B.Tech. 3rd Semester Mid Term Examination, 2021

Discrete Mathematical Structures

UCS03B10(NITA)/UCS03B04(IIITA)

Full Marks: 20 **Time: 1 hours**

1. Determine whether each of these statements is true or false.
   a) $0 \in \emptyset$
   b) $\emptyset \in \{0\}$
   c) $\{0\} \subset \emptyset$
   d) $\emptyset \subset \{0\}$ **(2)**
2. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.
   a) $A \cap (B \cup C)$
   b) $A \cap B \cap C$ **(2)**
3. Consider the following assumptions:
   S 1 : All dictionaries are useful.
   S 2 : Mary does not own any romance novel.
   S 3 : All romance novel are useful.
   Use a Venn diagram to determine the validity of each of the following conclusions:
   (a) Romance novels are not dictionaries.
   (b) Mary does not own a dictionary.
   (c) All useful books are dictionaries. **(3)**
4. Let R and S be the following relations on B = {a, b, c, d}:
   R = {(a, a), (a, c), (c, b), (c, d), (d, b)} and S = {(b, a), (c, c), (c, d), (d, a)}
   Find the following composition relations: (a) $R \circ S$; (b) $S \circ R$; (c) $R \circ R$; (d) $S \circ S$. **(2)**
5. Consider the universal set U = {1, 2, 3, ..., 8, 9} and sets A = {1, 2, 5, 6}, B = {2, 5, 7}, C = {1, 3, 5, 7, 9}. Find:
   i. $A \cap B$ and $A \cap C$
   ii. $A \oplus B$ and $A \oplus C$
   iii. $A \cup B$ and $B \cup C$
   iv. $A \backslash B$ and $A \backslash C$
   v. $(A \cup C) \backslash B$ and $(B \oplus C) \backslash A$ **(5)**
6. Determine whether the following relations are Reflexive, Symmetric, Antisymmetrc and Transitive
   i. Relation R on the positive integers N, such that xy is the square of an integer.
   ii. Set inclusion $\subseteq$ on a collection C of sets.
   iii. Relation (perpendicular) on the set L of lines in the plane.
   iv. Relation (parallel) on the set L of lines in the plane.
   v. Relation | of divisibility on the set N of positive integers.
   vi. Relations of congruencelet over set of triangles T in the Euclidean plane. **(6)**

Name : Aditya Kiran Pal Enrollment no : 20UCS119

Sec : A Exam : B.Tech 3rd Sem Mid-Term Examination

Subject Code : Subject : Discrete Mathematics

Q.1. (a) False. ($\phi = \{\}$ is an empty set has no elements)

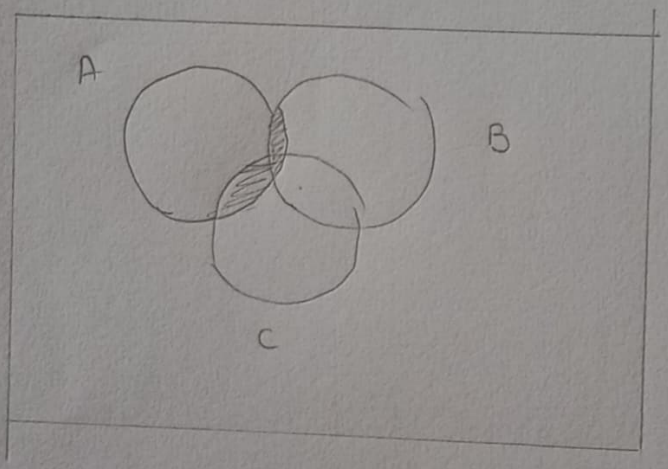
(b) ~~$\phi \in$~~ False, (The set $\{\phi\}$ does not have ϕ has element)

(c) $\{\phi\} \subset \phi$, ($\{\phi\}$ is not a subset of ϕ) (False).

(d) $\phi \subset \{\phi\}$, True, (ϕ is a subset of every set).

