A periodic function ~~a f(x)~~ $f(x)$ which can be express as a trigonometric series of the form

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}, \quad c \leq x \leq c+2l$$

is called a fourier series of $f(x)$

$$a_0 = \frac{1}{l}\int_{c}^{c+2l} f(x)\,dx,$$

$$a_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\cos\frac{n\pi x}{l}\,dx,$$

$$b_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\sin\frac{n\pi x}{l}\,dx.$$

where, a_0, a_n, b_n are fourier coefficient

$2l$ is period of function.

Cardinary - 1

If $c = 0$, $l = \pi$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\, dx.$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx\, dx.$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx\, dx$$

Cardinary - 2

If $c = -\pi$, $l = \pi$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$b_n = \int_{-\pi}^{\pi} f(x) \sin nx \, dx .$$

Cordinary-3

if $c = 0$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

$$a_0 = \frac{1}{l} \int_{0}^{2l} f(x) \, dx .$$

$$a_n = \frac{1}{l} \int_{0}^{2l} \cancel{\cos} \, f(x) \cos \frac{n\pi x}{l} \, dx$$

$$b_n = \frac{1}{l} \int_{0}^{2l} f(x) \sin \frac{n\pi x}{l} \, dx$$

Cor-4

If $\Rightarrow c = -l$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

$$a_0 = \frac{1}{l} \int_{-l}^{l} f(x) \, dx .$$

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} \, dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x) \sin \frac{n\pi x}{l}\, dx$$

Dirichlet's Condition

A function $f(x), x \in (c, c+2l)$ can be expanded into a fourier series if the following conditions are satisfied.

(i) $f(x)$ is periodic.

$$f(x) = f(x+2l)$$

where, $2l$ is the period of function.

(ii) $f(x)$ and it's integrous are finite and single valued.

(iii) $F(x)$ has a finite no. of discontinuties. i.e. $f(x)$ is piece wise continuous in interval $(c, c+2l)$

(iv) $f(x)$ has a finite no. of maxima and minima.

~~this condition~~

these above conditions are known as a dirichlets condition.

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Q:- Find fourier series of $f(x) = x$ $(0, 2\pi)$

~~$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n$~~

here $f(x)$ is a periodic function in period 2π. and $f(x)$ shatisfied all condition of dirichlets conditions. therefore, the fourier series expansion of $f(x)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} x\,dx = 2\pi$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} \cos nx \times x\,dx$$

$$= \frac{1}{\pi} \left[\left[\frac{x \sin nx}{n} \right]_0^{2\pi} + \left[\frac{\cos nx}{n^2} \right]_0^{2\pi} \right]$$

$$= \frac{1}{\pi} \left[\frac{1}{n^2} \cos 2\pi n - \frac{1}{n^2} \right]$$

$$= 0$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x \times \sin nx \, dx$$

$$b_n = \frac{1}{\pi}\left[-\frac{\cos nx \, x}{n} + \frac{\sin nx}{n^2}\right]_0^{2\pi}$$

$$b_n = \frac{1}{\pi}\left[0 - (2\pi \cos 2n\pi - 2\pi)\right]$$

$$b_n = \frac{-2}{n}$$

now substituting the value of

a_0, a_n, b_n

$$F(x) = \frac{2\pi}{2} + \sum_{n=1}^{\infty} 0 \times \cdot + \sum_{n=1}^{\infty} \frac{-2}{n} \sin nx$$

$$f(x) = \pi + \sum_{n=1}^{\infty} \frac{-2}{n} \sin nx$$

$$F(x) = \pi - \cancel{2}\left(\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \cdots\right)$$

$$f(x) = \pi - 2\left(\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \cdots\right)$$

Leibnitz's Rule for finding ~~equation~~ integration by part

$$\int uv\,dx = uv_1 + u'v_2 + u''v_3 - u'''v_4 + \cdots$$

Question

Find the fourier series of $f(x) = x^2$ ~~part~~ in $x \in (0, 2\pi)$ and hence deduce that

$\pi^2/12 = 1/1^2 + \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$

~~$f(x) = \frac{a_0}{2} + \sum_{n=1}$~~

here, $f(x)$ is periodic function of period 2π and $f(x)$ shatishfy all conditions of dirichlets conditions.

there the fourier series expansion of $f(x)$ will be

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$l = \pi$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} x^2\,dx$$

$$= \frac{1}{\pi} \times \frac{1}{3}(8\pi^3) = \frac{8\pi^2}{3}$$

$$a_0 = \frac{8\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} \cos nx \times x^2\,dx$$

$$a_n = \frac{1}{\pi}\left[x^2\frac{\sin nx}{n} - 2x\left(-\frac{\cos nx}{n^2}\right) + 2\left(-\frac{\sin nx}{n^3}\right)\right]_0^{2\pi}$$

$$a_n = \frac{1}{\pi}\left[0 + 4\pi\left(\frac{1}{n^2}\right) \pm (0)\right]$$

$$a_n = \frac{4}{n^2}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} \sin nx\; x^2\,dx$$

$$= \frac{1}{\pi}\left[-\frac{x^2\cos nx}{n} - \left(-\frac{2x\sin nx}{n^2}\right) + \left(\frac{+2\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[-\frac{4\pi^2}{n} + 2 \times 0\right]$$

$$= -\frac{4\pi}{n}$$

put values of a_0, a_n, b_n in eq^n (i)

$$\cancel{f(x) = \frac{8\pi^2}{6} + \sum_{n=1}^{\infty} \frac{4}{n^2} x^2 + \sum}$$

$$f(x) = \frac{8\pi^2}{6} + \sum_{n=1}^{\infty} \frac{4}{n^2} \cos nx + \sum_{n=1}^{\infty} \frac{-4\pi}{n} \sin nx.$$

$$f(x) = \frac{4\pi^2}{3} + \left(\frac{4}{1^2}\cos x + \frac{4}{2^2}\cos 2x + \frac{4}{3^2}\cos 3x + \cdots\right) + \left(-\frac{4\pi}{1}\sin x - \frac{4\pi}{2}\sin 2x - \frac{4\pi}{3}\sin 3x - \cdots\right)$$

to put $x = \pi$.

$$\cancel{\pi^2 = \frac{4\pi^2}{3} - \left(-4\right.}$$

$$\pi^2 = \frac{4\pi^2}{3} - 4\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots\right)$$

$$\frac{\pi^2}{12} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$

Ex - $f(x) = x\sin x \quad x \in (0, 2\pi)$

~~$a_0 = \frac{1}{}$~~

deduce that

$$\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{3}{4}$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} x\sin x \, dx$$

$$a_0 = \frac{1}{\pi}\left[-2\pi\right] = -2$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} \cos nx \; x\sin x \, dx$$

$$a_n = \frac{1}{\cancel{2}\pi} \int_0^{2\pi} x \sin x \cos nx \, dx$$

$$a_n = \frac{1}{2\pi} \int_0^{2\pi} x\left(\sin(n+1)x - \sin(n-1)x\right)dx$$

$$a_n = \frac{1}{2\pi}\left[-\frac{x\cos(n+1)x}{(n+1)} + \frac{\sin(n+1)x}{(n+1)^2}\right]_0^{2\pi}$$

$$+ \frac{1}{2\pi}\left[\frac{x\cos(n-1)x}{(n-1)} - \frac{\sin(n-1)x}{(n-1)^2}\right]_0^{2\pi}$$

$$a_n = \frac{1}{2\pi}\left[\frac{\sin(n+1)2\pi}{(n+1)^2} \rightarrow 0\right.$$

$$a_n = \frac{1}{2\pi}\left[-\frac{\pi}{n+1} - 0 + \frac{\pi}{(n-1)}\right.$$

$$a_n = \frac{1}{2\pi}\left[\frac{2\pi}{(n-1)} - \frac{2\pi}{(n+1)}\right]$$

$$a_n = \left[\frac{2}{(n-1)(n+1)}\right]$$

$$a_n = \frac{2}{n^2-1} \qquad \underline{\underline{n \neq 1}}$$

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} x \sin x \cos x \, dx$$

$$a_1 = \frac{1}{2\pi}\int_0^{2\pi} x \sin 2x \, dx$$

$$a_1 = \frac{1}{2\pi}\left[-\frac{x\cos 2x}{2} + \frac{\sin 2x}{4}\right]_0^{2\pi}$$

$$a_1 = \frac{1}{2\pi}\left[-\pi + 0\right]$$

$$a_1 = -\frac{1}{2}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x \sin x \sin nx \, dx$$

$$b_n = \frac{1}{2\pi}\int_0^{2\pi} x\left(\cos(n-1)x - \cos(n+1)x\right)dx$$

$$b_n = \frac{1}{2\pi}\left[\frac{x\sin(n-1)x}{n-1} + \frac{\cos(n-1)x}{(n-1)^2} - \frac{x\sin(n+1)x}{n+1} - \frac{\cos(n+1)x}{(n+1)^2}\right]_0^{2\pi}$$

$$\cancel{b_n = \frac{1}{2\pi}\left[\frac{1}{(n-1)^2} - \frac{1}{(n+1)^2}\right]}$$

$$b_n = 0$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$\cancel{x \sin x = -2 + a_1 + \sum_{n=2}^{\infty} \frac{2}{n^2-1}\sin nx}$$

