S$_3$ (ALL): ALL

B.Tech. 3rd Semester Mid-Term Examination - 2021

Name of Subject: Engineering Mathematics-III/Mathematics - III

Subject Code: UME03C12/UEE03C13/C16/UCS03B02/C10/ UEI03C13/UPE03C14/UCH03C17/UBE03C15 UCE03C14/ UEC03B07/UCS03C01

Full Marks: 20 **Time: 1 hour**

Sent the answer script PDF in this email: nita.ma.cse.a@gmail.com

Symbols used here have their usual meanings

Group A

Answer all the following questions **Marks: 10**

1. The probability mass function of a random variable X is zero except at the points $\boldsymbol{x = 0, 1, 2}$. At these points it has the values $\boldsymbol{p(0) = 3c^3}$, $\boldsymbol{p(1) = 4c - 10c^2}$ and $\boldsymbol{p(2) = 5c - 1}$ for some $\boldsymbol{c} > 0$. Describe the distribution function. **[2]**

2. Given the following table:

$\boldsymbol{x}$	$\boldsymbol{-3}$	$\boldsymbol{-2}$	$\boldsymbol{-1}$	$\boldsymbol{0}$	$\boldsymbol{1}$	$\boldsymbol{2}$	$\boldsymbol{3}$
$\boldsymbol{p(x)}$	$\boldsymbol{0.05}$	$\boldsymbol{0.10}$	$\boldsymbol{0.30}$	$\boldsymbol{0}$	$\boldsymbol{0.30}$	$\boldsymbol{0.15}$	$\boldsymbol{0.10}$

Find the values of $\boldsymbol{E(X)}$ and $\boldsymbol{E(4X+5)^2}$. **[2]**

3. In a factory machines A and B are producing springs of the same type. Of these production, machines A and B produces 5% and 10% defective springs, respectively. Machines A and B produces 40% and 60% of the total output of the factory. One spring is selected at random and it is found to be defective. What is the possibility that this defective spring was produced by machine A? **[3]**

4. A continuous random variable $\boldsymbol{X}$ follows the probability law: $\boldsymbol{f(x) = Ax^2}$, $\boldsymbol{0 \le x \le 1}$, then find the probability of $\boldsymbol{X} > \frac{3}{4}$ given $\boldsymbol{X} > \frac{1}{2}$ and $\boldsymbol{P}\left(\frac{1}{4} < X < \frac{1}{2}\right)$. **[3]**

Group B

Answer all the following questions **Marks: 10**

1. Form a partial differential equation by eliminating the arbitrary function $\boldsymbol{F}$ from $\boldsymbol{F\left(\frac{z}{x^2}, x - y\right) = 0}$. **[3]**

2. Expand $\boldsymbol{f(x)}$ in Fourier cosine series in $\boldsymbol{0 \le x \le \pi}$, where $\boldsymbol{f(x)} = \begin{cases} \frac{\pi x}{4}, & for\ \ 0 \le x \le \frac{\pi}{2} \\ \frac{\pi}{4}(\pi - x), & for\ \ \frac{\pi}{2} < x \le \pi \end{cases}$. **[3]**

3. Verify whether $\boldsymbol{x - x^3 = -\frac{12}{\pi^3}\left[\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \sin n\pi x\right]}$, where $\boldsymbol{x \in (-1, 1)}$. **[4]**

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National Institute of Technology, Agartala

Department of Mathematics

Name of Examination : 3rd Sem Mid-Term Date : 28/09/2021

Subject Name : Engineering Mathematics III Subject code : ~~UCS 0316~~ UCS03C16

Name of Student : Aditya Kiran Pal

Enrollment no : 20UCS119 Registration no. : 2012709

Branch (Section) Name : CSE A Semester : 3rd Sem.

Group A:

Q.3. Let E_1 = percent produced by A = 40 %

E_2 = percent produced by B = 60 %

Let X be event that produced item is defective.