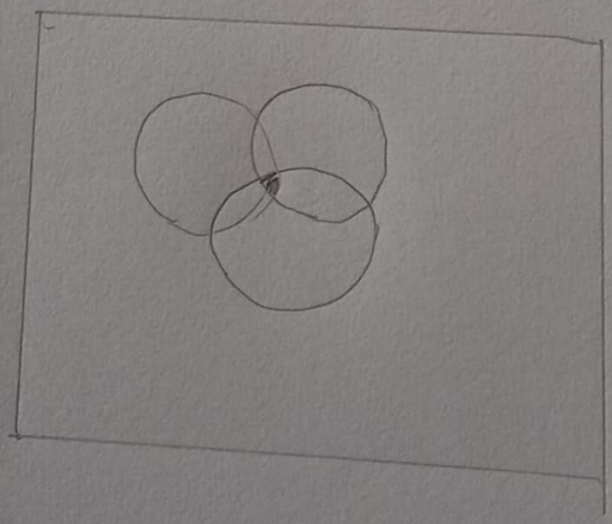
Q.2.

(a)



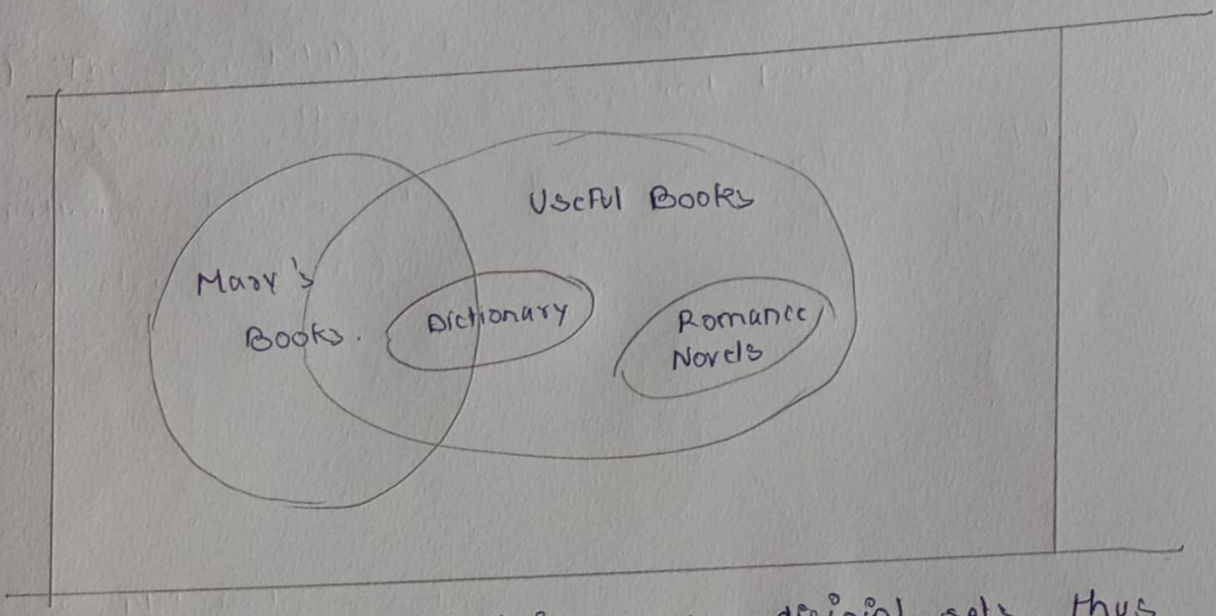
$A \cap (B \cup C)$

(b)



$A \cap B \cap C$

Q.3.



(a) Romance Novels, and dictionary are disjoint, sets, thus it is a valid statement.

(b) Mary might own a dictionary, thus given statement is valid.

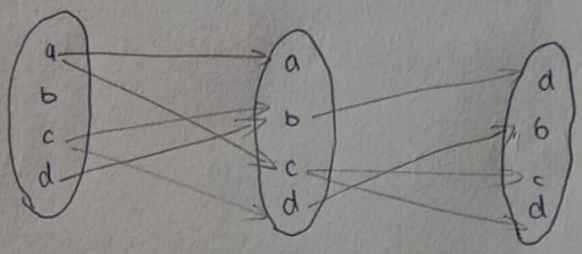
(c) All useful books are dictionaries ⟶ invalid

Q.4. $B = \{a, b, c, d\}$

$R = \{(a,a), (a,c), (c,b), (c,d), (d,b)\}$

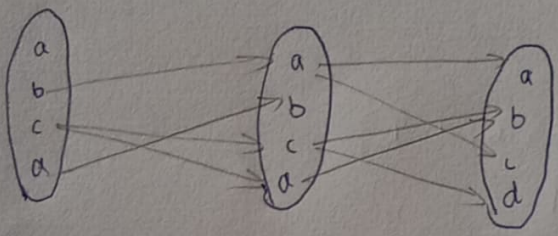
$S = \{(b,a), (c,c), (c,d), (d,a)\}$

(a)



