Binary Trees and Traversals

Lecture Question

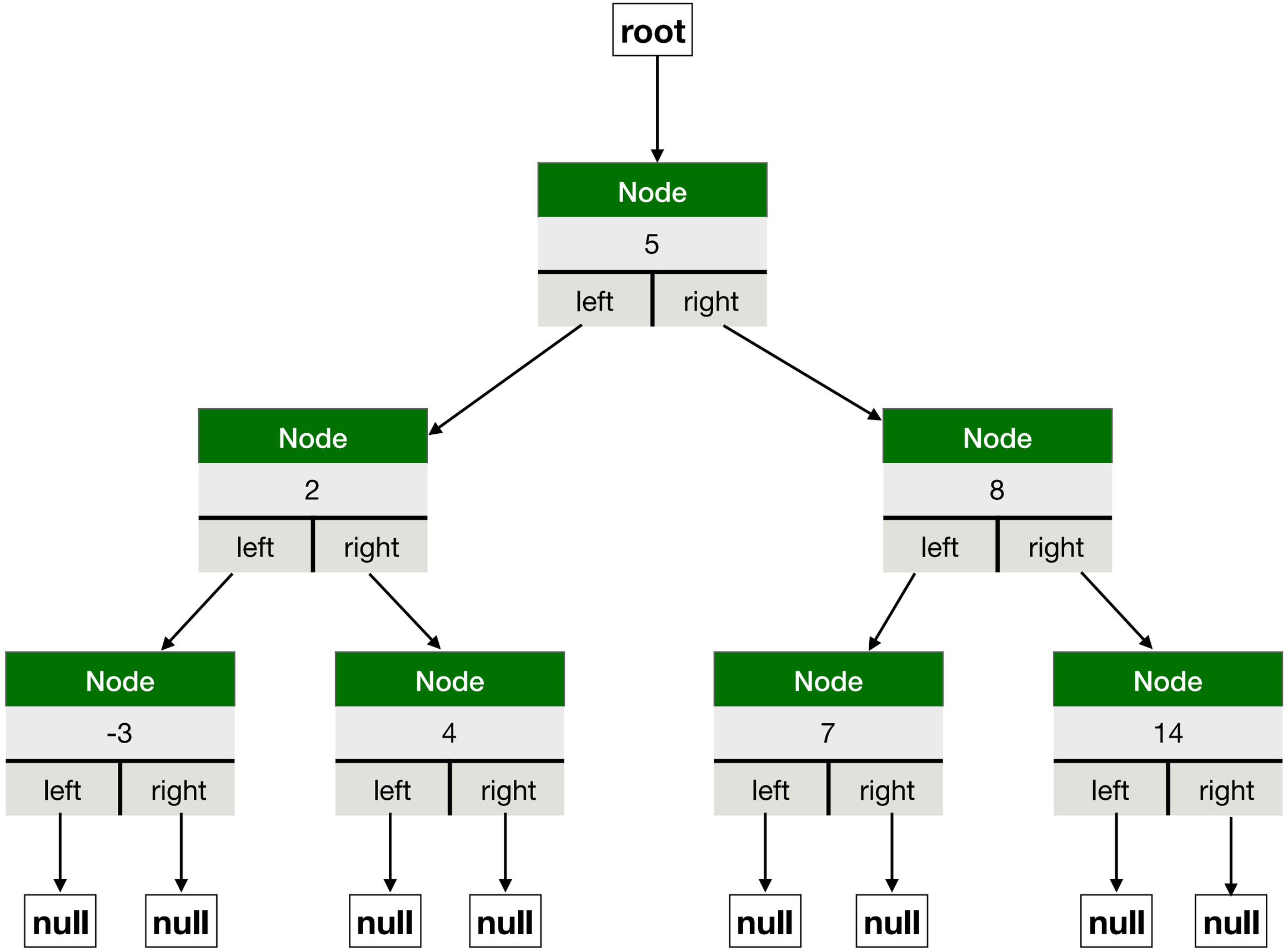
Task: Evaluate an expression tree

- In the week12.ExpressionTree object, write a method named evaluateTree that takes the root of an expression tree (BinaryTreeNode[String]) as a parameter and returns the evaluation of the tree as a Double
- The operators can be *, /, +, and -

* 20 points
* Due tomorrow night

Binary Trees

- Similar in structure to Linked List
  - Consists of Nodes
  - A Tree is only a reference to the first node (Called the root node)
- Trees have 2 references to nodes
  - Each node has left and right reference
  - Vocab: These are called its child nodes
  - Vocab: The node is the parent to these children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Node
14
left
right
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null
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null

The Code

```
class BinaryTreeNode[A](var value: A, var left: BinaryTreeNode[A], var right: BinaryTreeNode[A]) {

}
```

```
val root = new BinaryTreeNode[Int](5, null, null)
root.left = new BinaryTreeNode[Int](2, null, null)
root.right = new BinaryTreeNode[Int](8, null, null)
root.left.left = new BinaryTreeNode[Int](-3, null, null)
root.left.right = new BinaryTreeNode[Int](4, null, null)
root.right.left = new BinaryTreeNode[Int](7, null, null)
root.right.right = new BinaryTreeNode[Int](14, null, null)
```

- Binary Tree Nodes are very similar in structure to Linked List Nodes
- No simple prepend or append so we'll manually build a tree by setting left and right directly

Tree Traversals

- How do we compute with trees?
  - With linked lists we wrote several methods that recursively visited the next node to visit every value
- With trees, how do we visit both children of each node?
  - Recursive call on both child nodes
- We'll see 3 different approaches
  - Pre-Order Traversal
  - In-Order Traversal
  - Post-Order Traversal

Tree Traversals

- Pre-Order Traversal
  - Visit the node's value
  - Call pre-order on the left child
  - Call pre-order on the right child

```
def preOrderTraversal[A](node: BinaryTreeNode[A], f: A => Unit): Unit = {
  if(node != null) {
    f(node.value)
    preOrderTraversal(node.left, f)
    preOrderTraversal(node.right, f)
  }
}
```

```
preOrderTraversal(root, println)
```

Tree Traversals

```
preOrderTraversal(root, println)
```

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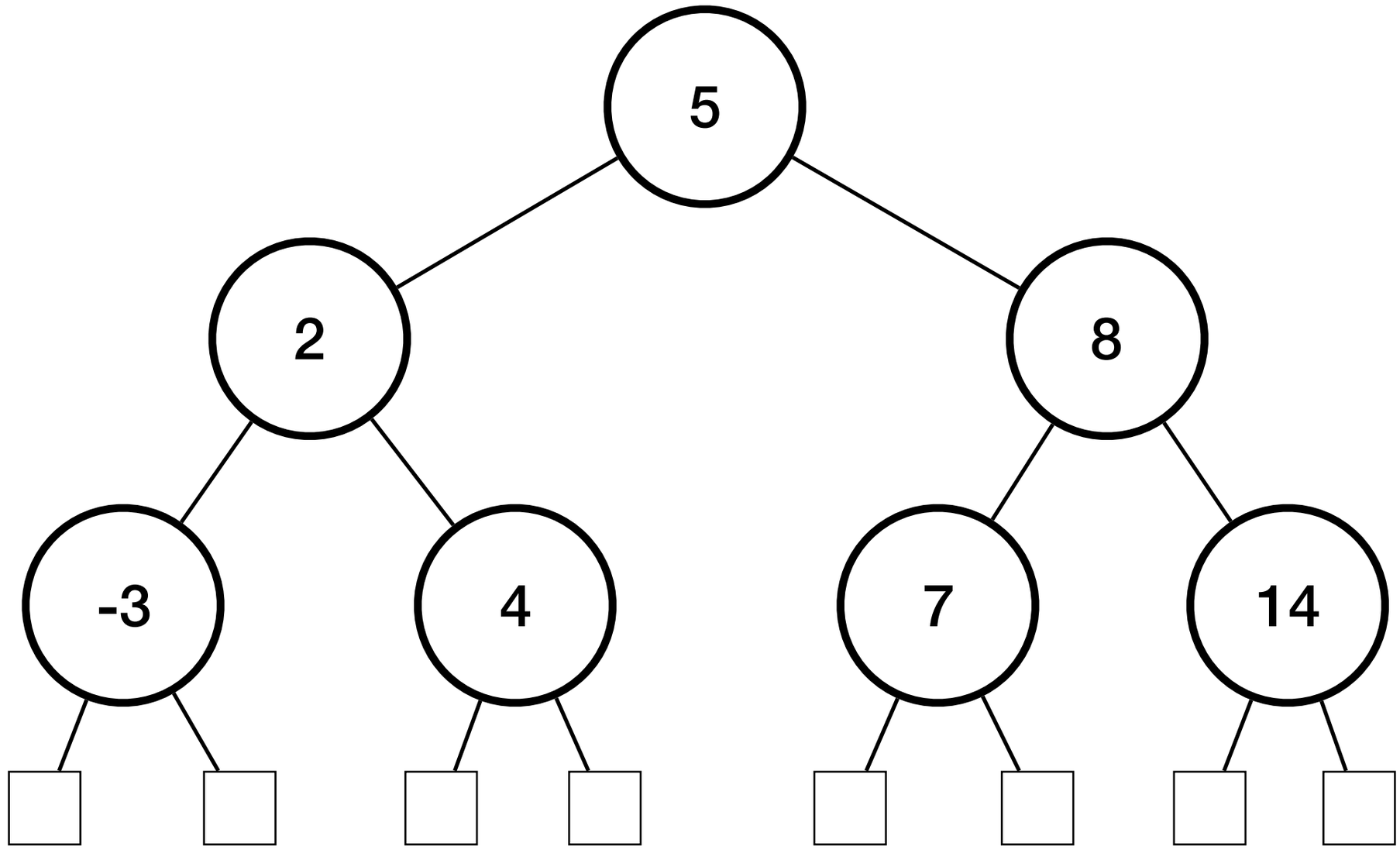

Tree Traversals

- Post-Order Traversal
  - Call post-order on the left child
  - Call post-order on the right child
  - Visit the node's value

```
def postOrderTraversal[A](node: BinaryTreeNode[A], f: A => Unit): Unit = {
  if(node != null) {
    postOrderTraversal(node.left, f)
    postOrderTraversal(node.right, f)
    f(node.value)
  }
}
```

```
postOrderTraversal(root, println)
```

Tree Traversals

```
postOrderTraversal(root, println)
```

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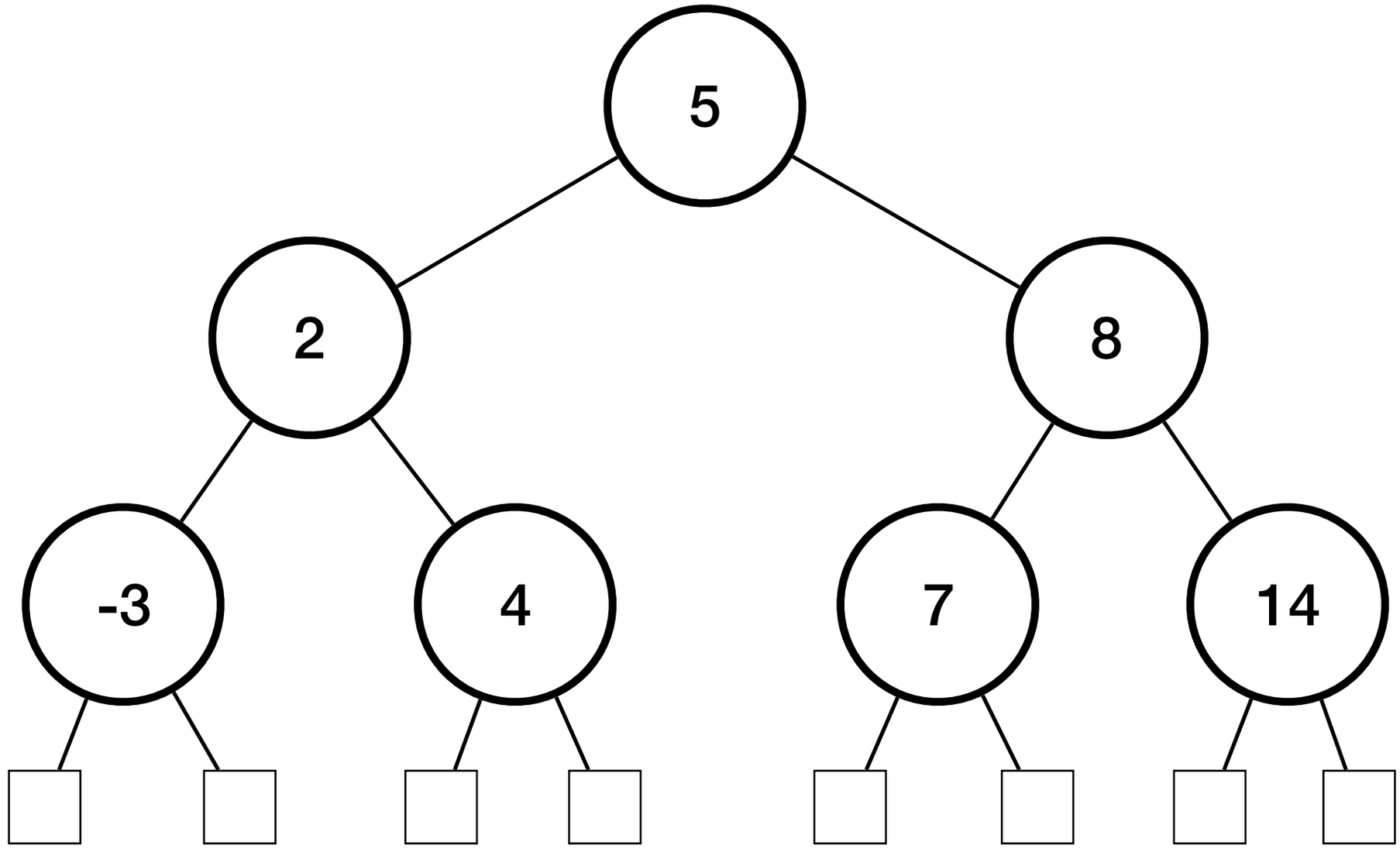

