19/07/19 Graph Theory

Nurshing dev

graph is a sketch of some specific points which follow any special arithmatic condition (relation).

$G(V, E)$

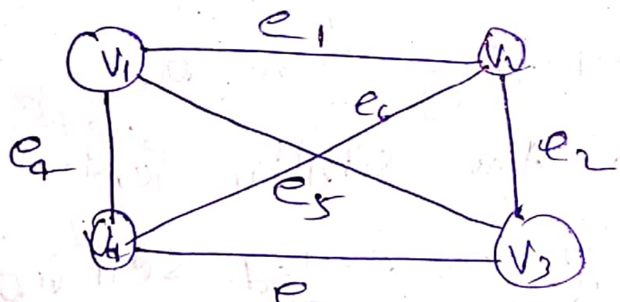
set of vertices. set of edges.

$V = (v_1, v_2, v_3 - - - - - - v_n)$

$E = (e_1, e_2, e_3 - - - - e_m)$

$\psi \rightarrow$ relational operator.

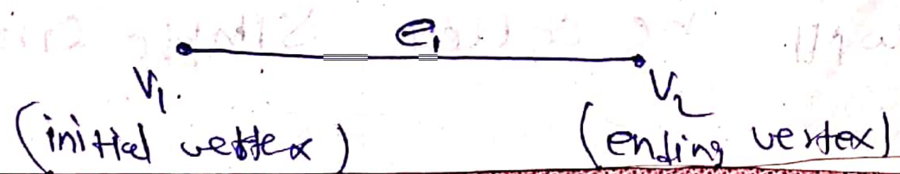
tells about relation b/w vertices and edges.



$V = \{v_1, v_2, v_3, v_4\}$

$E = \{e_1, - - - - e_6\}$

Eadges mapped to ~~Undoreder~~ unordered pair of vertices.



$$\psi = \begin{cases} e_1 \to (v_1, v_2) \\ e_2 \to (v_2, v_3) \\ \\ e_6 \to (v_4, v_2) \end{cases}$$

the vertex which is not connected ~~by~~ any edge with any other vertex, is called Isolated vertex.

<u>Null graph</u>

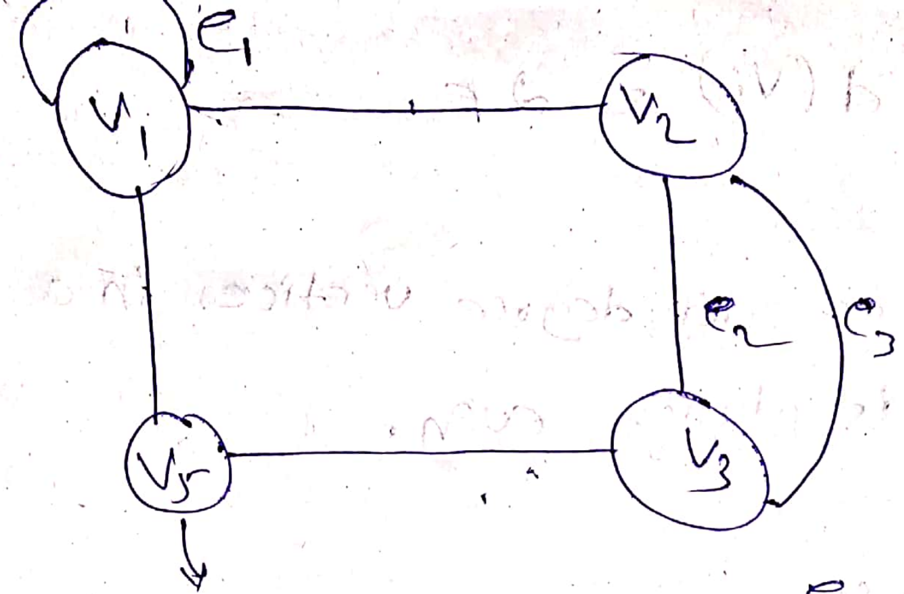
If a graph has all Isolated vertices then that will called Null Graph,

<u>Parallel eadges</u>

If two eadges are joining to the 2 same vertices then the eadges will be called Parallel ~~eadge~~ eadges.

"If a eadge is starting from a vertex and ends on the similar vertex then the eadge will be called self loop."

"If a graph does not has any parallel eadges and self loop then the graph will be called Simple Graph."



Pendent vertex

e_2, e_3 are parallel eadges

e_1 is called self loop.

the total no. of eadges by which a vertex is connected, is called degree.

$d(v_5) = 2$

$d(v_3) = 3$

$d(v_1) = 4$

$d(v_2) = 3$

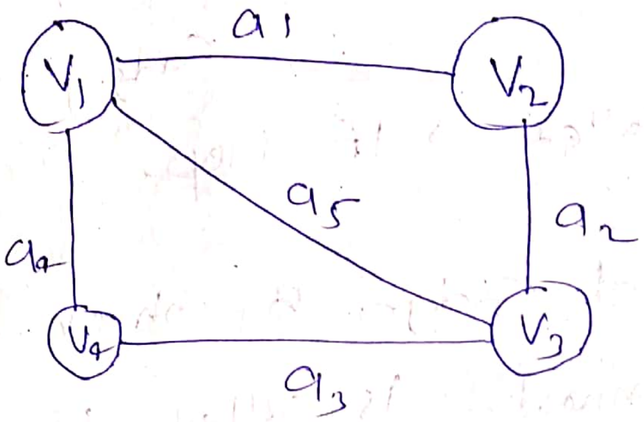
total degree $= 12$

total degree $= 2 \times$ total eadges.

$$\sum_{i=1}^{n} d(v_i) = 2e$$

the no. of odd degree vertices in a graph is always even.

<u>Adjurrency</u>



V_1, V_4 are adjurance.

a_1, a_2 " ".

two vertices are said to be adjucane if they have at least one common eadge.

two eadges said to be adjucane if they have at least on common vertex.

Scanned by CamScanner

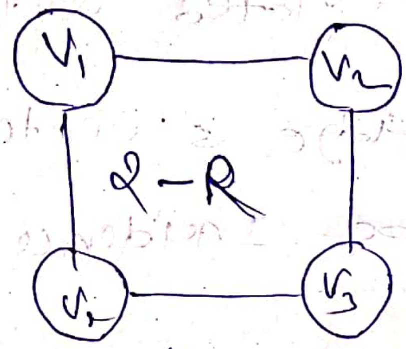
if any eadge is co-related with any vertex then eadge & vertex will be said to be Incidence.

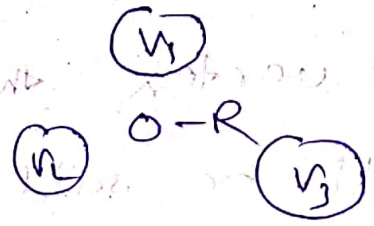
If a graph has no ~~mult~~ parallel eadge & no self loop is called ~~self graf~~ simple graph.

If a graph has parallel eadge but no self loop, will be called multi graph.

If ~~a g~~ every vertex is connected with rest of each vertices in a graph will be called complete graph. in complete graph, degree of graph will will be $(n-1)$. where n = total no. of vertices.

If Degree of each of the vertex are same then graph is called Regular graph.





all complete graph are regular graph but vise-versa is not corret

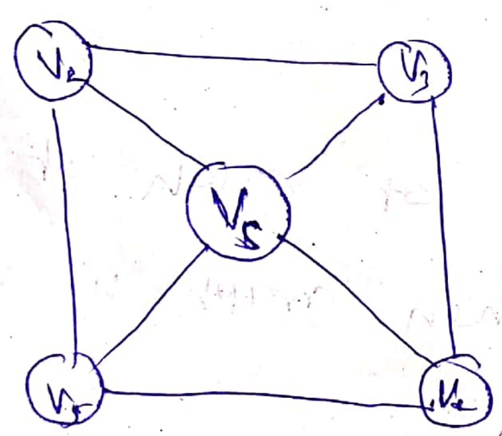
All complete will be

Ass-

Explain with proper example all complete graphs are regular graph but not all regular graph are complete graph.

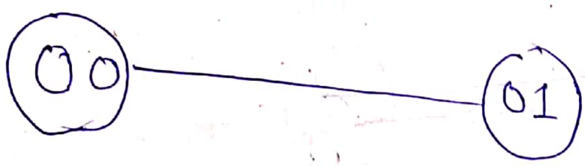
Wheel Graph

here, ~~exact~~ excactly one specific vertex is connected with rest of all vertices.

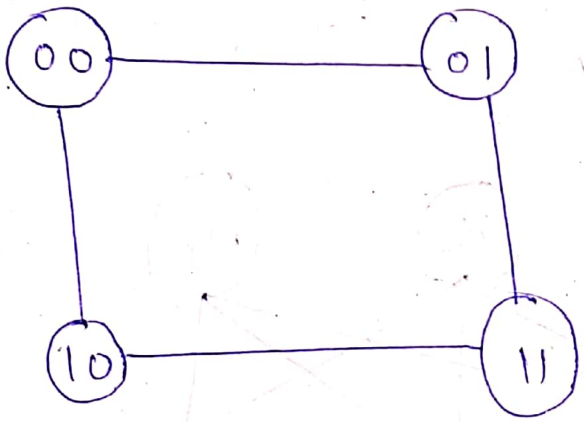


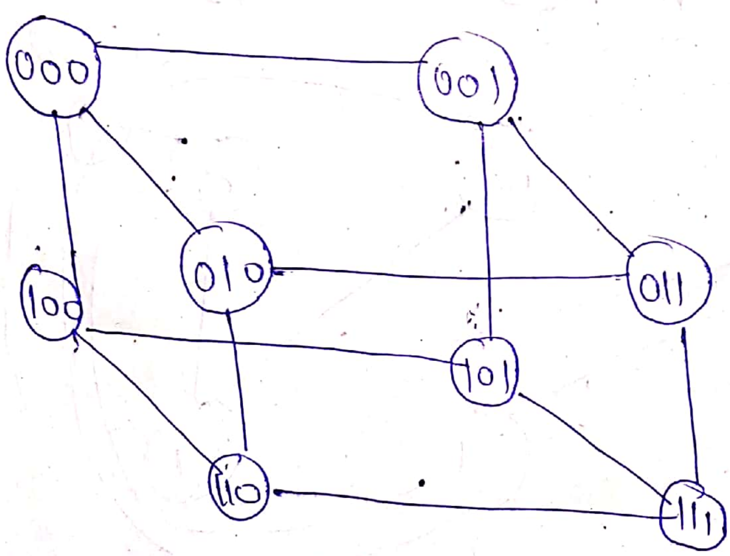
N-Cube graph

It is a special graph where the eadge is connected to two verticies where vertices are ~~diff~~ have 1 bit difference.



no. of verticy $= 2^n$

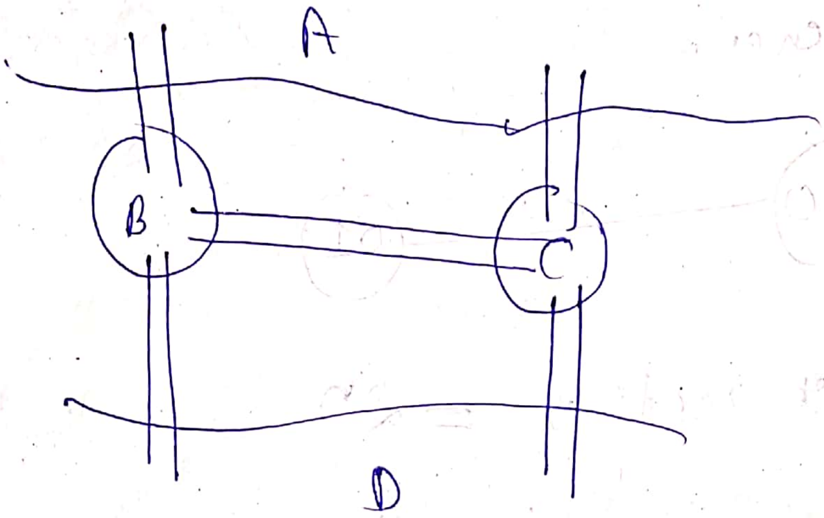




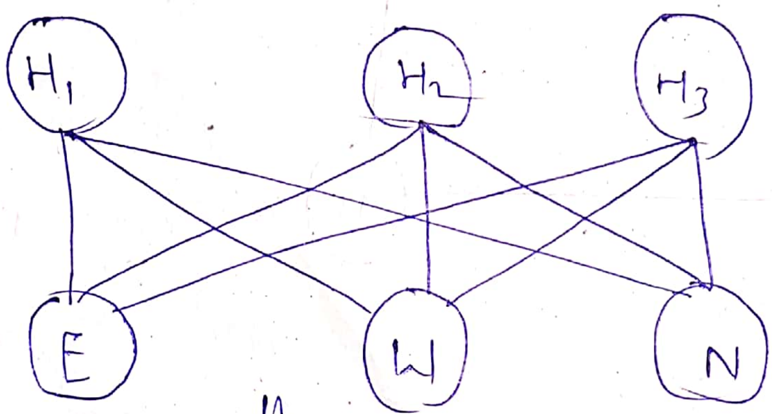
$2^3 = 8$

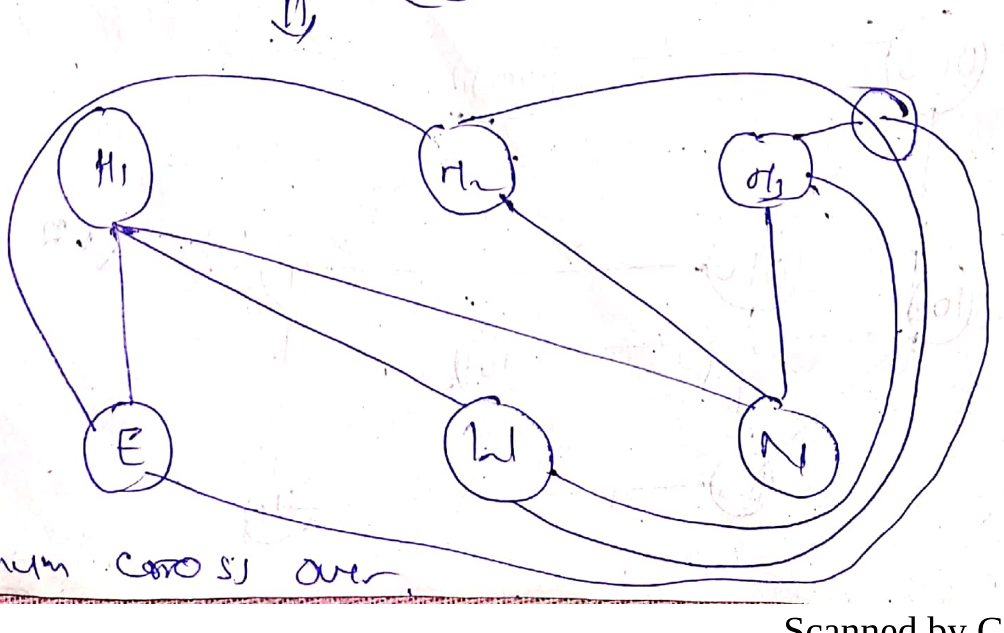
~~Draw~~

Konigsberg Bridge Problem



Bipartite Graph





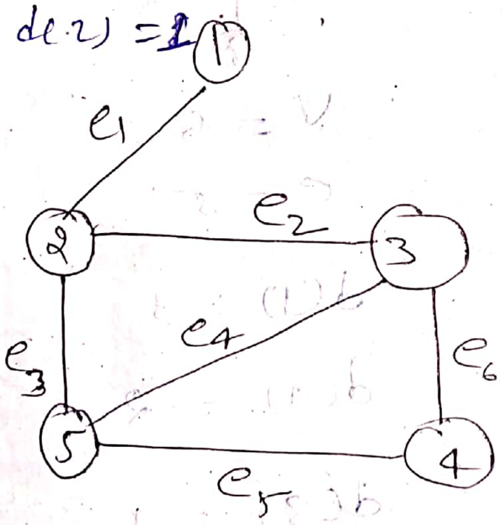
minimum cross over

Q1. Convince yourself that the maximum degree of any vertex in a simple graph with 'n' vertices is (n-1)

Ans → show that the maximum no. of edges in a simple graph with n- vertices is $\frac{n(n-1)}{2}$ nC_2 $n(n-1)$

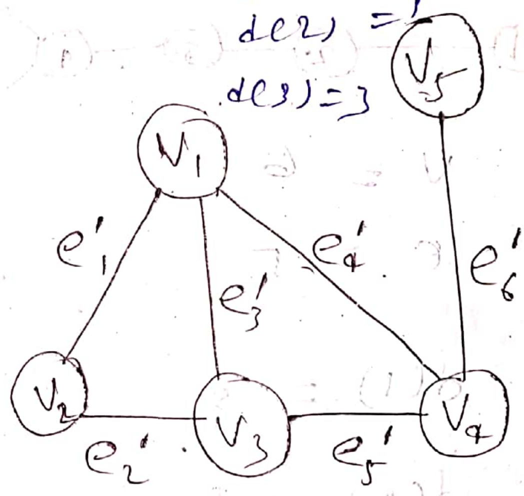
Isomorphism

d(1) = 1
d(2) = 1
d(3) = 3



G_1

d(1) = 1
d(2) = 1
d(3) = 3



correspondence

V	e
$1 \to V_5$	$e_1 \to e_6'$
$2 \to V_4$	$e_2 \to e_5'$
$3 \to V_3$	$e_3 \to e_4'$
$4 \to V_2$	$e_4 \to e_3'$
$5 \to V_1$	$e_5 \to e_1'$
	$e_6 \to e_2'$

check properties

(i) ~~verti~~ no. of vertices

(ii) no. of edges.

If ~~both~~ three properties are same ~~in~~ b/w two different graphs, will be called Isomorphism

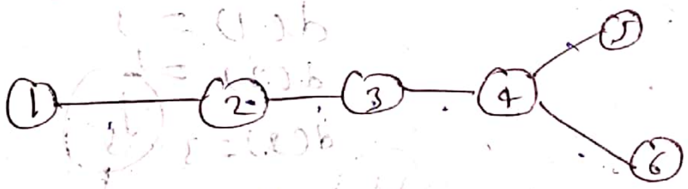
(i) $G_1(V) = G_2(V)$

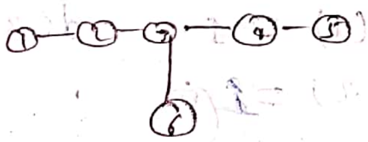
(ii) $G_1(e) = G_2(e)$

(iii) same no. of vertices with a fixed degree.

