

Problems for Recitation 1

1 Team Problem: Contrapositive

Prove by truth table that an implication is equivalent to its contrapositive.

2 Team Problem: A Mystery

A certain cabal within the 6.042 course staff is plotting to make the final exam *ridiculously hard*. (“Problem 1. Prove that the axioms of mathematics are complete and consistent. Express your answer in Mayan hieroglyphics.”) The only way to stop their evil plan is to determine exactly who is in the cabal. The course staff consists of nine people:

{Devin, Elizabeth, Emanuele, Hao, Henry, Hyungie, Michael, Patrick, Rachel}

The cabal is a subset of these nine. A membership roster has been found and appears below, but it is deviously encrypted in logic notation. The predicate *incabal* indicates who is in the cabal; that is, *incabal*(*x*) is true if and only if *x* is a member. Translate each statement below into English and deduce who is in the cabal.

$$(i) \exists x \exists y \exists z (x \neq y \wedge x \neq z \wedge y \neq z \wedge incabal(x) \wedge incabal(y) \wedge incabal(z))$$

$$(ii) \neg(incabal(\text{Michael}) \wedge incabal(\text{Henry}))$$

$$(iii) (incabal(\text{Hyungie}) \vee incabal(\text{Emanuele})) \rightarrow \forall x incabal(x)$$

$$(iv) incabal(\text{Michael}) \rightarrow incabal(\text{Henry})$$

$$(v) incabal(\text{Elizabeth}) \rightarrow incabal(\text{Hyungie})$$

$$(vi) (incabal(\text{Devin}) \vee incabal(\text{Hao})) \rightarrow \neg incabal(\text{Rachel})$$

$$(vii) (incabal(\text{Devin}) \vee incabal(\text{Henry})) \rightarrow \neg incabal(\text{Patrick})$$