17/7/19

Set Theory

Set - Unordered collection of objects.

$N \rightarrow$ set of +ve integers, $Z \rightarrow$ +ve Int, 0, -ve Int

$Q \rightarrow$ Rational no., $R \rightarrow$ Real No., $C \rightarrow$ complex no.

Universal sets can have variable size.

$$\boxed{C \supseteq R \supseteq Q \supseteq Z \supseteq N}$$

1. $\emptyset \subseteq \emptyset$ T   (Every set is a subset of itself)
2. $\emptyset \in \emptyset$ F   (Null set is the subset of all sets)
3. $\emptyset \subseteq \{\emptyset\}$ T
4. $\emptyset \in \{\emptyset\}$ ~~F~~ T
5. $\{\emptyset\} \in \emptyset$ F.
6. $\{\emptyset\} \subseteq \{\emptyset\}$ T.
7. $\{\emptyset\} \in \{\emptyset\}$ F.

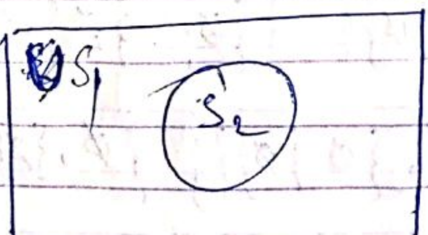
$E = \{x : x^2 - 3x + 2 = 0\}$ $F = \{2, 1\}$, $G = \{1, 2, 2, 1, 6/3\}$

$E \subset F$ (False) $E \subseteq F$

$\rightarrow$ E is proper subset of F

- S_1 contains all the students of NIT A
  S_2 contains all the students of 3rd sem of 19-20 batch

Draw a venn diagram that represents both these sets.



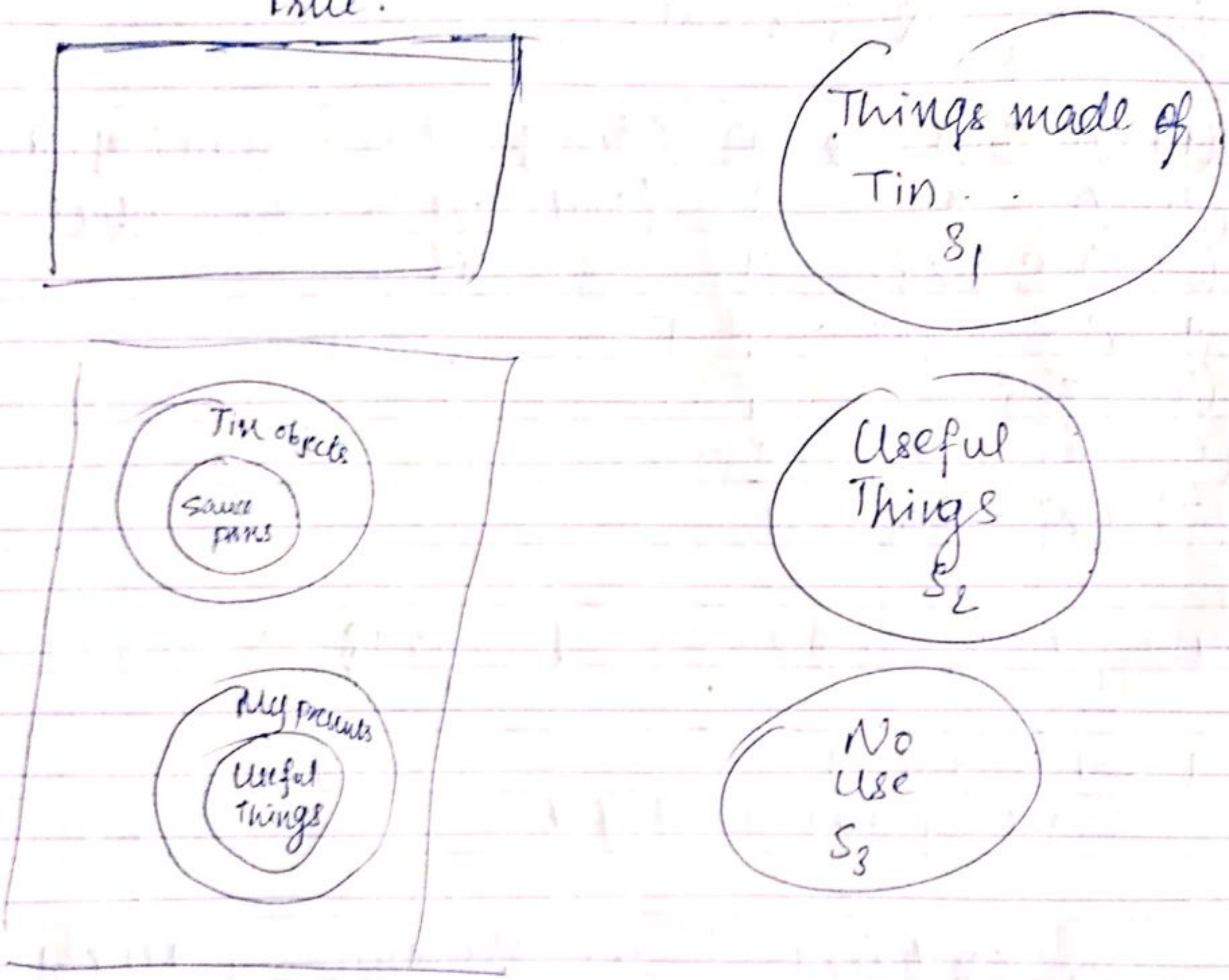
Q. Set S_1 is my sauce pans are only things I have that are made of tin

S_2: I find all my presents to be useful

S_3: Non of my saucepans are of slightest use

Conclusion : Your presents to me are not made of tin

True.



