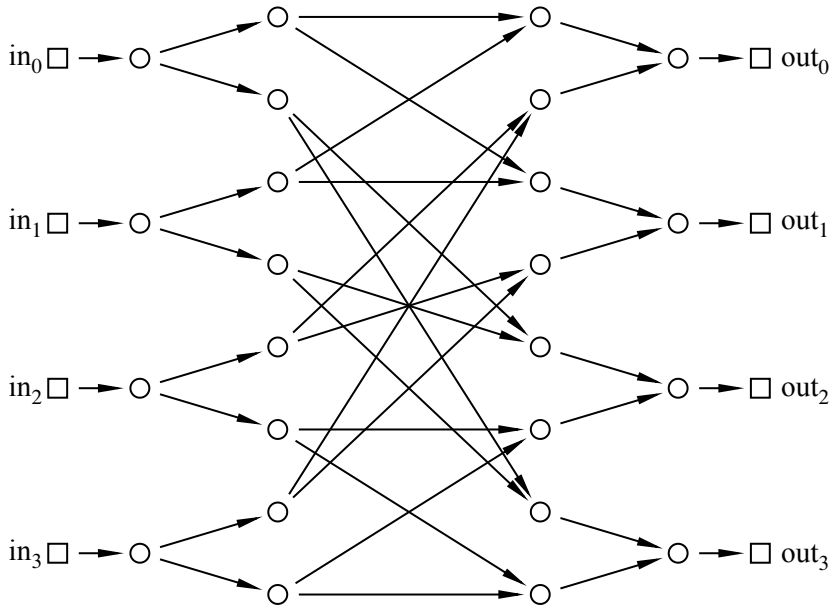
Problem 1. [**points**] A *multiple binary-tree network* has n inputs and n outputs, where n is a power of 2. Each input is connected to the root of a binary tree with $n/2$ leaves and with edges pointing away from the root. Likewise, each output is connected to the root of a binary tree with $n/2$ leaves and with edges pointing toward the root.

Two edges point from each leaf of an input tree, and each of these edges points to a leaf of an output tree. The matching of leaf edges is arranged so that for every input and output tree, there is an edge from a leaf of the input tree to a leaf of the output tree, and every output tree leaf has exactly two edges pointing to it.

(a) [pts] Draw such a multiple binary-tree net for $n = 4$.



Solution. ■

(b) [pts] Fill in the table, and explain your entries.

# switches	switch size	diameter	max congestion

Solution.

# switches	switch size	diameter	max congestion
$2n(n-1)$	$1 \times 2, 2 \times 1$	$1 + 2\log n$	1

These formulas were gotten as follows: a binary tree with $n/2$ leaves has $n-1$ nodes (switches), and there are $2n$ trees.

Each node of an input tree has one edge in and two out; the opposite for nodes of output trees.

The distance from any input to any output is 1 from input to tree root, $(\log n) - 1$ from root to leaf, 1 from input leaf to output leaf, $(\log n) - 1$ from output leaf to output root, and 1 to output, for a total of $1 + 2\log n$.

The path from any input to any output is unique, and paths from two inputs to different outputs don't overlap, so at most one packet goes through any switch. ■