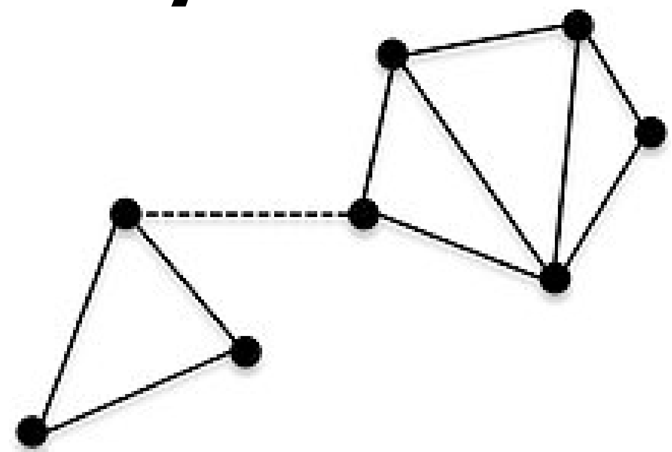
DAY 11

Connectivity

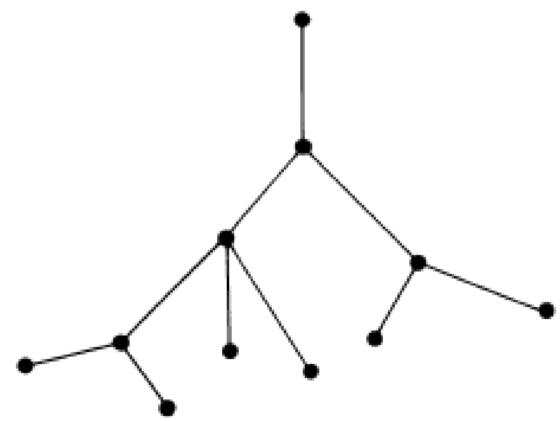
Edge Connectivity

- The Number of edges in the smallest cut-set is called the edge-connectivity of any graph.
- It is the minimum no. of edges whose removal reduces the rank of the graph by 1.
- The edge-connectivity of a tree is 1.

Notation – $\lambda(G)$



Edge connectivity = 1



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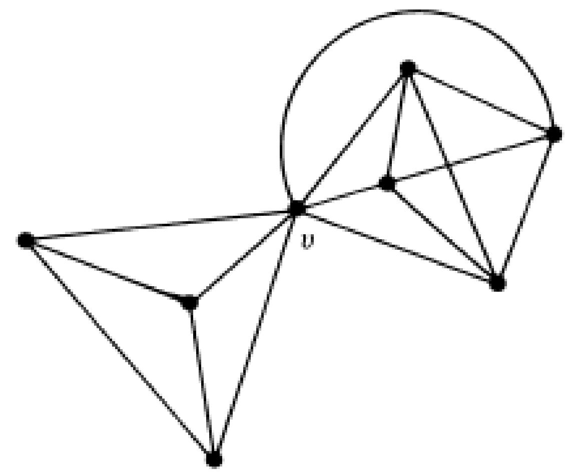
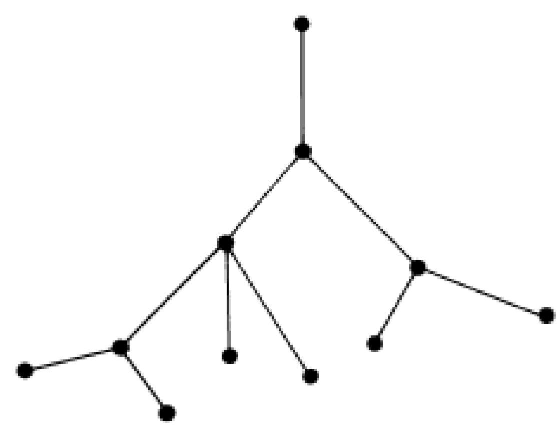
Vertex Connectivity



