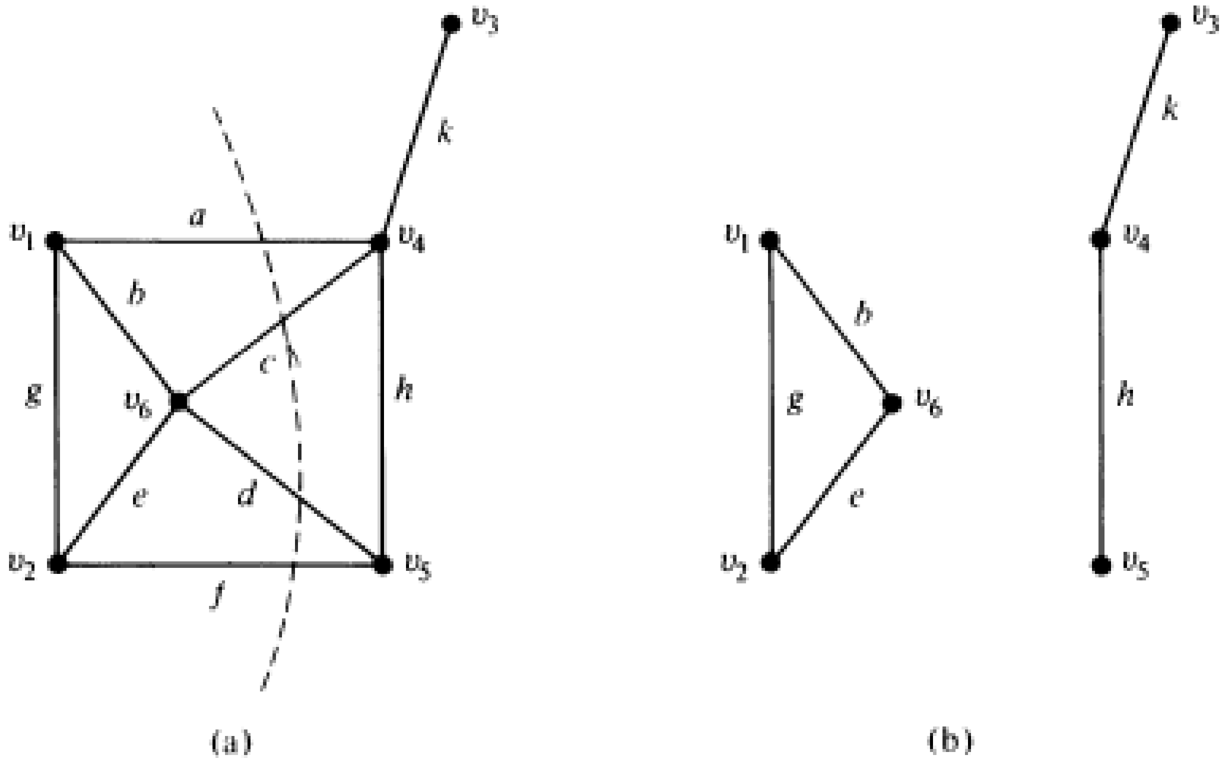
DAY 10

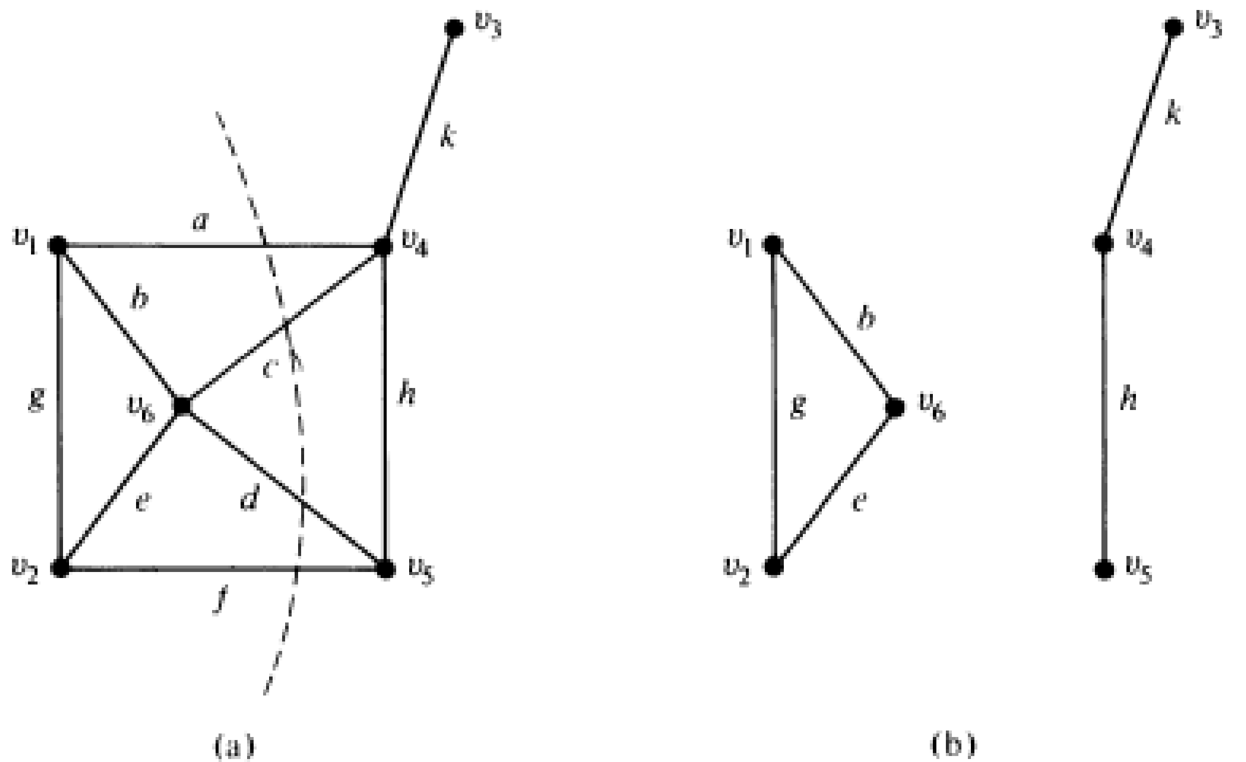
Cut Set

In a connected graph G, a *cut-set* is a set of edges† whose removal from G leaves G disconnected, provided removal of no proper subset of these edges disconnects G. For instance, in Fig. 4-1 the set of edges $\{a, c, d, f\}$ is a cut-set. There are many other cut-sets, such as $\{a, b, g\}$, $\{a, b, e, f\}$, and $\{d, h, f\}$. Edge $\{k\}$ alone is also a cut-set. The set of edges $\{a, c, h, d\}$, on the other hand, is *not* a cut-set, because one of its proper subsets, $\{a, c, h\}$, is a cut-set.



Cut Set Properties

1. A cut-set always cuts a graph into two.



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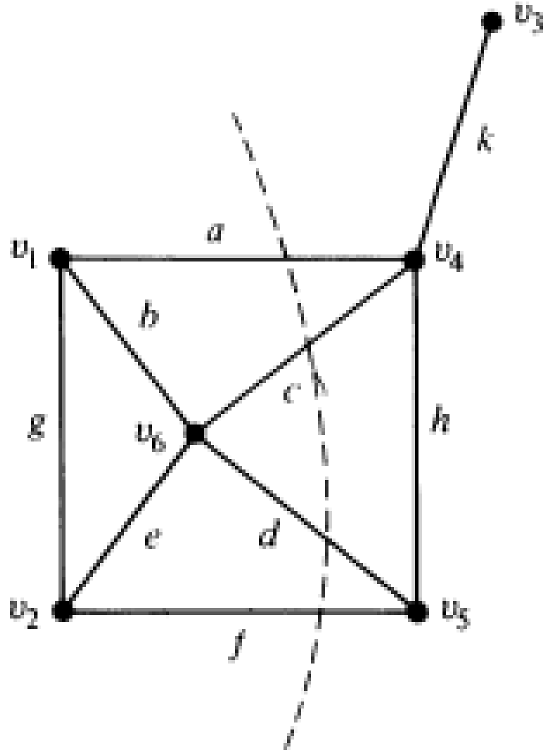
2. A cut-set must hold minimal number of edges.



