Albert R Meyer <meyer.csail@gmail.com>

Comments for MCS book.

1 message

Leonid Levin <lnd@bu.edu> Sun, Jan 19, 2014 at 1:24 PM
To: meyer@csail.mit.edu

: From meyer.csail@gmail.com Tue Dec 10 20:50:00 2013
: I plan to think over your suggestions and make changes
: in response in January after the term is over.

Dear Albert: In case you prefer text e-mail to Web file,
I duplicate the file http://www.cs.bu.edu/fac/lnd/.m/MCS.htm
in this email below. I added some clarifications, still trying not to
burden you with excessive elaboration. But please do not hesitate
to ask me if more details can save your time. Thanks, -Leonid.
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[From lnd@bu.edu Tue Dec 10 20:34:00 2013
Just finished teaching my class `Combinatorial Structures'
based on your book. Enjoyed your book a lot!
(Though `Math for CS' is a funny name, like `physics for housewives'. :-)]
Below I summarize my main points

A. ZFC axioms sec.7.3.2: too many, some minor, hard for kids to remember.
Plainly redundant are: the Paring axiom, the heavy family of Subset Axioms.
The Union axiom can be made redundant by extending Replacement from functions
to bounded relations (which may be even simpler, depending on taste).
Replacement also need not allow more than one free variable.
Also, axioms can be combined in three pairs to be more memorable.
I start with Infinity, the only axiom that unconditionally gives existence
of some sets. (Other axioms only construct them from each other.)
It is paired with its anti-dual Induction axiom.
Choice and Power set, too, seem related: one assures inverse functions
exist, the other assures they form a set. The last pair Replacement and
Extensionality give existence of formula-defined sets and their uniqueness.
(Btw, I do not use "=", only "\in".) All in all the full wording I use is:

1a Infinity: Nonempty S exist with no SOURCES
(members S does not share with any of its members).
1b (anti-dual to 1a) Induction:
Each nonempty S has a SINK (member not sharing members with S).
2a Power-set: For each set S, all functions f:S\to S form a set.
2b Choice: Each surjective function has an inverse.
3a Extensionality: (\forall t (t\in x\iff t\in y))
\then(\forall t (x\in t\iff y\in t).
3b Replacement family (for each ST-defined relation R_c(x,y)):
If each point x has a codomain (bound) Y: y\in Y\if R_c(x,y), then
each set A has an image Y=R(A):y\in Y\iff\exists x\in A R_c(x,y).

Not function-worded axioms (1a,1b,3a,3b) have only minor differences with your version. Here they are in LaTeX:

```
\documentclass[12pt]{article}\begin {document}\newcommand\ex\exists
\newcommand\all\forall\newcommand\then\Rightarrow
\renewcommand\iff\Leftrightarrow\begin{enumerate}\item\begin{description}
\item {Infinity:}\hspace{2pc}$\ex S,s\in S \all x\in S\ex y\in S (x\in y)$
\item {Induction:}     $\neg( \ex S,s\in S \all x\in S\ex y\in S (x\ni y) )$
 \end{description}\item\begin{description}\item {Extensionality:}
 $(\all t (t\in x\iff t\in y))\then (\all t (t\ni x\iff t\ni y)$
 \item {Replacement family} (for each ST-defined relation $R_c(x,y)$):\\
 $(\all x\ex Y(            R_c(x,y)\then y\in Y))\then
  (\all X\ex Y(\ex x{\in}X R_c(x,y)\iff  y\in Y))$
 \end{description}\end{enumerate}\end{document}
```

B. Theorem 11.6.8 need not depend on a specific Mating Ritual, can be stated simply as: "If A is optimal [or better] for B then B is pessimal [or worse] for A." The proof is even more straight from the definition. Then Theorem 11.6.7 follows immediately: While no men propose to sub-optimal women, no man is rejected by a feasible woman.

