Detailed analysis of f(x) is constant and f(x) is balanced cases.

f(x) is constant:

$$\sum_z \sum_x \frac{(-1)^{xz + f(x)}}{2^n} |z\rangle \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right).$$

$f(x) = c.$ when $z = 0^{\otimes N}$

$x \cdot z = x_1 \cdot 0 \oplus x_2 \cdot 0 \oplus \cdots \oplus x_n \cdot 0$

$= 0 \oplus 0 \oplus \cdots \oplus 0 = 0$

// Exclusive OR of dot products where each term is zero

$\therefore (-1)^{xz + f(x)} = (-1)^{0+c} = (-1)^c$ $xz = 1$ ~~$xz = 0$~~

For every value other than $z = 0^{\otimes N}$,

That means, the coefficient donot vanish but exists as $(-1)^c$.

coefficient exist for other functions which are not balanced. But the promise is f(x) is either const or balanced.

f(x) is balanced;

Half of the times $f(x) = 0$, Half of the times $f(x) = 1$

$(-1)^{xz + f(x)} = (-1)^{f(x)} \cdot (-1)^{xz}$

Let $N = 2 \therefore 2^N = 4. \Rightarrow$ For two values of x, $f(x) = 0$
For two values of x, $f(x) = 1$

$$\Rightarrow \sum_z \left[(-1)^0 + (-1)^0 + (-1)^1 + (-1)^1\right] (-1)^{xz}.$$

$$\sum_z \sum_x (-1)^{xz + f(x)}$$

$$= \sum_z \left[1 + 1 - 1 - 1\right] (-1)^{xz} = 0$$

$\Rightarrow$ ~~$(-1)^{xz + f(x)}$~~ $(-1)^{xz + f(x)}$ is zero for f(x) is balanced case