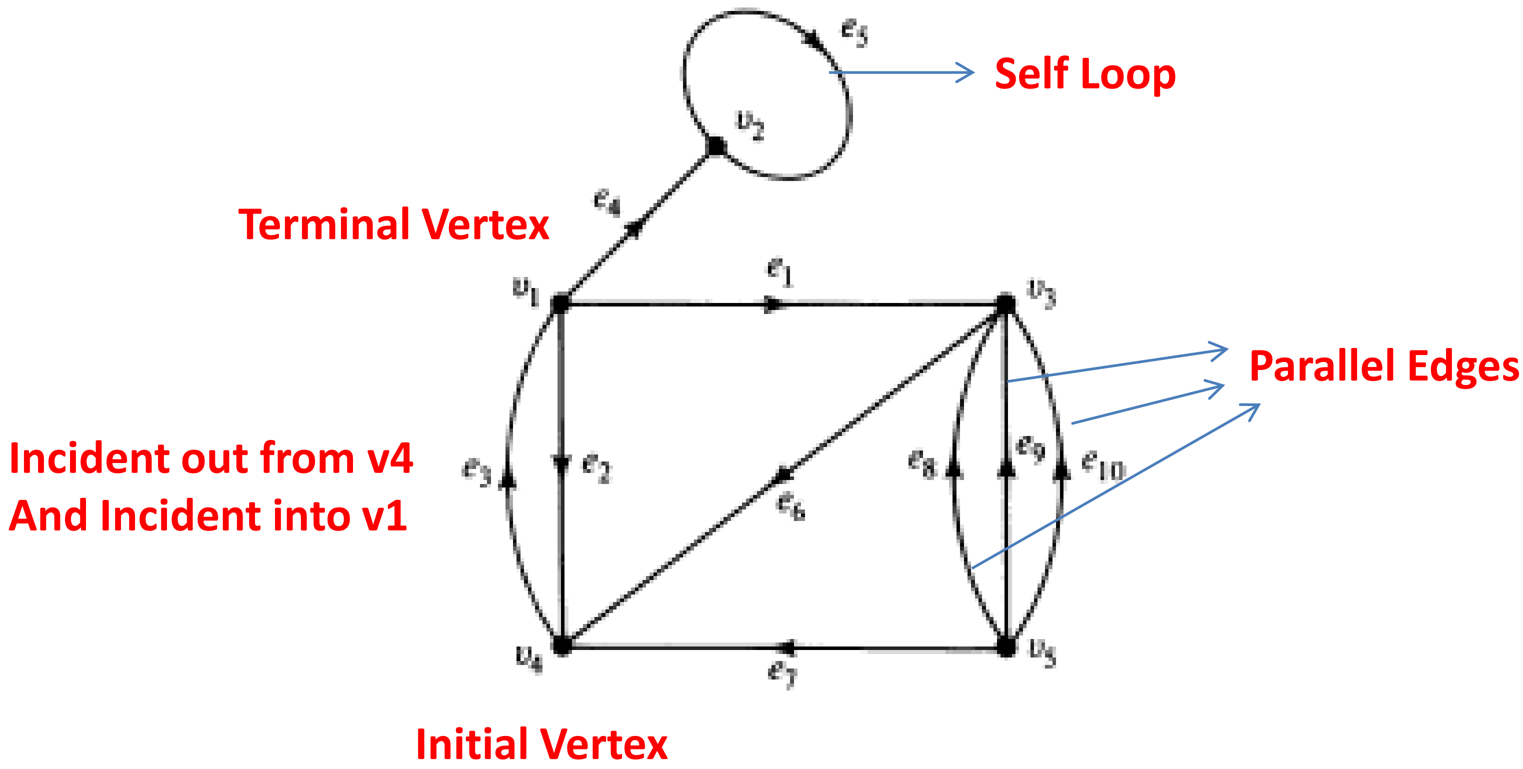
DAY 14

Directed Graph

A *directed graph* (or a *digraph* for short) G consists of a set of vertices $V = \{v_1, v_2, \ldots\}$, a set of edges $E = \{e_1, e_2, \ldots\}$, and a mapping Ψ that maps every edge onto some *ordered* pair of vertices (v_i, v_j).



