EXPT NO.4 :
STUDY OF BOOLEAN EXPRESSION SIMPLIFICATION

Objective :

To study the Boolean rules and Boolean Expression simplification.

Equipments :

Logic Circuit Simulator Pro.

Theory :

A set of rules or Laws of Boolean Algebra expressions have been invented to help reduce the number of logic gates needed to perform a particular logic operation resulting in a list of functions or theorems known commonly as the Laws of Boolean Algebra. Boolean Algebra is the mathematics we use to analyse digital gates and circuits. Boolean Algebra is therefore a system of mathematics based on logic that has its own set of rules or laws which are used to define and reduce Boolean expressions.

List of some of the Boolean Laws are given below :-

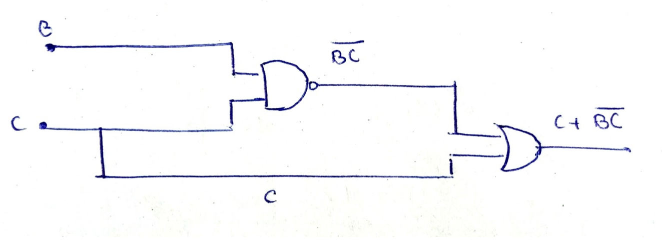
1. **The Idempotent Laws :**
   i) A.A=A ii) A+A=A
2. **The Associative Laws :**
   i) (AB)C = A(BC) ii) (A+B)+C = A+(B+C)
3. **The Commutative Laws :**
   i) AB = BA ii) A+B = B+A
4. **The Distributive Laws :**
   i) A(B+C) = AB + AC ii) A+BC = (A+B)(A+C)
5. **The Identity Laws :**
   i) AF = F, AT = A ii) A+F = A, A+T = T
6. **The Complement Laws :**
   i) A.$\overline{A}$ = 0 ii) A+$\overline{A}$ = 1
7. **The involution Law :**
   i) $\overline{\overline{A}}$ = A
8. **The De-Morgan's Laws :**
   i) $\overline{A+B} = \overline{A} + \overline{B}$ ii) $\overline{(A.B)} = \overline{A} + \overline{B}$

Simplification of some Boolean expressions and their verification :-

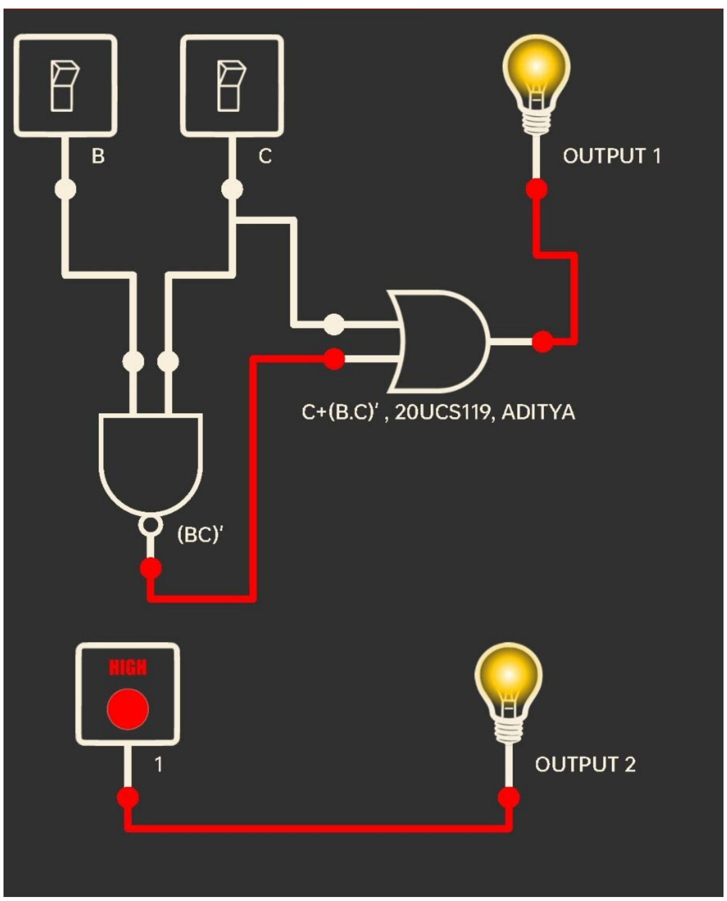
1. Simplify: C + $(BC)'$:

