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B24 241

1. a> RREF
b> REF
c> –
d> REF
e> –
f> RREF

2.

$$\begin{matrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{matrix}$$

• $R_2 \leftrightarrow R_1$

• $R_3 = R_3 - 3R_1$

• $R_1 = R_1/3$

$$\begin{matrix} 1 & -7/3 & 8/3 & -5/3 & 8/3 & 3 \\ 0 & 3 & -6 & 6 & 4 & -5 \\ 0 & -2 & 4 & -4 & -2 & 6 \end{matrix}$$

$R_2 = R_2/3$

$R_3 = R_3 + 2R_1$

$$\begin{matrix} 1 & 0 & -2 & 3 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{matrix}$$

Then make the elements above leading ones 0

3.

$$A|I = \left[\begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 4 & 7 & 2 & 0 & 1 & 0 \\ 6 & 18 & 5 & 0 & 0 & 1 \end{array}\right]$$

$R_1 = R_1/2$

$$\left[\begin{array}{ccc|ccc} 1 & 3/2 & 1/2 & 1/2 & 0 & 0 \\ 4 & 7 & 2 & 0 & 1 & 0 \\ 6 & 18 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & -5 & 0 & -2 & 1 & 0 \\ 6 & 18 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 3/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 2 & -3 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1/2 & 1/2 & -3/2 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 2 & 15 & -9 & 0 \end{array}\right]$$ $R_3 \to \frac{R_3}{2}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1/2 & 1/2 & -3/2 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 15/2 & -9/2 & 1/2 \end{array}\right]$$ $R_1 \to R_1 - \frac{R_3}{2}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/4 & 3/4 & -1/4 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 15/2 & -9/2 & 1/2 \end{array}\right]$$ A^{-1}

4.

$$\left[\begin{array}{ccc|ccc} 2 & 4 & 6 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 6 & 9 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1/2 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1/2 \\ 0 & 0 & 0 & -1/2 \\ 0 & 0 & 0 & 1/2 \end{array}\right]$$

Not I

$\therefore$ Inverse not possible.

5. $|A| = 2(27-40) - 4(9-35) + 6(8-21)$

$= 2(-13) - 4(-26) + 6(-13)$

$= -36 + 104 - 78$

$$\begin{vmatrix} \cancel{2} & \cancel{4} & \cancel{6} \\ 1 & 3 & 5 \\ 7 & 8 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 7 & 8 \end{vmatrix}$$

$= 8 - 21 = -13 \neq 0$

$\therefore$ Rank is 2 ($|A|$ of 2×2 is non zero)

6. $|A| = 0$

$d(6-2d) - 0(3-d) + 1(2-2) = 0$

$d(6-2d) = 0$

$d = 0$ or $d = 3$

for $d =$
$$\begin{bmatrix} 1 & 2 & d \\ d & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$
$R_1 \leftrightarrow R_2$

$R_2 \to R_2 - dR_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & d \\ 0 & -2d & 1-d^2 \\ 0 & 0 & 3-d \end{bmatrix}$$

if $(d = 3) \to$ Last row $\to [0\ 0\ 0]$

if $(d = 0) \to$ sec row $\to [0, 0, 0]$

$\hookrightarrow$ by doing op using $R_2 \to R_2 - R_3/3$

for $a \neq 3$ or $a \neq 0$ $\rightarrow$ Rank = 3

$a = 3$, $a = 0$ $\rightarrow$ Rank = 0

7.
$$\begin{bmatrix} 2 & 0 & 4 & 0 & 6 \\ 0 & 5 & -5 & 0 & 10 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 3 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \rightarrow R_1 / 2$

$R_2 \rightarrow R_2 / 5$

3 non zero rows $\rightarrow$ rank = 3

b) Col 1, 2, 4 are linearly independentdant

$\therefore$ Rank = 3

8. $$R\left(\frac{\pi}{2}\right) = \begin{vmatrix} \cos\left(\frac{\pi}{2}\right) & -\sin\left(\frac{\pi}{2}\right) \\ \sin\left(\frac{\pi}{2}\right) & \cos\left(\frac{\pi}{2}\right) \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$$

i>

$$\left[\begin{array}{cc|cc} 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

ii> $A^T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (Exchange Rows & Col's)