Q $S = \{ \phi, \{\phi\}, \{\phi\{\phi\}\} \}$

Power set of $S = \{ \{\phi\}, \{\{\phi\}\}, \{\{\phi\{\phi\}\}\},$

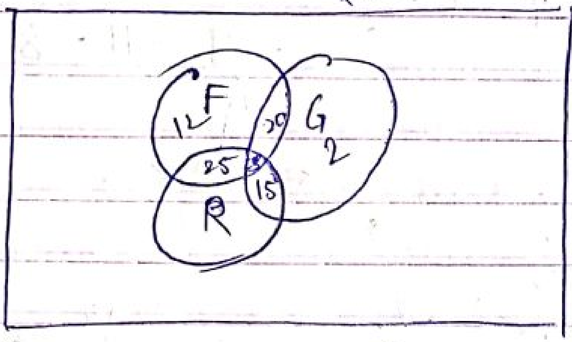
Cardinality = 3 $\quad \{\phi, \{\phi\}\} \; \{\{\phi\}, \{\phi\{\phi\}\}\},$

$\angle$ No. of elements in power set $= 2^3$ $\quad \{\phi, \{\phi\{\phi\}\}\} \; \{\phi, \{\phi\}, \{\phi\{\phi\}\}$

Q Find the no. of Mathematics students ~~in~~ at a college taking at least one of the languages French, German and Russian given the following data.

65 study French, 45 study German, 42 study Russian, 20 study French and German, 25 study French and Russian, 15 study German and Russian and 8 study all the 3 languages.

Principle of inclusion



$\cancel{A \cup B = A + B}$

$$A \cup B \cup C = A + B + C - A \cap B - A \cap C - B \cap C + A \cap B \cap C$$

$$= 65 + 45 + 42 - 20 - 25 - 15 + 8$$

$$= 100$$

1.3.2

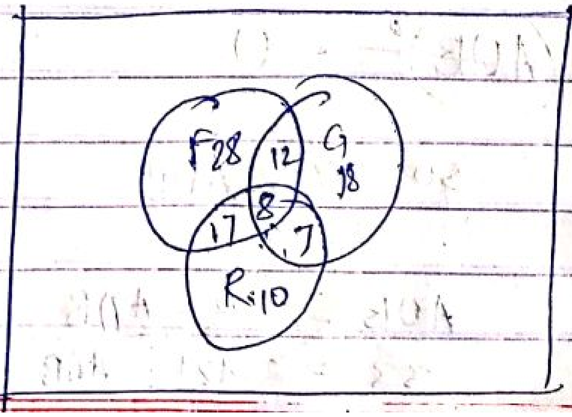
Q Consider the following data for 130 Mathematics students at a college concerning the languages French, German & Russian. 65 study French, 45 German, 42 Russian, 20 French and German, 25 French and Russian; 15 study G and R, 8 all langauges.

F − 28   65

G − 18   37

R − 10   28

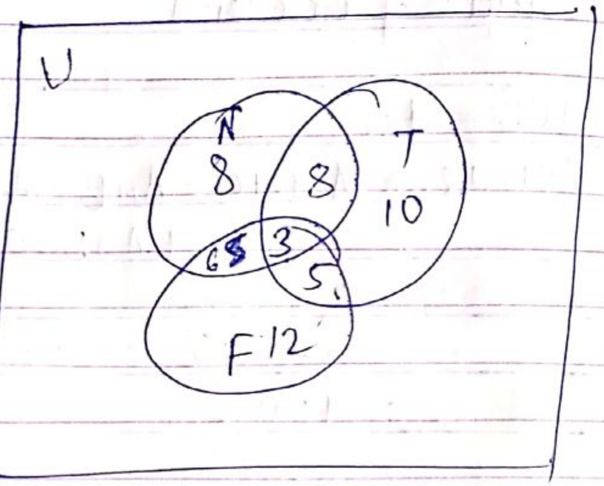
$\cancel{A \; F = (F \cup G)}$ 45 27 18



Q. In a survey of 60 people it was found that 25 read Newsly magazine, 26 read time 26 read fortune, 9 both newsly and fortune 11 read Newsly & Time, 8 both time & fortune and 3 all the 3 magazine.