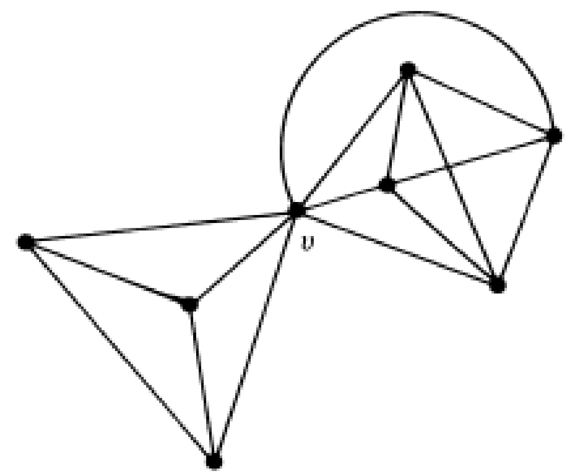
- It is the minimum no. of vertices whose removal from the graph makes the remaining graph disconnected.
- The vertex-connectivity of a tree is 1.



Separable Graph

- A connected graph is said to be separable if it’s vertex connectivity is 1. Otherwise it is non-separable.

- In a separable graph, a vertex whose removal disconnects the graph is called a cut-vertex or articulation point.



Vertex connectivity = 1
V is the cut-vertex.

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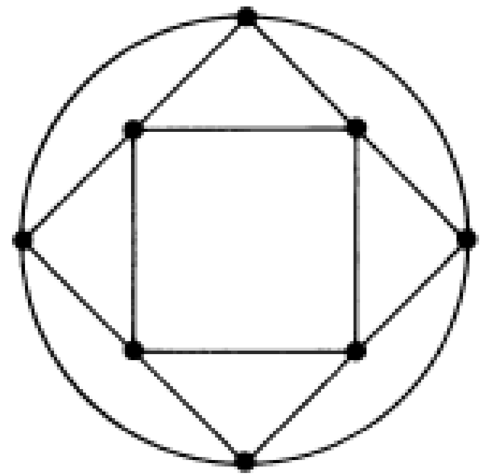

From the given graph, find edge-connectivity and vertex-connectivity.

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THEOREM 4-8

The edge connectivity of a graph G cannot exceed the degree of the vertex with the smallest degree in G.

THEOREM 4-9

The vertex connectivity of any graph G can never exceed the edge connectivity of G.

For any connected graph G,

$$K(G) \leq \lambda(G) \leq \delta(G)$$

Where,

Vertex connectivity ($K(G)$),
edge connectivity ($\lambda(G)$),
minimum number of degrees of G($\delta(G)$).

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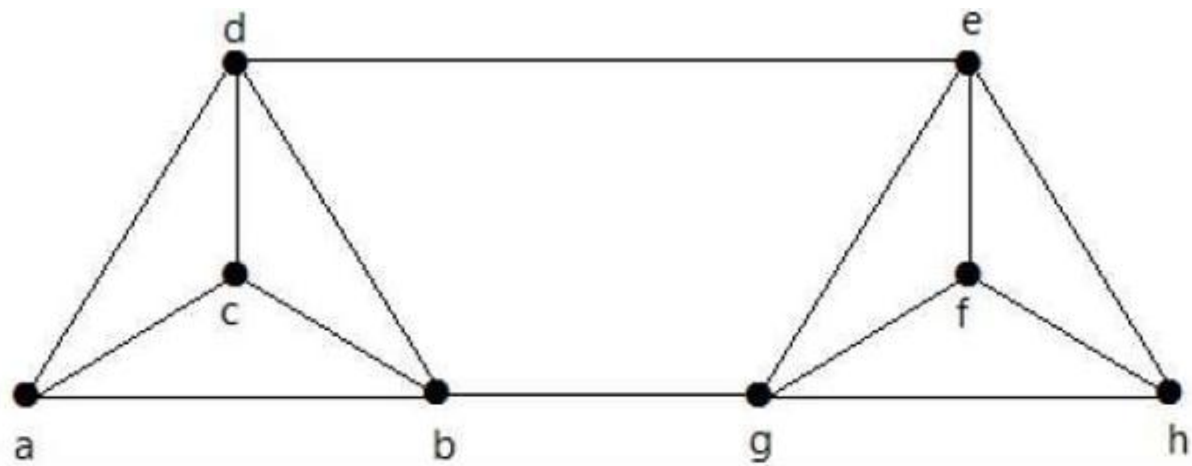