Expression	Rule(s) Used
C + (BC)′	Original Expression.
C+ $(B' + C')$	DeMorgan's Law.
(C+C') + B'	Commutative, Associative Laws.
T+ B'	Complement Law.
T	Identity Law.

Circuit Diagram :-



Logic diagram and simplified :-



TRUTH TABLE		
B	C	C+ (BC)’
0	0	1
1	0	1
0	1	1
1	1	1

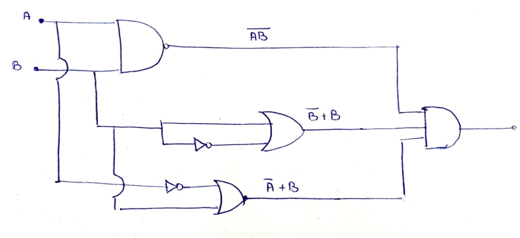
2. Simplify: $(AB)' + (A' + B)(B' + B)$

Expression	Rules used
$(AB)' + (A' + B)(B' + B)$	Original Expression
$A'B'(A' + B)$	Complement law, Identity law
$(A' + B')(A' + B)$	DeMorgan's Law
$A' + B'B$	Distributive law.

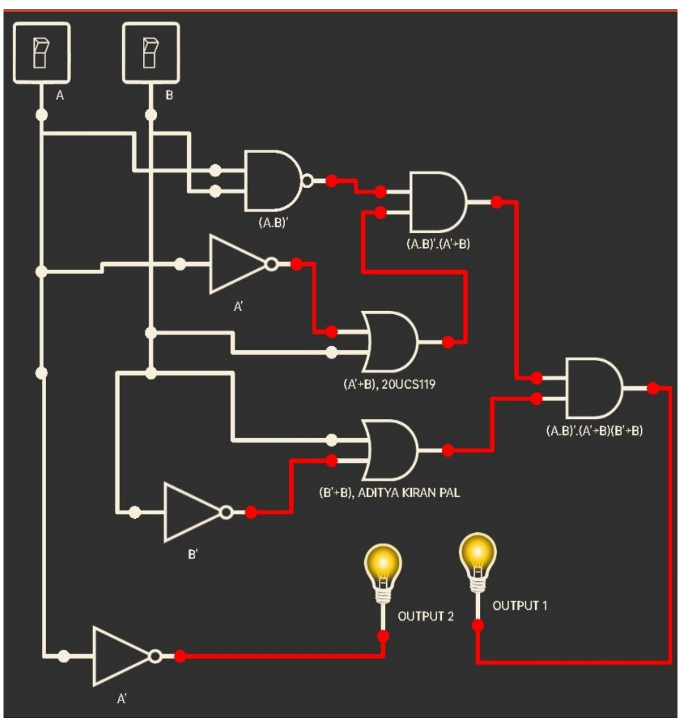
This step uses the fact that or distributes over and. It can look a bit strange since addition does not distribute over multiplication.

A	Complement. Identity.

