Discrete Mathematical Structure.
Text Book: Seymour Lipschutz , Marc Lipson ,Schaum's Outline of Discrete Mathematics, 3rd Edition
Recommended Book: Kenneth H.Rosen, “Discrete Mathematics and its Applications”, Fifth Edition, Tata McGraw –Hill

Reference Book Download Link: https://alas.matf.bg.ac.rs/~mi10164/Materijali/DS.pdf

Syllabus:
1. Set Theory
2. Functions
3. Relations
4. Propositional Logic
5. Predicate Logic
6. Group Theory

Topic:Set Theory
Set is a unordered collection of objects

A={1,2,3}
B={3,2,1}
C={1,1,1,1,1,1,1,1,1,1,2,2,3}

Block Points: Unordered elements are bounded by {braces} but the ordered collection of elements are bounded by (parenthesis)
(1,2)!=(2,1)!=(1,1,2)
{1,2}=={2,1}

N={0,1,2,3………..} Natural Number
Z={............-2, -1, 0, 1, 2……….}Integers
$\mathbf{Z}^{+}$={1, 2………}Positive Integers
$\mathbf{Z}^{-}$={-1,- 2………}Negative Integers
Q= Rational Number/ p/q
R= Real Numbers
C=Complex Numbers

- Observe that $N \subseteq Z \subseteq Q \subseteq R \subseteq C$.

Null Set, Subset , Proper subset, Belongs to Property, Exclusion-Inclusion Property.

Question 1. Find whether the the following are True or False

a. **{1,2}**⊆{1,2,2}True
b. **{1,2}**∈{1,2,2}False
c. **{1,2}**∈{1,2,2,{1,2}}True
d. **∅∈{1,2,2}False**
e. **∅⊆{1,2,2}True**
f. **∅∈∅False**
g. **∅⊆∅True**
h. **∅∈{∅}True**

i. $\emptyset \subseteq \{\emptyset\}$ True
Topic: Subset and superset

Set Builder: Representation of set in which the property that defines the elements of a set are given.
A={1,2,3}
A={x|x is positive number and is less than 4}

Two sets are said to be equal when all of the elements in both the sets are the same.
Universal Set: Set that contains everything (taking a particular topic in consideration)

Ø: empty set that contains nothing.
Eg of Ø : All sharks that are human.

Ø !={Ø}

Singleton set: set that contains only one element.

Subset : If all the elements of a set A in contained in set B , we say that A is a subset of B A$\subseteq$B

Proper subset:If A is a subset of B , but A is not equal to B , we write that A is a proper subset of B
A ⊂ **B**

- Observe that N ⊆ Z ⊆ Q ⊆ R ⊆ C.

Cardinality : the number of elements in a set.
Finite Set: Whenever the cardinality is finite , the set is finite
Infinite sets: Where the cardinality is infinite

Countably Infinite: When there is a property to get the next number in an infinite series, we say that set is countably infinite.
Uncountably Infinite: There are no such rules to define the next elements in the series.

Power set: is the set of all the subsets of the given set S.

A={1,2,3}
P({1,2,3})={Ø, {1},{2},{3},{1,2},{2,3},{1,3},{1,2,3}}
Cardinality of a power set =2^n where n is the cardinality of the set

Write the Power set of Ø and {Ø}
P(Ø)= {Ø}
P({Ø})={Ø,{Ø}}
Topic: Arguments and Venn Diagrams, The Inclusion Exclusion Principle

Q1. 3∈{1,2,3}True

Q2: $\{3\}\in\{1,2,3\}$False
Q3: $\{3\}\subset\{1,2,3\}$True
Q4: $3\subset\{1,2,3\}$False
Q5: $\emptyset\in\emptyset$ False

Q6: Show that the following argument (adapted from a book on logic by Lewis Carroll, the author of Alice in Wonderland) is valid using venn diagram:

S 2 : I find all your presents very useful.
S 3 : None of my saucepans is of the slightest use.
S : Your presents to me are not made of tin.

Since the present is a subset of Useful objects, and Tin is a subset of Saucepans and Saucepan add Useful objects are disjoint sets. So the Presents are not made of Tin.

