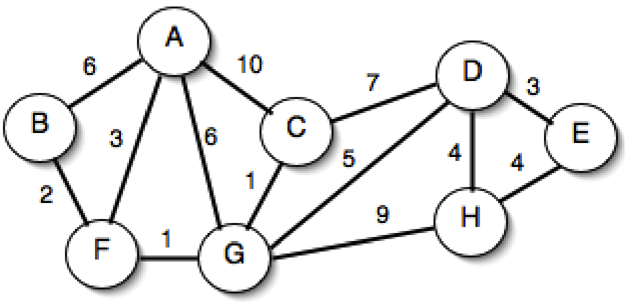
3^{rd} Semester Mid-Term Examination, 2021
Subject: - INTRODUCTION TO GRAPH THEORY
Paper Code: - UCS03B06 (UG) / UCS03B05 (IIITA)

Total Marks:-20 **Time: 1:00 hr.**

<u>Attempt all the questions.</u>

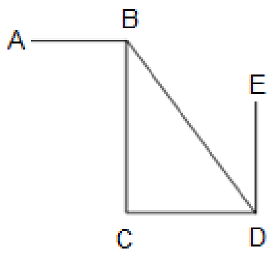
1. a) How Euler solved 'Konigsberg Bridge' problem?
   b) What is Fusion?
   c) Using Prim's algorithm find out the shortest spanning tree from the given graph. What is its weight?



[2 + 2 + (5 +1) = 10]

2. a) A tree has 2n vertices of degree 1, 3n vertices of degree 2 and n vertices of degree 3. Determine the no. of vertices and edges in that tree.
   b) Are the following two graphs isomorphic? Explain your answer.

   c) Using Kirchhoff's Matrix Tree Theorem find out the number of spanning trees in the given graph. Draw all those spanning trees.



[2 + 2+ (4 + 2) = 10]

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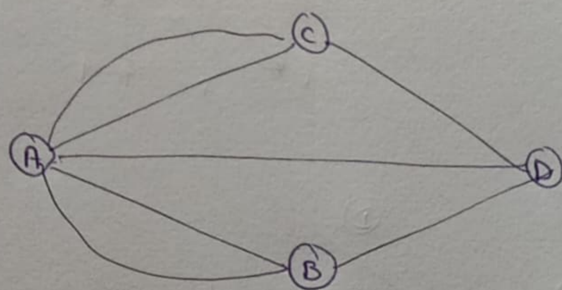
Section : A   Exam : 3rd Sem Mid-Term examination.

Subject : Graph Theory   Subject Code :

Q.1.(a) Euler states that, " In general, if the no. of bridges is any odd no. and if it is increased by one, then the no. of occurences of A is half of the result "



Graph Representation

In this graph,

1. vertices represent landmass.
2. Edges represent the bridges.

Euler's observation :

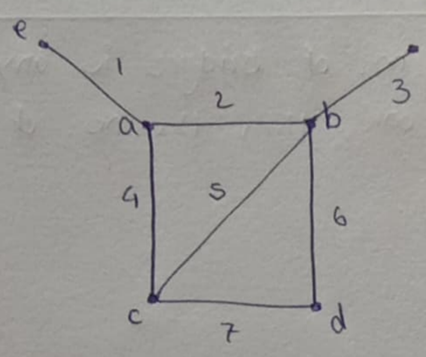
- There must be one edge that enters into the vertex
- There must be another edge that leaves the vertex.
- Therefore, order of the vertex must

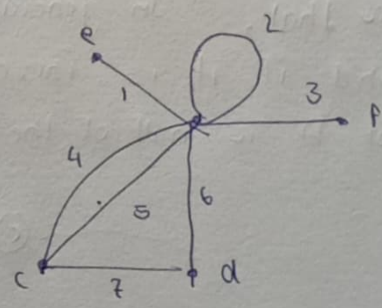
Based on this observation, Euler discovered that it depends on the no. ~~the no.~~ of odd vertices present in the network whether any network is transversable or not.

Euler found that, these networks are transversable, that have either,

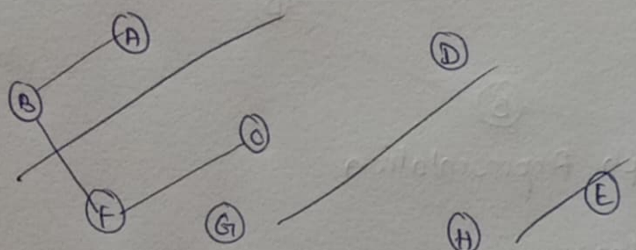
- No odd vertices
- OR exactly two odd vertices.

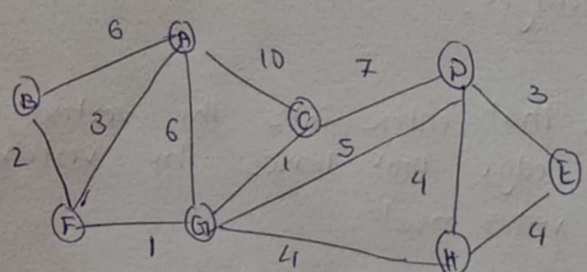
Q.1 (b) We can say that a graph is a fusion graph if any region A in the graph can always be merged within another region B, while preserving all other regions. e.g.





Q.1. (c)



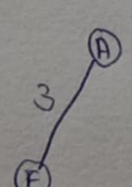


By Prim's Algorithm,

(i) We have to remove all self loops and parallel edges (none present)

(ii) Let us consider starting vertex = A, Now we will consider the minimum weight edge corresponding to A, which is not visited and does not form a cycle/circuit.