G_1 and G_2 are Isomorphism

G_3 ~~G_4~~ G_4





$V = 6$

$e = 5$

$d(1) = 3$

$d(2) = 2$

$d(3) = 1$

$V = 6$

$e = 5$

$d(1) = 3$

$d(2) = 2$

$d(3) = 1$

1) 1 to 1 correspondence between vertices and edges.

Correspondence

vertex	eadge
$1 \to V_1$	
~~$2 \to V_2$~~	
$2 \to$	
$3 \to$	
$4 \to$	
$5 \to$	
$6 \to$ V_6	

~~Corresp~~

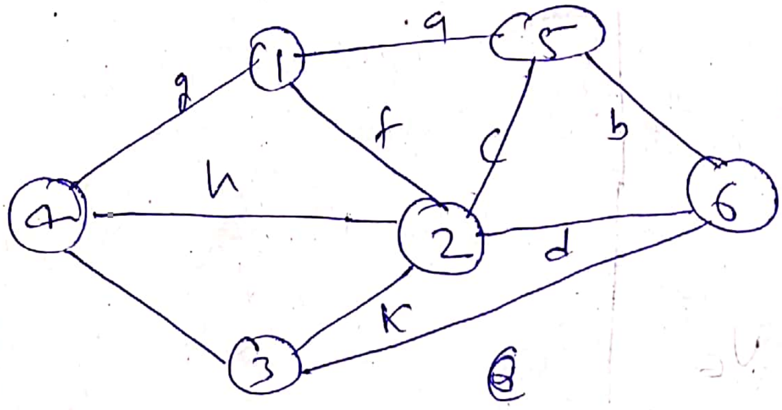
Correspondence ~~b/w~~ G_3 ; G_4 does not match hence G_3, G_4 are not Isomorphism.

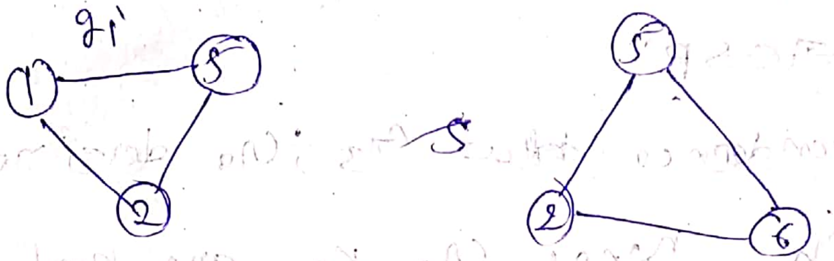
G_1 & G_2 are Isomorphism because they follow all 4 similar properties.

$\boxed{\text{Subgraph}}$

1. every graph is its own subgraph.
2. Any single vertex its own subgraph.
3. ~~Any~~

vertex disjoint subgraph   eadge disjoint subgraph.





It is not vertex disjoint & not eadge disjoint due to ② & C same vertex & eadge.

Multicolour Cube Problem

(Instant Instanity graph) Probl

R
R B G W   C-1
R

B
W G G R   C-2
W

W

R B W R

G

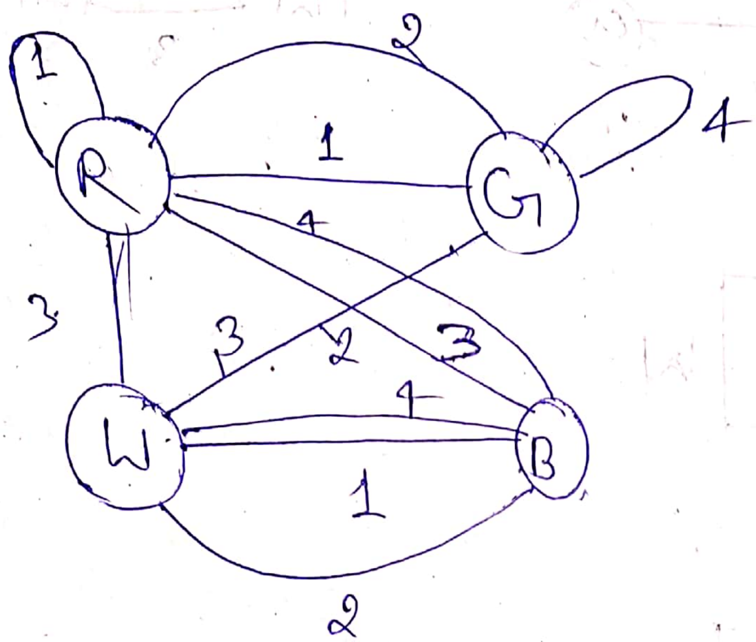
C-3

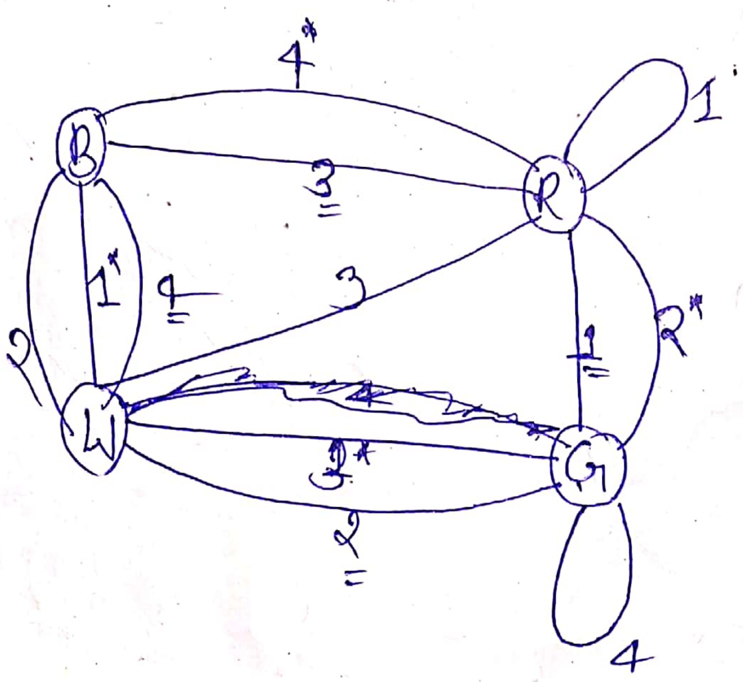
B

G B G W

R

C-4

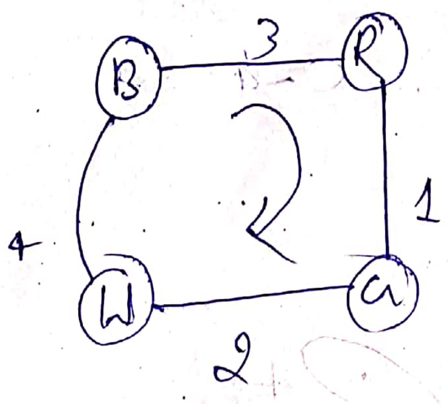


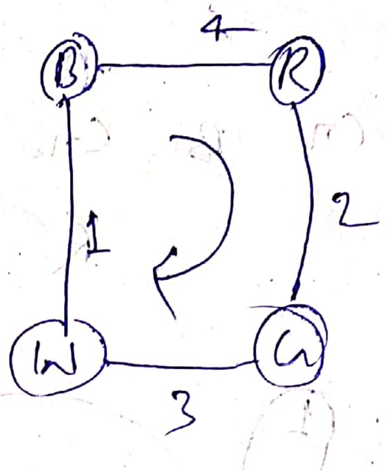


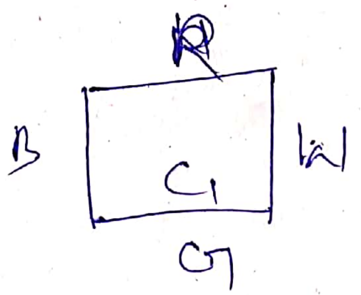
total 4X4 faces are visible
total of edges will be required for the

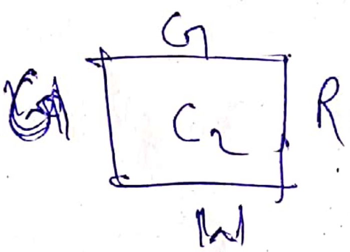
N-S

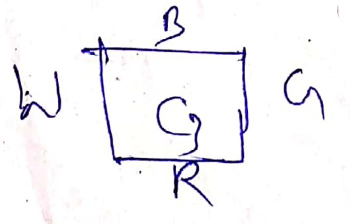
E-W

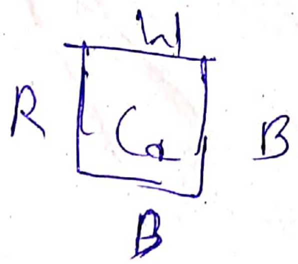












Q!- Given 4-cubes variously coloured B, R, G, W. is it possible to stack the cubes one on top of another to form a column such that no colour appearing twice on any of the four sides of this column.

Ans

(ii) is it possible to set the cubes such a way that each side shows only one colour.

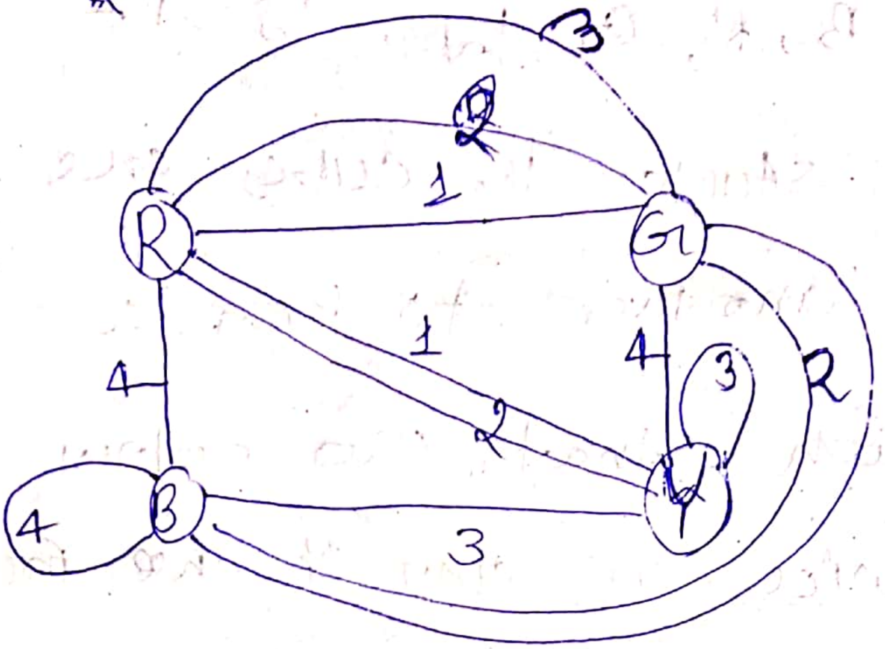
```
    G
Y   G   R   B
    R
```

```
    B
G   R   R   Y
    G
```

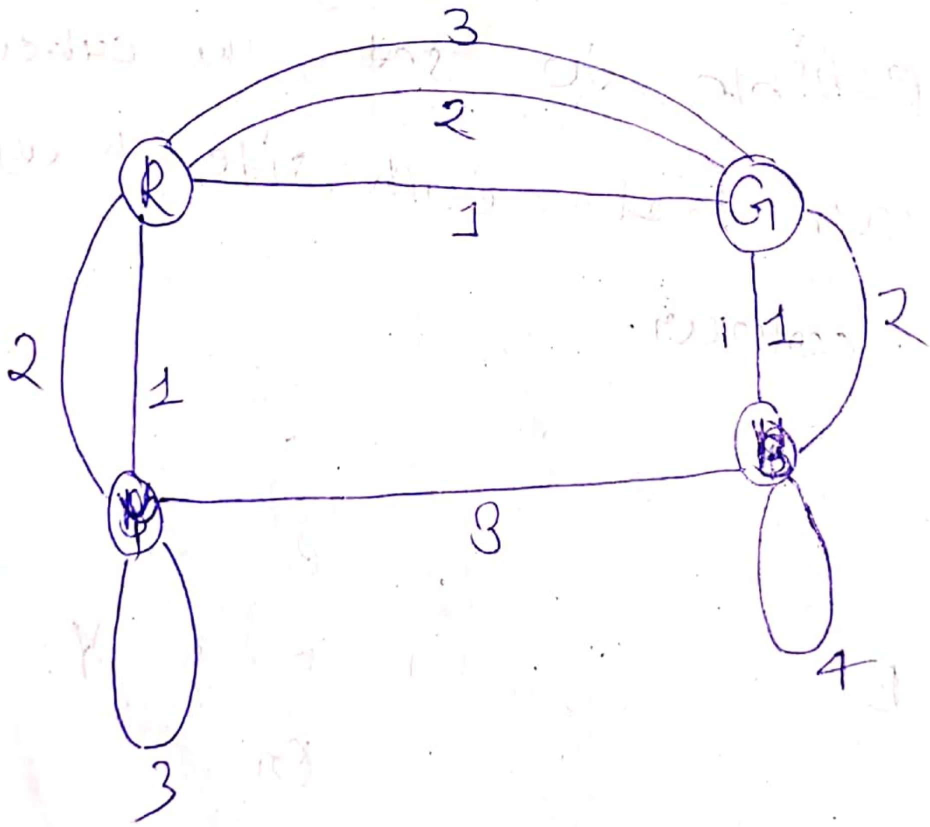
```
    Y
R   Y   G   B
    Y
```

```
    B
Y   B   G   R
    B
```

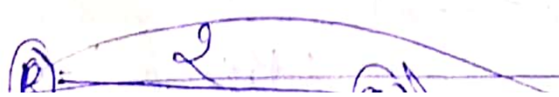
R



— C1

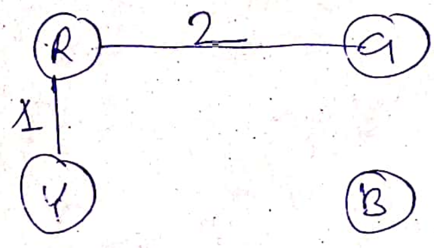


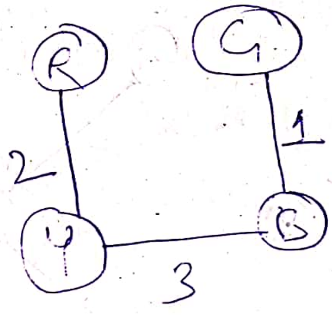
E-W



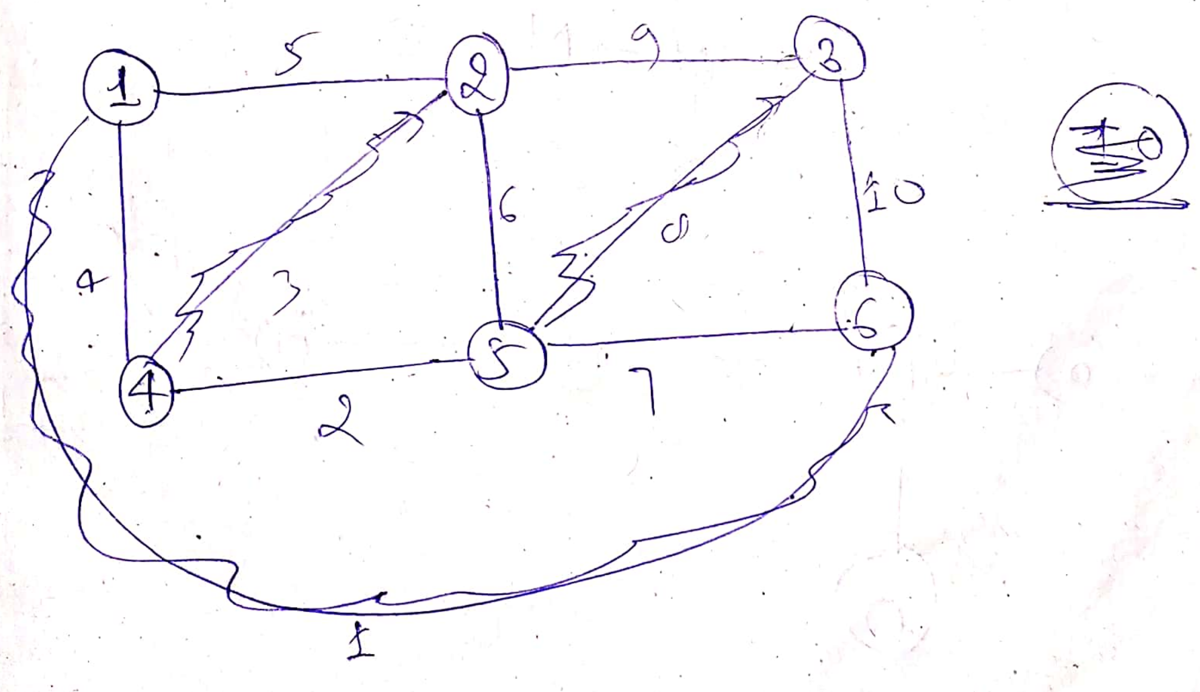
N-S

E – W

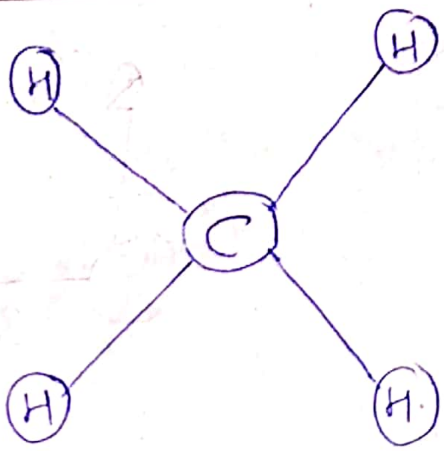




Q:- determine no. of edges in a graph with $V = 6$ out of which two of degree 4 and 4 of degree 2?