The Inclusion Exclusion Principle

$A\cup B=A+B-A\cap B$
$A\cup B\cup C=A+B+C-A\cap B-C\cap B-A\cap C+A\cap B\cap C$

Q7:Find the number of mathematics students at college taking at least one of the languages French, German, and Russian such that

1. French= 65
2. German= 45
3. Russian= 42
4. French and German= 20
5. French and Russian=25
6. German and Russian=15
7. All of the lang=8

2. Find the students taking single subjects only
3.

Topic:Classes of Sets, Partition of Sets.

A={1,2,3}
|Power(A)|=8
The set of sets/ Classes of sets
Classes of sets⊆Power(A)

Partition: Partition the power set

A={1,2,3}
P({1,2,3})={∅, {1},{2},{3},{1,2},{2,3},{1,3},{1,2,3}}

Partition of the set S is a collection of subsets of S such that:

1. Each a in S belongs to one of the A_i
2. The different partitions must be disjoint.

Example: Consider the set A={1,2,3}
Determine whether the following collection of subsets are partition of A or not:

1. [{1,2}, {2,3},{1}]
2. [{1},{2}, {3}]
3. [{1,2}, {2}]

Sol:1. This is not a partition because element 2 is common in {1,2}, {2,3}
2. This is a partition
3. This is not a partition since element 3 is not present and the subsets are not disjoint.

Que1. Determine whether or not each of the following is a partition of the set N of positive integers:
(a) [{n | n > 5}, {n | n < 5}];
(b) [{n | n > 6}, {1, 3, 5}, {2, 4}];
(c) [{n | n^2 > 11}, {n | n^2 < 11}].

Topic: Relation
S:All the people living in India
R=(a,b): a is a relative of b
R⊆SXS
A={1,2}
B={a,b,c}
AXB={(1,a),(1,b),..........}
R={(1,a),(1,b)}

C={All the cities in India}
S={All the states of India}
R is defined as (a,b) such that a∈C, and b∈S, and a is a capital of b
R={(Patna, Bihar), (Ranchi, Jharkhand),.. }

Q1: Consider a set of positive numbers less than 10. Relation R is define on(a,b) such that a=2b
Solution:R={(2,1), (4,2), (6,3), (8,4) }

Properties of a Relation:
Reflexive Relation:For all a∈S: aRa
Eg1: S={1,2,3,4}
R={(1,1), (1,2)} R is not reflexive
R1={(1,1), (2,2),(3,3), (4,4), (3,1), (4,1), (1,1)}
R=U ; R is reflexive

R=∅

Symmetric Relation: Whenever (a,b)∈R,or aRb: then (b,a)∈R or bRa
Eg2: S={1,2,3,4}
R={(1,1), (1,2)} R is not symmetric
R2={(1,1)} R2 is symmetric
R1={(1,1), (2,2),(3,3), (4,4), (3,1), (4,1), (1,1)} R1 is not symmetric

R=U ; R is Symmetric
R=∅; r is symmetric
Topic: Antisymmetric Relations and Transitive Relations
A relation R on a set A is antisymmetric if whenever aRb and bRa then a = b, that is, if a = b and aRb then b Ra. Thus R is not antisymmetric if there exist distinct elements a and b in A such that aRb and bRa.
R={(1,1),(2,2)} R is antisymmetric and symmetric
R1={(1,2)}R1 is antisymmetric but not symmetric
R2={(1,2),(2,1)} R2 is not antisymmetric
R=U ; R is not antisymmetric
R=∅;R is antisymmetric

Transitive: whenever (a,b) and (b,c) then (a,c)

R1={(1,2)} R1 is transitive
R2={(1,2),(2,4)} R2 is not transitive
R=U ; R is transitive
R=∅;R is transitive

Q1. Consider the following five relations:
(1) Relation ≤ (less than or equal) on the set Z of integers.: **Ref, Not Symmetric, Transitive**
(2) Set inclusion ⊆ on a collection C of sets.: **ReF, Not Sym, Transitive**
(3) Relation (perpendicular) on the set L of lines in the plane.: Not Ref, Syme, not Trans
(4) Relation (parallel) on the set L of lines in the plane. **Ref, Sym, Trans**
(5) Relation | of divisibility on the set N of positive integers. (Recall x | y if there exists z such that xz = y.)**Ref, Not Sym, Tra**