Tree Traversals

- In-Order Traversal
  - Call in-order on the left child
  - Visit the node's value
  - Call in-order on the right child

```
def inOrderTraversal[A](node: BinaryTreeNode[A], f: A => Unit): Unit = {
  if(node != null) {
    inOrderTraversal(node.left, f)
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  }
}
```

```
inOrderTraversal(root, println)
```

The Code

```
def inOrderTraversal[A](node: BinaryTreeNode[A], f: A => Unit): Unit = {
  if(node != null) {
    inOrderTraversal(node.left, f)
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  }
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def preOrderTraversal[A](node: BinaryTreeNode[A], f: A => Unit): Unit = {
  if(node != null) {
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    preOrderTraversal(node.right, f)
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}

def postOrderTraversal[A](node: BinaryTreeNode[A], f: A => Unit): Unit = {
  if(node != null) {
    postOrderTraversal(node.left, f)
    postOrderTraversal(node.right, f)
    f(node.value)
  }
}
```

- **Challenge: Write these with loops and no recursion**

Traversals

```
def inOrderTraversal[A](node: BinaryTreeNode[A], f: A => Unit): Unit = {
  if(node != null) {
    inOrderTraversal(node.left, f)
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}
```

```
inOrderTraversal(root, println)
```

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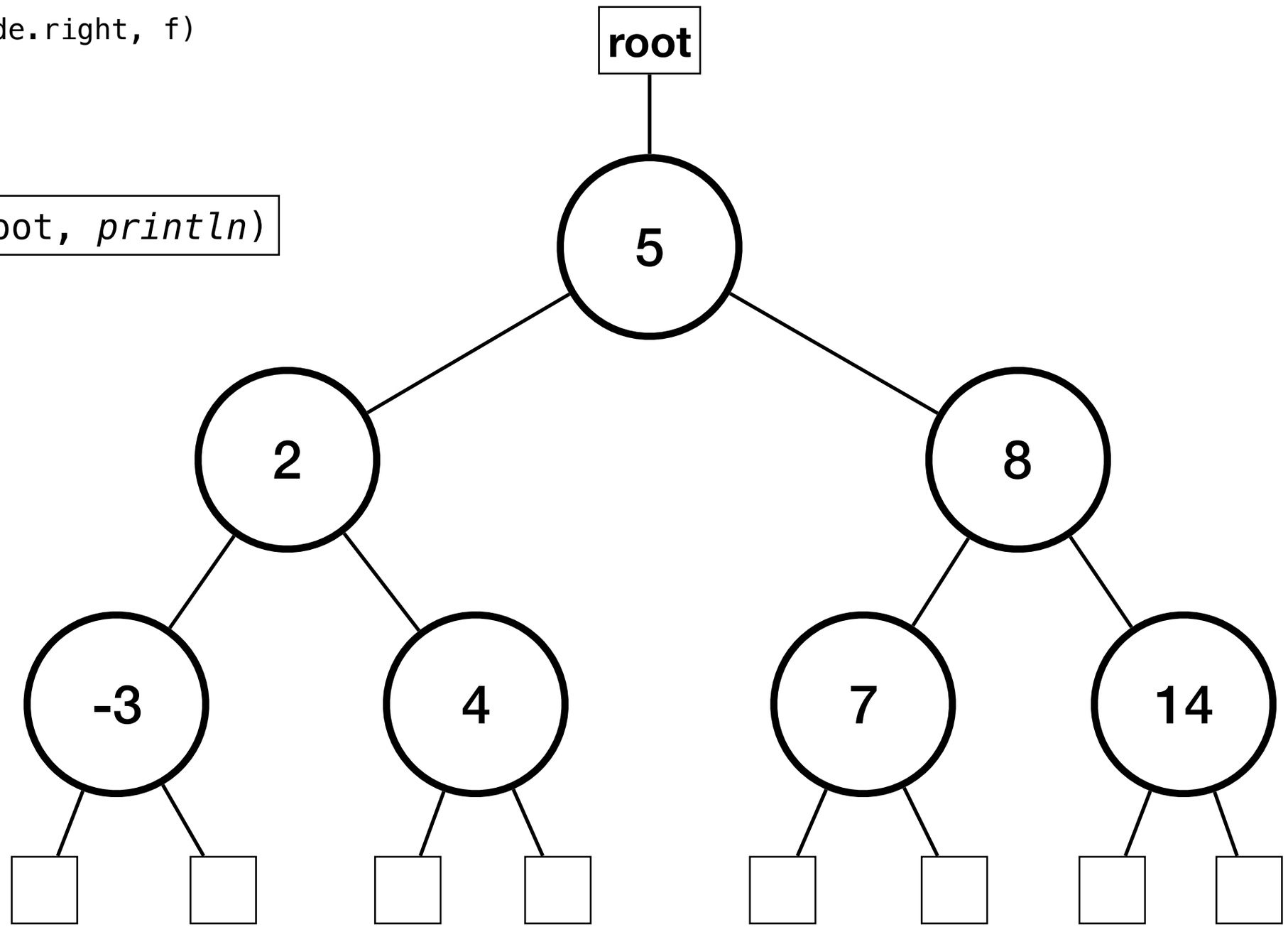

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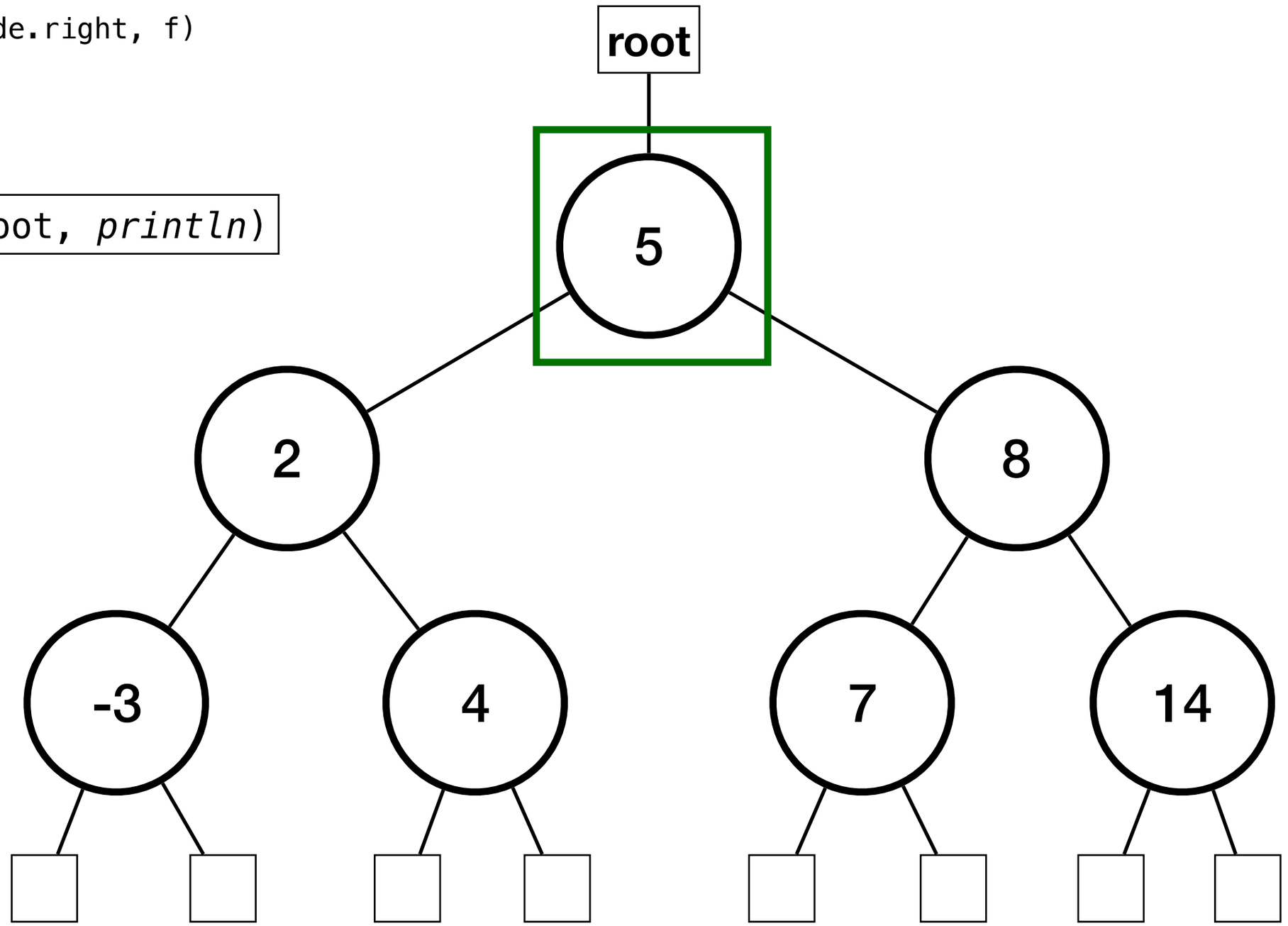

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`inOrderTraversal(root, println)`

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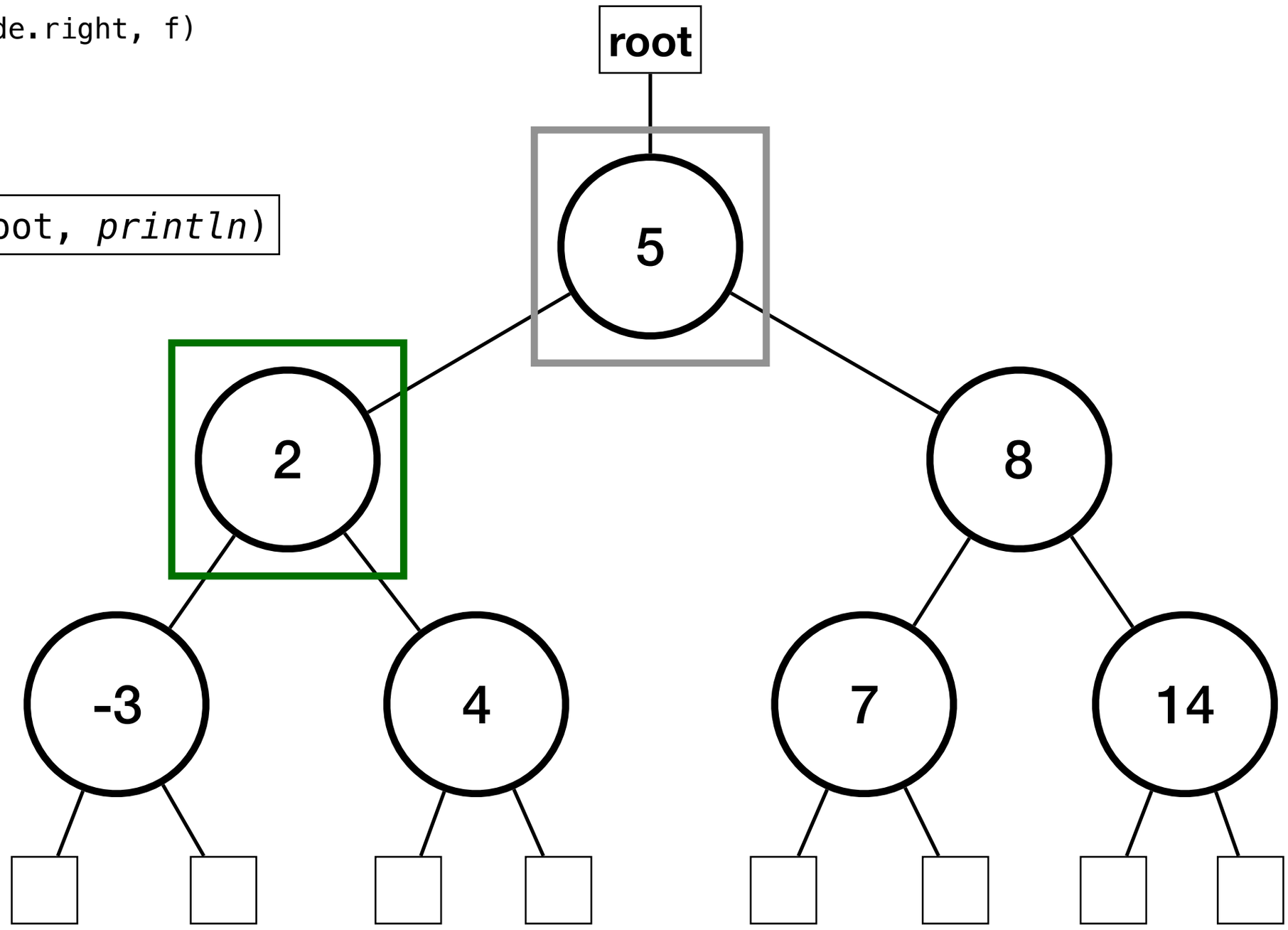