① find who read at least one of the three magazine.
② find people who read exactly one magazine
③ Depict all the 8 regions of the venn diagram

$N \cup T \cup F = 25+26+26$
$\quad -9-11-8$
$\quad +3$
$\quad = 52.$



-28

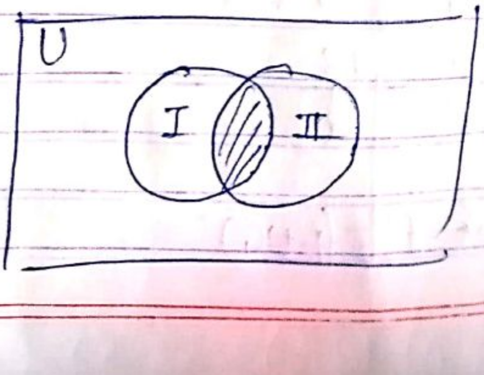
26
26
52
25
77
3
10
7
80
28

14
12

$N \rightarrow 8$
$T \rightarrow 10$
$F - 12$

Q. Among 50 students in a class 26 got A in first exam and 21 got A in second exam if 17 students did not get an A in any exam, how many got A in both

$\cancel{(A \cup B)^c = U}$



$50 - 17 = A \cup B$
$\quad 33$

$A \cup B = A + B - A \cap B$
$33 = 26 + 21 - A \cap B$

$A \cap B = 47 - 33$
$\quad = 14$

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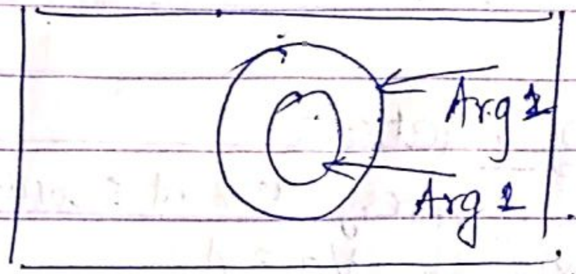
Q Draw a venn diagram to test the validity of the following arguement

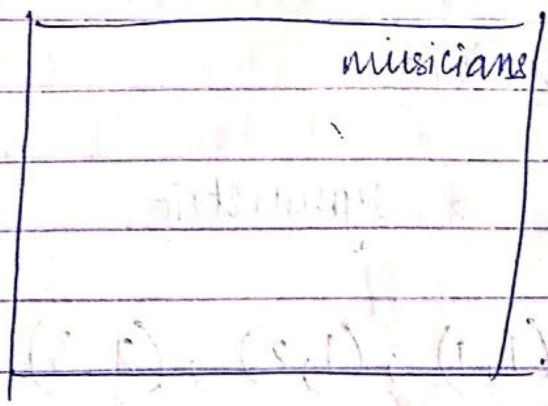
Arg 1 : All guilty people will be arrested
Arg 2 : All thieves are guilty people

Con : All the theives will be arrested. True



Q. Determine the validity of following arg.

Arg 1 : All my friends are musicians.
Arg 2 : John is my friends
Arg 3 : None of my neighbours are musician
Con : John is my neighbour. False.



Generally ϕ is symmetric & Transitive but not reflexive.

Relation

cartesian product: $A = \{1, 2\}$, $B = \{a, b, c\}$

$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$

A relation is subset of set of cartesian product of two sets

Types of relations

1) Reflexive $\rightarrow$ every element should be related to itself $\forall a \in A$
2) Symmetric $\forall (a,b) \in R : a, b \in R$ $(b,a) \in R$
3) Transitive $\forall (a,b) \in R$ & $(b,c) \in R$ $(a,c) \in R$

$A = \{1, 2, 3\}$ V ~~$R = \phi \rightarrow$ not reflexive.~~

$R = \{(1,1), (2,2), (2,1)\} \rightarrow$ not a reflexive relatio

$R = \{(1,1), (2,2)\} \rightarrow$ not a reflexive relation

Symmetric

$R = \phi \rightarrow$ symmetric.

Transitive

$R_1 = \{(1,1), (1,2), (2,3), (1,3), (4,4)\}$ over the set ($A = \{1, 2, 3, 4\}$

~~11 21 21 22~~

$R_2 = \{(1,1)\ (2,1)\ (1,2)\ (2\ 2)\ (3,3)\ (4,4)\}$

Reflexive, Transitive, Symmetric

$R_3 = \{(1,3)\ (2,1)\}$

$R_4 = \phi$, $R_5 = A \times A \rightarrow$ universal relⁿ.

↓
Symmetric
Transitive.

$R_5 = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,1), (2,2)$
$(2,3)\ (2,4)\ (3,1)\ (3,2)\ (3,3)\ (3,4)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\}$

Reflexive, Symmetric, Transitive.

Q1. ~~Relation~~ 1. R $\leq$ over the set Z of integer.

2. Set $\subseteq$ on a collection of 'C' sets.

3. R $\perp$ on the set 'L' of lines in the plane.

4. R $\parallel$ on the set of lines L in the plane.

5. R divisibility on a set 'n' of +ve integers

*1. Reflexive, Transitive
*2. ~~Reflexive~~, Transitive, ~~Symmet~~, Reflexive.
3. Symmetric, Transitive (only in 3D)
4. Symmetric, Transitive, Reflexive → (every line is parallel to itself).
*5. Reflexive, Transitive

Relations 1, 2, 5 are important

(8, 4) (4, 2)
(8, 2) (2

4,

$a < b$
$b < c$

Q. Let R be a relation defined on a set of +ve integers. ~~such that~~ such that $x, y \in R$ iff $(x-y)$ is divisible by 3. Prove that,

R is an equivalence relation.

→ Reflexive, Symmetric, Transitive

Sol^n Let the set be ~~Z~~ A

$A \cancel{Z} = \{x, y : (x-y) \text{ is divisible by } 3\}$

~~If (x-y) is d~~ $(x-y)/$

Let $(x-y)/3 = n \qquad n \in Z$.