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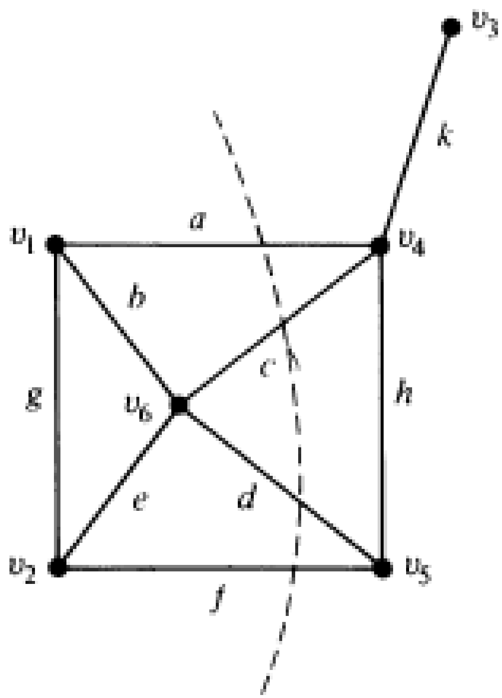
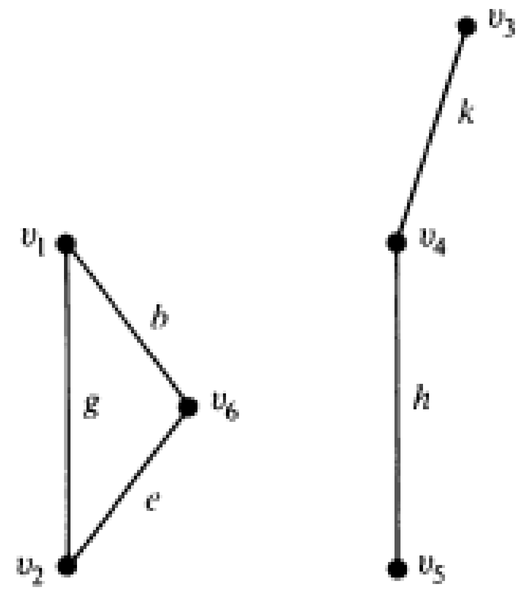
3. If all the vertices of a connected graph are partitioned into two mutually exclusive subsets, then a cut-set is the minimal no. of edges whose removal from the graph destroys all paths between these two sets of vertices.



(a)



(b)

