

Stack and Queue

Lecture Question

Task: Implement a backlog to track tasks that can't be complete immediately

- In the week11 package, write a class named Backlog with the following functionality
 - Takes a type parameter A
 - Takes a function in its constructor of type $A \Rightarrow \text{Unit}$
 - A method named addTask that takes a task of type A and returns Unit that adds the task to the backlog (A queue)
 - A method names completeTask that takes no parameters and returns Unit that called the function (from the constructor) on the oldest task in the backlog and removes that task from the backlog

Stack and Queue

- Data structures with specific purpose
 - Restricted features
- All operations are very efficient
 - Inefficient operations are restricted
- We'll see a stack and queue using linked lists
- *Scala has builtin Stack and Queue classes

Stack

- LIFO
 - Last in First out
 - The last element pushed onto the stack is the first element to be popped off the stack
- Only the element on the top of the stack can be accessed



Stack Methods

- Push
 - Add an element to the top of the stack
- Pop
 - Remove the top element of the stack

Stack Implementation

- Implement a Stack class by wrapping a linked list
- Stack uses the linked list and adapts its methods to implement push and pop

```
class Stack[A] {  
    var top: LinkedListNode[A] = null  
  
    def push(a: A): Unit = {  
        this.top = new LinkedListNode[A](a, this.top)  
    }  
  
    def pop(): A = {  
        val toReturn = this.top.value  
        this.top = this.top.next  
        toReturn  
    }  
}
```

Stack Usage

- Create a new empty Stack
- Call push to add an element to the top
- Call pop to remove an element
- Same exact usage when using Scala's builtin Stack

```
val stack = new Stack[Int]()  
stack.push(3)  
stack.push(7)  
stack.push(2)  
stack.push(-5)
```

```
val element = stack.pop()
```

Stack Usage

- We can use Scala's list as a Stack
 - The preferred way to use the concept of a stack in practice
- This is very efficient!
- But wait.. doesn't this create a new list each time an element is pushed or popped since List is immutable (cannot change)?
 - No.. well, kind of

```
var stack = List[Int]()  
stack = 3 :: stack  
stack = 7 :: stack  
stack = 2 :: stack  
stack = -5 :: stack
```

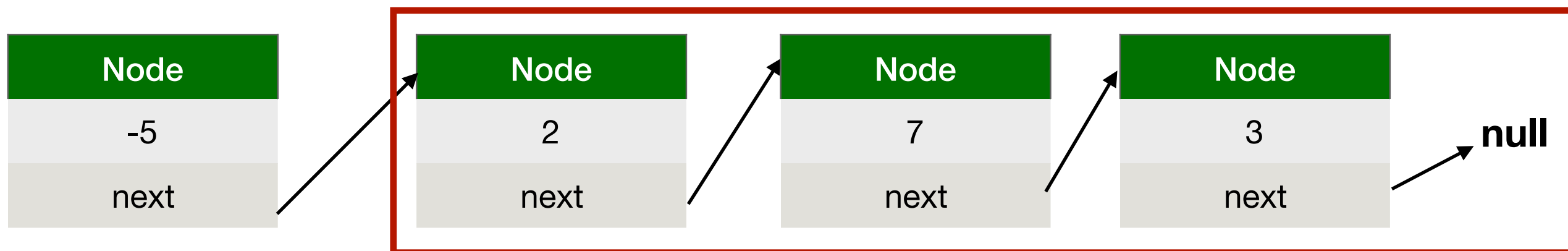
```
val element = stack.head  
stack = stack.drop(1)
```


Stack Usage

- Before -5 is pushed, the stack is equal to nodes in the red box
- After pushing -5, the red box is unchanged
- A new List **is** returned, but it reuses the old List
 - No need to recreate the entire List

```
var stack = List[Int]()  
stack = 3 :: stack  
stack = 7 :: stack  
stack = 2 :: stack  
stack = -5 :: stack
```

```
val element = stack.head  
stack = stack.drop(1)
```