Topic: Relation:Domain, Range, Inverse, Composition,
R={(1,2),(2,4)}
Domain: is a set of all the first elements of the ordered pair which belong to R={1,2}
Range: is a set of all the second elements of the ordered pair which belong to ={2,4}

Inverse:
R={(1,2),(2,4)}
R^{-1}={(2,1),(4,2)}

Composition of Relation:
A={1,2,3}, B={a,b}, C={x,y,z}
R is defined over A and B. R={(1,b), (2,a), (3,a)}
S={(a,y), (b,x)}

Find the composition of R and S: RoS

Set S={1,2,3,4,5}
R is defined on S and contains {(1,2), (2,3), (3,4)}
RoR, Composition of R and R

Q1: A={1,2,3,4}, B={a,b,c,d}, C={x,y,z}
R={(1,a),(2,d), (3,a), (3,b), (3,d)} and S={(b,x), (b,z), (c,y), (d,z)}
Find RoS and SoS
Topic: Equivalence Relation, Partial ordered sets.
Equivalence relation: Reflexive, symmetric and transitive
Partially Ordered set(POSET): Reflexive, antisymmetric and transitive

Q1. Consider the following five relations on the set A = {1, 2, 3, 4}:
R 1= {(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)}
R 2= {(1, 1)(1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}
R 3= {(1, 3), (2, 1)}
R 4= ∅, the empty relation
R 5= A × A, the universal relation
Determine whether they are equivalence relations and partial ordered sets or not.

Sol: R1: Not Reflexive, Not Symmetric, Transitive, Antisymmetric
R2: Reflexive, Symmetric, Not antisymmetric, Transive,
R3: Not Reflexive, Not Symmetric, Antisymmetric, not Transitive(for (2,1) and (1,3), there is not (2,3))
R4: not Refl, Symmetric, Antisymmetric, Transitive
R5: Ref, Syme, not antisym, transitive

R2 and R5 are equivalence relation, No partial ordered set present.

Q2: Consider the following five relations:
(1) Relation ≤ (less than or equal) on the set Z of integers.: **Ref, Not Symmetric, Transitive**
(2) Set inclusion ⊆ on a collection C of sets.: **ReF, Not Sym, Transitive**
(3) Relation (perpendicular) on the set L of lines in the plane.: Not Ref, Syme, not Trans
(4) Relation (parallel) on the set L of lines in the plane. **Ref, Sym, Trans**
(5) Relation | of divisibility on the set N of positive integers. (Recall x | y if there exists z such that xz = y.)**Ref, Not Sym, Tra**

Determine whether they are equivalence relations and partial ordered sets or not.
Topics: PROPOSITIONS AND COMPOUND STATEMENTS, BASIC LOGICAL OPERATIONS, PROPOSITIONS AND TRUTH TABLES, TAUTOLOGIES AND CONTRADICTIONS, Logical Equivalence

Proposition: is a statement which is either true or false.
Eg: sun rises in the east. True
Sun sets in the east. False
2+2=4: True
2+2=5 : False
Delhi is the capital of Pakistan: False

Cases which are not propositions: Commanding statements and interrogative statements are not propositions.
Shut the door.
Do you know me?
Where are you going?
Do your homework.
X+2>5 It won't be called a proposition, since the value changes with the different values of X

Operations:
Conjunction(∧): The result is true when all the propositions are true.
Disjunction(∨): The result is false when all the propositions are false.
Negation(¬):

Compound Proposition: joining simple propositions using connectives/Operations.

Conjunction(∧): AND

p	q	p∧q
0	0	0
0	1	0
1	0	0
1	1	1

P: sun rises in the east
Q: 2+2=5
Sun rises in the east and 2+2=5
p∧q=1∧0=0

Disjunction(∨) OR Suppose we have a joint account

p	q	p∨q
0	0	0
0	1	1
1	0	1
1	1	1

Sun rises in the east or 2+2=5
p=1 and q=0
p∨q=1

Negation(¬):

p	¬p

0	1
1	0

p=Sun rises in the east
Negate the proposition
Is is not the case that sun rises in the east /Sun doesn't rise in the east.