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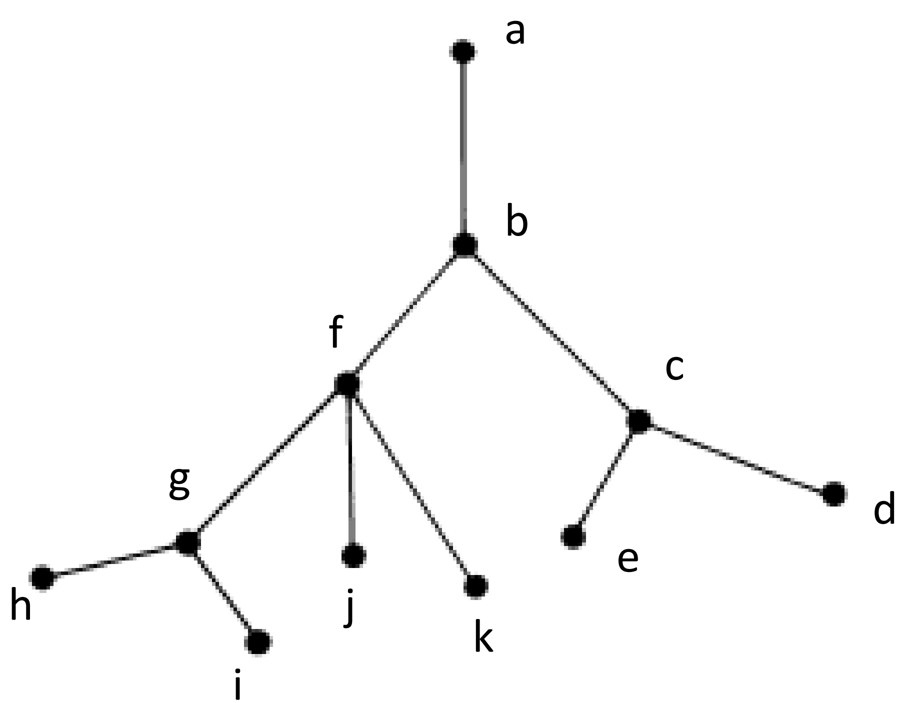
4. Every edge in a tree is a cut-set.



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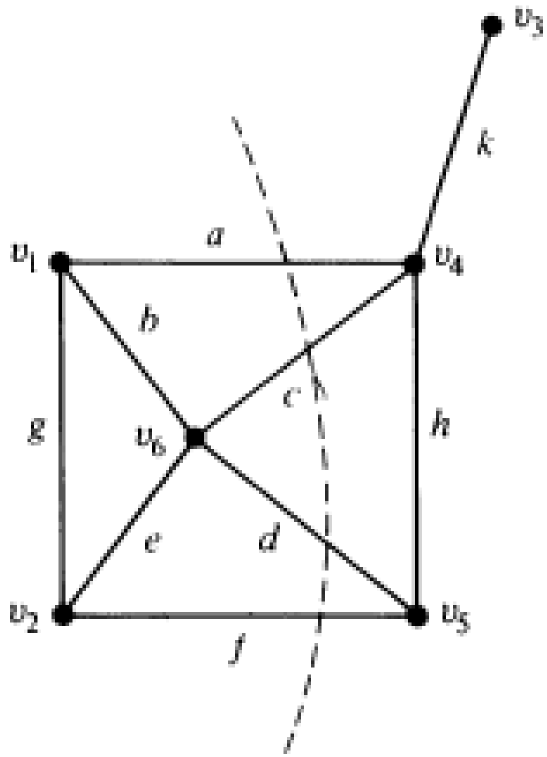
5. Every cut-set in a connected graph G must contain at least one branch of spanning tree of G.



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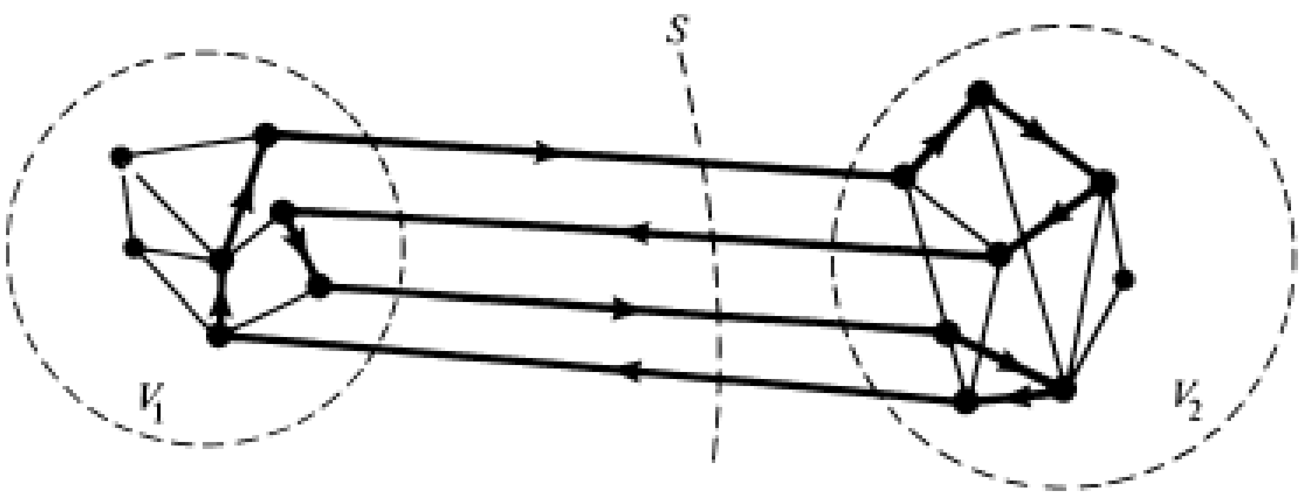
6. Every circuit has an even number of edges common with any cut-set.



