

Problems for Recitation 4

1 Problem: The Pulverizer!

There is a pond. Inside the pond there are n pebbles, arranged in a cycle. A frog is sitting on one of the pebbles. Whenever he jumps, he lands exactly k pebbles away in the clockwise direction, where $0 < k < n$. The frog's meal, a delicious worm, lies on the pebble right next to his, in the clockwise direction.

- (a) Describe a situation where the frog can't reach the worm.
- (b) In a situation where the frog can actually reach the worm, explain how to use the Pulverizer to find how many jumps the frog will need.
- (c) Compute the number of jumps if $n = 50$ and $k = 21$. Anything strange? Can you fix it?

2 Problem: The Fibonacci numbers.

The Fibonacci numbers are defined as follows:

$$F_0 = 0 \quad F_1 = 1 \quad F_n = F_{n-1} + F_{n-2} \quad (\text{for } n \geq 2).$$

Give an inductive proof that the Fibonacci numbers F_n and F_{n+1} are relatively prime for all $n \geq 0$.

3 Extra Problem: The power of 3.¹

Let N be a number whose decimal expansion consists of 3^n identical digits. Show by induction that $3^n \mid N$. For example:

$$3^2 \mid \underbrace{77777777}_{3^2 = 9 \text{ digits}}$$

Recall that 3 divides a number iff it divides the sum of its digits.

¹Try this if you have time!