C. Inductive definition of a planar graph is a bit heavy. It is used to "prove" Euler's Theorem, but leaves a gap anyway without Jordan Theorem (which would take some care even just for polygonal curves). I used the definition of embedding as given by cyclic order of semi-edges at each node (see footnote 1). I used Euler formula as a DEFINITION of planarity (after an intuitive exposition of why it makes sense. I also gave them the Fary's theorem: any such graph can be put in a plane with straight edges).
Footnote 1: E.g, Lando, Zvonkin, Graphs on Surfaces and their Applications:
Embedded graph given by two permutations on half-edges: M(x) =/= x = M^2(x) matching each semi-edge with its other half, and P (shifting semi-edges in each node circularly). Orbits of P are nodes, of M - edges, of P*M - faces.

D. Chapter 13 nicely written, but I gave them a slightly simpler form of Stirling's Formula: n!= (n/e)^n \sqrt {2\pi n + t_n}, where t_n is in [1, 1.05] for n>30. For all positive n, t_n=\pi/3 +o(1) is in [1, 1.11] (or precisely in [\pi/3, e^2-2\pi = t_1]).

E. On magic trick probl.14.42: Showing the hidden (turned back up) card H in the middle of the three open cards hints that the order of cards is informative, spoiling the surprise. I would allow the placement of the hidden card only at the end (left or right) of the three open cards, or showing no open cards at all: sometimes position of stars is so favorable to magic that the magician needs no information at all ! :-). (Then the trick seems nicer (and tight) than the one in the main text.)

These 12 allowed orders suffice to show the rank of H with a special treatment of kings (K). If chosen cards include K of spades (K0), then no open cards are shown. If H is K, its rank is shown as one of an open non-K card (and treated as its extra suit): K1 (of clubs) as their lowest (J, Junior) rank; K3 (hearts) as the highest (S Senior); K2 (diamond) as the Middle M or the rank of multiple open cards (including K1 in J, K3 in S). So, if J=M=S, it has 7 possible suits (including 3 K's).

The Magician only needs to guess the suit of H.
But this is perfect matching in a bipartite degree(4)-constrained graph.

The graph breaks into small connected components distinguished by
their sets R of different non-K ranks of the 4 chosen cards. Details:

a. For |R|=4: the relative rank of H reflects
the sum \mod 4 of the chosen non-K suits.
b. |R|=1: matching of (7\choose 4) 4-sets of 4 suits of J and 3 of K
to their (7\choose 3) 3-subsets (a cute exercise itself; e.g., one
can shrink each 4-subset of Z_7 to a reflection of its complement.).
c. |R|=3 breaks into 3 components: one for each two-suit rank, say J.
Then J has 5 possible suits (including K1) of which two are chosen.
I call them even if adjacent mod 5. They are open if parity matches
the one of the sum sm of S,M suits. Then H is S or M depending on sm.
Else H is the one to the left of two adjacent non-chosen cards in J.
d. |R|=2, both ranks two-suit: similar to the previous case.
e. |R|=2, one of the ranks is single-suit C: this is 4-edge-coloring
by C of (6\choose 3)\to (6\choose 2) subset graph, a cute exercise itself.
Colorings abound: (1): add blank edges to make uniform degree 4, and
alternate red/black on non-blank edges of any maximal Euler tour.
Then (2): alternate Major/Minor tint on each unicolor component.

(E.g., an edge is red if suit of H is to the left of the closest open
card, black if right; in a tie, red if in {0,1,2} of the Z_6, else black.
Tint is Minor in these patterns (in Z_6 or -Z_6 order of suits):
*O*O*K, *O*ON*, *OLH**, *O*LO*, *OR*H*. Here *s are all un-selected
suits in the rank, O: open, H: hidden, K: Kings (0,1 in Z_6), N: non-K,
L: left (1,2,3) in red edges and (4,5,0) in black, R: non-L.)

Footnote 2: I would also place the cards on the table, the assistant going
away before the magician can look. Then the viewers would not suspect
the information is conveyed in a way the assistant holds the cards :-).
Footnote 3: In Russia, the special treatment would be for Queen of Spades,
not King. She is considered to be especially malicious ... :-).