From the graph,
Minimum degree $\delta(G) = 3$

Deleting the edges {d, e} and {b, g}, we can disconnect G.

Therefore, edge-connectivity, $\lambda(G) = 2$

Since, $K(G) \leq \lambda(G) \leq \delta(G)$

vertex connectivity $K(G) = 2$ (delete vertex b and d)

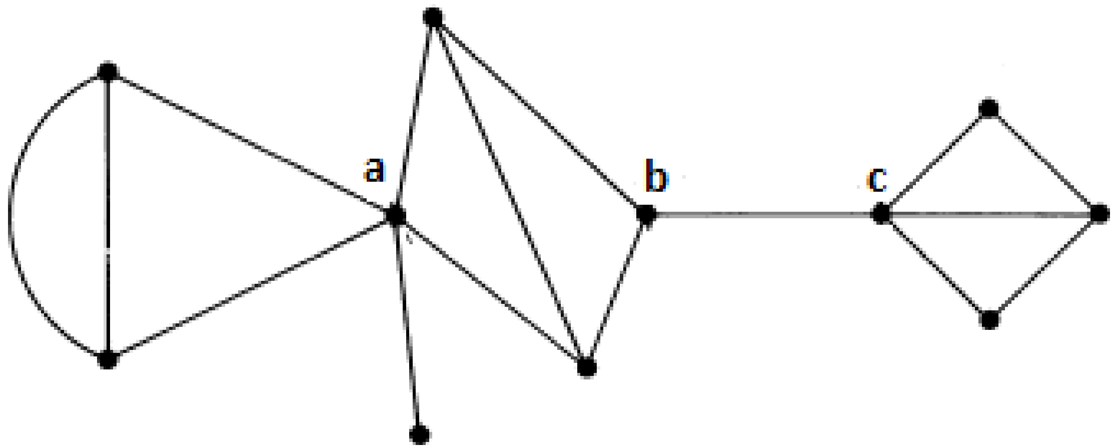
1-Isomorhism

A separable graph consists of two or more non-separable sub-graphs. Each of the largest non-separable sub-graph is called a block.

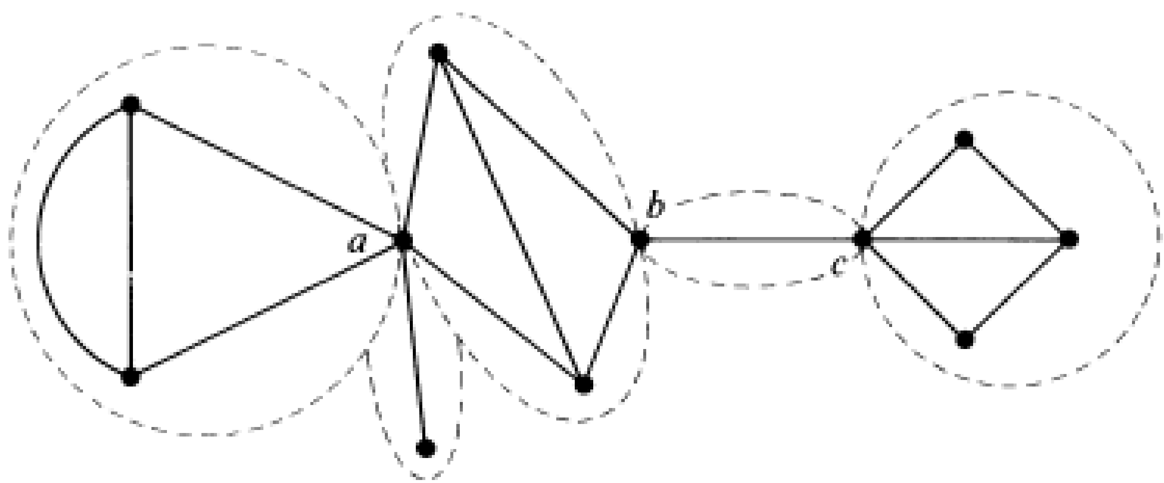
Whereas, in a disconnected graph, each of the connected sub-graphs are known as components.

