

Problems for Recitation 22

1 Properties of Variance

In this problem we will study some properties of the variance and the standard deviation of random variables.

- a. Show that for any random variable R , $\text{Var}[R] = \text{E}[R^2] - \text{E}^2[R]$.
- b. Show that for any random variable R and constants a and b , $\text{Var}[aR + b] = a^2 \text{Var}[R]$.
Conclude that the standard deviation of $aR + b$ is a times the standard deviation of R .
- c. Show that if R_1 and R_2 are independent random variables, then

$$\text{Var}[R_1 + R_2] = \text{Var}[R_1] + \text{Var}[R_2].$$

- d. Give an example of random variables R_1 and R_2 for which

$$\text{Var}[R_1 + R_2] \neq \text{Var}[R_1] + \text{Var}[R_2].$$

- e. Compute the variance and standard deviation of the Binomial distribution $H_{n,p}$ with parameters n and p .
- f. Let's say we have a random variable T such that $T = \sum_{j=1}^n T_j$, where all of the T_j 's are mutually independent and take values in the range $[0, 1]$. Prove that $\text{Var}(T) \leq \text{E}(T)$. We'll use this result in lecture tomorrow. *Hint: Upper bound $\text{Var}[T_j]$ with $\text{E}[T_j]$ using the definition of variance in part (a) and the rule for computing the expectation of a function of a random variable.*

2 Gambler's Ruin

A gambler is placing \$1 bets on the “1st dozen” in roulette. This bet wins when a number from one to twelve comes in, and then the gambler gets his \$1 back plus \$2 more. Recall that there are 38 numbers on the roulette wheel.

The gambler's initial stake is n and his target is T . He will keep betting until he runs out of money (“goes broke”) or reaches his target. Let w_n be the probability of the gambler winning, that is, reaching target T before going broke.

- a. Write a linear recurrence with boundary conditions for w_n . You need *not* solve the recurrence.
- b. Let e_n be the expected number of bets until the game ends. Write a linear recurrence with boundary conditions for e_n . You need *not* solve the recurrence.