S3 (UCS03B10(NITA)/UCS03B04(IIITA)) B.Tech

B.Tech. 3rd Semester End Term Examination, 2021

Discrete Mathematical Structures

UCS03B10(NITA)/UCS03B04(IIITA)

Full Marks: 50 **Time: 2 hours**

1. (a) Determine whether each of these statements is true or false.
   1. $x \in \{x\}$
   2. $\{x\} \subseteq \{x\}$
   3. $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ **(3)**

   (b) Use a Venn diagram to illustrate the following:
   1. Subset of odd integers in the set of all positive integers not exceeding 10.
   2. Use a Venn diagram to illustrate the relationship $A \subseteq B$ and $B \subseteq C$.
   3. $A \cap (B - C)$ for set A, B and C **(3)**

   (c) What is the cardinality of each of these sets?
   1. $\{\{a\}\}$
   2. $\{a, \{a\}, \{a, \{a\}\}\}$ **(2)**

   (d) In a survey of 120 people, it was found that 65 read Newsweek magazine, 20 read both Newsweek and Time, 45 read Time, 25 read both Newsweek and Fortune, 42 read Fortune, 15 read both Time and Fortune, 8 read all three magazines.
   1. Find the number of people who read at least one of the three magazines.
   2. Find the number of people who read exactly one magazine. **(2)**

2. (a) Determine whether the following relations are Reflexive, Symmetric, Antisymmetrc and Transitive
   1. Relation R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)} on S = {1, 2, 3}.
   2. Relation congruent modulo m, written $a \equiv b \pmod{m}$ for a fixed positive integer m over Set of positive integers **(4)**

   (b) Given A = {1, 2, 3, 4} and B = {x, y, z}. Let R be the following relation from A to B:

   R = {(1, y), (1, z), (3, y), (4, x), (4, z)}
   1. Determine the matrix of the relation.
   2. Draw the arrow diagram of R.
   3. Find the inverse relation R^{-1} of R.
   4. Determine the domain and range of R. **(4)**

   (c) Find the composition RoR for above relation **(2)**

3. (a) Express each of these statements using quantifiers and write the negation of the statement in English statements.
   1. All dogs have fleas.
   2. There is a horse that can add. **(2)**

   (b) Let P (x), Q(x), R(x), and S(x) be the statements "x is a baby," "x is logical," "x is able to manage a crocodile," and "x is despised," respectively. Suppose that the domain consists of all people. Express each of these statements using quantifiers; logical connectives; and P (x), Q(x), R(x), and S(x). a) Babies are illogical. b) Nobody is despised who can manage a crocodile. **(2)**

(c) Let $Q(x, y)$ be the statement "x has sent an e-mail message to y," where the domain for both x and y consists of all students in your class. Express each of these quantifications in English. a) $\exists y \forall x Q(x, y)$ b) $\forall y \exists x Q(x, y)$ **(2)**

(d) Let $S(x)$ be the predicate "x is a student," $F(x)$ the predicate "x is a faculty member," and $A(x, y)$ the predicate "x has asked y a question," where the domain consists of all people associated with your school. Use quantifiers to express each of these statements. a) Every student has asked Professor Gross a question. b) Every faculty member has either asked Professor Miller a question or been asked a question by Professor Miller. **(2)**

(e) Rewrite each of these statements so that negations appear only within predicates.

1. $\neg \forall y \exists x P(x, y)$
2. $\neg \forall x(\exists y \forall z P(x, y, z) \wedge \exists z \forall y P(x, y, z))$ **(2)**

4. Determine whether following algebraic structures are semigroups, monoids, group or abelian.
   i. The set of odd and even positive integers closed under multiplication
   ii. The set of 2 × 2 matrices with rational entries under the operation of matrix multiplication.
   iii. The set Q of rational numbers, and let $*$ be the operation on Q defined by $a * b = a+b+ab$
   iv. The set $G = \{1, 2, 4, 7, 8, 11, 13, 14\}$ under multiplication modulo 15.
   v. The Set N of positive integers, and let $*$ denote greatest common Divisor (gcd) operation on N. **(2*5=10)**

5. Answer these questions for the poset $(\{1,2,3,4,5,6,9,10,15,25,30,45,150\}, |)$
   i. Draw the Hasse diagram of the given poset.
   ii. Find the maximal and minimal element.
   iii. Find the greatest and least element.
   iv. Find the Upper bound of $\{2,10,15\}$
   v. Find the lower bound of $\{3,30,150\}$
   vi. Find Greatest Lower bound of $\{1,45\}$
   vii. Find the Least Upper Bound of $\{4,5\}$
   viii. Find Greatest Lower bound of $\{6,9\}$
   ix. Find the Least Upper Bound of $\{25,45\}$
   x. Is the poset Lattice? **(1*10=10)**

National Institute of Technology, Agartala.

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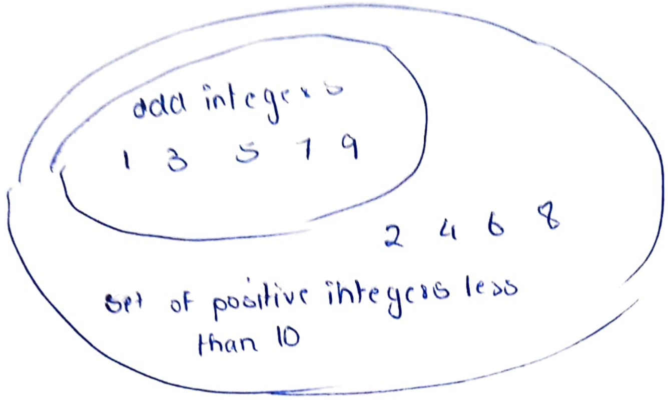
Q.1. (a)

(1) True

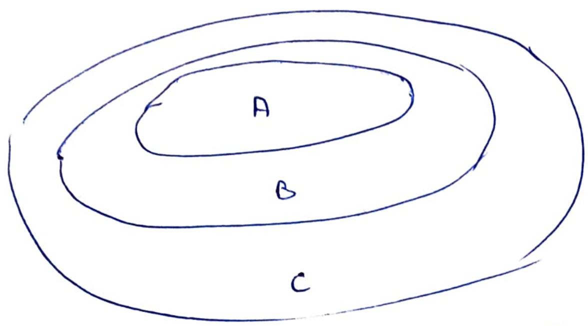
(2) ~~True~~ False

(3) True.

(b) 1.

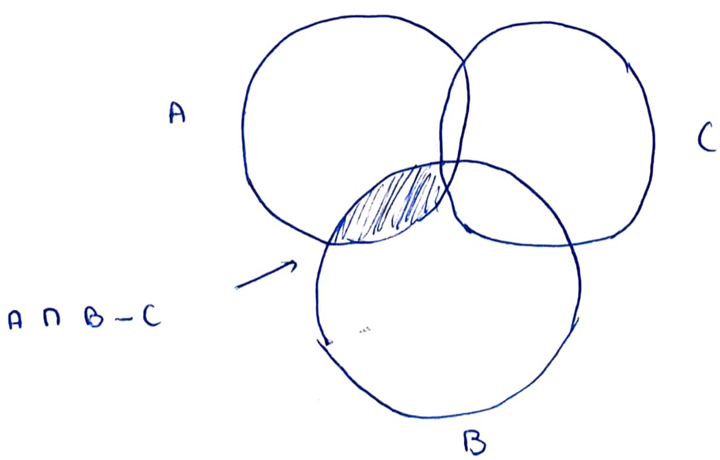


2.



$A \subseteq B \subseteq C$

3.



(c) 1. $n(\{\{a\}\}) = 1$

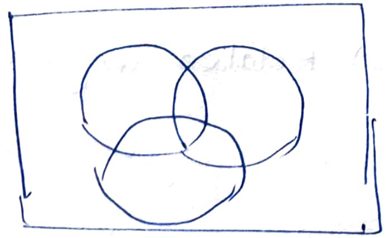
2. $n(\{a, \{a\}, \{a, \{a\}\}\}) = 3$

1. (a) Let people who read Newsweek be denoted by set A, people who

1.

$n(S) = 120$

$n(A) = 65$ $\quad n(A \cap B) = 20$



$n(B) = 45$

$n(A \cap C) = 25$ $\quad n(C) = 42$

$n(B \cap C) = 15$, $\quad n(A \cap B \cap C) = 8$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$= 65 + 45 + 42 - 20 - 25 - 15 + 8 = 100$$

2. People who read only Newsweek $= 65 - 12 - 8 - 17 = 28$

People who read only Time $= 45 - 12 - 8 - 7 = 18$

People who read only Fortune $= 42 - 17 - 8 - 7 = 10$

People who read only one magazine $= 56$

2. (a) Given

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$ on $S = \{1, 2, 3\}$

~~The~~ R is reflexive, $\left(\because \forall \text{ values of } x \in S,\ (a,a) \in R\right\}$

R is symmetric, $\left(\because (2,1) \in R,\ (1,2) \in R\right)$

R is not antisymmetric, $\left(\because (2,1) \in R, \text{ but } (1,2) \in R\right)$

R is transitive, $\left(\because (a,b) \in R, \text{ but } (b,c) \notin R\ \cancel{\text{for}}\right)$

Also

~~2 Abt~~

2. (a)

(2) Relation congruent modulo m written $a \equiv b \pmod m$

For congruent modulo ; $a - b = kn$ [n is the divisor, and k any value]

$\therefore$ R is reflexive, $\because$ $(a-a) = 0 = kn$

For (a,b), $a - b = kn$

then (b,a) $b - a = -kn$ $\because$ They have same divisor

R is symmetric.

