Tom Leighton and Ankur Moitra

Problems for Recitation 17

The ***(ordinary) generating function*** for a sequence $\langle a_0, a_1, a_2, a_3, \ldots \rangle$ is the power series:

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

Find closed-form generating functions for the following sequences. Do not concern yourself with issues of convergence.

(a) $\langle 2, 3, 5, 0, 0, 0, 0, \ldots \rangle$

(b) $\langle 1, 1, 1, 1, 1, 1, 1, \ldots \rangle$

(c) $\langle 1, 2, 4, 8, 16, 32, 64, \ldots \rangle$

(d) $\langle 1, 0, 1, 0, 1, 0, 1, 0, \ldots \rangle$

(e) $\langle 0, 0, 0, 1, 1, 1, 1, 1, \ldots \rangle$

(f) $\langle 1, 3, 5, 7, 9, 11, \ldots \rangle$

Problem 2

Suppose that:

$$\begin{aligned} f(x) &= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots \\ g(x) &= b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 + \cdots \end{aligned}$$

What sequences do the following functions generate?

(a) $f(x) + g(x)$

(b) $f(x) \cdot g(x)$

(c) $f(x)/(1 - x)$

Problem 3

There is a jar containing n different flavors of candy (and lots of each kind). I'd like to pick out a set of k candies.

(a) In how many different ways can this be done?

(b) Now let's approach the same problem using generating functions. Give a closed-form generating function for the sequence $\langle s_0, s_1, s_2, s_3, \ldots \rangle$ where s_k is the number of ways to select k candies when there is only $n = 1$ flavor available.

(c) Give a closed-form generating function for the sequence $\langle t_0, t_1, t_2, t_3, \ldots \rangle$ where t_k is the number of ways to select k candies when there are $n = 2$ flavors.

(d) Give a closed-form generating function for the sequence $\langle u_0, u_1, u_2, u_3, \ldots \rangle$ where u_k is the number of ways to select k candies when there are n flavors.