Problems for Recitation 10

1. Give a description of the equivalence classes associated with each of the following equivalence relations.

    (a) Integers x and y are equivalent if $x \equiv y \pmod 3$.

    (b) Real numbers x and y are equivalent if $\lceil x \rceil = \lceil y \rceil$, where $\lceil z \rceil$ denotes the smallest integer greater than or equal to z.

2. Show that neither of the following relations is an equivalence relation by identifying a missing property (reflexivity, symmetry, or transitivity).

    (a) The "divides" relation on the positive integers.

    (b) The "implies" relation on propositional formulas.

3. Here is prerequistite information for some MIT courses:

$$18.01 \to 6.042 \qquad\qquad 18.01 \to 18.02$$
$$18.01 \to 18.03 \qquad\qquad 6.046 \to 6.840$$
$$8.01 \to 8.02 \qquad\qquad 6.01 \to 6.034$$
$$6.042 \to 6.046 \qquad\qquad 18.03, 8.02 \to 6.02$$
$$6.01, 6.02 \to 6.003 \qquad\qquad 6.01, 6.02 \to 6.004$$
$$6.004 \to 6.033 \qquad\qquad 6.033 \to 6.857$$

(a) Draw a Hasse diagram for the corresponding partially-ordered set. (A **Hasse diagram** is a way of representing a poset $(A, \preceq)$ as a directed acyclic graph. The vertices are the element of A, and there is generally an edge $u \to v$ if $u \preceq v$. However, self-loops and edges implied by transitivity are omitted.) You'll need this diagram for all the subsequent problem parts, so be neat!

(b) Identify a largest chain. (A **chain** in a poset $(S, \preceq)$ is a subset $C \subseteq S$ such that for all $x, y \in C$, either $x \preceq y$ or $y \preceq x$.)

(c) Suppose that you want to take all the courses. What is the minimum number of terms required to graduate, if you can take as many courses as you want per term?

(d) Identify a largest **antichain**. (An **antichain** in a poset $(S, \preceq)$ is a subset $A \subseteq S$ such that for all $x, y \in A$ with $x \neq y$, neither $x \preceq y$ nor $y \preceq x$.)

(e) What is the maximum number of classes that you could possibly take at once?

(f) Identify a topological sort of the classes. (A ***topological sort*** of a poset $(A, \preceq)$ is a total order of all the elements such that if $a_i \preceq a_j$ in the partial order, then a_i precedes a_j in the total order.)

(g) Suppose that you want to take all of the courses, but can handle only two per term. How many terms are required to graduate?

(h) What if you could take three courses per term?

(i) Stanford's computer science department offers n courses, limits students to at most k classes per term, and has its own complicated prerequisite structure. Describe two different lower bounds on the number of terms required to complete all the courses. One should be based on your answers to parts (b) and (c) and a second should be based on your answer to part (g).