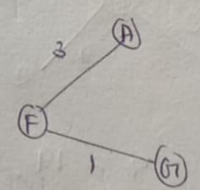
Tree becomes:



(iii) We will repeat the process for (n-1) edges to form minimum spanning Tree.

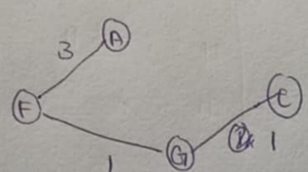
For vertex F, min wt. edge = F — G (1)

Tree becomes



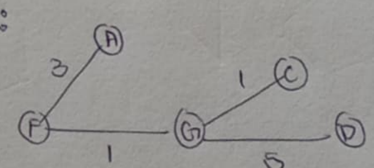
For vertex G, min wt edge = G — C (1)

∴ Tree becomes :-



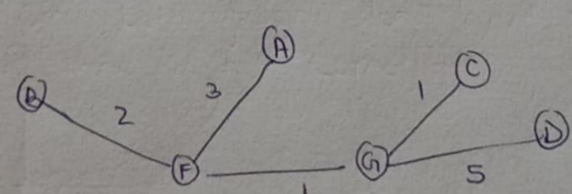
For edge D, min wt. edge = G — D (5)

Tree becomes :



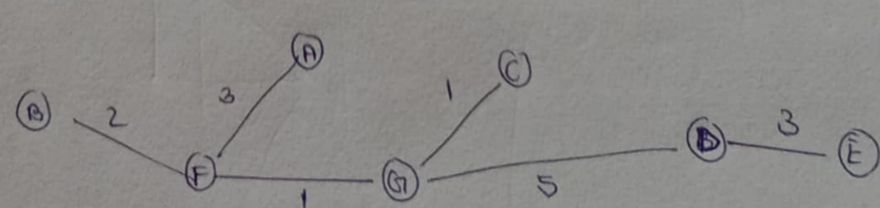
For edge B, min wt. edge = B — F

∴ Tree becomes :-



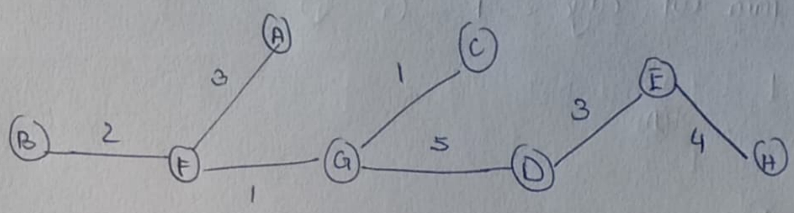
For edge E, min. wt. edge = D — E (3)

∴ Tree becomes



For edge H, min. wt. edge = H – F (4)

∴ Tree becomes :



Total wt. = 2 + 3 + 1 + 1 + 5 + 3 + 4 = 19.

Q.2. Given

(a) A tree has $2n$ vertices of degree 1, $3n$ vertices of degree 2, n vertices of degree 3.

$\Sigma d_i = 2|E|$ (Handshaking lemma)

$\Rightarrow 2|E| = 2n \times 1 + 3n \times 2 + n \times 3$

$|E| = 11n/2$

- For tree, $|V| - 1 = |E|$

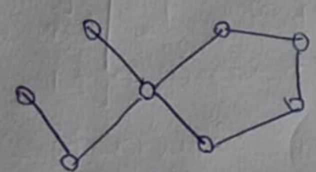
$\rightarrow 6n - 1 = \frac{11n}{2}$

$\rightarrow 12n - 2 = 11n \Rightarrow n = 2$

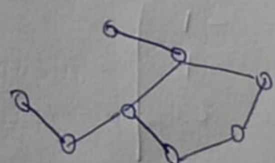
No. of edges $= \frac{11 \times 2}{2} = 11$

No of vertices $= 6n = 12$.

2. (b)



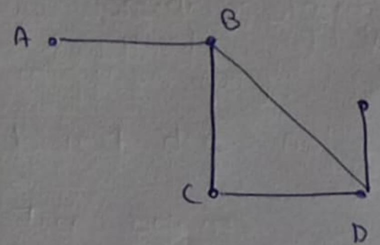
(a)



(b)

Both Graphs (a) & (b) have equal no. of vertices and edges. But Graph (a) has a vertex with degree 4 whereas graph (b) does not. Since both graphs don't equal no. of vertices with a given degree, they are not isomorphic.

Q.2. (c) Given ST,



Calculating Laplacian Matrix

$$Q = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & -1 \end{pmatrix}$$

Deleting row 1 and column 1

$$Q' = \begin{pmatrix} 3 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$$|Q'| = \begin{vmatrix} 3 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{vmatrix}$$

$$|Q'| = \begin{vmatrix} 3 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{vmatrix} \qquad R_3 \longrightarrow R_3 + R_4$$

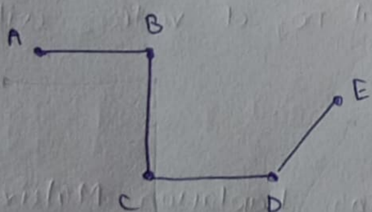
$$|Q'| = 1 \times \begin{vmatrix} 3 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix} \qquad \text{Finding determinant along } C_4$$

$$= 3(4-1) + 1(-2-1) - 1(1+2)$$

$$= 9 - 3 - 3 = 3$$

$\therefore$ There are 3 possible Spanning Trees.

①



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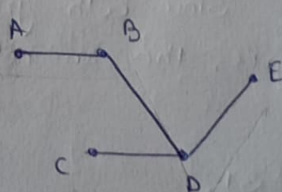

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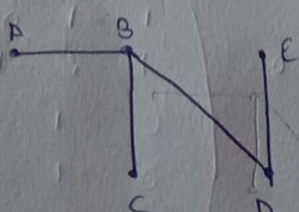

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