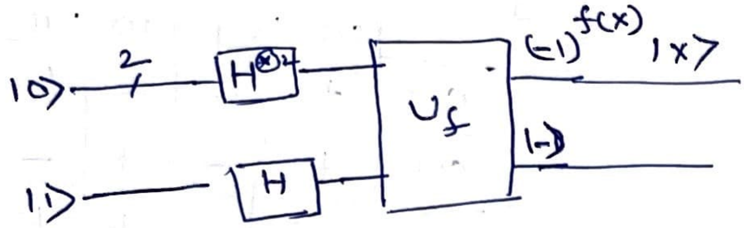
Example of Grover's algorithm

Given: Two bits as input one bit as output

$f: \{0,1\}^2 \rightarrow \{0,1\}$ find x such that $f(x) = 1$

Quantum circuit:



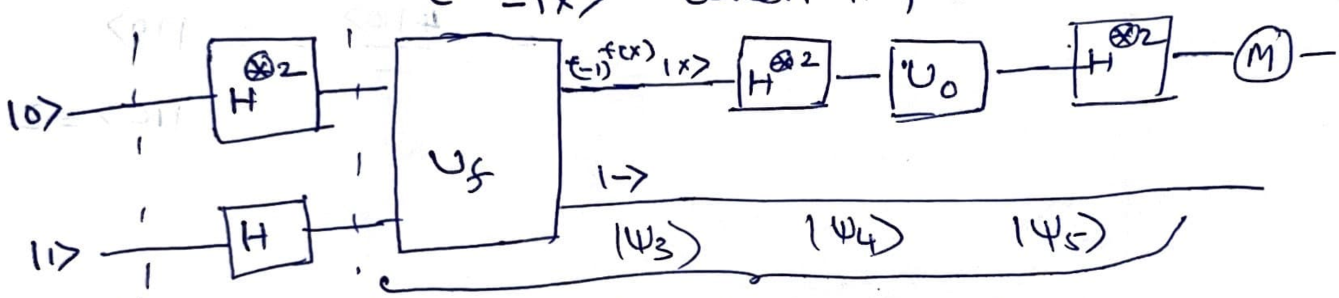
oracle knows

x	$f(x)$
00	0
01	0
10	1
11	0

U_f modifies the phase of the state.

$U_0|x\rangle$ is defined as

$$U_0|x\rangle = \begin{cases} |x\rangle & \text{when } x = 0^n \\ -|x\rangle & \text{when } x \neq 0^n \end{cases}$$



Lets circuit how many G. rotations are required for $n=2, a=1$, $N=4$

$$K = \left\lfloor \frac{\pi}{4}\sqrt{\frac{N}{a}} - \frac{1}{2} \right\rceil = \left\lfloor \frac{\pi}{4}\sqrt{\frac{4}{1}} - \frac{1}{2} \right\rceil = \lfloor 1.0707 \rceil = 1.$$

Hence one rotation is sufficient $\qquad N = 2^n$ $\quad n$: no of qubits

$a = 1$ as there is only one successful search element

$|\psi_1\rangle = |00\rangle|1\rangle$

$$|\psi_2\rangle = \frac{1}{\sqrt{2^2}} \sum_{x=0}^{3} |x\rangle |-\rangle$$

$$|\psi_3\rangle = \frac{1}{2}\left[|0\rangle + |1\rangle - |2\rangle + |3\rangle\right] |-\rangle$$

$$|\psi_4\rangle = H^{\otimes 2} |\psi_3\rangle = \frac{1}{2}\left[|0\rangle - |1\rangle + |2\rangle + |3\rangle\right]$$

after cancelation of 2/2 and then normalizing $\times |-\rangle$ we get the above

$|\psi_5\rangle =$

	00	01	10	11
00	+	+	+	+
01	+	−	+	−
10	−	−	+	+
11	+	−	−	+

$\frac{1}{\sqrt{2^2}}(2|00\rangle - 2|01\rangle + 2|10\rangle + 2|11\rangle)$

$|U_0\rangle$ $U_0 \rightarrow$ flips every state except the first state $|0\rangle$

Hence $|\psi_5\rangle = \frac{1}{2}\left[|0\rangle + |1\rangle - |2\rangle - |3\rangle\right] |-\rangle$

$|\psi_6\rangle = |2\rangle$

	00	01	10	11
00	+	+	+	+
01	+	−	+	−
−10	−	−	+	+
−11	−	+	+	−

$\frac{1}{\sqrt{2^2}}\left(0|00\rangle + 0|01\rangle + 4|10\rangle + 0|11\rangle\right)$

$= \frac{4|10\rangle}{2} = 2|10\rangle$

$|10\rangle = 2$ state