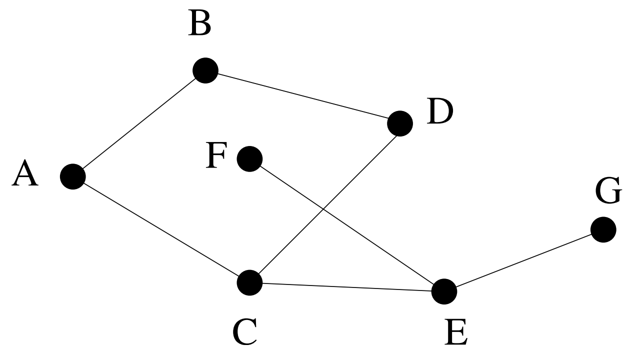
# Problems for Recitation 6

## 1 Graph Basics

Let $G = (V, E)$ be a graph. Here is a picture of a graph.




Recall that the elements of $V$ are called vertices, and those of $E$ are called edges. In this example the vertices are $\{A, B, C, D, E, F, G\}$ and the edges are

$$\{A—B, B—D, C—D, A—C, E—F, C—E, E—G\}.$$

Deleting some vertices or edges from a graph leaves a *subgraph*. Formally, a subgraph of $G = (V, E)$ is a graph $G' = (V', E')$ where $V'$ is a nonempty subset of $V$ and $E'$ is a subset of $E$. Since a subgraph is itself a graph, the endpoints of every edge in $E'$ must be vertices in $V'$. For example, $V' = \{A, B, C, D\}$ and $E' = \{A—B, B—D, C—D, A—C\}$ forms a subgraph of $G$.

In the special case where we only remove edges incident to removed nodes, we say that $G'$ is the *subgraph induced on* $V'$ if $E' = \{(x—y | x, y \in V' \text{ and } x—y \in E\}$. In other words, we keep all edges unless they are incident to a node not in $V'$. For instance, for a new set of vertices $V' = \{A, B, C, D\}$, the induced subgraph $G'$ has the set of edges $E' = \{A—B, B—D, C—D, A—C\}$.

## 2 Problem 1

An undirected graph $G$ has ***width*** $w$ if the vertices can be arranged in a sequence

$$v_1, \ v_2, \ v_3, \ \ldots, \ v_n$$

such that each vertex $v_i$ is joined by an edge to at most $w$ preceding vertices. (Vertex $v_j$ *precedes* $v_i$ if $j < i$.) Use induction to prove that every graph with width at most $w$ is $(w+1)$-colorable.

(Recall that a graph is $k$-*colorable* iff every vertex can be assigned one of $k$ colors so that adjacent vertices get different colors.)

# 3 Problem 2

A **planar graph** is a graph that can be drawn without any edges crossing.

1. First, show that any subgraph of a planar graph is planar.

2. Also, any planar graph has a node of degree at most 5. Now, prove by induction that any graph can be colored in at most 6 colors.