# Problems for Recitation 4

## 1 Problem: The Pulverizer!

There is a pond. Inside the pond there are $n$ pebbles, arranged in a cycle. A frog is sitting on one of the pebbles. Whenever he jumps, he lands exactly $k$ pebbles away in the clockwise direction, where $0 < k < n$. The frog's meal, a delicious worm, lies on the pebble right next to his, in the clockwise direction.

**(a)** Describe a situation where the frog can't reach the worm.

**(b)** In a situation where the frog can actually reach the worm, explain how to use the Pulverizer to find how many jumps the frog will need.

**(c)** Compute the number of jumps if $n = 50$ and $k = 21$. Anything strange? Can you fix it?

# 2 Problem: The Fibonacci numbers.

The Fibonacci numbers are defined as follows:

$$F_0 = 0 \qquad F_1 = 1 \qquad F_n = F_{n-1} + F_{n-2} \quad (\text{for } n \geq 2).$$

Give an inductive proof that the Fibonacci numbers $F_n$ and $F_{n+1}$ are relatively prime for all $n \geq 0$. (Two numbers $a$ and $b$ are *relatively prime* iff $\gcd(a, b) = 1$)

# 3 The power of 3.

Let $N$ be a number whose decimal expansion consists of $3^n$ identical digits. Show by induction that $3^n \mid N$. For example:

$$3^2 \mid \underbrace{777777777}_{3^2 = 9 \text{ digits}}$$

Recall that 3 divides a number iff it divides the sum of its digits.