$$\cancel{x \sin x = -2 + \frac{1}{2}} + \sum_{n=2}^{\infty} \frac{2}{n^2-1}\sin nx$$

$$x \sin x = \cancel{-\frac{5}{2} + \sum_{n=2}^{\infty}} \frac{2}{n^2-1}\sin nx$$

$$\boxed{x \sin x = -2 + \sum_{n=1}^{\infty} \frac{2}{n^2-1} \cos nx}$$

$$x \sin x = -1 + a_1 \cos x + \sum_{n=2}^{\infty} \frac{2}{n^2-1} \cos nx$$

$$+ \, b_1 \sin x + 0$$

$$x \sin x = -1 - \frac{\cos x}{2} + \sum_{n=2}^{\infty} \frac{2}{n^2-1} \cos nx$$

$$+ \, \pi \sin x$$

$\boxed{n\pi}$

$\boxed{x = \pi}$

$$0 = -2 + \frac{1}{2} + \sum_{n=2}^{\infty} \frac{2}{n^2-1} \times (+1)$$

$+0$

n is even

$$\frac{3}{2} = \sum_{n=2}^{\infty} \frac{2}{n^2-1}$$

$$\boxed{\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{3}{4}}$$

to put $x = 0$ in

$$0 = -\frac{3}{2} + \sum_{n=2}^{\infty} \frac{2}{n^2-1} + 0$$

$$\sum_{n=2}^{\infty} 1/n^2-1 = 3/4$$

ex- Find fourier series of

$$f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

hence deduce that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

f(x) is periodic function of period 2π. & f(x) satisfy all conditions of Dirichlet's condition.

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi \, dx + \int_{0}^{\pi} x \, dx\right]$$

$$a_0 = \frac{1}{\pi}\left[-\pi^2 + \frac{\pi^2}{2}\right]$$

$$a_0 = \frac{-\pi}{2}$$

$$\boxed{a_0 = \frac{-\pi}{2}}$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} \cos nx \cdot -\pi\, dx + \int_{0}^{\pi} \sin nx\, x\, dx\right]$$

$$a_n = \frac{1}{\pi}\left[-\pi^2 + \frac{-x \cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{\pi}$$

$$a_n = \frac{1}{\pi n^2}\left[(-1)^n - 1\right]$$

$$\hookrightarrow (\cos n\pi - 1)$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\cos nx\, \pi\, dx + \int_{0}^{\pi} \sin n\pi\, x\, dx\right]$$

$$b_n = \frac{1}{\pi}\left[-\left[\frac{\cos n\pi\, x}{n}\right]_0^{\pi} + \left[\frac{\sin n\pi}{n}\right]_0^{\pi}\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{-\pi \cos n\pi}{n} + \frac{\pi}{n}\right]$$

$$b_n = \frac{1}{n}\left(-2\cos n\pi + 1\right)$$

$$b_n = \frac{1}{n}\left(1 - 2(-1)^n\right)$$

$$f(x) = \frac{-\pi}{4} + \sum_{n=1}^{\infty} \frac{1}{\pi n^2}\left((-1)^n - 1\right)\cos nx$$

$$+ \sum_{n=1}^{\infty} \frac{1}{n}\left(1 - 2(-1)^n\right)\sin nx$$

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putting $x = 0$, we get

$$f(0) = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{1}{\pi n^2}\left((-1)^n - 1\right)$$

$$f(0) = \frac{1}{2}\left(f(0^-) + f(0^+)\right)$$

$$= \frac{1}{2}\left(-\pi + 0\right)$$

$$= -\frac{\pi}{2}$$

$$-\frac{\pi}{2} = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{1}{\pi n^2}\left((-1)^n - 1\right)$$

$$-\frac{\pi}{8}$$

$$-\frac{\pi}{4} = \frac{1}{\pi}(-2) + \frac{1}{\pi}(0) + \frac{1}{\pi}\left(\frac{-1}{3^2}\right)$$

$$-\frac{\pi^2}{4} = -2\pi$$

$$-\frac{\pi^2}{8} = -\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right)$$

$$\pi^2/8 = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2}$$

Find the Fourier series of

$$f(x) = \begin{cases} -x-\pi & , -\pi < x < 0 \\ x+\pi & , 0 < x < \pi \end{cases}$$

here $f(x)$ is a periodic function with the period of 2π & $f(x)$ satisfies all contition of Dritchlet's Condition

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{2l} +$$

$$\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{2l}$$

$l = \pi$

$$a_0 = \frac{1}{\pi} \times \int_{-\pi}^{0} (-x-\pi)\,dx + \frac{1}{\pi} \int_{0}^{\pi} (\pi + x)\,dx$$

$$a_0 = \frac{1}{\pi} \left[\frac{\pi^2}{2} \cancel{+} \pi^2 + \pi^2 + \frac{\pi^2}{2} \right]$$

$$\boxed{a_0 = \pi}$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} \cos\frac{n\pi x}{\pi}(-\pi-x)\,dx\right.$$

$$\left. + \int_{0}^{\pi} \cos\frac{n\pi x}{\pi}(\pi+x)\,dx\right]$$

$$a_n = \frac{1}{\pi}\left[\left[-\pi \times \frac{\sin nx}{n}\right]_{-\pi}^{0} + \left[\frac{\pi \sin nx}{n}\right]_{0}^{\pi}\right.$$

$$+ \left[\left[\frac{-x \sin nx}{n}\right]_{-\pi}^{0} - \left[\frac{\cos nx}{n^2}\right]_{-\pi}^{0}\right.$$

$$+ \left[\frac{x \sin nx}{n}\right]_{0}^{\pi} + \left[\frac{\cos nx}{n^2}\right]_{0}^{\pi}$$

$$\cancel{-\frac{1}{\pi}} \quad \frac{1}{\pi}\left[-\frac{\cos n\pi}{n^2} + \frac{1}{n^2} + \frac{\cos n\pi}{n^2}\right.$$

$$a_n = \frac{1}{\pi n^2}\left((-1)^n - 1\right)$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} \sin nx\,(-\pi - x)\,dx + \int_{0}^{\pi} \sin nx\,(\pi + x)\,dx\right]$$

$$b_n = -\frac{1}{\pi}\left[\frac{-\pi \cos nx}{n}\right]_{-\pi}^{0} + \left[\frac{\pi \cos nx}{n}\right]_{0}^{\pi}$$

$$-\frac{1}{\pi}\left[-\frac{\sin nx}{n^2}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[-\frac{\sin nx}{n^2}\right]_{0}^{\pi}$$

~~$b_n =$~~

$$\left[\frac{\cos nx}{n}\right]_{-\pi}^{0} - \left(\frac{\cos nx}{n}\right)_{0}^{\pi}$$

$$\frac{1}{n} - \frac{\cos(-n\pi)}{n} - \frac{\cos n\pi}{n} - \frac{1}{n}$$

$$\Rightarrow -\frac{2\cos n\pi}{n}$$

$$\Rightarrow -\frac{2}{n}(-1)^n$$

Find the fourier series of $f(x)$

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 < x < \pi \end{cases}$$

hence deduce that

$$\frac{1}{2} = \frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \cdots$$

$f(x)$ is a periodic function with period of 2π & it satisfies all conditions of Dirichlet's conditions.

hence fourier series of $f(x)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{\pi} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{\pi}$$

A

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} \sin x\,dx\right]$$

$$a_0 = \frac{1}{\pi} - [\cos x]_0^{\pi}$$

$$a_0 = \frac{2}{\pi}$$

$$a_0 = \frac{2}{\pi}$$

$$a_n = \frac{1}{\pi}\left[\int_0^{\pi} \cos nx \cdot \sin x \, dx\right]$$

$$a_n = \frac{1}{2\pi}\left[\int_0^{\pi}\left(\sin(n+1)x - \sin(n-1)x\right)dx\right].$$

$$a_n = \frac{1}{2\pi}\left[+\left[\frac{\cos(n-1)x}{(n-1)}\right]_0^{\pi} - \left[\frac{\cos(n+1)x}{(n+1)}\right]_0^{\pi}\right]$$

$$a_n = \frac{1}{2\pi}\left[\frac{\cos(n-1)\pi}{(n-1)} - \frac{1}{(n-1)} - \frac{\cos(n+1)\pi}{(n+1)} + \frac{1}{n+1}\right]$$

$$a_n = \frac{1}{2\pi}\left[\frac{1}{(n+1)} - \frac{1}{(n-1)} + \frac{\cos(n-1)\pi}{(n-1)} - \frac{\cos(n+1)\pi}{(n+1)}\right]$$

$$a_n = \frac{1}{2\pi}\left[\frac{1}{(n+1)} - \frac{1}{(n-1)} + \frac{(-1)^n}{n-1} - \frac{(-1)^n}{n+1}\right]$$

$$a_n = \frac{1}{2\pi}\left[\frac{1}{(n+1)}\left[1-(-1)^n\right] + \frac{1}{n-1}\left((-1)^n - 1\right)\right]$$

$$a_n = \frac{1}{2\pi}\left[\left(1-(-1)^n\right)\left[\frac{1}{(n+1)} - \frac{1}{n-1}\right]\right]$$

$$a_n = \frac{1}{2\pi}\left[1-(-1)^n\right]\left[\frac{2}{n^2-1}\right] \quad \{n \neq 1\}$$

