Oracle VM virtual box → terminal → ctrl+Alt+T.

- Richard Stallman.
- Linus Torvalds.

pwd → prints current directory.

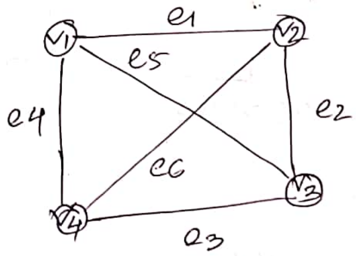
cd/ change directory.

Introduction to graph theory.

mathematical representation of a Graph $G(V, E)$ — V: set of vertices; E: set of edges

$V = \{v_1, v_2, \dots v_n\}$
$E = \{e_1, e_2 \dots e_n\}$] entities.

pictorial representation of some mathematical entities ~~with~~ the set of vertices & set of edges and set of v must be met with ~~som~~ the set of edges. / picture where some entities are

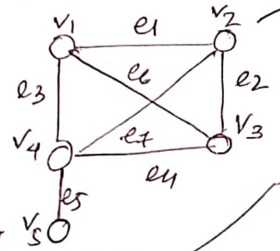


⇒ $V = \{v_1, v_2, v_3, v_4\}$
$E = \{e_1, e_2 \dots e_6\}$

G

3rd entity → ψ → says an edge is met to which pair of vertices

[relationship → edge mapped to an unordered vertices.]



→ undirected graph.

$\psi = \{e_1 \to (v_1, v_2)$ or (v_2, v_1)

$$\psi = \begin{cases} e_1 \to (v_1, v_2) \text{ or } (v_2, v_1) \\ e_2 \to (v_2, v_3) \\ e_3 \to (v_3, v_4) \\ e_4 \to (v_4, v_1) \\ e_5 \to (v_1, v_3) \\ e_6\ (v_4, v_2) \\ e_7\ (x_4, v_5) \end{cases}$$

example →

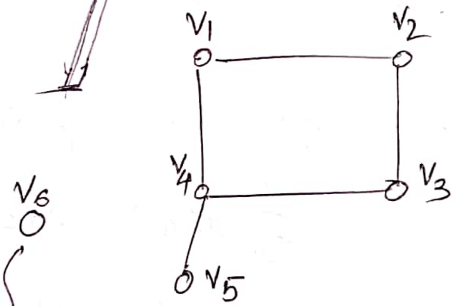
Teacher. Students.

question

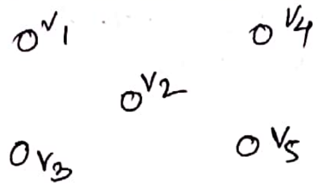
vertex.

edge (line of communicatⁿ)

minimum vertex for a graph ↓ 1



isolated vertex [not connected with any other vertices]

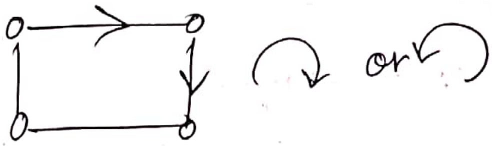


→ no comm, no edge // null graph

1 edge is not sufficient ⇓ 1 vertex is sufficient

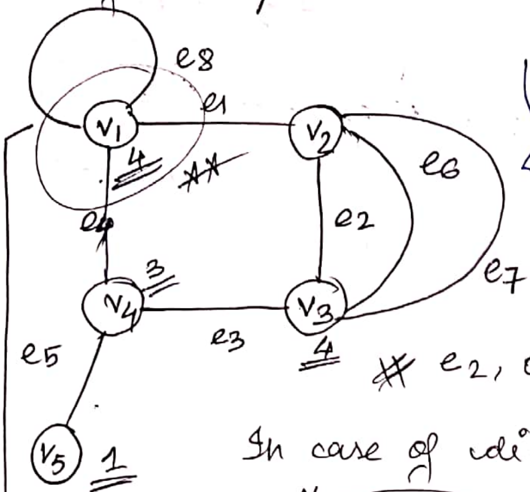
↓ smallest graph (X)

(*) Directed graph



or

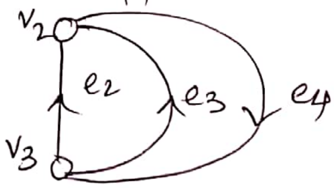
// If 1 edge is directed Q all are.



~~for every edge, there should~~
If for 2 or more edges, same pair of starting & ending vertex exist, then those 2 edges are known as parallel edges. → undirected graph.

e_2, e_6, e_7 → parallel edges.

In case of directed graph :



e_2, e_3 parallel.

e_8 → starting & ending vertex same ⇒ self loop.

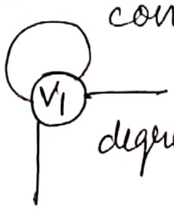
// simple graph → not having parallel edge,
not having ~~are~~ self loop.

p. ~~edge~~ s. ~~loop~~

1 vertex is connected to only 1 edge → pendant vertex

but self loop is not a pendent vertex.

Degree : The no of total edges connected to a vertex.

** 

degree → 4

// Handshaking Lemma

$$\sum_{i=1}^{n} d(v_i) = 2 \times e$$

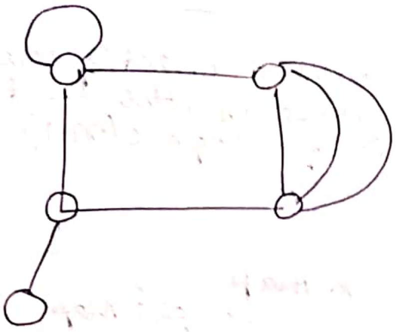
Total degree of all the vertices in a graph is twice the number of edges.

※ The no. of odd degree vertices in a graph is always even

$\sum_{i=1}^{n} \partial(v_i) = 2e$

$\Downarrow$

$\underbrace{\sum_{i=1}^{n} \partial_{odd}(v_i)}_{\text{has to be even}} + \underbrace{\partial_{even}(v_i)}_{\text{even no}} = \underbrace{2e}_{\text{even} \checkmark}$

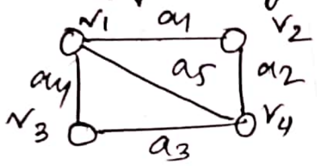


22/7

Adjacency : Two vertices are called adjacent to each other if they are having a common edge. → v_1, v_3 ; v_1, v_2 ; v_1, v_4

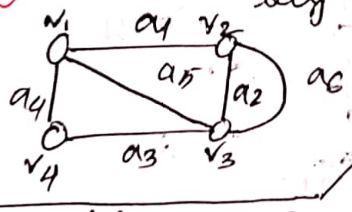
Two edges are also called adjacent if 1 vertex is common betn them.

The phenomenon is known as adjacency.



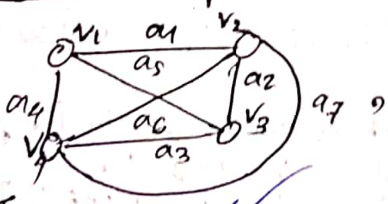
Incidence (fall upon) : Imp in case of directed graph
incident-in/out.

Multigraph : If a graph has only parallel edge but no self loop.



How many types of graphs are there?
Is there any specific name for a graph that is having only self loops.

Complete graph : If ~~one~~ each vertex is connected to each & every rest of all the vertices.



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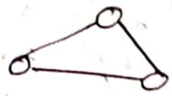
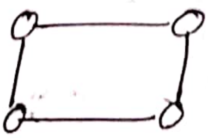


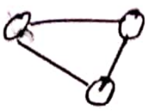
⇒ degree of each v. of c. graph : $(n-1)$.
n = total no. of ver.

Regular graph : If the degree of each of the vertices is same.

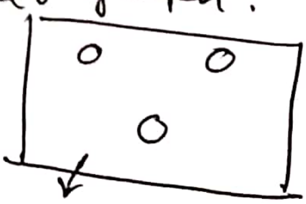


→ degree of each v. = 2
∴ known as 2r graph.

↳ 3r graph.



→ 2r graph.



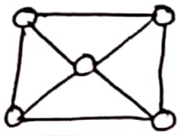
0r graph
[regular, null graph]

All complete graphs ~~can be~~ are regular graph.
But not all reg. graphs are complete g.

Q/ Assignment (1) : Explain w/ proper e.g that :

Wheel graph

A special type of graph where exactly one vertex is connected to rest of the vertices.
Exactly that 1 vertex is of degree (n-1)
Rest not known.



---

ec Binary code — Gorey Code

0 ← 00 ——— 00
1 ← 01 ——— 01
2 ← 10 ——— 11
3 ← 11 ——— 10

(*) minimisation of cost

⇓

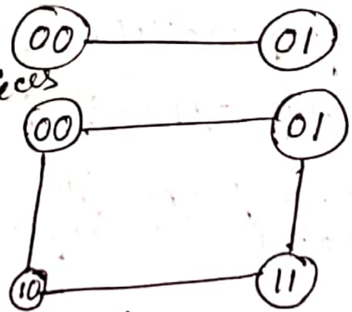
* circuit minimisation.
* Logic gates.

K-map
Karnough's Map.

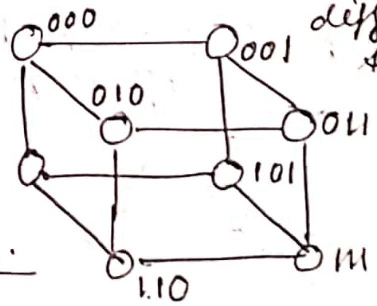
ab \ cd	00	01	11	10

N-cube graph → 1 edge exists betn 2 vertices if there is exactly 1 bit diff. betn them.

2^n → 2^1 vertices
→ 2^2 v.



2^3 →



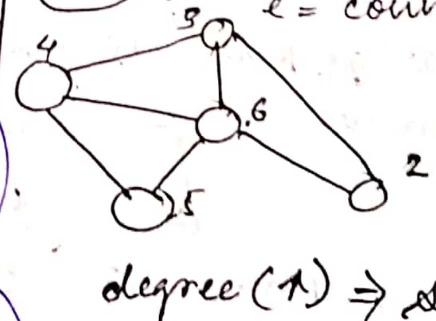
3 bits.

n = total no of bit.

Mention 5/10 different graphs in which real life situation is represented w/ vertices & edges.

[social media → one person is connected to another i.e friends]
[trains (V), movement (e)], (LAN) → V = locatns.
e = connectivity.

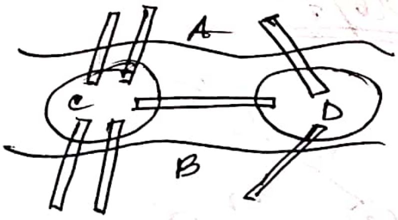
Airport, shops

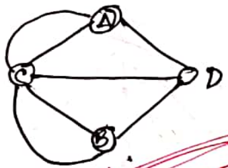


(*)

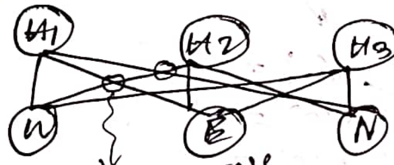
degree (↑) ⇒ stability (↑)
(↓) ⇒ vulnerable.

* Game Theory :
* Königsberg bridge problem (1736)
* Euler (Graph theory)





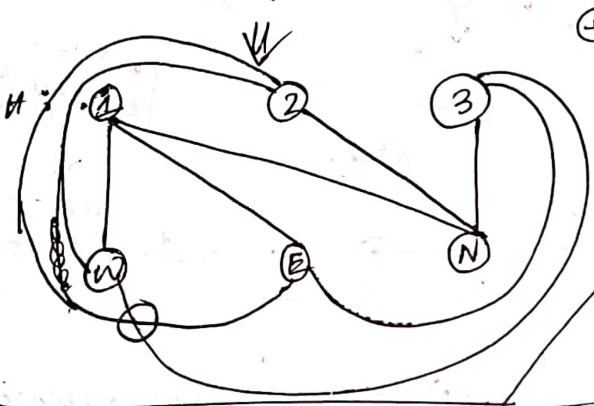
~~Housing complexes.~~



→ Bipartite graph.

V = { {H1, H2, H3}, {W, E, N} }

* minm no. of crossover.



minimizat^n of crossover.

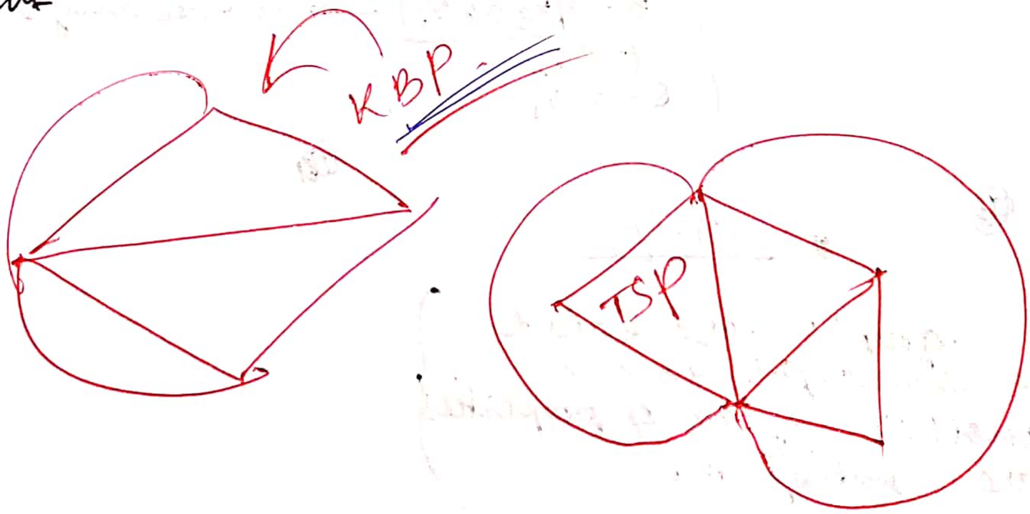
Google search.

* 1997 page-rank algorithm.
* no of references.

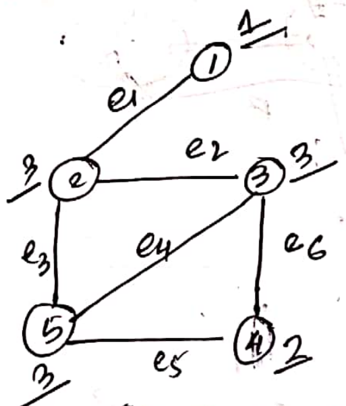
Q/ convince yourself that the max^m degree of any vertex in a simple graph with n vertices is (n-1).

Q3/ show that the max^m no of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$

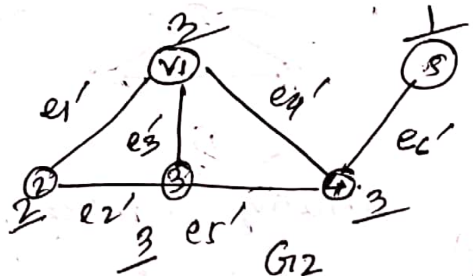
Assignment



26/7.



G_1



G_2

Isomorphism conditions :

1. same no. of vertices
2. same no. of edges.
3. same no. of vertices w/ a given degree

$d(1) = 01$
$d(2) = 01$
$d(3) = 03$

pendant vertices.

2°

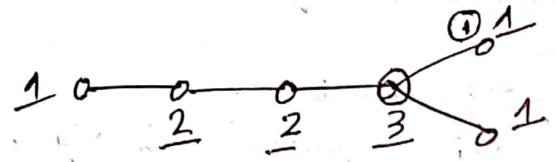
3°

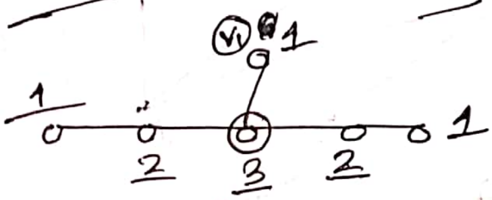
isomorphic

not correspondent

not isomorphic

G_3





6 vertices
5 edges

$d(1) = 3$
$d(2) = 2$
$d(3) = 1$

isomorphic

4th property : 1 to 1 correspondence. betn vertices & edges.

vertex | edge ⟹

$1 \rightarrow v_5$
$2 \rightarrow v_4$
$3 \rightarrow \boxed{v_3 \text{ or } v_1} \longrightarrow$ choose any : v_3
$4 \rightarrow v_2$
$5 \rightarrow v_1$

$e_1 - e_6'$, $e^6 - e_2'$
$e_2 - e_5'$
$e_3 - e_4'$
$e_4 - e_3'$
$e_5 - e_1'$

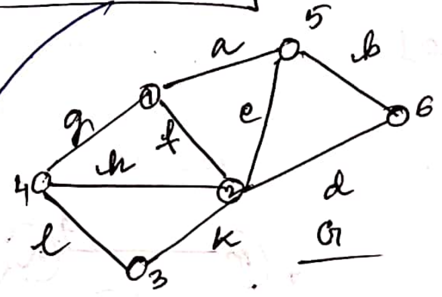
G_4

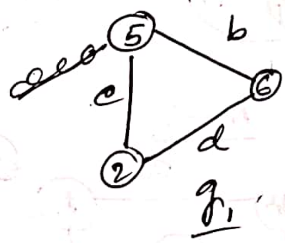
G_3

vertex

2 graphs are said to be isomorphic to one another if all 4 properties are satisfied.

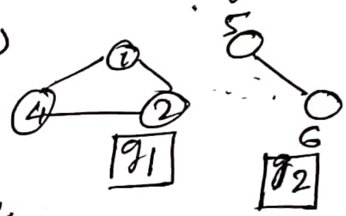
Subgraph





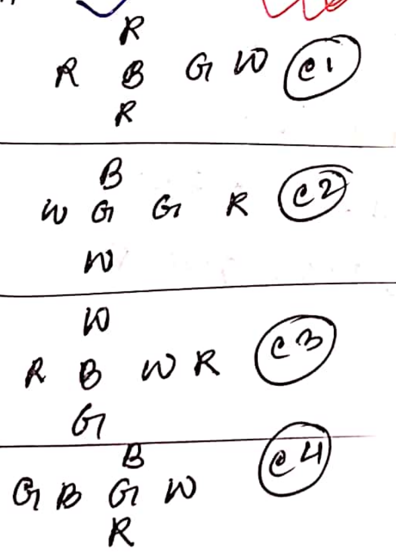
① Every graph is its own subgraph.
② Any single vertex is a subgraph.
③ Any single edge with its end vertices is a subgraph.
④ A S/Graph of a S/Graph is a S/G of the main graph.

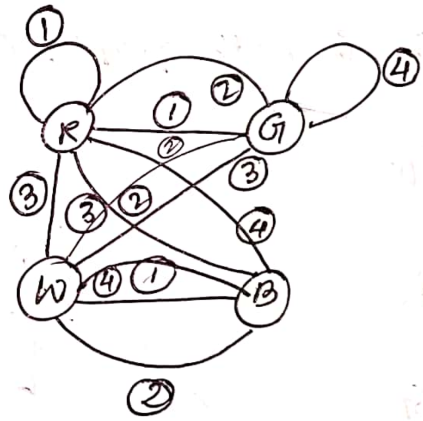
→ vertex disjoint subgraph (no vertex common)

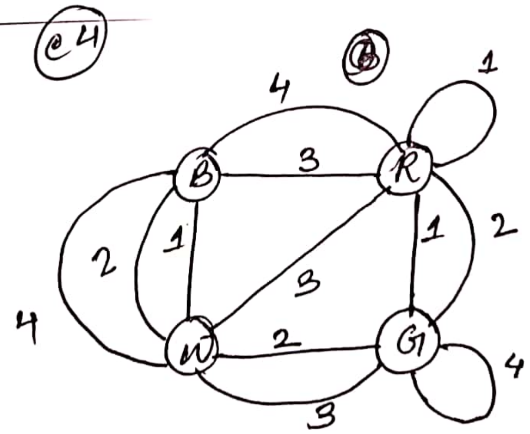


→ edge - disjoint subgraph
↳ no edge common

Multicolor cube Problem (3×4^3) combinations.