Circuit Γ shown in heavy lines, and is traversed along the direction of the arrows

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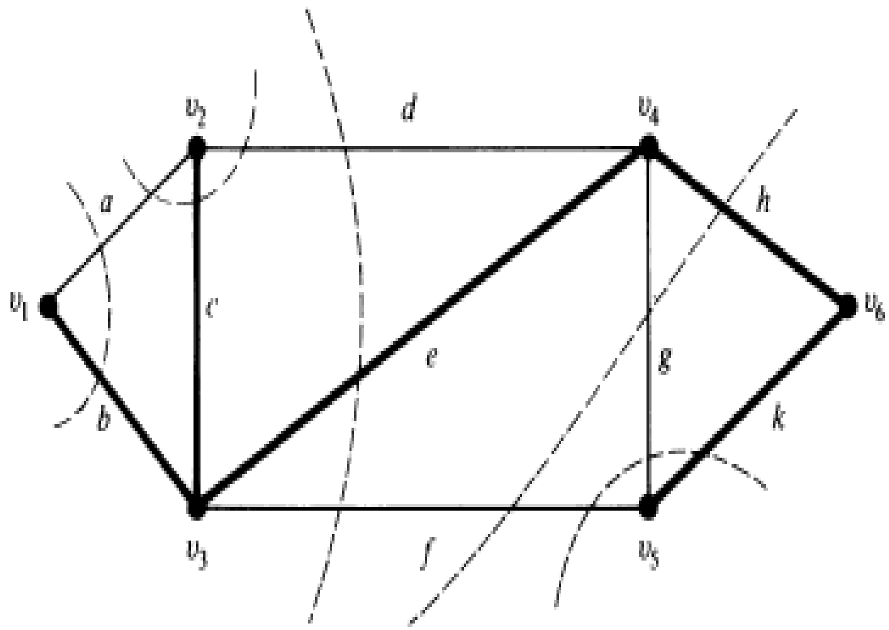
7. The ring sum of any two cut sets in a graph is either a third cut set or an edge-disjoint union of cut sets.



$$\{d, e, f\} \oplus \{f, g, h\} = \{d, e, g, h\}, \quad \text{another cut-set,}$$