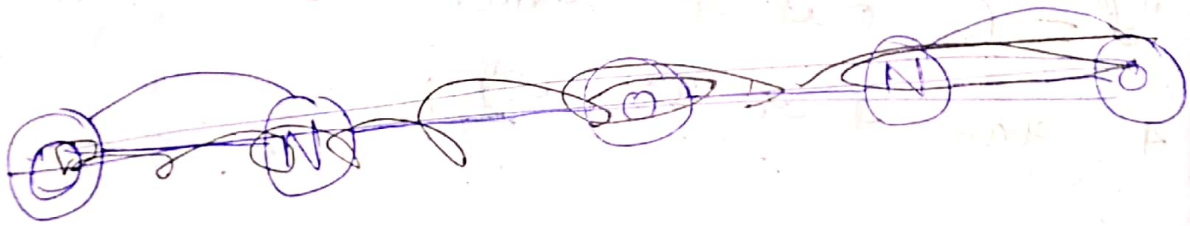
Draw a graph for CH_4

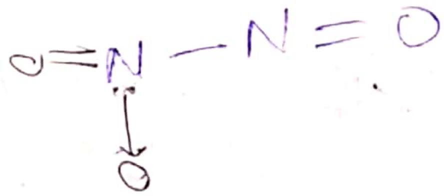


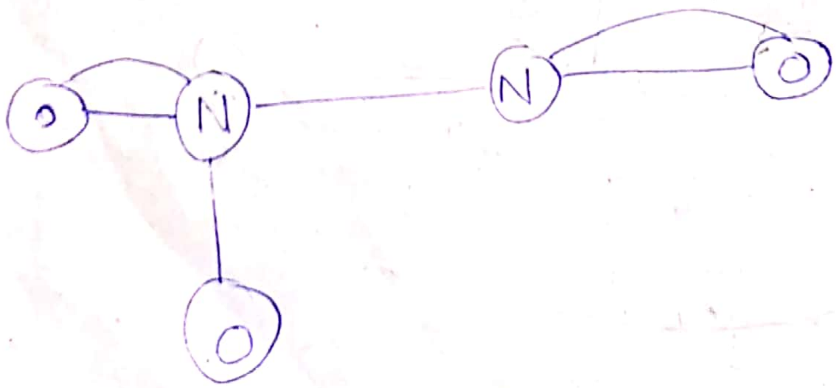
N = N

O = N — O — N =

(2) N_2O_3







$O=N-O-N=O$

Q1. Is it possible to construct a graph with 12 vertices such that two vertices have degree 3 and remaining have degree 4.

$$\frac{n(n-1)}{2} = \cancel{6\times 5} = \cancel{42/2}$$

Harish = 23

$6 \times 11 = \underline{66}$

$6 + 40 = \underline{46}$

max. no of ~~vertices~~ edges

$$\Rightarrow \frac{12 \times 11}{2} = 66$$

$$\Sigma d(v_i) = 2e$$

$$2\times 3 + 10\times 4 = 2e$$

$$\boxed{e = 23}$$

if e is fractional then no such graph is possible.

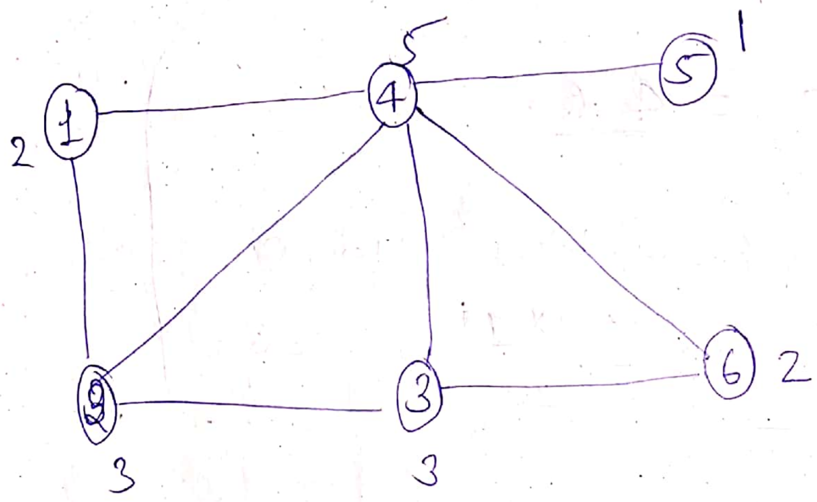
Scanned by CamScanner

Is it possible to draw a simple graph with 4 verties and 7 eadges.

$$\text{max. eadges} = \frac{4 \times 3}{2} = \underline{6} < 7$$

Not possible.

Degree sequence



$$\text{Degree seq} = \{1, 2, 2, 3, 3, 5\}$$

can there exist a simple graph with degree sequence $\{2, 2, 3, 4\}$

$$2 + 2 + 3 + 4 = 2e$$

e is not integer hence Not exist

$seq_2 = \{1, 2, 2, 3, 4, 5\}$

$e = 8.5$ / not possible.

$seq_3 = \{2, 2, 4, 6\}$

$e = 7$

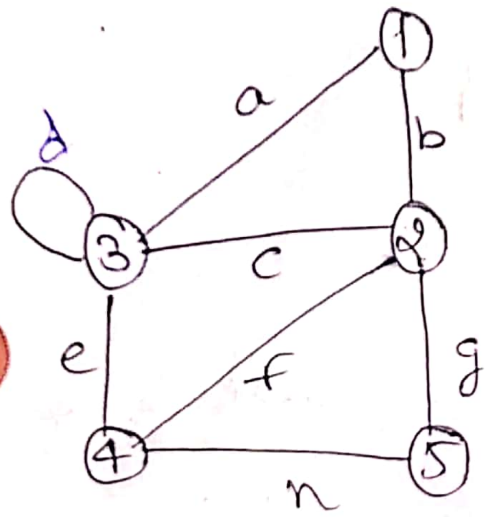
total no. of vertices = 4

max. degree of a simple graph = $n-1 = 3$

hence 4, 6 degree's vertex will not exist

hence such type of graph is not possible.

Walk, Path and circuit



G

walk is finite, alternating sequence of vertices and edges where edges and vertices can be repeated.

W1 : (1,5) → 1 b 2 g 5.

W2 : (1,5) → 1 a 3 d 3 c 2 g 5

W3 : (1,5) → 1 b 2 c 3 a 1 b 2 g 5

Trail

Trail is a walk where there is no repition of eadge ~~repett~~ but vertex can be repeated.

open → source & destination vertex diff.

close → same source & destination vertex.

Path

Path is a walk, where eadges and vertics will not repeated.

$P_1 (1,5) \longrightarrow 1\ 6\ 2\ 9\ 5$

length of Path :-

no. of eadges of path.

Path is always open walk.

Circuit

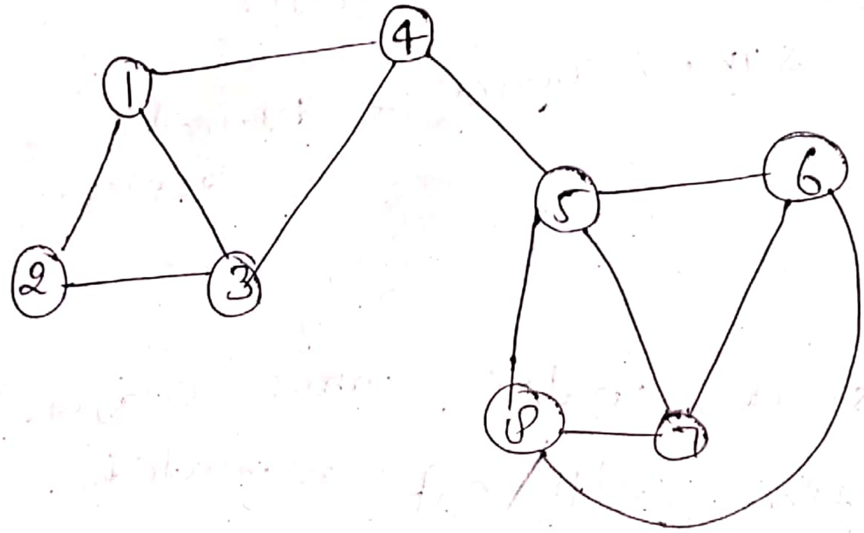
circuit is a closed trail where no vertex repetiona is not allowed excep ~~source and~~ starting & ending vertex

3 d3 → circuit

2 f 4 h 5 g 2 → circuit.

all selfloops are circuits

but all circuits are not selfloop.

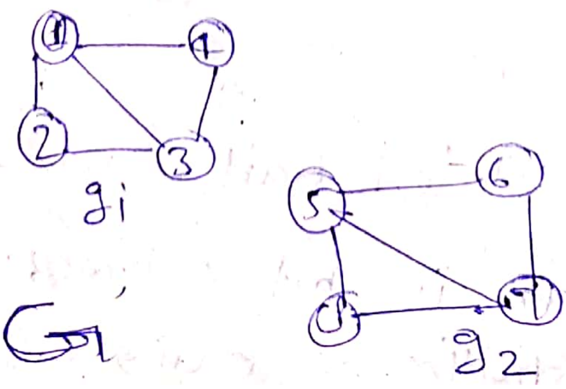


Connected graph

Connected Graph

In b/w every pair of vertices if there exist at least one path, then graph is called connected graph otherwise it will be disconnected graph.

ex_



G

it is a disconnected graph.

subgraphs of graph $G = G_1, G_2$

the connected subgraph in a disconnected graph are called <u>component (K)</u>:

A simple graph with n vertices and K components can have at most $(n-K)(n-K+1)/2$ eadge.

Let n_1 no. of verticy in component 1st

n_2 " " " " " " 2nd

n_K " " " " " " K^{th}

$n_1 + n_2 + n_3 - - - - + n_K = n.$

$$\left\{ \sum_{i=1}^{i=k} (n_i - 1) \right\}^2 = (n-K)^2$$

$$\sum_{i=1}^{k} (n_i^2 - 2n_i + 1) = n^2 + k^2 - 2nk$$

$$\sum_{i=1}^{k} n_i^2 - 2n + k = n^2 - 2nk + k^2$$

$$\sum_{i=1}^{k} n_i^2 = n^2 - 2nk + k^2 + 2n - k \longrightarrow (1)$$

max. no. of eadge for a simple graph of n verticy

$$= \frac{n(n-1)}{2}$$

then max. no. of eadge for n_i verticy $= \sum_{i=1}^{k} \frac{n_i(n_i-1)}{2}$

$$\Rightarrow \sum_{i=1}^{k} \frac{n_i^2}{2} = \frac{n}{2} \longleftarrow (2)$$

to put value of $\sum_{i=1}^{n} \frac{n_i^2}{2}$ from equation (1) to eqn (2)

Scanned by CamScanner

$$\Rightarrow \frac{n^2}{2} - nk + \frac{k^2}{2} + n - \frac{k}{2} - \frac{n}{2}$$

$$\Rightarrow \frac{n^2+k^2}{2} - nk - \frac{k}{2} + \frac{n}{2}$$

$$\Rightarrow \frac{1}{2}\left[(n-k)^2 - k + n\right]$$

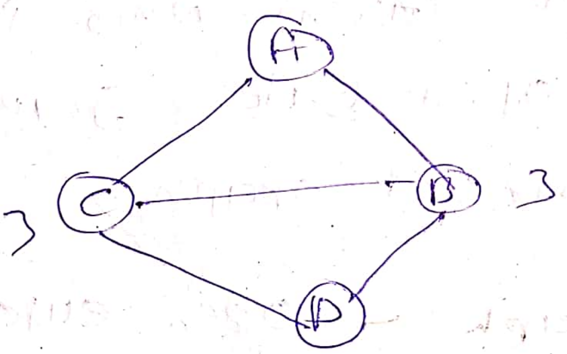
$$\Rightarrow \frac{1}{2}\left[(n-k)(n-k+1)\right]$$

Euler Graph

It is closed trail where every eadge is traversed only once.

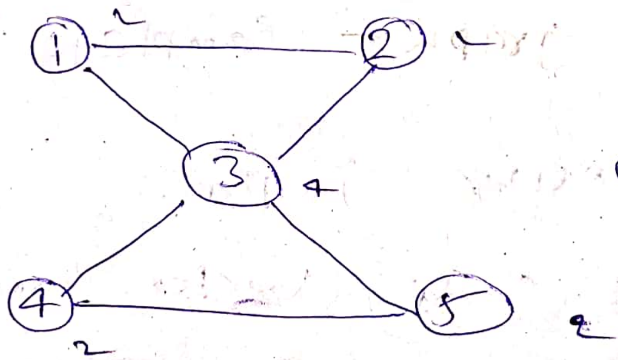
Degree at every vertex should be even.

Ex -



it can't be euler graph.

ex -

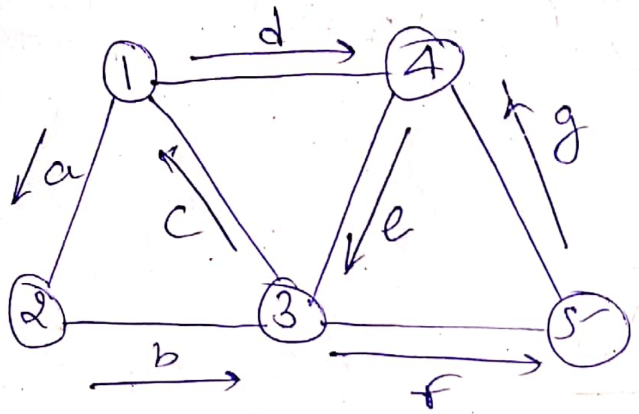


it is euler graph.

Explain, How Euler solve Konisberg bridge problem ?

If degree of every vertex is even then bridge is possible.

Open Euler Graph



G (1, 4)

all eadges are traversed exactly once and starting and ending vertex are different hence it will be open Euler graph. or Unicursal graph.

Unicursal graph – open euler graph.

Universal graph – Complete Graph.

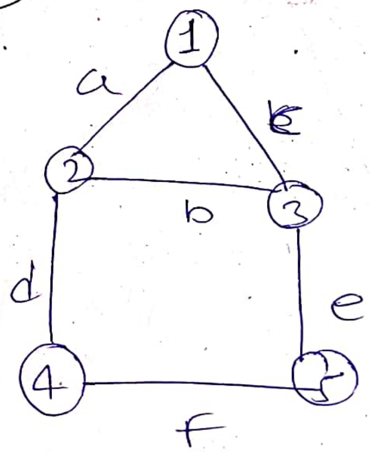
in open euler graph –

starting & ending vertex have odd

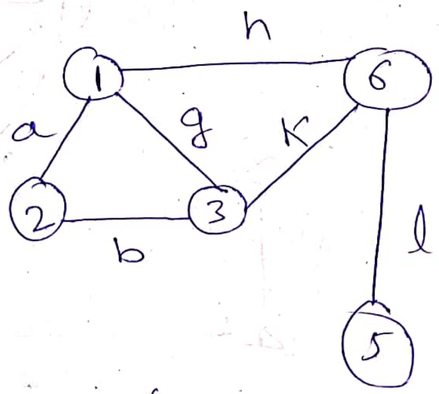
degree. remaining vertices will have even degree.

Operation on Graph

(1)

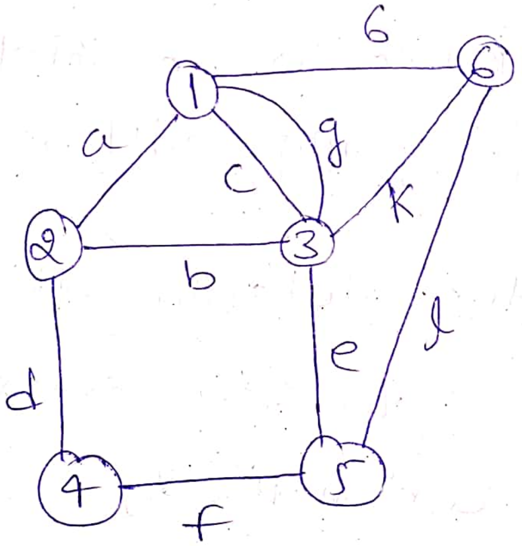


(g_1)

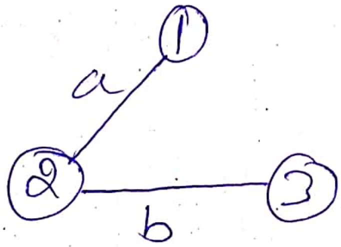


(g_2)

$g_1 \cup g_2$

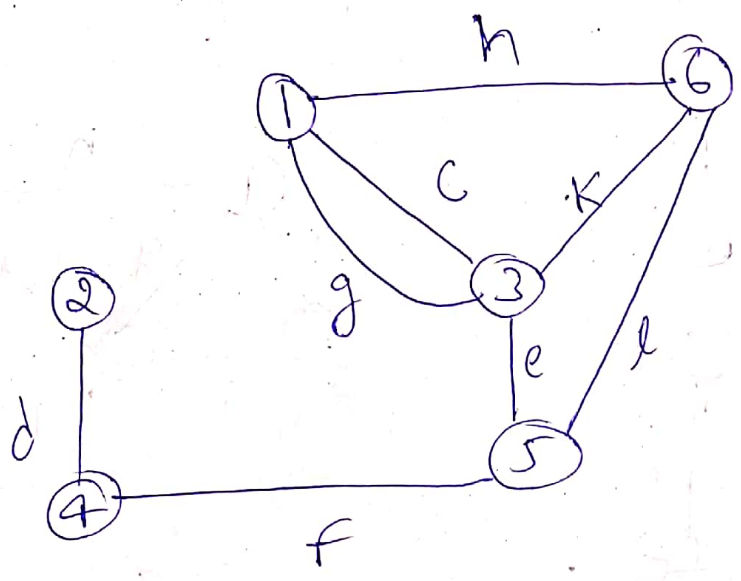


(2) $g_1 \cap g_2$



③ Ring sum

~~Common~~ all vertices and uncommon eadges



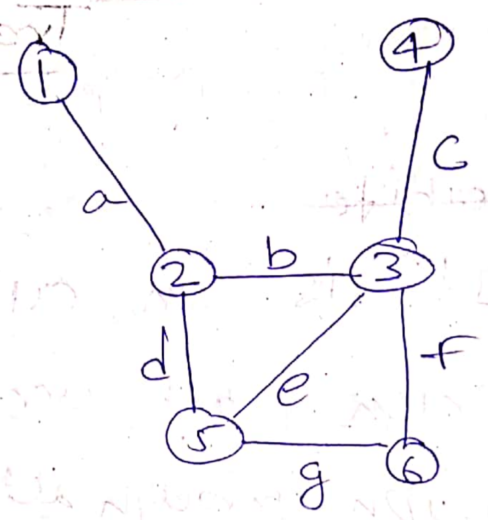
④ Deletion

to delete the element (vertex, eadge) which have to be delete and remaining graph will be deletion graph

in vertex deletion all eadges connected to the vertex will be deleted automatically.