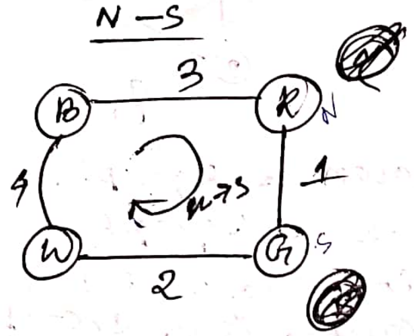


4 (N–S)

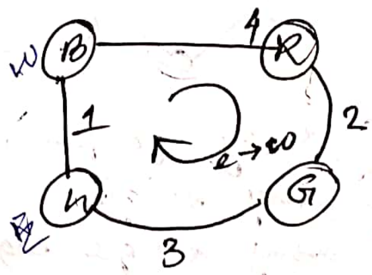
4 (E–W)

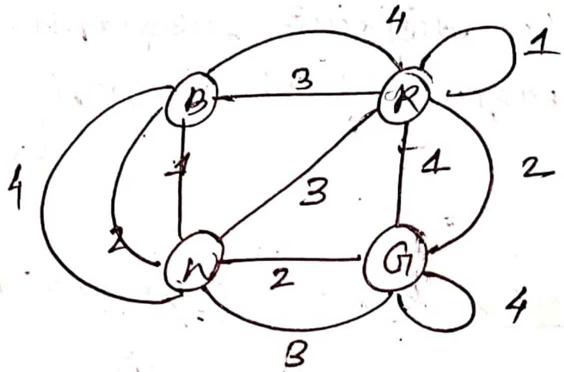
16 sides [visible]

N – S

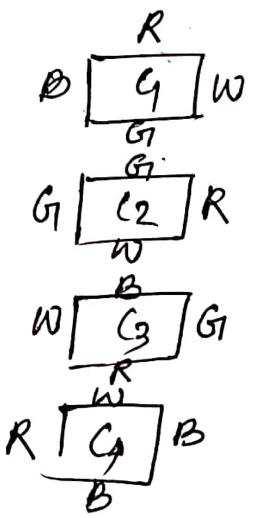


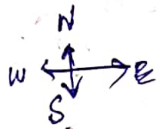
E – W





① same dirⁿ in both graphs (2)





2 subgraphs :

① 1 edge from each cube

② Every vertex has degree 2 .

Q Given 4 cubes variously coloured w/ B, R, G, W ① Is it possible to stack the cubes one on top of another to form a column such that no colour appears twice on any of the 4 sides of this column.

② Is it possible to stack the cubes in such a way the cubes in such a way that each side shows only 1 colour.

C-1:
R
B R G
W
G

C-2:
W
B G R
G
B

C-3:
G
B B R
R
W

C-4:
B
B W G
B
R

{Assignment} 4

29/7

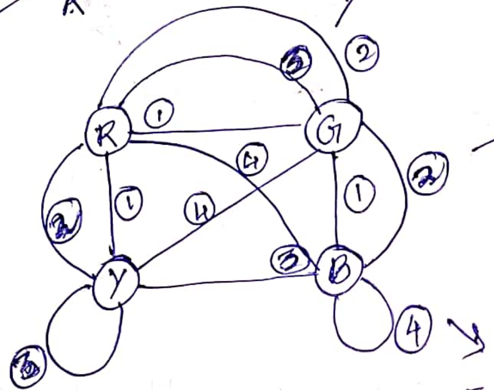
Q

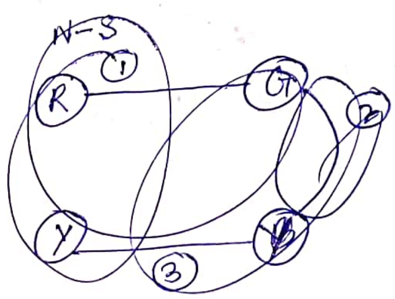
①
G
Y G R B
R

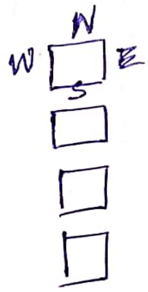
②
B Y
R Y G B
Y

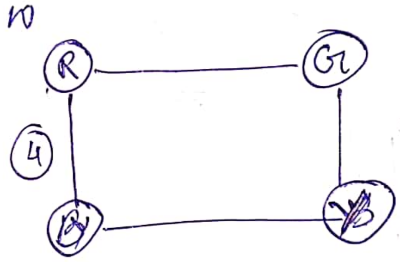
③
B
G R R Y
G

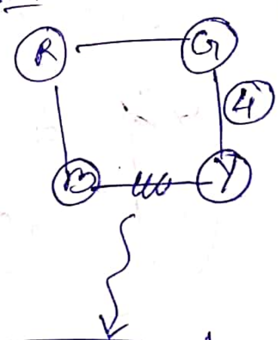
④
B
Y B G R
B



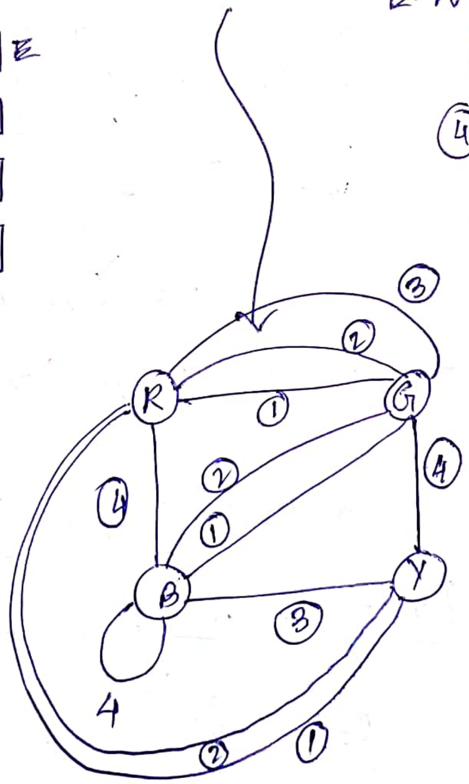








missing



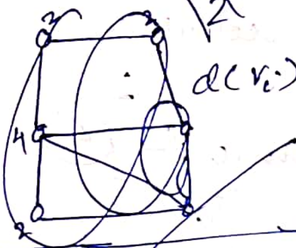
As per the arrangement of cubes given, N-S, E-W graphs aren't complete.

∴ It isn't possible to stack the cubes . . .

Q → { Determine the no. of edges in a graph w/ v = 6, out of which 2 of degree 4 & 4 of degree 2. Draw the graph. }

$\frac{2(2-1)}{2} = 1$ $\quad$ $\frac{2(4-1)}{2} = 3$

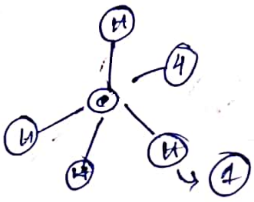
⇒ $\frac{n(n-1)}{2}$ $\quad$ $6(6-1)$ $\quad$ $\frac{2(2-1)}{2} = 1$ $\quad$ $\frac{4(3)}{2} = 6$



$\sum d(v_i) = 2e$ → $2\times4 + 4\times2 = 8+8$

$e = \frac{16}{2} = 8$

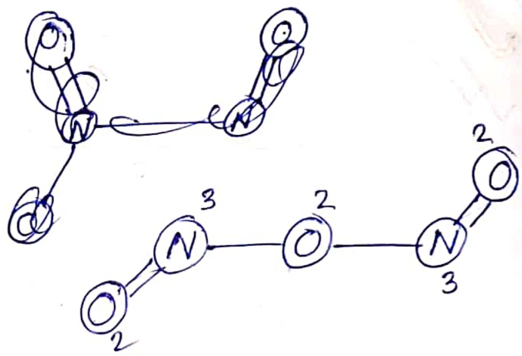
Q Draw a graph for this chemical combination CH_4



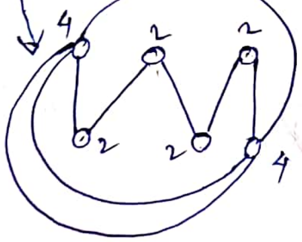
Q/ N_2O_3

⇓

degree (?)



Q/ Is it possible to construct a graph with 12 vertices such that 2 have degree 3 & remaining have degree 4.



⇒ $\underline{(2\times3) + (10\times4)} = \boxed{2e}$

⇓

46

$e = 23$

⇓

non-fractional no.

(*) If ans ⇒ fractional (graph) ✗

Q/ Is it possible to draw a simple graph (4 → v) (7 → e)

$\partial^{max}(n-1) = 3$

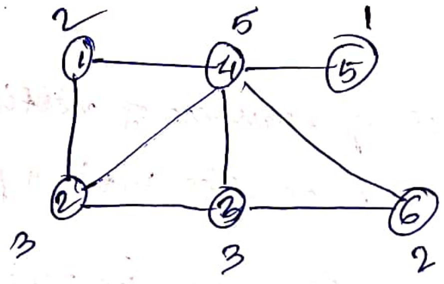
$\boxed{4\times3}$ → 12 (= 2e)

$2e = 14$ ✗

$\frac{n(n-1)}{2}$

(max^m edges)

Degree Sequence



$seq_G = \{1, 2, 2, 3, 3, 5\}$

can there exist a simple graph w/ degree seq:

2, 2, 3, 4 ? | $(4-1) \to 3$

$(1, 2, 2, 3, 4, 5)$ δ_2 Not possible. $17 = 2e$, $e = 8.5$

$(2, 2, 4, 6) \to 14 \to (4-1) \to 3$

$n = 6$

$n - 1 = 5$

$1\times1 + 2\times2 + 3\times1 + 4 + 5 = 2e$

$17 = 2e$

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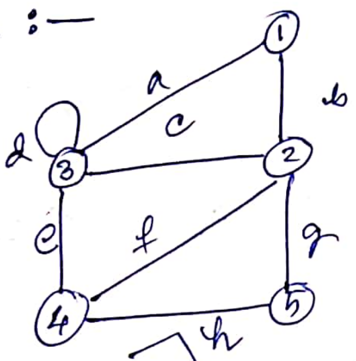
Walk, path, circuit :-
Trail

Walk :-



- Finite alternating sequence of vertices and edges,
- where every edge lies betn a pair of vertices,
- where edge & vertices can be repeated

W1 : (1,5) → 1 b 2 g 5

combinatn : vertex edge vertex

W2 : 1 a 3 d 3 e 4 h 5

W3 : $\boxed{1 b 2 c 3 a 1 b 2 g 5}$ not a trail

W1 (1,5) will be similar as W2 (5,1) as long as path is same?

- vertex / edge repeatation is allowed.

Types → closed walk (source & destination same)
→ open walk (source, destinatn different)

Trail :- A walk where no edge repeation is there. but, vertex can be repeated.

Trail WT1 : (1,5) → 1 b 2 g 5

→ open
→ closed.

All trails are walks but all walks are not trails.

WT2 : 1 a 3 d 3 e 4 h 5

Path :- A walk w/ no vertex, edge repeatation.
A trail w/ no vertex repeatation.

open ~~closed~~.

path is always open

W1 is a path, W2 and W3 are not.

- length of path = no. of edges.

P1 :- (1,5) → 1 b 2 g 5

→ length of P1 = 2

Closed trail is known as a circuit, where no vertex repeatation is allowed except the starting & the ending vertex.

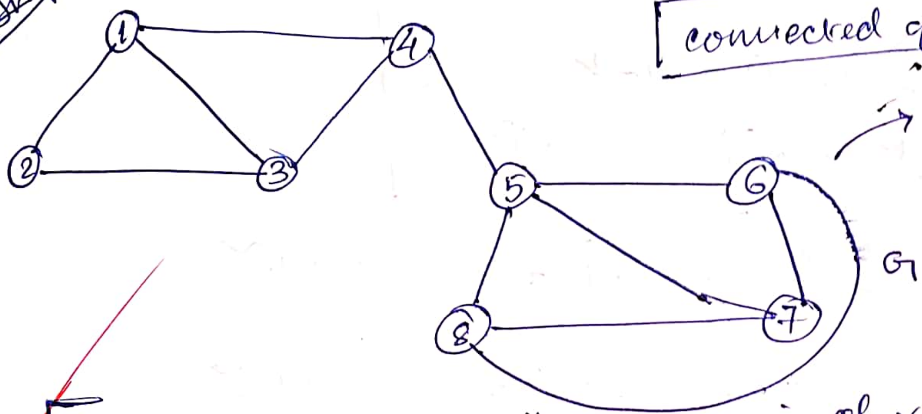
example : 3d3.

e2 : 2f4h5g2

walk → e ✓ v ✓
trail → e ✗ v ✓
path → e ✗ v ✗
ckt → closed trails v ✗

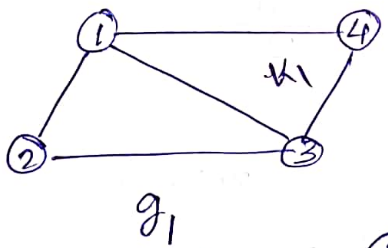
All self loops are circuits but not all circuits are self loops.

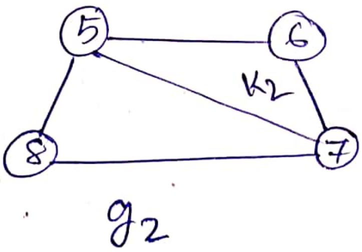
Imp



connected graph → at least 1 path in betⁿ every pair of vertices.

In a graph, in betⁿ every pair of vertices, if there exists ~~at least~~ 1 path, then its called connected graph, otherwise its called disconnected.





The connected subgraphs in a disconnected graph are called components. & are denoted as K of graph G

If a graph is connected → 1 component.
If a graph is disconnected → >1 component.

(*) A simple graph w/ n vertices & k components can have at most $(n-k)(n-k+1)/2$ edges.

→ Let us assume there are n_1 no. of vertices in component k_1

n_2 vertices in component k_2

⋮

n_k " " " k_n

$\left\{ \frac{(n-k)(n-k+1)}{2} \right\}$

$n_1 + n_2 + \dots n_k = n$

$n_i = n_1 + n_2 + \dots n_k$

$n_i = n$.

$$\sum_{i=1}^{k} n_i = n$$

$$\Rightarrow \sum_{i=1}^{k} (n_i - 1) = n - k.$$

$$\Rightarrow \left[\sum_{i=1}^{k} (n_i - 1)\right]^2 = (n-k)^2$$

$$\Rightarrow \sum_{i=1}^{k} (n_i^2 - 2n_i + 1) = n^2 - 2nk + k^2$$

$$\Rightarrow \sum_{i=1}^{k} (n_i^2) - 2n + k = n^2 - 2nk + k^2$$

$$\Rightarrow \sum_{i=1}^{k} n_i^2 = n^2 - 2nk + k^2 + 2n - k \quad \text{—— (1)}$$

If a graph is simple,

max^m edges $= \frac{n(n-1)}{2}$

$$\sum \frac{n_i(n_i - 1)}{2} = \frac{1}{2} \sum n_i(n_i - 1)$$

$$= \frac{1}{2} \sum_{i=1}^{k} n_i^2 - \frac{1}{2} \sum n_i$$

$$= \frac{1}{2} \sum_{i=1}^{k} n_i^2 - \frac{1}{2} n$$

$$= \frac{n^2}{2} - nk + \frac{k^2}{2} + n - \frac{k}{2} - \frac{n}{2}$$

$$\cancel{= \frac{n^2}{2} + \frac{k^2}{2} + \frac{n}{2} - k\left(n + \frac{1}{2}\right)}$$

$$\cancel{= \frac{n}{2}(n+1) + \frac{k}{2}(k-1) - nk}$$

$= \frac{1}{2}\{n^2 - 2nk + k^2 + 2n - k\} - \frac{n}{2}$

$= \frac{1}{2}\{n^2 - nk - nk + k^2 + 2n - k - n\}$

$= \frac{1}{2}\{n^2 - nk + n - nk + k^2 - k\}$

$= \frac{1}{2}\{n(n-k+1) - k(n-k+1)\}$

$= \frac{1}{2}(n-k+1)(n-k)$

#

$$= \cancel{\frac{n^2}{2}+\frac{n}{2}+\frac{k^2}{2}-\frac{k}{2}-kn}$$

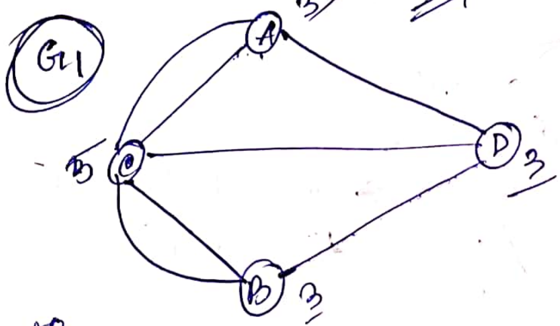
$$\cancel{= \frac{1}{2}\left[n^2+n+k^2-k\right]}$$

$$= \frac{1}{2}\left[\quad\right] - \frac{n}{2} \rightarrow \cancel{n^2-nk+n} \quad \cancel{nk+k^2-k}$$

$$= (n-k)(n-k+1)/2$$

Königsberg's bridge problem.

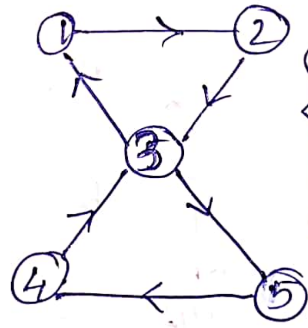
Euler graph : ⇒ closed trail where every edge will be traversed only once.



A graph is called a Euler graph, if there exists a closed trail ~~which traverses~~ where every edge will be traversed exactly once.

how to check → If ~~deg~~ degree of every vertex is even, then the graph is euler.

$\boxed{\text{euler graph}}$

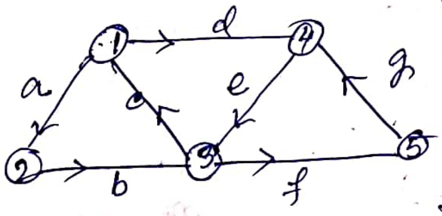


vertex repeatation possible, hence not circuit

Explain how Euler saw Königsberg bridge problem.

→ The königsberg bridge problem is represented in G1. ∵ all degrees of vertices aren't even, ∴ its not an euler graph & hence its not possible.

Open Euler graph / Unicursal graph



→ is a special version of euler graph where starting & ending vertices are different but all the edges are traversed exactly once.

Aka $\boxed{\text{unicursal graph}}$

Q/ Differentiate betⁿ $\boxed{\text{unicursal}}$ & $\boxed{\text{universal}}$ graph.

unicursal ↓ { open euler graph

universal ↓ complete graph. → aka complete graph.

{ starting & ending vertices must be of odd degree, rest all have even degree.

↳ If we add 1 line (edge) betⁿ start & end vertices → the open euler/ unicursal graph becomes a euler graph.

∴ those vertices must be of odd degree.

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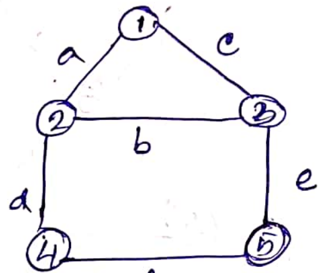
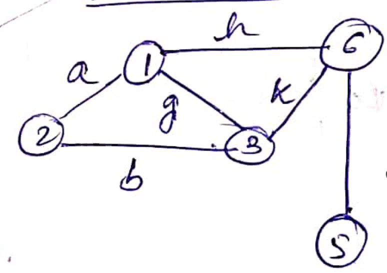
Operations on Graph

1. Union
2. Intersectⁿ
3. Ring Sum
4. Deletion
5. Fusion.

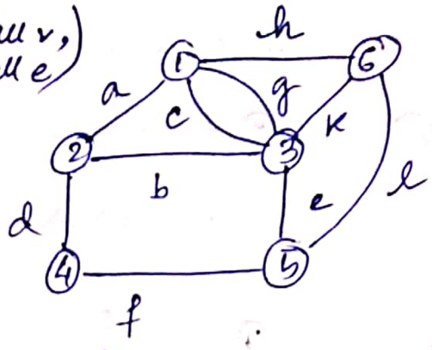


g_1



g_2

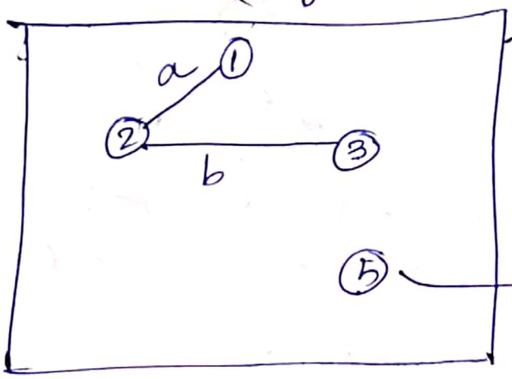
① union : (all v, all e)



$\boxed{g_1 \cup g_2}$

Includes all vertices & edges of both graphs.

② Intersection : (only common edges)



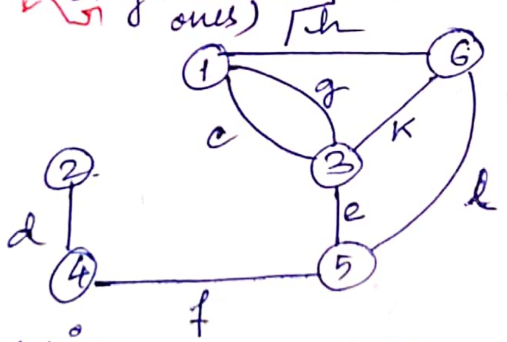
$\boxed{g_1 \cap g_2}$

Includes ~~all~~ only common edges & common V.

isolated vertex.

