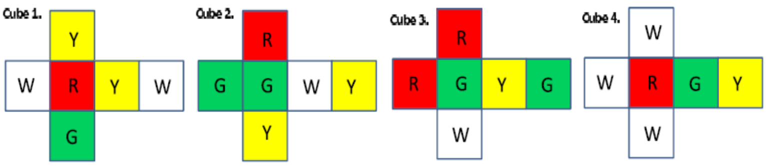
3rd Semester End-Term Examination, 2021
Subject: - INTRODUCTION TO GRAPH THEORY
Paper Code: - UCS03B06 (UG) / UCS03B05 (IIITA)

Total Marks:-50 **Time: 2:00 hrs**

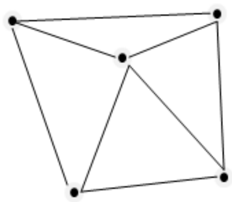
Attempt all the questions.

1. a) Discuss “handshaking di-lemma” with proper diagram.
   b) Show that – “a complete graph is a regular graph but not all regular graphs are complete”.
   c) Given 4 cubes whose 6 faces are coloured with R, G, Y, and W. Is it possible to stack the cubes one on top of another to form a column such that no colour appears twice on any of the 4 sides of this column? Explain your answer.



[2 + 2 + 6 = 10]

2. a) What is Chromatic Polynomial? Find out the Chromatic Polynomial for the given graph with $\lambda = 7$.



   b) Define 1-isomorphism and 2-isomorphism with proper diagram.

[(1 + 5) + (2 + 2) = 10]

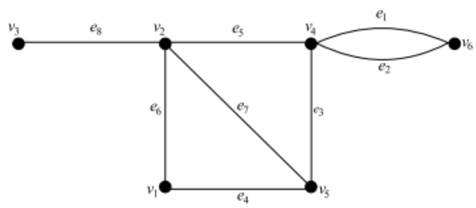
3. a) What are the steps to detect planarity?
   b) Define thickness and crossing.
   c) Show that a complete graph of four vertices is self-dual.

[3 + 3 + 4 = 10]

4. a) Define strongly connected and weakly connected digraphs with diagram.
   b) Show the relation matrix for X = {5, 2, 7, 9, 6, 3, 1} and R = ‘greater than equal to’.
   c) Find the Prufer decoding sequence of the following sequence and reconstruct the tree.
   (3,3,4,1,5,5,1,6,6,6,2,1,9,9,3)

[3 + 2 + (3 + 2) = 10]

5. a) Define: embedding and infinite region.
   b) Find the ***fundamental circuit matrix*** for the following graph:



   c) What is arborescence?

[(2 + 2) + 4 + 2 = 10]

National Institute of Technology

Name of Examination : End-Term Examination.

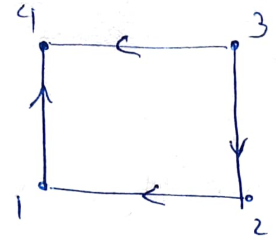
Name of Subject : Graph Theory. Subject Code : UC503B11

Name of Student : Aditya Kiran Pal

Enrollment no : 20UCS119 Section : A

Branch : Computer Science & Engineering Semester : 3rd Sem.

Q. 1. (a) For a digraph $G = (V(G), E(G))$, sum of all of the out-degrees in a graph is equal to the sum of the in-degrees in a graph

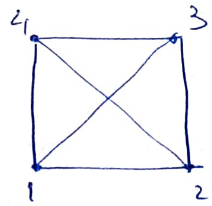


Σ out degree $= 1 + 1 + 2 + 0 = 4$

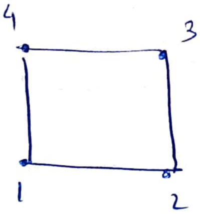
Σ in degree $= 1 + 1 + 0 + 2 = 4$

Q. 1. (b) In a complete graph, each vertex is connected to every other vertex. Degree of each vertex $= n - 1$

$\therefore$ Complete graph is a regular graph.



On the other hand, in a regular graph, degree of each vertex is same, but each edge might not be connected to every other edge.



Q.10 (c) Yes, it is possible to stack 4 cubes whose 6 faces are coloured with R, G, Y, W such that no colour appears twice on any of the 4 sides of the column.

Steps to solve the problem :

(i) Put every color as a separate vertex.