$$b_n = \frac{1}{\pi}\left[\int_0^{\pi} \sin nx \, \sin x \, dx\right.$$

$$\cancel{b_n} = \frac{1}{2\pi}\left[\int_0^{\pi} \cos(n-1)x - \cos(n+1)x\right]$$

$$\cancel{b_n} = \frac{1}{2\pi}$$

$$b_n = \frac{1}{2\pi}\left[\left[\frac{\sin(n-1)x}{(n-1)}\right]_0^{\pi} - \left[\frac{\sin(n+1)x}{n+1}\right]_0^{\pi}\right]$$

$$b_n = \frac{1}{2\pi}\left[0\right]$$

$$\boxed{b_n = 0}$$

$$a_1 = \frac{1}{\pi}\int_0^{\pi} \cos x \, \sin x \, dx$$

$$a_1 = \frac{1}{2\pi}\left[\sin 2x\right]_0^{\pi}$$

$$\boxed{a_1 = 0}$$

$$(n+1)$$

$$\boxed{b_n = 0}$$

$$b_1 = \frac{1}{\pi}\left[\int_0^{\pi} \sin x \sin x\, dx\right] \quad \begin{cases} \cos x = 1 - 2\sin^2 x \\ \cos 2x = 1 - 2\sin^2 x \end{cases}$$

$$b_1 = \frac{1}{2\pi}\left[\int_0^{\pi} \sin^2 x\, dx\right.$$

$$b_1 = \frac{1}{2\pi}\left[\int_0^{\pi} (1 - 2\cos 2x)\, dx\right]$$

$$b_1 = \frac{1}{\pi}\left[\pi - \left(\sin 2x\right]_0^{\pi}\right)$$

$$\boxed{b_1 = \frac{1}{2}}$$

to put value of a_0; $a_1 b_1$, $a_n (n>1)$

$b_n (n>1)$

$$f(x) = \frac{1}{\pi} + \sum_{n=2}^{\infty} -\frac{1}{2\pi}(1+(-1)^n)\left(\frac{2}{n^2-1}\right)\cos nx$$

$$\cos nx + \sum_{n=2}^{\infty} \frac{1}{20} \times 0 + \frac{1}{2}\sin x$$

$$f(x) = \frac{1}{\pi} - \frac{2}{\pi}\left[\frac{2\cos 2x}{2^2-1} + \frac{2\cos 4x}{4^2-1} + \frac{2\cos 6x}{6^2-1} + \cdots\right]$$

$$= \frac{1}{\pi} - \frac{2}{\pi}\left[\frac{\cos 2x}{1.3} + \frac{\cos 4x}{3.5} + \frac{\cos 6x}{5.7} + \cdots\right]$$

$$+ \frac{1}{2}\sin x$$

$f(0) = 0$

$$0 - \frac{1}{\pi} = -\frac{2}{\pi}\left(\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots\right)$$

$$\left[\frac{1}{2} = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots\right] \text{Ans}$$

Find Fourier series of $f(x) = x + x^2$ $(-\pi, \pi)$

hence deduce that

(i) $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$

(ii) $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} (x + x^2)\, dx\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2} - \frac{\pi^2}{2} + \frac{\pi^3}{3} + \frac{\pi^3}{3}\right]$$

$$a_0 = \frac{2\pi^2}{3}$$

$$a_1 = \frac{1}{\pi} \times 2(2\pi) = 4$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} \cos nx \,(x+x^2)\, dx\right]$$

$$a_n = \frac{1}{\pi}\left[\left[\frac{x\sin nx}{n}\right]_{-\pi}^{\pi} + \left[\frac{\cos nx}{n^2}\right]_{-\pi}^{\pi}\right]$$

$$\cancel{a_n = \frac{1}{\pi}\Big[}$$

$$+\frac{1}{\pi}\left[\left[\frac{x^2\sin nx}{n}\right]_{-\pi}^{\pi} + 2\left[\frac{x\sin nx}{n^2}\right]_{-\pi}^{\pi} + 2\left[\frac{\cos nx}{n^3}\right]_{-\pi}^{\pi}\right]$$

$$\Rightarrow \frac{2}{\pi}\left(\frac{\cos n\pi}{n^3} - \frac{\cos n\pi}{n^3}\right) + \frac{1}{\pi}\ \frac{\cos n\pi}{n^2} - \frac{\cos n\pi}{n^2}$$

$$a_n = \frac{1}{\pi} \times 2 \int_0^{\pi} x^2 \cos nx \, dx$$

$$a_n = \frac{4}{n^2}(-1)^n$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} \sin nx \,(x+x^2)\, dx\right]$$

$$\cancel{b_n = \frac{1}{\pi}\left[\left[x\sin nx\right]_{-\pi}^{\pi} + \left[\frac{x^2 \sin nx}{}\right]_{-\pi}^{\pi}\right]}$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} x \sin nx \, dx + x^2 \sin nx \, dx\right]$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} (x \sin nx) \, dx\right]$$

$$b_n = \frac{1}{\pi}\left[2 \times \int_{0}^{\pi} x \sin nx \, dx\right] + \cancel{\int_{0}^{\pi} x \sin nx \, dx}$$

$$b_n = \frac{1}{\pi}\left[-\left[\frac{x \cos nx}{n}\right]_0^{\pi} + \left(\sin nx\right]_0^{\pi}\right]$$

$$b_n = \frac{2}{\pi}\left[\frac{-\pi(-1)^n}{n} - 0\right]$$

$$b_n = \frac{-2}{n}(-1)^n$$

to put value of a_0, a_n, b_n in eqn (1)

$$f(x) = \frac{2\pi^2}{2 \times 3} + \sum_{n=1}^{\infty} \frac{4}{n^2}(-1)^n \cos nx$$

$$+ \sum_{n=1}^{\infty} \frac{-2}{n}(-1)^n \sin nx$$

$$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2}(-1)^n \cos nx$$

$$+ \sum_{n=1}^{\infty} \frac{-2}{n}(-1)^n \sin nx$$

$$f(x) = \frac{\pi^2}{3} + \frac{4}{n^2}\left[\frac{-\cos x}{1^2} + \frac{\cos 2x}{2^2} - \frac{\cos 3x}{3^2}\right.$$

$$\left. + \cdots\right] + \frac{2}{n}\left[-\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3}\right.$$

$$\left. + \cdots\right]$$

$$f(x) = \frac{\pi^2}{3}$$

to put $x = 0$

$$f(0) = \frac{\pi^2}{3} + \frac{4}{n^2}\left[-\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \cdots\right]$$

$$-\frac{2}{n}(0 + \cdots 0)$$

$$f(0) = \frac{\pi^2}{3} + \frac{4}{n^2}\left(-\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \cdots\right)$$

$$\underline{f(0) = 0}$$

$$\frac{-\pi^2}{3} = -\frac{4}{\cancel{n^2}}\left(\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots\right)$$

$$\frac{\pi^2}{12} = \left(\underline{\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots}\right)$$

to put $x = \pi$

~~$f(\pi) = \pi + \pi^2$~~

$\cancel{\pi + \pi^2} \Rightarrow \frac{\pi^2}{3} + \frac{4}{1}\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right]$

$f(\pi) = \frac{1}{2}\left(f(\pi^-) + f(\pi^+)\right)$

$f(\pi) = \frac{1}{2}\left(-\pi + \pi^2 + \pi + \pi^2\right)$

$= \underline{\pi^2}$

$\pi^2 = \frac{\pi^2}{3} + 4\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right)$

$$\left[\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

Pr

Q:- find fourier series of $f(x) =$

$f(x) = x^2 \quad (0, 4)$ hence deduce that

$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$

here f(x) is a periodic function of period 4 & f(x) satisfied all the conditions of Dirichlet's condition. therefor fourier series expansion of f(x) is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{2} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{2}$$

$$a_0 = \frac{1}{2}\int_0^4 x^2\,dx$$

$$a_0 = \frac{1}{2} \times \frac{1}{3}[64] = \frac{32}{3}$$

$$a_n = \frac{1}{2}\left[\int_0^4 \cos\frac{n\pi x}{2}\, x^2\,dx\right.$$

$$= \frac{1}{2}\left[\int_0^4 \cos\frac{n\pi x}{2} \cdot x^2\,dx\right.$$

$$= \frac{1}{2}\left[\left[\frac{\sin n\pi x}{2n\pi} \cdot x^2\right]_0^4 - \frac{2}{2n\pi}\int_0^4 \sin n\pi x \cdot x\,dx\right.$$

$$= \frac{1}{2}\left[\frac{\sin 4n\pi}{2n\pi} \times 16\right] - \frac{1}{n\pi}\left[-\frac{x\cos n\pi x}{n} + \frac{\sin n\pi x}{n^2}\right]_0^4$$

$$= \frac{4\sin 4n\pi}{n\pi} + \frac{1}{n\pi}\left[\frac{+4\cos 4n\pi}{n} - \frac{1}{n} + \frac{\sin 4n\pi}{n^2}\right]$$