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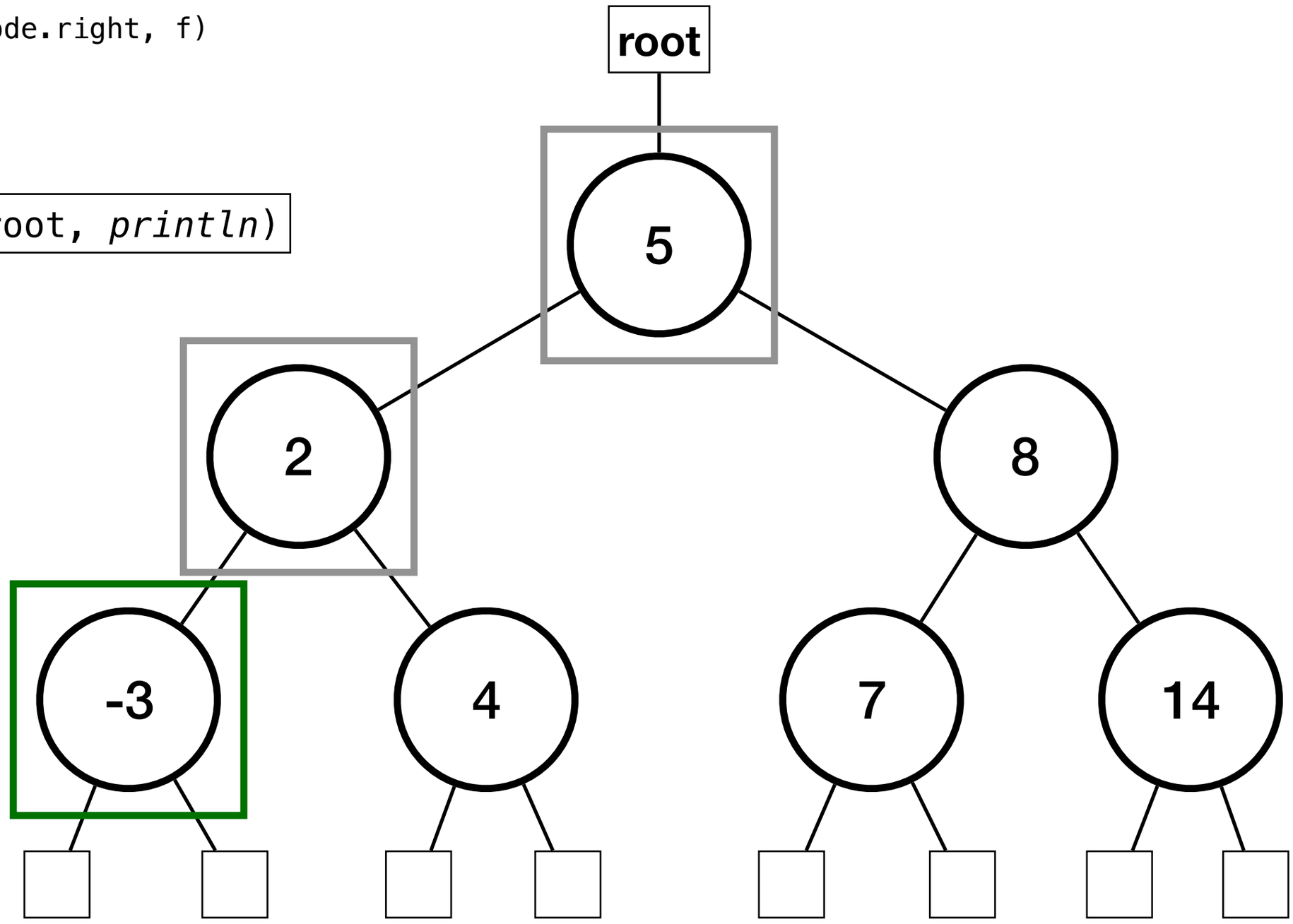

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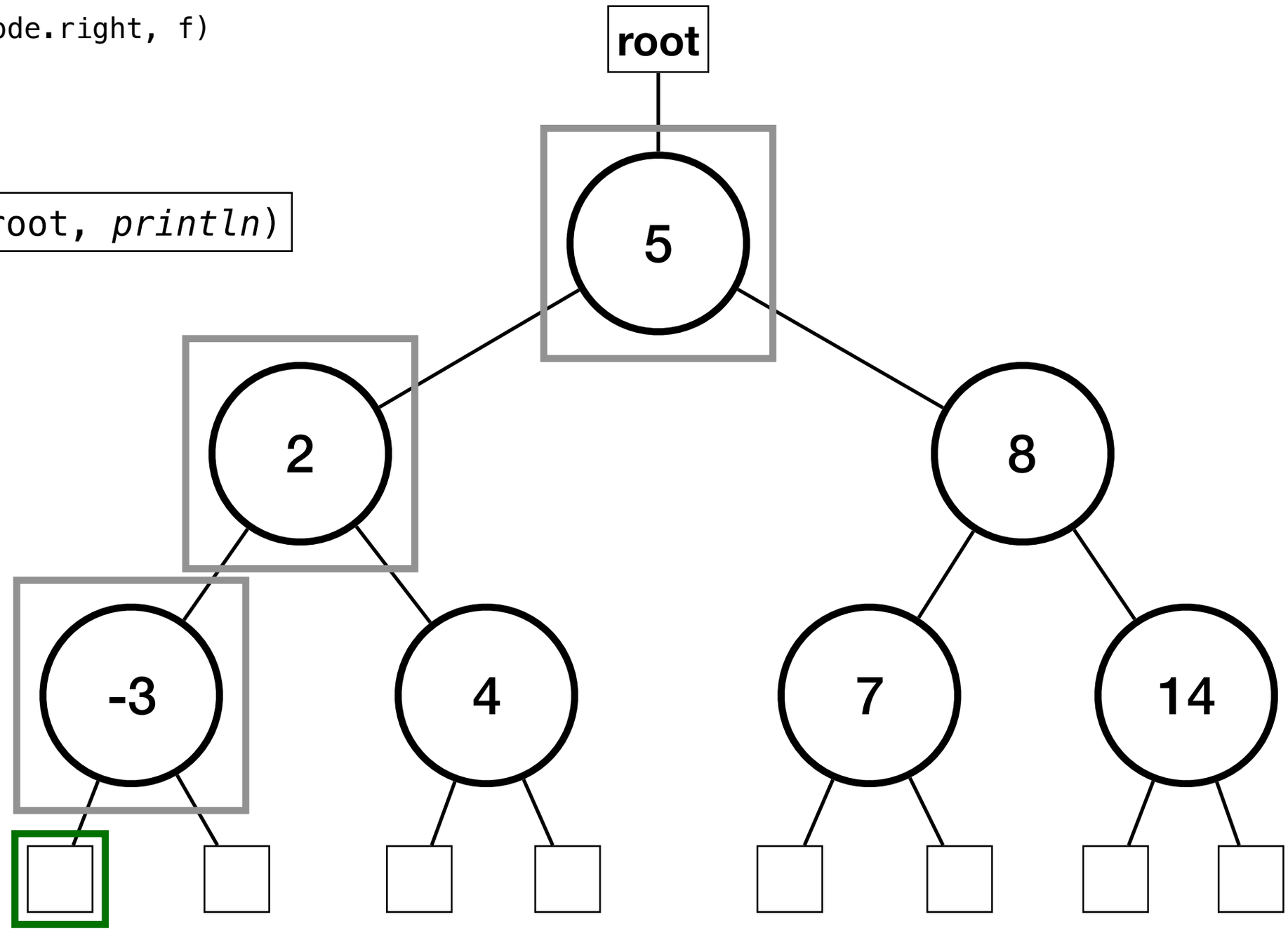

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  }
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```

`inOrderTraversal(root, println)`

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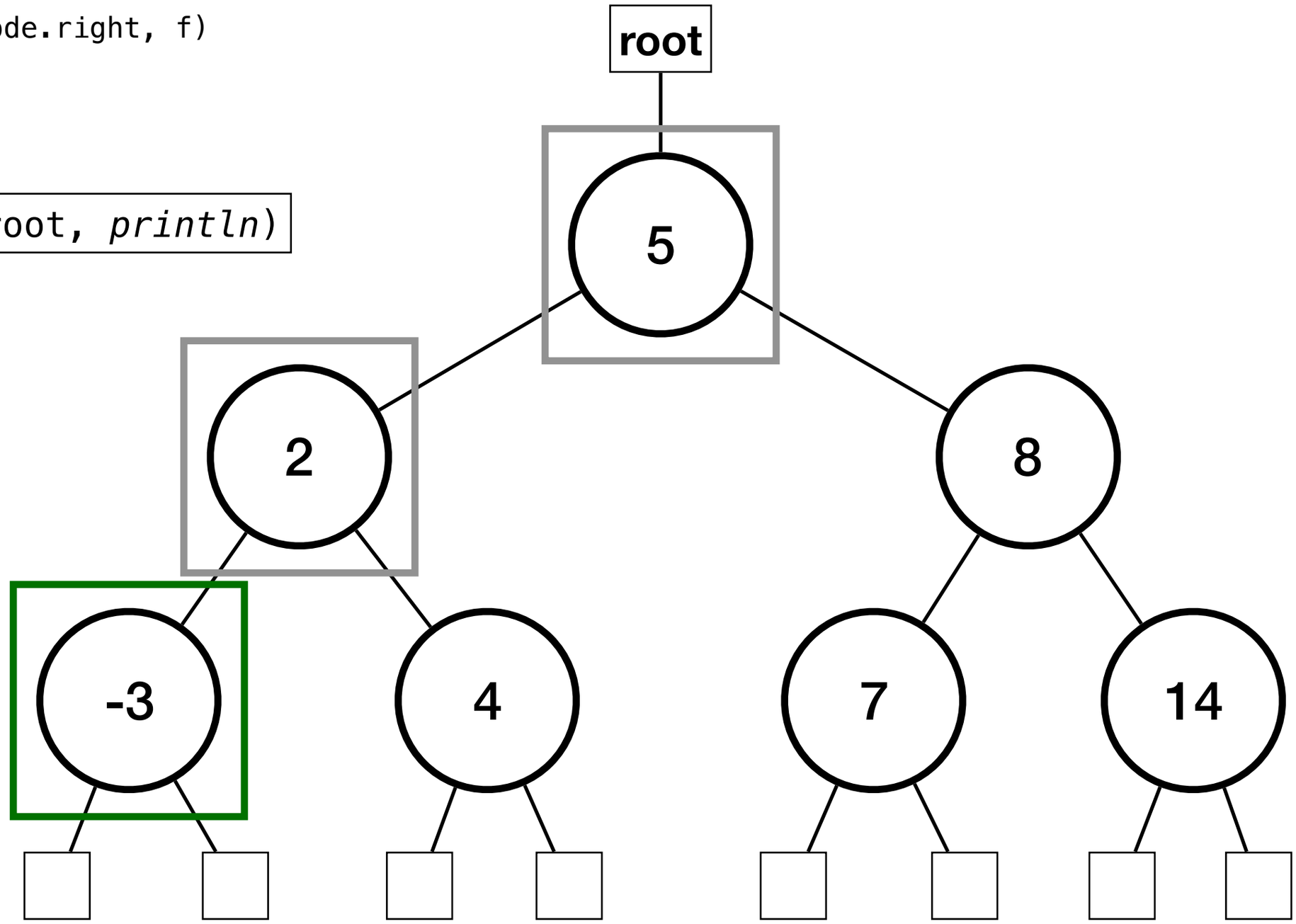

