Problem 1. [10 points]

(a) [5 pts] Prove or disprove $\neg(A \wedge B) \Leftrightarrow (\neg A \vee \neg B)$. *Hint*: Use a truth table.

Solution.

A	B	$A \wedge B$	$\neg(A \wedge B)$	$\neg A$	$\neg B$	$\neg A \vee \neg B$
`true`	`true`	`true`	`false`	`false`	`false`	`false`
`true`	`false`	`false`	`true`	`false`	`true`	`true`
`false`	`true`	`false`	`true`	`true`	`false`	`true`
`false`	`false`	`false`	`true`	`true`	`true`	`true`

Comparing the fourth and last colums, we see that the statement is true. *Note:* In fact, this is one of *DeMorgan's Laws.* ■

(b) [5 pts] Translate the following statements from English into propositional logic or vice versa.

1. If $n > 1$, then there is always at least one prime p such that $n < p < 2n$. *Hint*: Let $\text{Prime}(p) := p$ is a prime

   Solution. The domain is $\mathbb{Z}$.

   $$\forall n.(n > 1) \Rightarrow (\exists p.\text{Prime}(p) \wedge (n < p) \wedge (p < 2n))$$

   Note: This is known as *Bertrand's Postulate* ■

2. The domain is $\mathbb{N}$. $\forall m \exists p > m.\text{Prime}(p) \wedge \text{Prime}(p+2)$

   Solution. There are infinitely many primes p such that $p + 2$ is also prime. *Note:* This is known as the *Twin Prime Conjecture* ■

3. Let T be the set of TA's, S be the set of students, and $G(x, y) := x$ grades y's exam

   $$\exists t \in T \forall s \in S.G(t, s)$$

   Solution. One TA will grade all of the exams. *Note:* This is not a famous law, theorem, or conjecture, but would be quite impressive. ■