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IC-114 (Linear Algebra)

Tutorial-1

1. Find the inverse of the matrix $A = \begin{bmatrix} 4 & 7 & 2 \\ 3 & 6 & 1 \\ 5 & 9 & 4 \end{bmatrix}$ using the adjoint method.

2. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Compute the commutator $[A, B] = AB - BA$ and verify whether $[A, B]$ is nilpotent matrix of index 2 (i.e., $[A, B]^2 = 0$ but $[A, B] \neq 0$).

3. Prove or disprove: For any two square matrices A and B, the anti-commutator $\{A, B\} = AB + BA$ is always symmetric.

4. Let $D = \text{diag}(a, b, c)$ be a diagonal matrix. Compute D^3. What will be the general formula for D^3 and e^D?

5. Compute the direct product $A \otimes B$, where:
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}.$$

6. Prove that if $A(x)$ is an invertible matrix, the derivative of $A^{-1}(x)$ is given by:
$$\frac{d}{dx}(A^{-1}(x)) = -A^{-1}(x)\frac{dA}{dx}A^{-1}(x).$$

7. Evaluate the line integral of a matrix-valued function $A(x) = \begin{bmatrix} x^2 & \sin(x) \\ e^x & \cos(x) \end{bmatrix}$ along the path $x(t) = t^2, \ t \in [0, 1]$.

8. Show that the diagonal elements of a skew-symmetric matrix are equal to 0.

9. Let A and B be $n \times n$ Hermitian matrices. Answer true or false (also justify your answer):
   - If $AB = 0$, then $A = 0$ or $B = 0$.

10. Determine if the matrix A is singular or non-singular:
$$A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 7 & 2 \\ 1 & 5 & 3 \end{bmatrix}.$$

11. Find the real values of x and y such that A is an orthogonal matrix. Let matrix A be a 3x3 matrix:

$$A = \begin{pmatrix} x & 2 & 3 \\ 4 & y & 1 \\ 1 & 5 & 6 \end{pmatrix}.$$

12. If A be a unitary matrix of order 2 such that $|A| = 1$, show that it must be of the form $\begin{bmatrix} a & -b \\ \bar{b} & \bar{a} \end{bmatrix}$ where $a\bar{a} + b\bar{b} = 1$.

13. Show that if a triangular matrix is normal, then it is a diagonal matrix.

14. Find the eigenvalues and eigenvectors of the given matrix: $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 3 & 20 \end{bmatrix}.$