Recursion

Lecture Question

Restriction: No state is allowed in this question. Specifically, the keyword "var" is banned. (ie. You are expected to use a recursive solution)

Method: In a package named "functions" create an object named "Algebra" with a method named "factor" that takes an Int as a parameters and returns the prime factorization of that parameter as a List of Ints.

The following apply to this method:

- If the input is negative, 0, or 1, return an empty list
- Do not include 1 in the output for any inputs
- The order of the factors in the output List is undefined

Example: functions.Algebra.factor(12) can return List(2,2,3) -or- List(2,3,2) -or- List(3,2,2)

Hint: You can use a return statement inside a loop once a factor is found.

Unit Testing: Testing will not be checked by AutoLab since you've already written the tests for this problem. Use your tests from the last time we had this question to help your debugging process.

Recursion Example

```scala
def computeGeometricSum(n: Int): Int ={
  if(n <= 0){
    0
  }else{
    n + computeGeometricSum(n - 1)
  }
}


def main(args: Array[String]): Unit = {
  val result: Int = computeGeometricSum(3)
  println(result)
}
```

- Computes the geometric sum of the input
  - ex: if n == 3, geometric sum is 3+2+1 == 6

Recursion Example

```scala
def computeGeometricSum(n: Int): Int ={
  if(n <= 0){
    0
  }else{
    n + computeGeometricSum(n - 1)
  }
}


def main(args: Array[String]): Unit = {
  val result: Int = computeGeometricSum(3)
  println(result)
}
```

- Base Case:
  - An input with a trivial output
  - Geometric sum of 0 is defined as 0
  - We could also add 1 -> 1 as a base case

Recursion Example

```scala
def computeGeometricSum(n: Int): Int ={
  if(n <= 0){
    0
  }else{
    n + computeGeometricSum(n - 1)
  }
}


def main(args: Array[String]): Unit = {
  val result: Int = computeGeometricSum(3)
  println(result)
}
```

- Recursive Step:
  - Any input that is not a base case will put another recursive call on the stack
  - Write the recursive step with the assumption that the recursive call will return the correct value

Recursion Example

```scala
def computeGeometricSum(n: Int): Int ={
  if(n <= 0){
    0
  }else{
    n + computeGeometricSum(n - 1)
  }
}


def main(args: Array[String]): Unit = {
  val result: Int = computeGeometricSum(3)
  println(result)
}
```

- Recursive calls must get closer to the base case
  - All calls must eventually reach a base case or we'll go infinite
  - n-1 is closer to n<=0 than n
  - Regardless of the original value of n, it will eventually be decremented until the base case condition is true