③ Ring sum : (all vertices, only uncommon ones)

All the ~~common~~ vertices will be included. But in case of edges only uncommon ones i.e, those that are either in g_1 ~~and~~ or g_2.

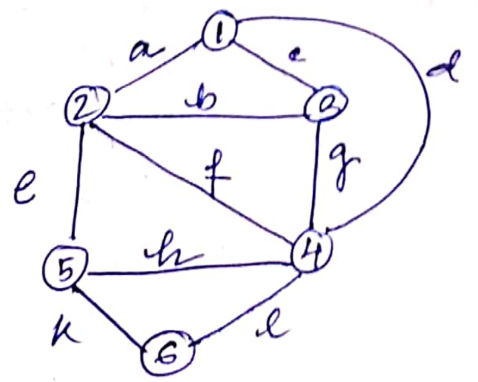


$\boxed{g_1 \oplus g_2}$

{ All the vertices & the uncommon edges }

④ Deletion → vertex
→ edge → G − e

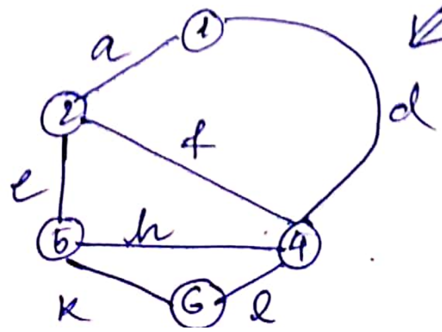
remove edge e without affecting any vertex.



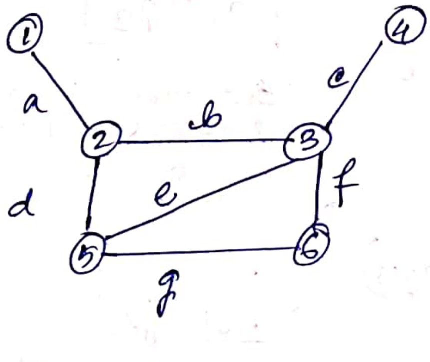
G

But for G − 3

all edges associated w/ vertex ③ will be removed.



5 Fusion (Merge)

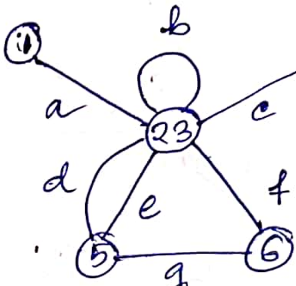


G

fuse 2 vertices :

* In one step only 2 vertices can be fused.

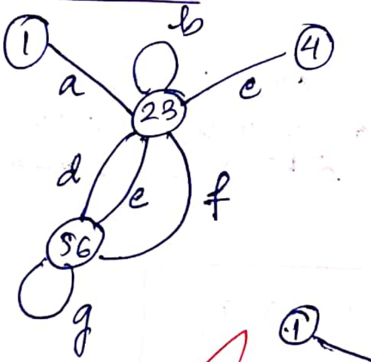
F(2,3) ⇒ 2 and 3 merged to a single vertex 23.



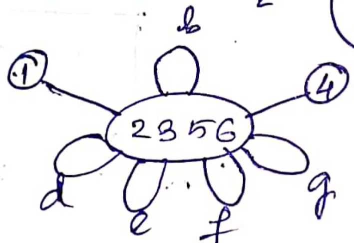
← fused graph

In a fused graph

- * no of edges are unchanged
- * but in every step 1 vertex is reduced.

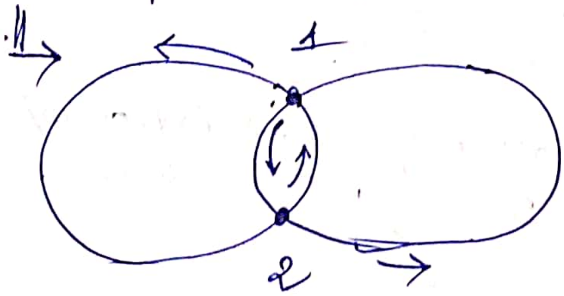


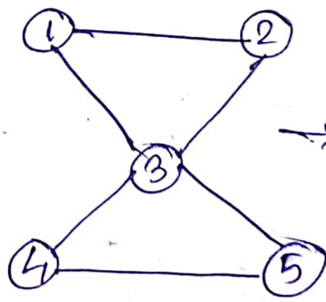
F(23, 56) ⇒



Arbitrarily Traceable Graph:

If a graph is eulerian with respect to every vertex, it is AT Graph.





→ It is eulerian graph only wrt 3.

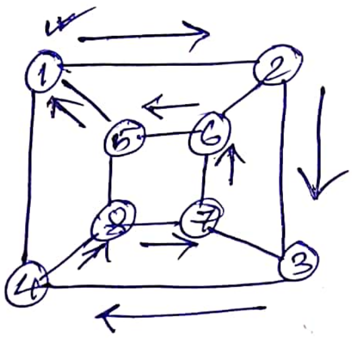
Hamiltonian Graph 1859 → Hamilton ✓

Q/ Diff betn Euler & Hamiltonian graph.

Euler Graph.	Hamiltonian graph.
(1) Every edge exactly once	(1) Every vertex exactly once.
(2) vertex can be traversed more than once/ About the vertex, how many times it's traversing we need not worry.	(2) About edge we need not to worry. [Every v traversed exactly once except the starting and ending vertex]
(3)	(3) Have to form a circuit, known as a Hamiltonian circuit ↳ defn.

In a graph, if we start from 1 v & reach to the same vertex by traversing every vertex exactly once, it is a Hamiltonian circuit (2) & the graph that has Hamilton. ckt is known as Hamiltonian graph.

(1) Hamiltonian path :



→ need not worry about edges.

→ 1 → 2 → 3 → 4 → 8 → 7 → 6 → 5 — 1

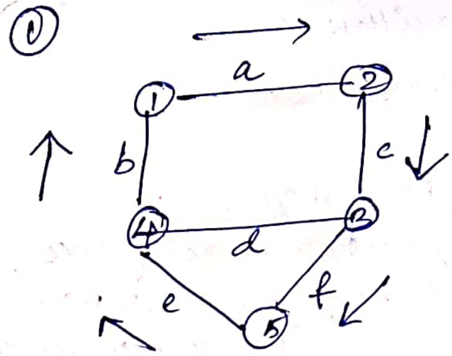
is a ckt ⇒ known as Hamiltonian circuit

↳ If 1 edge is deleted from a Ham. circuit, it will result in a path called Hamiltonian path.

If in a graph, a Hamiltonian circuit is there, the graph is called a H. graph,
But if ~~it a~~ a graph has only H. path, it will not be a H. graph.

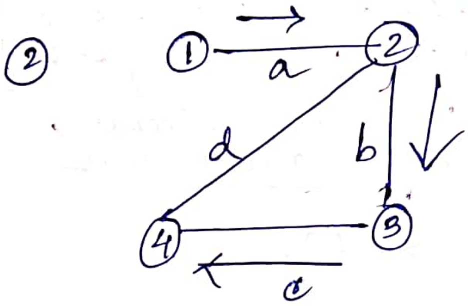
Q. 2.23 → has both H. ckt & path, hence is a H. graph.

(1)



→ has H. ckt
→ has H. path
⇒ is a H. graph.

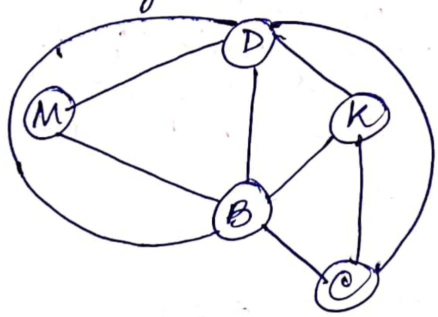
(2)



→ no H. ckt
→ has H. path ⇒ a b c or a d c

⇒ ∴ isn't a H. graph.

Travelling salesman problem (TSP)



{ minimum spanning tree }

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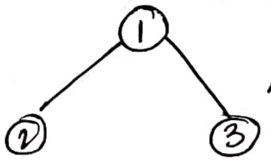
Tree → Graph (Not having any circuit)



Tree (a graph should have atleast 2 vertices to be considered as a tree)

1. simple graph.
2. No circuits

} Properties.

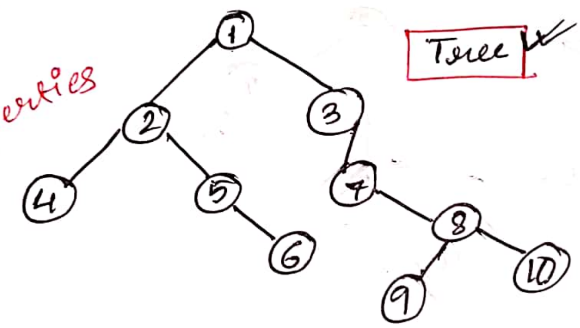
3. n vertices : must be (n-1) edges.

(*) (i) If a graph is having n vertices and (n-1) edges it's called a tree.

(*) (ii) There is exactly 1 path in between ~~a no. of~~ every pair of vertices.

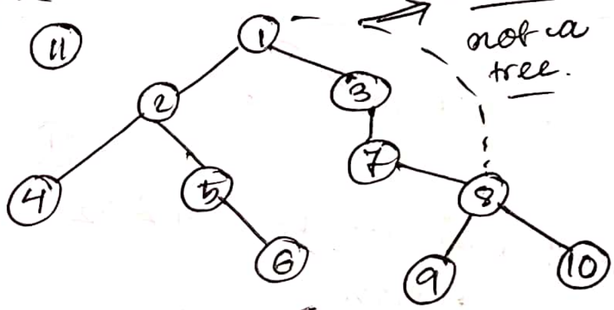
(*) (iii) Tree is minimally connected.

properties



Tree ✓

(ii)



→ circuit

not a tree.

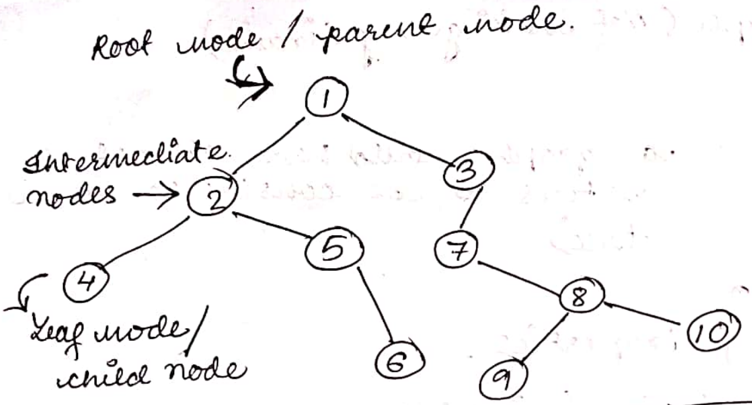
Q Explain w examples why/how a tree is minimally connected.

→ { should not have extra edges } e.g.
{ property (ii)

(iv) A tree should be connected.

(*) A tree is a simple, connected graph w n vertices & (n-1) edges, without any circuit, where there is exactly 1 path betn every pair of vertices.

Scanned by CamScanner



⊗ Decision tree.

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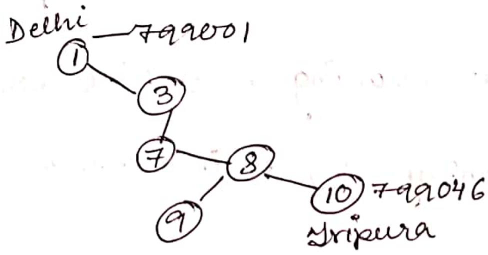
* Online shopping → delivery.

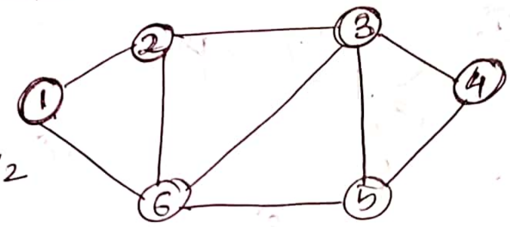


⟺ If there's connection betⁿ 2 places delivery ✓ otherwise delivery ✗

Distance :

distance betⁿ v_1 & v_2

$d(v_1, v_2)$



G

$d(1,3)$: Check how many paths are available.

1-2-3 → 2
1-6-3 → 2
1-6-5-4-3 → 4

for a graph { distance betⁿ a pair of vertices = length of the shortest path betⁿ the 2 vertices.

for a tree { distance betⁿ ~~the~~ a pair of vertices = length of the only path available betⁿ the vertices.

Metric :

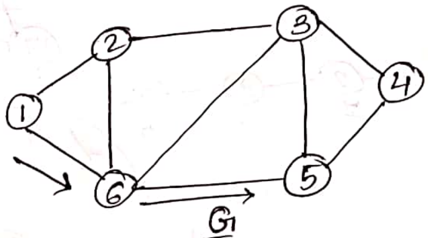
The distance between the vertices of a connected graph is a (metric) → a function.

$\boxed{f(x,y)}$ → ① Non-negativity. ② symmetry. ③ Triangle inequality.

} If a function has all these properties, it's a metric.

$f(x,y)$

① $d(1,3) = 02$ (non-negativity) $\therefore f(x,y) > 0$,



G

②

$f(x,y) = f(y,x)$

$d(1,3) = 2$

$d(3,1) = 2$

$\therefore f(x,y) = f(y,x)$

$\therefore$ Symmetry

③ triangle inequality says, $\cancel{f(x,z) \leq}$ $f(x,y) \leq f(x,z) + f(z,y)$.

$f(x,y) = d(1,3)$

$d(1,3) = 2$

$f(x,z) = d(1,5)$

$f(z,y) = d(5,3)$

$\therefore f(x,z) + f(z,y)$

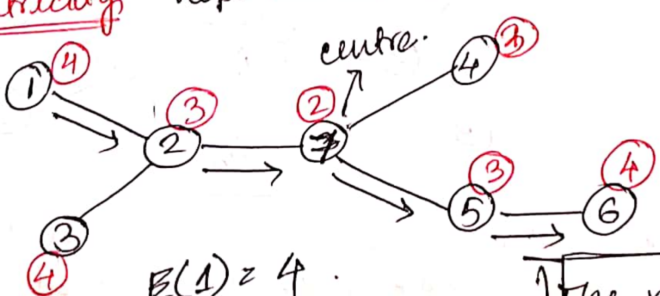
$= 2 + 1 = 3$ $\cancel{= f(x,y)}$

$\therefore f(x,y) \leq f(x,z) + f(z,y)$

Assignment Question : The distance bet n the $\boxed{\text{spanning trees}}$ of a graph is a metric. Prove it.

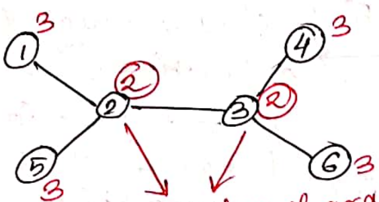
Eccentricity Represented as $E(v)$ Distance ⇒ $d(v_i, v_j)$

$\boxed{\text{Eccentricity}}$ of a vertex is the distance from that vertex to the farthest vertex.



$E(1) = 4$.

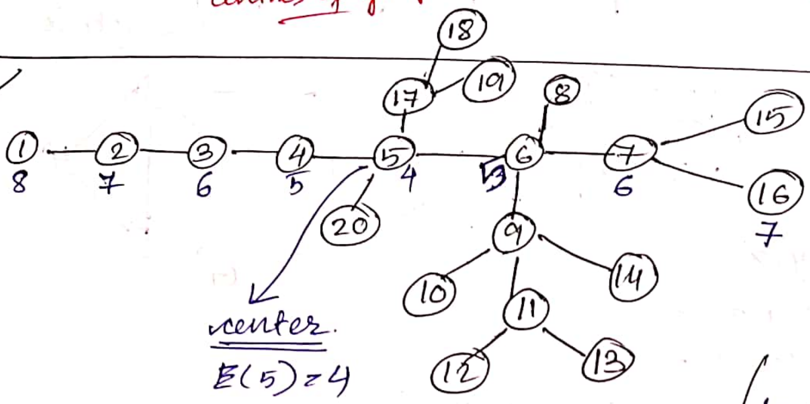
The vertex having lowest eccentricity is termed as the centre of the tree.



centres of graph.

∴ there can be 2 centres of a graph based on eccentricity.

Q. Find out the ecc values of all the vertices & locate the centre.



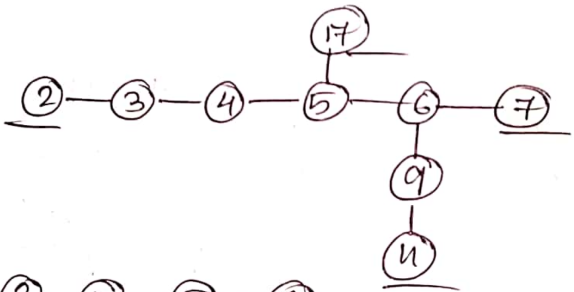
Leaf node → from which no more nodes come out.

pendant vertices

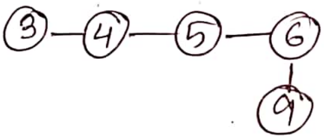
Shortcut ~~node~~ Method :

1. Remove all leaf nodes.

⇓



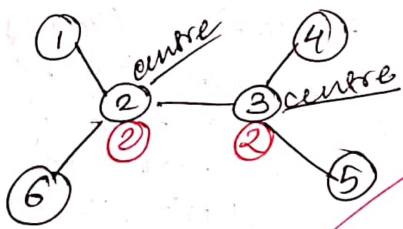
2. ↘

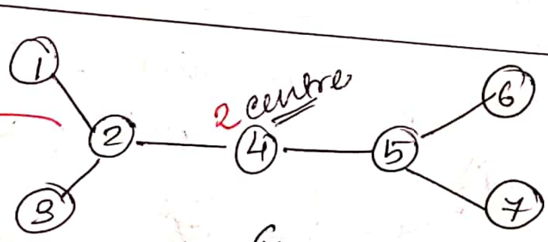


⇒

⇓ 5 ✓ centre

#





G_1 G_2

⇓

In a tree there can be either 1 or 2 centres, not more than that.

$\boxed{r = 2 \quad D = 3}$

$\boxed{r = 2 \quad D = 4}$ → If a tree is monocentric, $r = \frac{1}{2}D$.

But If its bicentric, $r \neq \frac{1}{2}D$.

Imp

Q// Prove that a tree is either monocentric or bicentric.

→ If there are 3 centres, outermost 2 will be deleted 1 remains.
Ully for 5 centres.
for 4 centres → ultimately 2 will be left.

radius Eccentricity value of the centre is called the radius.
diameter In a tree, the diameter is the length of largest path in the tree.

$d = 2r$

⇐ If a tree is monocentric, radius = $\frac{1}{2}$(Diameter)
If a tree is bicentric, radius ≠ $\frac{1}{2}$(Diameter)

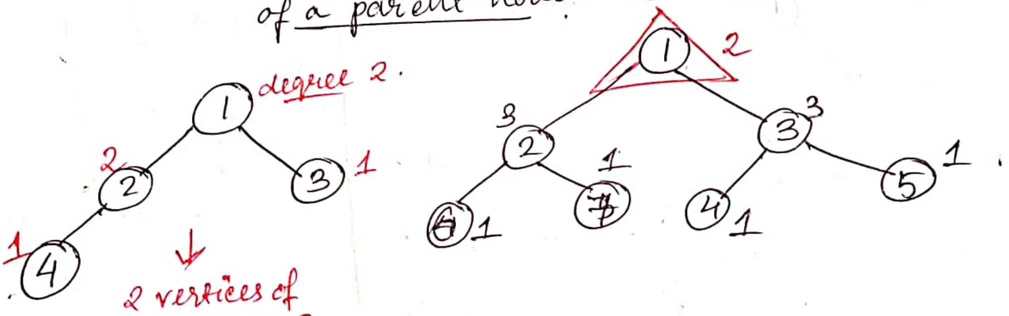
$d \neq 2r$

Q/ check whether $D = 2r$ or not in a monocentric tree.

Rooted binary tree → a special type of ~~tree~~ binary tree is called a rooted binary tree.

It is that tree, where there is exactly 1 vertex of degree 2. Rest of the vertices are of the vertices of degree 1 or 3.

(*) Binary tree ↓ max^m 2 ~~children~~ nodes can come out of a parent node.



∴ not a rooted binary tree.

(I) The number of vertices n in a rooted binary tree is odd.
There is exactly 1 vertex $\partial(2)$ → i.e even degree.
∴ $(n-1)$ vertices are either of ~~odd~~ $\partial(1)$ or $\partial(3)$ i.e, odd degree.

i.e, all are of odd degree.
$(n-1)$ = even.
There are even no. of odd degree vertices.

② ① $\frac{n+1}{2}$ no. of pendant vertices.