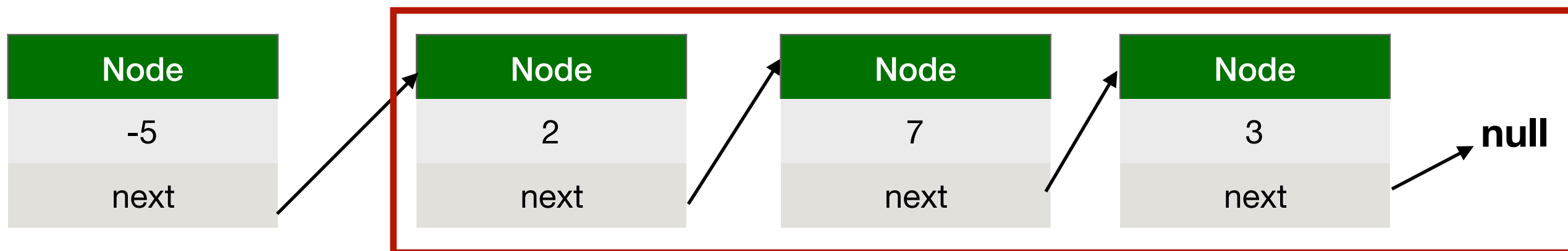


Stack Usage

- Same efficiency when -5 is popped
- The red box never changed, but we update the reference stored in the stack variable
- Other parts of the program can share parts of a List without having their changes affect each other

```
var stack = List[Int]()  
stack = 3 :: stack  
stack = 7 :: stack  
stack = 2 :: stack  
stack = -5 :: stack
```

```
val element = stack.head  
stack = stack.drop(1)
```



Queue

- FIFO
 - First in First out
 - The first element enqueued into the queue is the first element to be dequeued out of the queue
- Elements can only be added to the end of the queue
- Only the element at the front of the queue can be accessed



Queue Methods

- Enqueue
 - Add an element to the end of the queue
- Dequeue
 - Remove the front element in the queue

Queue Implementation

- Implement a Queue class by wrapping a linked list
- Queue needs a reference to the first and last element

```
class Queue[A] {  
  
  var front: LinkedListNode[A] = null  
  var back: LinkedListNode[A] = null  
  
  def enqueue(a: A): Unit = {  
    if (back == null) {  
      this.back = new LinkedListNode[A](a, null)  
      this.front = this.back  
    } else {  
      this.back.next = new LinkedListNode[A](a, null)  
      this.back = this.back.next  
    }  
  }  
  
  def dequeue(): A = {  
    val toReturn = this.front.value  
    this.front = this.front.next  
    if (this.front == null) {  
      this.back = null  
    }  
    toReturn  
  }  
}
```

Queue Usage

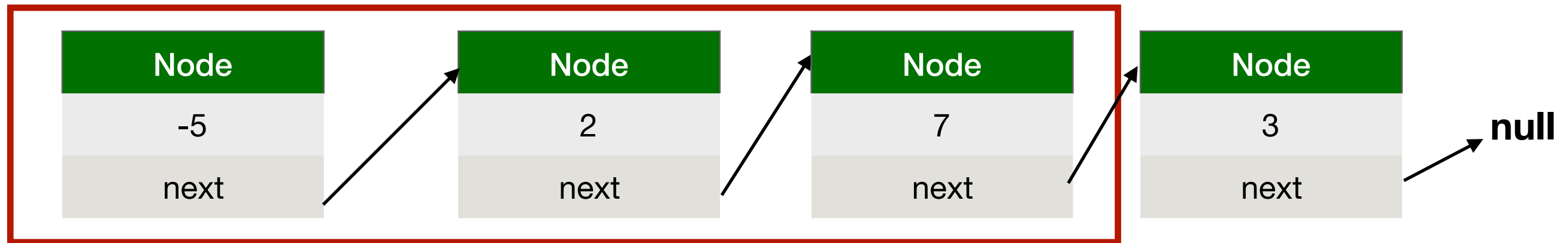
- Create a new empty Queue
- Call enqueue to add an element to the back
- Call dequeue to remove the element at the front
- Same exact usage when using Scala's builtin Queue
 - [based on mutable List just like our implementation]

```
val queue = new Queue[Int]()  
queue.enqueue(3)  
queue.enqueue(7)  
queue.enqueue(2)  
queue.enqueue(-5)
```

```
val element = queue.dequeue()
```

Queue Usage

- No efficient way to use an immutable List as a queue
- The enqueue 3 the list in the red box must change
 - The next reference of the node containing 7 has to be updated
- This List cannot be [should not be] used by other parts of the program since the List is changing