Recursive Example

```
1:   def computeGeometricSum(n: Int): Int ={
2:     if(n <= 0){
3:       0
4:     }else{
5:       n + computeGeometricSum(n - 1)
6:     }
7:   }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

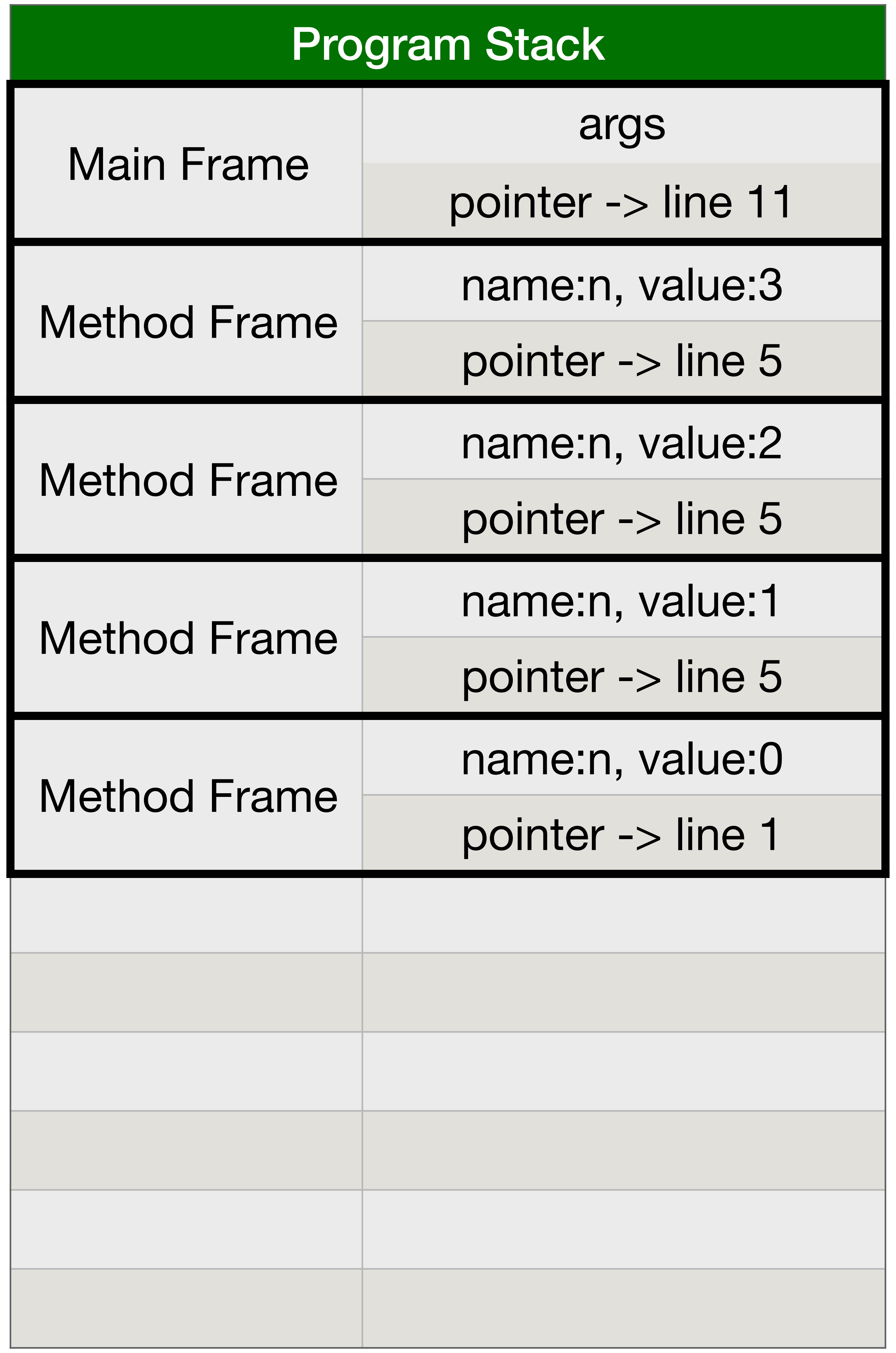
- Each recursive call creates a new stack frame
- Each frame remembers where it will resume running when it's on the top of the stack

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
Method Frame	name:n, value:1
	pointer -> line 5

Recursive Example

```
1:   def computeGeometricSum(n: Int): Int ={
2:     if(n <= 0){
3:       0
4:     }else{
5:       n + computeGeometricSum(n - 1)
6:     }
7:   }
8:
9:
10: def main(args: Array[String]): Unit = {
11:    val result: Int = computeGeometricSum(3)
12:    println(result)
13: }
```

- New frames start at the first line of the method
- Top frame on the stack executes the method one line at a time



Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
Method Frame	name:n, value:1
	pointer -> line 5
Method Frame	name:n, value:0
	pointer -> line 1

Recursive Example

```
1:   def computeGeometricSum(n: Int): Int ={
2:     if(n <= 0){
3:       0
4:     }else{
5:       n + computeGeometricSum(n - 1)
6:     }
7:   }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Recursive calls are added to the stack until a base case is reached

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
Method Frame	name:n, value:1
	pointer -> line 5
Method Frame	name:n, value:0
	pointer -> line 3

Recursive Example

```
1:   def computeGeometricSum(n: Int): Int ={
2:     if(n <= 0){
3:       0
4:     }else{
5:       n + computeGeometricSum(n - 1)
6:     }
7:   }
8:
9:
10:  def main(args: Array[String]): Unit = {
11:    val result: Int = computeGeometricSum(3)
12:    println(result)
13:  }
```

- When a method call returns, its frame is destroyed
- The calling frame resumes and uses the returned value

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
Method Frame	name:n, value:1
	pointer -> line 5
Method Frame	returning 0

Recursive Example

```
1:   def computeGeometricSum(n: Int): Int ={
2:     if(n <= 0){
3:       0
4:     }else{
5:       n + computeGeometricSum(n - 1)
6:     }
7:   }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Method continues after the recursive call
- Sums n + return value and returns this value

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
Method Frame	name:n, value:1
	pointer -> line 5
	gets return value of 0

Recursive Example

```
1:   def computeGeometricSum(n: Int): Int ={
2:     if(n <= 0){
3:       0
4:     }else{
5:       n + computeGeometricSum(n - 1)
6:     }
7:   }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
Method Frame	returning 1

- This frame reaches the end of the method
- Repeat the process

Recursive Example

```
1:   def computeGeometricSum(n: Int): Int ={
2:     if(n <= 0){
3:       0
4:     }else{
5:       n + computeGeometricSum(n - 1)
6:     }
7:   }
8:
9:
10: def main(args: Array[String]): Unit = {
11:    val result: Int = computeGeometricSum(3)
12:    println(result)
13: }
```