Circuit Diagram :-



Logic diagram and simplified :-



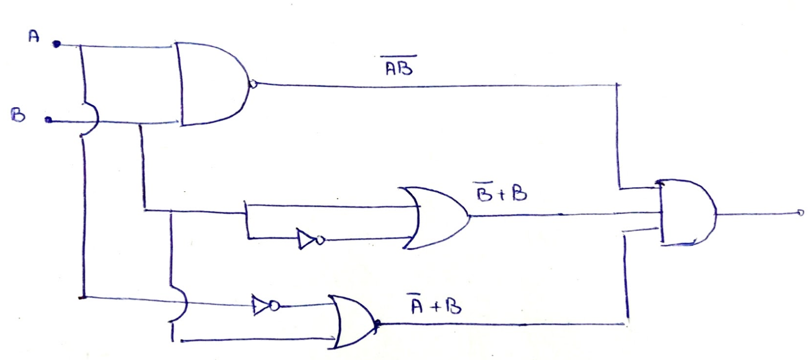
TRUTH TABLE			
A	B	A'	(AB)'(A' + B)(B'+B)
0	0	1	1
1	0	0	0
0	1	1	1
1	1	0	0

3. Simplify: (A+C)(AD +AD') + AC + C:

Expression	**Rule(s) Used**
(A+C)(AD +AD') + AC + C:	Original Expression
(A+C)A(D +D') + AC + C:	Distributive.
(A + C)A + AC + C	Complement, Identity.
A((A+C) + C) +C	Commutative. Distributive.
A(A+C) + C	Associative. Idempotent
AA+AC+C	Distributive
A+ (A + T)C	Idempotent, Identity, Distributive.
A+C	Identity, twice.

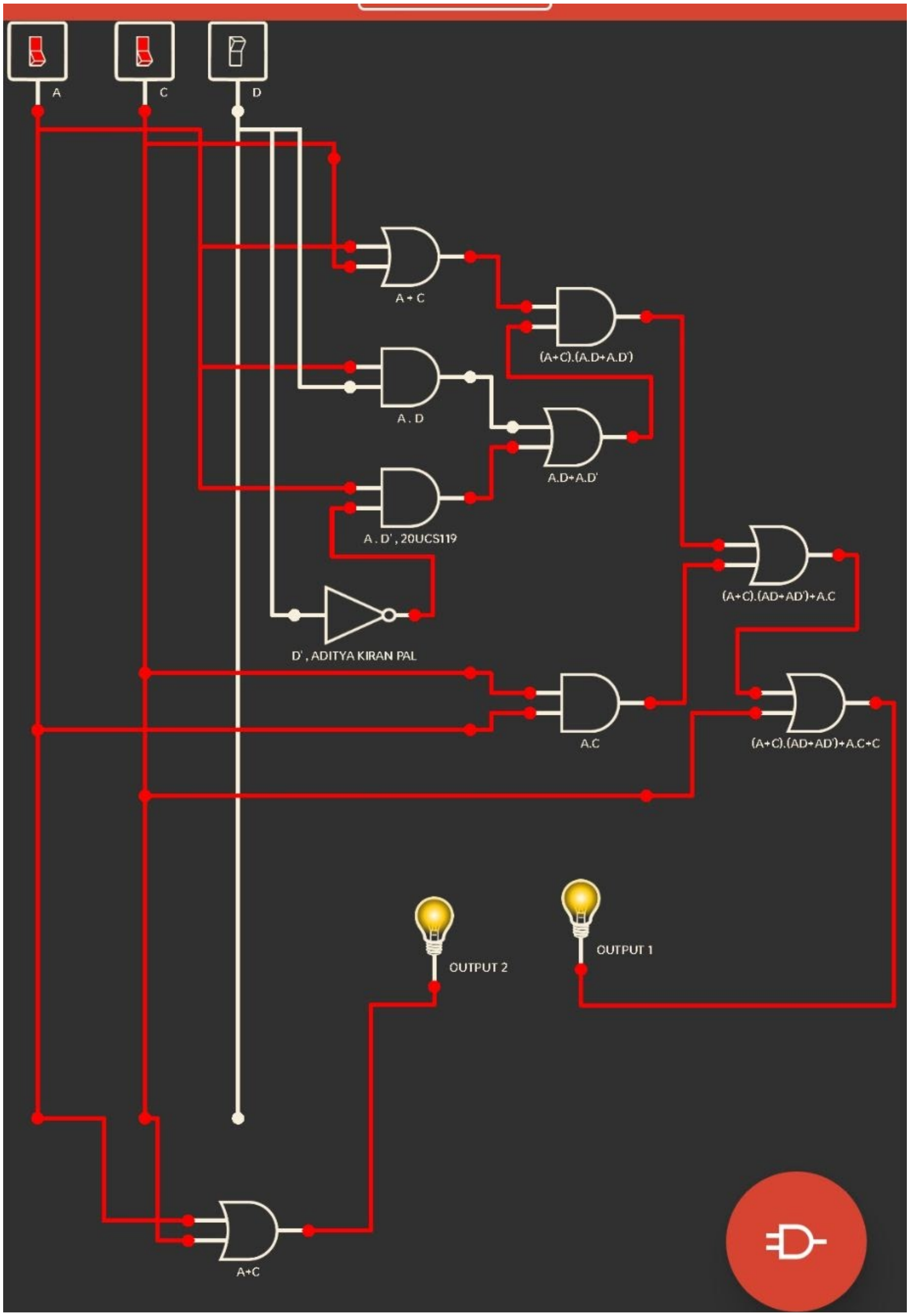
You can also use distribution of or over and starting from A(A+C) +C to reach the same result by another route.