$\therefore$

$E_1 = 40\% = \frac{2}{5}$

$E_2 = 80\% = \frac{3}{5}$

Probability A produces defective piece $= P\left(\frac{X}{E_1}\right) = 5\% = \frac{1}{20}$

Probability B produces defective piece $= P\left(\frac{X}{E_2}\right) = 10\% = \frac{1}{10}$

Applying Bayes theorem,

$$P\left(\frac{E_1}{X}\right) = \frac{P(E_1)\, P\left(\frac{X}{E_1}\right)}{P(E_1) \cdot P\left(\frac{X}{E_1}\right) + P(E_2)\, P\left(\frac{X}{E_2}\right)}$$

$$= \frac{\frac{2}{5} \times \frac{1}{20}}{\frac{2}{5} \times \frac{1}{20} + \frac{3}{5} \times \frac{1}{10}} = \frac{\frac{1}{5}}{\frac{1}{5} + \frac{3}{5}} = \frac{1}{4} \text{ Ans.}$$

Sol. $P(0) = 3c^3$, $P(1) = 4c - 10c^2$, $P(2) = 5c - 1$

$\therefore \quad \Sigma P_i = 1$

$\Rightarrow \quad P(0) + P(1) + P(2) = 1$

$\Rightarrow \quad 3c^3 + 4c - 10c^2 + 5c - 1 = 1$

$\Rightarrow \quad 3c^3 - 10c^2 + 9c - 2 = 0$

$\Rightarrow \quad (3c-1)(c-2)(c-1) = 0$

$\therefore$ Since c has to be less than 1

$\therefore c = \frac{1}{3}$

$P(0) = \cancel{3}\, 3\left(\frac{1}{3}\right)^3 = \frac{1}{9}$

$P(1) = \frac{4}{3} - \frac{10}{9} = \frac{12-10}{9} = \frac{2}{9}$

$P(2) = \frac{5}{3} - 1 = \frac{2}{3}$

$$P(x) = \begin{cases} \frac{1}{9}, & x = 0 \\ \frac{2}{9}, & x = 1 \\ \frac{2}{3}, & x = 2 \end{cases}$$

Ans.

Q.2.

$E(x) = \Sigma x P(x)$

$= (-3)(0.05) + (-2)(0.10) + (-1)(0.30) + 0 + 1(0.30)$
$+ 2(0.15) + 3(0.10)$

$= -0.15 - 0.20 - 0.30 + 0.30 + 0.30 + 0.30$

$E(x) = 0.25$

$E(x^2) = \Sigma x^2 P(x)$

$= (-3)^2(0.05) + (-2)^2(0.10) + (-1)^2(0.30) + 0$

$1(0.30) + 2^2(0.15) + 3^2(0.10)$

$= 0.45 + 0.40 + 0.30 + 0.30 + 0.60 + 0.90$

$= 2.95.$

$E(4x+5)^2$

$= E(16x^2 + 40x + 25)$

$= 16 \times E(x^2) + 40 E(x) + 25$

$= 16 \times 2.95 + 40 \times 0.25 + 25$

$= 47.20 + 10 + 25 = 82.20.$ Ans.

Q.4. $P(x) = Ax^2 \quad 0 \leq x \leq 1$

we know

$\int_{-\infty}^{\infty} f(x).dx = 1 \Rightarrow \int_0^1 Ax^2 dx = 1$

$\Rightarrow A\left[\frac{x^3}{3}\right]_0^1 = 1$

$\Rightarrow A\left[\frac{1}{3} - 0\right] = 1 \Rightarrow A = 3$

$f(x) = 3x^2 \quad 0 < x < 1$

To find

$P(x > 3/4 \,/\, x > \frac{1}{2})$ and $P(\frac{1}{4} < x < \frac{1}{2})$

Let

$E_1 : x > \frac{3}{4} \quad E_2 : x > \frac{1}{2}$

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

$$= \frac{P(E_1 \cap E_2)}{P(x > \frac{1}{2})} = \frac{\int_{3/4}^{1} 3x^2\,dx}{\cancel{P(x > \frac{1}{2})} \int_{1/2}^{1} 3x^2\,dx}$$

