Some useful facts about divisibility and GCDs

Divisibility

D1. If $a \mid b$, then $a \mid bc$ for all c.

D2. If $a \mid b$ and $b \mid c$, then $a \mid c$.

D3. If $a \mid b$ and $a \mid c$, then $a \mid sb + tc$ for all s and t.

D4. For all $c \neq 0$, $a \mid b$ if and only if $ca \mid cb$.

Greatest common divisor

G1. Every common divisor of a and b divides $\gcd(a, b)$.

G2. $\gcd(ka, kb) = k \cdot \gcd(a, b)$ for all $k > 0$.

G3. If $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$, then $\gcd(a, bc) = 1$.

G4. If $a \mid bc$ and $\gcd(a, b) = 1$, then $a \mid c$.

G5. If $m \mid a$ and $m \mid b$, then $m \mid \gcd(a, b)$.

G6. $\gcd(a, b) = \gcd(b, \text{rem } ab)$.