Tautology:The output of the truth table is 1 is all the cases/ If the TT contains T in the last column
Contradiction: The TT contains F in all the entries in the last column.

$1 \vee p$ is a tautology

p	1+p
0	1
1	1

$0 \wedge p$ is contradiction

p	$0 \wedge p$
0	0
1	0

Logical Equivalence:
Two compound propositions are logically equivalent if they have identical values in the TT
Demorgan's Law: $\neg(p \vee q) = \neg p \wedge \neg q$

Topic : Conditional and Biconditional Statements, Arguments.

Conditional Statements(→) if P then Q, p implies q,

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

Explanation: If P is true, Q has to be true, If P is false $p \to q$ is true
OR

$P \to q$ is false when p is true and q is false else it's true.

$P \to q$ is logically equivalent to $\neg p \lor q$

p	q	¬p	¬p∨q
0	0	1	1
0	1	1	1
1	0	0	0
1	1	0	1

Biconditional statements (↔) p if and only if q

p	q	p↔q
0	0	1
0	1	0
1	0	0
1	1	1

$P \leftrightarrow q$ is logically equivalent to $\neg((p \land \neg q) \lor (\neg p \land q))$

Arguments:
An argument is an assertion that a given set of propositions P1, P2…...called the premises yields another proposition Q that we call conclusion.

P1, P2….Pn ∴ Q

Law of Detachment:
Prove that P, p→q ∴ q is valid

p	q	p→q	p∧(p→q)	p∧(p→q))→q
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

When the Premises are True the conclusion has to be true

If the truth table of $(p \wedge (p \rightarrow q)) \rightarrow q$ is a tautology the argument is valid.

Topic: CONDITIONAL AND BICONDITIONAL STATEMENTS, CONVERSE, CONTRAPOSITIVE, AND INVERSE.

Conditional Statements($\rightarrow$) if P then Q, p implies q,

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

If I study, I will pass
If p, q
If P, then q
P is sufficient for Q
Q is necessary for p
Q whenever p

Conditional Statements($\rightarrow$) $p \rightarrow q$
Converse $q \rightarrow p$
Inverse : $\neg p \rightarrow \neg q$
Contrapositive: $\neg q \rightarrow \neg p$

Q: Make the truth table of the conditional , converse, inverse and contrapositive statements. Prove that Conditional statements are logically equivalent to contrapositive statements and converse statements are logically equivalent to inverse statements.

p	$\neg p$	q	$\neg q$	$p \rightarrow q$	$q \rightarrow p$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
0	1	0	1	1	1	1	1
0	1	1	0	1	0	0	1
1	0	0	1	0	1	1	0
1	0	1	0	1	1	1	1

Question: Write the negation of “if it rains, boys play”
$P \rightarrow q$ is logically equivalent to $\neg p \vee q$
So, “if it rains, boys play” can be written as “ it does not rain or boys play”
$\neg(\neg p \vee q)=$

Question: Negate the following statements:

1. Driving at 100km/h is sufficient for you to get arrested.
2. If you do not drive at 100km/h, you won't get arrested.
3. Whenever you get arrested, you are driving at 100km/h.

Topic:Rules of Inference

Addition Rule: $P \therefore P \vee Q$

p	q	pvq	p→(pvq)
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

Modus Tollens:

$$P \rightarrow Q$$
$$\neg Q$$
$$\therefore \neg P$$

Question: Prove the following rule of inference:
If p implies q and q implies r, then p implies r

$$P \rightarrow Q$$
$$Q \rightarrow R$$
$$\therefore P \rightarrow R$$

If Ram studies, he passes
If Ram passes, he goes to university
∴If ram studies, he goes to university

Home Work:

Determine the validity of the following argument:

S1. If 7 is less than 4, then 7 is not a prime number.

S2. 7 is not less than 4.

