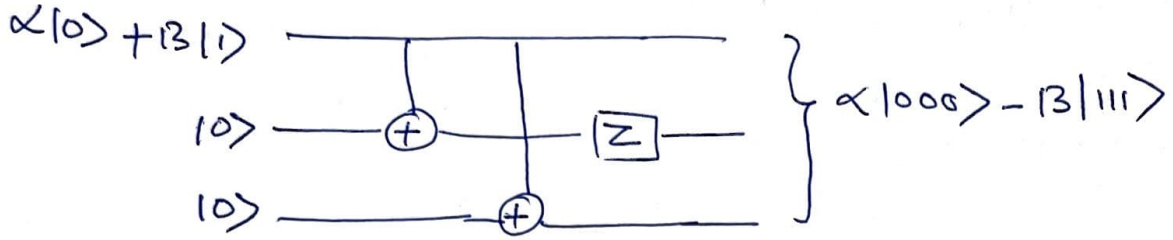
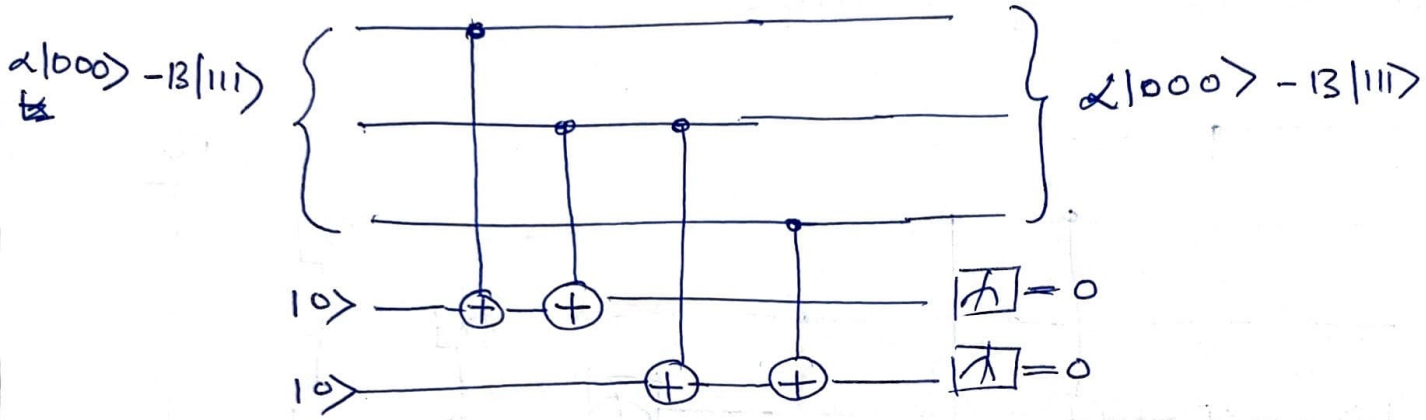
Phase flip errors

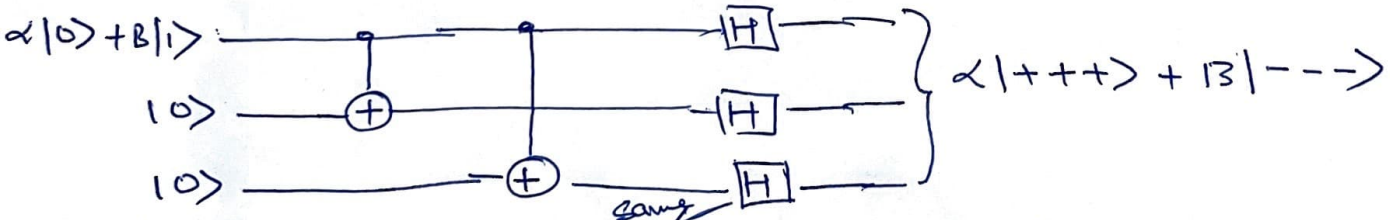
- Bit-flip errors are not the only errors we need to worry about. But we also have phase-flip errors described by Z-gate
- Unfortunately the 3-bit repetition code fails to detect phase-flip errors. (As shown below)





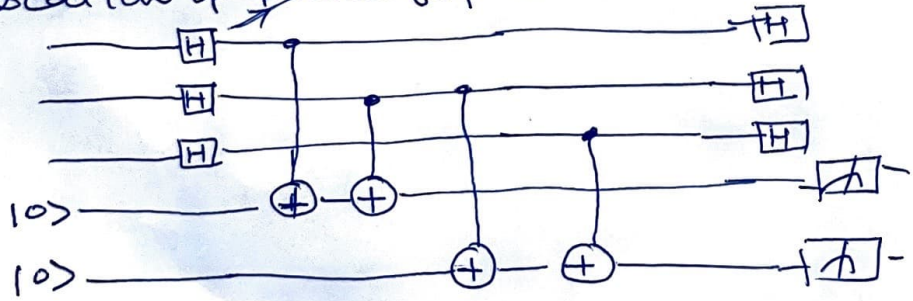
- which wrongly suggests no errors
- So three bit Repetition code helps to detect one bit bit-flip error but fails to detect phase-flip errors.