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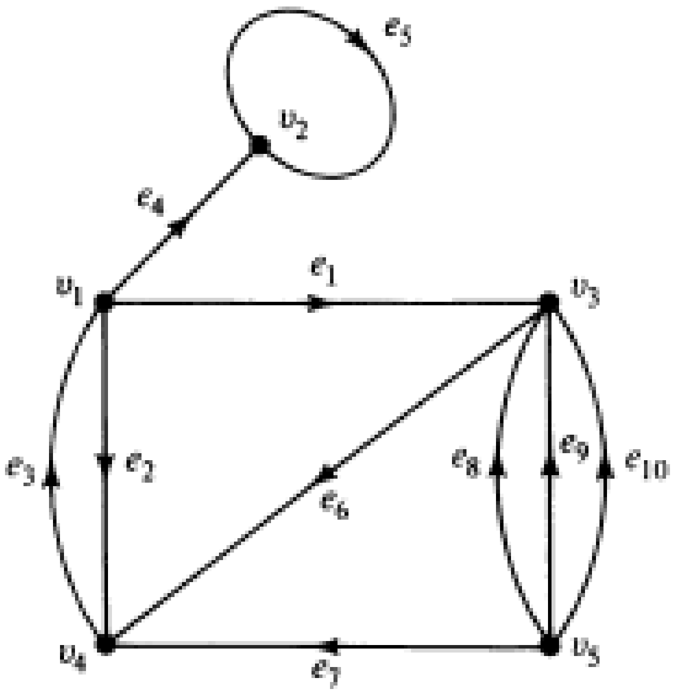


The no. of edges incident out from a vertex is called the out-degree of that vertex and is denoted as: $d^+(v_i)$.

The no. of edges incident into a vertex is called the in-degree of that vertex and is denoted as: $d^-(v_i)$.

d+(v1) = 3	d-(v1) = 1
d+(v2) = 1	d-(v2) = 2
d+(v3) = 1	d-(v3) = 4
d+(v4) = 1	d-(v4) = 3
d+(v5) = 4	d-(v5) = 0

$$\sum_{i=1}^{n} d^+(v_i) = \sum_{i=1}^{n} d^-(v_i).$$

Handshaking Di-lemma

Contd..

An *isolated vertex* is a vertex in which the in-degree and the out-degree are both equal to zero. A vertex v in a digraph is called *pendant* if it is of degree one, that is, if

$$d^+(v) + d^-(v) = 1.$$

Isomorphic Digraphs

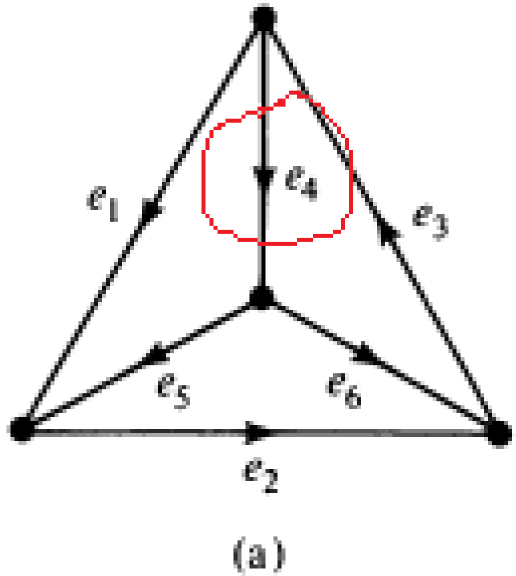


Fig. 9-2 Two nonisomorphic digraphs.

Types of Digraphs

Simple Digraphs: A digraph that has no self-loop or parallel edges is called a simple digraph

Asymmetric Digraphs: Digraphs that have at most one directed edge between a pair of vertices, but are allowed to have self-loops, are called *asymmetric* or *antisymmetric*.

Symmetric Digraphs: Digraphs in which for every edge (a, b) (i.e., from vertex a to b) there is also an edge (b, a).

A digraph is said to be *balanced* if for every vertex v_i the in-degree equals the out-degree; that is, $d^+(v_i) = d^-(v_i)$.

Contd..

Complete Digraphs: A complete undirected graph was defined as a simple graph in which every vertex is joined to every other vertex exactly by one edge. For digraphs we have two types of complete graphs. A *complete symmetric digraph* is a simple digraph in which there is exactly one edge directed from every vertex to every other vertex (Fig. 9-3), and a *complete asymmetric digraph* is an asymmetric digraph in which there is exactly one edge between every pair of vertices (Fig. 9-2).

