Quantum Error Correction

Repetition code

Quantum info is extremely fragile

① classical repetition code:

QC are highly susceptible to errors.

- Unwanted interactions with environment cause disturbance, including decoherence.

→ The idea is to simply repeat each bit multiple times

3 bit repetition code:

$0 \mapsto 000$

$1 \mapsto 111$

Decoding

$abc \to \text{majority}(a,b,c)$.

✓ This code corrects upto one bit flip on any of the three bits used for encoding

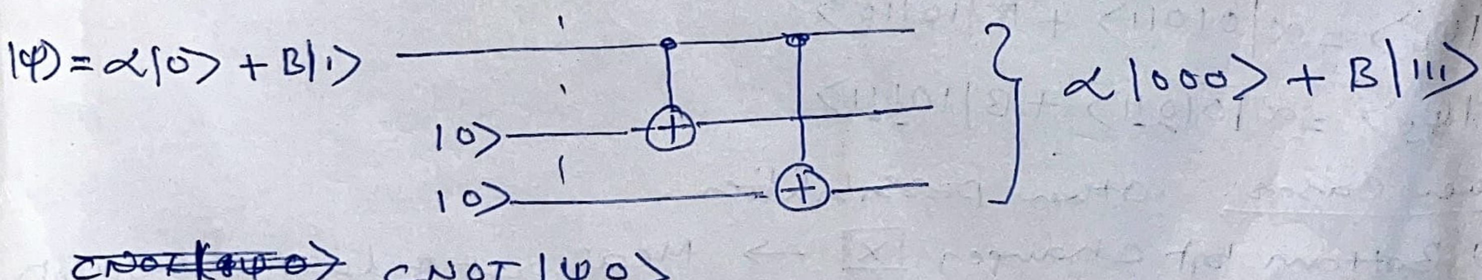
② Repetition code for qubits

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \to \alpha|000\rangle + \beta|111\rangle$

~~This is~~ Note that it is not same thing as $|\psi\rangle \to |\psi\rangle|\psi\rangle|\psi\rangle$

✓ Such an encoding can't be implemented for an unknown qubit state $|\psi\rangle$ by the no-cloning theorem

This circuit performs the encoding:—



~~CNOT|ψ0⟩~~ $\text{CNOT}|\psi 0\rangle$

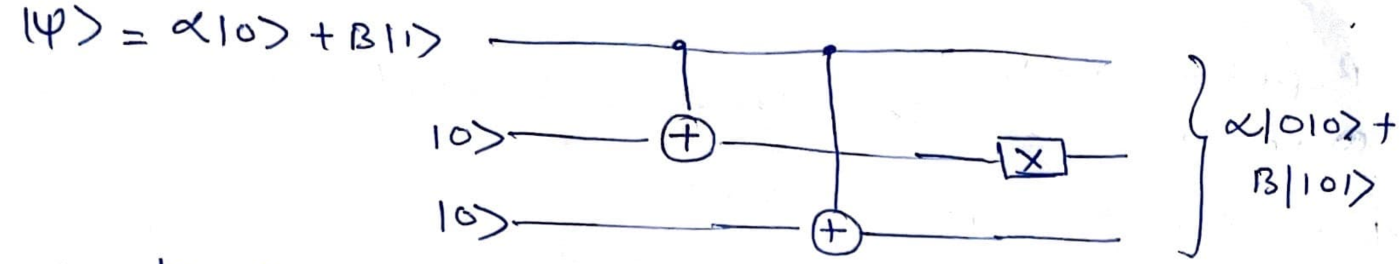
$\text{CNOT}|\alpha|00\rangle + \beta|10\rangle\rangle = \alpha|00\rangle + \beta|11\rangle$

Applying second CNOT gate on first qubit

$\text{CNOT}|(\alpha|00\rangle + \beta|11\rangle)|0\rangle\rangle =$

$\text{CNOT}|\alpha|\underline{0}00\rangle + \beta|\underline{1}10\rangle\rangle = \alpha|000\rangle + \beta|111\rangle$

Assume one qubit flipped.

