Figure 1: Integral bounds for two sums

Problem 1. [20 points] Bounding sums with integrals

Assume n is an integer larger than 1. Circle all the correct inequalities below.

Explanations are not required, but partial credit for wrong answers will not be given without them. You may find the graphs in Figure 1 helpful.

- $$\sum_{i=1}^{n} \ln(i+1) \leq \ln 2 + \int_{1}^{n} \ln(x+1)dx$$

- $$\sum_{i=1}^{n} \ln(i+1) \leq \int_{0}^{n} \ln(x+2)dx$$

- $$\sum_{i=1}^{n} \frac{1}{i} \geq \int_{0}^{n} \frac{1}{x+1}dx$$

- $$\sum_{i=1}^{n} \frac{1}{i} \leq 1.5 + \int_{3}^{n} \frac{1}{x}dx$$

Solution. The 2nd and 3rd inequalities hold. ∎

1