Properties of relations

Properties of a relation on A:

Reflexivity R is reflexive if

 $\forall x \in A. \ xRx.$

"Everyone likes themselves."

Every node in G has a loop.

Irreflexivity R is irreflexive if

 $\neg \exists x \in A. \ xRx.$

"No one likes themselves."

There are no loops in G.

Symmetry R is symmetric if

 $\forall x, y \in A. \ xRy \ \Rightarrow \ yRx.$

"If x likes y, then y likes x."

If there is an edge from x to y in G, then there is an edge from y to x in G as well.

Antisymmetry R is antisymmetric if

$$\forall x, y \in A. (xRy \land yRx) \Rightarrow x = y.$$

"No pair of distinct people like each other."

There is at most one directed edge between any pair of distinct nodes.

Transitivity R is transitive if

 $\forall x, y, z \in A. (xRy \land yRz) \Rightarrow xRz.$

"If x likes y and y likes z, then x likes z too."

For any walk v_0, v_1, \ldots, v_k in G where $k \geq 2$, $v_0 \rightarrow v_k$ is in G (and, hence, $v_i \rightarrow v_j$ is also in G for all i < j).