Correcting phase-flip errors:

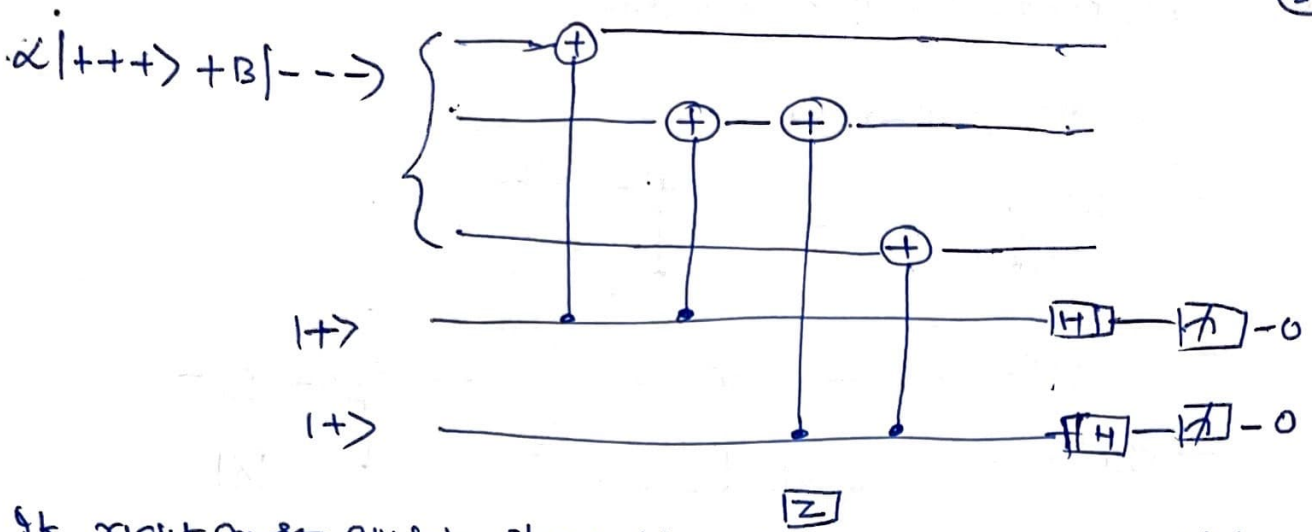


Hadamard gate converts 0 into +, 1 into −

Modifying the error detection circuit allows for the location of phase-flip error



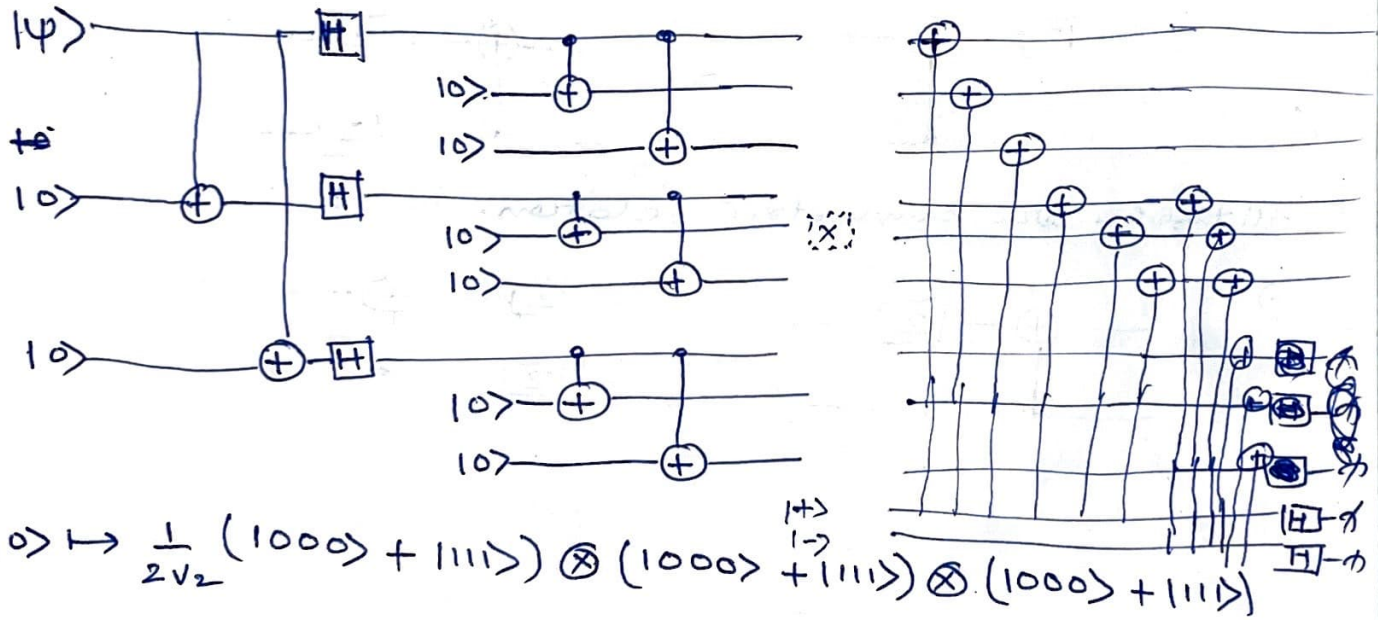
This Variant Circuit is shown in next page