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$$a_n = \frac{16}{n^2\pi^2}$$

$$b_n = \frac{1}{2}\int_0^4 x^2 \sin\frac{n\pi x}{2}\, dx$$

$$= \frac{1}{2}\left[\left[-\frac{x^2 \cos n\pi x}{2n\pi}\right]_0^4 + \frac{1}{n\pi}\int \frac{x^2 \cos n\pi x}{\cancel{n\pi}}\, dx\right]$$

$$= \frac{1}{2}\left[\frac{-x^2 \cos n\pi x}{2n\pi}\right]_0^4$$

$$= \frac{1}{2}\left[-\frac{16}{2n\pi} \times \cos 4n\pi\right]$$

$$= \frac{-4}{n\pi} \cos 4n\pi \quad +$$

$$\Rightarrow \frac{1}{n\pi}\left[\frac{x \sin n\pi x}{n\pi} + \frac{\cos n\pi x}{n\pi}\right]_0^2$$

$$\Rightarrow \frac{1}{(n\pi)^2}\left[\underbrace{4 \sin 4n\pi}_{0} + \underbrace{\cos 4n\pi}_{1}\right]$$

$$\Rightarrow \frac{1}{(n\pi)^2}$$

$$= \frac{-8}{(n\pi)} + \frac{1}{2(n\pi)} \Rightarrow -\frac{16}{n\pi}$$

$$f(x) = \frac{16}{3} + \sum_{n=1}^{\infty} \frac{16}{n^2\pi^2} \cos\frac{n\pi x}{2} +$$

$$\sum_{n=1}^{\infty} \frac{-16}{n\pi} \sin\frac{n\pi x}{2}.$$

to put $x=0$, in ② we get

$$f(0) = \frac{16}{3} + \frac{16}{\pi^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)$$

~~$f(0) = 0$~~

$$f(0) = \frac{1}{2}(0 + 16)$$

$$= 8$$

~~$8 = \frac{16}{3}$~~

$$8 - \frac{16}{3} = \frac{16}{\pi^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right)$$

$$\frac{8}{3} \times \frac{\pi^2}{16} = \frac{1}{1^2} + \frac{1}{2^2} + \cdots$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$$

Q1. Find fourier series of

$f(x) = 2x - x^2 \quad (0, 3)$

deduce that $\quad l = \frac{3}{2}$

$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \text{———}$

$a_0 = \cancel{\frac{1}{2}} \frac{2}{3} \int_0^3 (2x - x^2)\, dx$

$\frac{27}{3}$

$a_0 = \frac{2}{3}[9 - 9] = 0$

$a_n = \frac{2}{3} \int_0^3 \cos \frac{n\pi x}{3/2} \times (2x - x^2)\, dx$

$a_n = \frac{2}{3} \int_0^3 \left[2x \cos \frac{2n\pi x}{3}\, dx - x^2 \cos \frac{2n\pi x}{3}\, dx \right]$

~~$a_n = \frac{2}{3} \left[2 \times \frac{3}{2n\pi} \left[\sin \frac{2n\pi x}{3} \right]_0^3 \right.$~~

$a_n = \frac{2}{3} \left[2x \sin \frac{2n\pi x}{3} \times \frac{3}{2n\pi} \right]_0^3$

$a_n = \frac{2}{3} \times \frac{3}{n\pi} \left(x \sin \frac{2n\pi x}{3} \right]_0^3$

$a_n = \frac{2}{n\pi} \left[\sin 2n\pi \right]$

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$$-\frac{2}{3}\left[\left[x^2 \sin\frac{2n\pi x}{3} \times \frac{3}{2n\pi}\right]_0^3 - \int_0^3 2x \times \frac{3}{2n\pi} \sin\frac{2n\pi x}{3}\right.$$

$$-\frac{2}{3}\left[\left(\frac{27}{2n\pi}\sin 2n\pi\right) + \left[x\cos\frac{2n\pi x}{3} \times \frac{9}{4n^2\pi^2}\right.\right.$$

$$\left.- \sin\frac{2n\pi x}{3} \times \frac{27}{8n^3\pi^3}\right]_0^3$$

$$-\frac{2}{3}\left[\frac{27}{2n\pi}\sin 2n\pi + 3\cos 2n\pi \times \frac{9}{4n^2\pi^2}\right.$$

$$\left.- \underbrace{\sin 2n\pi}_{\to 0} \times \frac{27}{8n^3\pi^3}\right]$$

$$-\frac{9}{n\pi}\sin 2n\pi - \frac{27}{n\pi}\cos 2n\pi$$

$$\frac{-27}{n\pi} = a_n$$

$$-\frac{9}{2n}$$

$$\frac{3}{}\quad \frac{-9}{2n^2\pi^2}$$

$$a_n = \frac{-9}{2n^2\pi^2}$$

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$$b_n = \frac{2}{3}\int_0^3 \sin\frac{2n\pi x}{3} \times (2x - x^2)\,dx$$

$$b_n = \frac{2}{3}\int_0^3 \left[2x \sin\frac{2n\pi x}{3}\,dx - x^2 \sin\frac{2n\pi x}{3}\,dx \right]$$

$$b_n = \frac{2}{3}\int_0^3 \left[-\frac{3x}{n\pi} \cos\frac{2n\pi x}{3} \right.$$

$$b_n = \frac{2}{3}\left[\left[\frac{-3x}{n\pi} \cos\frac{2n\pi x}{3}\right]_0^3 + 2\times\frac{9}{4n^2\pi^2}\left[\sin\frac{2n\pi x}{3}\right.\right.$$

$$= \frac{2}{3}\left[\frac{-9}{n\pi} + 0 + \frac{10}{4n^2\pi^2} \times 0 \right]$$

$$= -\frac{6}{n\pi}$$

$$\int_0^3 \cancel{x^2 \sin\frac{2n\pi x}{3}}\,dx$$

$$\cancel{x^2} \qquad -\frac{2}{3}$$

$$-\frac{2}{3}\left[-\left[x^2 \cos\frac{2n\pi x}{3}\right]_0^3 \times \frac{3}{2n\pi} + \frac{3}{n\pi}\left[x \cos\frac{2n\pi x}{3}\right]_0^3 \right.$$

$$b_n = \frac{3}{n\pi}$$

$$f(x) = 0 + \sum_{n=1}^{\infty} \frac{-9}{n^2\pi^2} \times \cos\frac{2n\pi x}{3}$$

$$+ \sum_{n=1}^{\infty} \frac{3}{n\pi} \sin\frac{2n\pi x}{3}$$

$$f(x) = \frac{-9}{\pi^2} \cos\left(\frac{2}{3}\pi x\right) - \frac{9}{2^2\pi^2} \cos\left(\frac{2}{3} 2\pi x\right)$$

$$+ \frac{3}{\pi} \sin\frac{2\pi x}{3} + \frac{3}{2\pi} \sin\frac{2}{3} \times 2\pi x \text{---}$$

$$f(0) = \frac{1}{2}(0 - 6 + 9)$$

$$= \frac{3}{2}$$

$$\frac{3}{2} = \frac{-9}{\pi^2}\left[-\frac{1}{1^2} - \frac{1}{2^2} - \frac{1}{3^2} - \cdots\right]$$

$$\frac{-\pi^2}{6} = -\frac{1}{1^2} - \frac{1}{2^2} - \frac{1}{3^2} - \cdots$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \cdots$$

Q:- Find the fourier series of

$4-x^2$ $(0,2)$ hence deduce that

$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \cdots$ $\boxed{l=1}$

$a_0 = \frac{1}{1}\int_0^2 (4-x^2)\,dx$

$= 8 - \frac{8}{3} = \frac{16}{3} = a_0$

$a_n = \int_0^2 (4-x^2)\cos\frac{n\pi x}{2}\,dx$

$= \left[\frac{8}{n\pi}\sin\frac{n\pi x}{2}\right]_0^2 - \left[\frac{2x^2}{n\pi}\sin\frac{n\pi x}{2}\right]_0^2$

$2\left(-\cos\frac{n\pi x}{2} \times \frac{1}{(n\pi)^2} + \frac{\sin n\pi x}{(n\pi)^3}\right)_0^2$

$\Rightarrow \frac{8}{n\pi} \times \sin n\pi - \frac{2}{n\pi} \times 4\sin n\pi$

$+ 2\left(\frac{-1}{(n\pi)^2}\cos\right.$

$a_n = \frac{-4}{n^2\pi^2}$

$$b_n = \int_0^2 (4-x^2) \sin\frac{n\pi x}{2}\, dx$$

$$b_n = -4 \times \frac{2}{n\pi}\left[\cos\frac{n\pi x}{2}\right]_0^2$$

$$\Rightarrow +\frac{8}{n\pi} - \frac{8}{n\pi}\cos n\pi$$

$$-\int_0^2 x^2 \sin\frac{n\pi x}{2}\, dx$$

$$-\left[-x^2 \cos\frac{n\pi x}{2}\right]$$

$$b_n = \frac{4}{n\pi}$$

$$f(x) = \frac{8}{3} + \sum_{n=1}^{\infty} \cos\frac{n\pi x}{2} \frac{-4}{n^2\pi^2} + \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin\frac{n\pi x}{2}$$

$$f(x) = \frac{8}{3} + \frac{4}{n^2\pi^2} \sum_{n=1}^{\infty} \cos\frac{n\pi x}{2} + \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin\frac{n\pi x}{2}$$

so put $\underline{\underline{x=0}}$

Q:- find the fourier se

$4-x^2$ (0, 2) h

$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} +$

$a_0 = \frac{1}{1}\int_0^2 (4$

$= 8$

$+ \frac{1}{3^2} + \text{---}$

$a_n =$

$$2 - \frac{8}{3} = -\frac{4}{\pi^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right)$$

$$-\frac{2}{3} = -\frac{4}{\pi^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right)$$

$$\frac{1}{6} = \frac{1}{\pi^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \cdots\right)$$

$$\boxed{\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots}$$

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(1) find a_0, a_n, b_n for Fourier

$$a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\,dx$$

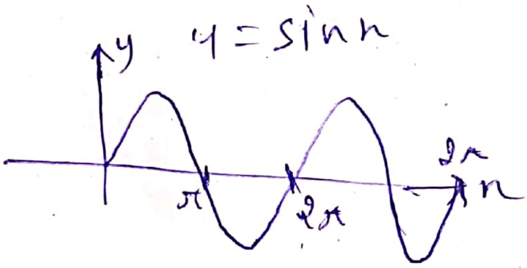
$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\sin\frac{n\pi x}{l}\,dx$$

(2) What is periodic function,

the function which varies such that ~~after a certain enterval~~ it is similar in a certain enterval. the fixed interval is called period of function,

ex -



its period is 2π.

