Grovers algorithm

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Advanced Maths ①

	Grover	Shor's
①	polynomial Speed	Exponential Speed
②	optimal	not optimal

↳ No classical alg can supercede Grover algorithm

③ ~~Eg~~ widespread application

↳ ML alg. We use Grover's algorithm or its variants.

$f: \{0,1\}^n \to \{0,1\}$

find ~~x~~ $X \in \{0,1\}^n$ s.t. $f(X)=1$

Let $n=3$

f-domain	f-range
000	0
001	0
010	0
~~011~~ 011	1
100	0
101	0
110	0
111	0

$f(000) = 0$

$f(011) = 1$

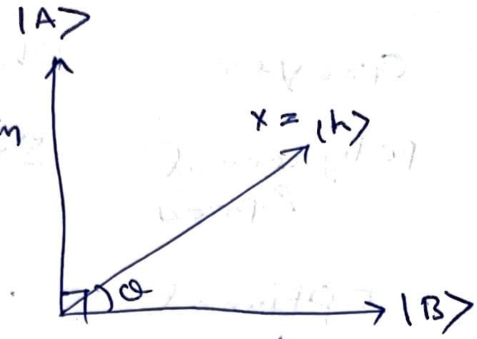
$f(111) = 0$

classical algorithm $O(2^n)$ complexity

Grovers alg. $O(\sqrt{2^n})$

Still exponential but polynomial improvement.

Key Idea



$|A\rangle$ represents equal superposition of all the inputs.

$\forall x$ s.t. $f(x) = 1$

Ex:

f-domain	f-range
000	1
001	0
010	0
011	1
100	0
101	0
110	1
111	0

Let $|A\rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

Let $|B\rangle = \frac{1}{\sqrt{5}} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$

Above: three o/ps yield true

remaining o/ps yield false

Vectors $|A\rangle$ and $|B\rangle$ are orthogonal to each other $\langle A|B\rangle = 0$

$|h\rangle$ Created using Hadamard gates.

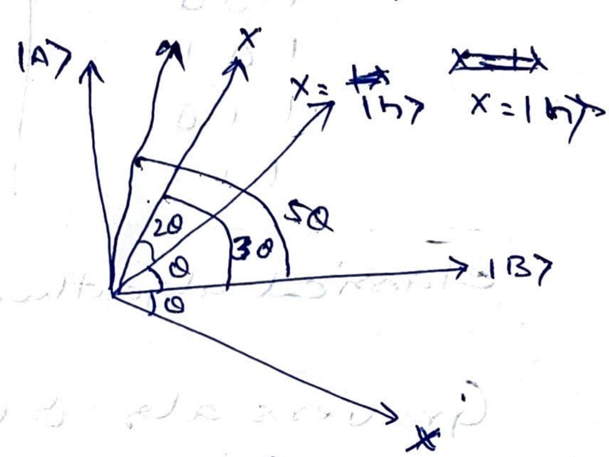
Let the angle between $|B\rangle$ and $|h\rangle$ is θ.

Our goal is to move Register x to move closer to $|A\rangle$

Action 1: Reflect x over $|B\rangle$

2. Reflect x over $|h\rangle$

Now x moved towards $|A\rangle$



Repeat Action 1 and Action 2

Steps in Grover's algorithm:

1. Create equal superposition of all possible inputs in $|h\rangle$
2. for (i = 1 to $\lfloor \frac{\pi}{4}\sqrt{N} \rfloor$) {

    X = Reflect X over $|B\rangle$

    X = Reflect X over $|h\rangle$

   }
3. Measure X // high prob to get the answer >99% accuracy

Yet to do

1. why we have $\lfloor \frac{\pi}{4}\sqrt{N} \rfloor$ Grover rotations.
2. Math of Grover's rotation
3. Quantum Circuit
4. Calculate Prob. of success
5. working Example