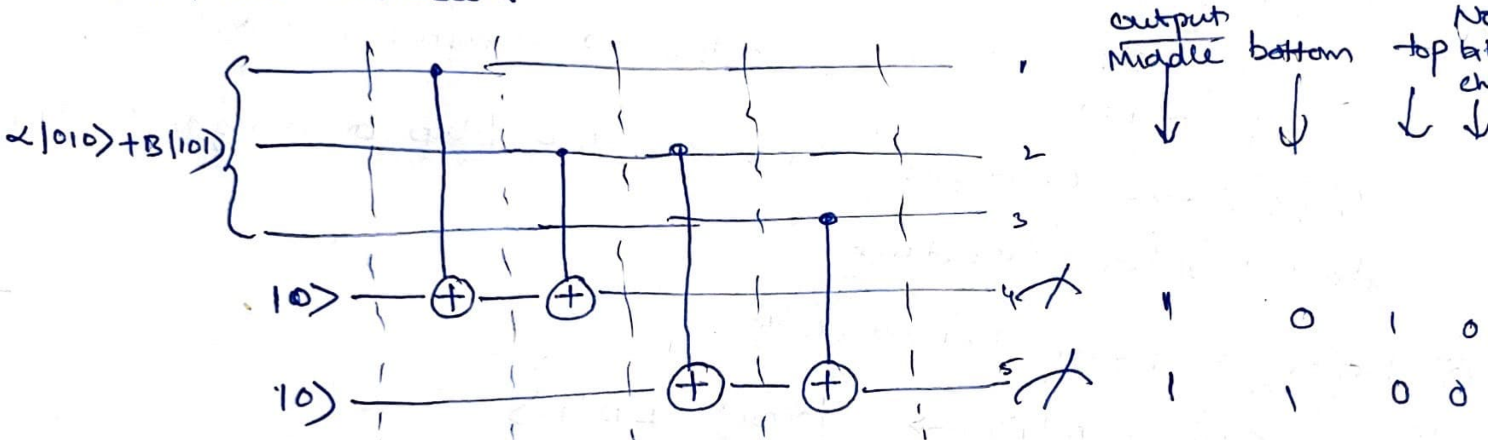


$CNOT(|\psi 0\rangle) = \alpha|00\rangle + \beta|11\rangle$

$CNOT(\alpha|000\rangle + \beta|110\rangle) = \alpha|000\rangle + \beta|111\rangle$

At stage 4, $|\psi_4\rangle = \alpha|010\rangle + \beta|101\rangle$

We can identify the location of a single bit-flip with this circuit:



$|\psi_1\rangle = \alpha|01000\rangle + \beta|10100\rangle$

$|\psi_2\rangle = \alpha|01000\rangle + \beta|10110\rangle$

$|\psi_3\rangle = \alpha|01010\rangle + \beta|10110\rangle$

$|\psi_4\rangle = \alpha|01011\rangle + \beta|10110\rangle$

$|\psi_5\rangle = \alpha|01011\rangle + \beta|10111\rangle$

Other cases: Other possibilities

① Bottom bit changes $\boxed{X}$ → Measurement of last two qubits
or leftmost qubit 0, 1

② Top bit or rightmost $\boxed{X}$ → Measurement 1, 0

③ No bit change $\boxed{X}$ → then 0, 0

Summary

State	Syndrome	Correction
$\alpha\|000\rangle + \beta\|111\rangle$	0 0	$1 \otimes 1 \otimes 1$
$\alpha\|100\rangle + \beta\|011\rangle$	1 0	$X \otimes 1 \otimes 1$
$\alpha\|010\rangle + \beta\|101\rangle$	1 1	$1 \otimes X \otimes 1$
$\alpha\|001\rangle + \beta\|110\rangle$	0 1	$1 \otimes 1 \otimes X$

Corresponding circuit: