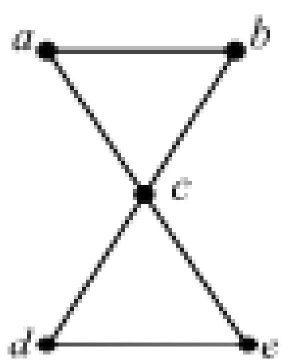
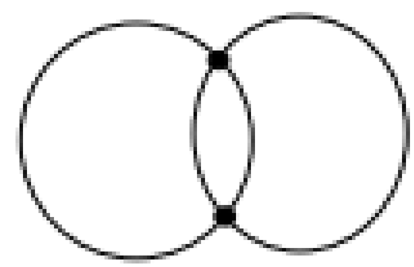
DAY 5

Arbitrarily Traceable Graph

An Eulerian graph G is said to be arbitrarily traceable (or randomly Eulerian) from a vertex v if every walk with initial vertex v can be extended to an Euler line of G. A graph is said to be arbitrarily traceable if it is arbitrarily traceable from every vertex



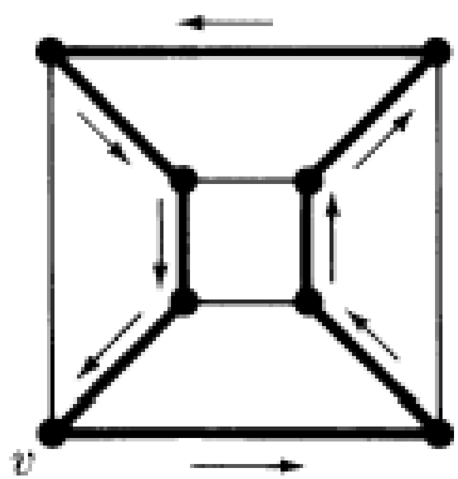
(a) Arbitrarily traceable graph from c



(b) Arbitrarily traceable graph from all vertices

Hamiltonian Graph

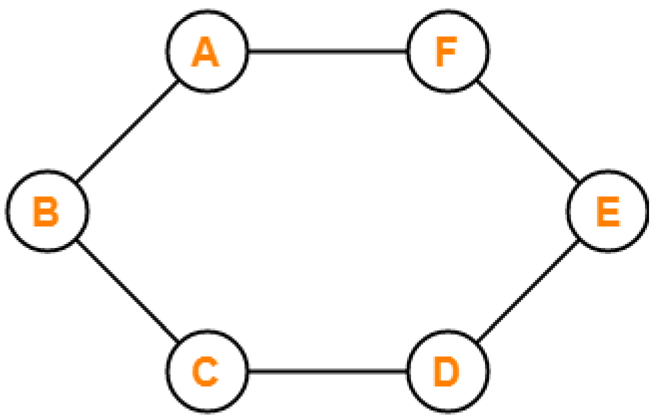
A connected graph G is called a Hamiltonian Graph if there exists a closed walk in that graph that visits every vertex of the graph exactly once (except starting vertex) without repeating the edges.



Hamiltonian Graph

Contd..

Alternatively, any connected graph that contains a Hamiltonian circuit is called as a Hamiltonian Graph.



- This graph contains a closed walk ABCDEFA.
- It visits every vertex of the graph exactly once except starting vertex.
- The edges are not repeated during the walk.

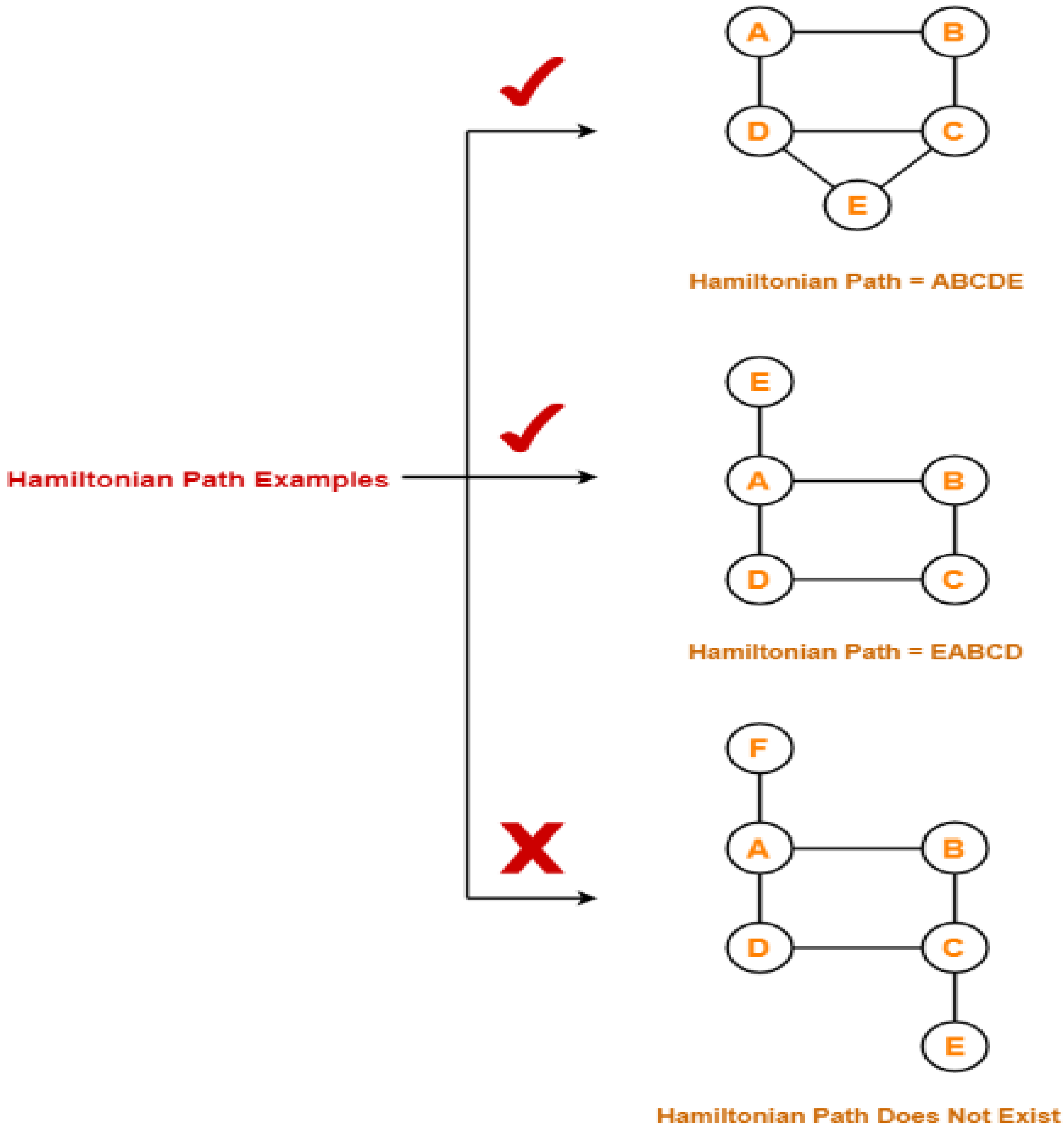
Therefore, it is a Hamiltonian graph.

Alternatively, there exists a Hamiltonian circuit ABCDEFA in the above graph, therefore it is a Hamiltonian graph.

Hamiltonian Path

If there exists a path in the connected graph that visits every vertex of the graph exactly once without repeating the edges, then it is called a Hamiltonian path.

In Hamiltonian path, all the edges may or may not be covered but edges must not repeat.

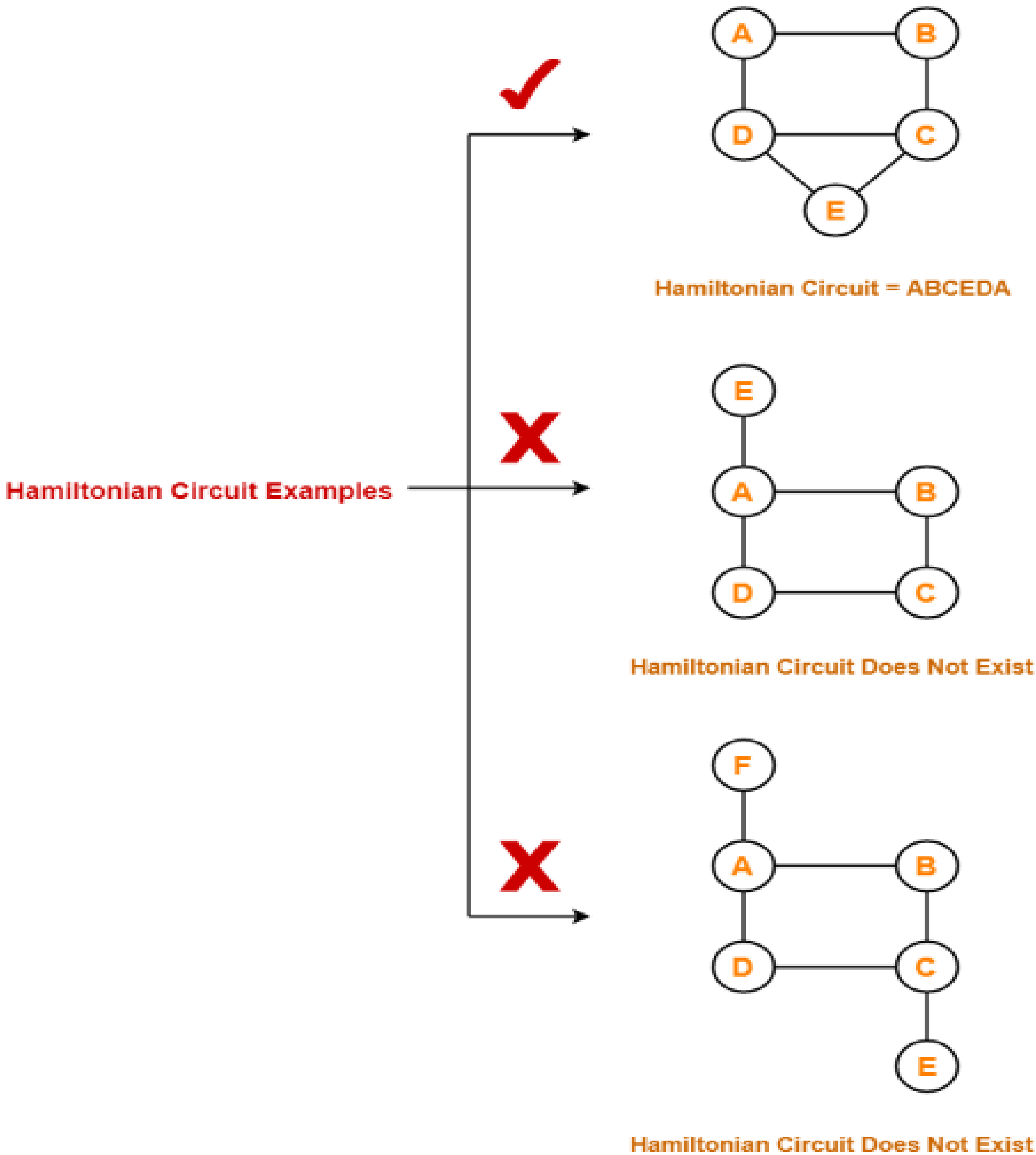

Hamiltonian Path Examples
A
B
D
C
E
Hamiltonian Path = ABCDE
E
A
B
D
C
Hamiltonian Path = EABCD
F
A
B
D
C
E
Hamiltonian Path Does Not Exist

Hamiltonian Circuit

If there exists a walk in the connected graph that visits every vertex of the graph exactly once (except starting vertex) without repeating the edges and returns to the starting vertex, then such a walk is called as a Hamiltonian circuit.

Any Hamiltonian circuit can be converted to a Hamiltonian path by removing one of its edges.

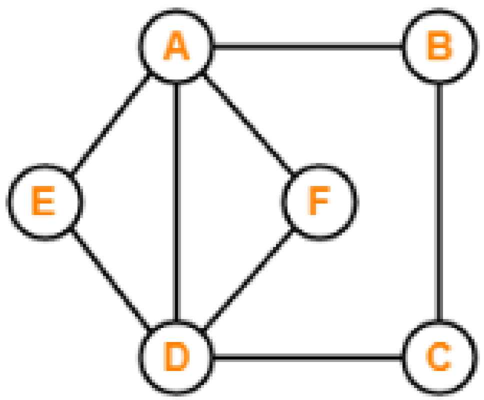
Every graph that contains a Hamiltonian circuit also contains a Hamiltonian path but vice versa is not true.


Hamiltonian Circuit Examples
A
B
D
C
E
Hamiltonian Circuit = ABCEDA
E
A
B
D
C
Hamiltonian Circuit Does Not Exist
F
A
B
D
C
E
Hamiltonian Circuit Does Not Exist

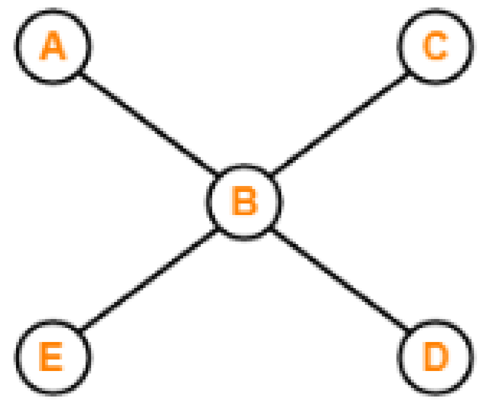
Solve

Are the following graphs Hamiltonian? Justify your answer.

A)

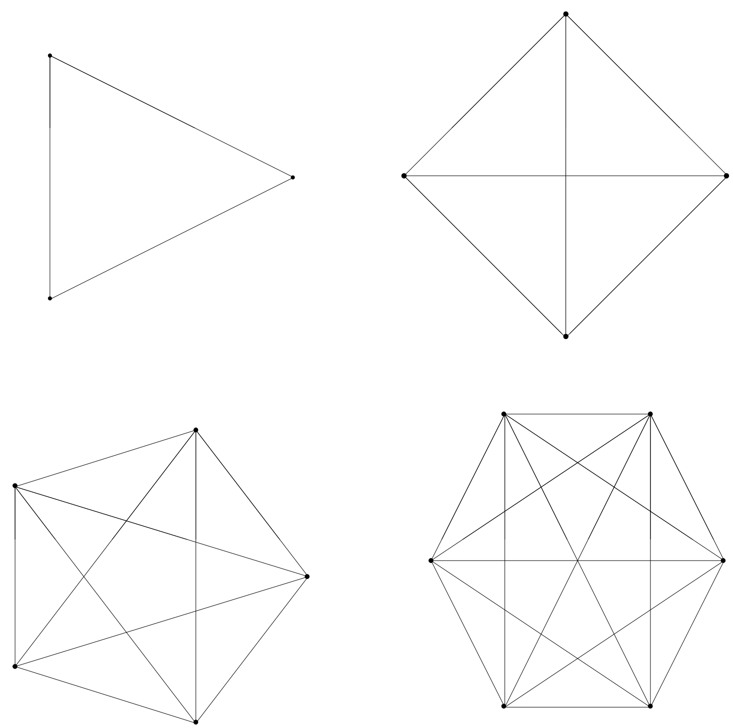


B)



Complete Graph K_n

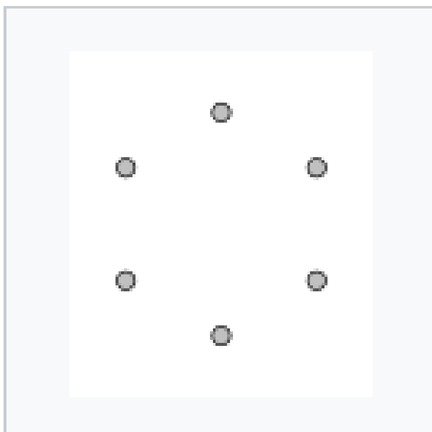
A graph with n vertices in which each vertex is adjacent to all other vertices is called a complete graph of n vertices, denoted by K_n.



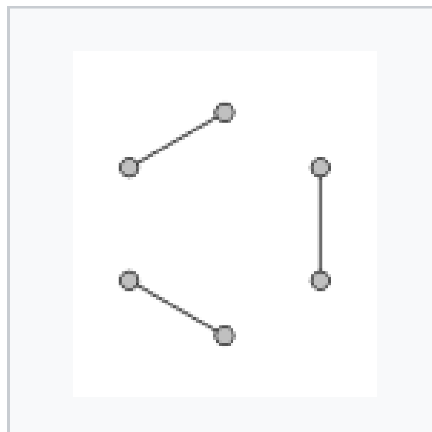
Degree of every vertex is (n-1).
It is also known as Universal Graph.

Regular Graph

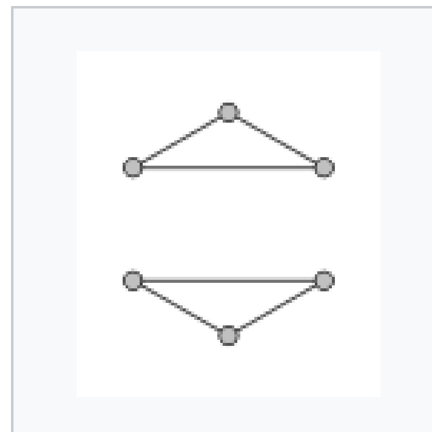
A graph is called a regular graph if each vertex has the same number of neighbors; i.e. every vertex has the same degree.



