GRAPH THEORY

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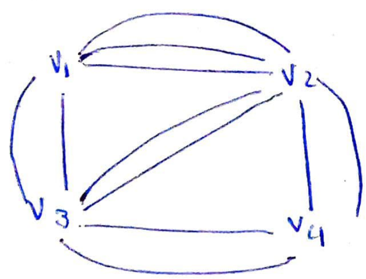
CLASS WORK - 1 :-

Q.1. With a proper diagram, show that an infinite graph with a finite number of vertices will have atleast one pair of vertices joined by infinite no. of parallel edges.

Ans :- Let $G(V, E)$ be an infinite graph with a finite no. of vertices and infinite edges

$V = \{v_1, v_2, v_3, v_4, \dots v_n\}$ $E = \{e_1, e_2, e_3, \dots \infty\}$

For example consider $n = 4$, four vertices



For the graph to be infinite atleast (v_1, v_2), (v_2, v_3), (v_3, v_4), or (v_4, v_1) or all four vertices must be joined by infinite number of parallel edges.

For 'n' such vertices, we will find atleast one pair of vertices joined by an infinite number of parallel edges

Q.2. With suitable diagram, show that the maximum number of edges in a simple graph with n vertices is : $\frac{n(n-1)}{2}$

Ans :- Let $G(V,E)$ be a simple graph with n vertices

Now,

Let v_1 be any arbitrary vertex. If we join v_1 with all the remaining vertices, the no. of edges will be atmost $(n-1)$.

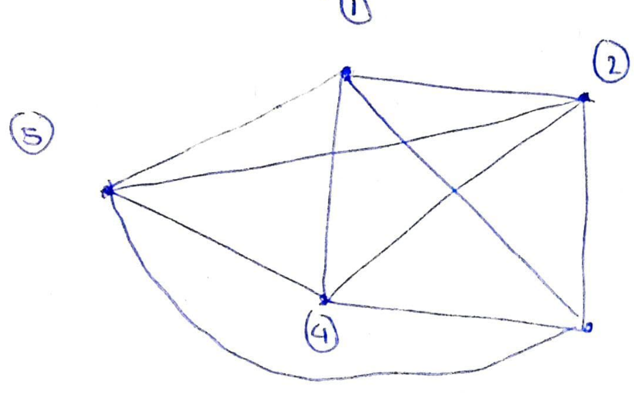
For V_2, if we join v_2 with remaining vertices except v_1, no. of edges will be atmost $(n-2)$.

And the process for the rest of the vertices

∴ Maximum no. of edges $= (n-1) + (n-2) \ldots 3+2+1$

$$= \frac{(n-1)(n-1+1)}{2} = \frac{n(n-1)}{2}$$

Considering a graph for $n=5$;



Here we can see

For 1st vertex $\Rightarrow n-1 = 4$ edges

For 2nd vertex $\Rightarrow n-2 = 3$ edges

For 3rd vertex $\Rightarrow n-3 = 2$ edges

For 4th vertex $\Rightarrow n-4 = 1$ edge

∴ Maximum no. of edges

$$= \frac{5(5-1)}{2} = 10 = 4+3+2+1$$

Q.3. Differentiate between :

(a) (i) Incidence matrix and Adjacency matrix

Ans → Incidence matrix :

(i) In this matrix, rows represent vertices and columns represent rows.

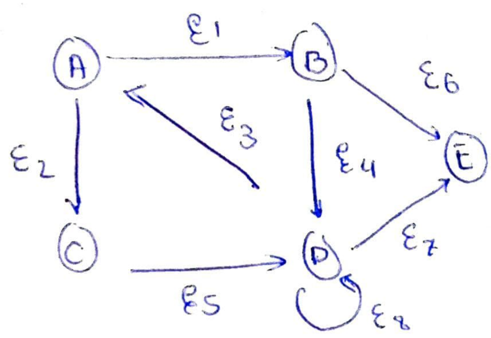
(ii) This matrix is filled with 1, −1, ~~2~~ or 0.

(iii) Here, 0 represents row edge is not connected to column vertex

(iv) 1 represents row edge is connected to outgoing edge to column vertex.

−1 represents row edge is connected to incoming edge to column vertex.

e.g.



$$
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{cccccccc} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_4 & \varepsilon_5 & \varepsilon_6 & \varepsilon_7 & \varepsilon_8 \end{array} \\
\begin{bmatrix}
1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & -1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & -1 & -1 & 0
\end{bmatrix}
\end{array}
$$

→ Adjacency Matrix :

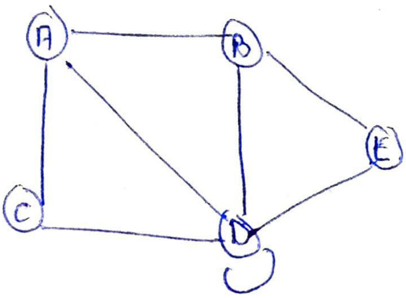
(i) A Graph G(V,E) where $V = \{0, 1, 2 \ldots n\}$ can be represen-ted using two-dimensional array of size $n \times n$.

(ii) The matrix filled with 0 and 1.

(iii) Here 1, indicates presence of edge between two vertices. and 0, indicates absence of edge between two vertices.

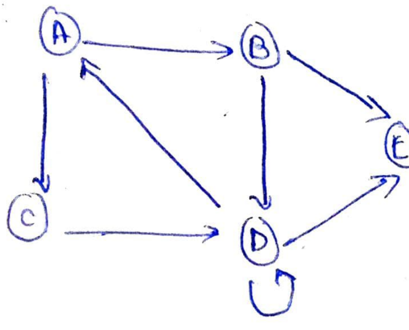
(iv) Adjacency matrix of an undefined graph is always a symmetric matrix. i.e. an edge (i,j) implies (j,i).

Adjacency matrix of a directed graph is never symmetric, then adj [i][j] = 1, represents directed edge from vertex i to j.



Undirected graph representation.

$$\Rightarrow \quad \begin{array}{c|c} \text{vertex} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

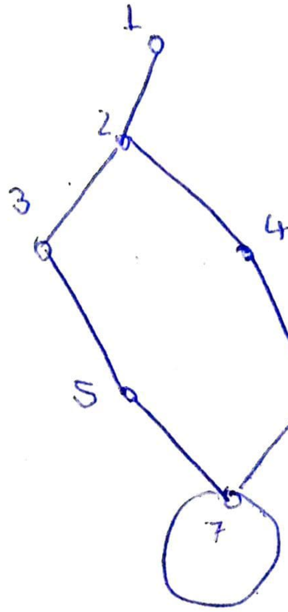


$$\Rightarrow \quad \begin{array}{c|c} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(b) **Series edge**

(i) Two adjacent edges are said to be in series if their common vertex is of degree two.

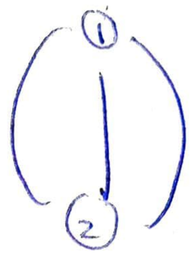
(ii)



Parallel edge

(i) Two edges with same end vertices are called parallel edges

(ii)



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