$$= \frac{3\left[\frac{x^3}{3}\right]_{3/4}^{1}}{3\left[\frac{x^3}{3}\right]_{1/2}^{1}} = \frac{1-\left(\frac{3}{4}\right)^3}{1-\left(\frac{1}{2}\right)^3}$$

$$= \frac{1-\frac{27}{64}}{1-\frac{1}{8}} = \frac{\frac{64-27}{64}}{\frac{8-1}{8}} = \frac{37}{56}$$

$$P(E_1/E_2) = \frac{37}{56} \text{ Ans.}$$

$$P\left(\frac{1}{4} < x < \frac{1}{2}\right) = \int_{1/4}^{1/2} 3x^2\,dx$$

$$= 3\left[\frac{x^3}{3}\right]_{1/4}^{1/2} \;\cancel{dx}$$

$$= \left(\frac{1}{2}\right)^3 - \left(\frac{1}{4}\right)^3$$

$$= \frac{1}{8} - \frac{1}{64} = \frac{7}{64} \text{ Ans}$$

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Group B:

Q.1. Given, $F\left(\frac{z}{x^2}, x-y\right) = 0$

Let $u = \frac{z}{x^2}$, $v = x - y$ —— (1)

Differentiating Eq (1) partially wrt x

$$\frac{\partial F}{\partial u}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial x}\right) + \frac{\partial F}{\partial v}\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z}\cdot\frac{\partial z}{\partial x}\right) = 0$$

$$\Rightarrow \frac{\partial F}{\partial u}\left(\frac{-2z}{x^3} + \frac{p}{x^2}\right) + \frac{\partial F}{\partial v}(1) = 0$$

$$\rightarrow \frac{\frac{\partial F}{\partial v}}{\frac{\partial F}{\partial u}} = \left(-\frac{2z}{x^3} + \frac{p}{x^2}\right) \quad \text{—— (2)}$$

Differentiating Eq (1) partially wrt y

$$\cancel{\frac{\partial F}{\partial u}\left(\frac{q}{x^2}\right) + \frac{\partial F}{\partial v}}$$

$$\frac{\partial F}{\partial u}\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial y}\right) + \frac{\partial F}{\partial v}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}\cdot\frac{\partial z}{\partial y}\right) = 0$$

$$\frac{\partial F}{\partial u}\left(\frac{q}{x^2}\right) + \frac{\partial F}{\partial v}(-1) = 0$$

$$\frac{\frac{\partial F}{\partial v}}{\frac{\partial F}{\partial u}} = \frac{q}{x^2} \quad \text{—— (3)}$$

Equating (2) and (3)

$$\frac{q}{x^2} = \left(\frac{2z}{x^3} - \frac{p}{x^2}\right)$$

$$\Rightarrow qx = 2z - px$$

$$\rightarrow 2z = (p+q)x$$

which is a partial differential equation of First order

Ans.

Group B

Q.2. $f(x) = \begin{cases} \frac{\pi x}{4} & 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{4}(\pi - x & \text{for } \frac{\pi}{2} < x \leq \pi \end{cases}$

$$f(x) = \sum_{n=1}^{\infty} a_n \cos n x$$

$$a_n = \frac{1}{\pi} \cancel{2} \; \frac{1}{\pi}\int_0^{\pi} f(x) \cos (n\pi x)\, dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2} \frac{\pi}{4} x \cos n\pi x\, dx + \int_{\pi/2}^{\pi} \frac{\pi}{4}(\pi - 4) \cos n\pi x\, dx\right.$$

