Tom Leighton and Ankur Moitra

Problem Set 7

Due: Tuesday, November 1

Problem 1. [15 points] Express

$$\sum_{i=0}^{n} i^2 x^i$$

as a closed-form function of n.

Problem 2. [20 points]

(a) [5 pts] What is the product of the first n odd powers of two: $\prod_{k=1}^{n} 2^{2k-1}$?

(b) [5 pts] Find a closed expression for

$$\sum_{i=0}^{n} \sum_{j=0}^{m} 3^{i+j}$$

(c) [5 pts] Find a closed expression for

$$\sum_{i=1}^{n} \sum_{j=1}^{n} (i+j)$$

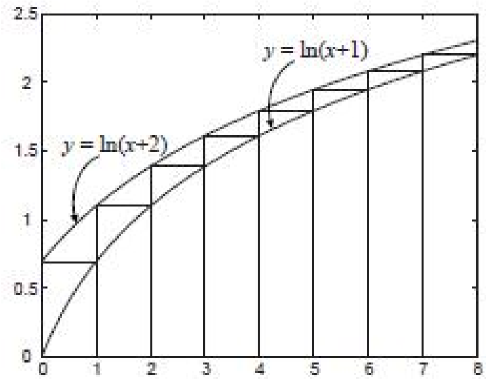
(d) [5 pts] Find a closed expression for

$$\prod_{i=1}^{n} \prod_{j=1}^{n} 2^i \cdot 3^j$$

Problem 3. [10 points]

(a) [6 pts] Use integration to find upper and lower bounds that differ by at most 0.1 for the following sum. (You may need to add the first few terms explicitly and then use integrals to bound the sum of the remaining terms.)

$$\sum_{i=1}^{\infty} \frac{1}{(2i+1)^2}$$



(b) [4 pts] Assume n is an integer larger than 1. Which of the following inequalities, if any, hold. You may find the graph helpful.

1. $\displaystyle\sum_{i=1}^{n} \ln(i+1) \leq \int_0^n \ln(x+2)dx$
2. $\displaystyle\sum_{i=1}^{n} \ln(i+1) \leq \ln 2 + \int_1^n \ln(x+1)dx$

Problem 4. [20 points] For each of the following six pairs of functions f and g (parts (a) through (f)), state which of these order-of-growth relations hold (more than one may hold, or none may hold):

$$f = o(g) \qquad f = O(g) \qquad f = \omega(g) \qquad f = \Omega(g) \qquad f = \Theta(g) \qquad f \sim g$$

(a)	$f(n) = \log_2 n$	$g(n) = \log_{10} n$
(b)	$f(n) = 2^n$	$g(n) = 10^n$
(c)	$f(n) = 0$	$g(n) = 17$
(d)	$f(n) = 1 + \cos\left(\frac{\pi n}{2}\right)$	$g(n) = 1 + \sin\left(\frac{\pi n}{2}\right)$
(e)	$f(n) = 1.0000000001^n$	$g(n) = n^{10000000000}$