Q:- Find a Fourier series expansion for the function $f(x) = x + x^2$

$-\pi < x < \pi$

hence deduce, $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$

$\cancel{a_0 = \frac{1}{\quad}}$ $\quad \underline{l = \pi}$

$\cancel{a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi}}$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} (x + x^2)\, dx$$

$$a_0 = \frac{1}{\pi} \left[\left[\frac{x^2}{2}\right]_{-\pi}^{\pi} + \left[\frac{x^3}{3}\right]_{-\pi}^{\pi} \right]$$

$$a_0 = \frac{1}{\pi} \left[0 + \frac{2\pi^3}{3} \right]$$

$$\boxed{a_0 = \frac{2\pi^2}{3}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{\cos n\pi x}{\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} \cos nx \,(x+x^2)\, dx\right]$$

$$\cancel{a_n = \frac{1}{\pi}\left[\left[\frac{x \sin nx}{n}\right]_{-\pi}^{\pi} + \left[\frac{\cos nx}{n^2}\right]_{-\pi}^{\pi}\right.}$$

$$\cancel{+\left[\frac{x^2 \sin nx}{n}\right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{2x \sin nx}{n}\, dx}$$

$$\cancel{a_n = \frac{1}{\pi} \times \frac{1}{n^2}\left(\cos n\pi - \cos -n\pi\right)}$$

$$a_n = \frac{1}{\pi}\left[\left[\frac{x \sin nx}{n}\right]_{-\pi}^{0} + \left[\frac{x \sin nx}{n}\right]_{0}^{\pi}\right.$$

$$+\left[\frac{\cos nx}{n^2}\right]_{-\pi}^{0} + \left[\frac{\cos nx}{n^2}\right]_{0}^{\pi}$$

$$-\left(\left[-\frac{2x}{n^2}\cos nx + \frac{2}{n^3}\sin nx\right]_{-\pi}^{\pi}\right)$$

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$$a_n = 0 + 0\left(1 - \frac{\cos n\pi}{n^2}\right) + \left(\frac{\cos n\pi}{n^2} - 1\right)$$

$$- \left(0 + \frac{-2\pi}{n^2}\cos n\pi + \frac{-2\pi}{n^2}\cos n\pi - 0\right]$$

$$\frac{4\pi}{n^2}\cos n\pi$$

$$a_n = \frac{4\pi}{n^2}\cos n\pi \times \frac{1}{\pi}$$

$$a_n = \frac{4}{n^2}\cos n\pi$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin \frac{n\pi x}{\pi} \times (x + x^2)\,dx$$

$$\cancel{b_n = \frac{1}{\pi}\left[x \sin x\right.}$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} \sin nx\,(x+x^2)\,dx + \int_{0}^{\pi} \sin nx\,(x+x^2)\,dx\right.$$

$$b_n = \frac{1}{\pi}\left[\left[-\frac{x\cos nx}{n}\right]_{-\pi}^{0} + \left[-\frac{x\cos nx}{n}\right]_{0}^{\pi} + \right.$$

$$\left[\frac{2x\cos nx}{n}\right]_{-\pi}^{0} + \left[-\frac{2x\cos nx}{n}\right]_{0}^{\pi}$$

$$+ \left(\frac{2x\sin nx}{n^2}\right]_{-\pi}^{\pi} + \frac{2}{n^3}\left[\cos nx\right]_{-\pi}^{0} + \frac{2}{n^3}\left[\cos nx\right]_{0}^{\pi}$$

~~$b_n = \frac{1}{\pi}\left[\frac{\pi}{n} + \frac{\pi}{n} + \right.$~~

~~$b_n = \frac{1}{\pi}\left[\frac{\pi \cos n\pi}{n}\right.$~~

$$b_n = \frac{1}{\pi}\left[\frac{-\pi\cos\pi n}{n} - \frac{\pi\cos\pi n}{n}\right.$$

$$+\left(-\frac{2\pi}{n}\cos n\pi - \frac{2\pi}{n}\cos n\pi\right)$$

$$\left. + \left(-\frac{2}{n^3}\cos n\pi + \frac{2}{n^3}\cos n\pi\right)\right]$$

$$b_n = -\frac{6\pi}{n}\cos n\pi \times \frac{1}{\pi}$$

$$= -\frac{6}{n}\cos n\pi$$

$$f(n) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi n}{l} +$$

$$\sum_{n=1}^{\infty} b_n \sin\frac{n\pi n}{l}$$

$$f(n) = \frac{2\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} \times \cos^2 n\pi$$

$$+ \sum_{n=1}^{\infty} -\frac{6}{n}\cos n\pi \sin n\pi .$$

$f(x) = x + x^2$

~~to put x = 0~~

$\cancel{-\frac{2\pi^2}{3} = \sum_{n=1}^{\infty} \frac{4}{n^2}}$

$\cancel{-\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}}$

to put $x = \pi$

$$\frac{f(\pi) + f(-\pi)}{2}$$

$\pi + \pi^2 + (-\pi + \pi^2)$

2π

$\pi + \pi^2 + \pi$

$$\frac{2\pi^2}{2} = \pi^2$$

$$\pi^2 = \frac{2\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} \times (+1)$$

$$\frac{\pi^2}{3} = \frac{2\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2}$$

$$\frac{\pi^2}{3} = 4 \sum_{n=1}^{\infty} \frac{1}{n^2}$$

Q- find fourier series of

$$f(x) = \begin{cases} 4-x & , 3 < x < 4 \\ x-4 & , 4 < x < 5 \end{cases}$$

here $f(x)$ is a periodic function of period $= 2$ and $f(x)$ satisfies all conditions of Dirichlet's condition therefore fourier series of $f(x)$ will be -

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \frac{\sin n\pi x}{l}$$

$$\boxed{l = 1}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x + \sum_{n=1}^{\infty} b_n \sin n\pi x$$

~~$a_0 = \sum_3$~~

$$a_0 = \int_3^4 (4-x)\,dx + \int_4^5 (x-4)\,dx$$

$$a_0 = 4 - \left(\frac{16}{2} - \frac{9}{2}\right) + \frac{25}{2} - \frac{16}{2} - 4$$

$$= \frac{25}{2} + \frac{9}{2} - 16$$

$$= \frac{34}{2} - 16 = \underline{1}$$

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$$a_n = \int_3^4 (4-x)\cos n\pi x \, dx$$

$$+ \int_4^5 (x-4)\cos n\pi x \, dx$$

$$= 4\left[\frac{\sin n\pi x}{n\pi}\right]_3^4 - \left[\frac{x\sin n\pi x}{n\pi}\right]_3^4 + \left[\frac{\cos n\pi x}{(n\pi)^2}\right]_3^4$$

$$+ \left[\frac{x\sin n\pi x}{n\pi}\right]_4^5 + \left[\frac{\cos n\pi x}{(n\pi)^2}\right]_4^5 - 4\left[\frac{\sin n\pi x}{n\pi}\right]_4^5$$

$$= 4\left[\frac{\sin 4n\pi - \sin 3n\pi}{n\pi} - \frac{\sin 5n\pi}{n\pi} + \frac{\sin 4n\pi}{n\pi}\right]$$

$$+ 5\times\frac{\sin 5n\pi}{n\pi} - 4\times\frac{\sin 4n\pi}{n\pi} - \frac{4\sin 4n\pi}{n\pi}$$

$$+ \frac{3\sin 3n\pi}{n\pi} +$$

$$\underline{a_n = -1}$$

$$b_n = \int_3^4 (4-x)\sin n\pi x \, dx + \int_4^5 (x-4)\sin n\pi x \, dx$$

$$= -\left[\frac{4\cos n\pi x}{n\pi}\right]_3^4 + \left[\frac{x\cos n\pi x}{n\pi}\right]_3^4 - \left[\frac{\sin n\pi x}{n\pi}\right]_3^4$$

$$+ \left[\frac{\sin n\pi x}{n\pi}\right]_4^5 - \left[\frac{x\cos n\pi x}{n\pi}\right]_4^5 + 4\times\left[\frac{\cos n\pi x}{n\pi}\right]_4^5$$

$$\Rightarrow \frac{4\cos 3n\pi}{n\pi} - \cancel{\left(\frac{4\cos 4n\pi}{n\pi}\right)}$$

$$+ \left(\cancel{\frac{4\cos 4n\pi}{n\pi}} - \frac{3\cos 3n\pi}{n\pi}\right)$$

$$- \left(\frac{\sin 4n\pi}{n\pi} - \frac{\sin 3n\pi}{n\pi}\right)$$

$$+ \frac{\sin 5n\pi}{n\pi} - \frac{\sin 4n\pi}{n\pi} - \left(\frac{5\cos 5n\pi}{n\pi}\right)$$

$$+ \cancel{\frac{4\cos 4n\pi}{n\pi}} + \frac{4\cos 5n\pi}{n\pi} - \cancel{\frac{4\cos 4n\pi}{n\pi}}$$