Lecture Question

Task: Implement a backlog to track tasks that can't be complete immediately

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Lecture Question Example

```
class Email {  
  var checked = false  
}  
  
def checkEmail(email: Email): Unit = {  
  email.checked = true  
  println("Checked an email")  
}  
  
val backlog = new Backlog[Email](checkEmail)  
  
// 7 new emails hit the inbox  
backlog.addTask(new Email) // 1  
backlog.addTask(new Email) // 2  
backlog.addTask(new Email)  
backlog.addTask(new Email)  
backlog.addTask(new Email)  
backlog.addTask(new Email)  
backlog.addTask(new Email)  
  
// Only time to check 2 emails  
backlog.completeTask() // checks the email marked 1  
backlog.completeTask() // checks the email marked 2
```

**Good News
Approaching**

Bonus Homework

Task: Evaluate general infix expressions

- **Worth:** 50 points
- **Calculator Recovery:** If you complete this assignment you will also recover unto half of the points you missed on the calculator assignment, excluding the bonus points
 - An 80 would become a 90
 - A 0 would become a 50
- Grades are all or nothing and the assignment has a single objective
 - Primary objective. No testing required by the grader, but nearly required to be successful
- **Due:** ??? (You decide)
- **PhysicsEngine Recovery:** The last homework will contain a recovery for the first HW of the semester

Infix Expressions

$$(12-4) - (8+9/3)$$

- The standard way to write an expression
- Operators placed **between** two operands
- Order of operations must be considered
- Parentheses used to override order of operations

Evaluating Infix Expressions

- PEMDAS
 - Parentheses -> Exponentiation -> Multiplication/Division -> Addition/Subtraction
- $(12-4) - (8+9/3)$
- $8 - (8+9/3)$
- $8 - (8+3)$
- $8 - 11$
- -3

Postfix Expressions

- 12 4 - 8 9 3 / + -
- Advantages:
 - No parentheses needed
 - No order of operations to consider
 - Easy for computers to read
- Disadvantages
 - Hard for humans to read (Without practice)

Evaluating Postfix Expressions

- Find the first operator and evaluate it using the previous 2 operands
- Repeat until there are no operators
- **12 4 -** 8 9 3 / + -
- 8 8 **9 3 /** + -
- 8 **8 3 +** -
- **8 11 -**
- **-3**

Infix -> Postfix

- Shunting Yard
 - Convert infix to postfix
- Read expression left to right
- Copy operands to the output
- Push operators and parentheses on a stack
 - If reading `)`, move top of stack to output until `(` is popped
 - If reading an operator, first move top of stack to output until a lower precedent operator is on top or the stack is empty
- After reading the entire input, copy the rest of the stack to the output

$(12-4) - (8+9/3)$ \rightarrow 12 4 - 8 9 3 / + -

Infix -> Postfix

- Read expression left to right
- Copy operands to the output
- Push operators and parentheses on a stack
 - If reading), move top of stack to output until (is popped
 - If reading an operator, first move top of stack to output until a lower precedent operator is on top or the stack is empty

Output

Input

(12-4) - (8+9/3)

Stack

Infix -> Postfix

- Read expression left to right
- Copy operands to the output
- Push operators and parentheses on a stack
 - If reading), move top of stack to output until (is popped
 - If reading an operator, first move top of stack to output until a lower precedent operator is on top or the stack is empty

Output

Input

12-4) - (8+9/3)

Stack

(

Infix -> Postfix

- Read expression left to right
- Copy operands to the output
- Push operators and parentheses on a stack
 - If reading `)`, move top of stack to output until `(` is popped
 - If reading an operator, first move top of stack to output until a lower precedent operator is on top or the stack is empty

Output

12

Input

-4) - (8+9/3)

Stack

(

Infix -> Postfix

- Read expression left to right
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 - If reading), move top of stack to output until (is popped
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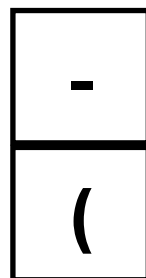
Output

12

Input

4) - (8+9/3)

Stack



Infix -> Postfix

- Read expression left to right
- Copy operands to the output
- Push operators and parentheses on a stack
 - If reading `)`, move top of stack to output until `(` is popped
 - If reading an operator, first move top of stack to output until a lower precedent operator is on top or the stack is empty

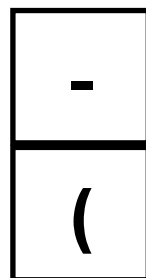
Output

12 4

Input

) - (8+9/3)

Stack



Infix -> Postfix

- Read expression left to right
- Copy operands to the output
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 - If reading `)`, move top of stack to output until `(` is popped
 - If reading an operator, first move top of stack to output until a lower precedent operator is on top or the stack is empty

Output

12 4 -

Input

) - (8+9/3)