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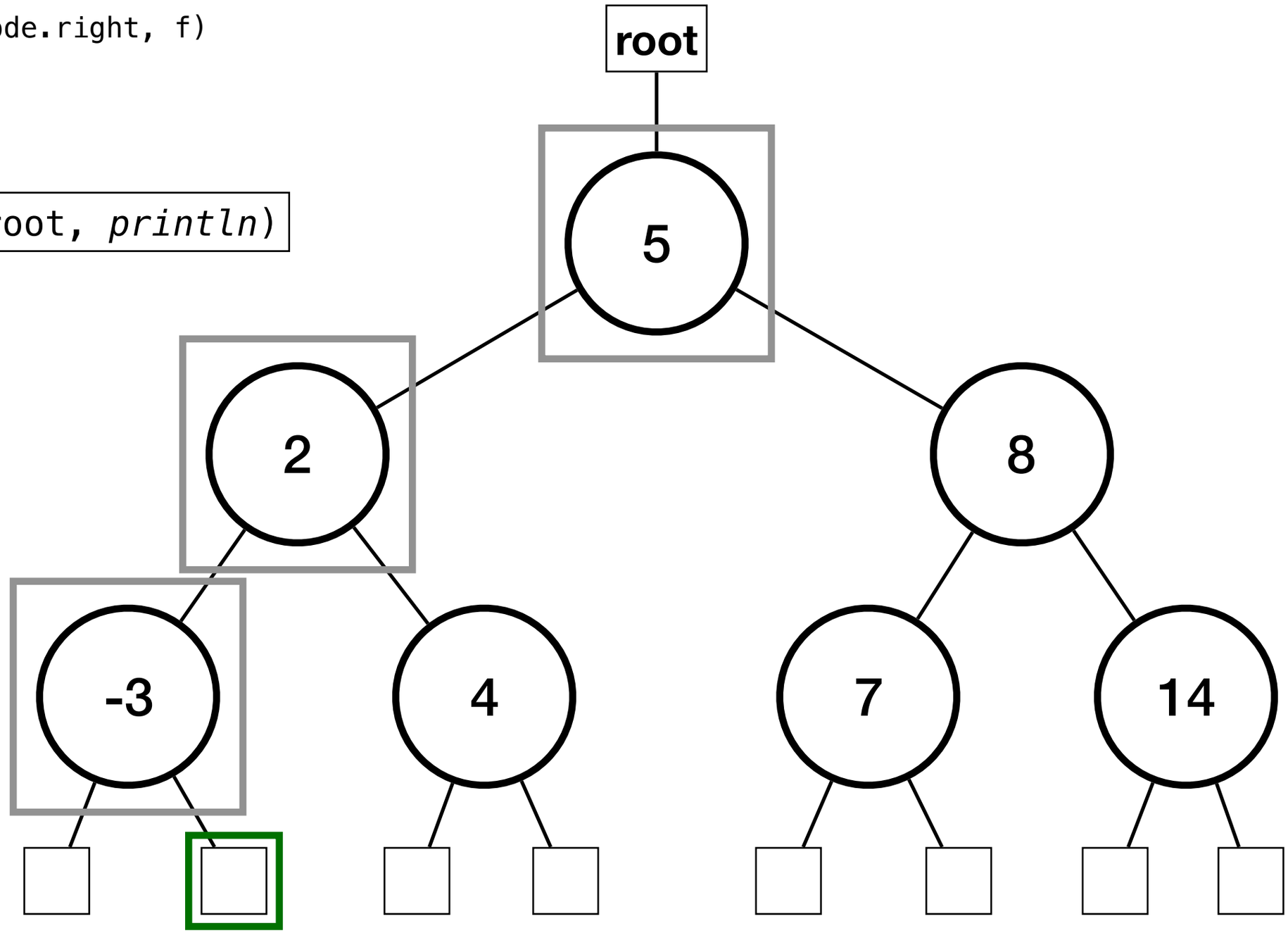