Circuit Diagram :-



Truth table				
A	C	D	A+C	(A + C)(AD + AD’) + AC + C
0	0	0	0	0
0	0	1	0	0
0	1	0	1	1
1	0	0	1	1
1	1	0	1	1
1	0	1	1	1
0	1	1	1	1
1	1	1	1	1

Logic diagram and simplified :-



4. Simplify: A' (A+B) + (B + AA) (A + B'):

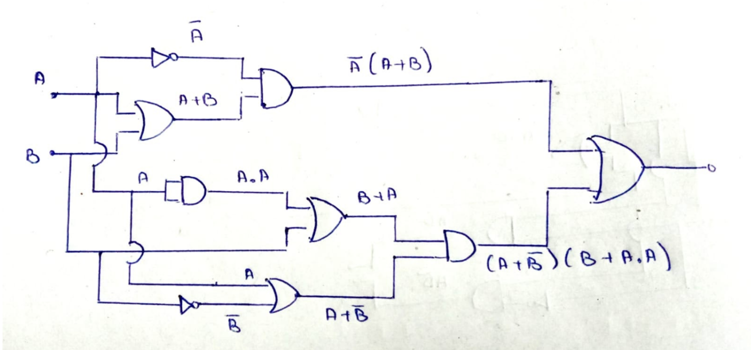
Expression	**Rule(s) Used**
A'(A+B) + (B + AA) (A + B'):	Original Expression
A'A+A'B+ (B +A) A+(B+A) B'	Idempotent (AA to A), then Distributive, used twice.

AB + (B + A) A+ (B + A) B Complement, then Identity. (Strictly speaking, we also used the Commutative Law for each of these applications.)

A'B+BA+AA + B B'+ AB' Distributive, two places AB+BA+A+AB Idempotent (for the A's), then Complement and Identity to remove BB.