Conclusion. 7 is a prime number.

Topic: Predicate and Quantifiers.

3+2>5 False

x>3 is not a proposition.
X is greater than 3:
First part is X and it is the variable or subject of the statement.
Second part is “is greater than 3” is called the predicate, and it refers to a property that the subject of the statement can have.

The statement “X is greater than 3” is also called a propositional function over x.
Eg: P(x) denotes Mobile x is infected by Virus.

x+y=5 can be denoted by P(x,y)
Find the truth value of propositions P(1,2), P(10,0)....
P(1,2) is false, P(10,0) is also false.

Quantifiers: Universal Quantifier(∀ for All), Existential Quantifier(∃ Some).
∃x P(x) P(x):x>3 given the domain is set of the positive numbers.
∀x P(x): where P(x) denotes x <5 over the domain of negative integers.
x+y=5 can be denoted by P(x,y) and x and y are integers
The
∃x ∃y P(x,y) x=2, y=3, 2+3=5
∃x ∀y P(x,y) is not true
∀x ∃y P(x,y) is true
∀x ∀y P(x,y) false.

Question: What is the truth value of ∀x (x^2>=x), if the domain consists of all the real numbers?
Sol: $(½)^2$ is not greater than equal to ½
SO it is false.

1. Let A = {1, 2, 3, 4, 5}. Determine the truth value of each of the following statements:
   (a) (∃x ∈ A)(x + 3 = 10) (c) (∃x ∈ A)(x + 3 < 5)
   (b) (∀x ∈ A)(x + 3 < 10) (d) (∀x ∈ A)(x + 3 ≤ 7)

Topic: **Negation of Quantifier**

∃x ∃y P(x,y) x=2, y=3, 2+3=5
∃x ∀y P(x,y) is not true
∀y ∃x P(x,y) is not true

∀x ∃y P(x,y) is true
∀x ∀y P(x,y) false.

Note:
1. The sequence of the quantifier is important.

2. We can distribute a universal quantifier over conjunction and existential quantifier over disjunction, but the vice versa is not true.
   $\exists x(P(x) \vee Q(x))$ is logically equivalent to $\exists x P(x) \vee \exists x Q(x))$
   $\forall x(P(x) \wedge Q(x))$ and $\forall x P(x) \wedge \forall x Q(x)$ are logically equivalent

3. Quantifiers have got more precedence than the other logical connectives. For Eg: $\forall x P(x) \vee Q(x)$ is logically different from $\forall x(P(x) \vee Q(x))$
   Eg 2: $\forall x P(x) \vee Q(x)$ is logically equivalent to $(\forall x P(x)) \vee Q(x)$
4. $\exists y\ (x*y>10)$, x is called free variable and y is called bounded variable.

Question 1: Negate the following statement using quantifiers:
a. All Indians love Sachin Tendulkar.
b. There is a boy who doesn't like chocolate.

Sol1. All Indians love Sachin Tendulkar. $\forall x\ L(x)$
Let L(x) denote that x loves Sachin Tendulkar, where the domain consists of all the Indians.
Negation: $\neg \forall x\ L(x) = \exists x\ \neg L(x)$: These is some Indian that doesn't love Sachin/ Some Indians Dont love Sachin.

Sol2. There is a boy who doesn't like chocolate.
Let P(x) denote that x does not like chocolate, where the domain consists of all the boys. So it can be expressed as $\exists x\ P(x)$
Negation: $\neg\exists x\ P(x)=\forall x \neg P(x)$. All boys like chocolate.

Nesting the Quantifiers: $\exists x(\forall y\ x>y)$

Question1. Write the negation of $\forall x\ (x^2>x)$
Negation of $\forall x\ (x^2>x)$ is $\neg\forall x\ (x^2>x) = \exists x\ \neg(x^2>x) = \exists x\ (x^2<=x)$

Question2. Write the negation of $\exists x(x^2 = 2)$
Negation of $\exists x(x^2 = 2)$ is $\neg\exists x(x^2 = 2) = \forall x\ (x^2 != 2)$

Topic: Algebraic Structure, Group Theory

Algebraic Structure=(Set,Operation)
Group, Monoid….
Properties:
1. Closure
2. ASSOCIATIVE
3. Identity
4. Inverse
5. Commutative

Operation: We perform operations on the operands. Eg: add, sub, mul, div, Union, Intersection………

We can define new operations such as a*b=a+b+ab

Closure: The output of the operation on any subset of elements within the set will belong to the same set.

