Quantum Computing

Assignment-2

1. Prove that the matrix $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ is both Unitary and Hermition matrix

2. Let $|\psi\rangle = i|0\rangle + 7|1\rangle$ and $|\phi\rangle = |00\rangle + 3|10\rangle + 7|11\rangle$ Find $|\psi\phi\rangle$ and also find its equivalent column matrix representation (Ket-Ket notation)

3. (Bra-Bra notation) Let $\langle\psi| = 3\langle 0| + 7\langle 1|$ and $\langle\phi| = \langle 0| + i\langle 1|$ then find $\langle\psi\phi|$.

4. (Ket-Bra notation) Let $|\alpha\rangle = 3|0\rangle + i|1\rangle$ and $|B\rangle = |00\rangle + 2|10\rangle + 7|11\rangle$ then find $|\alpha\rangle\langle B|$

5. (Bra-ket notation) Let $|\alpha\rangle = i|0\rangle + 7|1\rangle$ and $|B\rangle = 3|0\rangle + |1\rangle$ then find $\langle\alpha|B\rangle$

6. Let $|\psi\rangle = -\frac{4}{5}i|0\rangle + \frac{3}{5}|1\rangle$ and $|\phi\rangle = \frac{|0\rangle}{\sqrt{2}} + \frac{|1\rangle}{\sqrt{2}}$ then find $|\psi\phi\rangle$ and find the Probability of $|00\rangle$

7. Let $|\psi\rangle = \frac{1}{\sqrt{5}}|0000\rangle - \sqrt{\frac{2}{5}}|0100\rangle + \sqrt{\frac{1}{5}}|0110\rangle + \frac{1}{\sqrt{5}}|1111\rangle$, find the Probability and the resultant (normalised) state, if first and fourth ~~bit~~ cubits are zeros

8. Prove that $|+\rangle$ is a Unit vector.

9. Prove that $\langle +|-\rangle$ is orthonormal.

10. Let $|\psi\rangle = |0\rangle$, then find the Probability of measuring $|+\rangle$. Hint: compute $|\langle\psi|+\rangle|$.

11. Let $|\psi\rangle = \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}}\right)|+\rangle + \left(\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{3}}\right)|-\rangle$
then find the probability of measuring $|\psi\rangle$ on $|0\rangle$

12. Let $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ then find the resultant quantum state after a global phase shift of $\frac{\pi}{2}$

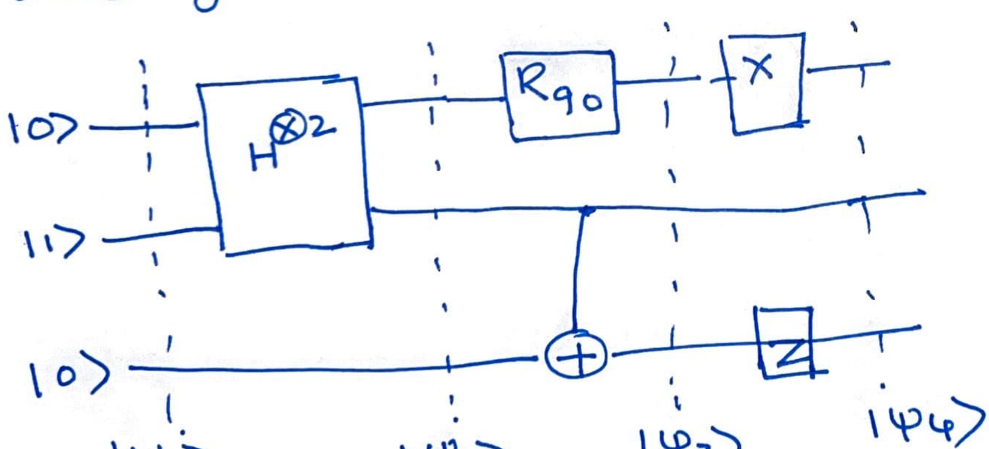
~~13.~~ Hint: $e^{i\frac{\pi}{2}}|\psi\rangle$

13. Find $H^{\otimes 2}|00\rangle$ using Tensor Product rep of $H^{\otimes 2}$

14. ~~Find~~ Prove $CNOT|-+\rangle = |-+\rangle$

and $CNOT|+-\rangle = |--\rangle$

15. The given circuit is



Find the following:

a) Make the reverse circuit

b) Find the output of the circuit

c) Write the complete circuit as Unitary Matrix

d) Write the reverse circuit as Unitary Matrix

e) Compute output using Unitary matrix