$\Rightarrow$ 1 vertex of $\partial(2)$

Let p vertices of $\partial(1)$ are there

$\therefore$ $(n-p-1)$ are of $\partial(3)$ $\to x$

∵ In a tree of n vertices there are $n-1$ edges

$\frac{n+1}{2} = p$

$\Rightarrow -2p+n+1=0$

$\Rightarrow p+2+3n-3p-3 = 2n-2$

Handshaking lemma :

$\Rightarrow p\times 1 + 1\times 2 + (n-p-1)\times 3 = 2(n-1)$

$\Rightarrow p + \cancel{2} + 3n - 3p - 3 = 2n - \cancel{2}$

$\Rightarrow$ ~~$-2p+3n=3$~~ $\Rightarrow$ ~~$2p-3n=-3$~~

$\Rightarrow -2p+3n = 2n+3$ ~~$\Rightarrow n = \frac{3+2p}{3}$~~ ~~$\Rightarrow 2p = 3n-3$~~

$\Rightarrow -2p+n = 3$ ~~$\Rightarrow p =$~~

$\Rightarrow 2p - n = -3$ $\Rightarrow 2p = n-3$

$\boxed{\cancel{p = \frac{n-3}{2}}}$

⑪ $\frac{n-3}{2}$ no. of 3 degree vertices.

$\therefore \frac{n+1}{2} + 2 + 3x = 2n-2$

$\Rightarrow \frac{n+1}{2} - 2n + 4 = -3x \Rightarrow \frac{n+1-4n+8}{2} = -3x$

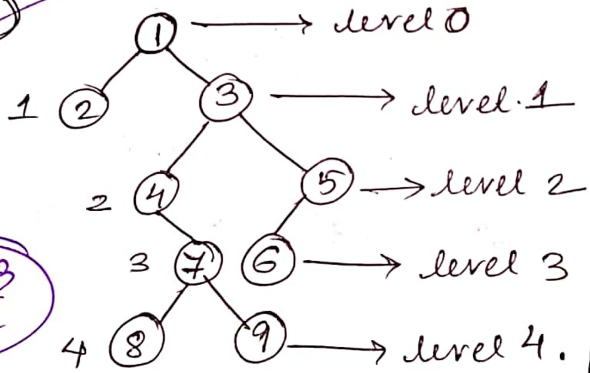
$\frac{-3n+9}{2} = -3x$

$\Rightarrow \frac{-n+3}{2} = -x$

$\Rightarrow x = \frac{n-3}{2}$

---

height (level)



height

↓ minimum height

↓ maximum height

⑩ Q/ what is the min^m & max^m height of the given tree.

① Maximum height is given as $\frac{n-1}{2}$

② Minimum height is given as $[\log_2(n+1) - 1]$

Let $n = 11$, Max h. $= \frac{11-1}{2} = 5$

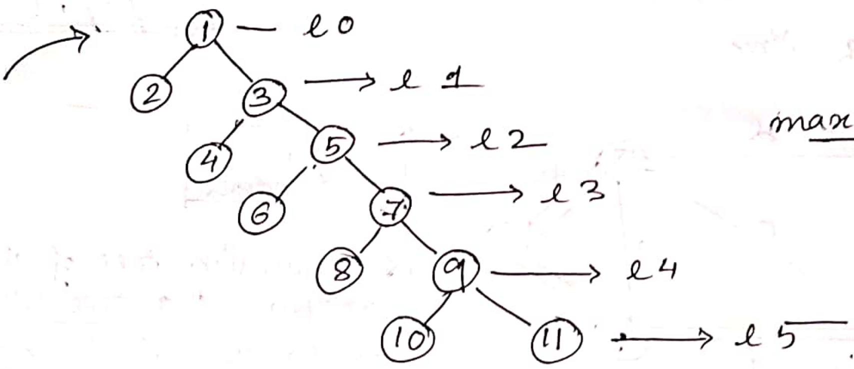
min h. $= \log_2(12) - 1$

$= 3.58 - 1$

$= 2.58$

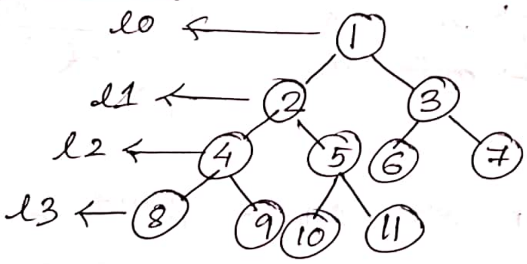
$\simeq 3$

If $2.4 \Downarrow \simeq 2$.

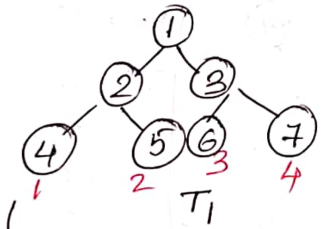


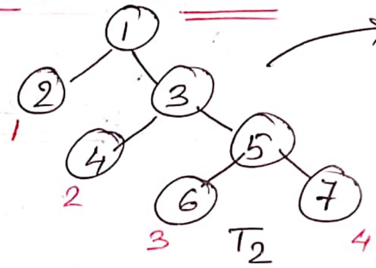
max^m height

min^m height



Weighted path length :- $\sum w_i l_i$





weighted path length
$= (1\times1) + (2\times2) + (3\times3) + (4\times3)$
$= 1 + 4 + 9 + 12$
$= 26$

for vertex (4), weight associated $= 1$
path length (root to leaf node) $= 2$.
for v_5, $w = 2$, path length $= 2$
$\therefore$ Total : $(2\times1) + (2\times2) + (2\times3) + (2\times4) = 20$

Q $\rightarrow$ Weight set will be given, & asked which is the weight set for which tree.

---

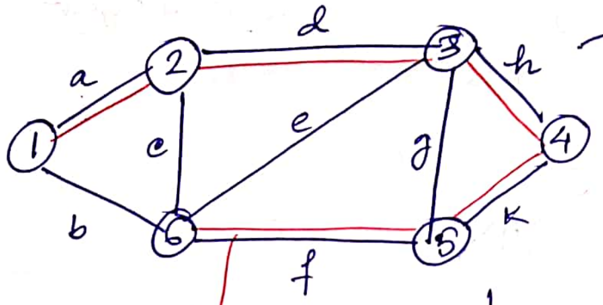
next class :- spanning tree.

~~Imp~~

- latex format
- websites { overleaf, sharelatex }

19/8/19

Spanning Tree



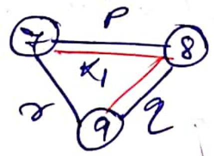
n vertex
$n-1$ edges

A spanning tree of the graph is a tree which contains all the vertices.

↳ does not contain any circuit.
↳ minimally connected (minm number of edges)

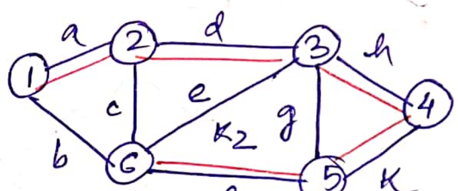
A spanning tree
or: a, c, d, h, k

{ It is also known as the skeleton/scaffending of a graph.
A graph must be connected to find out the spanning tree.



component K_1

$T_1 = \{p, q\}$



component K_2

$T_2 = \{a, d, h, k, f\}$

$G = \{V, E\}$
$V = \{1, 2 \cdots 6\}$
$E = \{a, b, c, d, e, f, g, h, k\}$

{ The collection of spanning tree is known as a spanning forest.

↳ [If there are 2 components in a disconnected graph, A spanning forest is the collection of ~~col~~ spanning trees found from every component of a disconnected graph.

branches : ⇒ no. of branches is always $(n-1)$ // n vertices
The edges of the graph present in the spanning tree are known as the branches.

chords :
The rest of the edges [{b, c, e, g}] not present in the spanning tree are called chords.

no of chords $= e - (n-1)$
$= e - n + 1$

$G = \{V, E\}$
$V = \{1, 2, 3, 4, 5, 6\}$
$E = \{a, b, c, d, e, f, g, h, k\}$
branches $= \{a, d, h, k, f\}$ // $(n-1)$
chords $= \{b, c, e, g\}$ // $e-(n-1) = e+1-n$

Assign-ment

Q: The dist betn spanning trees in a graph is a metric.

Proof:

$T_1 = \{a, d, h, k, f\}$
$T_2 = \{a, d, e, g, h\}$

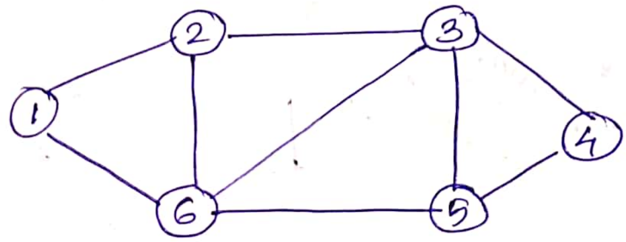
Distance between T_1 and T_2: No of edges present in T_1 but not in T_2 is the distance betn T_1 and T_2.

$d(T_1, T_2)$

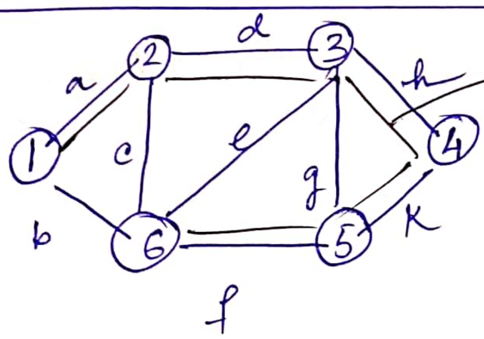
$\therefore \begin{cases} d(T_1, T_2) = 2 \\ d(T_2, T_1) = 2 \end{cases}$

$\therefore$ $\boxed{\text{symmetric}}$, $\boxed{\text{non-negative}}$.

$T_3 = \{$



$d(T_1, T_2) \leq d(T_1, T_3) + d(T_3, T_2)$



$b_r = \{a, d, h, k, f\}$
$c_h = \{b, c, e, g\}$

4 ckts → abc, cde, egf, ghk.
crops production
→ c_h cut → water logged in flood will leak out.

to apply spanning tree concept in real life.

rank and nullity

question {
1. Multicolor cube
2. MST
3. counting tree
4. Metric tree

25 → from tree

$\boxed{\text{Rank} = n - k}$ denoted as r

Rank of a graph = difference betⁿ the no. of vertices and the number of components.

If a graph is connected, $k = 1$

$\therefore$ $\boxed{\text{Rank}} = n - 1$ = no. of branches in a graph
= no. of edges in a tree.

(*) $\boxed{\text{Rank}}$ of a connected graph = no. of branches of its spanning tree.

$\boxed{\text{Nullity}}$ ⟹ denoted as μ.
Nullity of a graph is the difference betⁿ no. of edges of that graph & its rank.

cyclomatic no, 1st Betti number

$\boxed{\mu = e - r}$

$\mu = e - r = e - (n-k)$

$\therefore \mu = e - n + k$

for a connected graph = $\mu = e - n + 1$

Nullity of a graph = no. of chords of its spanning tree.

i.e $\mu_{(\text{connected graph})}$ = no. of chords.

Q/ How to find out the minᵐ spanning tree (MST).

G

rank = no. of branches of any spanning tree of G

nullity of G = no. of chords in G

rank + nullity = no. of edges in G.

Minimum Spanning Tree

1. Kruskal's algorithm.
2. Prim's algorithm.

TSP

either wrt cost / time / distance MST is found out so that the gt. can be minimum.

1. Kruskal's Algorithm:

steps: (1) List all edges of the graph G in increasing order of weight.

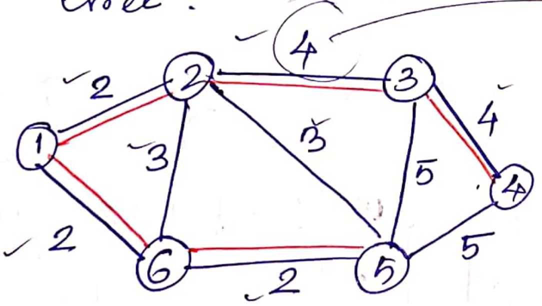
(2) Select the smallest edge of G.

(3) For each successive step, ~~ste~~ select another smallest edge that makes no circuit with the previously selected edges.

(4) Continue until (n-1) edges have been selected to construct the desired minimum spanning tree.

weights associated to the edge.

Q



selected edge — edges		weights
1	2	→ 2
1	6	→ 2
6	5	→ 2
2	5	→ 3
2	6	→ 3
2	3	→ 4
3	4	→ 4
3	5	→ 5
4	5	→ 5

increasing order.

more than 1 smallest edge ↓ select any one.

both will make a ckt so not selected.

MST found. ⇐ (n-1) selected ⇐ 5 edges done

weight of MST = 2+2+2+4+4 = 14.

Prim's Algorithm :

Steps ① Draw n no. of isolated vertices, and label them as $v_1, v_2 \dots v_n$.

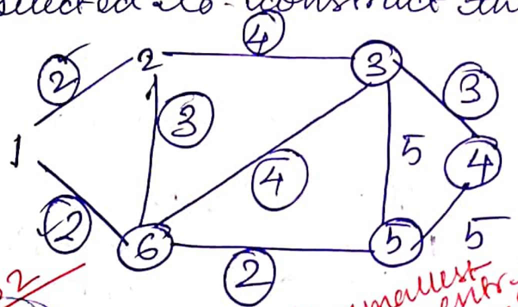
② Tabulate the given wts of the edges of G in an $n \times n$ table.

③ Set the weights of non-existent edges as very large. → ∞

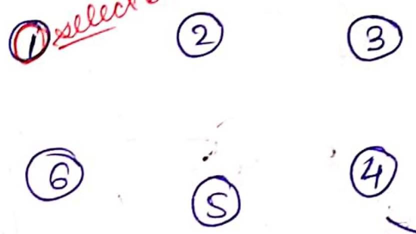
④ Start from vertex v_1 & connect it to its nearest neighbour, say, v_k.

⑤ Consider v_1 and v_k as one subgraph and connect these subgraph to its nearest neighbour, say v_i.

⑥ Now consider v_1, v_k and v_i be the subgraph & continue the process until $n-1$ edges have been selected to construct the desired spanning tree.



step 1



① selected

step 2

smallest entry selected

	1	2	3	4	5	6
1	—	2	∞	∞	∞	2
2	2	—	4	∞	∞	3
3	∞	4	—	3	5	4
4	∞	∞	3	—	5	∞
5	∞	∞	5	5	—	2
6	2	3	4	∞	2	—

not selected.

why self loops aren't considered

1 1 → self loop keep blank

[tree = simple graph, no self loops]

If there's self loop, ignore → blank.

$\boxed{\text{all diagonal values = null}}$

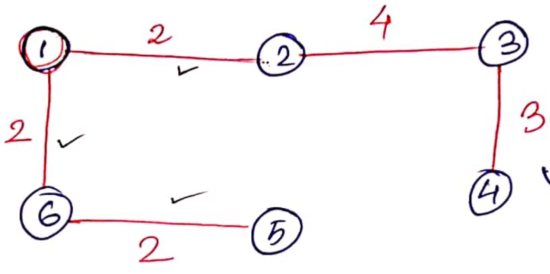
∵ no self loops.

12 taken

∴ 21 same ✓ ↳ next step consider both rows 1 & 2.

step 4

nearest neighbour (say, v_1) → smallest entry in 1st row. either $\frac{12}{2}$ or $\frac{16}{2}$.

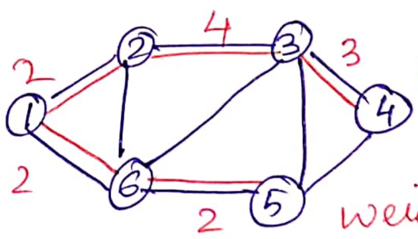


weight = 13

- now subgraph = 12
  consider both rows 1 & 2
  ↳ smallest (2) → row 1 ∴ (16), mark 61 too.
  now rows = 1, 2, 6.
  ↳ 65 selected. → (2)
  1, 2, 6, 5 → If 26 is selected → ckt ?? skip
  dont select.
  (4) - selected. (2 3)
  1, 2, 3, 5, 6 → 34 → (3) — n-1 → done.

Kruskal's :

12 → (2)
16 → 2
65 → 2
26 → 3 ✗
34 → 3
23 → 4
63 → 4
35 → 5
4·5 → 5



(n-1)

weight = 2+2+2+4+3
= 13.

23/8

Chirchoff's ~~matrix~~ Tree Theorem

$$\boxed{\begin{aligned} br &= n-1 \\ ch &= e-n+1 \end{aligned}}$$

fundamental ckt

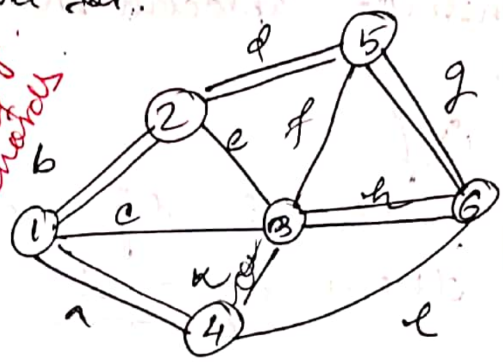
↓

find out the spanning tree 1st.

$br = \{a, b, d, g, h\}$

$ch = \{c, e, f, k, l\}$

μ = nullity = no. of chords



In a graph, there are exactly $(e-n+1)$ i.e, $2d-n+1$ fundamental circuits.

$ch = \{c, e, f, k, l\}$

$\{b, c, h, g, d\}$ → $\{d, e, h, g\}$

$\{f, g, h\}$

$\{k, a, b, d, g, h\}$

→ $\{l, a, b, d, g\}$

$f_1 = \{k, a, b, d, g, h\}$

$(f_1 - a) = \{k, b, d, g, h\} \Rightarrow$ Another spanning tree.

$= T_2$

$T_1 = \{a, b, d, g, h\}$, $br = \{a, b, d, g, h\}$
$ch = \{c, e, f, k, l\}$

* **Elementary tree transformation :-** ~~the~~ from a given graph, find out the spanning tree, find out the f. ckt, then make another S. Tree.

→ The phenomenon using which a S. Tree can be formulated from a given graph & one chord can be added to this S. Tree to form 1 f. ckt.

Again, from the f. ckt, an edge can be deleted to formulate another S. Tree.

Chirchoff's Matrix Tree Theorem

→ used to find out no. of S. Trees in a graph.

Steps:

① construct Laplacian matrix Q for the given graph G.

② Form $i \neq j$

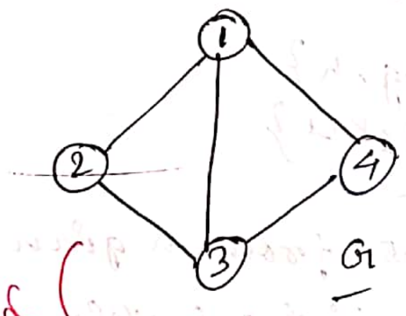
— if vertex i and j are adjacent in G.
then, $q_{ij} = -1$

otherwise, $q_{ij} = 0$

③ for $i = j$, q_{ij} = degree of vertex i in G.

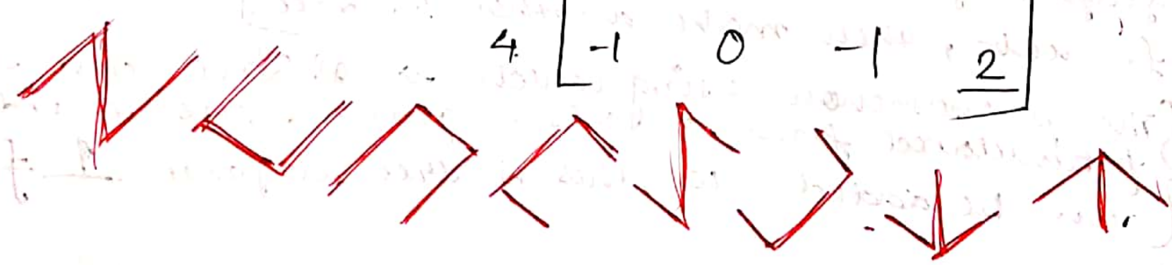
④ from matrix Q, construct matrix Q' by deleting any one row & any one column from Q.

⑤ Finally calculate the determinant of Q' to get the total no. of S. Tree in G.