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}
```

`inOrderTraversal(root, println)`

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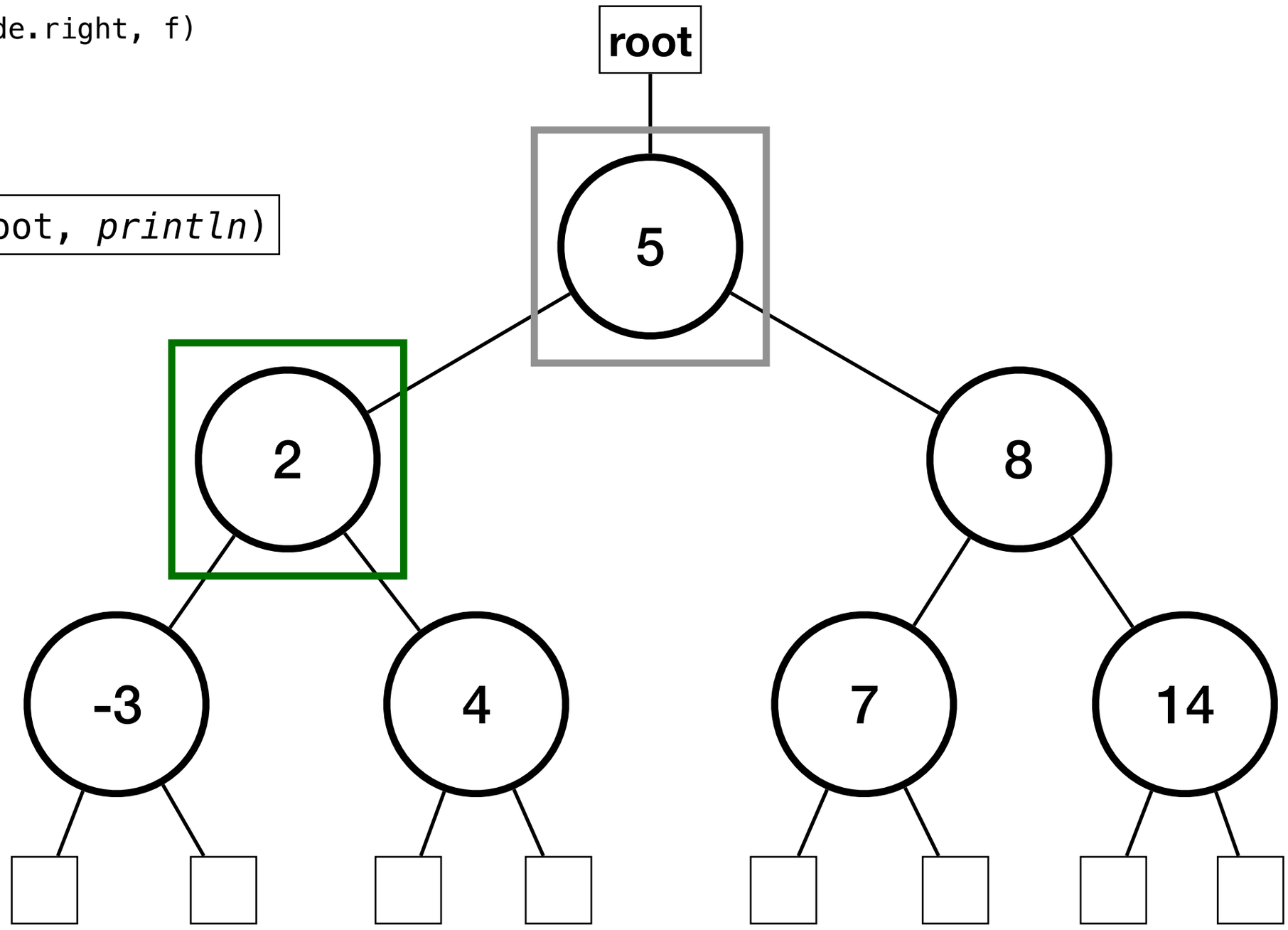

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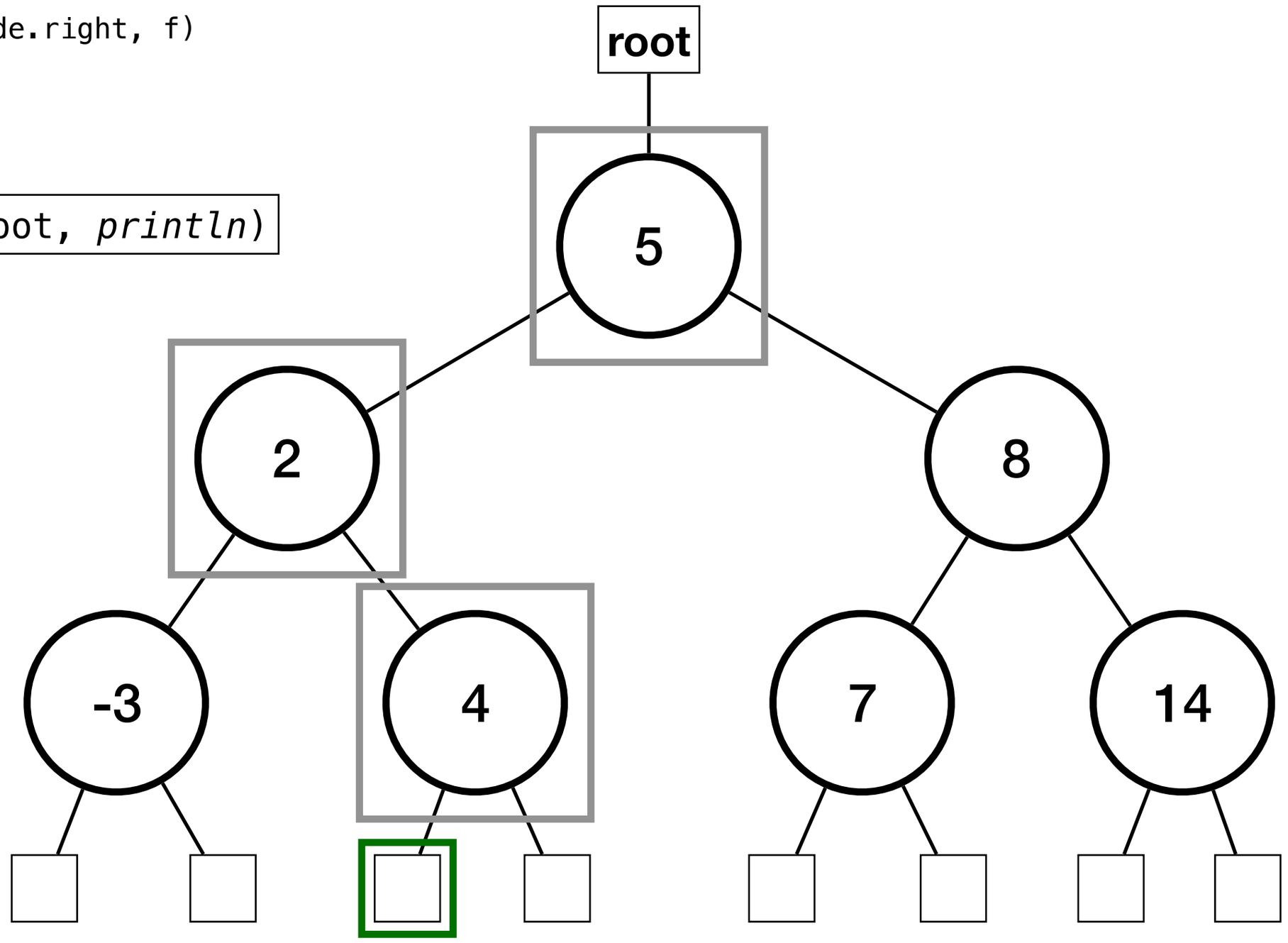

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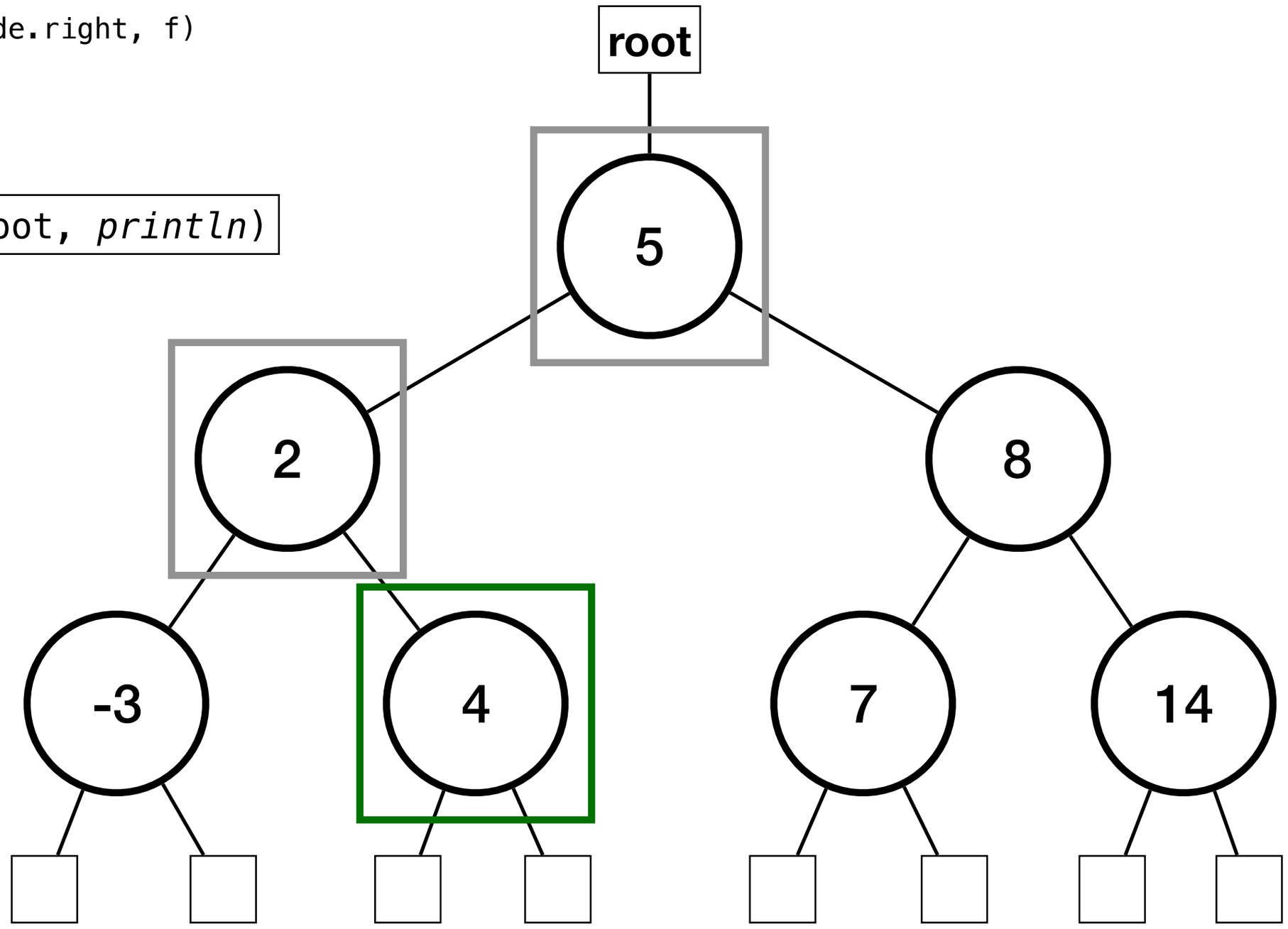

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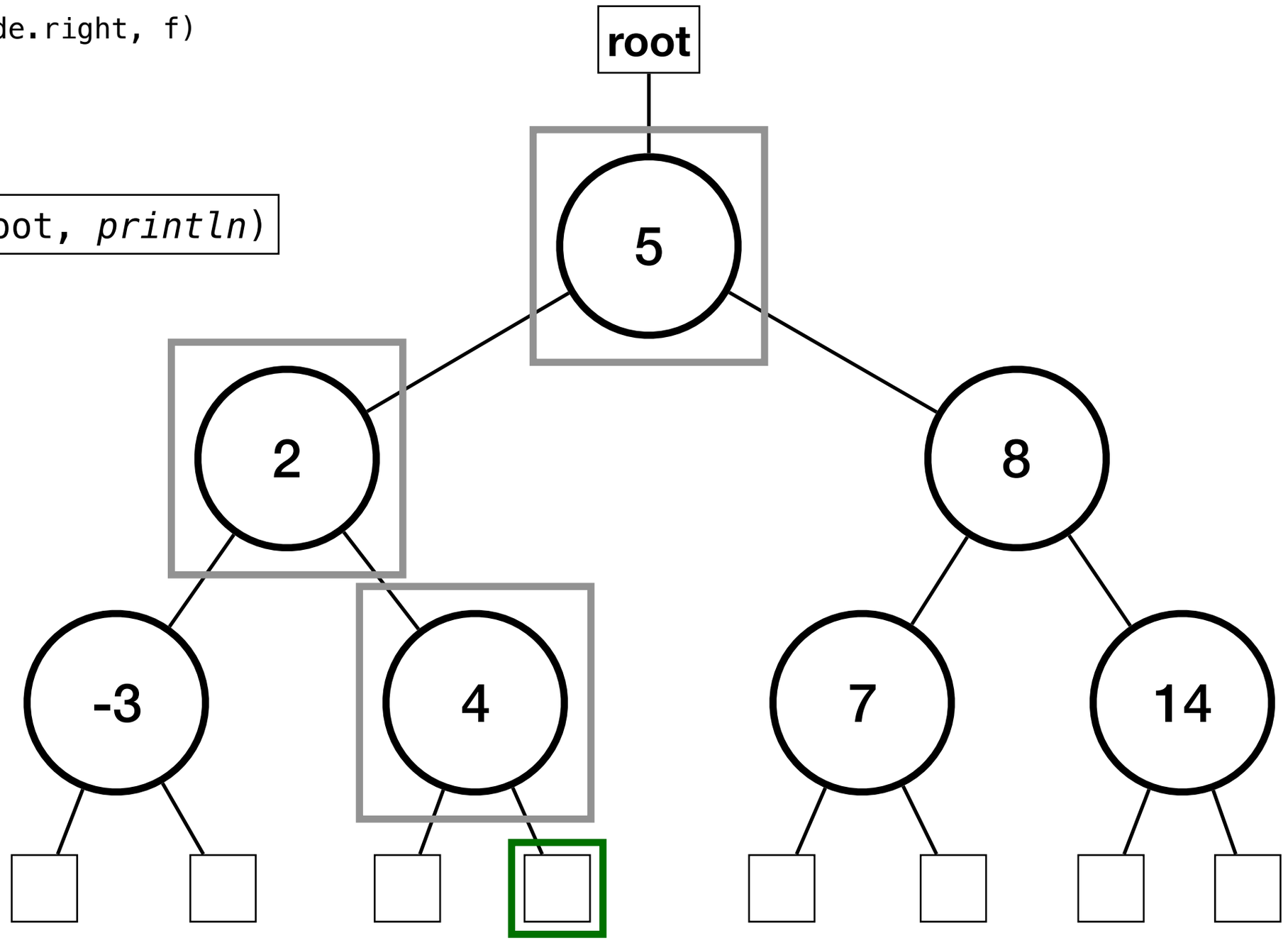

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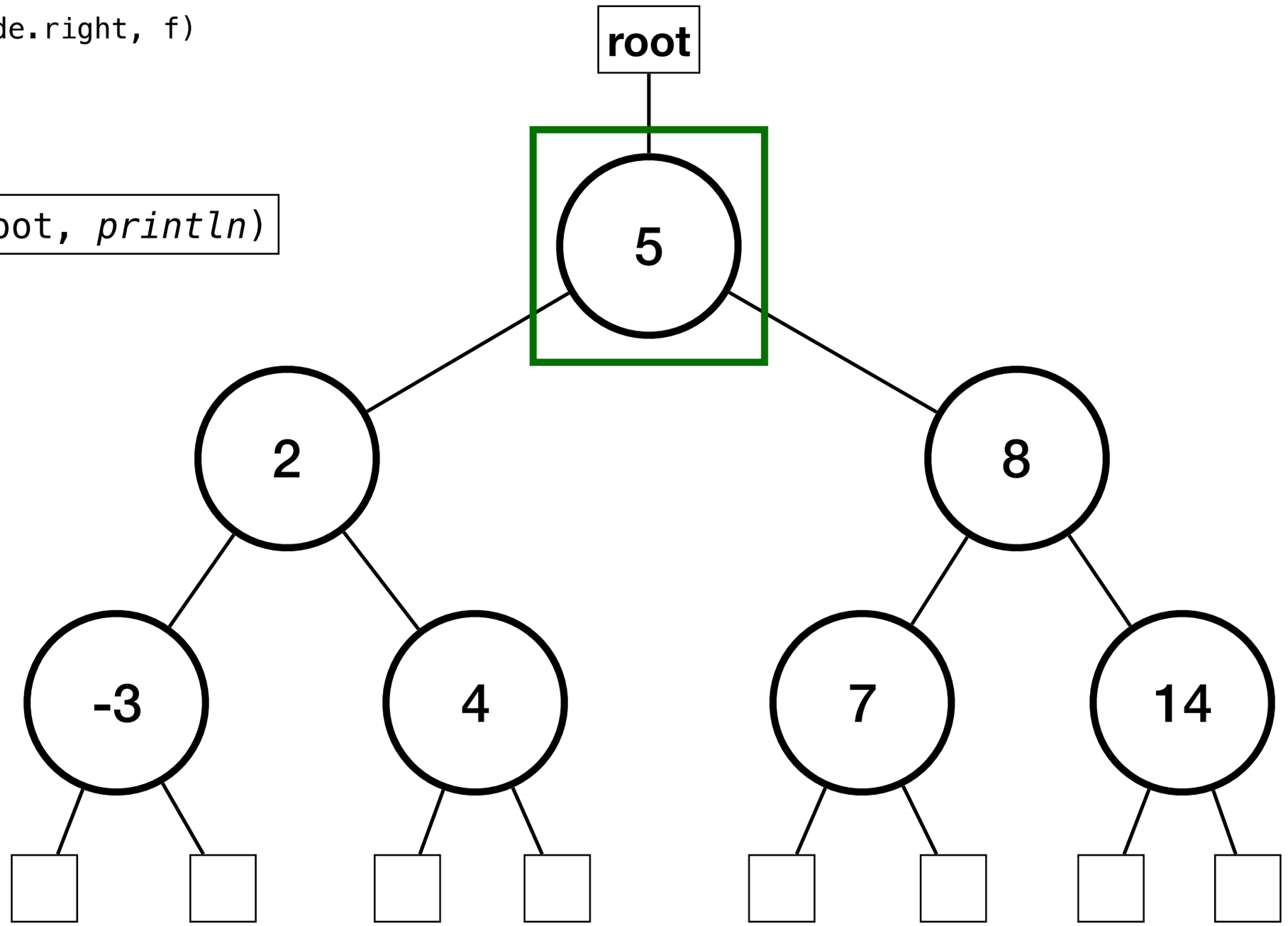

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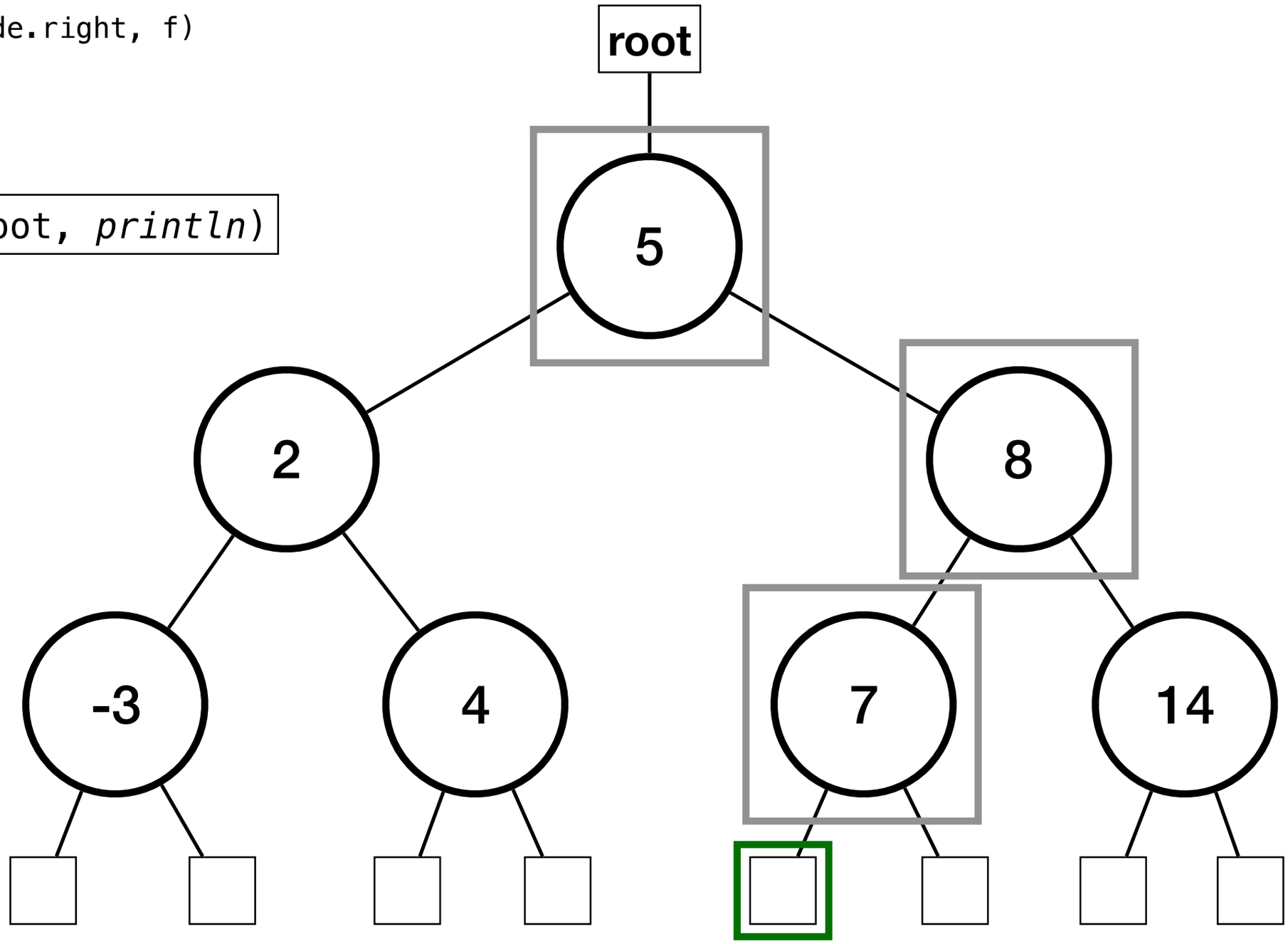

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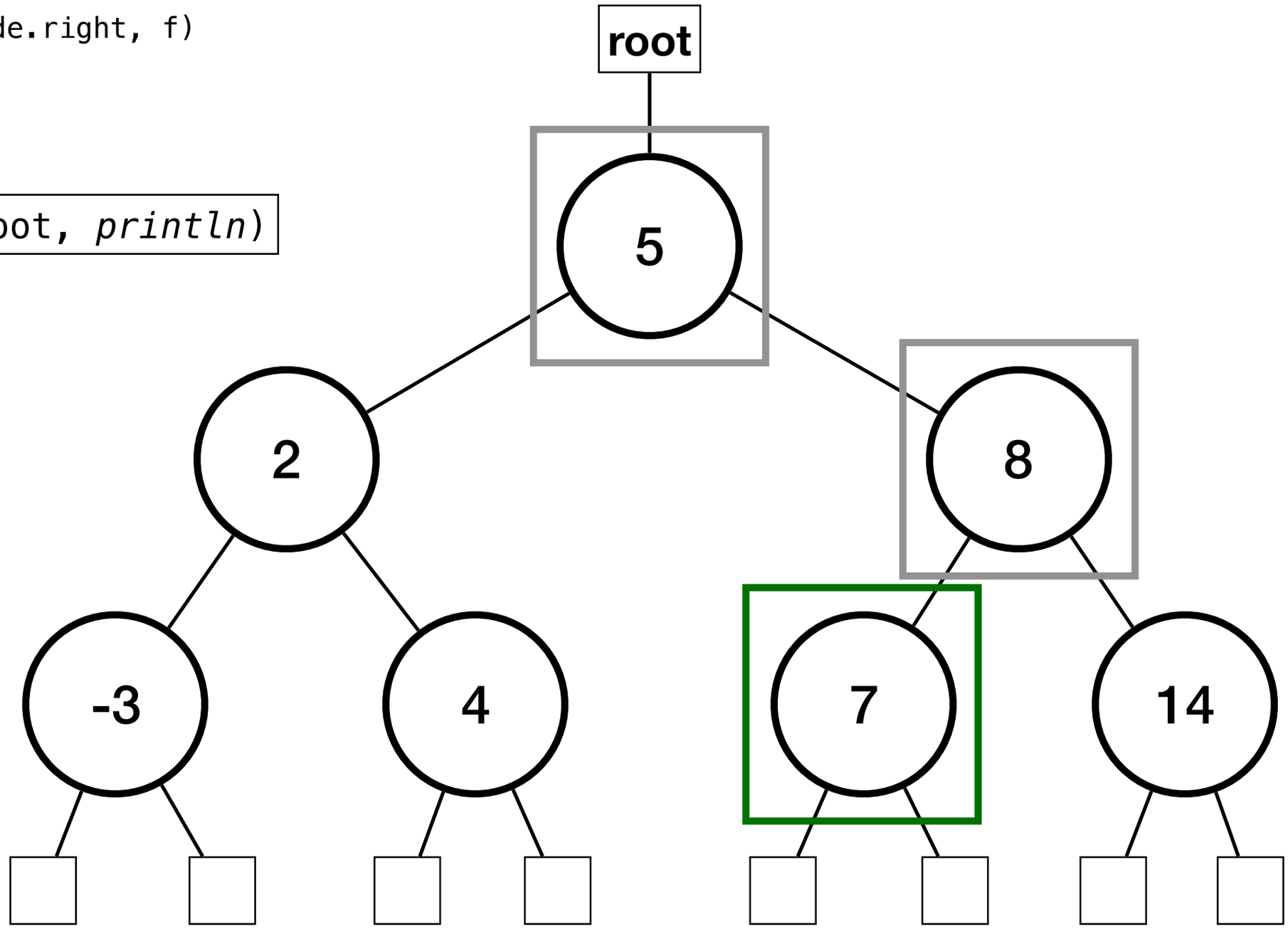

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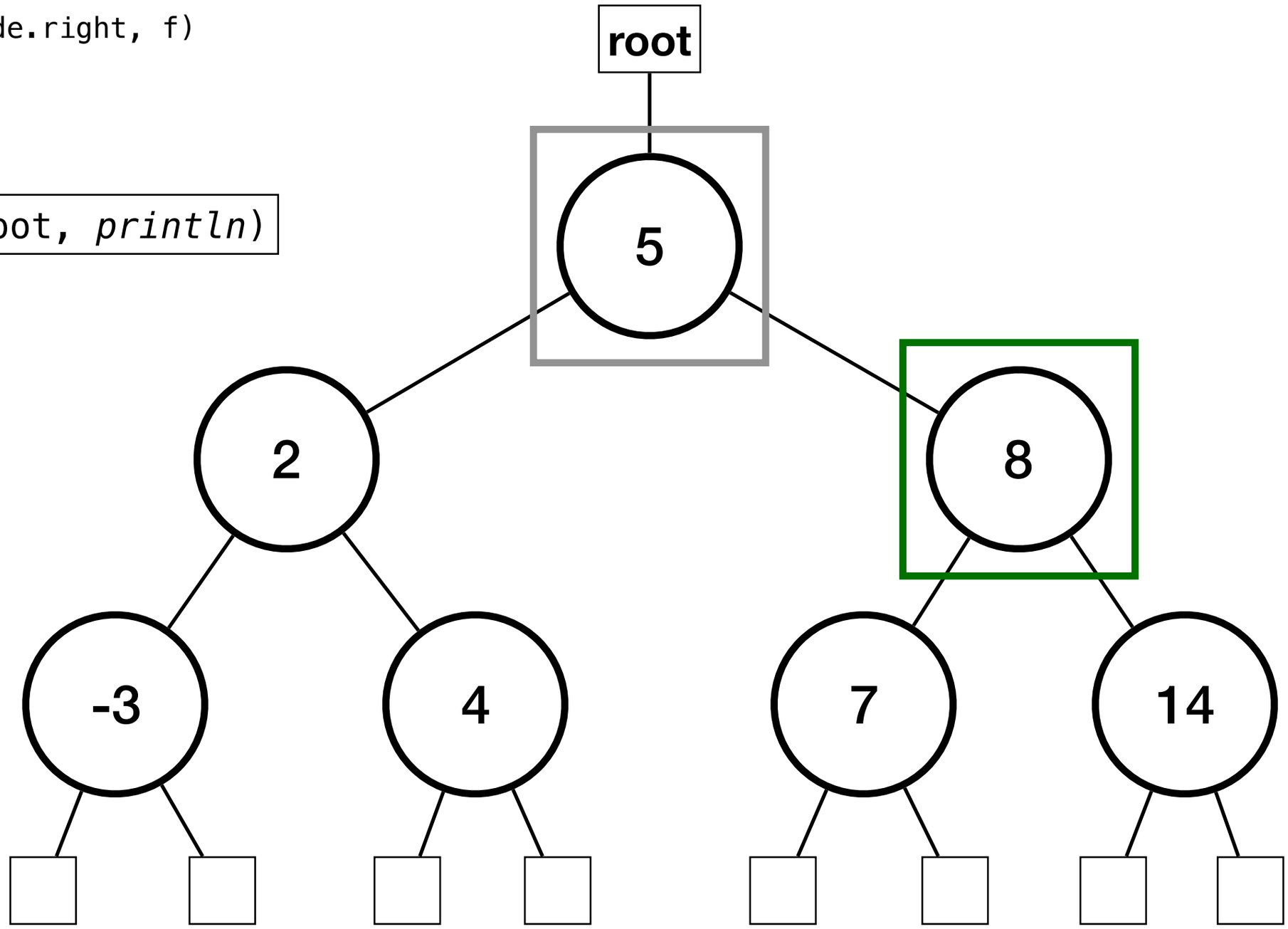

Traversals

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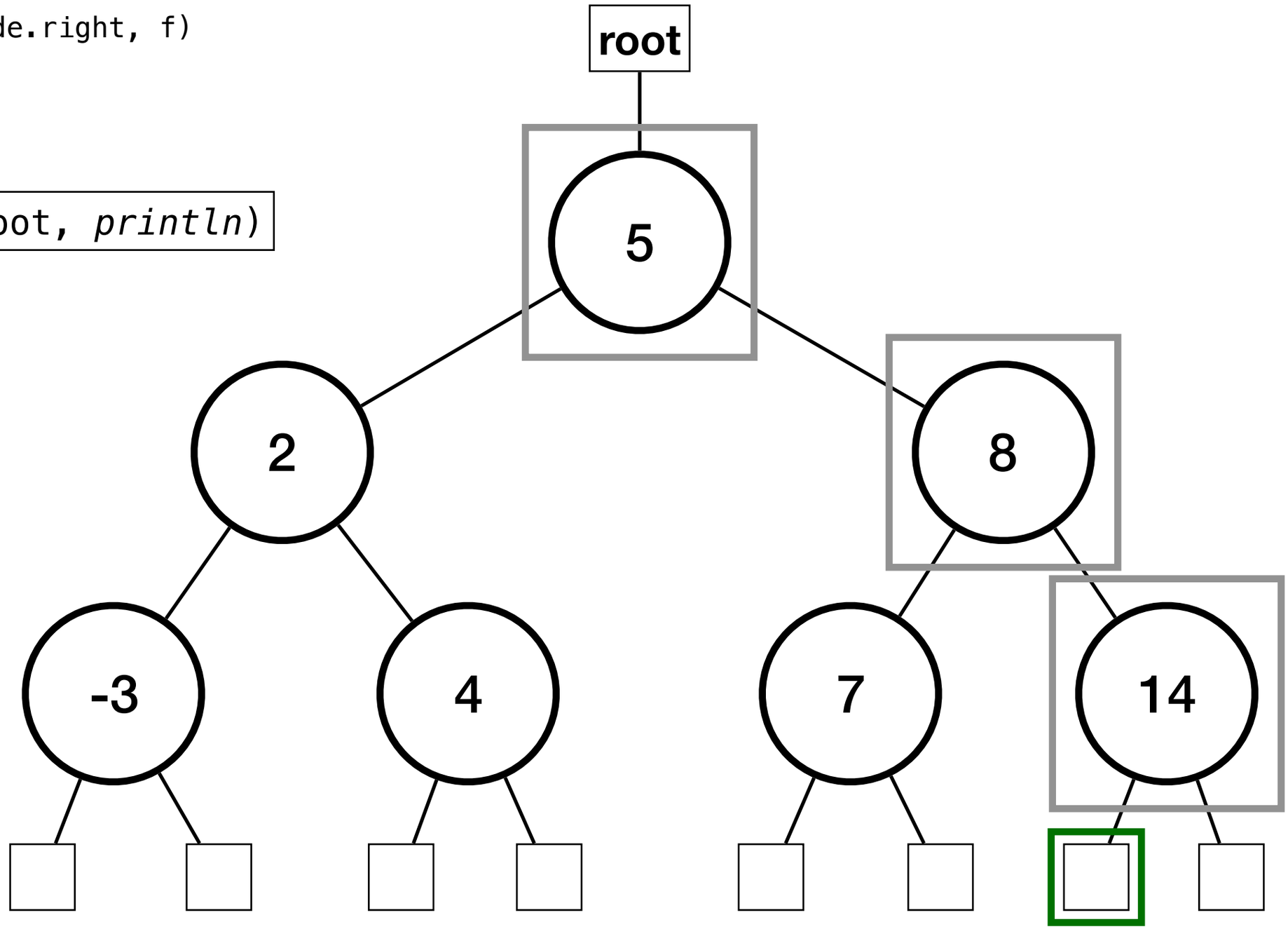

Traversals

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}
```

```
inOrderTraversal(root, println)
```