(ii) Join the vertices in such a way that each edge represents the opposite faces of a cube and the name of edge is represented by the number of the cube.

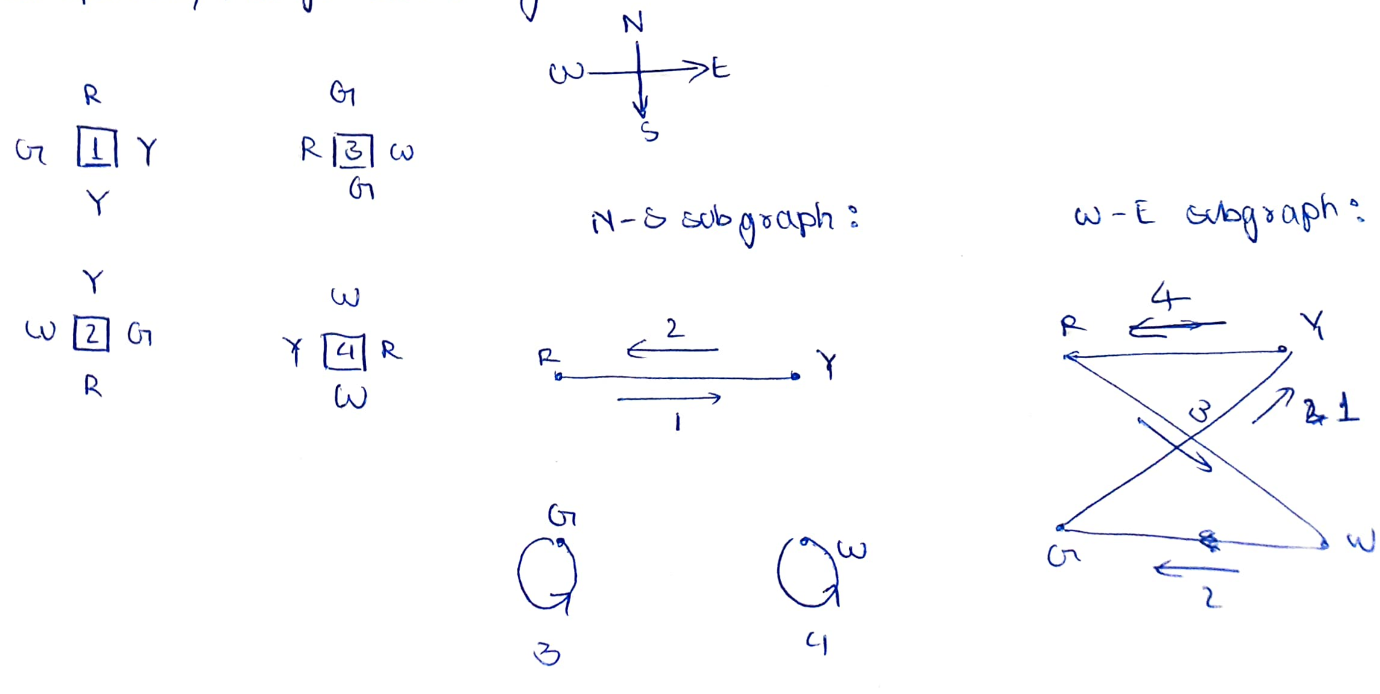
(iii) Check for the degree of a vertex which must be equal to the total occurences of a colour in all four cubes.

(iv) From the completed graph find two edge-disjoint sub-graphs such that : one is facing north-south and other is facing east-west.

(v) Since every edge represents two opp faces ∴ N-S subgraph will represent 8 faces and E-W sub graph will represent 8 faces.

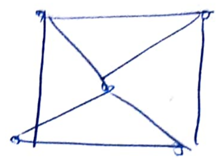
(vi) Each of the 4 edges in a sub-graph has a label 1, 2, 3, or 4, and no colour will appear in any of the four sides if and only if every edge in the subgraph has degree 2.