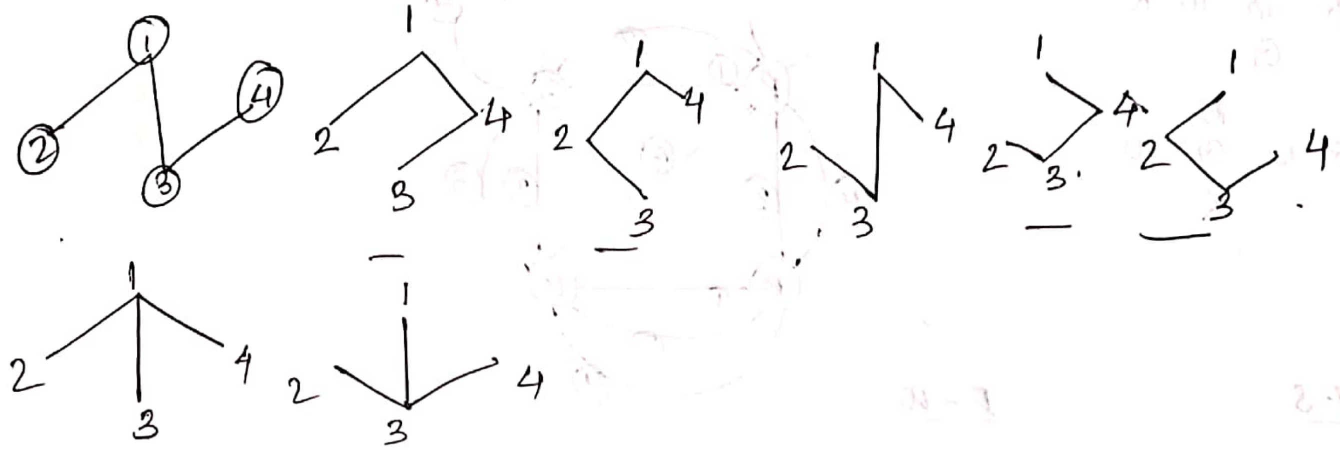
$$Q = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix} \end{array}$$

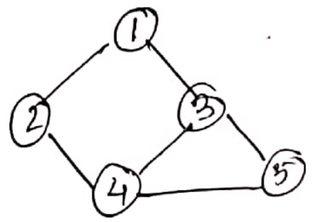
spanning trees



$$Q' = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\det(Q') = 2(6-1) + 1(-2) = 10 - 2 = 8$$





G

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ -1 & 0 & 3 & -1 & -1 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{bmatrix} \end{array}$$

$$Q' = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} = 2\begin{bmatrix} 2 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & 2 \end{bmatrix} + 1\begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$= 2[2(-2-1)] + 1[-1(-2-1) + 1(-2)]$$

$$= 2(-6) + 1[3-2]$$

$$= -12 + 1 = -11$$

26/8

~~II~~ Counting Tree

1857 . Arthur . Cayley

$\boxed{C_k H_{2k+2}}$ hydrocarbons.

No of vertices $n = \cancel{C_k H_{2k+2}} = k + 2k + 2$
$= 3k + 2$

As per handshaking Lemma,

$$2e = \sum d(v_i)$$

$$e = \frac{1}{2} \sum d(v_i)$$

$$= \frac{1}{2}\left[4k + 1(2k+2)\right]$$

$$= \frac{1}{2}\left[6k+2\right]$$

$$= 3k+1.$$

$\Rightarrow$ $v = 3k+2$

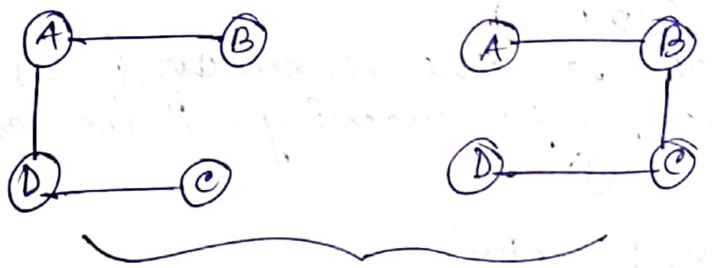
$e = 3k+1$

i.e, n vertices & $(n-1)$ no. of edges

$\therefore$ graph = tree.

Q/ Describe how C. Tree originated?

If n vertices are given, how many $\boxed{\text{labelled trees}}$ can be drawn to these n vertices.



unlabelled tree

without label both will look like

$\left\{ n^{(n-2)} \text{ labelled trees can be drawn.} \right\}$

Let $n = 4$

$4^{4-2} = 16$ trees.

P-53

Draw 16 trees.

→ As n (↑) no. of trees (↑) → ~~not feasible~~ (drawing all trees)

(*) **Prufer** : {Theorem by Cayley; Proof by prufer}

encoding | decoding

: Encoding sequence : (PES)

Steps:

1. Let the n no. of vertices of a tree T be labelled as 1, 2, 3, ... n. (continuous sequence)
2. ~~Let the n vertices of a tre~~ Remove the pendant vertex, say a_1
3. Assume b_1 is the vertex adjacent to a_1
4. Among the remaining (n-1) vertices, let a_2 be the pendant vertex with the smallest label and b_2 be the vertex adjacent to a_2
5. Remove the edge $a_2 b_2$.
6. Repeat the operatⁿ on the remaining (n-2) vertices to get the Prufer encoding sequence as

$\boxed{b_1, b_2 \dots b_n}$ —— (1)

: Prufer decoding sequence : (PDS)

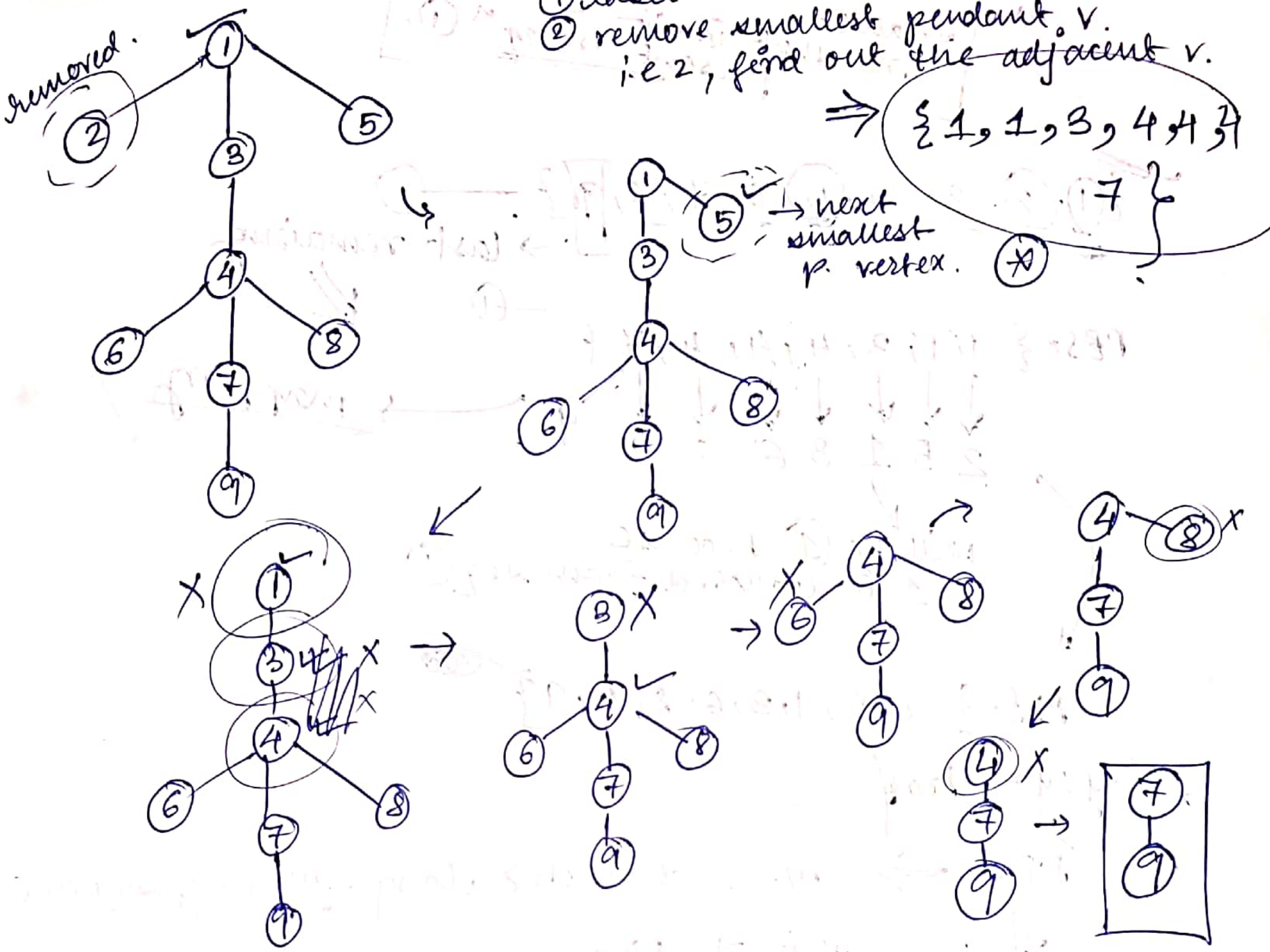
Steps :

1. Determine the 1st no. in the sequence 1, 2, 3, ... n. ← (2)
~~2~~ that does not appear in eqⁿ (1).
2. The number clearly, is a_1 & thus the edge $a_1 b_1$ is defined.
3. Remove b_1 from eqⁿ (1) and a_1 from eqⁿ (ii).
4. In the remaining sequence of eqⁿ (ii) find the 1st no. that doesn't appear in the reminder of eqⁿ (1).

⑤ The construction is continued till the sequence of eqn ① has no element left and finally the last two vertices remaining in eqn ② are joined.

① label.
② remove smallest pendant v. i.e 2, find out ~~the adjacent~~ v.

$\Rightarrow \{1, 1, 3, 4, 4, 4, 7\}$



$\{1, 1, 3, 4, 4, 4, 7\} \longrightarrow$ 7 character long.

i.e, $n = 9$.

∴ $n-2$ character long sequence.

PDS.

Decoding sequence : PES = {1, 1, 3, 4, 4, 4, 7} —— (i)

↓ n−2 characters.

{1, 2, 3, 4, 5, 6, 7, 8, 9} → n character sequence —— (ii)

[smallest no. that isn't present in eq^n (i).]

{1, 2, 3, 4, 5, 6, 7, 8, $\boxed{9}$} —— (ii)

→ last remaining.

PES = {~~1~~, ~~1~~, 3, 4, ~~4~~, ~~4~~, 7} —— (i)

↓ ↓ ↓ ↓ ↓ ↓ ↓

2 5 1 3 6 8 4 9 ⟶ mapping

will be (1) because 1 is deducted from seq (i).

PDS = {2, 5, 1, 3, 6, 8, 4, 9}

Cayley ⇒ Proof.

PES ⟹ $n-2$ character long encoding seque

⇓ from this ⇒ PDS.

⇓

tree If the tree ≡ to the given one, there is a 1 to 1 mapping from Tree to sequence.

⇓

As there are n vertices in tree,

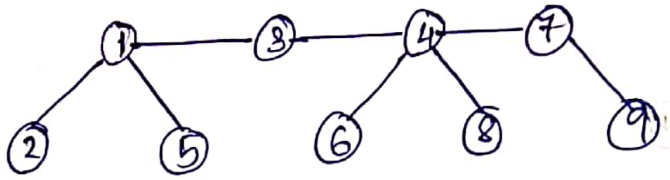
∴ n^{n-2} trees found.

[Indirect proof]

⇒ PDS = {2, 5, 1, 3, 6, 8, 4, 9} → rem. vertices

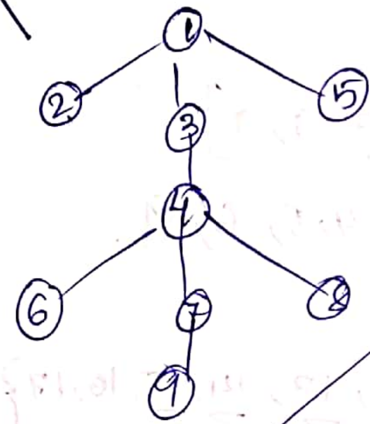
PES = {1, 1, 3, 4, 4, 4, 7}

↳ vertices = 1, 3, 4, 7 . 2, 5, 6, 8, 9.

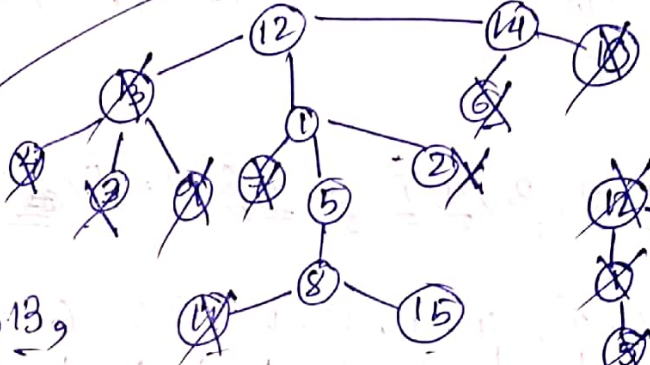


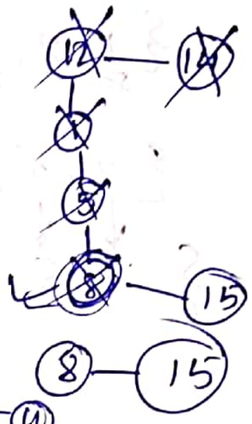
check mapping.

similar: check if both are similar, if yes ⇒ proved.



⇒ proved.



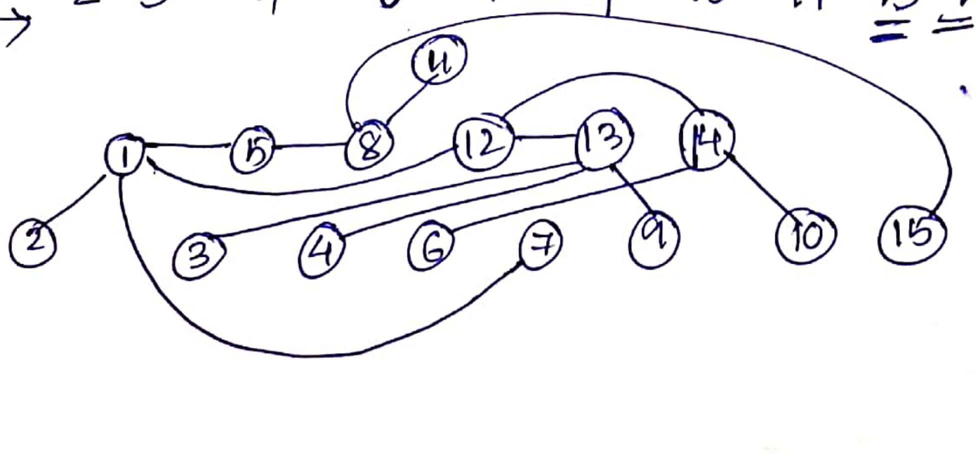


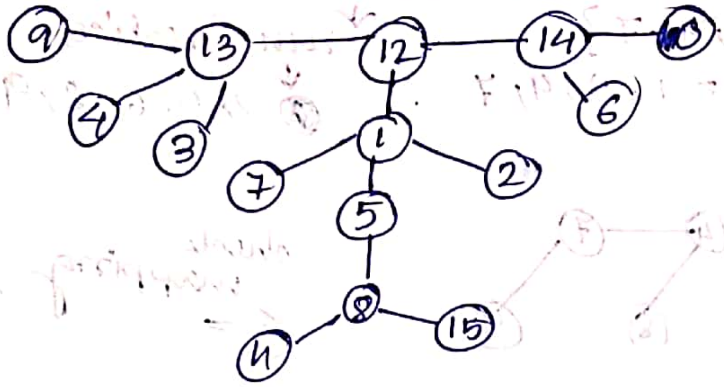
PES = {1, 13, 13, 14, 1, 13,
14, 8, 12, 12, 1, 5, 8, 13}

PDS {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} — 11

PES {1, 13, 13, 14, 1, 13, 14, 8, 12, 12, 1, 5, 8} — 1

PDS → 2 3 4 6 7 9 10 11 13 14 12 1 5 15





---

② from the given PES find out the PDS & gen. the tree.

$\{3, 3, 4, 1, 5, 5, 1, 6, 6, 6, 2, 1, 3, 9, 9\}$ → 1, 2, 3, 4, 5, 6, 9.

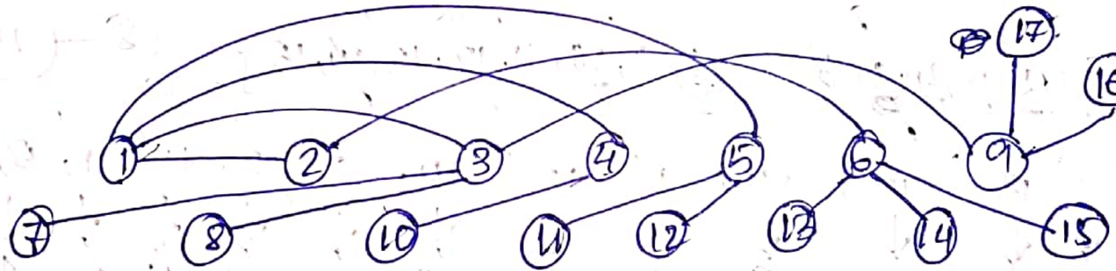
$n-2 = 15$

$n = 17$

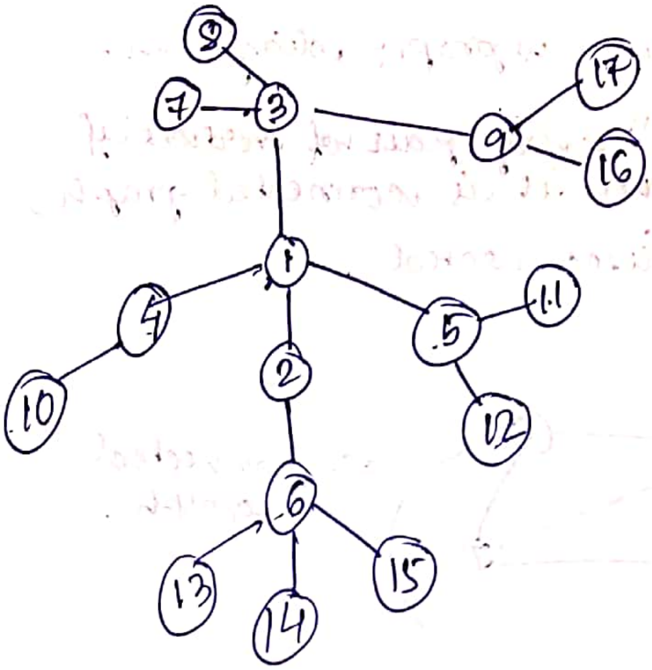
$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17\}$

$\{3, 3, 4, 1, 5, 5, 1, 6, 6, 6, 2, 1, 3, 9, 9\}$

PDS = $\{7, 8, 10, 4, 11, 12, 5, 13, 14, 15, 6, 2, 1, 3, 16, 17\}$



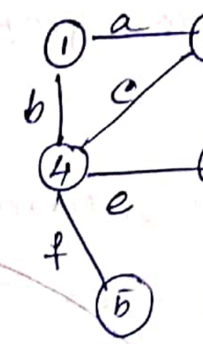
Palanisenan



Colouring

Planar Graph

$$\psi = \begin{cases} a \to (1,2) \\ b \to (1,4) \\ \vdots \\ f \to (4,5) \end{cases}$$



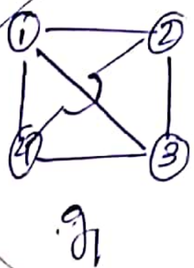
G

$G = (V, E, \psi)$

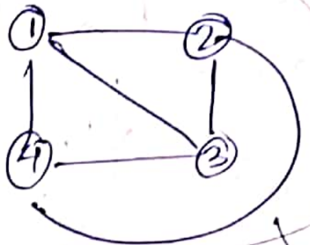
$V = \{1, 2, \dots 5\}$

$E = \{a, b \dots f\}$

If a geometrical representation of a graph is possible over a plane without any cross over → planar graph.



g_1



g_2

The technique of representing a graph on a single plane w/out cross over → embedding.

4 vertex ~~plan~~ complete graph is a planar graph

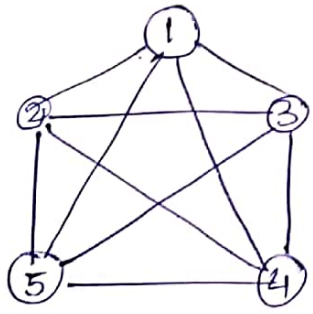
// • housing complexes → bipartite graph.

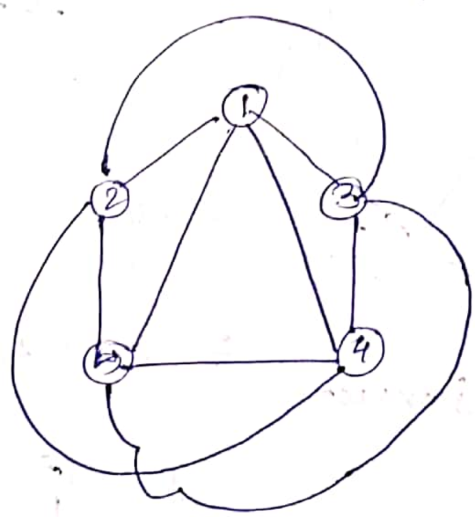
= Kuratowski

1. K_5
2. K_{33}

→ baseline graphs / reference graphs.