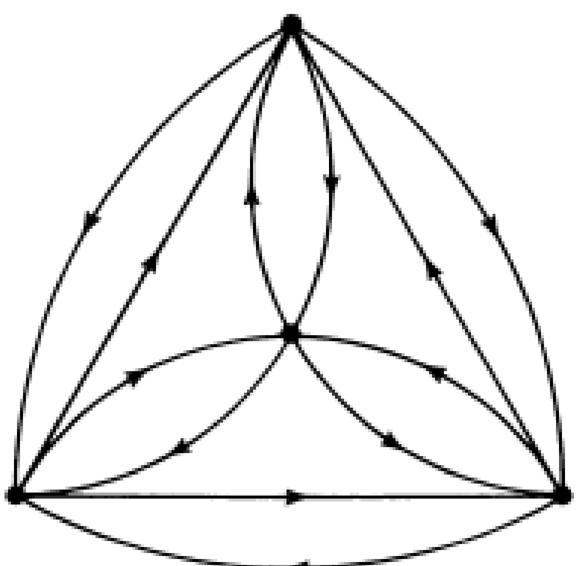


Fig. 9-3

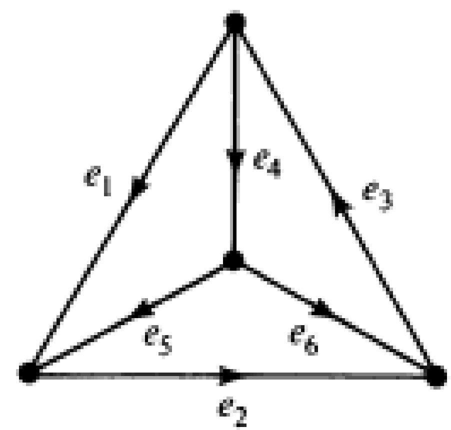


Fig. 9-2

Relation Matrix

Relation Matrices: A binary relation R on a set can also be represented by a matrix, called a *relation matrix*. It is a (0, 1), n by n matrix, where n is the number of elements in the set. The i, jth entry in the matrix is 1 if $x_i R x_j$ is true, and is 0, otherwise. For example, the relation matrix of the relation "is greater than" on the set of integers {3, 4, 7, 5, 8} is

$$\begin{array}{c c} & \begin{array}{ccccc} 3 & 4 & 7 & 5 & 8 \end{array} \\ \begin{array}{c} 3 \\ 4 \\ 7 \\ 5 \\ 8 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}.$$

Question

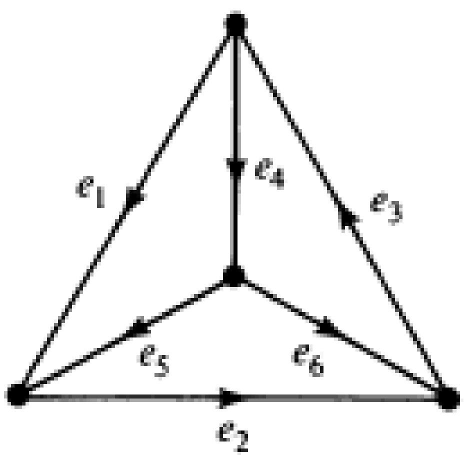
Define with example:

Directed Walk vs. Semi walk

Directed Path vs Semi Path

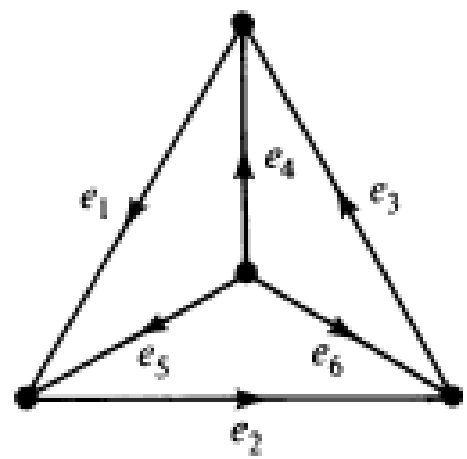
Connected Digraph

In a digraph there are two different types of paths. Consequently, we have two different types of connectedness in digraphs. A digraph G is said to be *strongly connected* if there is at least one directed path from every vertex to every other vertex. A digraph G is said to be *weakly connected* if its corresponding undirected graph is connected but G is not strongly connected.