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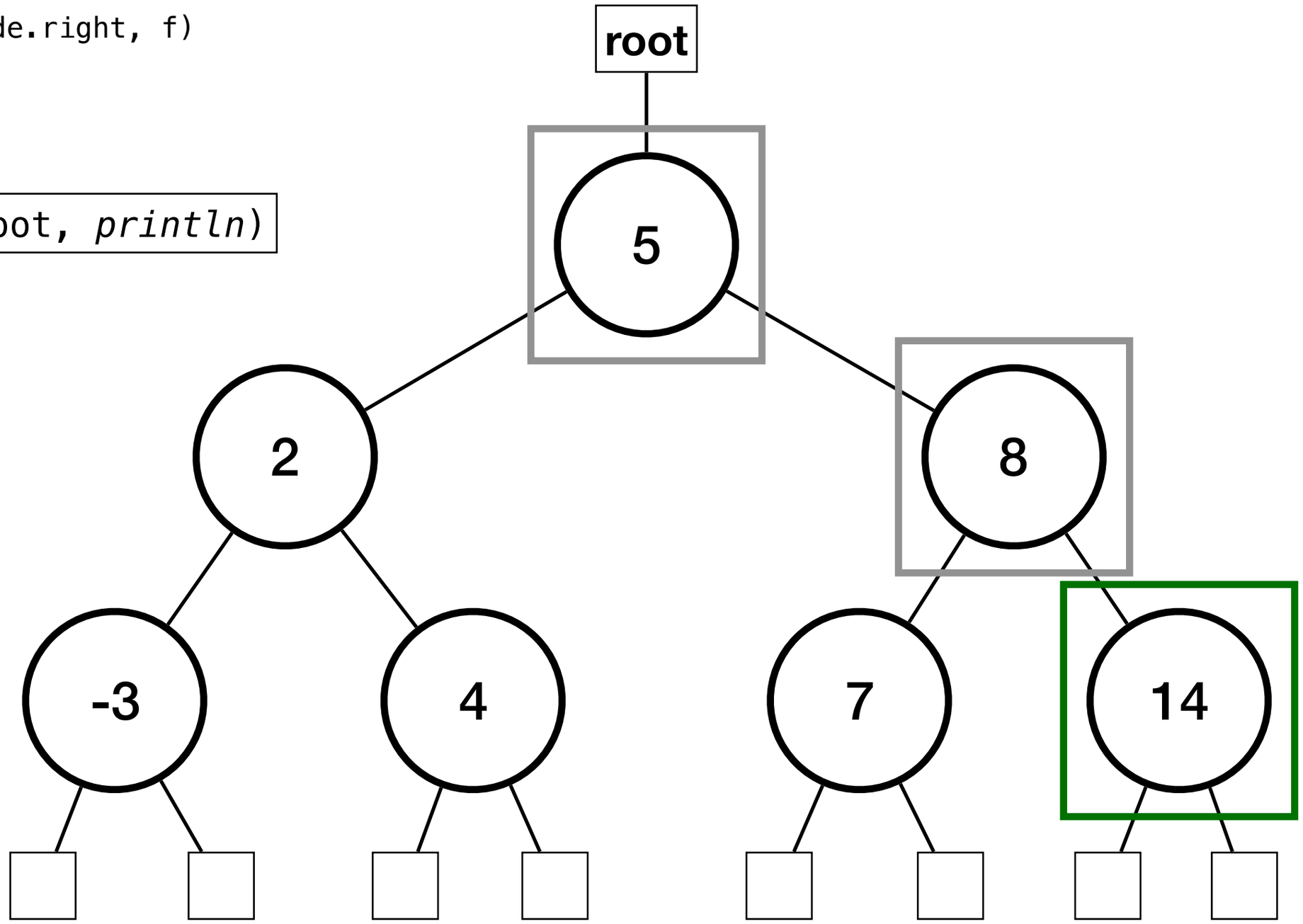