1-Isomorhism

G

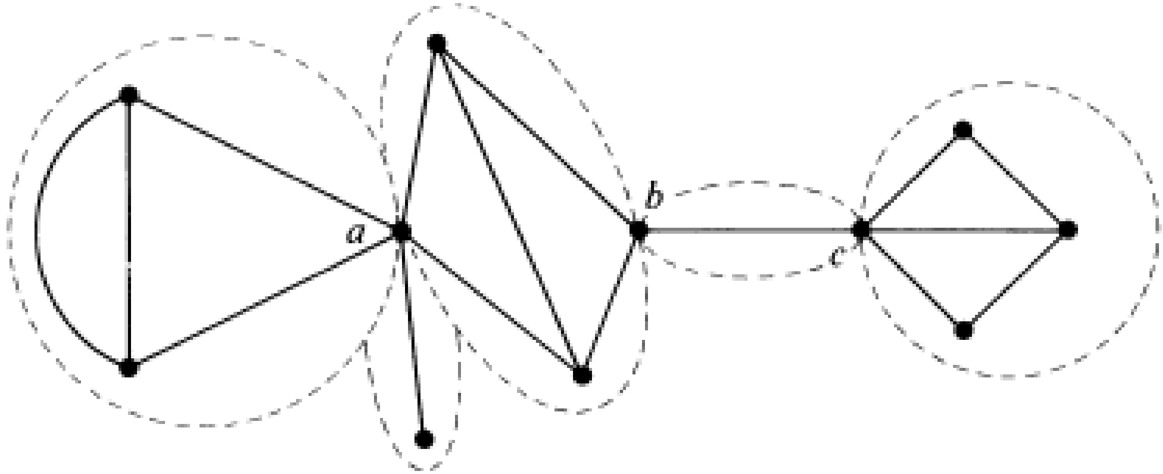


G1

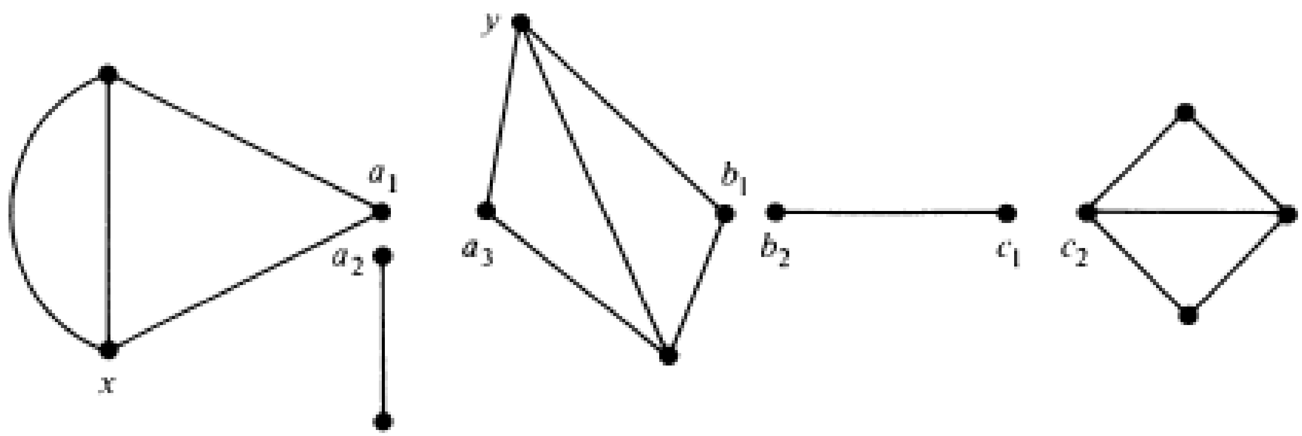


1-Isomorhism

G1



G2



1-Isomorhism

Now, if we visually compare the disconnected graph G3 with G2, we will find that:

- They are not isomorphic to each other (Do not have same no. of vertices)

- But the blocks of G2 are isomorphic to the components of G3.

Two graphs G_1 and G_2 are said to be *1-isomorphic* if they become isomorphic to each other under repeated application of the following operation.

Operation 1: "Split" a cut-vertex into two vertices to produce two disjoint subgraphs.

1-Isomorhism

THEOREM 4-14

If G_1 and G_2 are two 1-isomorphic graphs, the rank of G_1 equals the rank of G_2 and the nullity of G_1 equals the nullity of G_2.

Please check the above theorem with appropriate diagram

2-Isomorhism

In case of 2-connected graphs (i.e. graphs having vertex connectivity as 2), two graphs are said to be 2-isomorphic after undergoing operation 1, or operation 2 or both operations any number of times.

Operation 2: "Split" the vertex x into x_1 and x_2 and the vertex y into y_1 and y_2 such that G is split into g_1 and $\bar{g}_1$. Let vertices x_1 and y_1 go with g_1 and x_2 and y_2 with $\bar{g}_1$. Now rejoin the graphs g_1 and $\bar{g}_1$ by merging x_1 with y_2 and x_2 with y_1. (Clearly, edges whose end vertices were x and y in G could have gone with g_1 or $\bar{g}_1$, without affecting the final graph.)