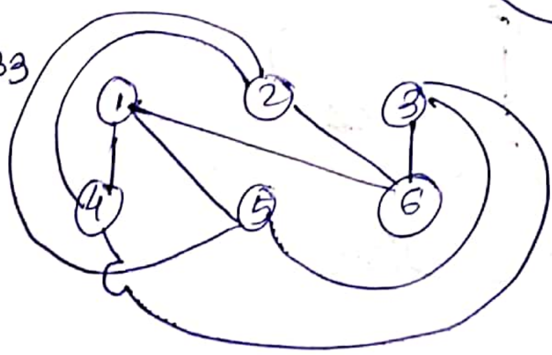
K_5





$v = 5$
$e = 10$

K_{33}



$v = 6$
$e = 9$

→ 1 cross over.

→ housing.

common properties of K_5, K_{33}

- both of them are → non-planar graphs.
  - ↳ regular graph [In K_5 → ∂ of all v =
    K_{33} →

- from K_5 → if an
  - edge is deleted → its planar.
  - v → deleted ⇒ planar.

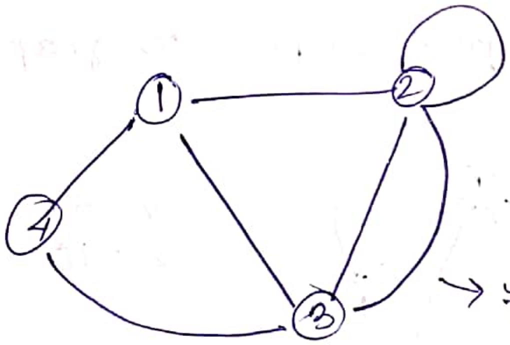
K_{33} → delete → v → planar
↳ e

K_5 is the simplest non planar graph in the smallest no of vertices.

bipartite
V_2 {1 . . . 6}
V_1 {1, 2, 3}
V_2 = {4, 5, 6}

K_{33} → sim n.p. g — smallest no of edges.

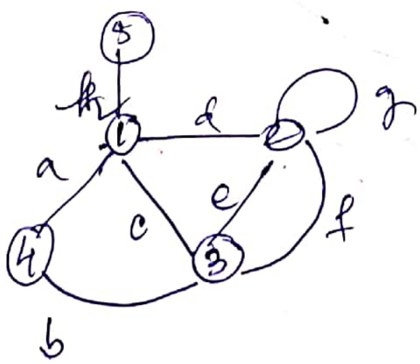
Region
→ Internal
→ External / infinite.

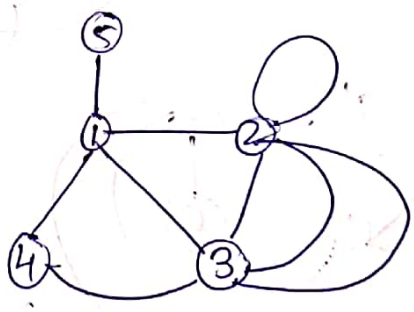


$f = e - n + 2$ → no of regions.

→ 7 − 4 + 2
3 + 2 = 5

$f_{disconnected} = e - n + (k+1)$





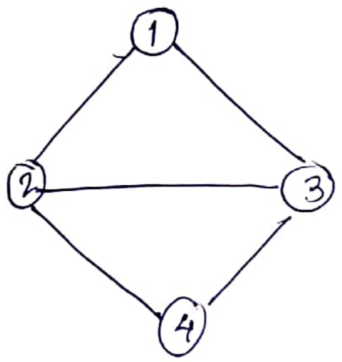
Colouring

(A) In a simple connected planar graph with f regions, e edges & n vertices, the ~~2 ineq~~ 2 inequ- must hold :

① $e \geq \frac{3}{2} f$

② $e \leq 3n - 6$.

$f = e - n + 2$



$2e \geq 3f$

{ 3 edges atleast req for a region, 1 e is common betn 2 regions }

$e \geq \frac{3}{2}(e - n + 2)$

$2e \geq 3e - 3n + 6$

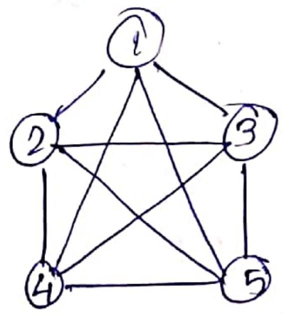
$-e \geq -3n + 6$

$$\boxed{e \leq 3n - 6}$$

If this inequality is true → graph is planar

→ false → non planar.

K_5



$n = 5$

$e = 10$

$10 \leq 15 - 6$

$10 \leq 9$

$K_4 \to$ 4 vertex.

$K_{33} \to v = 6$, $e = 9$ → $9 \leq 18 - 6$, $9 \leq 12$ → true → must be planar ✗

↓

also ⇒ no regions

(*) $2e \geq 4f$

$e \geq 2f$

$e \geq 2(e-n+1)$

$e \geq 2e - 2n + 2$

$-e \geq -2n + 2$

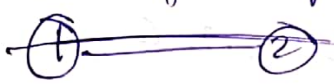
$\boxed{e \leq 2n - 2}$ → $9 \leq 12 - 2$

$9 \leq 10$

Detection of planarity

① $G = \{g_1, g_2 \cdots g_x\}$

② self loops do not effect planarity.



② remove all self loops

③ " " parallel edges

not all ↳ (all except 1)

④ " " series edges. → merge those edges i.e delete the v having degree 2.

series edges → 2 edges sharing a vertex & d of the vertex is 2.

outcome → 1 st line.

⑤ I/p graph → G; after applying 4 steps resultant graph G_4 → may have 3 types ⇒

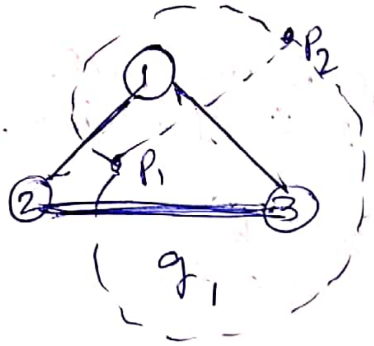
Colouring

Detection of planarity. $G_0 \longrightarrow$ ① a single edge $\to$ planar

② A vertex complete planar graph (K_4)

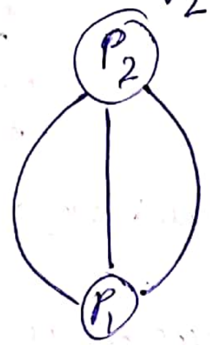
③ $v \geq 5$, $e \geq 7$ $\longrightarrow$ apply eulers inequations.

Geometric Dual



$n = 3$
$e = 3$
$f = 2$

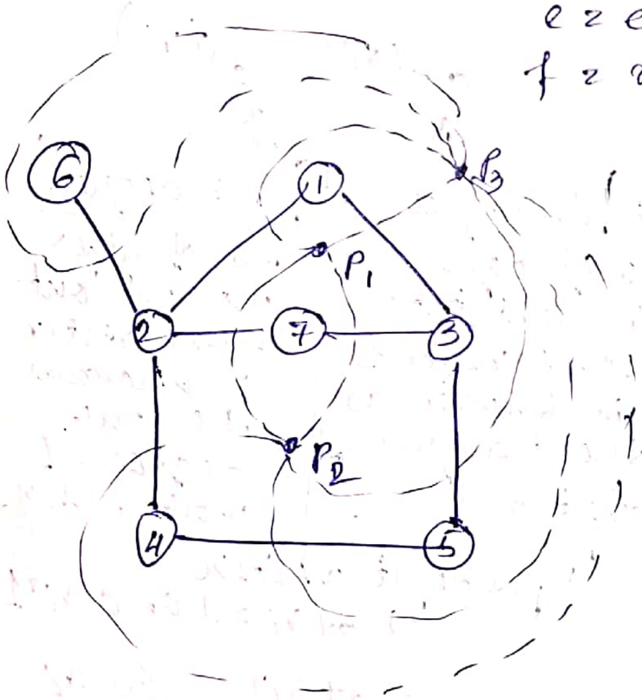
g_2



$n^* = 2$
$e^* = 3$
$f^* = 3$

relation $\leftarrow g_1 \,\&\, g_2$

$n = f^*$
$e = e^*$
$f = n^*$



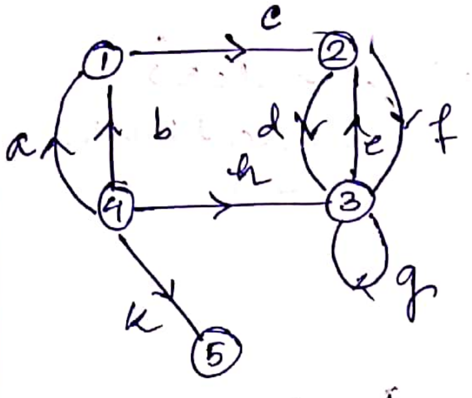
- Thickness
- Crossing numbers.

$\boxed{\text{Directed graph}}$ Digraph

$G(V, E, \psi)$

undirected graph

$\psi = \{ h \to (4,3) \text{ or } (3,4) \}$



G

→ A set of mapping where every edge is mapped to an ordered pair of vertex.

$h \begin{cases} 4 \to \text{initial vertex} \\ \boxed{\text{incident-} \cancel{\text{vertex}} \text{ out}} \\ 3 \to \text{terminal vertex} \end{cases}$

$\psi = \{ h \to (4,3)$
$g \to (3,3)$
$d \to (2,3)$
$e \to (3,2) \ldots \}$

without direction → parallel edges ⇒ a, b ; d, e, f

parallel edges w dirⁿ :

[2 or more edges are ∥, if they are having same pair of initial & terminal vertices

→ a, b → $\boxed{\begin{array}{l} v_i = 4 \\ v_t = 1 \end{array}}$

Degree of some vertex : $\boxed{\text{In degree}}$ | $\boxed{\text{out degree}}$

$\partial(v_i)$

$\partial^-(v_i)$ — In degree: no. of edges incident into some vertex is known as the indegree of that vertex.

$\partial^+(v_i)$ — out degree: no of edges incident out of some vertex is known as the out-degree of that vertex.

→ pendent vertex
→ $\partial^+ v_i = 1$ or $\partial^- v_i = 1$

indegree	out degree
$\partial^- v_1 = 2$	$\partial^+ v_1 = 1$
$\partial^- v_2 = 2$	$\partial^+ v_2 = 2$
$\partial^- v_3 = 4$	$\partial^+ v_3 = 2$
$\partial^- v_4 = 0$	$\partial^+ v_4 = 4$
$\partial^- v_5 = 1$	$\partial^+ v_5 = 0$
$\Sigma \partial^\pm v_i =$ 9	9

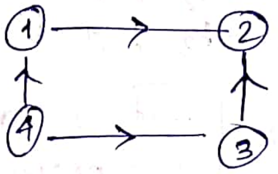
$\Sigma \partial^+(v_i) = \Sigma \partial^-(v_i)$

→ handshaking lemma

* { ← ∂ lemma

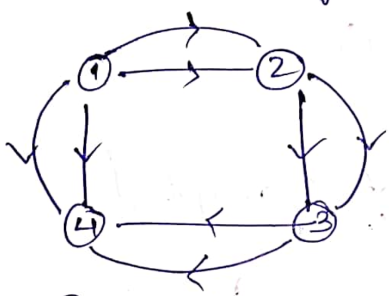
Colouring

Asymmetric ∂ graph is a special ∂ graph where in between every pair of vertices. at max 1 directed edge would be there



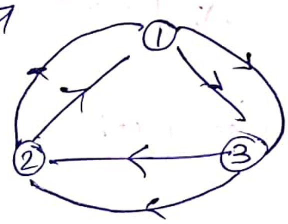
- parallel edges ✗ not allowed
- self loops ✓

Symmetric ∂ graph is a special ∂ graph. where in bet^n every pair of vertices, 1 indegree & another out degree must be there



complete ∂ graph

Q.→

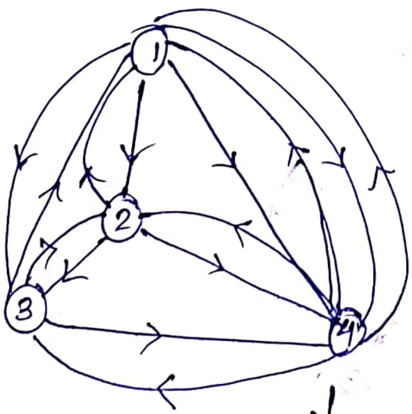


$\partial+1 = 2$	$\partial-1 = 2$
$\partial+2 = 2$	$\partial-2 = 2$
$\partial+3 = 2$	$\partial-3 = 2$

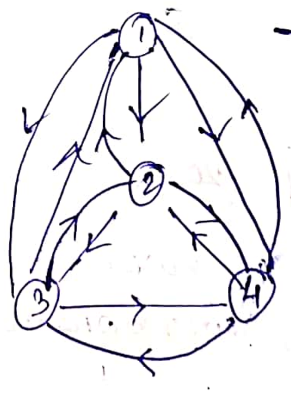
Indegree & out degree of every vertex $= n-1$

- Balanced ∂ graph

Indegree of every vertex = outdegree of every vertex



Complete ✗
Sym ✗
Balanced ✓

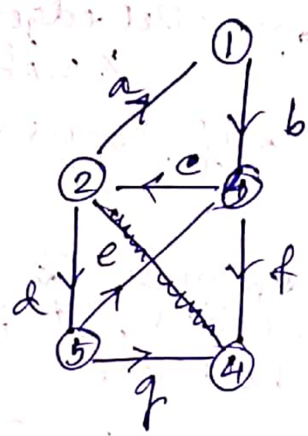


→ 4 vertex Sym. Comp. ∂ graph.

Walk path Circuit

P1 → semi-path.

P1 : 1-b-3-e-5 → un dir. graph



P_2 : 1-a-2-d-5

P_3 : 1-b-3-c-2-d-5.

dir. path.. vert. & edge rep isn't there in a proper dir^n.

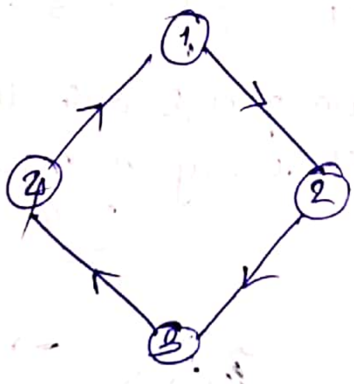
semi-path. If a path is there but dir^n isn't ~~provided~~ proper.

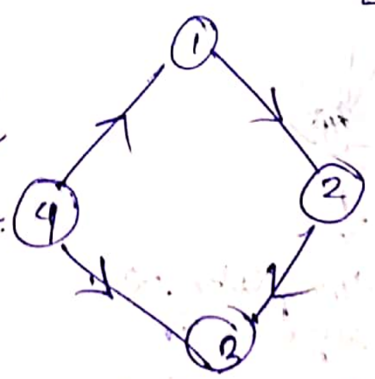
(X) p-202 (ch-9)

- Disconnected, connected graph.

strongly connected. → If in bet^n every pair of vertices there exists 1 directed path.

otherwise → weakly connected





- p-20.3

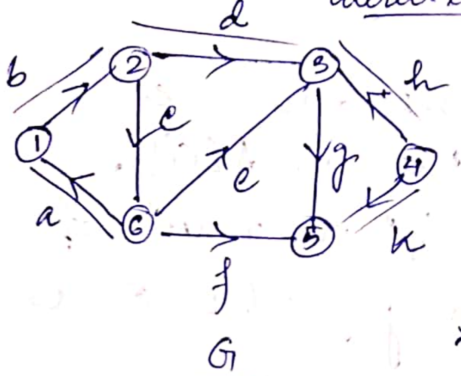
- Euler D-graph
- Arborescence.

. Colouring

P-203

E circuit

directed — semi



G

$b_r = \{a, b, d, h, k\}$

$c_h \{c, e, g, f\}$

spanning tree

$\{a, b, d, h, k\}$

Matrix representatⁿ of graphs

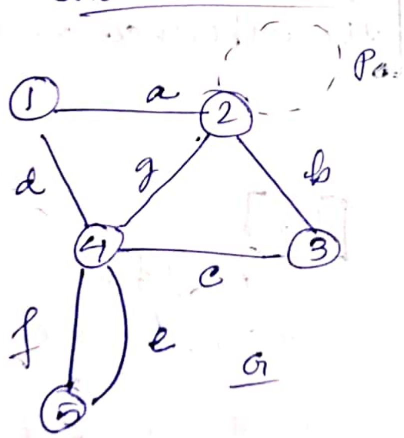
• Incidence matrix :

$[A(G)]$

$\hookrightarrow [a_{ij}]$

$[a_{ij}] = 1$, jth edge is inc. on ith vertex

$= 0$, otherwise.



G

$$\begin{array}{c|c} & \begin{array}{cccccccc} a & b & c & d & e & f & g & p \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

every column must have 2 1s.

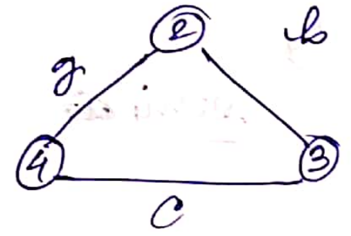
only one 1, self loop isn't included

• incident matr does not allow self loops.

• A row w all zeroes

↳ isolated vertex.

parallel edges generate identical columns in case of inc. mat g



$$\begin{array}{c|ccc} & g & b & a \\ \hline 2 & 1 & 1 & 0 \\ 3 & 0 & 1 & 1 \\ 4 & 1 & 0 & 1 \end{array}$$

if same values are there in A[G], ∴ A[g] is the matrix of graph g, which is a sub graph of G.

- Reduced incident matrix. A(f)

$A(G) = (n-1) \times e$

let ③ is removed

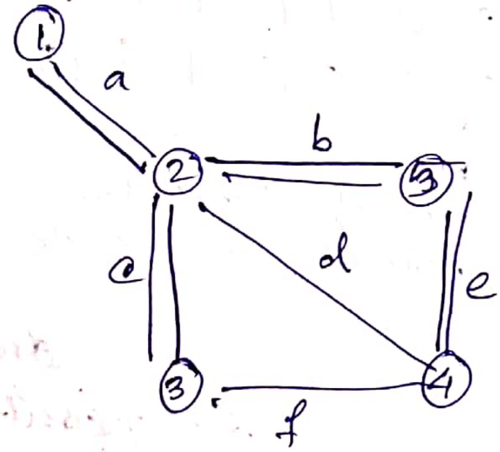
$$\begin{array}{c|c} & a\ b\ c\ d\ e\ f\ g \\ \hline 1 & \\ 2 & \\ 4 & \\ 5 & \end{array}$$

⇒ In case of red inc mat the vertex whose row is deleted is the reference vertex.

$q_1 = \{b, c, d\}$
$q_2 = \{d, e, f\}$
$q_3 = \{b, c, f, a\}$

$B(G) = [b_{ij}]$
$q \times e$

$= 1$, if ..
$= 0$, otherwise



$$\begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 1 & 1 \\ 3 & 0 & 1 & 1 & 0 & 1 & 1 \end{array}$$

- fundamental ckt matrix.
  - S. Tree

$br = \{a, b, c, e\}$
$C_h = \{d, f\} \to \{a, b, c, f\} = c_2$
$\to \{b, c, d\} = c_1$

Colouring

$$B_f(G) = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{bmatrix} d & f & a & b & c & e \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$\to I_\mu$

Identity matrix of order μ

$\therefore$ 1st part is I_μ

$\therefore \mu = e - n + 1$

= no. of chords.

$[I_\mu \; B_t]$

$(e-n+1) \times (e-n+1)$

$(n-1) \times (e-n+1)$

9/ $A B^T = B \cdot A^T = 0 \pmod 2$ p-143

A → inc. matrix

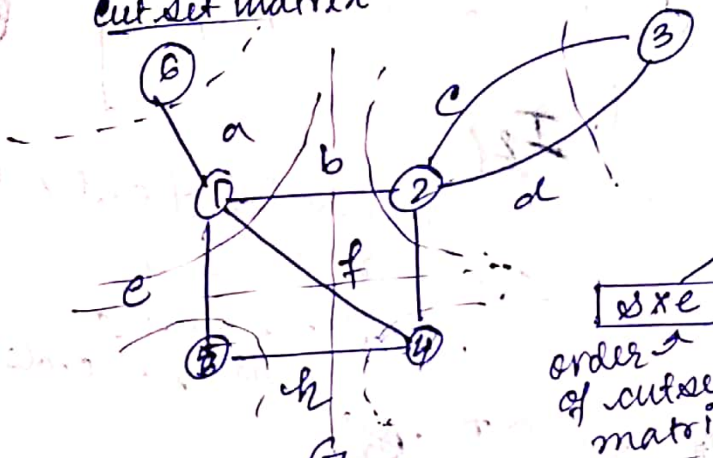
B → ckt matrix

wed
$\boxed{11}$ (Soft Engg Lab)

check

- $A B^T = 0 \pmod 2$
- $A_f B_f^T = 0 \pmod 2$
- $C_f \cdot B_f^T = 0 \pmod 2$

Cut set matrix



G

$c(G) = [c_{ij}]$

$\boxed{s \times e}$ order of cutset matrix

$= 1$, jth edge is included in its cutset
$= 0$, otherwise.