Q1. Does the set A={1,2,3 } defined over integer addition satisfies the Closure property?
Sol: 1+3 =4 which doesn't belong to the set. So it doesn't satisfy the closure property.

Q2: Does the set N of natural number defined over integer addition satisfy the Closure property?
Q3: Does the set N of natural numbers defined over integer subtraction satisfy the Closure property?
Q4: Does the set N of natural numbers defined over integer multiplication satisfy the Closure property?
Q5: Does the set N of natural numbers defined over integer division satisfy the Closure property?
Q6: Multiplication of 2X2 matrix, is it closed?

ASSOCIATIVE: a*(b*c)=(a*b)*c

Q7: Does the set A={1,2,3 } defined over integer addition satisfies the Associative property?
Q8: Does the set N of natural numbers defined over integer subtraction satisfy the Associative property?
2-(1-1)=2
(2-1)-1=0 So Subtraction is not associative.

Q9: Let A and B denote, respectively, the set of even and odd positive integers. Is A as well as B closed under addition? Is A as well as B closed under product operation?
Q10: Consider the following table :

*	**a**	**b**	**c**	**d**
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

Does it satisfy the closure property? Is it associative?
Q11: Consider the following table :

*	**a**	**b**	**c**	**d**

a	a	a	a	a
b	b	b	b	b
c	c	c	c	c
d	d	d	d	d

Does it satisfy the closure property? Is it associative?

Topic: Identity and Inverse , Monoid, Groups and abelian.
Q1. Does the set A={1,2,3 } defined over integer multiplication satisfy the Identity property?
Sol: The identity is 1 because 1*1=1, 1*2=2, 1*3=3.
Q2. Does the set A={1,2,3 } defined over integer addition satisfy the Identity property?
Sol: We don't have an identity element.

Identity: e is called identity when e*a=a for all the elements of the set iver operation *
1*a=a, we say that 1 is identity w.r.t multiplication

0+a=a ., so 0 is the identity element in case of addition.

Note: There has to be a single identity for the set and unique inverse for every element.
Q3: Consider the following table and decide whether the identity element exists :

*	**a**	**b**	**c**	**d**
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

a is the identity element, because a*a=a. a*b=b. a*c=c and a*d=d.

Q4: Consider the following table and decide whether the identity element exists :

*	**a**	**b**	**c**	**d**
a	a	a	a	a
b	b	b	b	b
c	c	c	c	c
d	d	d	d	d

Sol: all the elements can be said as identity, but it's not unique so identity property does not exist for the said algebraic structure.

*	**a**	**b**	**c**	**d**
a	a	b	c	d
b	a	b	c	d
c	a	b	c	d
d	a	b	c	d

Inverse Property: Inverse property exists only when we have a unique identity.
Inverse of 2=½
$a*a^{-1}=e$

Q5. For the set of positive integers {1,2,3,4………..}, do we have an inverse property over the multiplication operation?
Sol: 1 is the identity w.r.t multiplication , there is no inverse becz ½ is not a positive integer and so on.
Q6. For the set of positive even integers,does the inverse property exist for multiplication?
Sol: Since the identity doesn't exist
Q7: For the set of rational numbers, do we have inverse property over multiplication operations?
Sol: Identity =1. 1*a=a, $a*a^{-1}=1$, For 0 inverse does not exist.

Groupoid:Closure
Semigroup:Closure and associative
Monoid: Closure, associative and identity
Group:Closure, associative, identity and inverse
Abelian: Closure, associative, identity, inverse and commutative

Q8: Is the set of positive integers over addition operation a group?
Sol; Closure exists bcz adding positive integers we will get positive integers. Associative property satisfies a+(b+c)=(a+b)+c for all positive numbers.
Identity?No bcz 0 is not present here, w.r.t addition zero is the additive identity.
Inverse?No bcz identity is not there.

