Problems for Recitation 8

Routing in a Beneš Network

In lecture, we saw that the Beneš network has a max congestion of 1; that is, every permutation can be routed in such a way that a single packet passes through each switch. Let's work through an example. A Beneš network of size $N = 8$ is attached.

1. Within the Beneš network of size $N = 8$, there are two subnetworks of size $N = 4$. Put boxes around these. Hereafter, we'll refer to these as the *upper* and *lower* subnetworks.

2. Now consider the following permutation routing problem:

$$\pi(0) = 3 \qquad \pi(4) = 2$$
$$\pi(1) = 1 \qquad \pi(5) = 0$$
$$\pi(2) = 6 \qquad \pi(6) = 7$$
$$\pi(3) = 5 \qquad \pi(7) = 4$$

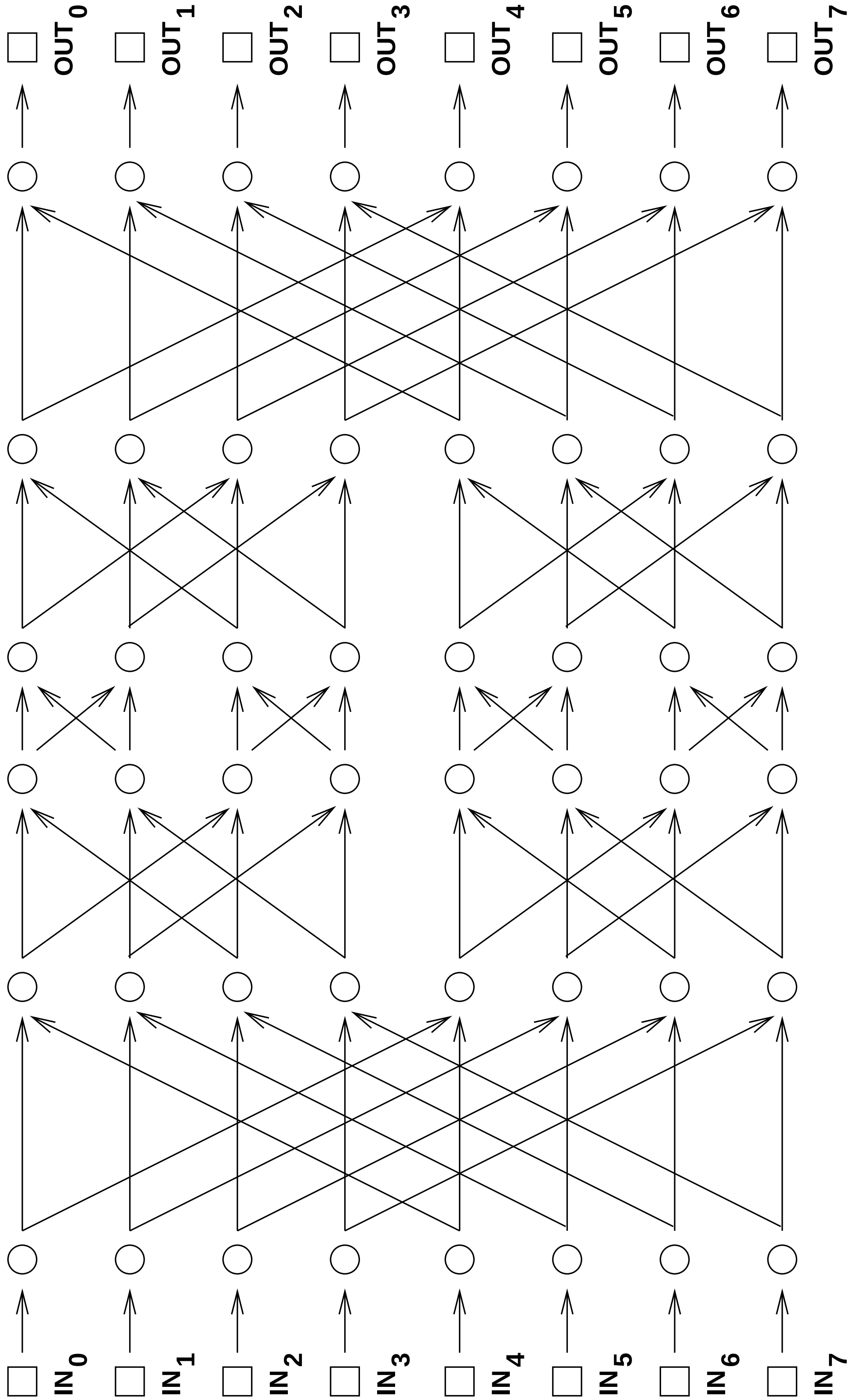
   Each packet must be routed through either the upper subnetwork or the lower subnetwork. Construct a graph with vertices 0, 1, ..., 7 and draw a *dashed* edge between each pair of packets that can not go through the same subnetwork because a collision would occur in the second column of switches.

3. Add a *solid* edge in your graph between each pair of packets that can not go through the same subnetwork because a collision would occur in the next-to-last column of switches.

4. Color (i.e., label) the vertices of your graph red and blue so that adjacent vertices get different colors. Why must this be possible, regardless of the permutation π?

5. Suppose that red vertices correspond to packets routed through the upper subnetwork and blue vertices correspond to packets routed through the lower subnetwork. On the attached copy of the Beneš network, highlight the first and last edge traversed by each packet.

6. All that remains is to route packets through the upper and lower subnetworks. One way to do this is by applying the procedure described above recursively on each subnetwork. However, since the remaining problems are small, see if you can complete all the paths on your own.


OUT_0
OUT_1
OUT_2
OUT_3
OUT_4
OUT_5
OUT_6
OUT_7
IN_0
IN_1
IN_2
IN_3
IN_4
IN_5
IN_6
IN_7

1 Euler tours

(a) Prove that a graph G has an Euler tour if and only if: i) every vertex of G has even degree, and ii) the subgraph obtained after removing all isolated vertices is connected. (An *isolated vertex* is a vertex of degree 0.)

Note that there are two directions to prove!

(b) Come up with a necessary and sufficient condition for the existence of an Euler tour in a *directed* graph. Adapt your proof above to prove that your condition is the right one.