Traversals

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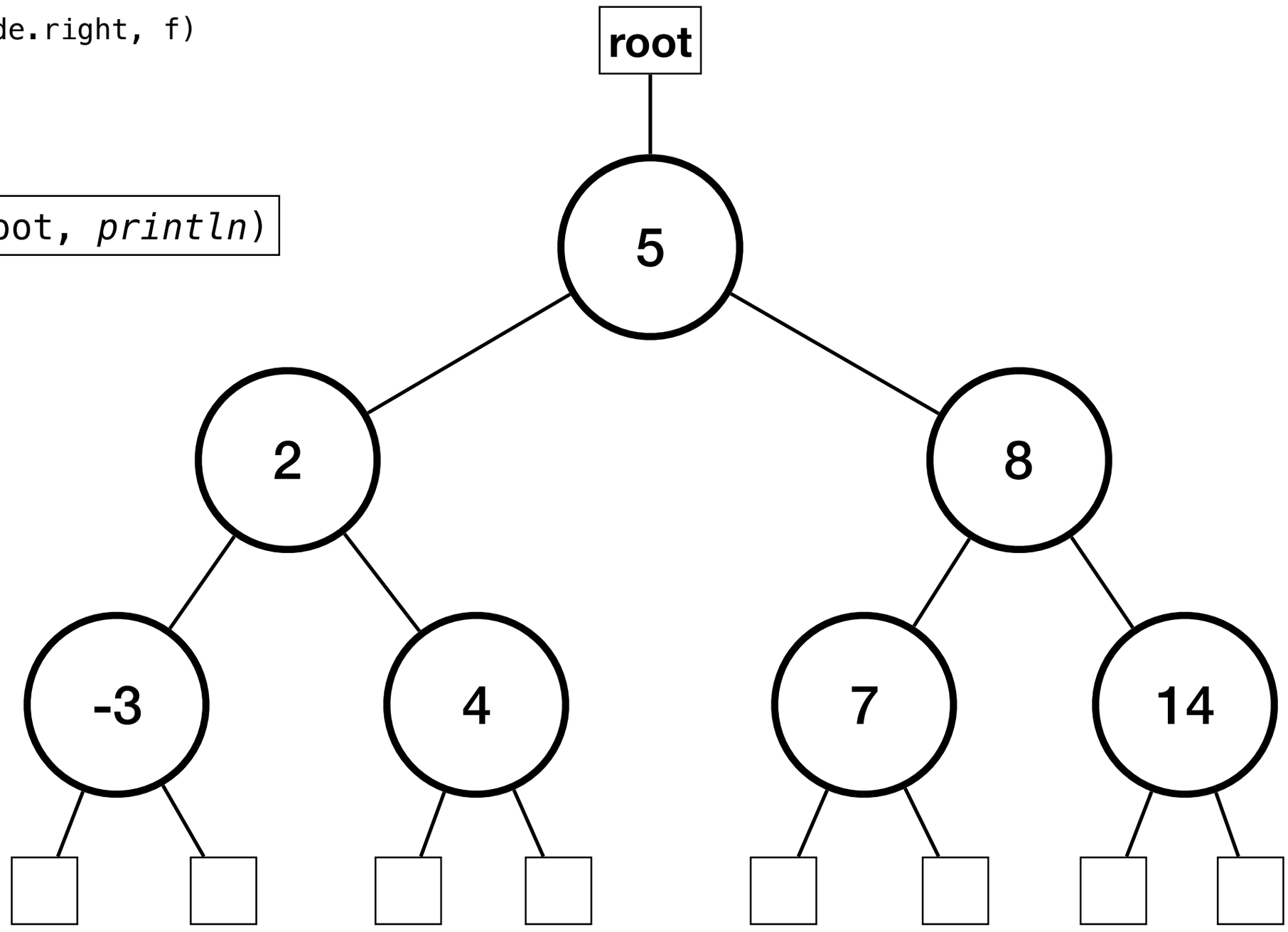


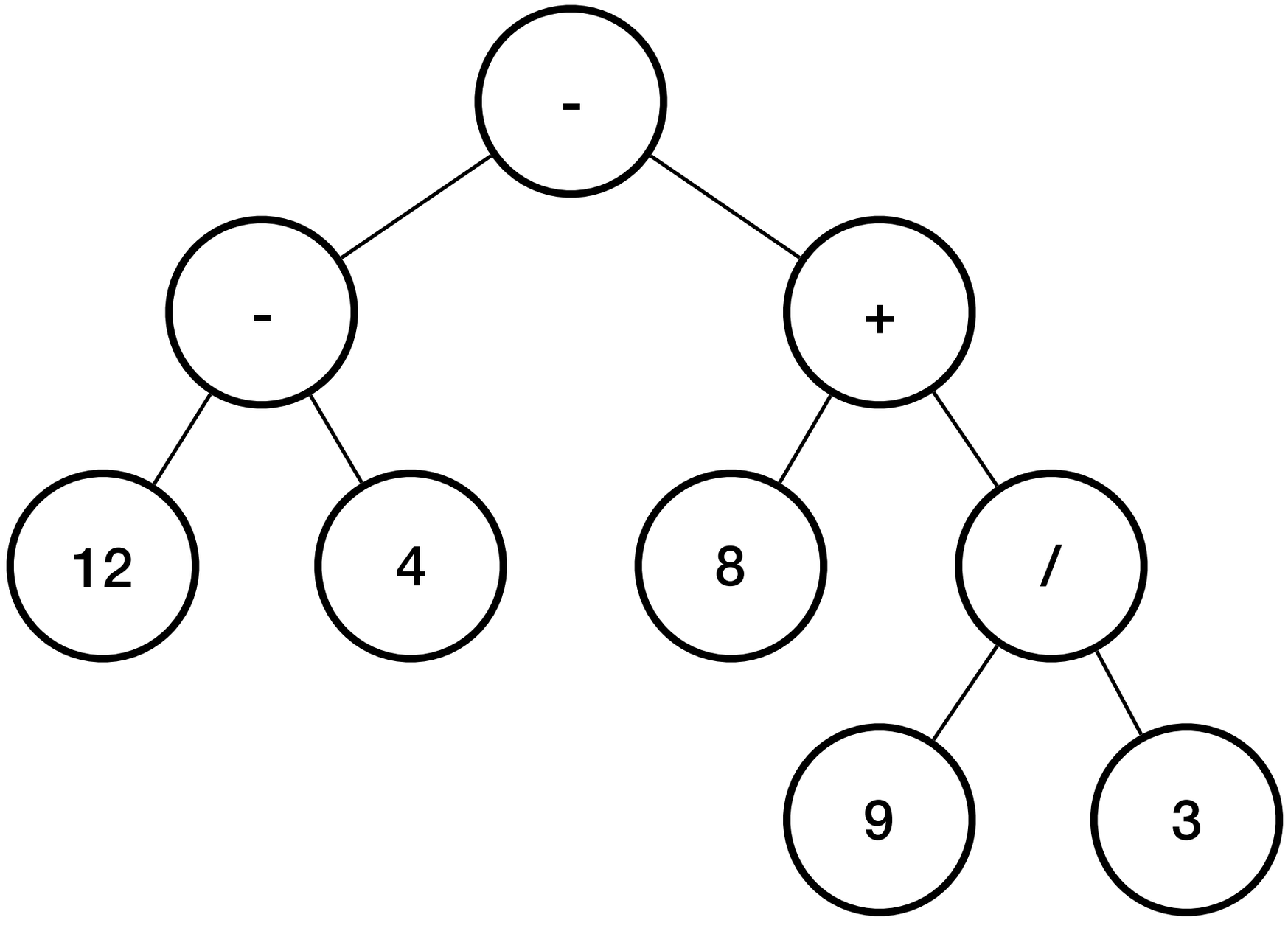
Expression Trees

Expression Trees

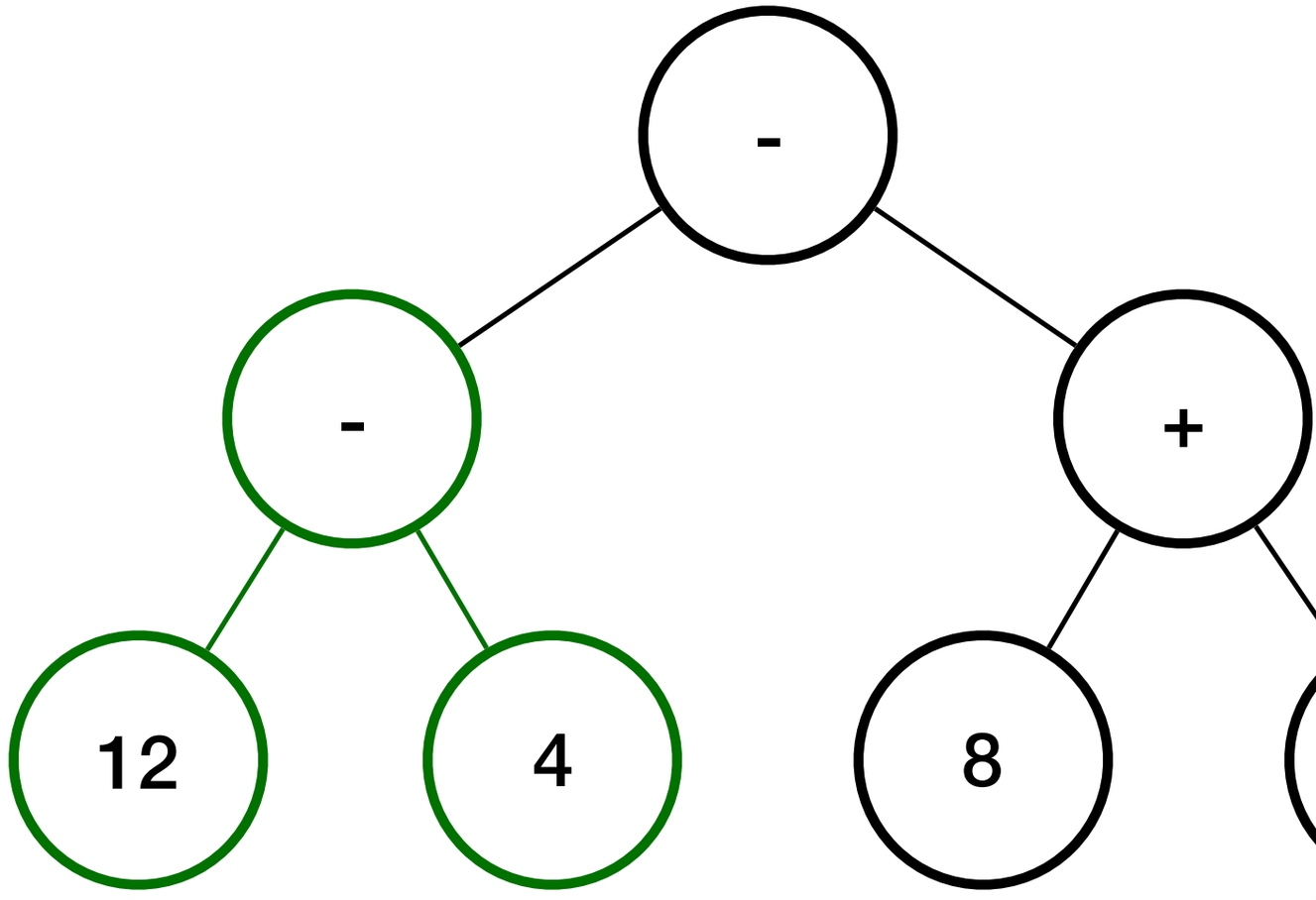
- Represent an expression as a binary tree
- Nodes can be
  - Operands
  - Operators
- An operand is a literal value
- An operator is evaluated by using its left and right children as operands
  - Operands can be operators