✓ If right most qubit phase flip error then output would be $\alpha|++-\rangle + \beta|--+\rangle$ and the measurement be 1

Combining both bit-flip & Phase-flip ⇒ 9-Qubit Shor Code

9-qubit Shore code:



$$|0\rangle \mapsto \frac{1}{2\sqrt{2}}(|000\rangle + |111\rangle) \otimes (|000\rangle + |111\rangle) \otimes (|000\rangle + |111\rangle)$$

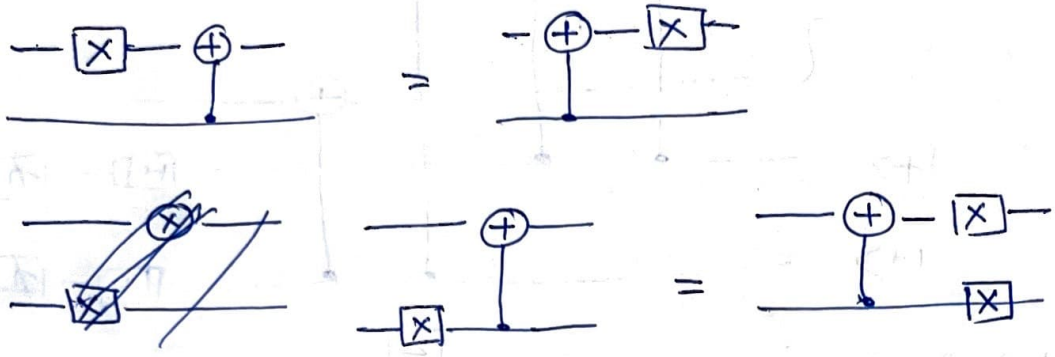
$$|1\rangle \mapsto \frac{1}{2\sqrt{2}}(|000\rangle - |111\rangle) \otimes (|000\rangle - |111\rangle) \otimes (|000\rangle - |111\rangle)$$

Bit-flip errors can be detected and corrected independently on each block by means of the inner code.

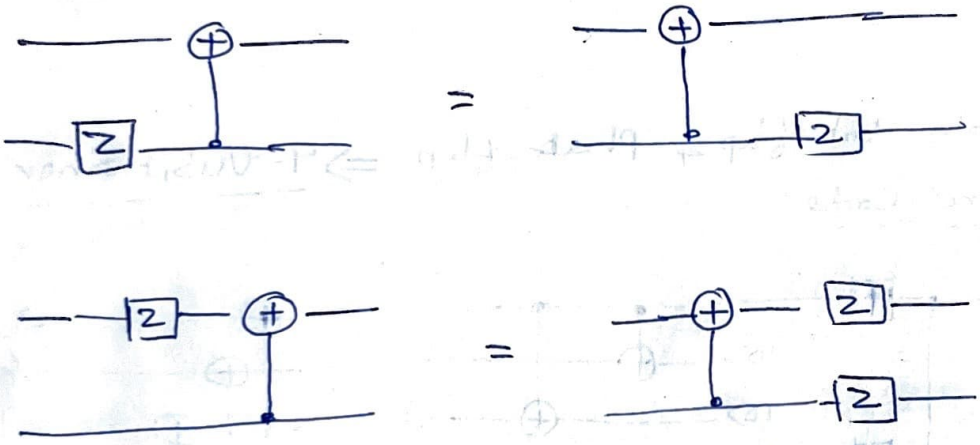
- We can correct even multiple bit-flips, provided there is only one bit flip in each block
- Bit-flips and phase-flip can occur at same qubit, corrected completely independently

Errors and C-NOTs relation.

X and C-NOT relations



Z and C-NOT relations



All these are symmetric relation. like