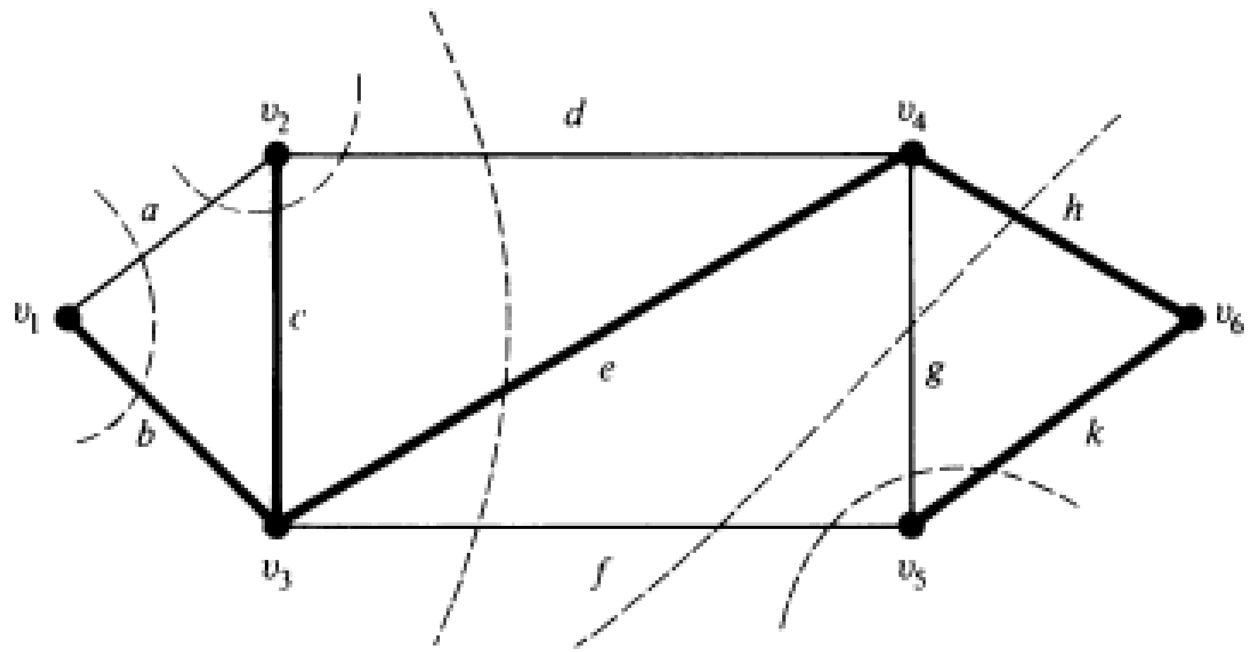
$$\{a, b\} \oplus \{b, c, e, f\} = \{a, c, e, f\}, \quad \text{another cut-set,}$$

$$\{d, e, g, h\} \oplus \{f, g, k\} = \{d, e, f, h, k\}$$

$$= \{d, e, f\} \cup \{h, k\}, \text{ an edge-disjoint union of cut-sets.}$$

Fundamental Cut Set

A cut-set containing exactly one branch of a spanning tree is called a fundamental cut set. No. of such fundamental cut sets in a graph is (n-1).



Relation between Fundamental Cut Set and Fundamental Circuit

A cut-set containing exactly one branch of a spanning tree is called a fundamental cut set. Number of such fundamental cut sets in a graph is (n-1).

A circuit, containing exactly one chord with respect to a spanning tree, is called a fundamental circuit. Number of such fundamental circuit in a graph is (e-n+1).

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Consider a spanning tree T in a given connected graph G. Let c_i be a chord with respect to T, and let the fundamental circuit made by c_i be called Γ, consisting of k branches $b_1, b_2, \ldots, b_k$ in addition to the chord c_i; that is,

$$\Gamma = \{c_i, b_1, b_2, \ldots, b_k\} \quad \text{is a fundamental circuit with respect to } T.$$

Let S_1 be the fundamental cut-set associated with b_1, consisting of q chords in addition to the branch b_1; that is,

$$S_1 = \{b_1, c_1, c_2, \ldots, c_q\} \quad \text{is a fundamental cut-set with respect to } T.$$