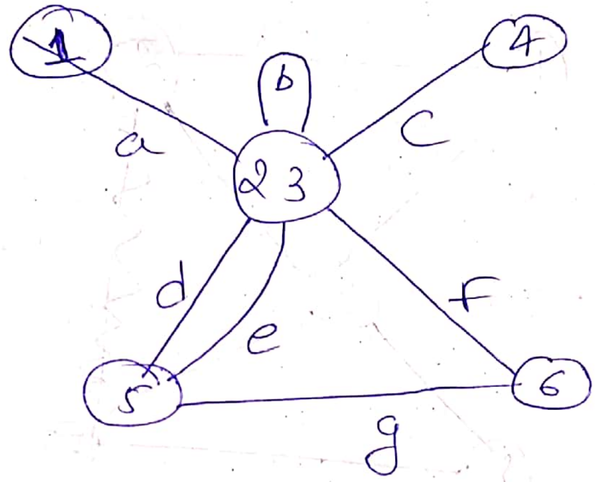
(5) Fusion (merge).

ex. G →



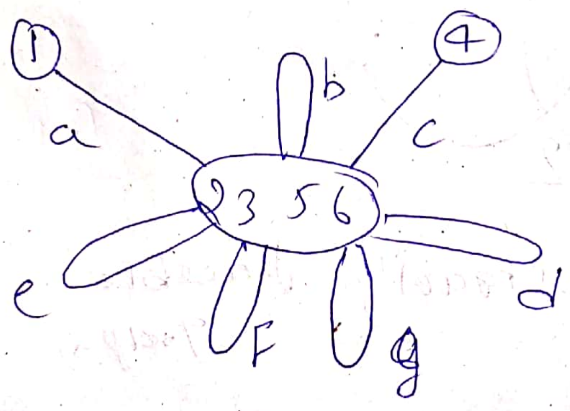
G' = G(2,3)

G' →



in case of fusion of 3 vertices first ~~any~~ two vertices will be fused and then 3rd vertex will be fused with previous one.

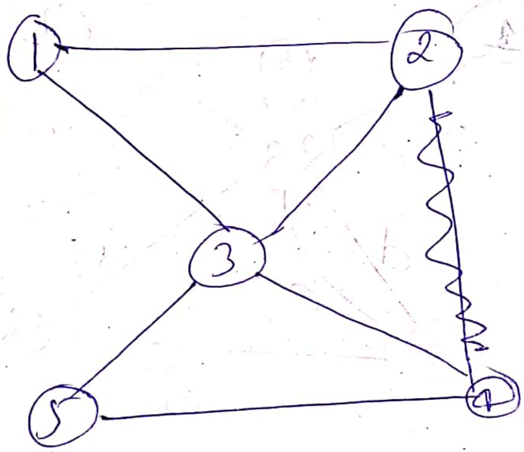
G'' = G'(23, 5, 6)



Arbitrarily ~~Tracebble~~ Traceable graph

~~Arbitra~~

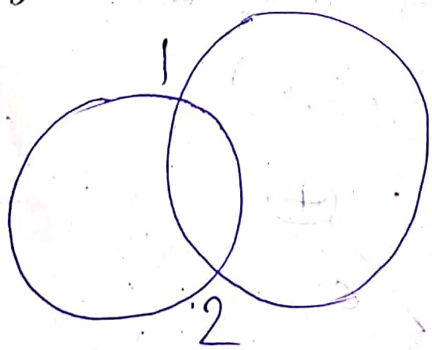
It is an eulerian graph on which all vertex show eulerian graph through all paths.



it will be eulerian only After vertex(3)

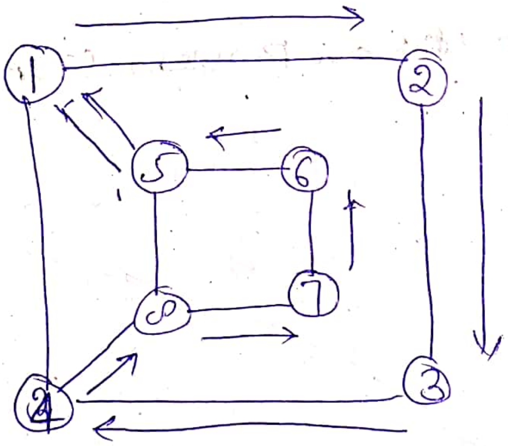
hence it will not be Arbitarality traceable graph,

~~Hamilton gro~~



it will be arbitrarily traceble graph

Euler Graph	Hamilton ian graph
every ~~vertex~~ eadge should be traversed exactly once.	every vertex traversal exactly once.
vertex mere than once	eadge mere than once traversed.
	starting vertex and ending vertex would be same, circuit will be formed. called Hamiltonian circuit.

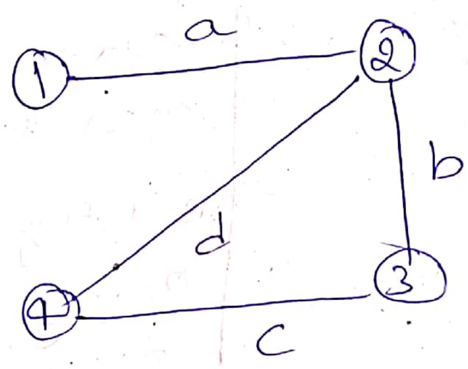


(1 – 2 – 3 – 4 – 8 – 7 – 6 – 5 – 1)
is hamiltonian circuit.

Q1- Draw a hamiltonian graph which has hamiltonian path but not hamiltonian circuit.

hamiltonian

Q1-



abc → is hamiltonian path

but there is no hamiltonian circuit

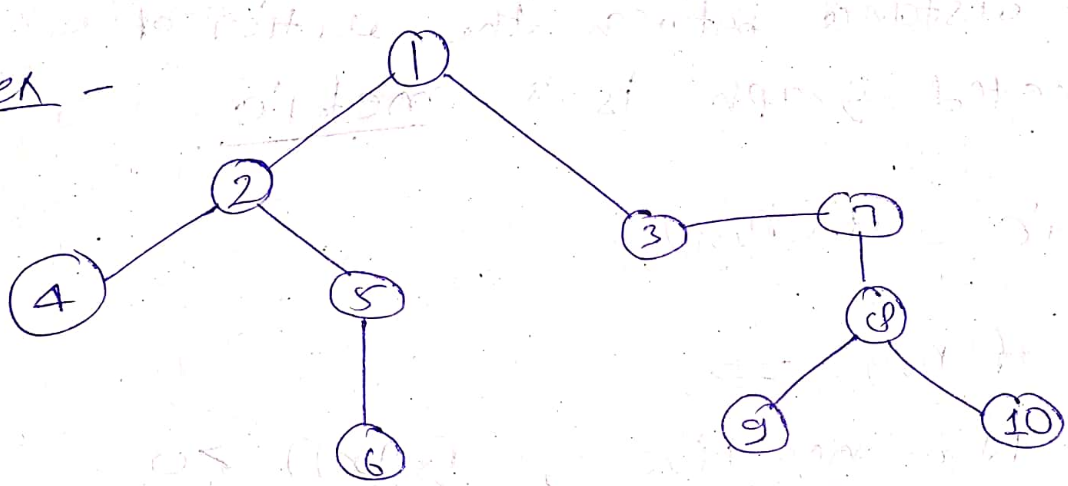
Q1- travelling salesman Problem. (TSP)

Scanned by CamScanner

Tree

① tree is a simple graph which has no circuit.
② and if tree has 'n' no. of vertices then it should has (n-1) eadges.
③ ~~tree is a tree~~
there is exactly one path in between every pair of vertices.
④ ~~the~~ tree is minimally connected.

ex -

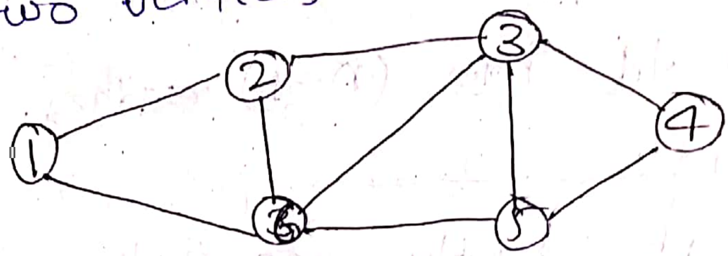


Final definition of tree

Tree is a simple connected graph with 'n' vertices and (n-1) eadges where there is exactly one path between every pair of vertices without any circuit.

Distence $d(V_i, V_j)$

Length of shortest path (mini no. of edges) b/w two vertices.



$d(1,3) = 2$, $(1-2-3)$

the distance between the vertices of a connected graph is a metric

metric $= f(x,y)$

~~f(x)~~ $f(x,y) \Rightarrow$

(i) Non-negative, $f(x,y) > 0$

(ii) symmetry, $f(x,y) = f(y,x)$

(iii) triangle inequality.

$\downarrow$

$f(x,y) \leq f(x,z) + f(z,y)$.

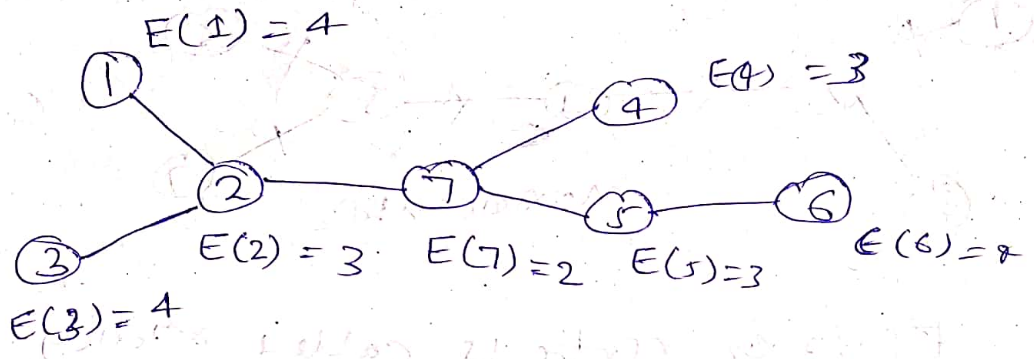
$d(1,3) \leq d(1,5) + d(5,3)$

$2 \leq 2 + 1$

$2 \leq 3$

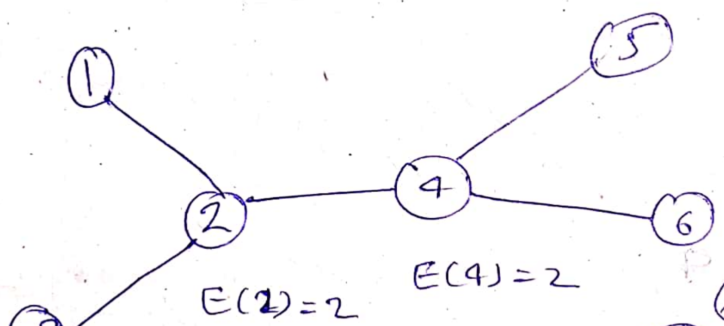
Ass ~~quest~~ The distance between two spanning trees of a graph is a metric

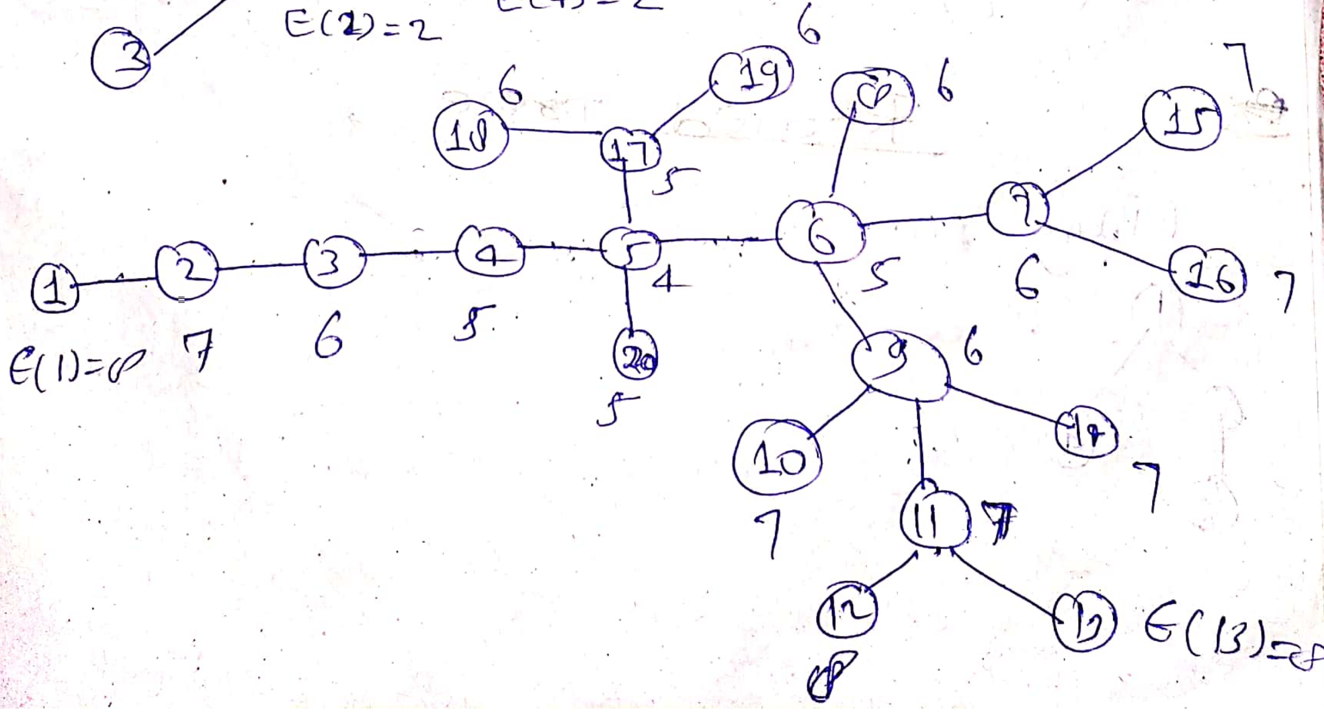
Eccentricity E(v)



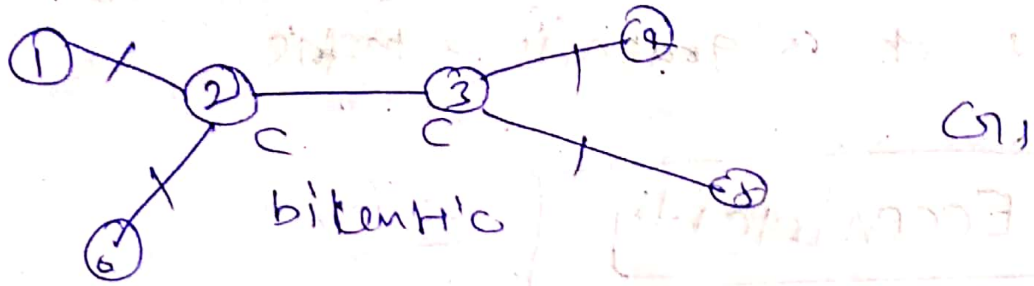
distance to farthest vertex.

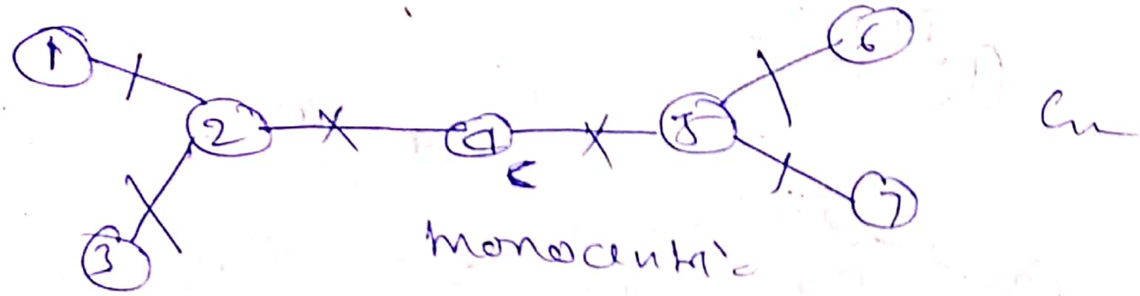
which may lowest eccentricity will be called centre of the tree.





Imp. T: a tree is monocentric or bicentric.





E(V) of centre is called radius.

Length of largest path is diameter.

$r(G_1) = 2$

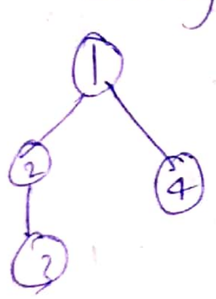
$D(G_1) = 3$

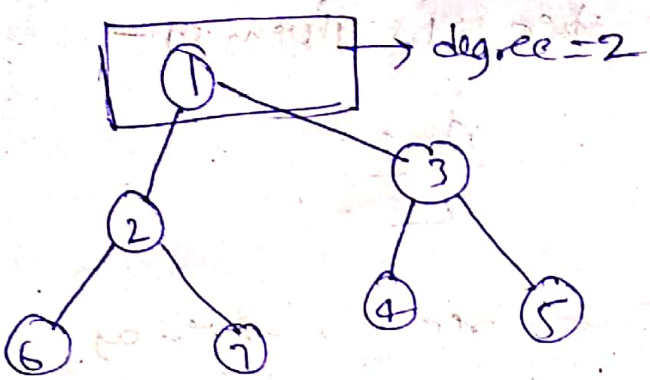
$r(G_2) = 2$

$D(G_2) = 4$

Rooted tree

Binary tree →





⇒ rooted binary tree.

rooted binary tree →

1. no. of verticy 'n' is odd.
2. $\frac{n+1}{2}$ no. of pendant vertices.