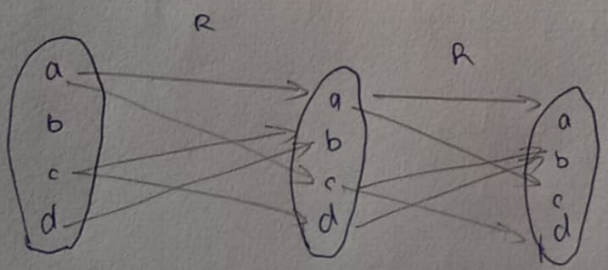
$R \circ S = \{(a,d), (c,a), (d,a), (a,c)\}$

(b)



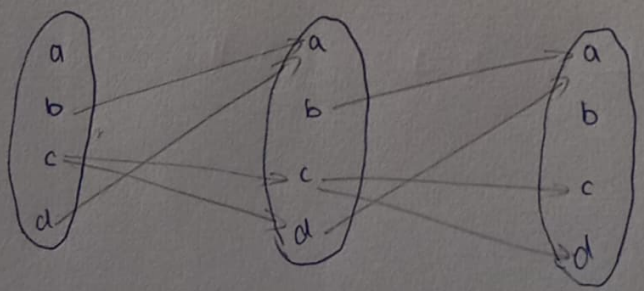
$S \circ R = \{(b,a), (c,d), (a,c)\}$

(c)



$R \circ R = \{(a,a), (b,b), (a,c), (a,d), (c,b)\}$

(d)



$S \circ S = \{(c,a), (c,c), (c,d)\}$

0.5. $U = \{1, 2, 3, 4 \ldots 8, 9\}$

$A = \{1, 2, 5, 6\}$, $B = \{2, 5, 7\}$

$C = \{1, 3, 5, 7, 9\}$

(i) $A \cap B = \{2, 5\}$

$A \cap C = \{1, 5\}$

(ii) $A \oplus B = \{1, 6, 7\}$

$A \oplus C = \{2, 3, 6, 7, 9\}$

(iii) $A \cup B = \{1, 2, 5, 6, 7\}$

$B \cup C = \{1, 2, 3, 5, 7, 9\}$

(iv) $A \setminus B = \{1, 6\}$

v) $A \setminus C = \{2, 6\}$

(v) $(A \cup C) \setminus B$ =

$A \cup C = \{1, 2, 3, 5, 6, 7, 9\}$

$B = \{2, 5, 7\}$

$A \cup C \setminus B = \{1, 3, 6, 9\}$

$B \oplus C = \{1, 2, 3, 9\}$

$A = \{1, 2, 5, 6\}$

$(B \oplus C) \setminus A = \{3, 9\}$

Q.6.

(i) The given Relation R on +ve integers : " xy is square of +ve integer " is reflexive, symmetric and transitive.

Reflexive : 'a,a' is always the square of integer 'a'. (True)

Symmetric : if (a,b) is square of some integer, then (b,a) is also there.

Transitive : If (a,b), (b,c) belongs to R, then (a,c) is also there.

Anti-symmetric : If (a,b) and (b,a) $\in$ R, it does not necessarily mean $a = b$.

(ii) The Relation is Reflexive, Anti-Symmetric & Transitive

(iii) The Relation is

- Not Reflexive
  A line is not perpendicular to itself.
- It is symmetric,
  $a \perp b$, $b \perp a$.
- It is not antisymmetric,
  If (a,b) and (b,a) both exist,
  $b \neq a$.
- It is not transitive,
  IF $a \perp b$, $b \perp c$, $a \parallel c$, not perpendicular,

(iv) The Relation is

- Symmetric,
  $(a \parallel b) \implies (b \parallel a)$
- Transitive
  $a \parallel b,\ b \parallel c,\ c \parallel a$
- Reflexive,
  as every line is parallel to itself
- Not Antisymmetric

(v) The Relation,

- ~~Symm~~ Not symmetric
- Reflexive
- Transitive
- Anti-symmetric

(vi) The Relation,

- Reflexive
  A triangle is congruent to itself
- Symmetric
  $A \cong B \implies B \cong A$
- Transitive
  $A \cong B,\ B \cong C \implies A \cong C$
- Not Anti-symmetric
  $A \cong B,\ B \cong A,\ A \neq B$ (Not necessarily)