Theorem 1. *If* $p_1 + p_2 + \cdots = 1$ *and all* $p_i \geq 0$*, then the sum*

$$\Omega = \sum_k \frac{p_k}{p_{k+1} + p_{k+2} + \cdots}$$

diverges.

Proof. We simplify the problem by substituting variables twice. First, let

$$S_k = p_k + p_{k+1} + \ldots$$

Note that $S_k - S_{k+1} = p_k$, $S_1 = 1$, and $\lim_{k\to\infty} S_k = 0$. Then, the sum is

$$\Omega = \sum_k \frac{S_k - S_{k+1}}{S_{k+1}} = \sum_k \left(\frac{S_k}{S_{k+1}} - 1 \right)$$

Now, let $a_k = \frac{S_k}{S_{k+1}} - 1$. We have by telescoping that

$$\prod (a_k + 1) = \lim_{k\to\infty} \frac{S_1}{S_k} = \lim_{k\to\infty} \frac{1}{S_k} = \infty$$

then, the exponential of the sum

$$\begin{aligned} e^\Omega = \exp\left[\sum_k \left(\frac{S_k}{S_{k+1}} - 1 \right) \right] &= \exp\left[\sum_k a_k \right] \\ &= \prod e^{a_k} \\ &\geq \prod (a_k + 1) \\ &= \infty \end{aligned}$$

so $\Omega = \infty$. $\square$