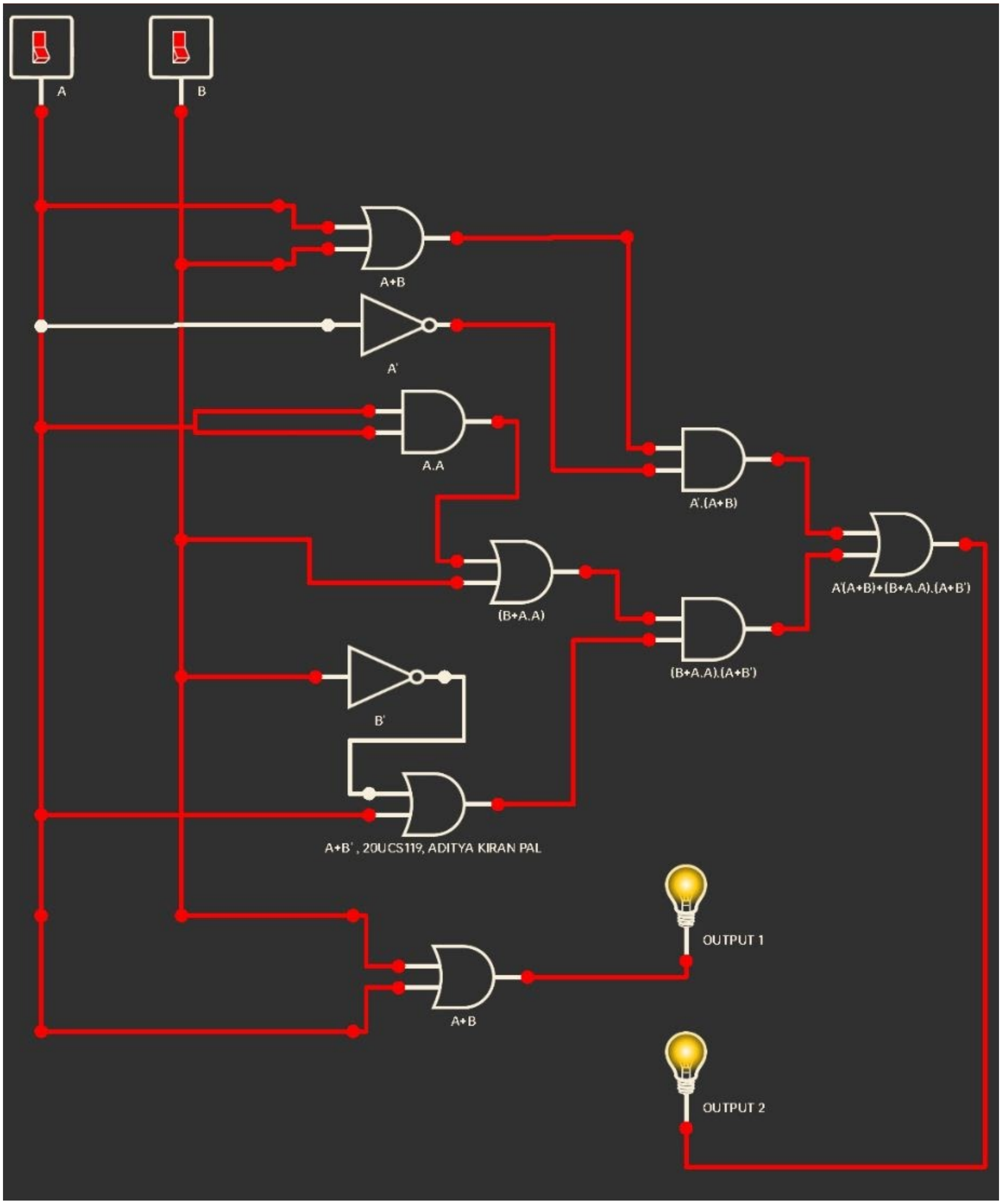
A'B + AB+AT+AB'	Commutative, Identity; setting up for the next
A'B + A(B+T+B')	Distributive.
A'B+A	Identity, twice (depending how you count it).
A+ A'B	Commutative.
(A+ A')(A+B)	Distributive.
A+B	Complement, Identity