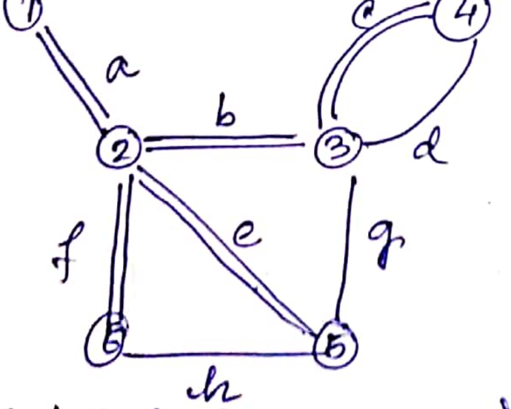
$S_1 = \{a\}$
$S_2 = \{b, f, e\}$
$S_3 = \{c, d\}$
$S_4 = \{b, g\}$
$S_5 = \{e, h\}$
$S_6 = \{h, f, g\}$
$S_7 = \{e, f, g\}$
$S_8 = \{b, f, h\}$

$$c(G) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ & & & & & & & \\ & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{array}\right] \end{array}$$

Find out
(*) fundamental cutset matrix

$b_r = \{a, b, c, e, f\}$

~~$f(G) =$~~



$b_r = \{a, b, c, e, f\}$
$c_h = \{d, g, h\}$

5 fundamental cutsets.

wrt b_r $a \longrightarrow \{a\}$
$b \longrightarrow \{b, g\}$
$c \longrightarrow \{c, d\}$
$e \longrightarrow \{e, g, h\}$
$f \longrightarrow \{f, h\}$

Colouring

$C_f(G)$:

fundamental cutset matrix

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{cccccccc} d & g & h & a & b & c & e & f \end{array} \\ \left[\begin{array}{ccc|ccccc} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$$C_f = [\,\underbrace{C_c}_{(n-1)\times(e-n+1)} \mid \underbrace{I_{n-1}}_{(n-1)\times(n-1)}\,]$$

Here, $n = 6$

~~Relatn betn~~

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccccc} d & g & h & a & b & c & e & f \end{array} \\ \left[\begin{array}{ccc|ccccc} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

→ incidence matrix of G.

↳ on deleting row 6, dimn → $(n-1)\times e$

$$A_f = [\,\underbrace{A_c}_{(n-1)\times(e-n+1)} \mid A_t\,] \rightarrow (n-1)\times(n-1)$$

$$B_f = [\,\underbrace{I_\mu}_{(e-n+1)\times(e-n+1)} \mid B_t\,] \rightarrow (n-1)\times(e-n+1).$$

$A_f \cdot B_f^T = 0$

$$[A_c \mid A_t] \cdot \left[\frac{I_M}{B_t^T}\right] = 0$$

$A_f = [A_c \mid A_t]$

$B_f = [I_M \mid B_t]$

$$A_c + A_t \cdot B_t^T = 0$$

$$A_c \cdot A_t^{-1} + B_t^T = 0$$

$$A_c \cdot A_t^{-1} = -B_t^T$$

$$\boxed{A_c \cdot A_t^{-1} = B_t^T}$$

$$C_f \cdot B_f^T = 0$$

$$[C_c \mid I_{n-1}] \left[\frac{I_M}{B_t^T}\right] = 0$$

$$C_c + B_t^T = 0$$

$$C_c = -B_t^T$$

$$\boxed{C_c = B_t^T}$$

$$A_c A_t^{-1} = B_t^T = C_c$$

Path matrix

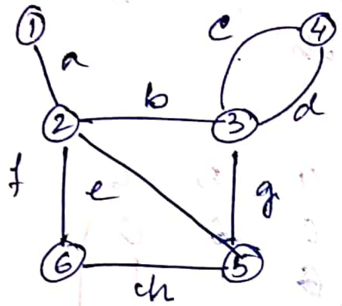
$P(x,y)$ — x = ①, y = ⑤

$P(1,5)$:

$P_1 = \{a, b, g\}$

$P_2 = \{a, e\}$

$P_3 = \{a, f, h\}$



$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \left[\begin{array}{cccccccc} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{array}\right] \end{array}$$

$P(x,y) = P[p_{ij}]$

$= 1$, j^{th} edge is included in i^{th} path.

$= 0$

Check whether:

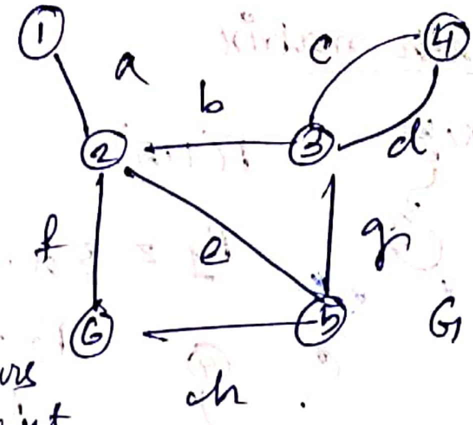
$$\boxed{A \cdot P^{T}_{(x,y)} = (M)^3}$$

M will be having all 1s in row x and y. rest of all (n−2) rows will be having 0.

for this graph of for $P_{(x,y)}$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \left[\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Adjacency matrix



$X(G) = [x_{ij}]$
$n \times n$ $= 1$ if ith vertex and jth vertex are neighbours or adjacent
$= 0$

name of edge → irrelevant.

here, $X =$ (for G)

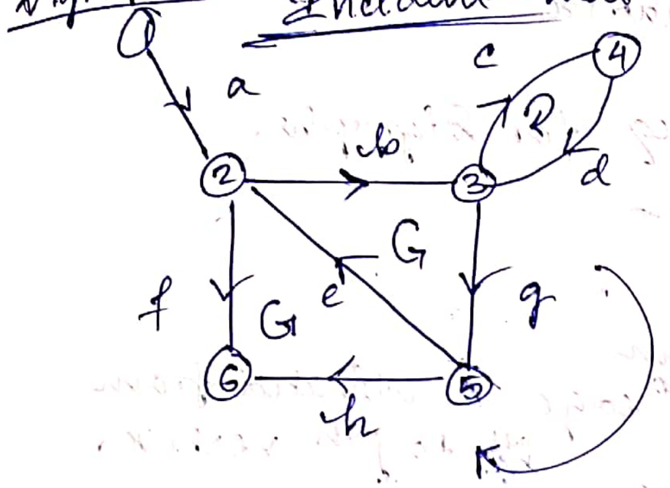
$$\begin{array}{c|c} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

- for parallel edges → $\cancel{2}$ 1
  i.e ~~for~~ parallel edges can't be shown by adjacency matrix but self loops can be shown.

- If any diagonal element is 1 → self loop.

- which matrix can show self loops but not parallel edges? → Adjacency matrix.
- which matrix can show parallel edges but not self loops?
  → Incidence matrix

Digraphs Incident mat: Colouring ... colouring a graph so that.



$A(G) = [a_{ij}]$

$= +1$, j^{th} edge is incident out from i^{th} vertex

$= -1$, j^{th} edge incident into i^{th} vertex.

$= 0$,

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & +1 & -1 & 0 & 0 & +1 & 0 \\ 0 & 0 & -1 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & 0 & -1 & +1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 \end{bmatrix} \end{array}$$

- cant accomodate self loop.

Ckt matrix

$B(G) = [a_{ij}]$

$q \times e$

4×8 matrix.

$= +1$, j^{th} edge is included in i^{th} ckt & dir^n of edge & ckt is same

$= -1$, j^{th} edge is included in i^{th} ckt but dir^n of edge & ckt aren't same.

$= 0$.

R $q_1 = \{c, d\}$
L $q_2 = \{b, e, g\}$
L $q_3 = \{e, f, h\}$
R $q_4 = \{b, g, h, f\}$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} 0 & 0 & +1 & +1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & +1 & +1 & 0 & -1 \\ 0 & +1 & 0 & 0 & 0 & -1 & +1 & +1 \end{bmatrix} \end{array}$$

* 2° fundamental ckt matrix

* Cutset matrix not req for Digraphs.

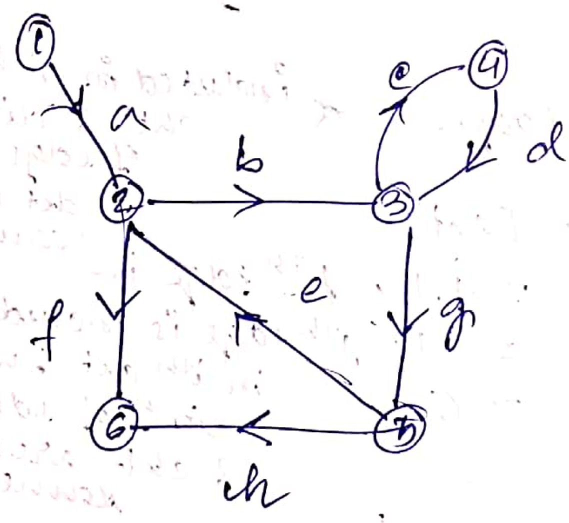
Adjacency matrix

$X = [x_{ij}]$

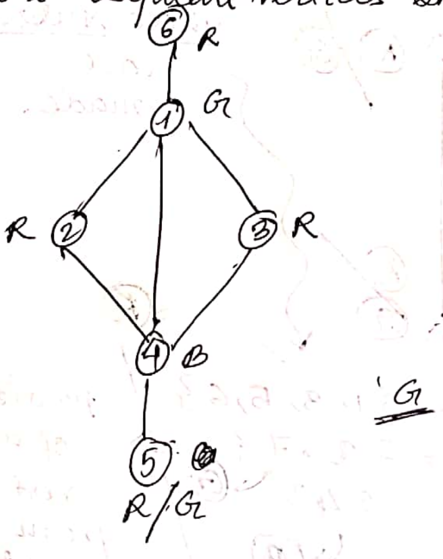
$= 1$, if ~~ith~~ an edge is directed from ith to jth vertex,

$= 0$, otherwise.

$$
\begin{array}{c|c}
 & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} &
\begin{bmatrix}
0 & +1 & 0 & 0 & 0 & 0 \\
0 & 0 & +1 & 0 & 0 & +1 \\
0 & 0 & 0 & +1 & +1 & 0 \\
0 & 0 & +1 & 0 & 0 & 0 \\
0 & +1 & 0 & 0 & 0 & +1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$



proper coloring is a techique of coloring (Colouring) a graph so that,
→ no 2 adjacent vertices should have the same color.



Chromatic no. of a graph.
→ min^m no. of colours used to color a graph properly.

→ denoted as $\boxed{K_\gamma}$.

'Chroma' = color.

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here, $K_\gamma = 03$

① for a null graph, $K_\gamma = 1$.
for a isolated graph, i.e all isolated vertices

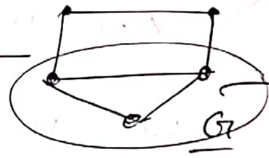
② A graph w 1 or more edges but not a self loop is atleast $K_\gamma = 2$ i.e 2-chromatic.

③ A complete graph w n vertices → $K_\gamma = n$.
↳ each vertex connected to each other.

④ v vertices, with r vertices making up a complete subgraph → graph will be atleast r-chromatic.
$K_\gamma \geq r$.

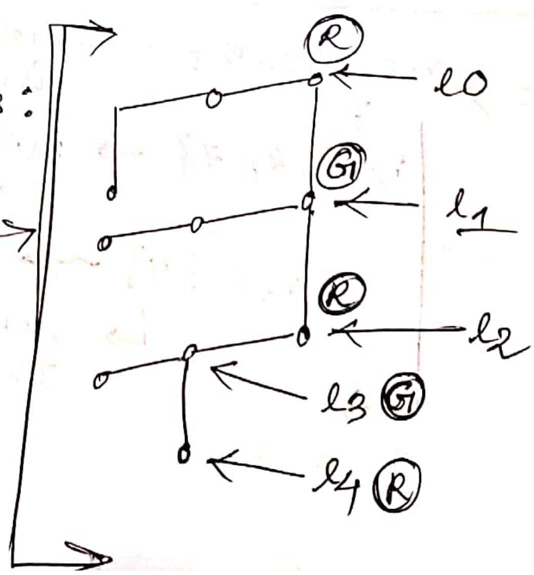
⑤ Graph has exactly 1 ckt:
2-chromatic if n is even
3-chromatic if n is odd.

$K_\gamma \geq 3$

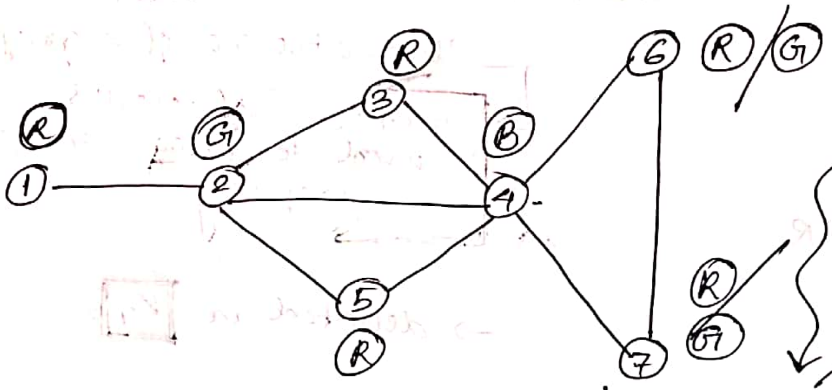


→ 3 v comp. graph

⑥ A tree w 2 or more vertices: is always $K_\gamma = 2$



Chromatic partitioning → Technique by which these subsets are made.



$V = \{1, 2 \ldots 7\}$

$V_1 = \{1, 3, 5, 6\}$ → R

$V_2 = \{2, 7\}$ → G

$V_3 = \{4\}$ → B

⟹ partition of the vertices from universal set V.

I.S.

Each of these subset is known as independent set. These independent subsets depends on a property known as independence.

- property 1: In a subset / independent set: none of the vertices in one subset to another. That's why they are assigned the same colour first of all.

Maximal Independent set (M.I.S) → An independent set from where no vertex can be added without destroying its independence property.

→ Aka internally stable set.

here V_1, ~~V_2, V_3 are both~~ is a both I.S & M.I.S.

If $V_1 = \{1, 3, 5\}$ → not a M.I.S ✗ → bcz 1 more vertex either 6 or 7 can be added.
is an I.S ✓

$V_2 = \{2, 7\}$ → M.I.S ✓
I.S ✓

$V_3 = \{4\}$ → 1 can be added.
↳ not a M.I.S ✗
I.S ✓

(*) largest Maximal Independent set : Among the M.I.Ss the subset that has the max^m no. of vertices.

here, v_1 is largest

(*) Independence number = the no. of vertices in the larges M.I.S.

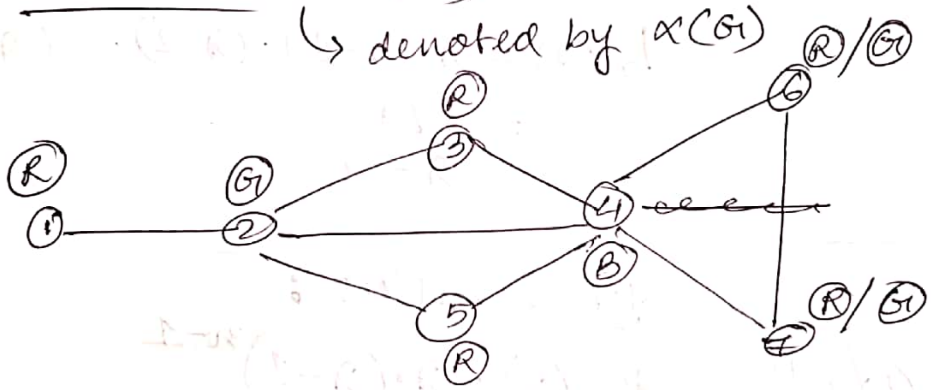
denoted by $\beta(G)$

here, I.N = 4

property (2) Domination

- Dominating set
- minimal Dominating set — (v_1, v_2) → no more vertices can be removed without destroying its domination property.
- smallest M.D.S. (v_1) → M.D.S containing minimum no of vertices
- Domination no. (04) → no of vertices in the smallest M.D.S.

↳ denoted by $\alpha(G)$



(*) Any single vertex in a graph is a trivial Indep. set

(*) All the vertices of the graph ⇒ is a trivial dominating set

$V = \{1, 2, 3 \ldots 7\}$

* Dominating set :— Either all vertices will be in the set or some of the vertices will be present in such a way that rest of the vertices are adjacent to someone of the vertices.

e.g → $v_1 = \{1, 3, 4\}$ X M.D.S ?? check.

e.g → $v_2 = \{1, 4\}$ M.D.S.

$v_1 = \{1, 3, 5, 6\}$ I.S + D.S

$v_2 = \{2, 7\}$ I.S + D.S

$v_3 = \{4\}$

→ minimal Dominating set.

$S = \{1, 2, 3, 5, 6\}$

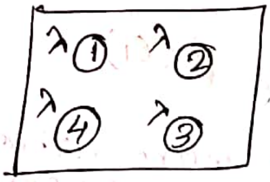
D.S ✓

M.D.S X

chromatic polynomial $\lambda = 10$

5 vertices
10 colours

At a time we'll be able to use only 5 colors.



G

4 vertices
λ colours | λ^4

Each vertex can be coloured in λ ways.

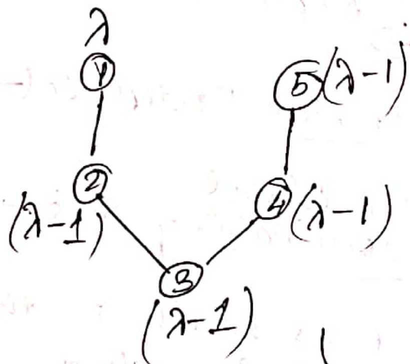
n vertices $\Rightarrow \lambda^n$ ways

n v, n colours $\Rightarrow n^n$ ways / check

graph?

$$\boxed{P_n(\lambda) = \lambda^n}$$

$$P_n(\lambda) = \lambda \cdot (\lambda-1) \cdot (\lambda-2) \cdots (\lambda-n+1)$$
$$= \frac{\lfloor \lambda}{\lambda - n}$$



for n vertices:

$$P_n(\lambda) = \lambda \cdot (\lambda-1)^{n-1}$$

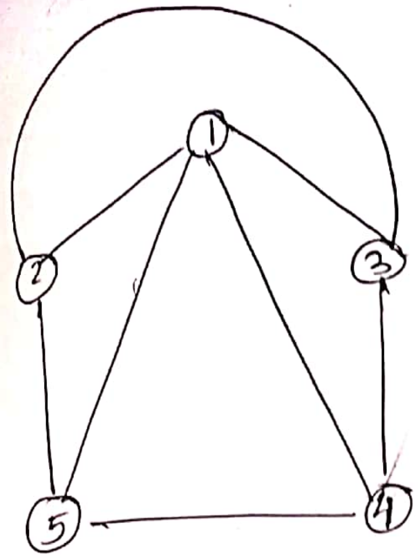
$$\boxed{\lambda(\lambda-1)^4}$$

path graph

given λ no. of colours

At a time $\Rightarrow$ i colours are used.

$\therefore$ there are C_i ways of coloring the graph properly.



G_1

$$C_i\binom{\lambda}{i}$$

chromatic polynomial :

$$P_n(\lambda) = \sum_{i=1}^{n} C_i\binom{\lambda}{i}$$

$$P_n(\lambda) = \sum_{i=1}^{n} C_i\binom{\lambda}{i}$$

$$= \frac{C_1\lambda}{\lfloor 1} + \frac{C_2\cdot\lambda(\lambda-1)}{\lfloor 2} + \cdots \frac{C_n\lambda\cdot(\lambda-1)\cdots(\lambda-n+1)}{\lfloor n}$$

Q/ find out $P_n(\lambda)$ for given λ = how many ways a graph can be coloured. given P_n

λ given, find $P_n(\lambda)$

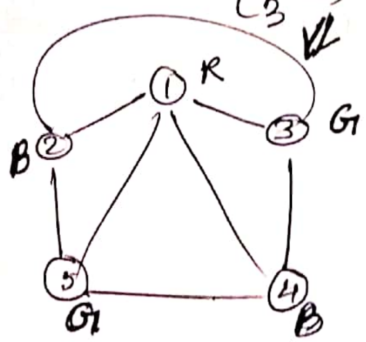
$C_1 \rightarrow$ In how many ways the given graph can be coloured by using 1 colour.