Q9: Is the following example a group?

*	**a**	**b**	**c**	**d**
a	a	b	c	d
b	d	a	b	c

c	c	d	a	b
d	b	c	d	a

Sol: Closure, Assosistive, Identity=a,

Inverse: inverse of a=a, inverse of b=b,inverse of c=c, inverse of d=d

Q10: Is the following example a group?

*	**a**	**b**	**c**	**d**
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

Sol: Closure, Assosistive, Identity=a,

Inverse: inverse of a=a, inverse of b=d,inverse of c=c, inverse of d=b.

Q11. Prove that 1,-1,i,-i forms a group under multiplication operation.

*	**1**	**-1**	**i**	**-i**
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-1	i	1	-1

Sol: Close, associative, Identity =1,

Inverse of 1=1, Inverse of -1=-1, Inverse of i=-i, Inverse of -i=i

Q12. Prove that 1, w, w^2 forms a group under multiplication operation.

*	**1**	**w**	w^2
1	1	w	w^2
w	w	w^2	1

w^2	w^2	1	w

Sol: Closure, Associative, Identity= 1, Inverse of 1=1,Inverse of w=w^2, Inverse of w^2 =w

Addition modulo n, Z_n, such that Z_n={0,1,2…..n-1}, Operation is addition modulo n

Q13 Prove that**Addition modulo 3 Z_3 is a group.**

+	**0**	**1**	**2**
0	0	1	2
1	1	2	0
2	2	0	1

Sol: Closure, Associative, Identity= 0, Inverse of 0=0,Inverse of 1=2, Inverse of 2 =1

Multiplication modulo n, Z_n, such that Z_n={1,2…..n-1}, Operation is multiplication modulo n

Q14. Prove that multiplication modulo 4 is a group.

Sol:

*	**1**	**2**	**3**
1	1	2	3
2	2	0	2
3	3	2	1

Sol: closure property is not satisfied

Q15. Prove that multiplication modulo 5 is a group.

*	**1**	**2**	**3**	**4**
1				
2				
3				
4				

Topic: **Hasse Diagram, Maximal and Minimal Elements, Greatest and Least Elements, Upper Bound and Lower Bound,** Lattices.

Relations: Reflexive, Symmetric, Antisymmetric and Transitive.

Equivalent: Reflexive, Symmetricand Transitive.

POSET: Reflexive, Antisymmetric and Transitive.

Lattices are defined on the POSETs

Eg. R=({1, 2, 3, 4, 6, 8, 12},|).

R={(1,2), (1,3), (1,4), (1,4), ….(2,4), (2, 6)………..(3,6)…., (4, 8), (4,12).........} is a POSET

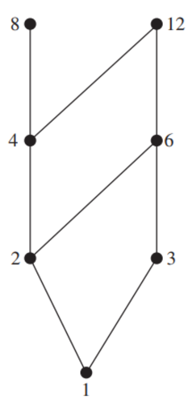
If we draw the diagram of the POSET and remove the Reflexive and Transitive relation we get a hasse diagram

Maximal and Minimal Elements:The elements in the subset of the hasse diagram that don't have any other greater element is called maximal. The elements in the subset of the hasse diagram that don't have any other lesser element is called minimal

, Greatest and Least Elements: are unique maximal and minimal elements

There can be more than one maximal and minimal element but there is only one greatest and least elemnst.

Upper Bound and Lower Bound:



Upper Bound(1,2,3)=6,12,LUB=6,, Lower Bound(1,2,3)=1, GLB=1

Upper Bound(2,4,6)=12,LUB=12 Lower Bound(2,4,6)=1,2, GLB=2

Upper Bound(1,2,4)=4,8,12, LUB:4, Lower Bound(1,2,4)=1, GLB=1

Upper Bound(8,12)=Empty Set, LUB IS NOT DEFINED

The poset is not a lattice.

Lattice is a poset which has Greatest Lower bound and Least Upper bound for all the subsets.

Greatest Lower bound: Greatest of the lower bound

Least Upper bound: Least of the Upper bound