

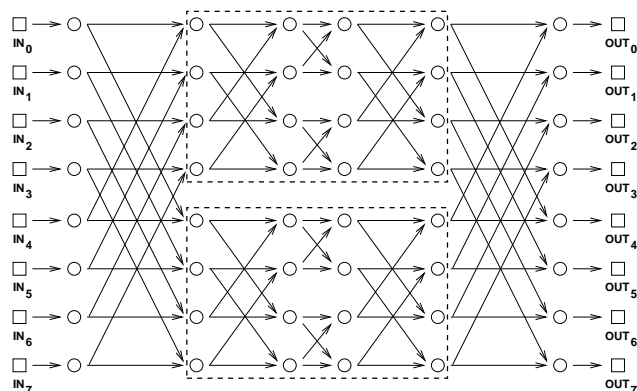
## Notes for Recitation 8

### Routing in a Beneš Network

In lecture, we saw that the Beneš network has a max congestion of 1; that is, every permutation can be routed in such a way that a single packet passes through each switch. Let's work through an example. A Beneš network of size  $N = 8$  is attached.

1. Within the Beneš network of size  $N = 8$ , there are two subnetworks of size  $N = 4$ . Put boxes around these. Hereafter, we'll refer to these as the *upper* and *lower* subnetworks.

**Solution.**



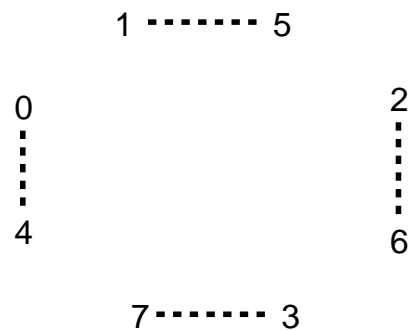
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2. Now consider the following permutation routing problem:

$$\begin{array}{ll}
 \pi(0) = 3 & \pi(4) = 2 \\
 \pi(1) = 1 & \pi(5) = 0 \\
 \pi(2) = 6 & \pi(6) = 7 \\
 \pi(3) = 5 & \pi(7) = 4
 \end{array}$$

Each packet must be routed through either the upper subnetwork or the lower subnetwork. Construct a graph with vertices  $0, 1, \dots, 7$  and draw a *dashed* edge between each pair of packets that can not go through the same subnetwork because a collision would occur in the second column of switches.

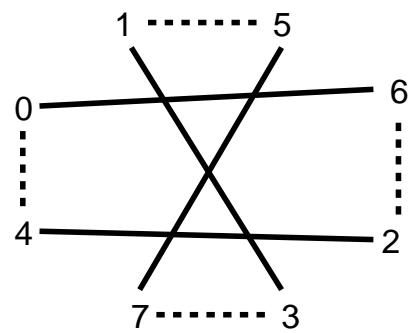
**Solution.**



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3. Add a *solid* edge in your graph between each pair of packets that can not go through the same subnetwork because a collision would occur in the next-to-last column of switches.