2-Isomorhism

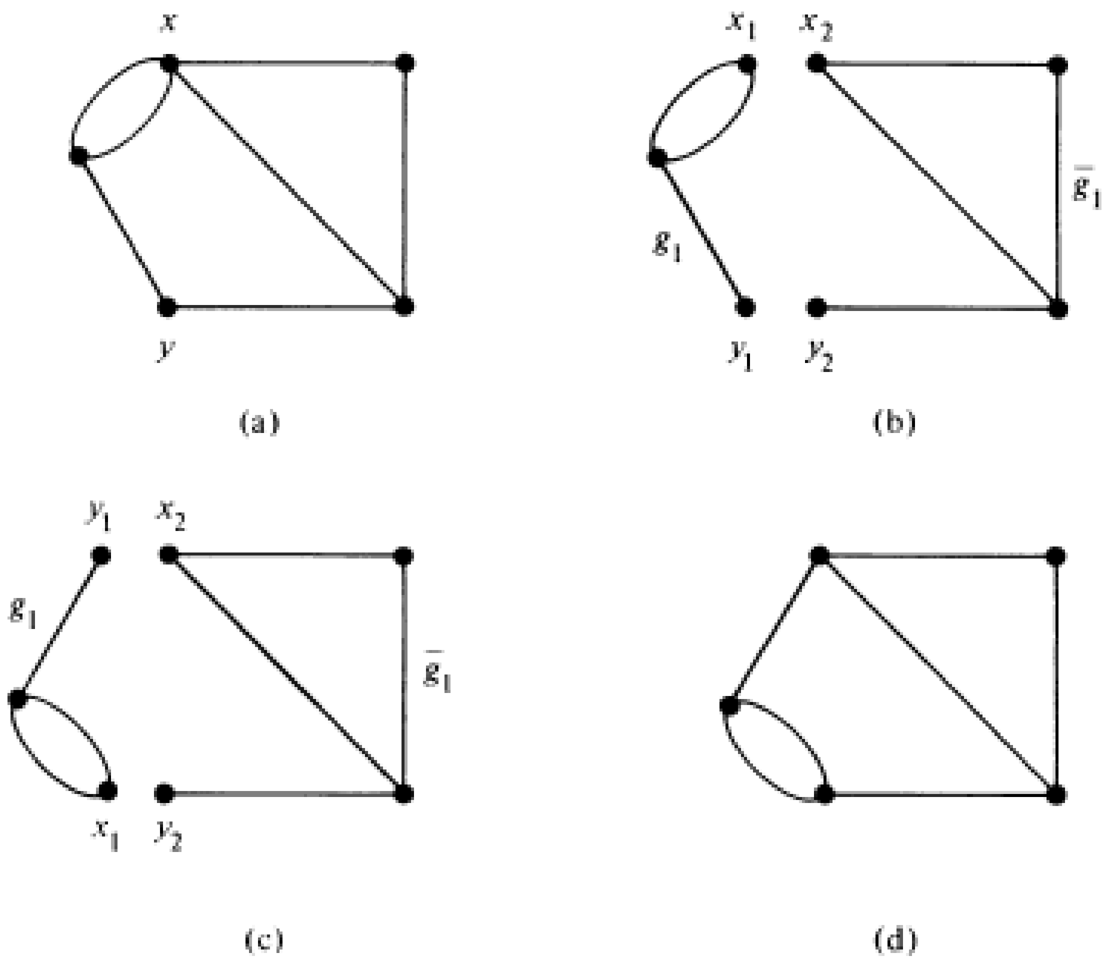
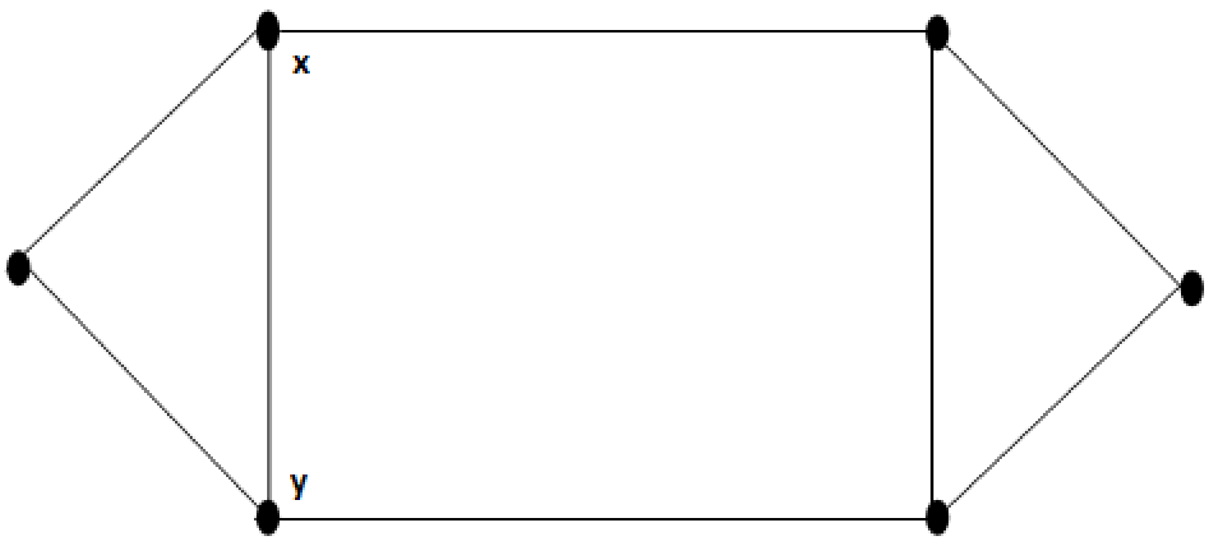


Fig. 4-11 2-isomorphic graphs (a) and (d).

2-Isomorhism



Please check the 2-Isomorhism concept in the above diagram