on the input String with 1 character removed
    - newString.length == input.length -1
  - This always gets us closer to the base case

# Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Always reach a base case
  - When the base case is reached and returned, our logic starts working for us
  - If this code does append the removed character to each returned anagram, output is generated starting at the base case and built up as the stack frames return

# Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
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  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
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    }).toList

    output.flatten.distinct
  }
}
```

- Example:
  - input == "at"
  - Makes 2 recursive calls to the base case
    - "a" and "t" are returned
  - Append "t" to "a" and "a" to "t" (The removed characters)
  - Return ["at", "ta"] to the next recursive call with an input of length 3

# Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
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      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Functional Programming notes (More detail Later)
  - yield: Creates a data structure containing the last expression that was evaluated on each iteration of a loop

# Anagrams Example

```
def anagrams(input: String): List[String] = {
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    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Functional Programming notes (More detail Monday)
  - map: Creates a new data structure by applying a function to each element
    - The _ is shorthand syntax we can use instead of naming the parameters of a function when the types can be inferred, and each input is only used once

# Anagrams Example

```scala
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Functional Programming notes (More detail Monday)
  - Scala data structures come with many helpful FP style methods
  - Flatten: Creates a single List from a List of Lists containing all the elements from each List
  - Distinct: Creates a new List with all duplicate values removed

# Lecture Task

## - Genetic Algorithm: Lecture Task 3 -

**Functionality**: In the statistics.Statistics object, implement the following method:

- A method named “bayesianAverage” with:
  - A type Parameter T
  - Has parameters of type List of T, a function of type T to Double, an Int representing the number of extra “fake” ratings, and an Int representing the value of the extra ratings
  - Returns the bayesian average of the elements in the list after the input function is applied to them

**Testing**: In the tests package, complete the test suite named LectureTask3 that tests this functionality.