Graphically, we get something like this :



Q. 2. (a) For a given graph G, the number of ways of colouring the vertices with x or fewer colors is denoted by $P(G, x)$ is called chromatic polynomial of G (x)

$$P_5(7) = \sum_{i=1}^{5} c_i\,(7_i)$$



$$\Rightarrow c_1\,{}^7C_1 + c_2\,{}^7C_2 + c_3\,{}^7C_3 + c_4\,{}^7C_4 + c_5\,{}^7C_5$$

$$= c_1\binom{7}{1} + c_2\binom{7}{2} + c_3\binom{7}{3} + c_4\binom{7}{4} + c_5\binom{7}{5}$$

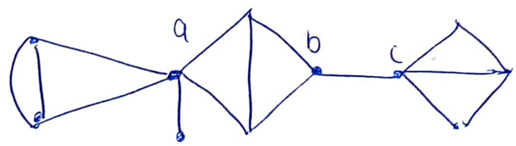
$$= \lambda(\lambda-1)(\lambda-2) + 2\lambda(\lambda-1)(\lambda-2)(\lambda-3) + \lambda(\lambda-1)(\lambda-2)(\lambda-3)(\lambda-4)$$

$$= \lambda(\lambda-1)(\lambda-2)(\lambda^2 - 5\lambda + 7)$$

$$= 7 \times 6 \times 5\,(49 - 35 + 7) = 4410$$

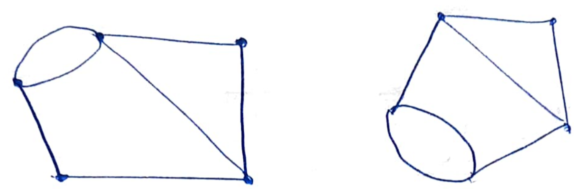
Q. 2(b) 1 - Isomorhism :

A separate graph consists of two or more non-separable sub-graph's each of the largest non-separable sub-graph is called a called a block whereas in a disconnected graph, each of the connected sub-graph are known as component.



2 - Isomorphism :

In case of 2-connected graphs, two graphs are said to be 2-isomorphic after undergoing operation 1 or operation 2 or both operations any number of times.



2-isomorphic graphs.

Q.3. (a) Steps to Detect Planarity :

Step 1 : Since a disconnected graph is planar if and only if each of its components is planar, we need to consider only one component at a time. Also, a separable graph is planar if and only if each of its blocks is planar.

$$G = \{G_1, G_2, \ldots G_k\}$$

where G_i is a non-separable block of G, we have to test each G_i for planarity.

Step 2 : Remove all self-loops

Step 3 : Eliminate all parallel

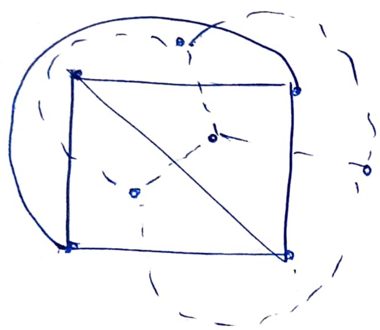
Step 4 : Eliminate all edges in series.

Q.3.(b) The least no. of planar sub-graphs whose union is the given graph G, is called the thickness of a graph.

e.g. In a printed circuit board, the number of insulation layers necessary is the thickness of the corresponding graph.

Crossing : Crossing is the edge intersection in a planar representation of a graph, and crossing number is the number of edge intersection of a graph.

Q.3.(c)



G^*

4 vertices

6 edges

Now, no. of degrees of every vertex is 3

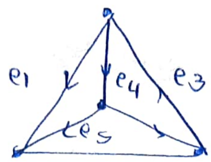
i.e, the graph is isomorphic

Also, For being self dual $e = 2V - 2$

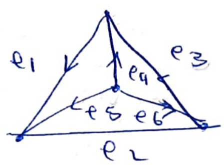
$6 = 2(4) - 2$

Hence proved.

Q.4.(a) A digraph G is said to be strongly connected if there is at least one directed path from every vertex to every other vertex.



A digraph G is said to be ~~strongly~~ weakly connected if ~~there is at least on~~ its corresponding undirected graph is connected but G is not strongly connected.