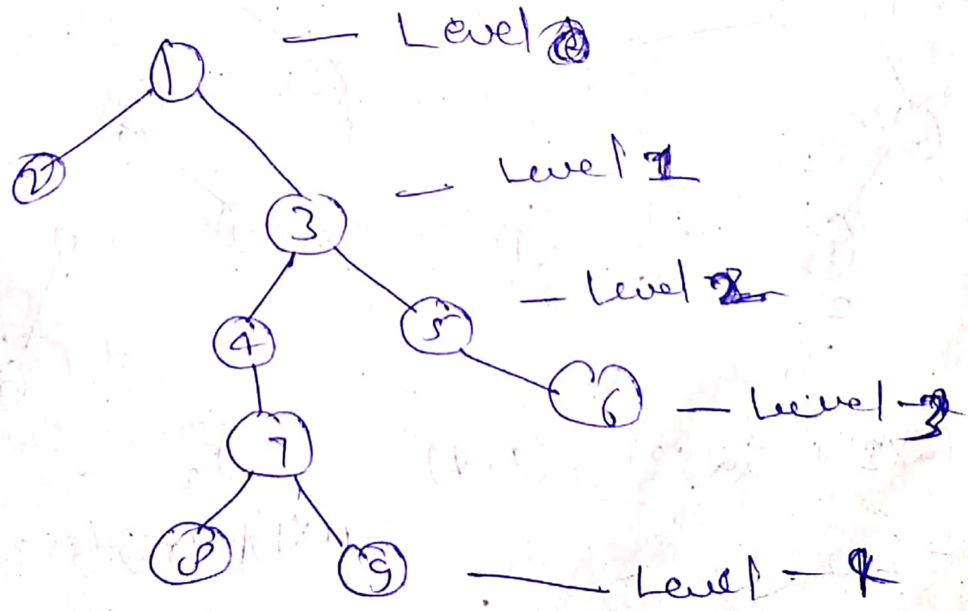
1 vertex of ___ d(2)

let p vertices of ___ d(1)

(n-1-p) vertices of ___ d(3)

$$(p \times 1) + (2 \times 1) + 3 \times (n-p-1) = 2(n-1).$$

<u>Hight</u>



max. hight of tree is given as –

$$\text{max. hight} = \frac{n-1}{2}$$

mini hight of tree is given as

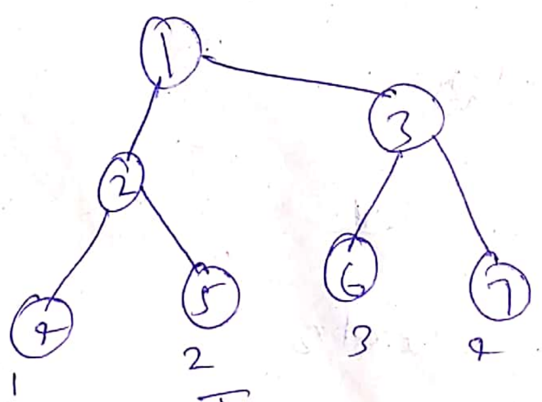
$$\text{min hight} = \log_2(n+1) - 1$$

let $n = 11$

max hight $= 5$

$$\text{min hight} = (\log_2(12) - 1)$$
$$= 2.58$$
$$\simeq 3$$

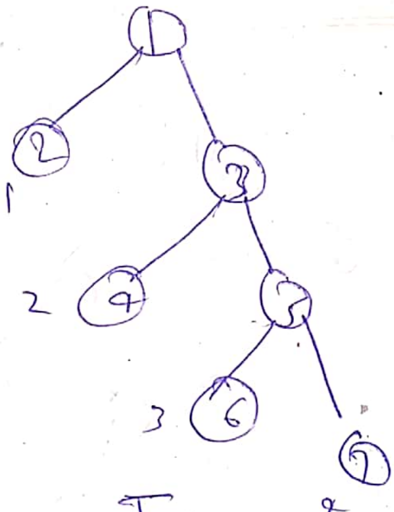
Weighted Path length $\sum W_{ij}$



T_1

$(2\times1) + (2\times2) + (2\times3) + (2\times4)$

$W_{ij} = 20$

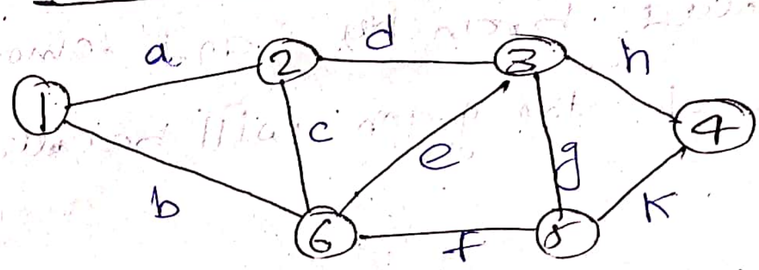


T_2

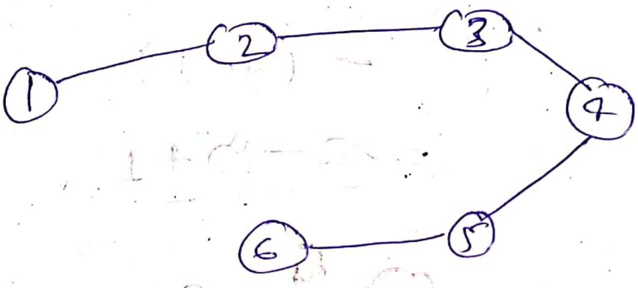
$(1\times1)+(2\times2)+(3\times3)+(4\times3)$

$W_{ij} = 26$

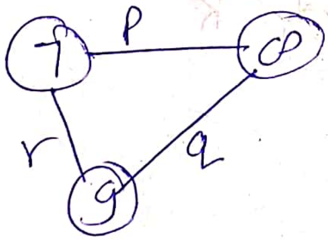
Spanning tree



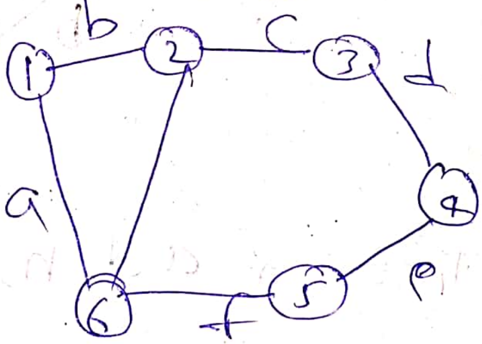
is a tree, which contains all the vertices of a graph



G_1,



G_2



$ST_1 = \{p, q\}$ $ST_2 = \{a, b, c, d, e\}$

Spanning forest $\Rightarrow \{ST_1, ST_2\}$

spanning forest of a collection of spanning trees of disconnected graph.

the eadges of the spanning tree are called branches, and remaining eadges of the graph will be called chords.

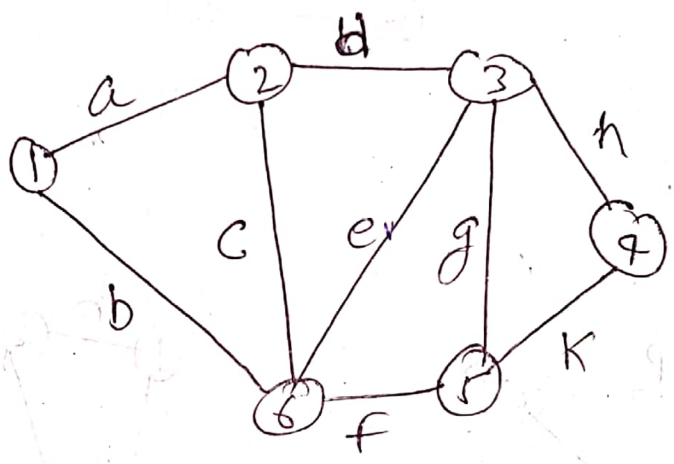
no. of branches is $(n-1)$

$n \to$ no. of vertices

no. of chords = total eadges in graph $-(n-1)$

$= \underline{e-n+1}$.

G →



$ST_1 = \{a, d, h, k, f\}$

$ST_2 = \{a, d, e, g, h\}$

$d(T_1, T_2) = 2$ $\{h, k\}$

$d(T_2, T_1) = 2$ $\{e, g\}$

$T_3 = \{b, c, d, h, k\}$

Rank

is difference between the no. of vertices and the no. of component (K) of graph.

rank = $n - K$

if connected graph-

rank = $n - 1$

the rank of graph is equal to branches of spanning tree.

Nullity

Nullity of a graph is the difference between the no. of edges of graph and its rank.

$$\text{Nullity} = e - r$$
$$= e - n + k$$

the nullity of a connected graph = no. of chords of its spanning tree.

~~Imp~~ Minimum spanning tree

1. Kruskal's algorithm
2. Prim's algorithm

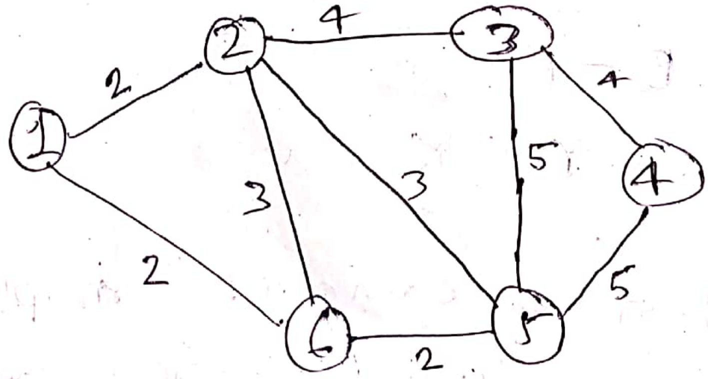
① Kruskal's algorithm.

Step 1 → List all eadges of the graph G in increasing order of weight.

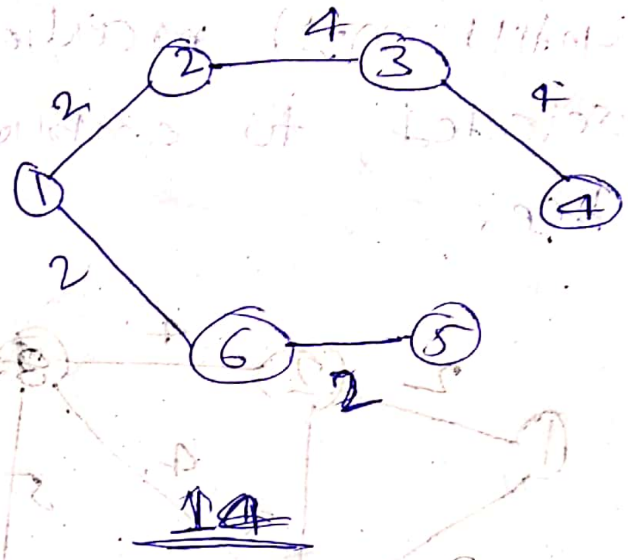
Step 2 → select the smallest eadge of 'G'.

Step-3 → for each successive step select another smallest eadge that makes no circuit with previously selected eadges.

Step-4 → Continue untill (n-1) eadges have been selected to construct ~~minimal~~ desired spanning tree.



12 → 2
16 → 2
65 → 2
26 → 3
25 → 3
34 → 4
23 → 4
35 → 5
45 → 5



① Prim's algorith

(i) Draw 'n' of isolated vertices and level that $V_1, V_2, V_3 \ldots V_n$.

(ii) tabulet the given weights of the edges of 'G' in an $n \times n$ table

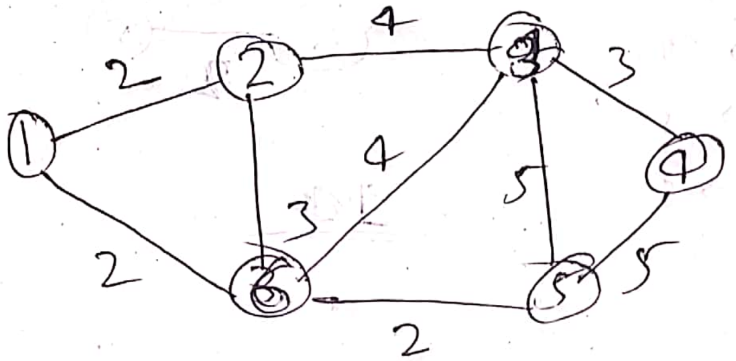
(iii) set the weights of non-existing edges as very large.

(iv) Start from vertex V_1 and connect it to its nearest neighbour set V_k.

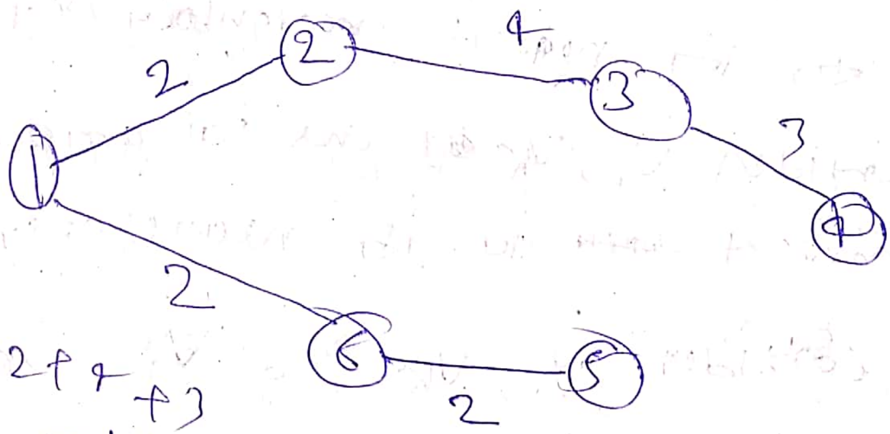
(v) Consider V_1, V_k as an subgraph and connect to its nearest neighbour.

(vi) Consider V_1, V_k and V_i be the

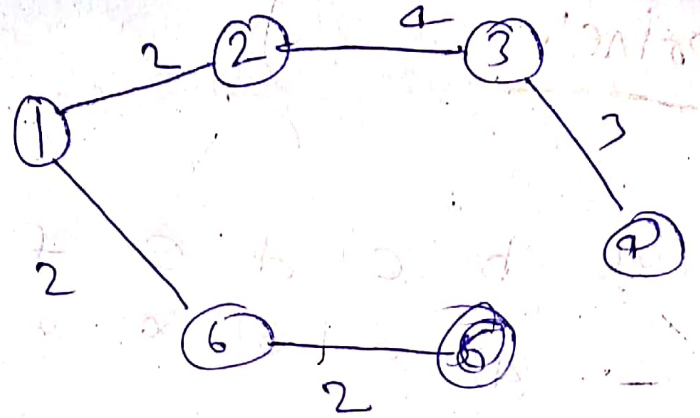
Sub graph and continue the process untill (n-1) ~~no~~ edges have been selected, to construct desired spanning tree.



	1	2	3	4	5	6
1	–	2	∞	∞	∞	2
2	2	–	4	∞	∞	3
3	∞	4	–	3	~~5~~	4
4	∞	∞	3	[illegible]	5	∞
5	∞	∞	5	5	–	2
6	2	3	4	∞	2	–