Fourier series expansion for odd & even function -

$f(x) \longrightarrow$ odd

$$a_0 = \int_{-\pi}^{\pi} \underbrace{f(x)}_{odd} \, dx = 0$$

$$a_n = \int_{-\pi}^{\pi} \underbrace{f(x)}_{odd} \underbrace{\cos nx}_{even} \, dx = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{f(x)}_{odd} \underbrace{\sin nx}_{odd} \, dx \Rightarrow$$

$$= \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx \, dx$$

* then

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx \, dx$$

$f(x) = \text{even}$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{f(x)\cos n\pi x}_{\text{even}}\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \underbrace{f(x)}_{\text{even}}\ \underbrace{\cos nx}_{\text{even}}\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{f(x)}_{\text{even}}\ \underbrace{\sin nx}_{\text{odd}}\,dx = 0$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx.$$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx = \frac{2}{l}\int_0^{l} f(x)\,dx.$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\int_0^{l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$b_n = 0$$

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find fourier series of $f(x) = x^3 \ (-\pi, \pi)$

$F(x)$ is periodic function of period $= 2\pi$ & $f(x)$ satisfy all condition of diritchlet's conditions. ~~Therefore fourier series~~

here $f(x)$ is odd function

$a_0 = a_n = 0$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{1}{\pi} \times 2 \int_0^{\pi} x^3 \sin nx \, dx$$

$$= \frac{2}{\pi} \left[-\frac{x^3 \cos nx}{n} + 3x^2 \frac{\sin nx}{n^2} + 6x \frac{\cos nx}{n^3} - 6 \frac{\sin nx}{n^4} \right]_0^{\pi}$$

$$b_n = \frac{2}{\pi} \left[\frac{-\pi^3 \cos n\pi}{n} + \frac{6\pi \cos n\pi}{n^3} \right]$$

$$= 2 \left(\frac{-\pi^2}{n} \cos n\pi + 6 \frac{\cos n\pi}{n^3} \right)$$

$$= 2 \left(\frac{-\pi^2}{n} (-1)^n + \frac{6}{n^3} (-1)^n \right)$$

$$f(x) = \sum_{n=1}^{\infty} 2\left(\frac{-\pi^2}{n}(-1)^n + \frac{6}{n^3}(-1)^n\right) \sin nx$$

Let $f(x) = x^2$

x^2 is an even function.

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x^2\,dx$$

$$= \frac{2}{3\pi} \times \pi^3 = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \cos\frac{n\pi x}{\pi}\, x^2\,dx$$

$$a_n = \frac{2}{\pi}\left(\left[x^2\frac{\sin nx}{n}\right]_0^{\pi} - 2\int_0^{\pi}\frac{x\sin nx}{n}\,dx\right)$$

$$a_n = \frac{2}{\pi}\left(\frac{2}{n^2}x\cos nx + \frac{2\sin nx}{n^3}\right)_0^{\pi}$$

$$a_n = \frac{2}{\pi}\left(\frac{2}{n^2}\right)\pi(-1)^n$$

$$= \frac{4}{n^2}(-1)^n$$

$$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4}{n^2}(-1)^n \cos nx$$

ex- Find fourier series of $|\cos x|$

$x \in (-\pi, \pi)$

$f(x)$ is periodic function of period $= 2\pi$, and $f(x)$ satishfied all conditions of dirichlet's conditions.

$f(x)$ is an even function $x \in (-\pi, \pi)$

there fore Fourier expansion will be

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$f(x) = \begin{cases} \cos x & 0 < x < \frac{\pi}{2} \\ -\cos x & \frac{\pi}{2} < x < \pi \end{cases}$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\, dx$$

$$= \frac{2}{\pi} \left[\int_0^{\frac{\pi}{2}} \cos x\, dx + \int_{\pi/2}^{\pi} -\cos x\, dx \right]$$

$$= \frac{2}{\pi} \left[[\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi} \right]$$

$$= \frac{2}{\pi} [1 + 1] = \frac{4}{\pi}$$

$$a_n = \frac{2}{\pi}\left[\int_0^{\pi/2} \cos x \cos nx \, dx \right.$$

$$\left. + \int_{\pi/2}^{\pi} -\cos x \cos nx \, dx \right]$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2} (\cos(n+1)x - \cos(n-1)x)\, dx + \int_{\pi/2}^{\pi} \right.$$

$$\left. (\cos(n-1)x - \cos(n+1)x)\, dx \right]$$

$$= \frac{2}{\pi}\left[\left[\frac{\sin(n+1)x}{n+1}\right]_0^{\pi/2} - \left[\frac{\sin(n-1)}{(n-1)}\right]_0^{\pi/2} \right.$$

$$\left. + \left[\frac{\sin(n-1)x}{n-1}\right]_{\pi/2}^{\pi} - \left[\frac{\sin(n+1)x}{n+1}\right]_{\pi/2}^{\pi} \right.$$

$$\cancel{\Rightarrow \frac{2}{\pi}\left[\cos n+1)\right.}$$

$$\Rightarrow \frac{2}{\pi}\left[\frac{\cos\frac{n\pi}{2}}{n+1} + \frac{\cos\frac{n\pi}{2}}{(n-1)} + \frac{\cos\frac{n\pi}{2}}{n-1} + \frac{\cos\frac{n\pi}{2}}{n+1}\right]$$

$$= \frac{-2}{\pi} \times \cancel{2} \cos\frac{n\pi}{2} \times \frac{1}{(n^2-1)}$$

$$= \frac{-4}{\pi(n^2-1)} \cos\frac{n\pi}{2} \qquad (n \neq 1)$$

$$\underline{a_1 = 0}$$

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$$\cancel{f(x) = \frac{2}{\pi} + \sum_{n=1}^{\infty}}$$

$$a_1 = \frac{2}{\pi}\left[\int_0^{\pi/2} \cos x \cos x \, dx + \int_{\pi/2}^{\pi} -\cos x \cos x \, dx\right]$$

$$a_1 = \frac{2}{\pi}\left[\int_0^{\pi/2} \cos^2 x \, dx - \int_{\pi/2}^{\pi} \cos^2 x \, dx\right]$$

$$a_1 = \frac{2}{\pi}\left[\int_0^{\pi/2}\left(\frac{\cos 2x + 1}{2}\right) dx - \int_{\pi/2}^{\pi} \frac{\cos 2x + 1}{2} \, dx\right]$$

$$a_1 = \frac{2}{\pi}\left[\frac{\pi}{4} - \frac{\pi}{4} + \left(\frac{\sin 2x}{2}\right)_0^{\pi/2}\right]$$

$$a_1 = 0$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$f(x) = \frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{-2}{\pi(n^2-1)} \cos \frac{n\pi}{2} \cos nx.$$

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Find fourier series of

$f(x) = x|x| \quad -1 < x < 1$

$f(x)$ is periodic function with period of 2 and $f(x)$ satisfy all conditions of dirichlet's conditions.

therefore $f(x)$ is an odd function in $-1 < x < 1$

therefore fourier series expansion of

$$f(x) = \sum_{n=1}^{\infty} b_n \sin n\pi x \quad \text{—— (1)}$$

$$f(x) = \begin{cases} -x^2, & -1 < x < 0 \\ x^2, & 0 < x < 1 \end{cases}$$

$$b_n = \frac{2}{1} \int_0^1 f(x) \sin n\pi x \, dx$$

$$= 2 \int_0^1 x^2 \sin n\pi x \, dx$$

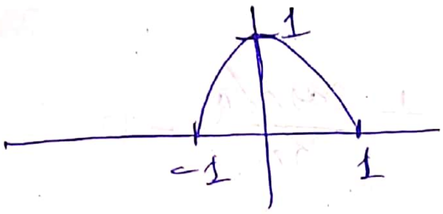
$$= 2 \left[x^2 - \frac{\sin n\pi x}{n\pi} - 2x \left(-\frac{\sin n\pi x}{\pi^2 n^2} \right) + 2 \cdot \frac{\cos n\pi x}{n^3\pi^3} \right]$$

$$= 2\left[-\frac{1}{n\pi}\cos n\pi + \frac{2}{n^3\pi^3}(\cos n\pi - 1)\right]$$

$$= 2\left[\frac{-1}{n\pi}(-1)^n - \frac{2}{n^3\pi^2} + \frac{2}{n^3\pi^3}(-1)^n\right]$$

find fourier series of $f(x) = 1 - x^2$

$-1 < x < 1$



$f(x)$ is an even function. a

~~$a_0 = 0$~~

$f(x)$ is periodic function with period $= 2$ and $f(x)$ satisfy all conditions of dirichlet's conditions