- Return value
- Pop off the stack
- Resume execution of the top frame

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	name:n, value:3
	pointer -> line 5
Method Frame	name:n, value:2
	pointer -> line 5
	gets return value of 1

Recursive Example

```
1:   def computeGeometricSum(n: Int): Int ={
2:     if(n <= 0){
3:       0
4:     }else{
5:       n + computeGeometricSum(n - 1)
6:     }
7:   }
8:
9:
10: def main(args: Array[String]): Unit = {
11:    val result: Int = computeGeometricSum(3)
12:    println(result)
13: }
```

Program Stack	
Main Frame	args
	pointer -> line 11
Method Frame	returning 6

- Process continues until all recursive calls resolve
- Last frame returns to main

Recursive Example

```
1:   def computeGeometricSum(n: Int): Int ={
2:     if(n <= 0){
3:       0
4:     }else{
5:       n + computeGeometricSum(n - 1)
6:     }
7:   }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

Program Stack	
Main Frame	args
	pointer -> line 11
	gets return value of 6

- Main continues with the result from the recursive calls

Recursive Example

```
1:   def computeGeometricSum(n: Int): Int ={
2:     if(n <= 0){
3:       0
4:     }else{
5:       n + computeGeometricSum(n - 1)
6:     }
7:   }
8:
9:
10: def main(args: Array[String]): Unit = {
11:   val result: Int = computeGeometricSum(3)
12:   println(result)
13: }
```

- Main continues with the result from the recursive calls

Program Stack	
Main Frame	args
	name:result, value:6
	pointer -> line 12

Anagrams Revisited

Anagrams Example

```scala
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Recall anagrams
  - Rewritten to use functional programming and no vars
  - The syntax may not fully make sense until the next Functional Programming lecture

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Base Case
  - A String of length 1 is itself its only anagram
  - If the length is 1, return a new list containing only that String

Anagrams Example

```scala
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Base Case Note
  - We will eventually return a list containing all anagrams from the top level call
  - The base case is the only time we create a new List

Anagrams Example

```scala
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Recursive Step
  - For each character in the input String
    - Remove that character and make a recursive call with the remaining characters
    - Append the removed character to all the returned anagrams

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Recursive Step
  - We write this code with the assumption that our recursive calls will return all the anagrams of the new Strings
  - If our logic is sound, this assumption will be true through the power of recursion!

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Always reach a base case
  - We always make recursive calls on the input String with 1 character removed
    - newString.length == input.length -1
  - This always gets us closer to the base case

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Always reach a base case
  - When the base case is reached and returned, our logic starts working for us
  - If this code does append the removed character to each returned anagram, output is generated starting at the base case and built up as the stack frames return

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Example:
  - input == "at"
  - Makes 2 recursive calls to the base case
    - "a" and "t" are returned
  - Append "t" to "a" and "a" to "t" (The removed characters)
  - Return ["at", "ta"] to the next recursive call with an input of length 3

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Functional Programming notes (More detail Later)
    - yield: Creates a data structure containing the last expression that was evaluated on each iteration of a loop

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
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  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Functional Programming notes (More detail Monday)
  - map: Creates a new data structure by applying a function to each element
    - The _ is shorthand syntax we can use instead of naming the parameters of a function when the types can be inferred, and each input is only used once

Anagrams Example

```
def anagrams(input: String): List[String] = {
  if (input.length == 1) {
    List(input)
  } else {
    val output: List[List[String]] = (for (i <- 0 until input.length) yield {
      val newString: String = input.substring(0, i) + input.substring(i + 1, input.length)
      anagrams(newString).map(_ + input.charAt(i))
    }).toList

    output.flatten.distinct
  }
}
```

- Functional Programming notes (More detail Monday)
  - Scala data structures come with many helpful FP style methods
  - Flatten: Creates a single List from a List of Lists containing all the elements from each List
  - Distinct: Creates a new List with all duplicate values removed

Anagrams in the Debugger

Lecture Question

Restriction: No state is allowed in this question. Specifically, the keyword "var" is banned. (ie. You are expected to use a recursive solution)

Method: In a package named "functions" create an object named "Algebra" with a method named "factor" that takes an Int as a parameters and returns the prime factorization of that parameter as a List of Ints.

The following apply to this method:

- If the input is negative, 0, or 1, return an empty list
- Do not include 1 in the output for any inputs
- The order of the factors in the output List is undefined

Example: functions.Algebra.factor(12) can return List(2,2,3) -or- List(2,3,2) -or- List(3,2,2)

Hint: You can use a return statement inside a loop once a factor is found.

Unit Testing: Testing will not be checked by AutoLab since you've already written the tests for this problem. Use your tests from the last time we had this question to help your debugging process.