Problem 1. [10 points] Consider the following relation:

$$R = \{(x, y) : x + y = 0, x, y \in \mathbb{R}\}.$$

Which of the following properties holds for R? If it has the property, prove it. If not, provide a counterexample.

(a) [2 pts] Symmetry.

Solution. Yes. $x + y = 0 \Leftrightarrow y + x = 0$ ■

(b) [2 pts] Antisymmetry.

Solution. No. It is symmetric. ■

(c) [2 pts] Irreflexivity.

Solution. No. Counterexample: $x = 0$ ■

(d) [2 pts] Transitivity.

Solution. No. Let $x = 1, y = -1$ and $z = 1$. Then xRy and yRz but it is not true that xRz. ■

(e) [2 pts] The property of being an equivalence relation.

Solution. No. It is not reflexive, since $x + x \neq 0$ for $x \neq 0$ ■