$$= \frac{1}{\pi}\left[\frac{\pi}{4}\left(x\left(\frac{\sin n\pi x}{n}\right) - \left(\frac{-\cos n\pi x}{n^2}\right)\right]_0^{\pi/2}\right.$$

$$+ \left[\frac{\pi}{4}\left\{(\pi - x)\frac{\sin n\pi x}{n} - (-1)\left(\frac{-\cos n x}{n^2}\right)\right]_{\pi/2}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{4}\left[\frac{x \sin n x}{n} + \frac{\cos n x}{n^2}\right]_0^{\pi/2} + \frac{\pi}{4}\left[(\pi - x)\frac{\sin (n x)}{n} - \frac{\cos n\pi}{n^2}\right]_{\pi/2}^{\pi}\right.$$

$$= \frac{1}{\pi}\left[\frac{\pi}{4}\left[\frac{\pi/2 \sin(n\pi/2)}{n} + \frac{\cos\left(\frac{n\pi}{2}\right)}{n^2} - \frac{1}{n^2}\right]\right.$$

$$\left.+ \frac{\pi}{4}\left[0 - \frac{(-1)^n}{n^2} - \left(\frac{\pi/2 \sin\left(\frac{n\pi}{2}\right)}{n} - \frac{\cos\left(\frac{n\pi}{3}\right)}{n^2}\right)\right]\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{2\pi} \sin\left(\frac{n\pi}{2}\right) + \frac{\cos\left(\frac{n\pi}{2}\right)}{n^2} - \frac{1}{n^2} - \frac{(-1)^n}{n^2}\right.$$

$$\left. - \frac{\pi}{2n} \sin\left(\frac{n\pi}{2}\right) + \frac{\cos\left(\frac{n\pi}{2}\right)}{n^2}\right]$$

$$= \frac{1}{4}\left[\frac{2\cos\left(\frac{n\pi}{2}\right) - 1 - (-1)^n}{n^2}\right]$$

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{4}\left[\frac{2\cos\left(\frac{n\pi}{2}\right) - 1 - (-1)^n}{n^2}\right] \cos n x$$

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Q. 3. $f(x) = x - x^3 \quad x \in (-1, 1)$

$f(-x) = -x - (-x)^3$

$= -x + x^3 = -f(x)$

Thus $f(x)$ is an odd function.

$\therefore$ Fourier series of $f(x)$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin(n\pi x)$$

$\therefore$

$$b_n = \frac{2}{1}\int_0^1 f(x)\sin(n\pi x)\,dx$$

$$= 2\int_0^1 (x - x^3)\sin(n\pi x)\,dx$$

$$= 2\left[-(x - x^3)\frac{\cos n\pi x}{n\pi} - (1 - 3x^2)\left(\frac{-\sin(n\pi x)}{n^2\pi^2}\right) + (-6x)\left(\frac{\cos n\pi x}{n^3\pi^3}\right) - (-6)\left(\frac{\sin n\pi x}{n^4\pi^4}\right)\right]$$

$$= 2\left[-(1-1)\frac{\cos n\pi}{n\pi} - (1-3)\left(\frac{-\sin n\pi}{n^2\pi^2}\right) - 6\frac{\cos(n\pi)}{n^3\pi^3} + 6\frac{\sin(n\pi)}{n^4\pi^4}\right]$$

$$= 2\left[-6\frac{\cos n\pi}{n^3\pi^3}\right] = \frac{-12}{\pi^3}\left[\frac{(-1)^n}{n^3}\right]$$

$$\therefore f(x) = \sum_{n=1}^{\infty} b_n \sin(n\pi x)$$

$$= \sum_{n=1}^{\infty}\left(-\frac{12}{n^3\pi^3}\right)(-1)^n \sin(n\pi x)$$

$$x - x^3 = \frac{-12}{\pi^3}\sum \frac{(-1)^n}{n^3}\sin(n\pi x)$$

Ans.