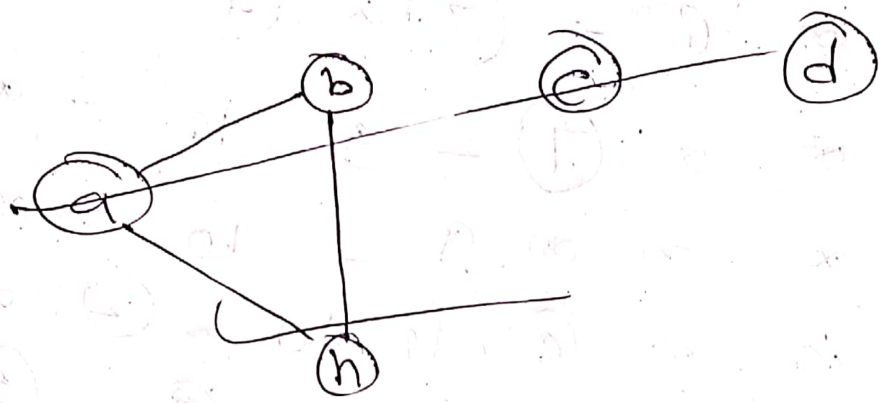


2+2+2+4+3
=13

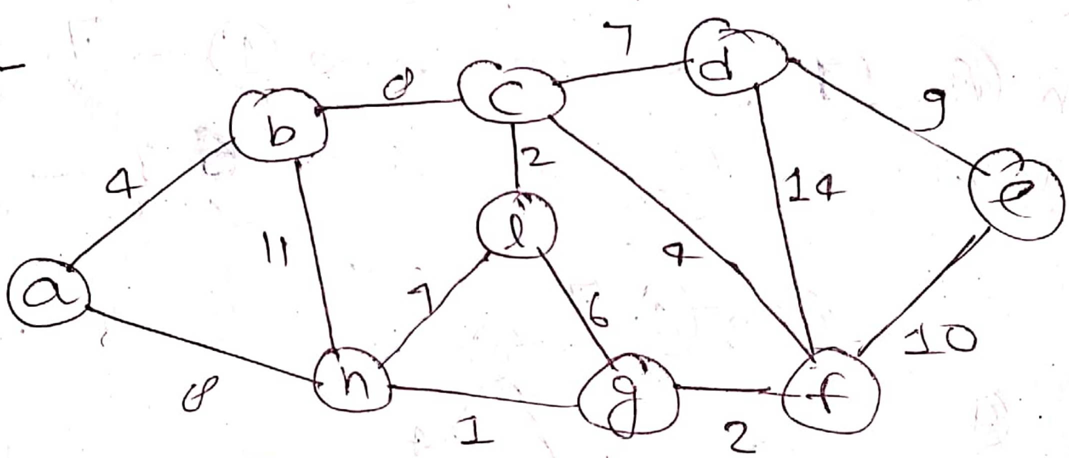


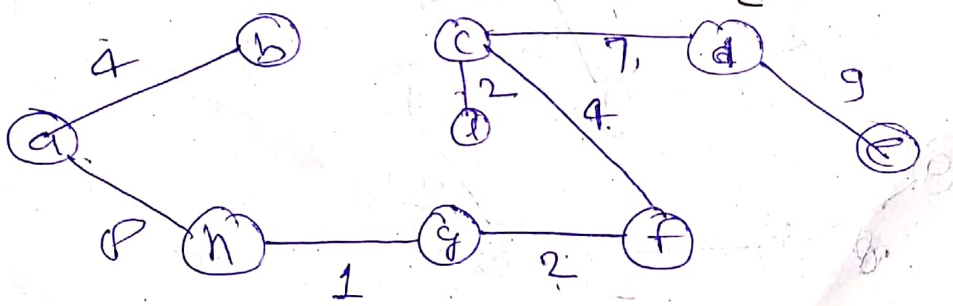
⇒ 13

Q →



Q -



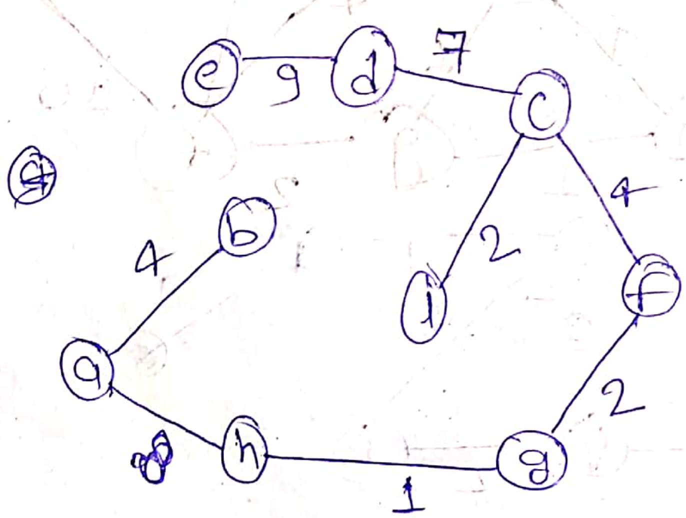


4+8+1+2+2+4+7+9

37

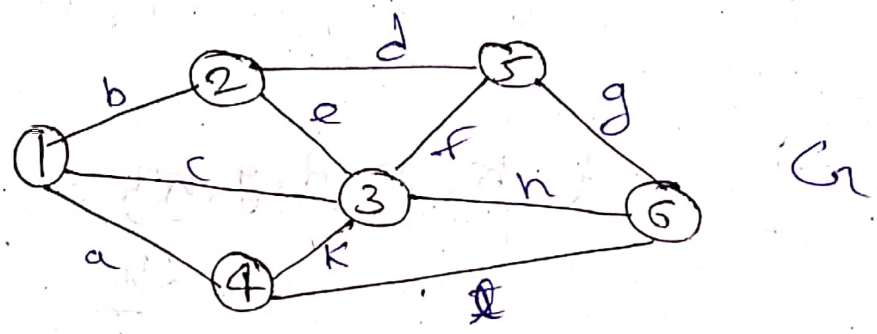
prine's

	a	b	c	d	e	f	g	h	i
a	—	4	∞	∞	∞	∞	∞	8	∞
b	4	—	8	∞	∞	∞	∞	11	∞
c	∞	8	—	7	∞	4	∞	∞	2
d	∞	∞	7	—	9	14	∞	∞	∞
e	∞	∞	∞	9	—	10	∞	∞	∞
f	∞	∞	4	14	10	—	2	∞	∞
g	∞	∞	∞	∞	∞	2	—	1	6
h	8	11	∞	∞	∞	∞	1	—	7
i	∞	∞	2	∞	∞	∞	6	7	—



9+7+4+2+1+8+4+2 = 37

Chirchoff's - matrix tree theorem



<u>Fundamental circuit</u>

spanning tree

$(n-1) \longrightarrow$ $b_n = \{a, b, d, g, h\}$ $\rightarrow T_1$

$(e-n+1) \rightarrow$ $C_n = \{c, e, f, k, l\}$

in a graph there are $(e-n+1)$ ~~graph~~ fundamental circuits.

with $c \rightarrow$ chord, fundamental circuit is

(b, c, h, g, d)

$e \rightarrow$ (d, e, h, g)

$f \rightarrow$ $\{f, g, h\}$

$k \rightarrow$ $\{k, a, b, g, h\}$

$l \rightarrow$ $\{l, a, b, d, g\}$

$i = \{K, a, b, d, g, h\}$ → fundamental circuit.

$(f_1 - a) \Rightarrow \{K, b, d, g, h\}$

$\Downarrow$
T_2

this whole process is called "elementary tree transformation".

~~from~~ it is a phenomena using which a spanning tree is formulated from a given graph and one chord can be added with this spanning tree to form fundamental circuit again ~~from~~ from fundamental circuit an edge is been deleted to formulated another spanning tree.

Scanned by CamScanner

Chirchoff's matrix Tree Theorem
(No. of spanning trees in a 'G')

(i) construct laplasian matrix 'Q' for the given graph 'G'.

(ii) for $(i \neq j)$

If (vertex 'i' and 'j' are adjacent in 'G')

then $q_{ij} = -1$

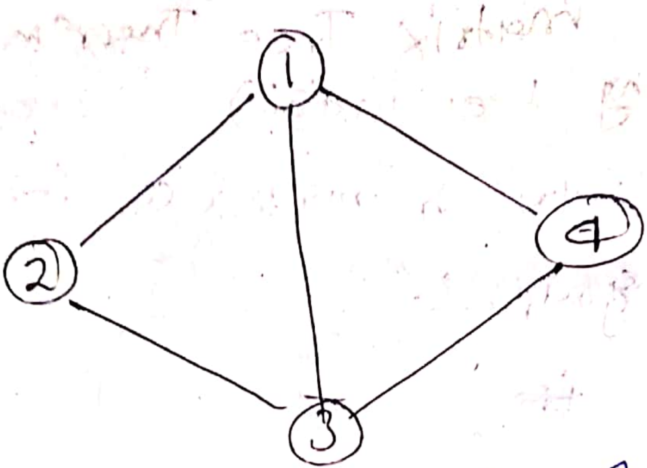
Otherwise $q_{ij} = 0$

(iii) for $(i = j)$

q_{ij} = degree of vertex i in G.

from matrix Q construct matrix Q' by deleting anyone row and anyone column from Q.

(iv) finally calculate determinant of Q' to get total no. of spanning trees in 'G'.



G

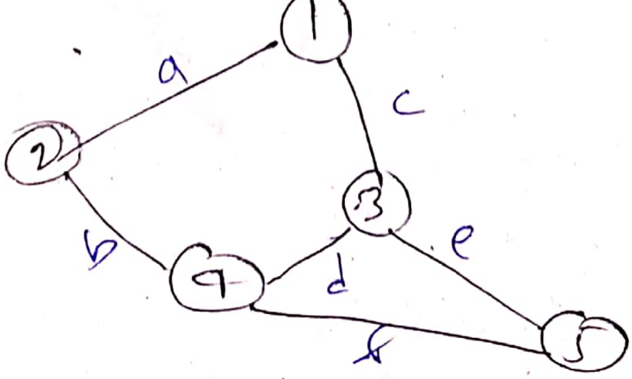
$$Q = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix} \end{array}$$

$$Q' = \begin{vmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{vmatrix}$$

$$= 10 - 2 = \underline{8}$$

total no. of spanning trees $= \underline{8}$

G →



G

$$
Q = \begin{array}{c|c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
2 & -1 & -1 & 0 & 0 \\
-1 & 2 & 0 & -1 & 0 \\
-1 & 0 & 3 & -1 & -1 \\
0 & -1 & -1 & 3 & -1 \\
0 & 0 & -1 & -1 & 2
\end{bmatrix}
\end{array}
$$

$$
Q' = \begin{bmatrix}
2 & -1 & 0 & 0 \\
-1 & 0 & -1 & -1 \\
0 & -1 & 3 & -1 \\
0 & 0 & -1 & 2
\end{bmatrix}
$$

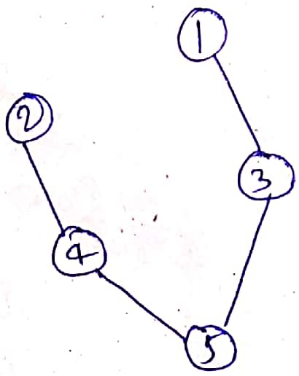
$$= 2\left[+1\left(+1+\cancel{1}\,2\right)\right] + 1\left[-1(6-1)\right]$$

$$= 2\left[+3\right] + 1\left[+5\right]$$

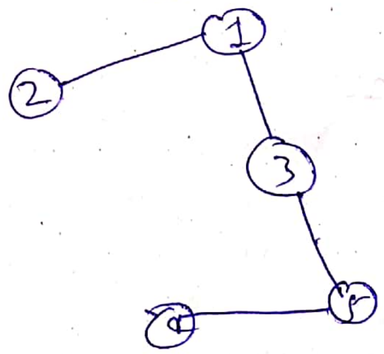
$$= +6 + 5 = \boxed{-11}$$

$$= \underline{11}$$

1



2



3

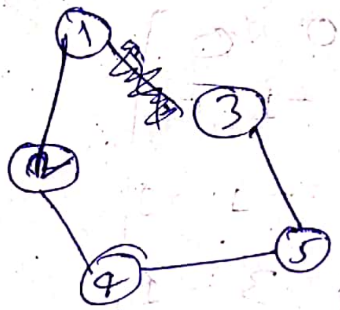
9

$1 \to 4$

$2 \to 7$

$3 \to 11$

$3n+1$



Counting tree

(Arthur calley) → 1857

Hydrocarbons →

total no. of vertices $= 3K+2$

CH_4

C_2H_6

$C_K H_{2K+2}$

$2e = \sum d v_i$

$$2e = (4K + 1\cdot(2K+2))$$

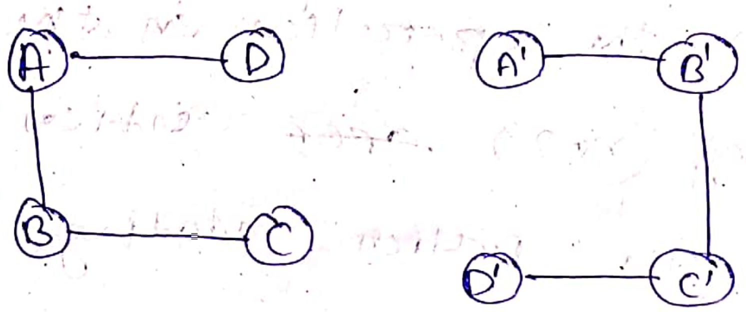
$$e = 2K + K + 1$$

$$\boxed{e = 3K+1}$$

$$\boxed{V = 3K+2}$$

~~If n~~

Lebelled trees



of the same lebells.

Ques, with 'n' vertices how many lebelled trees are possible?

Ans → n^{n-2}

Prufer's theorem

it has two parts

(i) Encoding ~~system~~ sequence

1. let 'n' no. of vertices of a tree ~~of~~ 'T' be labelled as 1, 2, 3 -- n.
2. Remove the pandent vertex say a_1
3. Assume small b_1 is the vertex adjacent to a_1
4. Among the remaining $n-1$ vertices, let a_2 be the pandent vertex with the smallest lebell and b_2 be the vertex adjacent to a_2.

5) Remove the eadge $a_2 b_2$

6) Repeat the operation on the remaining (n-2) ~~oper~~ vertices to get the prufer endoding sequence of $b_1, b_2 \ldots b_n$

$b_1, b_2 \ldots b_n \longrightarrow (1)$

(ii) Prufer Decoding sequence -

1) Determine the 1st number in the sequence $1, 2, 3 \ldots n$.

$1, 2, 3 \ldots n \longrightarrow (2)$

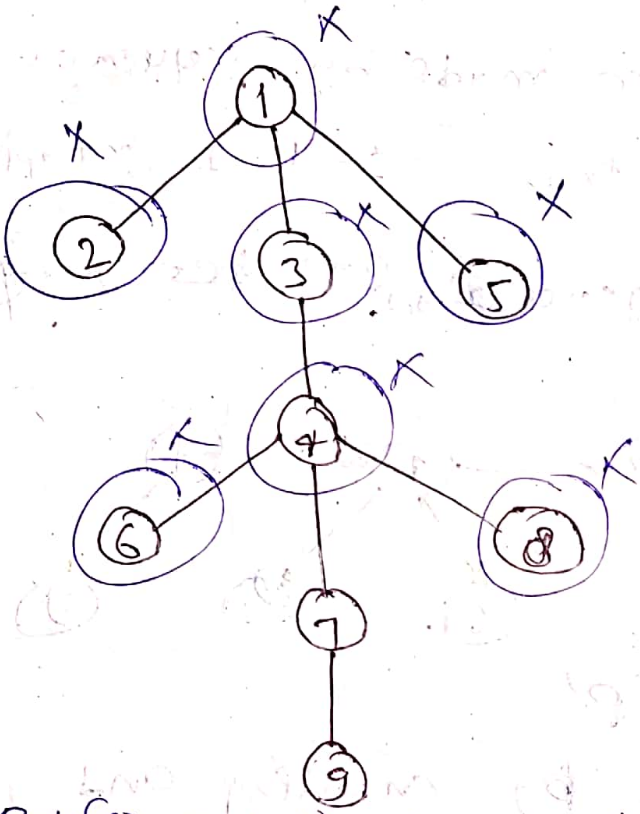
that does not apper in eqn (1).

2) → the number clearly is a_1 and thus the eadge $a_1 b_1$ is define.

3) Remove b_1 from equation (1) and a_1 from eqn (2)

4) in the remaining sequence of eqn (1) (2) find the first number that does not appear in the remainder of equation (1)

5) the construction is continue till the sequence of equation ① has no element left and finally the last two vertices remaining in equation ② are joined.



{1, 1, 3, 4, ~~4~~, ~~4~~, ~~7~~}

{untill last one edge is left or (n-2) long character seqn.}

Prufer encoding sequence →

{1, 1, 3, 4, 4, 4, 7} ⟶ ①

{1, 2, 3, 4, 5, 6, 7, 8, 9}

$$\{\cancel{1}, \cancel{1}, \cancel{3}, \cancel{4}, \cancel{4}, \cancel{4}, \cancel{7}\}$$

$$\begin{array}{cccccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \searrow \\ 2 & 5 & 1 & 3 & 6 & 8 & 4 & 9 \end{array}$$

P.D is = {2, 5, 1, 3, 6, 8, 4}

From one tree 'T'

(n-2) character long encoding ~~a~~

sequence can be found. then

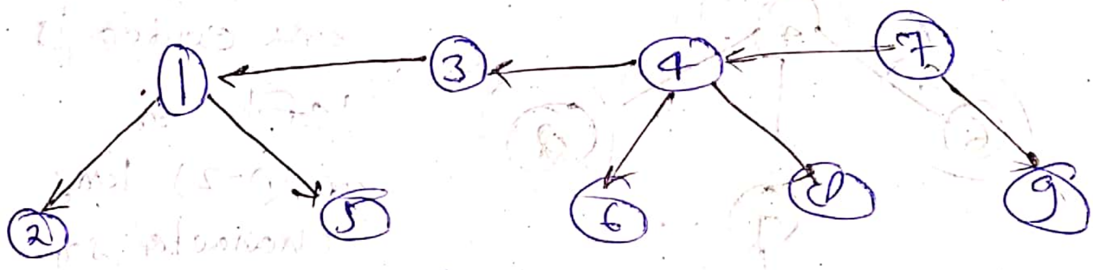
~~a to~~ decoding sequence is found

from sequence ① encoding.

if the tree made by sequence

decoding ①, ② then 1-to 1 mapping

between seqence ①, ② and ~~tess~~ tree



'P'

the tree 'P' made by encoding and

decoding ~~sy~~ sequence and original

tree T ~~DO~~ is similar.