(a)

Strongly Connected



(b)

Weakly Connected

Directed Tree: Arborescence

Arborescence: A digraph G is said to be an arborescence if

1. G contains no circuit—neither directed nor semicircuit.
2. In G there is precisely one vertex v of zero in-degree.

This vertex v is called the *root of the arborescence*. An arborescence is shown in Fig. 9-11.

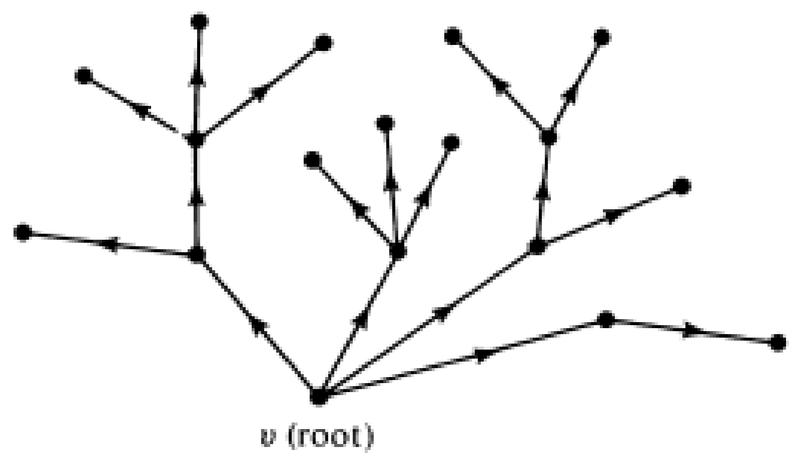


Fig. 9-11 Arborescence.

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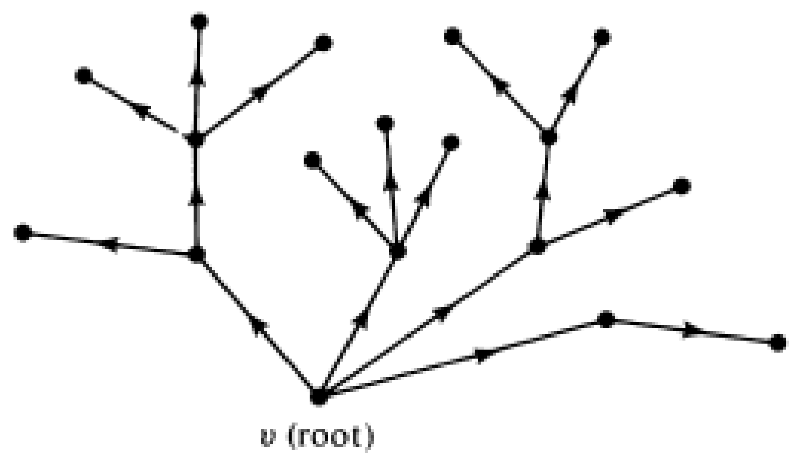
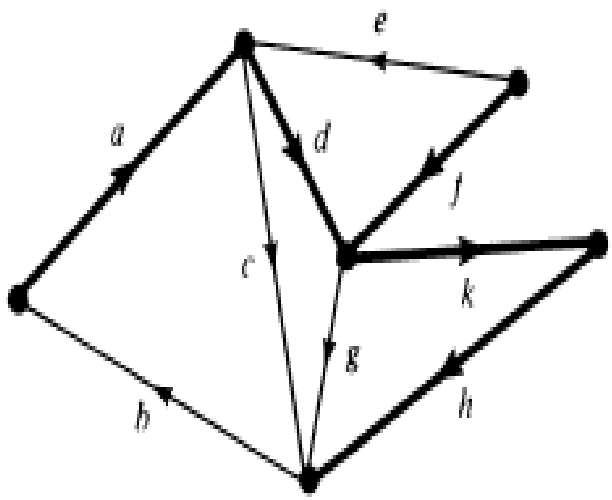


Fig. 9-11 Arborescence.

Fundamental circuit in Digraph



Rank $r = 5$

Nullity $\mu = 4$

Spanning tree T: $\{a, d, f, h, k\}$

Chord-set with respect to T: $\{b, c, e, g\}$

Fundamental circuits with respect to T:

- $d\ f\ e$ (semicircuit)
- $d\ k\ h\ c$ (semicircuit)
- $k\ h\ g$ (semicircuit)
- $a\ d\ k\ h\ b$ (directed circuit)

Fundamental cut-sets with respect to T:

- $a\ b$
- $b\ c\ d\ e$
- $e\ f$
- $b\ c\ g\ k$
- $b\ c\ g\ h$