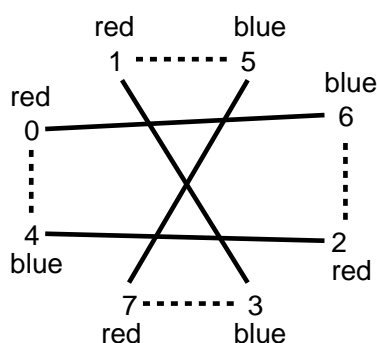
**Solution.**



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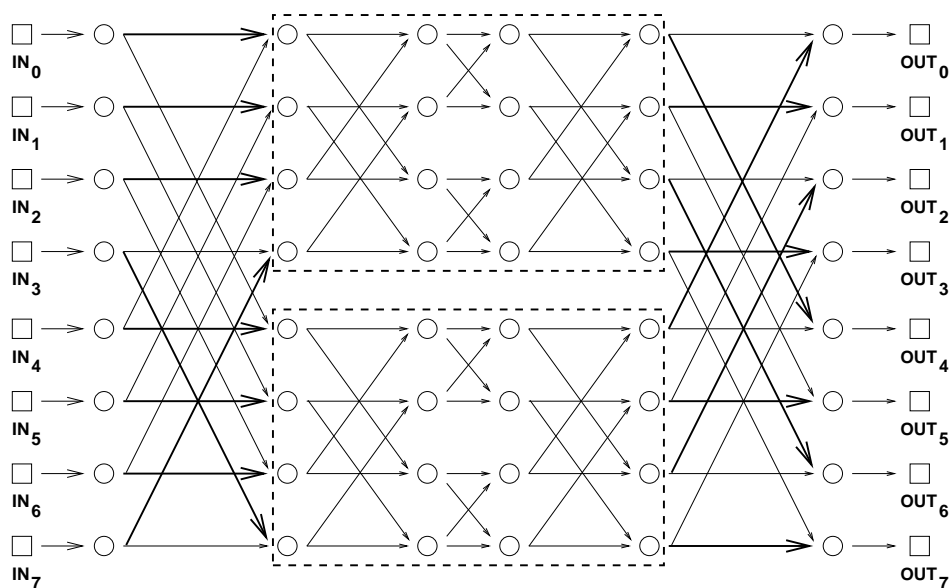
4. Color (i.e., label) the vertices of your graph red and blue so that adjacent vertices get different colors. Why must this be possible, regardless of the permutation  $\pi$ ?

**Solution.** This must be possible, because the dashed edges form a matching and the solid edges form another matching. Because of the result you proved in homework, when you combine the edges, the result is a bipartite graph, which must be 2-colorable.



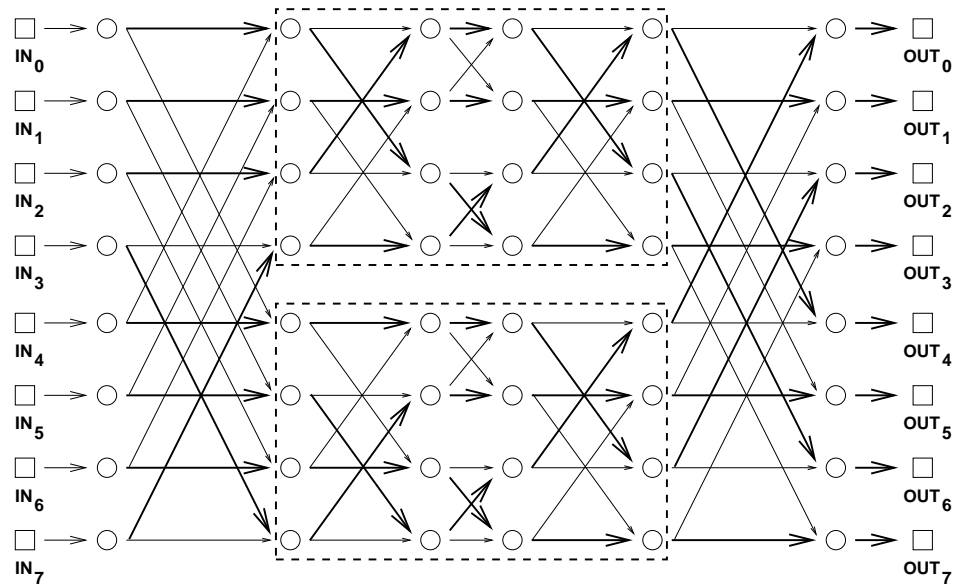
5. Suppose that red vertices correspond to packets routed through the upper subnetwork and blue vertices correspond to packets routed through the lower subnetwork. On the attached copy of the Beneš network, highlight the first and last edge traversed by each packet.

**Solution.**



6. All that remains is to route packets through the upper and lower subnetworks. One way to do this is by applying the procedure described above recursively on each subnetwork. However, since the remaining problems are small, see if you can complete all the paths on your own.

Solution.



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