Proof

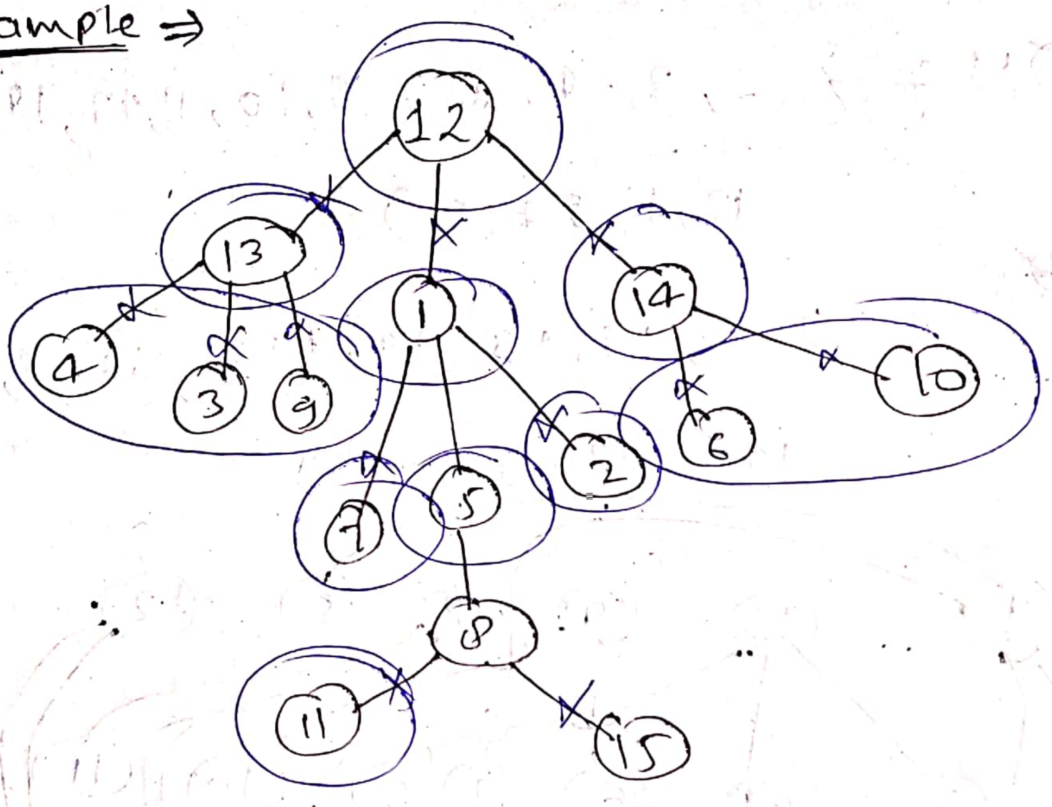
a seqence of (n-2) vertices is formed

by 'n' vertices then total no. of

sequence $= (n)^{n-2}$

~~12~~ ↓14 ~~7~~ ↓12 8 ↓1 9 ↓5 ↘15

example ⇒



~~P E S = { 13, 13, 13, 14, 14, 1, 1, 1,~~

~~P.E.S. = { 13,~~

P.E.S = { 1, 13, 13, 14, 1, 13, 14, 8, 12, 12, 1, 8

P.E.S. ⇒

{ 1, 13, 13, 14, 1, 13, 14, 8, 12, 12, 1, 5, 8 }

P.D.S. ⇒ { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 }

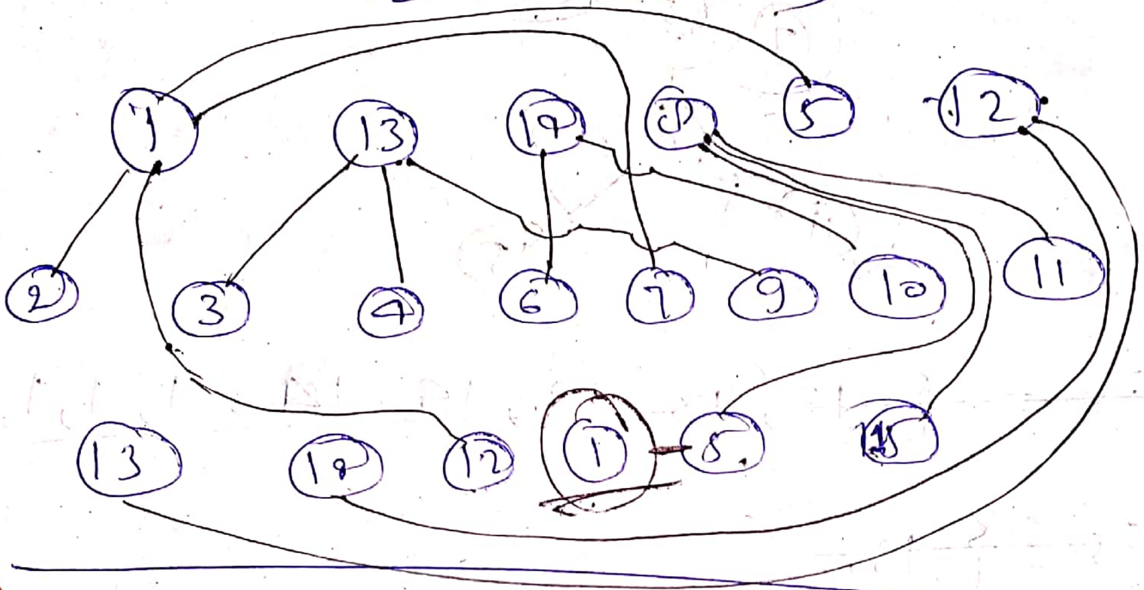
{ ~~1~~ ↓ 2, ~~13~~ ↓ 3, ~~13~~ ↓ 4, ~~14~~ ↓ 6, ~~1~~ ↓ 7, ~~13~~ ↓ 9, ~~14~~ ↓ 10, ~~8~~ ↓ 11, ~~12~~ ↓ 13

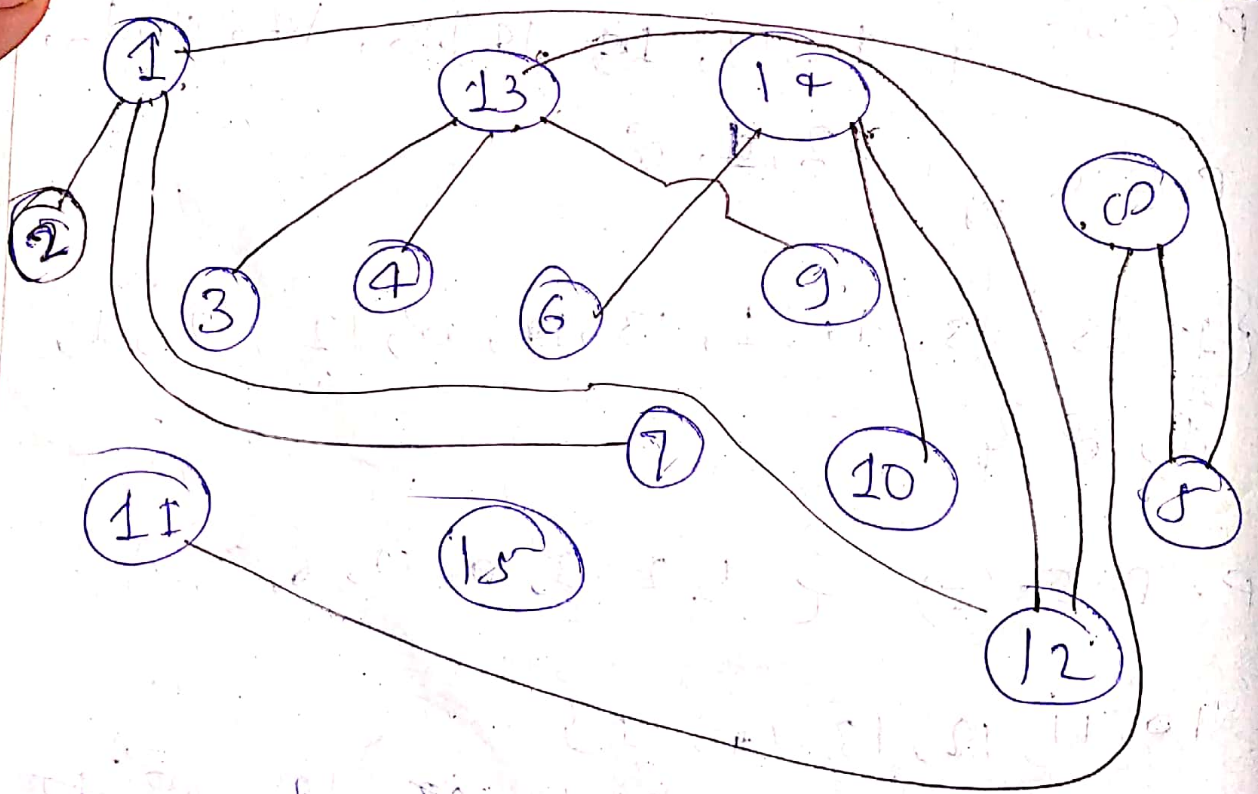
PDFS = { 2, 3, 4, 6, 7, 9, 10, 11, 13, 14,

12, 1, 8, 15 }

DFS = { 1, 13, 13, 14, 1, 13, 14, 8,

12, 12, 1, 5, 8 }





Q) from the given prufer encoding sequence find out prufer decoding sequence and generate the tree

A: P.E.S. ⇒

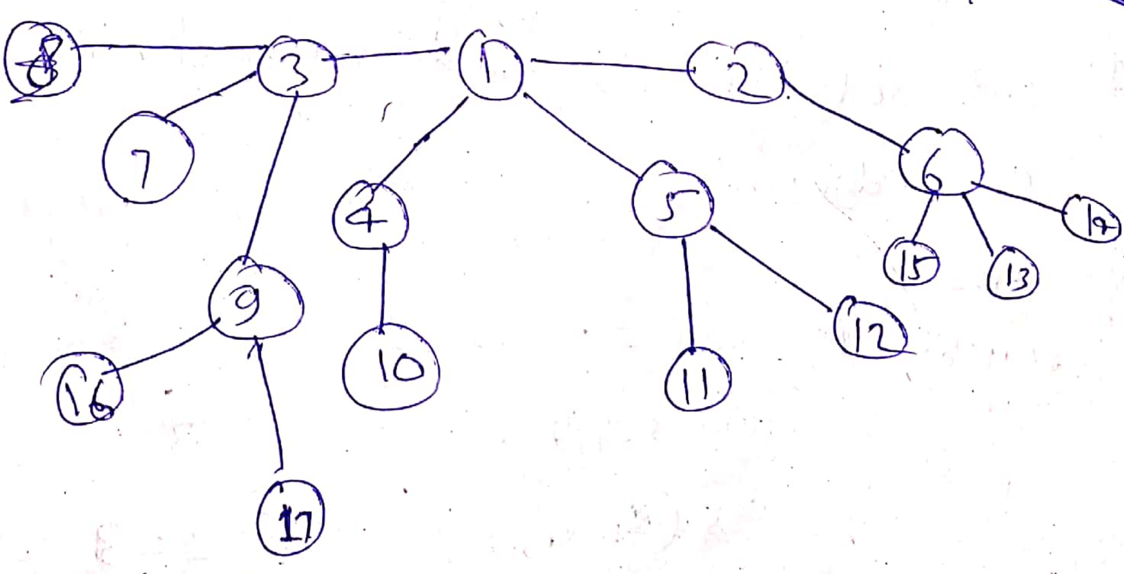
{ 3, 3, 4, 4, 5, 5, 1, 6, 6, 6, 2, 1, 3, 9, 9, 10 }

{1, 2, 3, 4, 5, 6, 7, 8, 9}

$$\Rightarrow \{ \begin{array}{ccccccc} 3 & 3 & 4 & 4 & 5 & 5 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 8 & 10 & 4 & 11 & 12 & 5 \end{array}$$

$$\begin{array}{ccccccccc} 6 & 6 & 6 & 2 & 1 & 3 & 9 & 9 & 10 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 13 & 14 & 15 & 6 & 2 & 1 & 3 & 16 & 9 \end{array}$$

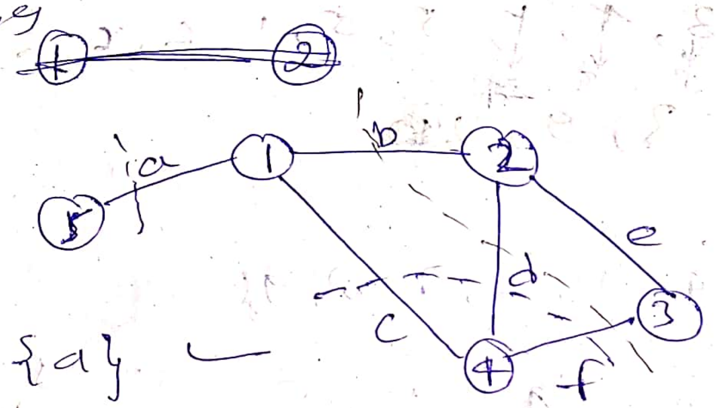
17



Cut-set

(set of edges)

is set of edges ~~so that~~ ~~whose~~ removal from the graph makes remaining graph disconnected provided no-subset of edges



G_1 {a} ✓

G_2 {c, d, f} ✓ all cutset

G_3 {b, d, f} ✓

G_4 {b, d, e, f} (X) Rank of G_4 = 5−3 = 2

Properties

① cut set should ~~own~~ hold minimal no. of edges.

② ~~in~~

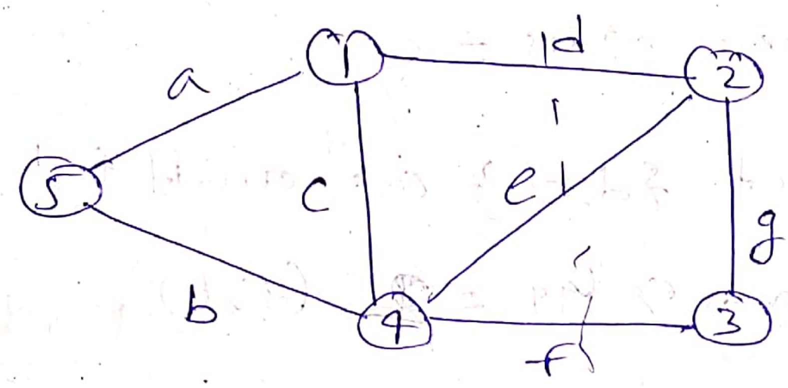
rank of G = 5−1 = 4

" " G_1 {a} = 5−2 = 3

" " G_3 (b, d, f) = 5−2 = 3

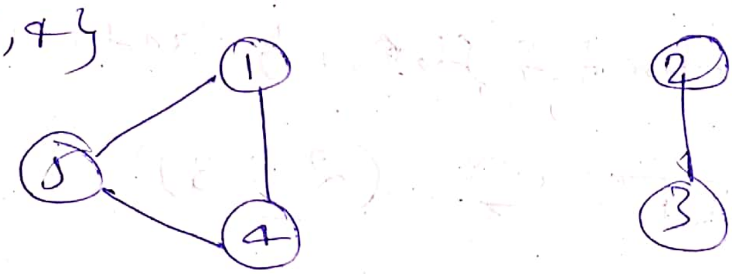
② Rank of graph reduces by 1 after cutset applying.

③ Cut set cuts the graph into two ~~parts~~ set of vertices such that all the paths in b/w these two vertices are distroyed.



V_1 {1, 5, 4}

V_2 {2, 3}

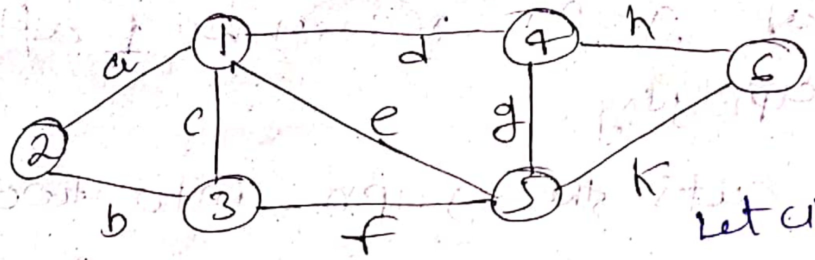


④ every edge in a tree is a cutset.

⑤ every cut set in a connected graph must contain at least one branch of a spanning tree.

⑥ every circuit has an even no. of edges in common with ~~every~~ any ~~circuit~~ cutset.

Scanned by CamScanner



Let a

~~For {d,e,f}~~

For cutset {d, c, f} and circuit {4, 5, 6}

the common edges = 0

for cutset {d, e, f} and circuit {1, 3, 5}

the common edges = 2 (e, f)

For cutset {d, e, f} and circuit

~~(2, 1, 3, 5, 4)~~ (2, 1, 3)

· the common edge = 0

7

The ring sum of ~~two cut~~ any two subset in any graph is either a third cutset ~~and~~ or an edge disjoint union of cut set.

$\{d, e, f\} \oplus \{f, g, h\}$

$= \{d, e, g, h\}$

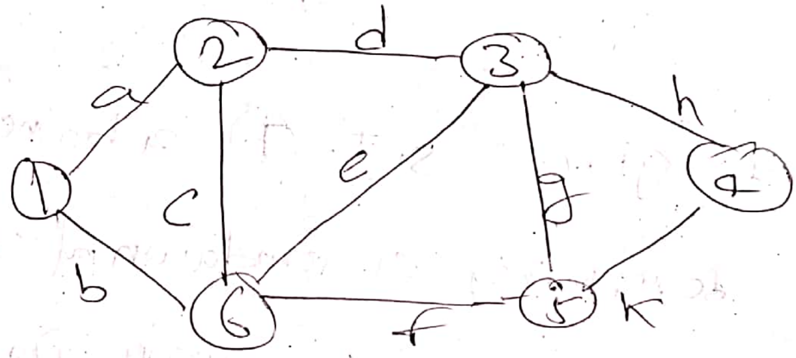
Scanned by CamScanner

$\{d, e, g, h\} \oplus \{f, g, k\}$

$= \{d, e, h, f, k\}$

$= \{d, e, f\} \cup \{h, k\}$

Fundamentall cutset | circuit



S.T. (T) ⟶

bn $= \{b, a, d, h, k\}$

cn $= \{c, e, g, f\}$

fundamental cut sets ~~cut~~ ~~is~~ specific cutset which chords exactly one branch of spanning tree. and graph will have $(n-1)$ no. of ~~cutsets~~ cutsets.

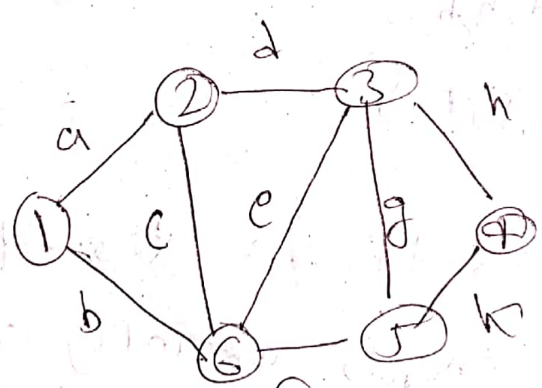
$b_n = \{b, a, d, h, k\}$

$\{b, c, e, f\}$ $\{a, c, d\}$, $\{d, e, f\}$

Relation b/w fundamental cutset and fundamental circuit.

① w.r.t. a given s.tree (T) a chord 'Ci' that determine a fundamental circuit 'F' occurs in every fundamental cutset associate in the branches in 'F'.

② w.r.t. given S.T. (T) a branch bi' that determines a fundamental cutset 'S' is contained in every fundamental circuit associated with the chords in 'S'.



①

T_1 $b_n = \{a, b, d, h, k\}$

$c_n = \{c, e, g, f\}$ → circuit

take edge e from fundamental circuit

chord 'e' $\rightarrow$ circuit {b, a, d, e}

b $\rightarrow$ {b, c, e, f}

a $\rightarrow$ {a, c, e, f}

d $\rightarrow$ {d, e, f}

② branch d $\rightarrow$ cutset {d, e, f}

d $\rightarrow$ {b, a, d, e}

f $\rightarrow$ {b, a, d, h, m, f}

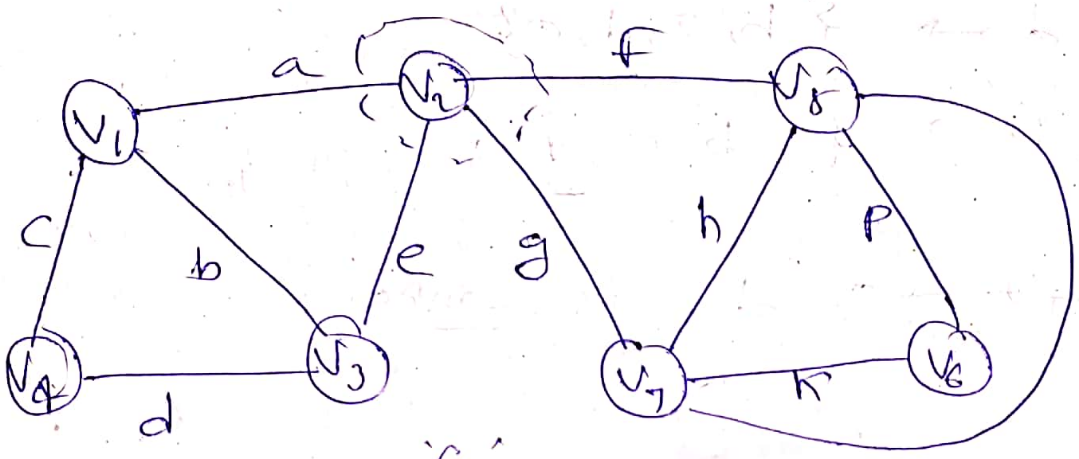
~~d is~~ d is common.