R is not antisymmetric, ($\because$ $(a,b) \in R$, $(b,a) \in R$)

R is not transitive.

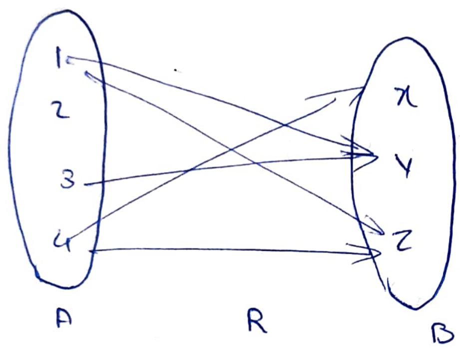
2. (b) $A = \{1, 2, 3, 4\}$ to $B = \{x, y, z\}$

$R = \{(1,y), (1,z), (3,y), (4,x), (4,z)\}$

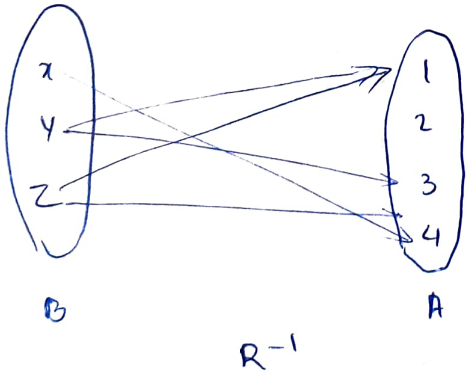
Relation matrix :

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

2.



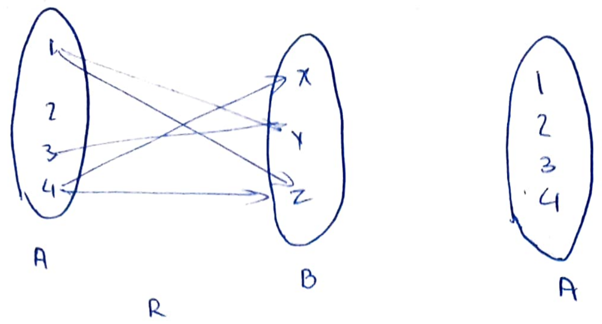
3. $R^{-1} = \{(y,1), (y,3), (z,1), (z,4), (x,4)\}$



4. Domain $(R) = \{1, 3, 4\}$

Range $(R) = \{x, y, z\}$

2. (c) $R = \{(1,y), (1,z), (3,y), (4,x), (4,z)\}$



$R \circ R = \{\ \}$ is a Null Relation because R is relation from $A \to B$, ~~b~~ $R \circ R$ is relation from $A \to A$, ~~∴ RoR is~~ But for ~~each~~ A but no relation from B to A exist.

Q.3. (a)

1. Let $P(x)$: dogs have fleas

All dogs have fleas $\Rightarrow \forall x \, (P(x))$

Now, Negation :-

$\sim \forall x \, P(x) \Rightarrow \exists_x \sim P(x)$

There are some dogs which do not have fleas.

2. Let $P(x)$: x is a horse that can add.

$\Rightarrow$ ~~E~~ $\exists x \, P(x)$

Negation :- $\sim \exists x \, P(x)$

$\Rightarrow \forall x \sim P(x)$

$\Rightarrow$ All horse can not add.

(b) $P(x)$: x is a baby

$Q(x)$: x is logical

$R(x)$: x is able to manage a crocodile.

$S(x)$: x is despised.

a) Babies are illogical

$\forall x \, (P(x) \rightarrow x \, Q(x))$

0.3. (b) Nobody is despised who can manage a crocodile:

$\sim \exists x \, (S(x) \wedge R(x))$

(c) $Q(x,y)$: 'x has sent an email message to y:'

(a) $\exists y \, \forall x \, Q(x,y)$

=> There is a student in your class who has been sent a message by every student in your class.

(b) $\forall y \, \exists x \, Q(x,y)$

=> Every student in your class has been sent a message from atleast one student in your class.

3.

(d) $S(x)$: x is a student.

$F(x)$: x is a faculty member.

(a) Every student has asked Proff Gross a question.