$C_2 \rightarrow$ 2 colours.

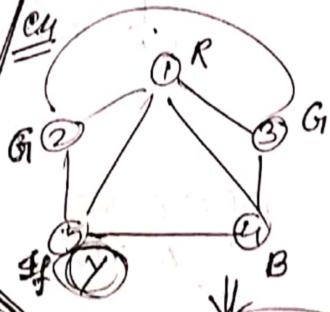
$C_4 \rightarrow$ 4 colours $\rightarrow$ $4! \times 2$

here, for G_1, $C_1 = C_2 = 0$.

$C_3 \rightarrow$ 3 colours. $\rightarrow$ $3!$ ways.



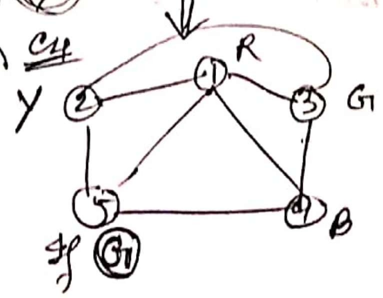
C_4



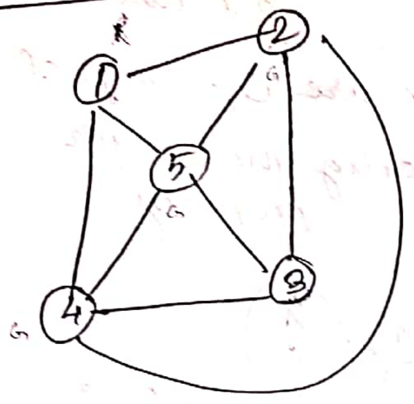
$C_5 \rightarrow$ 5 colours $= 5!$

$C_n \rightarrow n!$

C_4



chromatic polyn. → How many ways a graph can be coloured if λ colours are given.



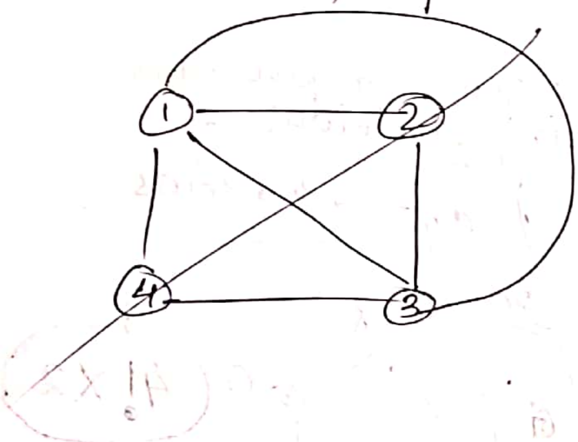
$C_1 = 0$
$C_2 = 0$
$C_3 =$

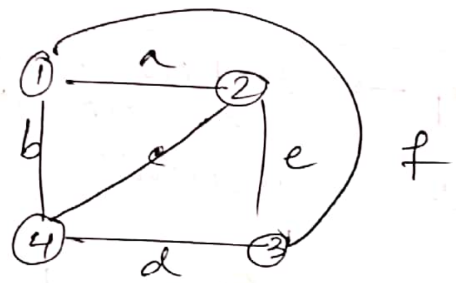
Matching → ~~p~~ p – 178

→ find out prop of matching w e.g.
- maximal matching
- largest maximal matching
- matching number.

Not associated with v, but with edges

→ set of edges where no 2 edges are adjacent.



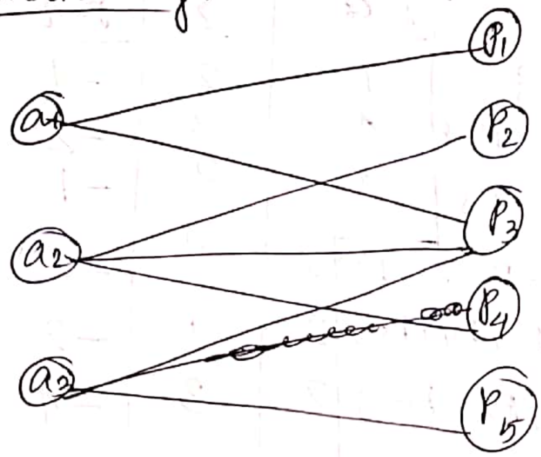


$m_1 = \{b, e\}$ → matching ✓

M.M → where no. more edge can be added.

LM.M – · – M.N

Complete Matching



Phillip Hall

$V_2 = \{P_1, P_2, P_3, P_4, P_5\}$

$V_1 = \{a_1, a_2, a_3\}$

not connected among themselves → same

Complete matching → Every vertex of V_1 should be matched to some vertex of V_2.

matching is wrt V_1 not V_2

a → set of applicants

- whether every applicant is fixed
- not whether all posts are filled.

P → set of posts.

"Hall's marriage problem".

Hall's Theorem

A complete matching of set V_1 to set V_2 in a bipartite graph exists iff every subset of r vertices in set V_1 is collectively adjacent to r or more vertices in set V_2 for all values of r.

P–Q

value of r	V_1	V_2
r = 1	$\{a_1\}$	$\{P_1, P_3\}$
	$\{a_2\}$	$\{P_2, P_3, P_4\}$
	$\{a_3\}$	$\{P_3, P_5\}$
r = 2	$\{a_1, a_2\}$	$\{P_1, P_2, P_3, P_4\}$
	$\{a_2, a_3\}$	$\{P_2, P_3, P_4, P_5\}$
	$\{a_1, a_3\}$	$\{P_1, P_3, P_5\}$
r = 3	$\{a_1, a_2, a_3\}$	$\{P_1, P_2, P_3, P_4, P_5\}$

value of r	V_1	V_2	p	q	$p-q$
$r=1$	$\{a_1\}$	$\{P_1, P_3\}$	1	2	-1
	$\{a_2\}$	$\{P_2, P_3, P_4\}$	1	3	-2
	$\{a_3\}$	$\{P_3, P_5\}$	1	2	-1
$r=2$	$\{a_1, a_2\}$	$\{P_1, P_3, P_2, P_4\}$	2	4	-2
	$\{a_2, a_3\}$	$\{P_2, P_3, P_4, P_5\}$	2	4	-2
	$\{a_1, a_3\}$	$\{P_1, P_3, P_5\}$	2	3	-1
$r=3$	$\{a_1, a_2, a_3\}$	$\{P_1, P_2, P_3, P_4, P_5\}$	3	5	-2

p = no. of vertices in set V_1
q = no of vertices in set V_2

largest value $= -1$.
$\Downarrow$
denotes deficiency.
$\delta(G)$

Hall's theorem (2)

↳ if $\delta(G) \leq 0$
⇒ then complete matching exists.

If complete matching &

maximal matching → [no. of vertices in set V_1 $- \delta(G)$]

$= 3 - 1$

$= 2$ //check.

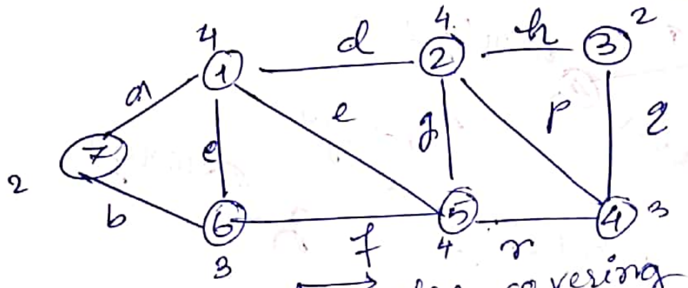
Q/ { graph drawn
or rel^n given } $a_1 \rightarrow (P_1, P_3)$
$a_2 \rightarrow (P_2, P_3, P_4)$

p-182

Covering

edge covering

vertex covering.



set of edges such that all the vertices of the graph are covered.

- edge covering
- minimal edge covering
- smallest minimal edge covering
- edge covering or covering no.

~~{a, f, h, q}~~ → This is a M.E.C because no edge can be removed without disturbing the covering.

a → 1, 7; f → 6, 5; h → 2, 3; q → 3, 4

- spanning tree
- ham. circuit

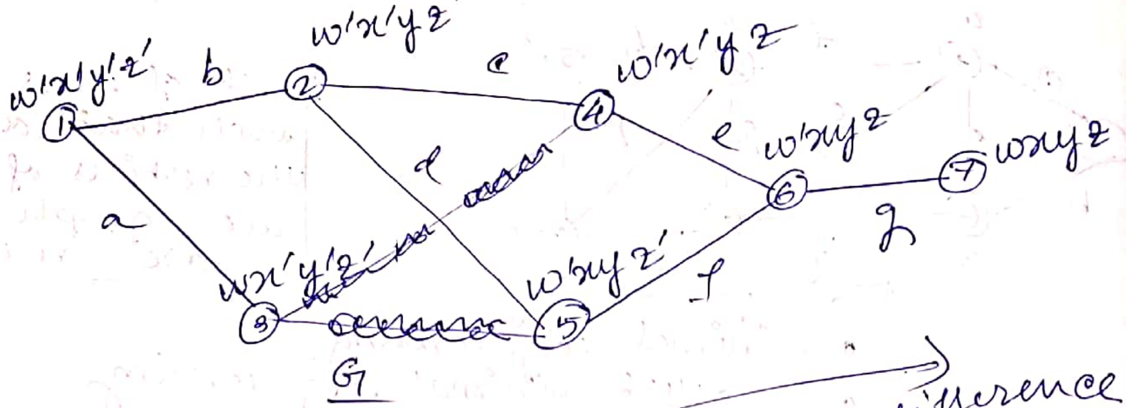
vertex covering → set of vertex such that all the edges of the graph are covered.

- Minimal vertex covering.
- smallest minimal vertex covering.

e.g: {1, 3, 4, 5, 6}

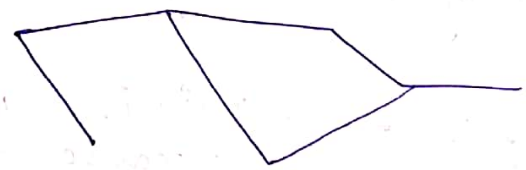
(8) SOP → ~~POS~~

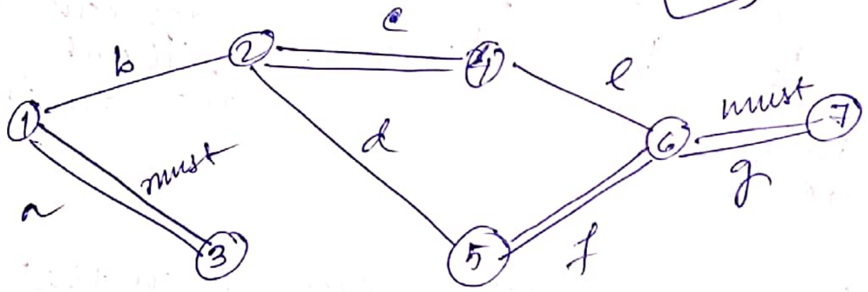
$F = w'x'y'z' + w'x'yz' + wx'y'z' + \cancel{wxy'z} + w'xyz' + w'xyz + wxyz + w'x'yz$

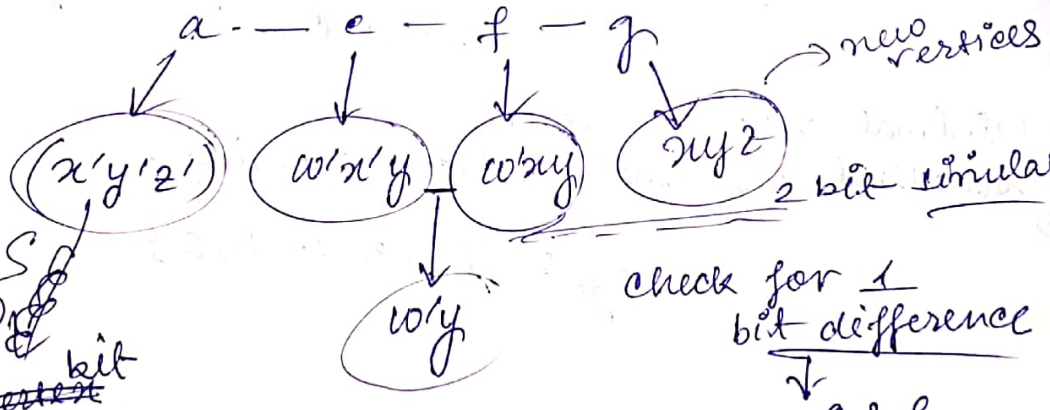


G

1 bit difference → connected by edges.







2 bit similarity

check for 1 bit difference → e & f.

1 bit diff ⇒ 3 ~~vertex~~ bit similarity.

Conjecture

4 color problem conjecture

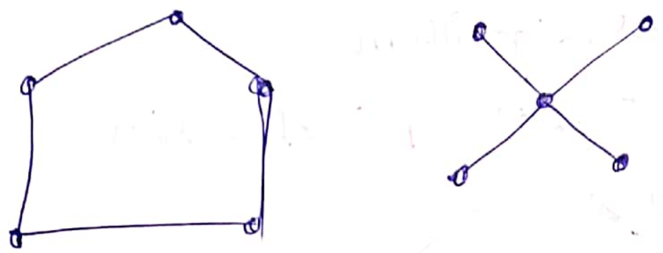
map region coloring

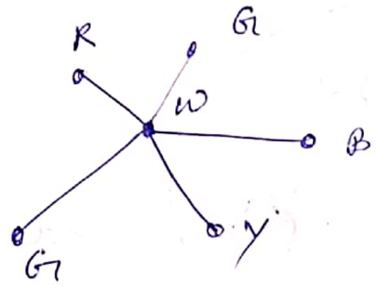
• planar

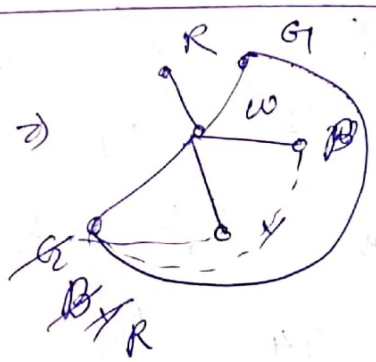
• 5 color problem

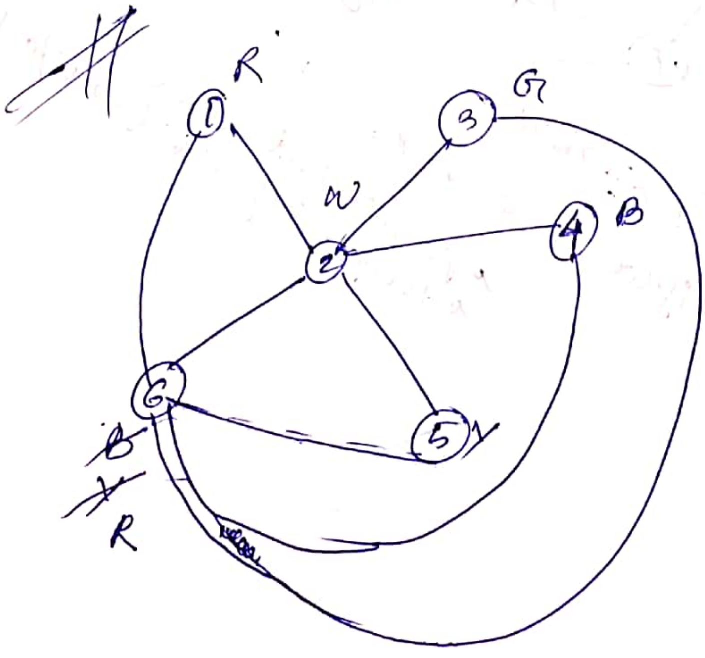
If you take a planar graph only 5 colours at most 5 colors will be enough to colour its vertices properly.

To make a graph planar, 5 v will be req, out of which 1 vertex should have $\partial(5)$.

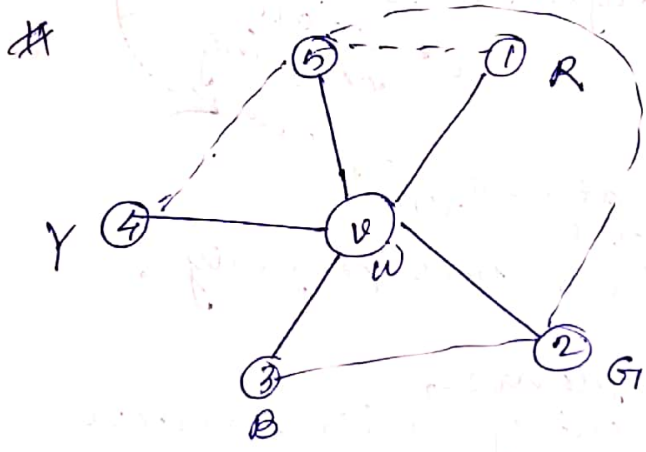






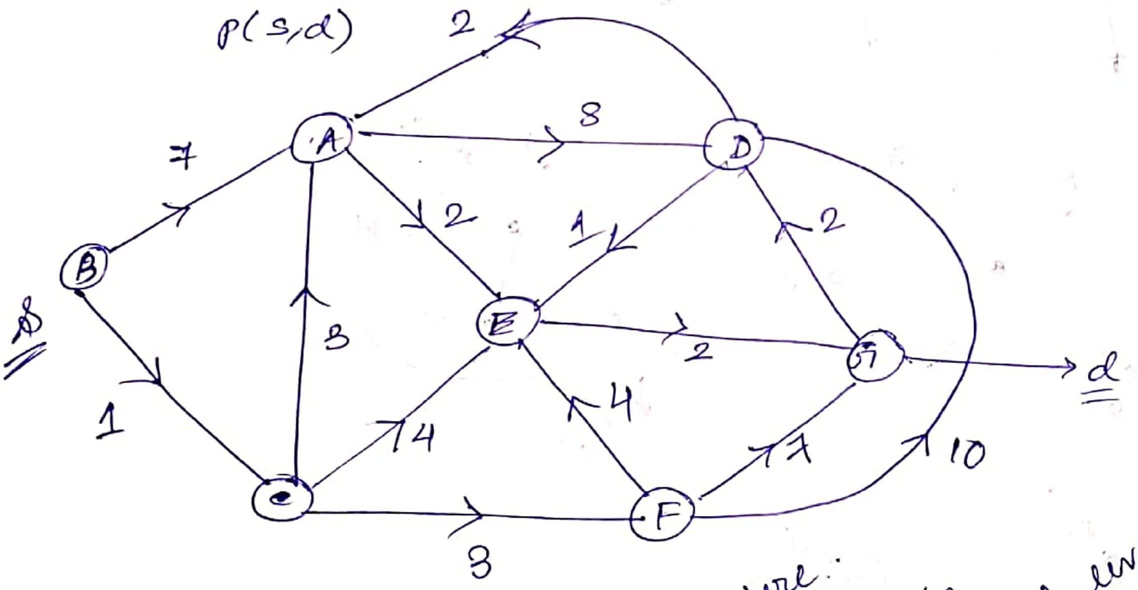


• 5 color problem



Dijkstra's Algorithm

— shortest path in betn a pair of vertices.



S → source

1st → source vertex = 0 → here.

• identified in a box.

• permanent level

• temporary level → w/o box

7, 1 boxed bec 1 < 7 and else < ∞.

→

	A	B	C	D	E	F	G
	∞	$\boxed{0}$	∞	∞	∞	∞	∞
wrt A ←	7	$\boxed{0}$	$\boxed{1}$	∝	∝	∝	∝
wrt C ←	4	$\boxed{0}$	$\boxed{1}$	∝	5	$\boxed{4}$	∝
wrt F ←	4	$\boxed{0}$	$\boxed{1}$	14	5	$\boxed{4}$	11
	$\boxed{4}$	$\boxed{0}$	$\boxed{1}$	12	5	$\boxed{4}$	11
	$\boxed{4}$	$\boxed{0}$	$\boxed{1}$	12	$\boxed{5}$	$\boxed{4}$	$\boxed{\cancel{8}\ 7}$
	$\boxed{4}$	$\boxed{0}$	$\boxed{1}$	12	$\boxed{5}$	$\boxed{4}$	$\boxed{7}$

A (∝ , 7+0)
A (∝ , 7)
↑ min

level of A → 7.

B – C – E – G

$\boxed{\text{not considering -ve path}}$

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100 → 10 Questions → 1 applicatn each / (Midterm – Questions of midterm)

- Kayley's theorem.
- Prufer
- Kruskal's.

- tree
- matrix ↓ graph given → make matrix → show soln A_F, B_F, C_F.

- colouring ↓ chromatic polynomial.
- Hall's theorem.

- undirected
- or directed

- short Questions.

→ diagram must.