Problems for Recitation 22

Problem 1. [**points**] Here's yet another fun 6.042 game! You pick a number between 1 and 6. Then you roll three fair, independent dice.

- If your number never comes up, then you lose a dollar.

- If your number comes up once, then you win a dollar.

- If your number comes up twice, then you win two dollars.

- If your number comes up three times, you win *four* dollars!

What is your expected payoff? Is playing this game likely to be profitable for you or not?

Problem 2. [**points**] The number of squares that a piece advances in one turn of the game Monopoly is determined as follows:

- Roll two dice, take the sum of the numbers that come up, and advance that number of squares.

- If you roll *doubles* (that is, the same number comes up on both dice), then you roll a second time, take the sum, and advance that number of additional squares.

- If you roll doubles a second time, then you roll a third time, take the sum, and advance that number of additional squares.

- However, as a special case, if you roll doubles a third time, then you go to jail. Regard this as advancing zero squares overall for the turn.

(a) [pts] What is the expected sum of two dice, given that the same number comes up on both?

(b) [pts] What is the expected sum of two dice, given that different numbers come up? (Use your previous answer and the Total Expectation Theorem.)

(c) [pts] To simplify the analysis, suppose that we always roll the dice three times, but may ignore the second or third rolls if we didn't previously get doubles. Let the random variable X_i be the sum of the dice on the i-th roll, and let E_i be the event that the i-th roll is doubles. Write the expected number of squares a piece advances in these terms.

(d) [pts] What is the expected number of squares that a piece advances in Monopoly?