Circuit Diagram :-



TRUTH TABLE			
A	B	A+B	A' (A + B) + (B +AA)(A+B')
0	0	0	0
1	0	1	1
0	1	1	1
1	1	1	1

Logic diagram and simplified :-

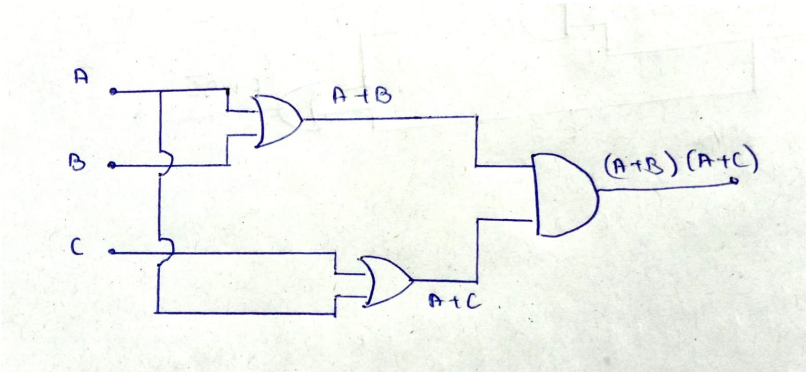


5. Simplify: (A+B). (A+C)

Expression	**Rules Used**
A. A+A.C + A.B + B.C	Distributive law
A+A.C + A.B +B.C	Idempotent AND law
(A.A = A) A(1+C) + A. B+B.C	Distributive law
A.1 + A. B+B.C	Identity OR law
(1+C = 1) A(1+B) + B.C	Distributive law
A.1 +B.C	Identity OR law
(1+ B = 1) Q= A + (B.C)	Identity AND law
(A.1 = A)	

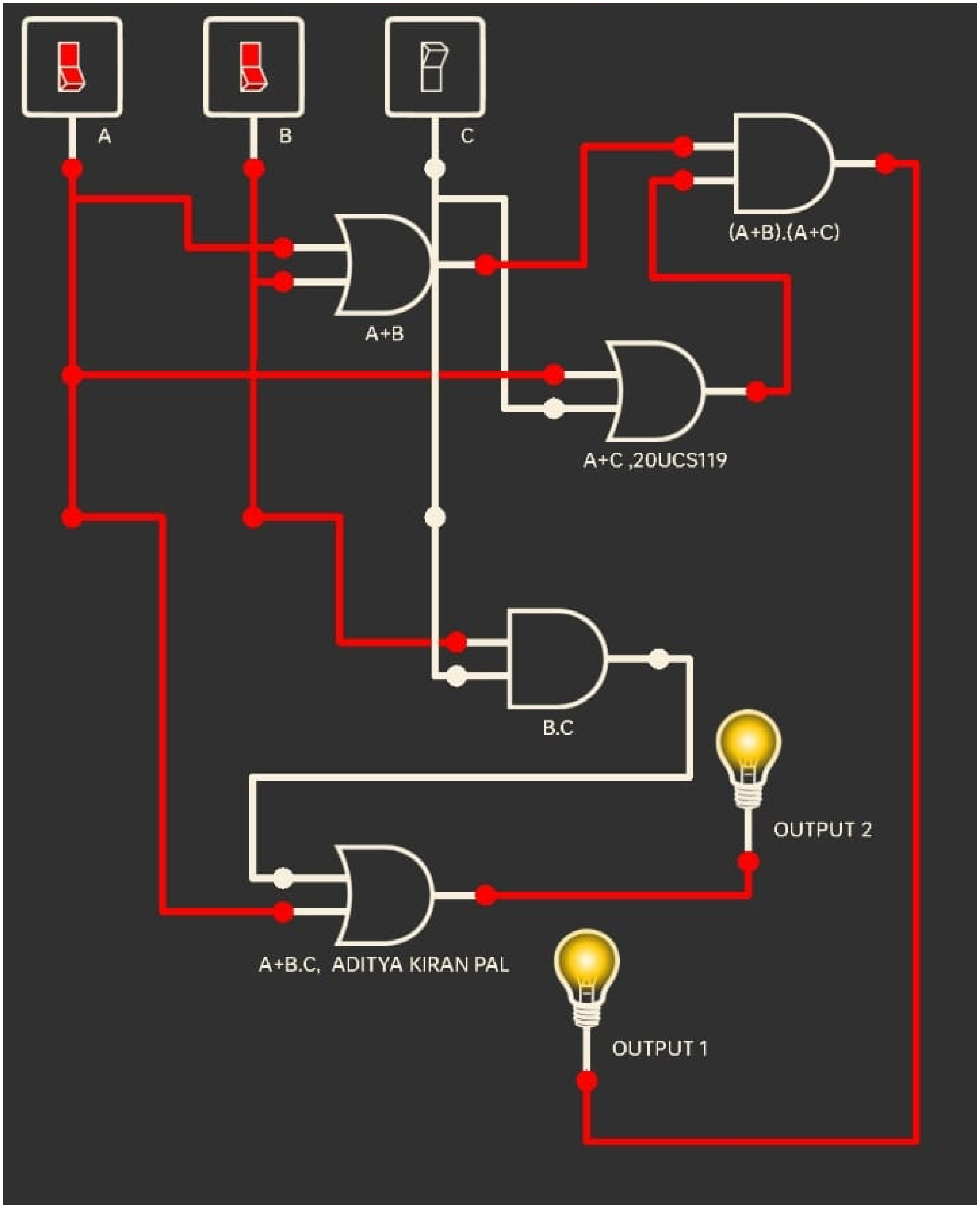
Expression: (A + B) (A+C) can be simplified to A+ (B.C) as in the Distributive law.