=> $\forall x \, (S(x) \rightarrow P(x, \text{Professor Gross}))$

(b) $\forall x \, (F(x) \rightarrow (A(x, \text{Professor Miller})$

$\vee \, A(\text{Professor Miller}, x)))$

(e) 1. $\sim \forall y \, \exists x \, P(x,y)$

=> $\exists y \, \forall x \sim P(x,y)$

2. $\sim \forall x \, (\exists y \, \forall z \, P(x,y,z) \wedge \exists x \, \forall y \, P(x,y,z))$

=> $\exists x \, (\forall y \, \exists z \sim P(x,y,z) \vee \forall z \, \exists y \sim P(x,y,z))$

Q.4 (i) Given,

set of odd and even positive integers closed under multiplication.

→ Closure exists,

multiplication will result in either positive even or odd integer

→ Associative :

$(a*b)*c = a*(b*c)$ are positive.

→ Identity does ~~not~~ exist, because. 1 is ~~not~~ present, as it ~~neither~~ odd or ~~even~~.

Inverse ~~is~~ does not exist.

∴ ~~It is a semigroup.~~

∴ It is a monoid.

(ii) The set of 2x2 matrices with rational entries under the operation of matrix multiplication.

Closure : For every pair, a and b, $a*b$ must be a rational number, hence it is closure.

Associative. ~~(a)~~ Matrix multiplication is associative.

Identity : $a*e = e*a$. It is identity.

Commutative : It is not communicative

~~Hence it is a group~~.

$a * a^{-1} = 0$

$\Rightarrow a + a^{-1} + a a^{-1} = 0$

$a^{-1} = \frac{-a}{1+a}$.

~~Uniqu~~ Unique inverse does not exists for $\forall a \in Q$

It is monoid.

4.(iv) = $G = \{1, 2, 4, 7, 8, \cancel{13}\ 11, 13, 14\}$ under multiplication modulo 15

	1	2	4	7	8	11	13	14
1	1	2	4	7	8	11	13	14
2	2	4	8	14	1	7	11	13
4	4	8	1	13	2	14	7	11
7	7	14	13	4	11	2	1	8
8	8	16	2	13	4	13	14	7
11	11	7	24	2	13	7	8	4
13	13	11	7	1	14	8	4	2
14	14	13	13	8	7	4	2	1

Cleary it is closure. It is associative.

Identity = 1

Inverse of 1 = 1
" " 2 = 8
" " 4 = 4
" " 7 = 13
" " 8 = 2
" " 11 = 11
" " 13 = 7
" " 14 = 14

Hence, there exits unique inverse.

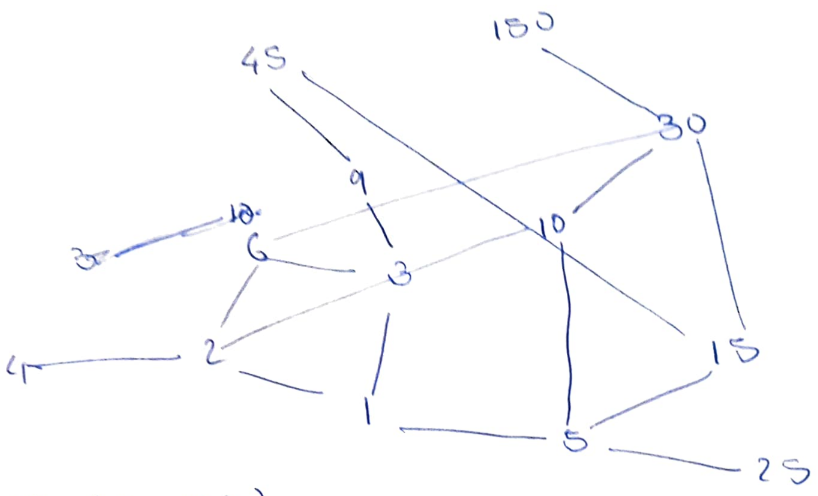
Hence, it is a group.

Hence, there exists unique inverse

Henc it is a group.

Q.5. {1, 2, 3, 4, 5, 6, 9, 10, 15, 25, 30, 45, 150}

(a) (i)



(ii) Maximal = {4, 25, 45, 150}

Minimal = {1, 2}

(iii) greatest = 150 , least = 1

(iv) up {2, 10, 15} : {10, 30, 150}

(v) LB {3, 30, 150} : {3, 15}

(vi) LB {1, 45} : {1, 3, 9}

GLB = {9}

(vii) UB {4, 5} = ϕ

LUB {4, 5} = does not exist.

(viii) LB {6, 9} = {1, 3}

GLB = {3}

(ix) UB {25, 45} = does not exist.

(x) As UB {4, 5} and UB {25, 45} is null., thus it is not a lattice

—— x ——