Stack

(

Infix -> Postfix

- Read expression left to right
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 - If reading), move top of stack to output until (is popped
 - If reading an operator, first move top of stack to output until a lower precedent operator is on top or the stack is empty

Output

12 4 -

Input

- (8+9/3)

Stack

Infix -> Postfix

- Read expression left to right
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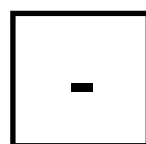
Output

12 4 -

Input

(8+9/3)

Stack



Infix -> Postfix

- Read expression left to right
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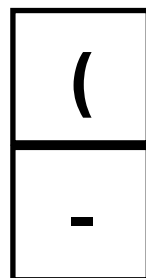
Output

12 4 -

Input

8+9/3)

Stack



Infix -> Postfix

- Read expression left to right
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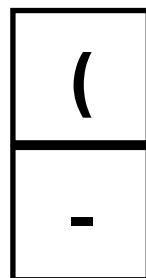
Output

12 4 - 8

Input

+9/3)

Stack



Infix -> Postfix

- Read expression left to right
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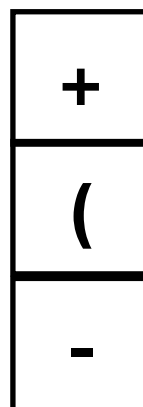
Output

12 4 - 8

Input

9/3)

Stack



Infix -> Postfix

- Read expression left to right
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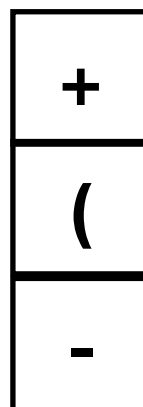
Output

12 4 - 8 9

Input

/3)

Stack



Infix -> Postfix

- Read expression left to right
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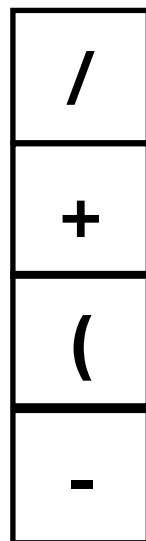
Output

12 4 - 8 9

Input

3)

Stack



Infix -> Postfix

- Read expression left to right
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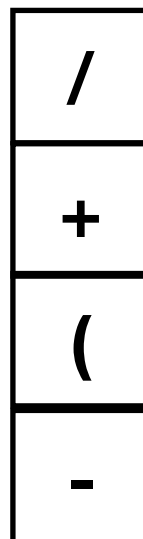
Output

12 4 - 8 9 3

Input

)

Stack



Infix -> Postfix

- Read expression left to right
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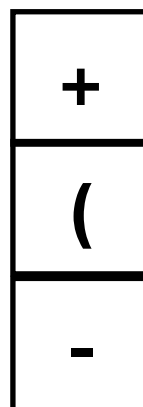
Output

12 4 - 8 9 3 /

Input

)

Stack



Infix -> Postfix

- Read expression left to right
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 - If reading an operator, first move top of stack to output until a lower precedent operator is on top or the stack is empty

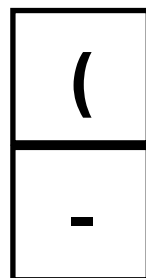
Output

12 4 - 8 9 3 / +

Input

)

Stack



Infix -> Postfix

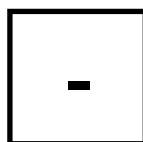
- Read expression left to right
- Copy operands to the output
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 - If reading), move top of stack to output until (is popped
 - If reading an operator, first move top of stack to output until a lower precedent operator is on top or the stack is empty

Output

12 4 - 8 9 3 / +

Input

Stack



Infix -> Postfix

- Read expression left to right
- Copy operands to the output
- Push operators and parentheses on a stack
 - If reading), move top of stack to output until (is popped
 - If reading an operator, first move top of stack to output until a lower precedent operator is on top or the stack is empty

Output

Input

1 2 4 - 8 9 3 / + -

Stack

Bonus Homework

- Evaluate any expression with any order of operations
- The example was arithmetic using PEMDAS
- Solution should also work with boolean logic
 - `"(true -> false) <> (false && true)"` evaluates to true
- Solution should work regardless of the domain given the following:
 - The type of the operands and result of the expression as a type parameter
 - The expression to be evaluated
 - A function to convert strings into operands (ex. `toDouble`, `toBoolean`)
 - A map of operator symbols to their functions
 - The order of operations as a list of lists
 - Ex. `List(List("^"), List("*", "/"), List("+", "-"))` for PEMDAS