Circuit Diagram :-



TRUTH TABLE				
A	B	C	A + BC	(A+ B)(A+C)
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
1	0	0	1	1
1	1	0	1	1
1	0	1	1	1
0	1	1	1	1
1	1	1	1	1

Logic diagram and simplified :-



6. Simplify: AB+A(B+C) +B(B+C)

Step 1: Apply the distributive law

AB + AB + AC + BB + BC {Distributive law; A (B+C) = AB+AC, B (B+C) = BB+BC}

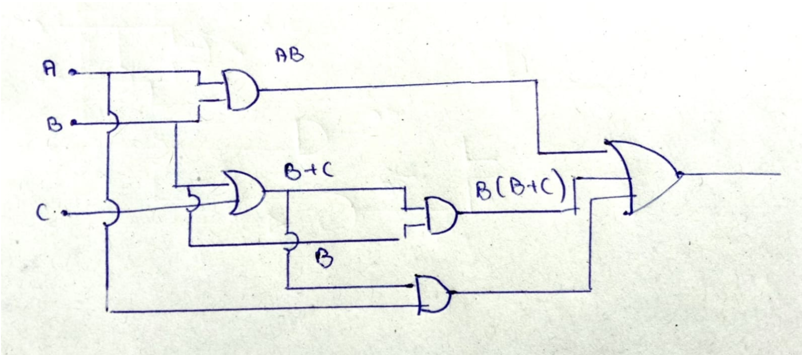
Step 2: Apply the idempotent law AB + AB + AC + B + BC {Idempotent law; BB = B}

