# Properties of relations

## Properties of a relation on A:

#### Reflexivity R is reflexive if

 $\forall x \in A. \ xRx.$ 

"Everyone likes themselves."

Every node in G has a loop.

#### Irreflexivity R is *irreflexive* if

 $\neg \exists x \in A. \ xRx.$ 

"No one likes themselves."

There are no loops in G.

#### Symmetry R is symmetric if

 $\forall x, y \in A. \ xRy \ \Rightarrow \ yRx.$ 

"If x likes y, then y likes x."

If there is an edge from x to y in G, then there is an edge from y to x in G as well.

### Antisymmetry R is antisymmetric if

$$\forall x, y \in A. (xRy \land yRx) \Rightarrow x = y.$$

"No pair of distinct people like each other."

There is at most one directed edge between any pair of distinct nodes.

#### Transitivity R is transitive if

 $\forall x, y, z \in A. (xRy \land yRz) \Rightarrow xRz.$ 

"If x likes y and y likes z, then x likes z too."

For any walk  $v_0, v_1, \ldots, v_k$  in G where  $k \geq 2$ ,  $v_0 \rightarrow v_k$  is in G (and, hence,  $v_i \rightarrow v_j$  is also in G for all i < j).