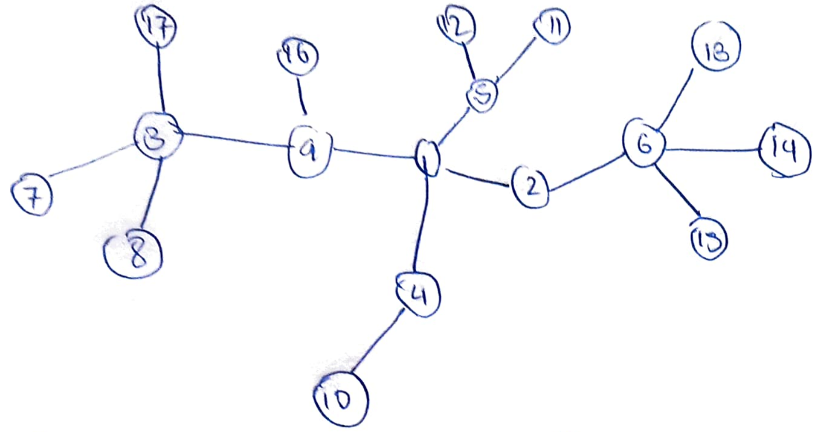
Q.4.(b) ~~A drawing of a geometri~~

$X = \{5, 2, 7, 9, 6, 3, 1\}$

R = greater than equal to

$$\begin{array}{c|c} & \begin{matrix} 5 & 2 & 7 & 9 & 6 & 3 & 1 \end{matrix} \\ \begin{matrix} 5 \\ 2 \\ 7 \\ 9 \\ 6 \\ 3 \\ 1 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Q.4. (c) The tree for the above encoding sequence
(3, 3, 4, 1, 5, 5, 1, 6, 6, 6, 2, 9, 9, 3)



~~{3, 3, 3, 9, 4, 1, 5~~

length = 15, vertices = 17

n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17

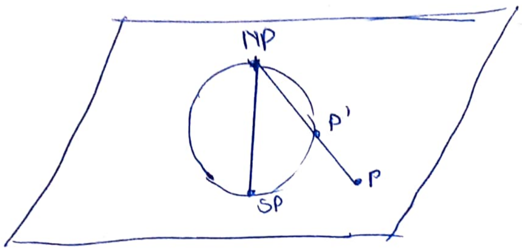
PE = {3, 3, 4, 1, 5, 5, 1, 6, 6, 6, 2, 1, 9, 9, 3}

7 8 10 4 11 12 5 13 14 15 6 2 10 5 7

Decoding sequence
(7, 8, 10, 9, 11, 12, 5, 13, 14, 15, 6, 2, 1, 16, 9, 7)

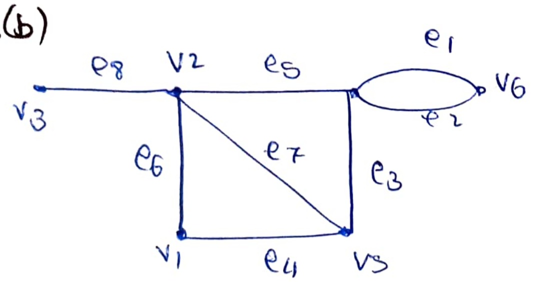
Q. 5. (a) A drawing of a geometric representation of the graph on any surface such that no edges intersect is called embedding.

Embedding on a sphere is accomplished by stereographic projection of a sphere on a plane. We will then have a construction as follows :-



From the construction, it is clear that any graph that can be embedded on a plane can also be embedded in a sphere.

Q.5.(b)

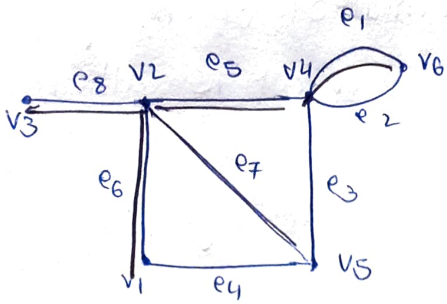


$B = [B_{ij}]$

Here, 4 circuit are $\{e_1, e_2\}$, $\{e_3, e_5, e_7\}$, $\{e_6, e_4, e_7\}$, $\{e_3, e_4, e_6, e_5\}$

Thus, its circuit matrix is 4×8 matrix.

$$B(G) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$



The black line shows the spanning tree

Fundamental circuit matrix :

$$
\begin{array}{cccccccc}
e_2 & e_3 & e_4 & e_1 & e_5 & e_6 & e_7 & e_8
\end{array}
$$
$$
\left[\begin{array}{ccc|ccccc}
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 6 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 0
\end{array}\right]
$$

Q. 5. (c) Arborescence : A digraph G is said to be an aborescence if

1. G contains no circuit - either directed nor semicircuit.
2. In G there is precisely one vertex v of zero in-degree