Step 3: Apply the idempotent law AB + AC + B + BC {Idempotent law; AB+AB = AB}

Step 4: Apply the absorption law AB + AC +B {Absorption law; B+BC = B}

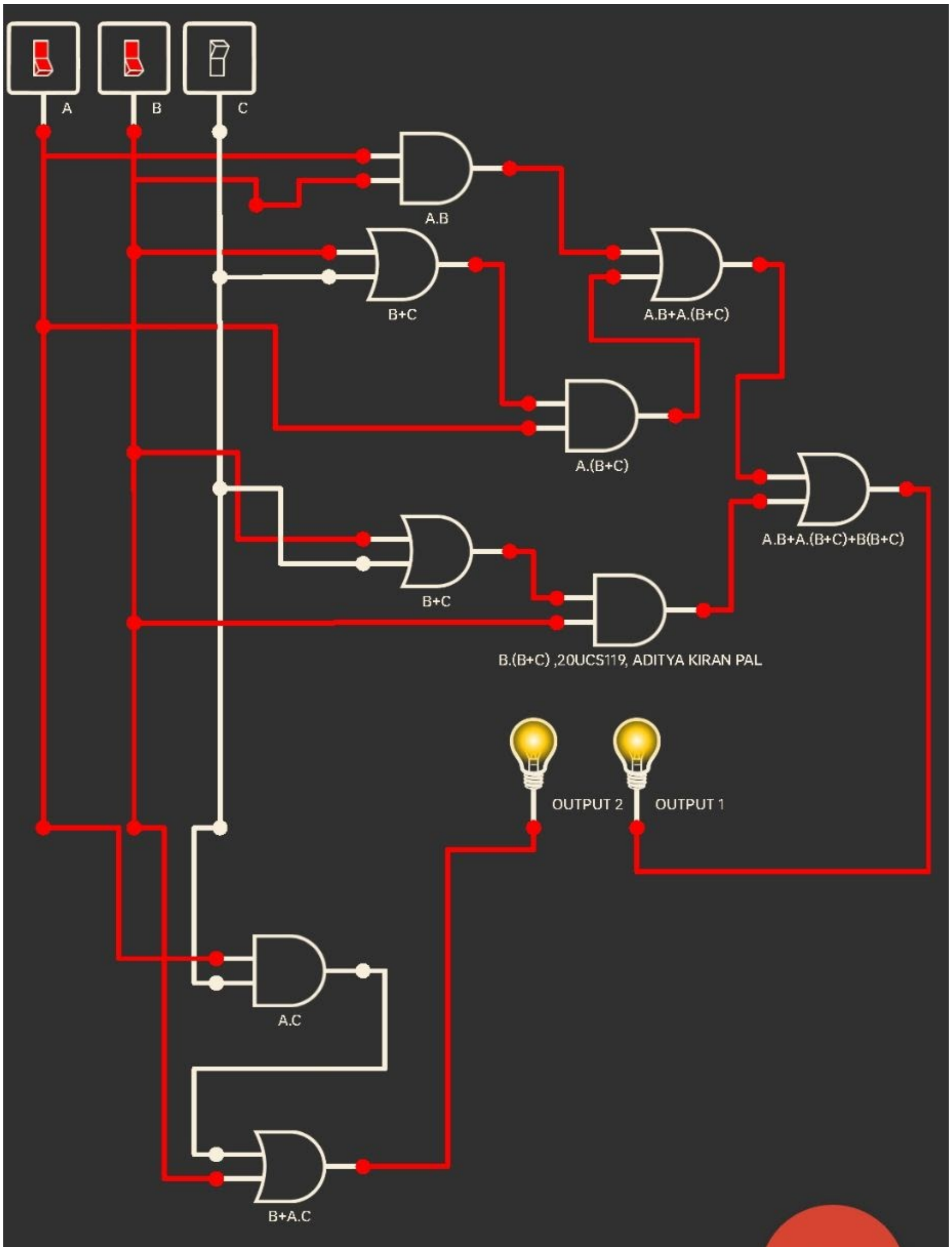
Step 5: Apply the absorption law B + AC {Absorption law; AB+B = B} Hence, the simplified Boolean function will be B + AC.

Circuit Diagram :-



TRUTH TABLE				
A	B	C	B+AC	AB + A(B + C) + B(B+C)
0	0	0	0	0
0	0	1	0	0
0	1	0	1	1
1	0	0	0	0
1	1	0	1	1
1	0	1	1	1
0	1	1	1	1
1	1	1	1	1

Logic diagram and simplified :-



Conclusion :

All the basic rules of boolean algebra are verified.