0-regular graph



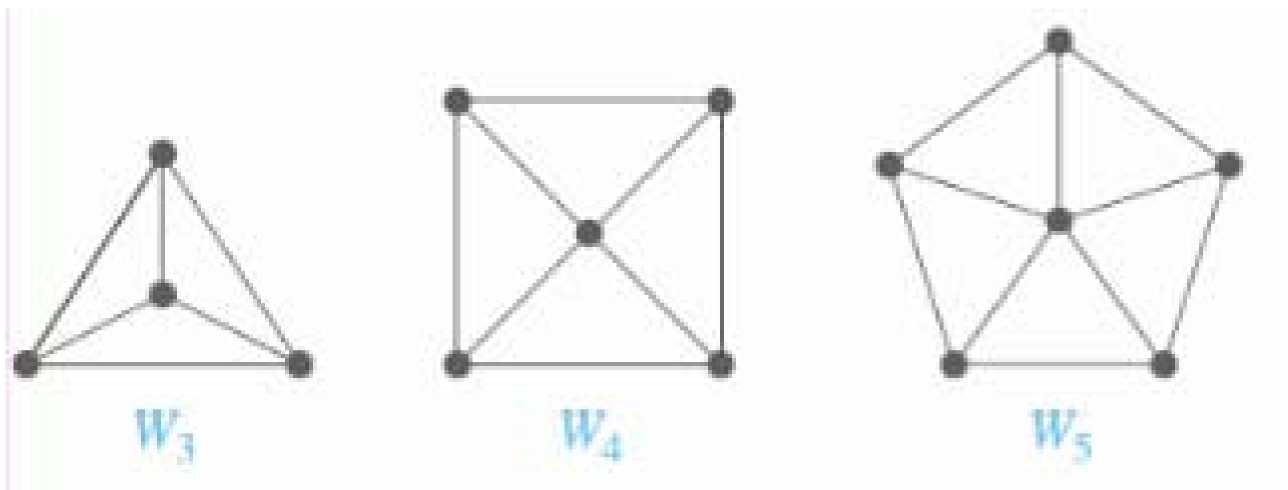
1-regular graph



2-regular graph

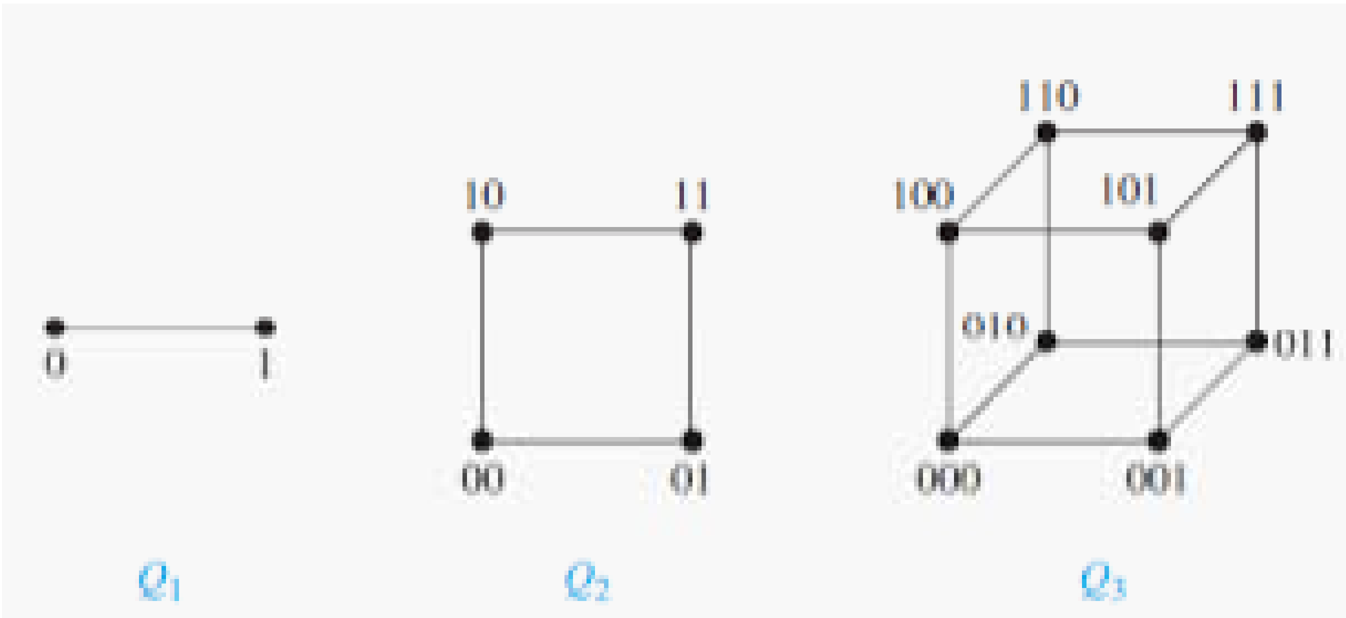
Wheel Graph

In a wheel graph all (n-1) vertices of a graph will be connected with one single vertex which is known as the center of that wheel graph. Degree of center in a wheel graph is (n-1)



N-cube Graph

In N-cube graph, two vertices are adjacent if and only if those two vertices differ in only one bit position.



Solve

“Every complete graph is regular but not all regular graphs are complete” - define the statement with proper diagram.