Expression Tree

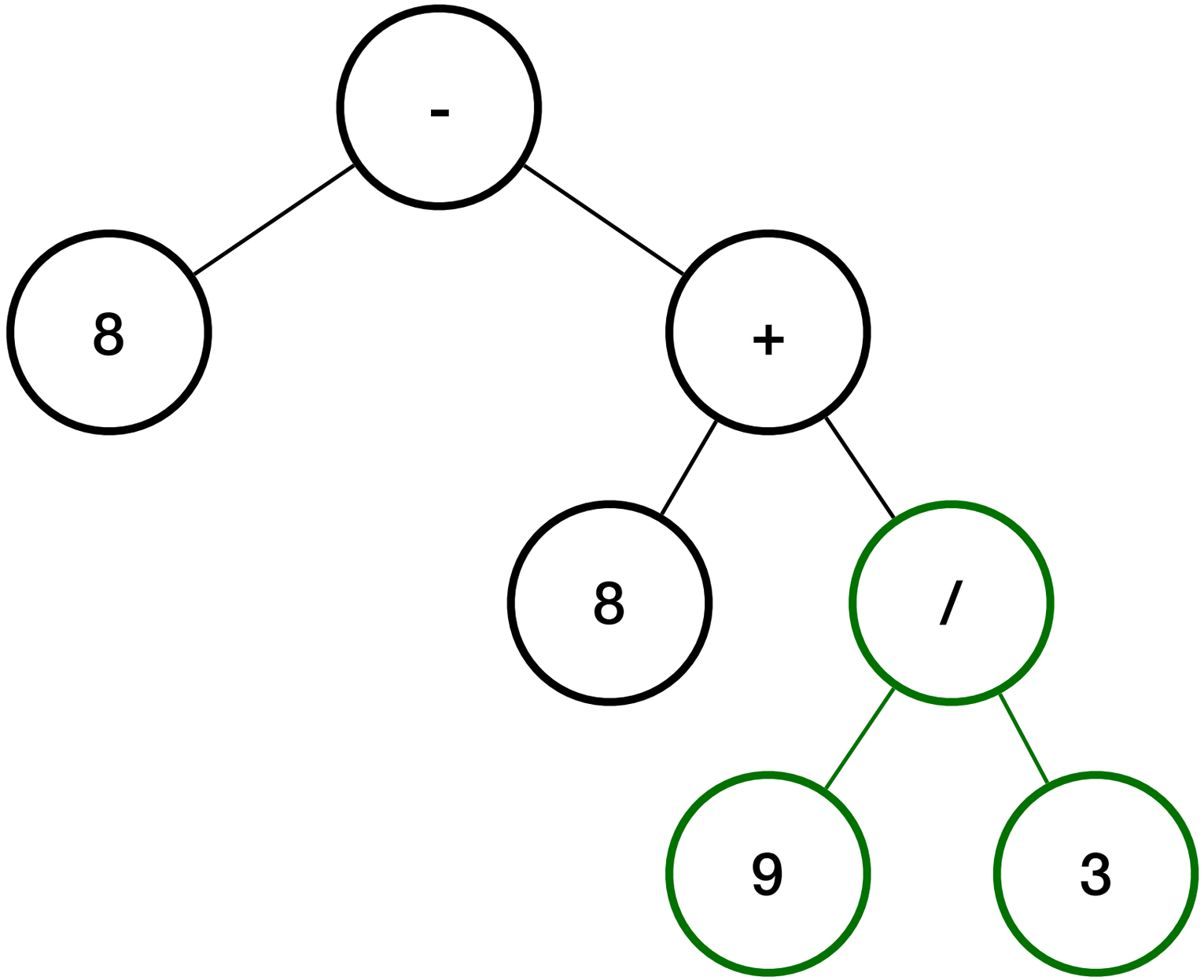
- (12-4) - (8+9/3) as an expression tree



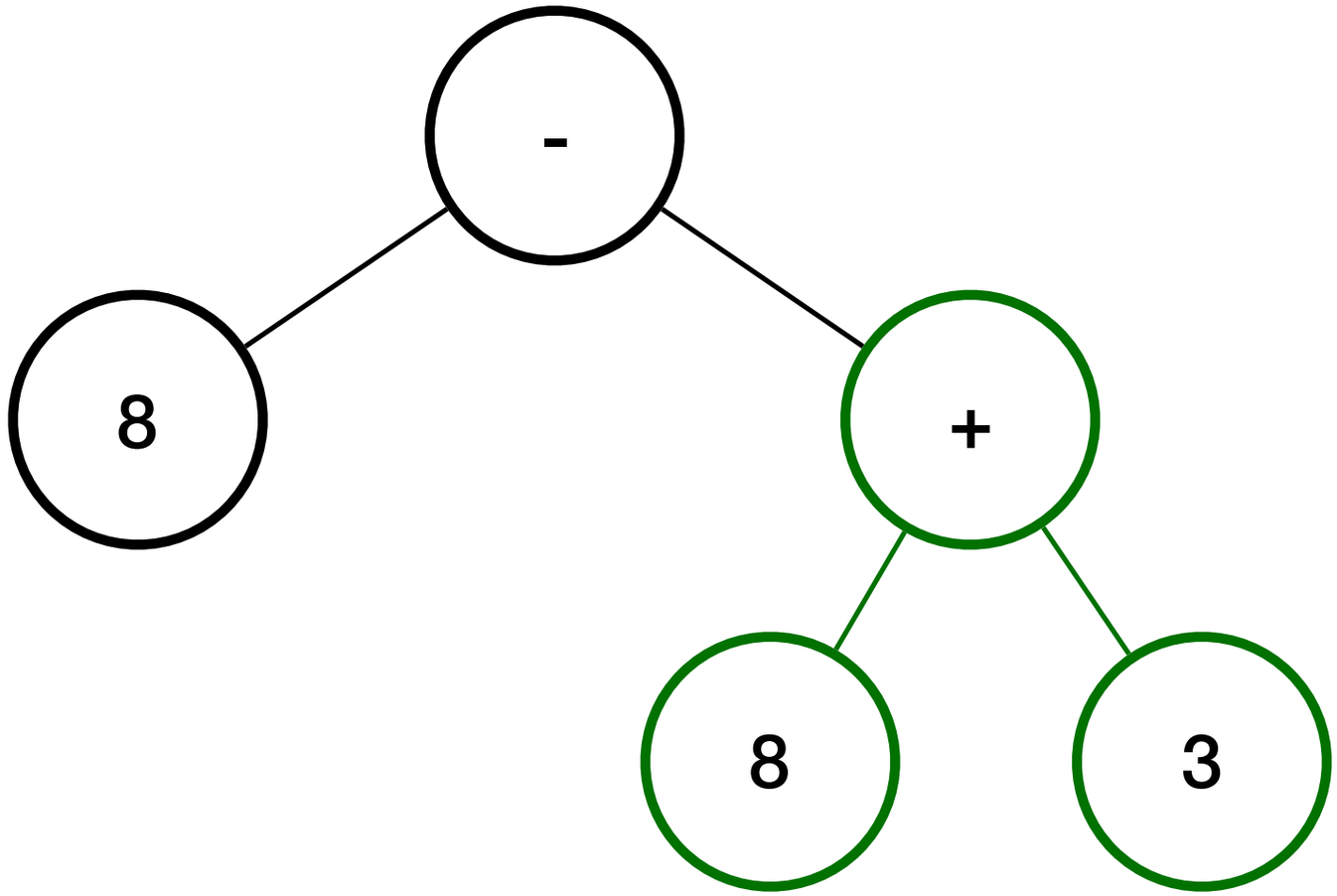
Evaluating Expression Tree



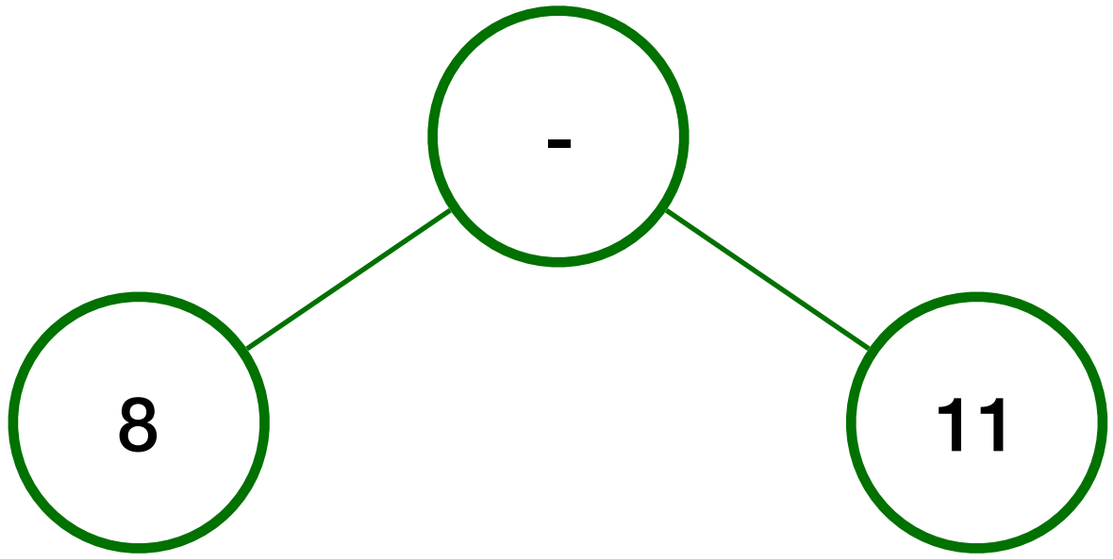
Evaluating Expression Tree



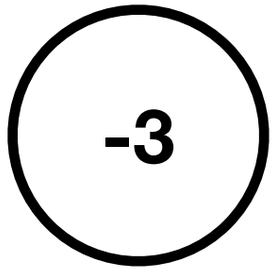
Evaluating Expression Tree



Evaluating Expression Tree



Evaluating Expression Tree



Expression Tree Traversals

- Modified in-order traversal that adds parentheses around each operator
- Generates a fully parenthesized infix expression
- ((12-4)-(8+(9/3)))

```
def fullyParenthesizedInOrderTraversal[A](node: BinaryTreeNode[A], f: A => Unit): Unit = {
  if (node != null) {
    val operator = List("^", "*", "/", "+", "-").contains(node.value)
    if (operator) {
      print("(")
    }
    fullyParenthesizedInOrderTraversal(node.left, f)
    f(node.value)
    fullyParenthesizedInOrderTraversal(node.right, f)
    if (operator) {
      print(")")
    }
  }
}
```

Expression Tree Traversals

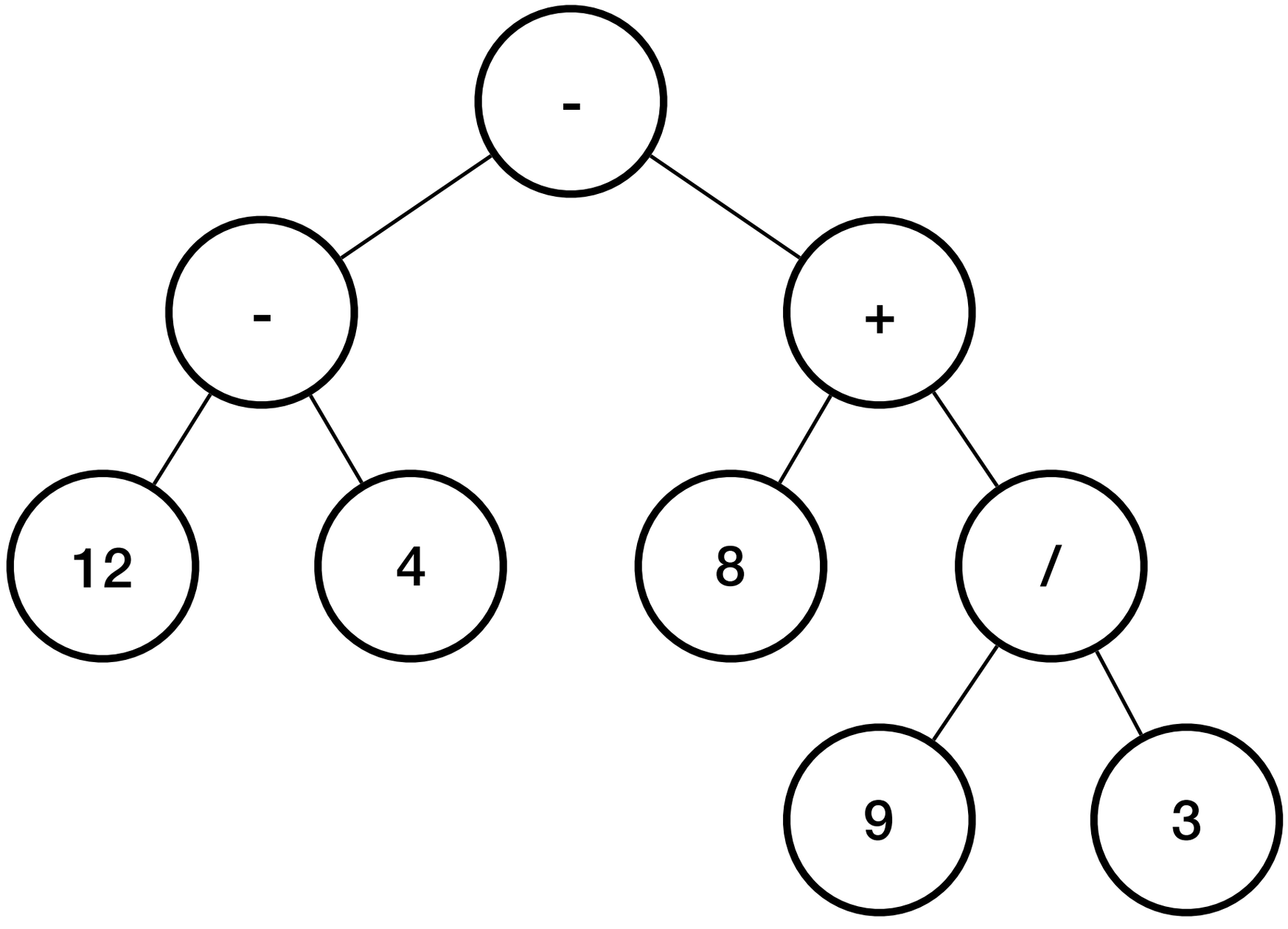
- Unmodified post-order traversal generates a postfix express (Reverse Polish Notation)

- 12 4 - 8 9 3 / + -

```
postOrderTraversal(root, (token: String) => print(token + " "))
```

Expression Tree Traversals

- 12 4 - 8 9 3 / + -



Lecture Question

Task: Evaluate an expression tree

- In the week12.ExpressionTree object, write a method named evaluateTree that takes the root of an expression tree (BinaryTreeNode[String]) as a parameter and returns the evaluation of the tree as a Double
- The operators can be *, /, +, and -

* 20 points
* Due tomorrow night