$a_0 = a_n = 0$

$f(x) = \begin{cases} 1 - x^2 & x < \ldots \end{cases}$

~~$b_n = \int_0^1$~~

$$b_n = \frac{2}{1}\int_0^1 (1-x^2)\sin n\pi x\,dx$$

$$b_n = 2\left[\left[-\frac{\cos n\pi x}{n\pi}\right]_0^1 + \left[\frac{x^2\cos n\pi x}{n\pi}\right]_0^1\right.$$

$$-\int 2x\frac{\cos n\pi x}{n\pi}$$

$$\Downarrow$$

$$-2\frac{x\sin n\pi x}{(n\pi)^2} + 2\int_0^1 \frac{\sin n\pi x}{(n\pi)^2}$$

$$b_n = 2\left[-\frac{\cos n\pi}{\cancel{n\pi}} + \frac{1}{n\pi} + \frac{\cos n\pi}{\cancel{n\pi}} - 0\right.$$

$$-\frac{2\sin n\pi}{(n\pi)^2} - 2\left(\frac{\cos n\pi x}{(n\pi)^3}\right)_0^1$$

~~$b_n = 2\left[-\frac{\cos n\pi}{n\pi} + \frac{1}{n\pi} + \cos\right.$~~

$$b_n = 2\left[\frac{1}{n\pi} + \frac{2}{(n\pi)^3} - \frac{2\sin n\pi}{(n\pi)^2} \to 0\right.$$

$$\left.-\frac{2\cos n\pi}{(n\pi)^3}\right]$$

$$= 2\left[\frac{1}{n\pi} + \frac{2}{(n\pi)^3} - \frac{2\cdot(-1)^n}{(n\pi)^3}\right]$$

$$a_n = \frac{4}{n^2\pi^2}(-1)^n$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin n\pi x$$

~~Half wave~~

Half ~~kalf~~ fourier series

any arbitary function ~~h(f)~~ ~~which~~ f(n)

with period '2l' which is difined on ~~half~~ half at interval $0<n<l$ can also be represented in terms of sine and cosine functions.

A half kalf expansion containing only cosine term is known as cosine series.

containing only sine term

to represent any function f(n) in the half ralf fourier series $0<n<l$ we extend the function by reflecting it in vertical axis (y-axis) so that $f(-n) = f(n)$

the extended function is even function. $-l<n<l$ and is periodic with period $2l$. the half ralf cosine series of f(n) is given by -

~~$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$~~

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$$

$$a_0 = \frac{2}{l} \int_0^l f(x)\, dx$$

$$a_n = \frac{2}{l} \int_0^l f(x) \cos \frac{n\pi x}{l}\, dx$$

if any function period $= 2\pi$ is defined in interval $0 < x < \pi$ in the ~~half~~ half cosine series;

half half cosine series is given by -

$$f(x) = \sum_{n=1}^{\infty} a_n \cos nx$$

where,

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\, dx$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\, dx$$

Similarly to represent any function $f(x)$ in half half sine series in interval $0 < x < l$, we extend the function by reflecting it in origin so that

$$f(-x) = -f(x)$$

the extended function is odd function with open interval $-l < x < l$. and is periodic with period $2l$. the half ralf sine series of such function is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

$$b_n = \frac{2}{\pi} \int_0^l f(x) \sin \frac{n\pi x}{l} \, dx$$

if any function with period 2π is define in interval $0 < x < \pi$, half ralf sine series of such a function is given by

$$\cancel{f(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} a_n}$$

$$\cancel{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n}$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx.$$

~~sin nx =~~

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

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Find half ralf cosine series of

$f(x) = x \quad (0, \pi)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \quad \text{--- (1)}$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} x \, dx = \pi$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} x \cos nx \, dx =$$

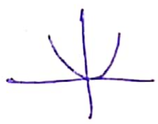
$$= \frac{2}{\pi} \left[\pi \times 0 + \left(\frac{\cos nx}{n^2} \right)_0^{\pi} \right]$$

$$= \frac{2}{\pi} \left(\frac{(-1)^n - 1}{n^2} \right)$$

$$= \frac{2}{\pi n^2} \left((-1)^n - 1 \right)$$

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2} \left((-1)^n - 1 \right)$$

$f(x) = x^2$, $\boxed{-\pi < x < \pi}$



<u>Odd</u>

$$b_n = \frac{2}{\pi} \int_0^{\pi} x^2 \sin \frac{n\pi x}{\cancel{\pi}} \, dx$$

$$b_n = \frac{2}{\pi} \left[-x^2 \frac{\cos n\pi x}{n} + \int 2x \frac{\cos n\pi x}{n} \right.$$

$$\left. + 2x \frac{\sin n\pi x}{n^2} + \frac{\cos n\pi}{n^3} \right]$$

$$= \frac{2}{\pi}\left[-\frac{(-1)^n}{n^3} + \frac{(-1)^n}{n^3} + \frac{1}{n^3}\right]$$

$$= \frac{2}{\pi}\left[\right.$$

$$\frac{2}{\pi}\left(\frac{-\pi^2}{n}(-1)^n + \frac{2}{n^3}\left((-1)^n - 1\right)\right)$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{\pi}\left(\frac{-\pi^2}{n}(-1)^n + \frac{2}{n^3}\left((-1)^n - 1\right)\right)$$

$\sin nx$.

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Find the half range sine series of

$f(x) = lx - x^2 \quad (0, l)$

Let the half range sine series of $f(x)$ will be

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \quad \text{--- (1)}$$

and hence deduce that

$$\frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$$

$$b_n = \frac{2}{l} \int_0^l (lx - x^2) \sin \frac{n\pi x}{l} \, dx$$

$$= \frac{2}{l} \left[(lx - x^2) \left(-\cos \frac{n\pi x}{l} \right) \frac{l}{n\pi} - (l - 2x) \left(-\sin \frac{n\pi x}{l} \right) \left(\frac{l}{n\pi} \right)^2 + (-2) \cos \frac{n\pi x}{l} \left(\frac{l}{n\pi} \right)^3 \right]_0^l$$

$$= \frac{2}{l} \left[0 - (-l - l) \left(-\sin n\pi \right) \times \left(\frac{1}{n} \right)^2 \times + (-2) \left(\cos n\pi - 1 \right) \frac{l^2}{n^3} \right]$$

$$= \frac{4 l^2}{n^3 \pi^3} \left(1 - (-1)^n \right)$$

$$f(x) = \sum_{n=1}^{\infty} \frac{4l^2}{n^3\pi^3}\left(1-(-1)^n\right) \sin\frac{n\pi x}{l}$$

$$f(x) = \frac{8l^2}{\pi^3}\left[\frac{\sin\frac{\pi x}{l}}{(1)^3} + \frac{\sin\frac{3\pi x}{l}}{(3)^3} + \frac{\sin\frac{5\pi x}{l}}{(5)^3} + \cdots \right]$$

to put $x = \frac{l}{2}$

$$\frac{l^2}{2} - \frac{l^2}{4} = \frac{8l^2}{\pi^3}\left[\frac{1}{(1)^3} - \frac{1}{(3)^3} + \frac{1}{(5)^3} - \cdots\right]$$

$$\frac{l^2}{4 \times 8 l^2} \times \pi^3 = \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \cdots$$

$$\frac{1}{32}\pi^3 = \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \cdots$$

Proves

Find half range cosine series for

$$f(x) = -\frac{x}{l} + 1 \qquad 0 \le x \le l$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$$

$$a_0 = \frac{2}{l}\int_0^l \left(-\frac{x}{l} + 1\right) dx$$

$$a_0 = \frac{2}{l}\left[\frac{-x^2}{2l} + x\right]_0^l$$

$$a_0 = \frac{2}{l}\left[-\frac{1}{2} + 1\right]$$

$$\underline{a_0 = 1}$$

$$a_n = \frac{2}{l}\int_0^l \cos\frac{n\pi x}{l}\left(\frac{-x}{l}+1\right)dx$$

$$a_n = \frac{2}{l}\left[\left(\frac{-x}{l}+1\right)\left(\sin\frac{n\pi x}{l}\right)\left(\frac{l}{n\pi}\right) - \left[\frac{-1}{l}\times -\cos\frac{n\pi x}{l}\times\frac{l^2}{(n\pi)^2}\right]\right]_0^l$$

$$a_n = \frac{2}{l}\left[0 - \left[\frac{l^2}{(n\pi)^2}\left((-1)^n - 1\right)\right]\right]$$

$$a_n = \frac{2l}{(n\pi)^2}\left(1-(-1)^n\right)$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{2l}{(n\pi)^2}\left(1-(-1)^n\right)\cos\frac{n\pi x}{l}$$

Q:- Find half range sine series of

$$f(x) = \begin{cases} \frac{1}{4} - x, & 0<x<\frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2}<x<1 \end{cases}$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}.$$