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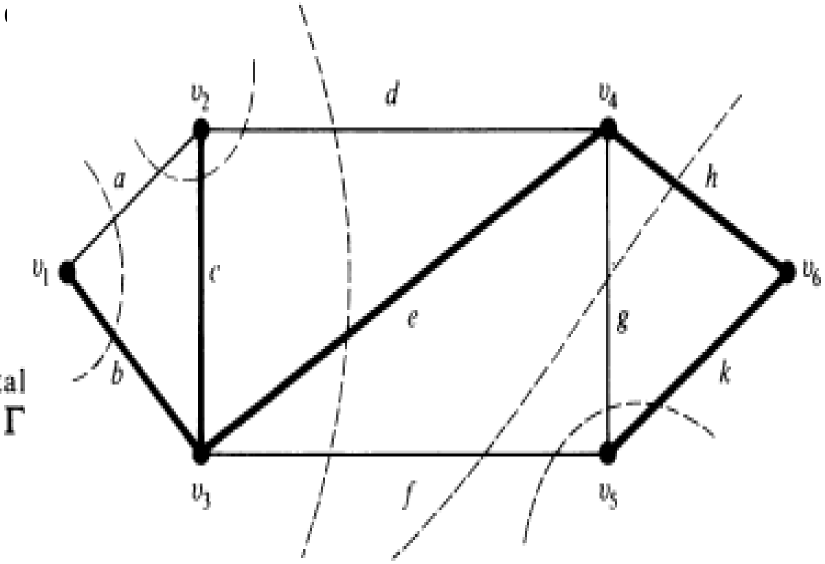

THEOREM 4-5

With respect to a given spanning tree T, a chord c_i that determines a fundamental circuit Γ occurs in every fundamental cut-set associated with the branches in Γ and in no other.

As an example, consider the spanning tree $\{b, c, e, h, k\}$, shown in heavy lines, in Fig. 4-3. The fundamental circuit made by chord f is

$$\{f, e, h, k\}.$$

The three fundamental cut-sets determined by the three branches e, h, and k are

determined by branch e: $\{d, e, f\}$,

determined by branch h: $\{f, g, h\}$,

determined by branch k: $\{f, g, k\}$.

Chord f occurs in each of these three fundamental cut-sets, and there is no other fundamental cut-set that contains f. The converse of Theorem 4-5 is also true.

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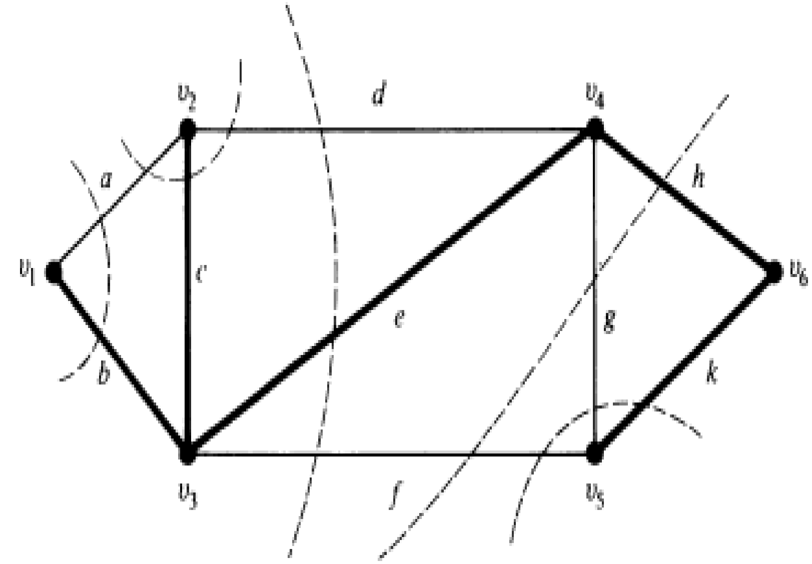

THEOREM 4-6

With respect to a given spanning tree T, a branch b_i that determines a fundamental cut-set S is contained in every fundamental circuit associated with the chords in S, and in no others.

Turning again for illustration to the graph in Fig. 4-3, consider branch e of spanning tree $\{b, c, e, h, k\}$. The fundamental cut-set determined by e is

$$\{e, d, f\}.$$

The two fundamental circuits determined by chords d and f are

determined by chord d: $\{d, c, e\}$,

determined by chord f: $\{f, e, h, k\}$.

Branch e is contained in both these fundamental circuits, and none of the remaining three fundamental circuits contains branch e.