Then $(y-x)/3 = -(x-y)/3 = -n \in Z$

⇒ Thus set A is symmetric.

Suppose $x = y$

Then $(x-y) = 0$

∴ 0 is divisible by any number.

⇒ ∴ Set A is Reflexive.

$(x-y)$ is divisible by 3, $(y-z)$ is divisible by 3.

∴ $(x-z)$ is also divisible by 3.

∴ Transitive.

∴ Relation is equivalence

Anti Symmetric $\forall (a,b) \in R$ $(b,a) \in R$ iff $a = b$.

Find a relation which is symmetric but not antisymmetric and vice-versa.

* Assymmetric Property : If $(a,b) \in R$, $(b,a) \notin R$.

* Partially ordered set or POSET

→ Ref + Antisymm + Transitive

Q. Let R be a relation defined on a set of ordered pair of +ve integers such that $\forall$ $(x,y), (u,v) \in Z^+ \times Z^+$ iff $\frac{u}{x} = \frac{v}{y}$

Let the set be A

$$A = \left\{ (x,y), (u,v) : \frac{u}{x} = \frac{v}{y} \right\}.$$

$$\frac{x}{x} = \frac{y}{y} \rightarrow \text{Reflexive}$$

$(x, y) \quad (u, v) \qquad\qquad (u, v) \quad (x, y)$

$$\frac{u}{x} = \frac{v}{y} \qquad\qquad \frac{y}{v} = \frac{x}{u}$$

$\therefore$ It is symmetric.

~~$\therefore$ The relation is an equivalence.~~

$(x, y) \quad (u, v) \quad (w, z)$

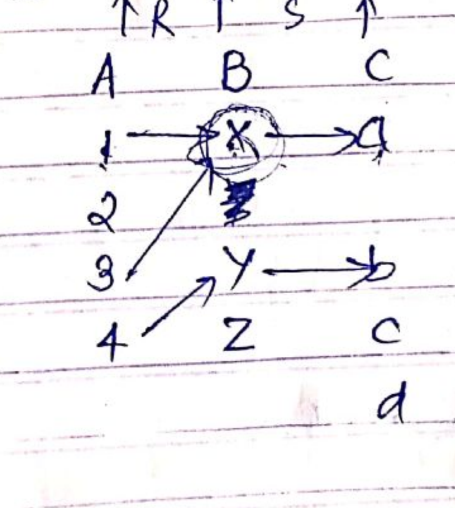
$$\frac{u}{x} = \frac{v}{y} = n \qquad\qquad \frac{w}{u} = \frac{z}{v}$$

$$\frac{w}{\cancel{n}x} = \frac{z}{\cancel{n}y}$$

$\therefore$ It is a transitive relation

Thus A is an equivalence relation

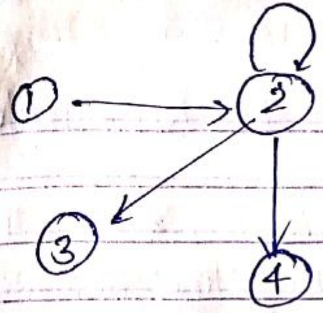
24/7/19 Composition of Relation



Composition of R and S or

ROS = { (1,a) (3,a) (4,b) }

Follows the transitive property.



(1, 2) (2, 4) (2, 2) (2, 3)
~~(2, 4~~

All elements are not related to themselves - Not reflexive
Not symmetric neither transitive

R = {(a, b) (c, d) (b, b)

Q R = { (a, b) (c, d) (b, b)}

S = { (d, b) (b, e) (c, a)}, (a, c)}
d b b e c, a a c

Find ROS, SOR, (SOR) O R, RO(SOR), ROR
SOS, ROROR

~~ROS~~

Soln : ROS = { (a, e) (c, b) (b e)}

SOR = { (d, b) (c, b) (a, d)}.

(SOR) OR = {(d, b) (c, b)}

RO (SOR) = { (c, b)}

~~ROR = {(a, a) (c, c) (b, b)}~~ ROR = { (a, b) (b, b)}

~~SOS = {(d, d) (b, b) (c, c) (a, a)}~~

~~ROROR = {(a, b) (c, d) (b, b)}~~

SOS = { (d, e) (c, c) (a, a)}

ROROR = { (a, b) (b, b)}

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Q**   congruence relation - ?

~~A = B|M|~~ $a = b \pmod{m}$

$$a = b \pmod{m}$$

a → Dividend

b → Remainder

m → divisor.

$$m \overline{) \; a \; (} \atop b$$

Q. Let 'm' be a fixed positive integer. Two integers a and b are said to be congruent modulo written as $a = b \pmod{m}$. Find whether it is an equivalent relation

$$a = m(\underset{\text{quotient}}{K}) + b.$$

~~Symmetric~~. Reflexive.

~~$a = m(k) + c$~~.

$(a-b) = mk$

$b - a = -mk$

$= m(-k)$ → k can be -ve also as ~~also~~ relation does not talk abt k.

$a = m(k) + b$

+

~~b~~ $= m(k_1) + c$.

$a = m(k+k_1) + c.$ → again the relation does not talk abt $(k+k_1)$

∴ Equivalence set.

25/7/19 Equivalence Class & Partition

Q. Consider the following relation R on set S = {1, 2, 3}. R = {(1,1) (1,2) (2,1) (3,3)}

(1) Show whether it is an equivalence relation or not.

→ Since every element is not related to itself the given relation is not an equivalence relation. ~~For a relation to be reflexive~~ ~~∀ a ∈ A c~~

Q.(2) Let R be the following equivalence relation on the set A = {1, 2, 3, 4, 5, 6}

R = {(1,1) (1,5) (2,2) (2,3) (2,6) (3,2) (3,3) (3,6) (4,4) (5,1) (5,5) (6,2) (6,3) (6,6)}.

Find the partition of A/R → A induced by R and find the equivalence class of R.

Refeexive, Symmetric, Transitive ⇒ Equivalence reln.

Equivalence classes

[1] = {1, 5}
[2] = {2, 3, 6}
[3] = {2, 3, 6}
[4] = {4}
[5] = {1, 5}
[6] = {2, 3, 6}

Partition = [{1, 5}, {2, 3, 6}, {4}]

If a relation is equivalence then the equivalence class containing a particular element would be present in only & only ~~one~~ that ~~on~~ equivalence class and no other.

Q. Let R_5 is a relation on the set Z of integers defined by $x \equiv y \bmod 5$

x ↙ dividend, y ↓ remainder, 5 → divisor

Find the ~~pa~~ equivalence class of the given relation

$x = y \bmod 5$

$x = 5k + y$

$\underbrace{(x-y)}_{\text{remainder}} = 5k.$

These are equivalence classes of the remainders

$[0] = \{0, 5, 10, 15, 20, -5, -10 \dots\}$

$[1] = \{1, 6, 11, 16, \dots\}$

$[2] = \{2, 7, 12, 17, \dots\}$

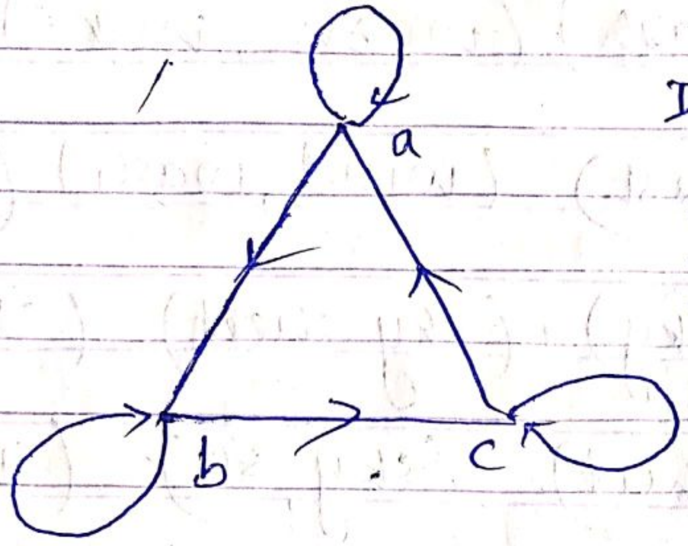
$[3] = \{3, 8, 13, 18, \dots\}$

$[4] = \{4, 9, 14, 19, \dots\}$

Partition class = $[\{0, 5, 10, 15, 20, -5, -10, \dots\}, \{1, 6, 11, 16, \dots\}, \{2, 7, 12, 17, \dots\}, \{3, 8, 13, \dots\}, \{4, 9, 14, 19, \dots\}]$

↓

W/R (W induced by R).



Directed Graph

$\{(a,a)\ (a,b)$

$(b,b), (b,c), (c,c)$

$(c,a)\}$

The relation is only reflexive neither symmetric nor transitive.

$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 0 & 0 & 1 \\ b & 1 & 0 & 1 \\ c & 0 & 1 & 1 \end{array}$$

$R = \{(a,c), (b,a), (b,c), (c,b), (c,c)\}$

Form relation from row to column.

Not an equivalence relation.

$\because$ It is not reflexive. Not symmetric, neither transitive

For a relation to be symmetric the matrix above and below the leading diagonal should be the same.

Q

$$M_R = \begin{array}{c|cccc} & a & b & c & d \\ \hline 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}_{(4\times 4)} \quad M_S = \begin{array}{c|ccc} & x & y & z \\ \hline a & 0 & 0 & 0 \\ b & 1 & 0 & 1 \\ c & 0 & 1 & 0 \\ d & 0 & 0 & 0 \end{array}_{(4,3)}$$

$$M_{ROS} = M_R M_S = \begin{array}{c|ccc} & x & y & z \\ \hline 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 3 & 1 & 0 & 1 \\ 4 & 0 & 0 & 0 \end{array}$$

$(M_{R_{11}} \times M_{S_{11}}) +$
$(M_{R_{12}} \wedge M_{S_{21}})$
$+ (M_{R_{13}} \wedge M_{S_{31}})$
$+ (M_{R_{14}} \times M_{S_{41}})$

* Closure

$S = \{1, 2, 3\}$

$R = \{(1,1)\ (2,2)\ (1,2)\}$

Relation R is not ~~under~~ closed under reflexive property.

Q. Let $A = \{1, 2, 3, 4\}$. Let $R = \{(1,3) (2,2), (1,4) (3,4)\}$

Find Reflexive, Symmetric and Transitive closure of this relation

Reflexive closure : $\{(1,3)\ (2,2)\ (1,4)\ (3,4),$
$+ \{(1,1)\ (3,3)\ (4,4)\}\}$

Symmetric closure $= \{(1,3)\ (2,2)\ (1,4)\ (3,4)\}$

$+ \{(3,1)\ (4,1)\ (4,3)\}$

This relation is closed under transitive property.

For Symmetric : we add the inverse

Q Let $X = \{1, 2, 3, 4\}$. Let R be a relation such that $X-Y$ is divisible by 3. Find whether it is equivalence.

$X \neq Y \bmod 3 \rightarrow$ divisor

dividend Remainder

$X = Y \bmod 3 \rightarrow$ divisor.

dividend Remainder

Q. Show that the relation $R = \{(a,b) : (a+b) = \text{even int}\}$

is an equivalence relation

Solⁿ: i) ~~∀ a ∈~~ $\forall (a,a) \in R$ $(a-a) = 0$ i.e an even integer

$\therefore$ R is a reflexive relation

ii) ~~∀ (a,b) ∈ R (b~~

let $a-b = k$ $\forall (a,b) \in R$ where k is even integer

$\therefore$ $b-a = -k$ where $-k$ is also even integer

$\therefore$ R is symmetric relation

iii) let $(a-b) = k_1$ and $(b-c) = k_2$ ~~where~~ ~~k_1, k_2~~

$\forall$ (a,b) and $(b,c) \in R$, k_1, k_2 are even integers

$a-b = k_1$
$b-c = k_2$

$\therefore$ $a-c = k_1 + k_2$

where $k_1 + k_2$ is also an even integer as K_1 & K_2 are both even integers.

$\therefore$ $(a,c) \forall R$ $\therefore$ R is also transitive.

$\therefore$ R is an equivalence relation.

Q. Let $A = \{1, 2, 3, 4, 5\}$. Find the equivalence relation generated by the partition $\{\{1,3,5\}, \{2,4\}\}$

$R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)$

—

Q. Find whether the following relations over positive integers 'n' reflexive, symmetric, anti symmetric and transitive

i) $x > y$ ii) xy is the square of an integer

iii) $x + y = 10$

iv) $x + 4y = 10$

v) $x + y = 5$

ϕ is antisymmetric, Antisymmetric, Transitive

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i) Transitive, ~~antisymmetric~~

ii) Reflexive, Symmetric, Transitive

iii) ~~Reflexive~~, Symmetric

iv) ~~Reflexive~~

v) ~~Reflexive~~, Symmetric.

Q. Let R be a relation on a set of all people. Determine whether the following relation are equivalence or not

i) A is related to B when A and B are of same age. Reflexive, symmetric, Transitive.

ii) A and B have same parents. Reflexive, Symmetric, Transitive

iii) A and B share a common parent
Reflexive, Symmetric

iv) A and B have met. Reflexive, Symmetric

v) A is brother of B. Transitive, ~~Reflexive~~ (A girl can't be her brother)

vi) A and B speak a common language.
~~Reflexi~~ Reflexive, Symmetric, ~~Transitive~~

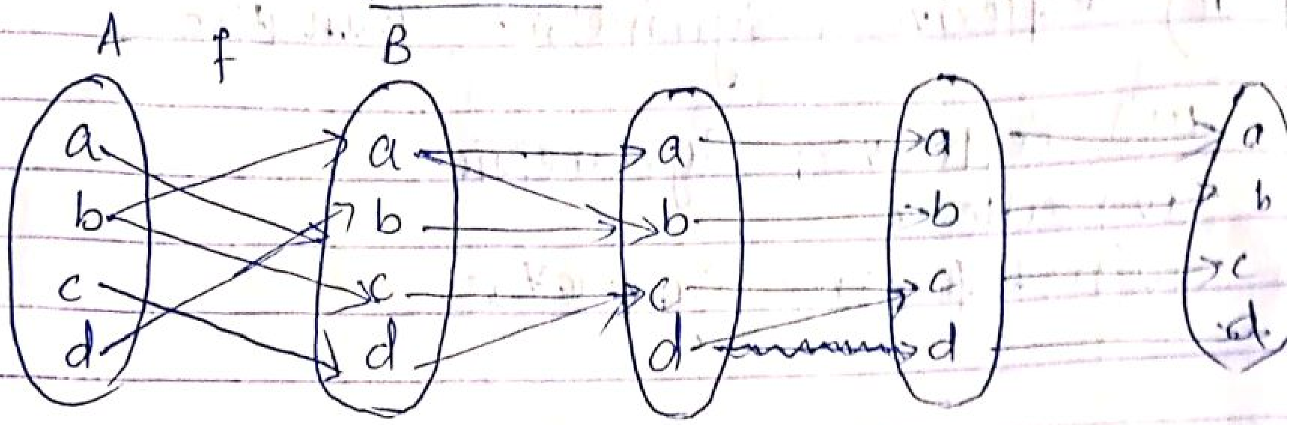
Domain = ? Range = ? , Co-domain = ?

~~Anti symmetric = ? ⇒ If ac (a,b) ∈ R ⇒ (b,a) ∉ R~~

31/7/19 Proper subset = ? $A = \{1, 3, 5\}$, $B = \{1, 5\}$

B is a proper subset of A

Function



~~f : A → B~~ Not a function. This is a function. Function

f is not a function

One to One , Onto — One to one correspondence (if both onetoone & onto seen)

Injective , Surjective — also called Bijective also called invertible function

Every relation is not a function. &

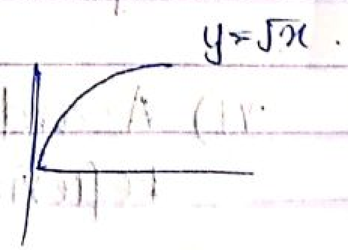
Every function is a relation.

Is relation a subset of function? No

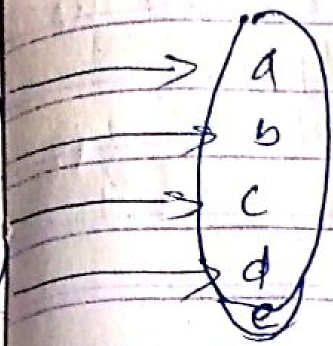
Is function a subset of relation? Yes

Q. Consider the validity of following statements as function.

1. $f(x) = x^2$ → function
2. $f(x) = \sqrt{x}$ → Not a function



Q. Let A be a set of students in the school. Determine which of the following assignments defines a funⁿ on A



Function

✓ i) To each student assign his age

X ii) Each student assigned his teacher

✓ iii) To each student his gender

X iv) To each student assign his friend

Q. Determine if e

① To each person on the earth assign the no. which corresponds to his age. Not one to one

② To each country of this world assign the latitude and longitude of its capital.

One to one function

③ To each book written by only one author assign author. Not one to one

④ To each country with a PM assign its PM

one to one.

1/8/19 Composition of functions

f , g

$g \circ f$

Q. let f ~~be~~, g, h be functions from $N \to N$ where N is the set of natural numbers so that $f_n = n+1$

$g_n = 2n$, $h_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ 1 & \text{if } n \text{ is odd} \end{cases}$

$f(f(x))$

determine $f \circ f$, $f \circ g$, $g \circ f$ & $g \circ h$.

$f \circ f = (n+1)+1 = n+2$, $f \circ g = 2n+1$, $g \circ f = 2(n+1) = 2n+2$

$g(h(x))$

$g \circ h = \begin{cases} 0 & \text{if } n \text{ is even} \\ 2 & \text{if } n \text{ is odd.} \end{cases}$

Special Functions

*) Floor *) Ceiling *

Floor or GIF $\lfloor 7.5 \rfloor = 7$ $\lfloor -7.5 \rfloor = -8$

Ceiling or $\lceil 7.5 \rceil = 8$ $\lceil -7.5 \rceil = -7$

Remainder function / Mod $25 \bmod 7$ → divisor $= 4 \bmod 7$
↓
remainder

$-25 \bmod 7 = 3 \bmod 7$

→ Read theory & write a short note.

* Hash functions, Recursive function, Permutation function

~~Hash~~

Recursive function: The function that calls Itself.

Eg: Factorial, Fibonacci, Ackermann

Ackermann (m, n)

i) If $m=0$, $A(m,n) = n+1$

ii) If $m \neq 0$, but $n=0$ then $A(m,n) = A(m-1, 1)$

iii) If $m \neq 0$ and $n \neq 0$, $A(m,n) = A(m-1, A(n, n-1))$

Calculate $A(1,1)$ ← A

$A(1,1) = A(0, A(1,0))$

$A(1,0) = A(0,1) = 1+1 = 2.$

$A(1,1) = A(0,2)$

$A(1,1) = 2+1 = 3.$

Q. Let a and b be +ve integers and suppose 'Q' is defined recursively as follows;

$$Q(a,b) = \begin{cases} 0 & \text{if } a<b \\ Q(a-b,b)+1 & \text{if } b \le a \end{cases}$$

i) Find the value of $Q(2,5) = 0$; $Q(12,15) = 0$.

ii) What does this Q function do? What is its physical significance?

$\cancel{Q(5,2)}$ $Q(12,5) = Q(12-5, 5) + 1$

$= Q(7,5) + 1$

$= Q(7-5, 5) + 1 + 1$

$= Q(2,5) + 2$

$= 0 + 2 = 2$

20/8/19

Logic & Propositional Calculus

P: Today is Monday ✓
Q: 2+2=4 ✓
R: Do your homework × not a proposition

$\wedge$ Conjunction $\neg$ Negation
$\vee$ Disjunction

p	q	$p \wedge q$
0	0	0
1	0	0
0	1	0
1	1	1

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

p	$\bar{p}$
0	1
1	0

Tautology : If the final value is 'T' ~~irrespecti~~ always for all the conditions.

Contradiction or Fallacy: If the final value is 'F' always for all the conditions.

$\neg(p \vee q) \equiv \neg p \wedge \neg q$ these are logically equivalent statements

Implication : if p then q $p \rightarrow q$
if p, q
p is sufficient for q
q if p
q when p
a necessary condition for p is q
q unless $\neg p$ (negation of p)
q is necessary for p
q whenever p

Biimplications iff p, q $p \leftrightarrow q$

p	q	p→q	p↔q
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

$P_1 : p \to q$

$P_2 : q \to r$

$Q : p \to r$

$P_1 \wedge P_2 \vdash Q$ is a tautology

$P_1 \wedge P_2 \to Q$

p	q	r	P_1: p→q	P_2: q→r	$P_1 \wedge P_2$	Q: p→r	$P_1 \wedge P_2 \to Q$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

This is tautology. ∴ The given statement is always true.

Consider the following

1. If a man is bachelor, he is unhappy
2. If he is unhappy he dies young
3. Bachelor dies young.

This is a tautology.

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p : Man is bachelor
q : He is unhappy
r : He dies young

$p \to q$ → Condition / Implication
$q \to p$ → Converse
$\neg p \to \neg q$ → Inverse
$\neg q \to \neg p$ → Contrapositive

Q: Determine the contrapositive of each statement.

1) If John is a poet then he is poor
2) Only if Mark studies will he pass the test
q p $p \to q$

Contrapositive of St 1: If John is not poor then he is not a poet

— " — St 2: ~~If Mark does not pass the test then he does not study~~

If Mark does not study then he will not pass the test.

		Implication	Converse			Inverse	Contrapositive
p	q	$p \to q$	$q \to p$	$\neg p$	$\neg q$	$\neg p \to \neg q$	$\neg q \to \neg p$
0	0	1	1	1	1	1	1
0	1	1	0	1	0	0	1
1	0	0	1	0	1	1	0
1	1	1	1	0	0	1	1

Implication and contrapositive

$p \to q$, p is sufficient for q

Q. Determine the validity of following arguement

St 1: If two sides of a Δ are equal (p) then the opposite angles are equal. (q)

St 2: Two sides of a Δ are not equal ($\neg p$), the opposite angles are not equal. ($\neg q$)

$p \mid q$

p : Two sides of a Δ are equal

q : Opposite angles are equal

¬p : Two sides of Δ are not equal

¬q : The opposite angles are not equal

p	q	P_1: $p \to q$	$\neg p$	$\neg q$	P_2: $\neg p \to \neg q$	$P_1 \wedge P_2$
0	0	1	1	1	1	1
0	1	1	1	0	0	0
1	0	0	0	1	1	0
1	1	1	0	0	1	1

* $p \leftrightarrow q$ p is necessary and sufficient for q and q is necessary & sufficient for p.

Q You can access the internet from campus only if you are a CS Major or you are not a freshman.

S. Convert it into logical expression

Sol^n p : You can access the internet

q : You are a CS major

r : You are ~~not a~~ a freshman

$\cancel{(q \vee \neg r) \to p}$ $p \to (q \vee \neg r)$

~~p~~ ~~$(q \vee \neg r) \longrightarrow$~~ p

q if p.

Q. You cannot ~~write the~~ ride the rollercoster if you are under 5 feet tall unless you are older than 16 yrs

p : You can~~not~~ ride the roller coster

q : You are under five feet tall

r : You are ~~16 yrs old~~, older than 16 yrs.

~~$\neg p \rightarrow (q$~~ $\quad q \vee \quad$ ~~$\rightarrow \neg p$~~ $\quad (q \vee \neg r) \rightarrow \neg p$

q	r	$q \rightarrow r$
1	1	1
0	1	1
1	0	0
0	0	1

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Q. Write down the following statements using quantifiers, propositional functions

i) Everyone enrolled in the university has lived in a dormitory.

ii) All students in this class understand logic

iii) All movies directed by Quintin Tarentino are wonderful.

iv) Someone in Mumbai has never seen Amitabh Bachan

v) Every student who has taken a course in Discrete Mathematics can take a course in Algorithms.

Universal quantifier : $\forall x \; P(x) \rightarrow B(x)$

$(A \rightarrow B \equiv A' + B)$

i) ~~$P(x) \rightarrow B(x)$~~ $\forall x \;\; P(x) \rightarrow B(x)$

ii) $\forall x \;\; P(x) \longrightarrow B(x)$

iii) $\forall x \;\; P(x) \wedge B(x)$

iv) $\exists x \;\; P(x) \wedge B(x)$

v) $\exists x \;\; P(x) \wedge B(x)$

vi) All tigers$^{p(x)}$ and lions$^{q(x)}$ hunt when threatened$^{r(x)}$ or hungry$_{s(x)}$

$\rightarrow \exists x, y \;\; p(x), q(y) \longrightarrow r(z) \wedge s(z)$

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★ Permutation Function

Q. Let the universe of discourse of x is the set $A = \{1, 2, 3, 4\}$ and for y is the set $B = \{5, 6, 7, 8\}$ and the predicate $P(x,y)$ is defined as follows $P(x,y)$ is such that x is less than y. Find the truth values of the ~~4~~ 8 propositions involving existantial and universal quantifiers

→ $\forall x \; \forall y \; P(x,y)$ T

→ $\forall x \; \exists y \; P(x,y)$ ~~F~~ T

→ $\exists x \; \forall y \; P(x,y)$ ~~F~~ T

→ $\exists x \; \exists y \; P(x,y)$ ~~F~~ T

→ $\exists y \; \forall x \; P(x,y)$ F

→ $\forall y \; \exists x \; P(x,y)$

→ $\forall y \; \forall x \; P(x,y)$

→ $\exists y \; \exists x \; P(x,y)$