Connectivity

the cut set with mini. no. edges

edge connectivity of graph is no. of edges in smallest cutset.

cutset {c,d}, edge connectivity = 2



G

the mini. no. of vertices whose removal makes a connected graph into a disconnected is called vertex connectivity of the graph.

vertex connectivity of G = 1 (V_2)

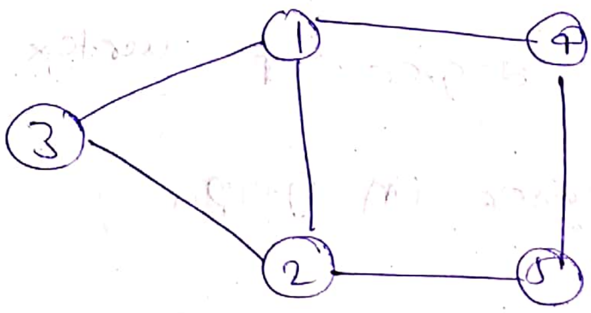
if a graph has vertex connectivity is equal to 1, graph is known as separable graph.

V_2 is cut vertex.

If ~~ee~~ vertex connectivity is greater than 1 then the graph is called non- separable or n- separable.

n ~~is~~ no. of vertices to remove for making graph ~~dis separa~~ dis connected

in separable graph the vertex, ~~is~~ which is deleted the vertex is called cut vertex or Articulation point

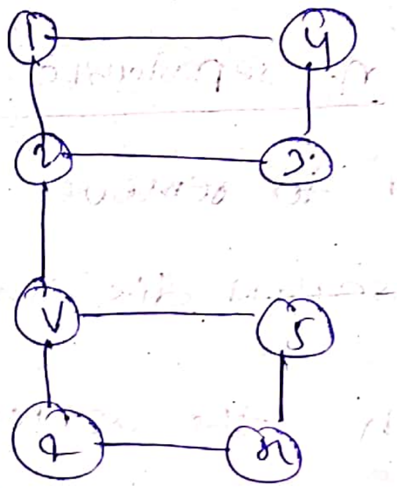


non-separable graph
2 - separable graph.

theorem-1

A vertex 'v' in a connected graph 'G' is a cut vertex ~~if and on~~ iff there exist two vertices 'x' and 'y' in G such that every path b/w

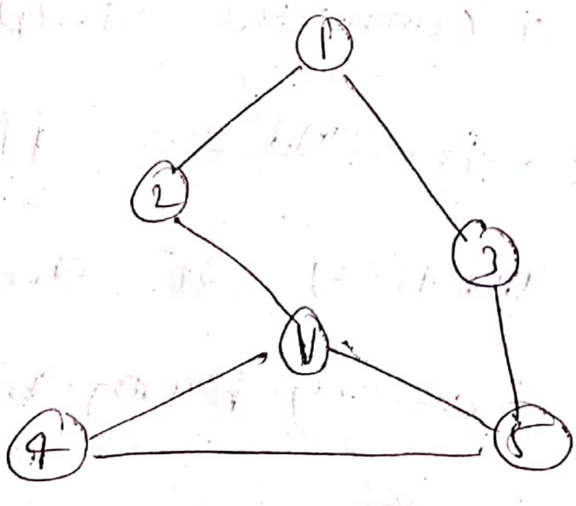
~~Pa~~ n and y passes through 'V'



Theorem - 2

the edge connectivity of graph can't exceed the degree of vertex with smallest degree in graph

the vertex connectivity of a graph can never exceed the edge connectivity of graph



V'C' = 1

e.c = 2

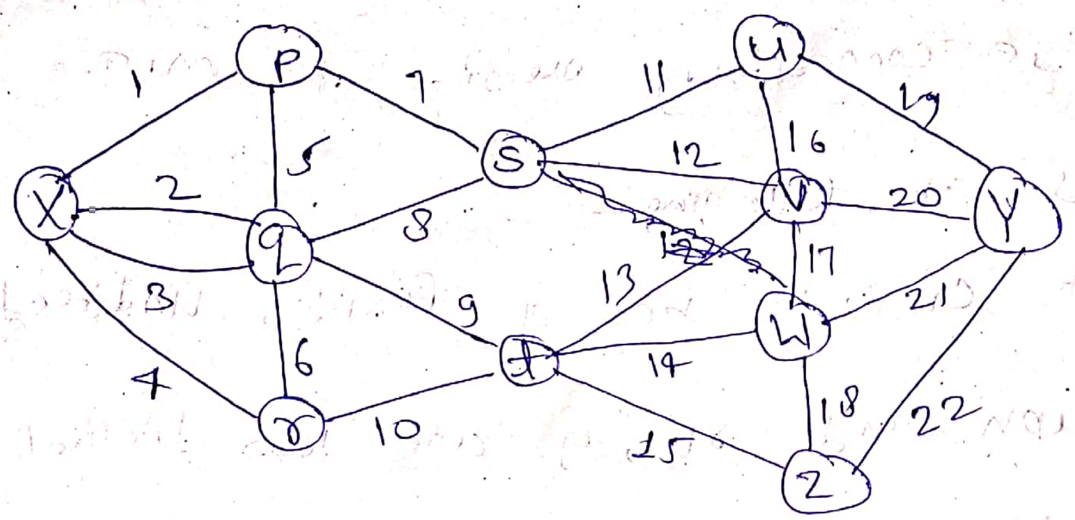
⇒ edge connectivity version of M.T. is as following.-

Let 'G' ~~is~~ be a finite, undirected graph and x, y are two distinct vertices, theorem states that the size of min. edge cut for x and y is equal to max. no. of pairwise edge disjoint paths from x to y.

⇒ vertex connectivity version of M.T. is as following -

Let G be a finite, undirected graph and x, y are two distinct vertices, theorem states that -

the size of min. vertex cut is equal to for x and y is equal to the max. no. of pairwise ~~dis~~ vertex disjoint paths from x to y.



edge connectivity for x and y

= 4 {7, 8, 9, 10}

P_1 : 1 - 7 - 11 - 19

P_2 : 2, 8, 12, 20

P_3 : 3, 9, 14, 21

P_4 : 4, 10, 15, 22

vertex connectivity = 2 {s, t}

P_1 : X, P, S, U, Y

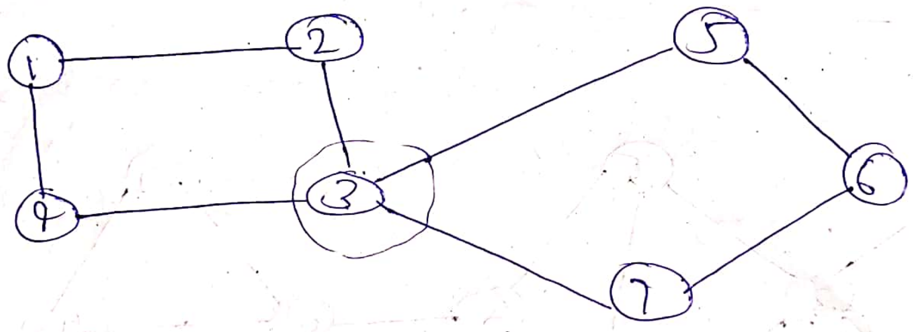
P_2 : X, r, t, W, Y

1- Isomorphism → separable graph

Component v/s Blocks

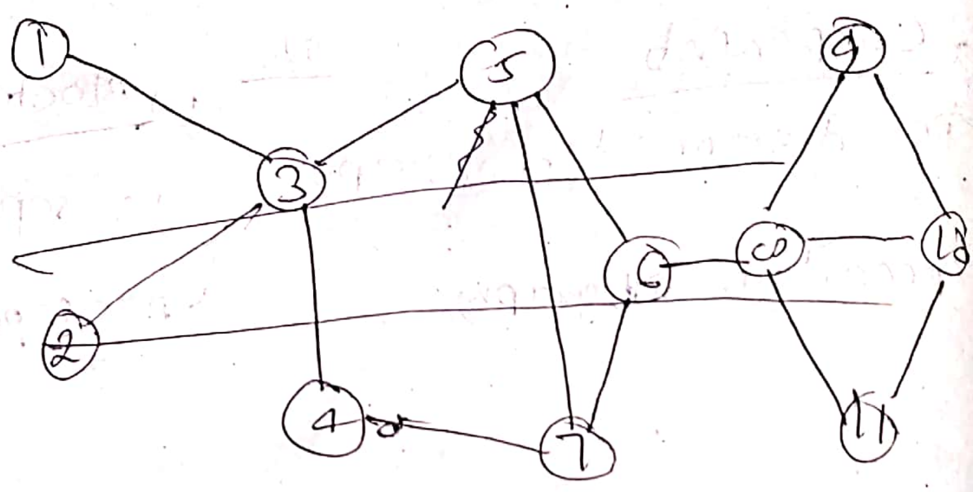
Component: in disconnected graph, connected subgraph

Blocks: In separable graph, nonseparable parts

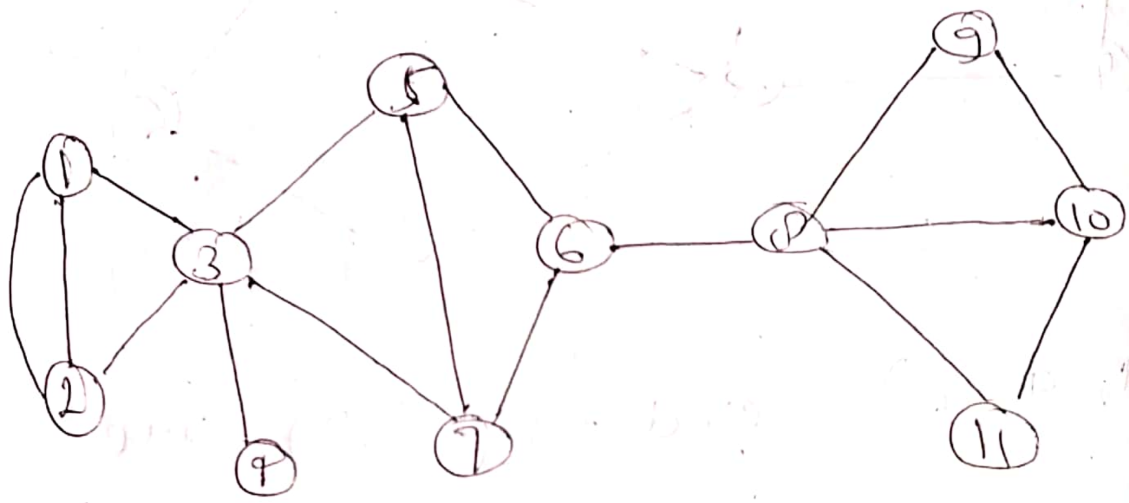


(1, 4, 2) and (5, 6, 7) are component of 'G' on deleting 3.

(4, 1, 2, 3) and (3, 5, 6, 7) are blocks w.r.t. 3 in G.



ex



G

3 or 6 or 8 are cut vertex

~~ele~~ on taking 3 as cut vertex

(1, 2, 3), ~~→ block~~ (3, 4) are blocks

on taking 6 as cut vertex

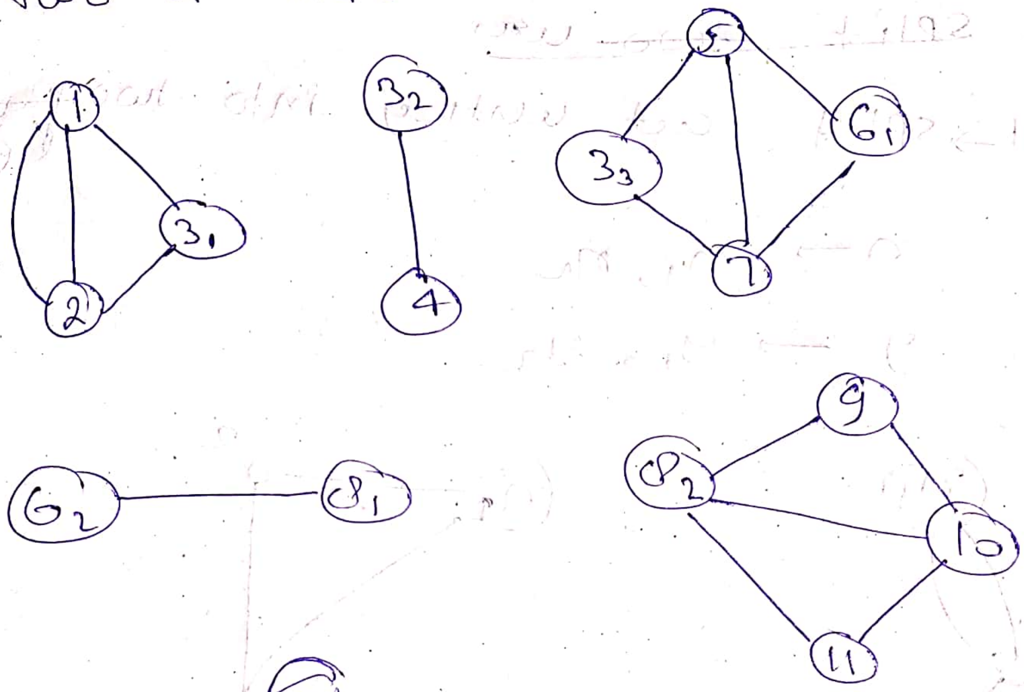
(8, 9, 10, 11) is block

turning 8 as cut vertex

(8, 9, 10, 11) is block.

<u>operation</u> ⇒

split the cut vertices into two or more vertices.



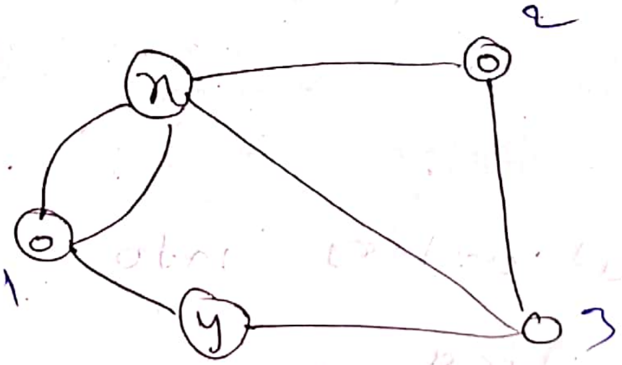
G_1

each of the block of G_1 is isomorphic to each component of G.

this phenomena is called 1-isomorphism

Rank of G = Rank of G_1 = 10

2- Isomorphism : non separable graph,



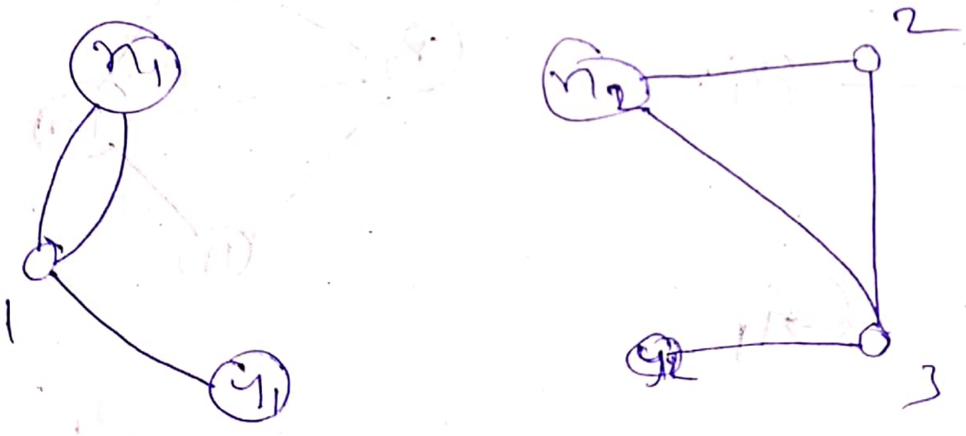
G_1

<u>split ~~two~~ way</u>

1 → split cut vertices into two ~~parts~~ side,

$x \rightarrow x_1, x_2$

$y \rightarrow y_1, y_2$

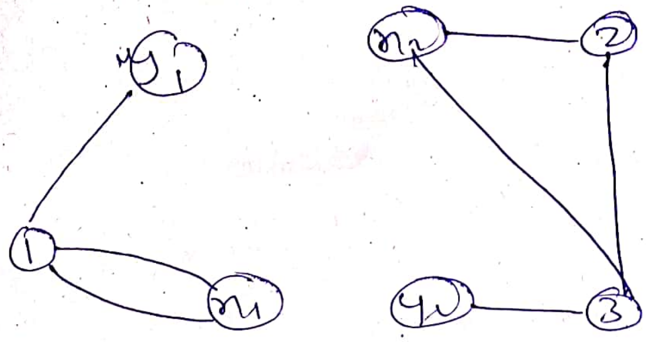


G_2

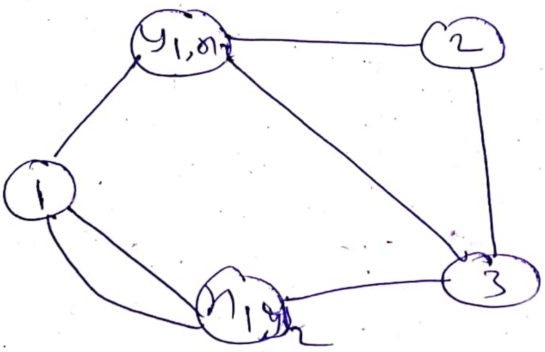
2 → twist the first part of graph in such way that $x_1 \Rightarrow y_2$

$x_2 \Rightarrow y_1$

3. and rejoin the graph



G_1:



G_2

G_1, G_2 are called two-isomorphic graph

and this phenomena is 2-isomorphism