# Problem Set 2

*Due*: March 3

**Reading:**

- Chapter 3.6. *Predicate Formulas*,
- Chapter 4. *Mathematical Data Types* through 4.4. *Binary Relations*,
- Chapter 5. *Induction*.

These assigned readings do **not** include the Problem sections. (Many of the problems in the text will appear as class or homework problems.)

**Reminder**:

- Instructions for PSet submission are on the class Stellar page. Remember that each problem should prefaced with a *collaboration statement*.

**Problem 1.**

Express each of the following predicates and propositions in formal logic notation. The domain of discourse is the nonnegative integers $\mathbb{N}$. Moreover, in addition to the propositional operators, variables and quantifiers, you may define predicates using addition, multiplication, and equality symbols, but no *exponentiation* (like $x^y$) and no use of integer *constants* like $\texttt{0}$ or $\texttt{1}$ (until you justify using them in part (a)).

For example, the predicate "$x \geq y$" could be expressed by the following logical formula.

$$\exists w. (y + w = x).$$

Now that we can express $\geq$, it's OK to use it to express other predicates. For example, the predicate $x < y$ can now be expressed as

$$y \geq x \text{ AND NOT}(x = y).$$

For each of the predicates below, describe a logical formula to express it. It is OK to use in the logical formula any of the predicates previously expressed.

**(a)** $x = \texttt{1}$.

**(b)** $m$ is a divisor of $n$ (notation: $m \mid n$)

**(c)** $n$ is a prime number.

**(d)** $n$ is a power of a prime.

**Problem 2.**
Let $A$, $B$ and $C$ be sets. Prove that:

$$A \cup B \cup C = (A - B) \cup (B - C) \cup (C - A) \cup (A \cap B \cap C). \tag{1}$$

*Hint:* $P$ OR $Q$ OR $R$ is equivalent to

$$(P \text{ AND } \overline{Q}) \text{ OR } (Q \text{ AND } \overline{R}) \text{ OR } (R \text{ AND } \overline{P}) \text{ OR } (P \text{ AND } Q \text{ AND } R).$$

**Problem 3.** **(a)** Write predicates that express the following assertions.

- $R$ is a surjection [>= 1 in].
- $R$ is a function [<= 1 out].
- $x = y - z$.
- $x = \{\}$.

**Problem 4.**
The Fibonacci numbers $F(0), F(1), F(2), \ldots$ are defined as follows:

$$\begin{aligned} F(0) &::= 0, \\ F(1) &::= 1, \\ F(n) &::= F(n-1) + F(n-2) \qquad \text{for } n \geq 2. \end{aligned}$$

Thus, the first few Fibonacci numbers are 0, 1, 1, 2, 3, 5, 8, 13, and 21. Prove by induction that for all $n \geq 1$,

$$F(n-1) \cdot F(n+1) - F(n)^2 = (-1)^n. \tag{2}$$