$\underline{l = 1}$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin n\pi x$$

$$b_n = \frac{2}{1}\left[\int_0^{\frac{1}{2}}\left(\frac{1}{4}-x\right)\sin n\pi x\,dx + \int_{1/2}^{1}\left(x-\frac{3}{4}\right)\sin n\pi x\,dx\right]$$

$$b_n = 2\left[\left(\frac{1}{4}-x\right)\left(-\cos n\pi x \times \frac{1}{n\pi}\right) - (-1)\left(-\sin n\pi x \times \left(\frac{1}{n\pi}\right)^2\right)\right]_0^{1/2}$$

$$+2\left[\left(x-\frac{3}{4}\right)\left(-\cos n\pi x \times \frac{1}{n\pi}\right) - \left(-\frac{\sin n\pi x}{(n\pi)^2}\right)\right]_{1/2}^{1}$$

$$b_n = 2\left[\left(-\frac{1}{4}\right)\left(-\cos\frac{n\pi}{2} \times \frac{1}{n\pi}\right) - \left(\frac{1}{4}\right)\left(-\frac{1}{n\pi}\right) - \left(\sin\frac{n\pi}{2}\left(1/n\pi\right)^2 - 0\right)\right]$$

$$+2\left[\frac{-3}{4}\left(-\cos\frac{n\pi}{2} \times \frac{1}{n\pi}\right) + 0 - \sin\frac{n\pi}{2}\left(\frac{1}{n\pi}\right)^2\right]$$

$$\frac{1}{8} + \frac{1}{n\pi} \qquad 0$$

$$\cancel{b_n = 2 \times \frac{1}{4(n\pi)^2} + 2 (\quad s}$$

$$b_n = \underline{2 \times \frac{1}{4} \times \frac{1}{n\pi}} - 2 \times \frac{1}{(n\pi)^2} \sin\frac{n\pi}{2}$$

$$\left. - \underline{\frac{3 \times 2}{2} \times \frac{1}{n\pi}} - 2 \times \frac{1}{(n\pi)^2} \sin\frac{n\pi}{2} \right]$$

$$b_n = -\frac{2}{n\pi} - \frac{4}{(n\pi)^2} \sin\left(\frac{n\pi}{2}\right) \quad (-1)^{n+1}$$

$$b_n = 2\left(\frac{1}{4n\pi} (1-(-1)^n) - \frac{2 \sin\frac{n\pi}{2}}{n^2\pi^2} \right)$$

$$f(x) = \sum_{n=1}^{\infty} 2 \left\{ \frac{1}{4n\pi} (1-(-1)^n) - \frac{2 \sin\frac{n\pi}{2}}{n^2\pi^2} \right\}$$

Find half range cosine serie for

$$f(x) = \begin{cases} 2x, & 0<x<1 \\ 2(2-x), & 1<x\,2 \end{cases} \quad l=2$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{2}$$

$$a_0 = \frac{2}{2} \left\{ \int_0^1 2x\,dx + \int_1^2 (4-2x)\,dx \right.$$

$$= [x^2]_0^1 + [4x - x^2]_1^2$$

$$= 1 + 4 - 4 + 1$$

$$a_0 = \underline{\underline{2}}$$

$$a_n = \int_0^1 2x \cos\frac{n\pi x}{2}\,dx + \int_1^2 (4-2x)\cos\frac{n\pi x}{2}\,dx$$

$$a_n = \left[2x \sin\frac{n\pi x}{2} \times \frac{n\pi}{2} + 2\cos\frac{n\pi x}{2} \times \left(\frac{n\pi}{2}\right)^2\right]_0^1$$

$$+ \left[(4-2x)\sin\frac{n\pi x}{2} \times \frac{n\pi}{2} - 2\cos\frac{n\pi x}{2}\left(\frac{n\pi}{2}\right)^2\right]_1^2$$

$$a_n = 2\sin\frac{n\pi}{2} \times \frac{n\pi}{2} - 2\left(\frac{n\pi}{2}\right)^2 + \left(-2\frac{n\pi}{2}\sin\frac{n\pi}{2}\right)$$

$$-2\left(\cos n\pi\left(\frac{n\pi}{2}\right)^2 - 0\right)$$

$$a_n = n\pi \sin\frac{n\pi}{2} - \frac{(n\pi)^2}{2} - n\pi\sin\frac{n\pi}{2}$$

$$- \frac{(n\pi)^2}{2}\cos n\pi$$

$$a_n = -\frac{(n\pi)^2}{2}\left(1+\cos n\pi\right)$$

$$a_n = -\frac{(n\pi)^2}{2}\left(1+(-1)^n\right)$$

$$a_n = \frac{16}{(n\pi)^2}\cos\left(\frac{n\pi}{2}\right). \quad \frac{-8}{n^2\pi^2}\left(1+(-1)^n\right)$$

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(Cont $\frac{\pi}{2}$)

Q! (a) find fourier series of $f(x) = |x|$ if $(-\pi, \pi)$

(b) $f(x) = \begin{cases} -\left(\frac{\pi + x}{2}\right), & -\pi < x < 0 \\ \frac{\pi - x}{2}, & 0 < x < \pi \end{cases}$

(c) $f(x) = |\sin x|$ in $(-\pi, \pi)$

(d) $f(x) = |\cos x|$ in $(-\pi, \pi)$

(e) $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 < x < \pi \end{cases}$

(f) $f(x) = x\sin x$ in $(-\pi, x)$ hence deduce that

$$\frac{\pi - 2}{4} = \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \cdots$$

(g) $f(x) = x^2$ in $(-\pi, \pi)$ hence deduce that

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}$$

Change of interval

(a) $f(x) = \begin{cases} 2, & -2 < x < 0 \\ x, & 0 < x < 2 \end{cases}$

(b) $f(x) = e^{-x}$ in $(-l, l)$

(c) $f(x) = 2x - x^2$ in $(0,3)$

hence deduce that

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$

(d) $f(x) = \begin{cases} 0, & -2 < x < 1 \\ 1+x, & -1 < x < 0 \\ 1-x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$

(e) $f(x) = x \cos \frac{\pi x}{l}$ in $(-l, l)$

(f) $f(x) = \begin{cases} x, & -1 < x < 0 \\ x+2, & 0 < x < 1 \end{cases}$

deduce that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

(g) $f(x) = \begin{cases} \pi x & 0 < x < 1 \\ \pi(2-x) & 1 < x < 2 \end{cases}$

(h) $f(x) = \begin{cases} \frac{1}{2} + x, & -1 < x < 0 \\ \frac{1}{2} - x, & 0 < x < 1 \end{cases}$

(i) $f(x) = \sin ax$ in $(-l, l)$

(a) half range cosine series for

$f(x) = \sin \frac{\pi x}{l}, \quad 0 \le x \le l$

(b) Half range cosine series of $f(x)$

$$f(x) = \begin{cases} kx, & 0 \le x \le l/2 \\ k(l-x) & l/2 \le x \le l \end{cases}$$

Hence deduce that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

(c) find the half-range sine series of

$$f(x) = \begin{cases} \frac{2x}{l}, & 0 \le x \le l/2 \\ \frac{2(l-x)}{l}, & l/2 \le x \le l \end{cases}$$

(d) find half range sine series of

$f(x) = x, \quad 0 < x < 1$

$= 2 - x, \quad 1 < x < 2$

hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

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find the fourier series of periodic function $f(x)$ with period 2π

$$f(x) = \begin{cases} 0, & -\pi < x \le 0 \\ x, & 0 < x < \pi \end{cases}$$

what is sum of series at ~~$x = 0, \pm\pi$~~,

at $x = 0, \pm\pi, 4\pi, -5\pi$

here $f(x)$ is a periodic function of period 2π and $f(x)$ shatisfy all the conditions of diritchlet's conditions.

therefore the fourier serie expansion of

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{\pi} + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0 + \int_{0}^{\pi} x\,dx\right]$$

$$a_0 = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\left[\int_{0}^{\pi} x \cos\frac{n\pi x}{\pi}\,dx\right]$$

$$a_n = \frac{1}{\pi}\left[x \sin nx \times \frac{1}{n} + \cos nx \times \frac{1}{n^2}\right]_0^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{1}{n^2}\right]$$

$$a_n = \frac{1}{\pi n^2}\left[(-1)^n - 1\right]$$

$$b_n = \frac{1}{\pi}\left[\int_0^{\pi} x \sin nx \, dx\right]$$

$$b_n = \frac{1}{\pi}\left[-x \cos nx \times \frac{1}{n} + \sin nx \frac{1}{n^2}\right]_0^{\pi}$$

$$b_n = \frac{1}{\pi}\left[-\frac{\pi}{n}\cos n\pi - 0\right]$$

$$b_n = \cancel{\frac{1}{\pi}} \; -\frac{(-1)^n}{n} = \frac{(-1)^{n+1}}{n}$$

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{1}{\pi n^2}\left((-1)^n - 1\right)\cos nx$$

$$+ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx \, .$$

~~at x = 0~~

$$\cancel{f(x) = \frac{\pi}{4} + \left(\frac{-2}{\pi n^2}\right)}$$

$$f(x) = \frac{\pi}{4} + \frac{-2}{\pi} \times \left(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots\right)$$

$$+ \frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \frac{1}{} \text{———}$$

$$- \frac{1}{2}\sin 2x - \frac{1}{4}\sin 4x - \frac{1}{6}\sin 6x$$

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because of periodicity value of fourier series at $x = 2\pi$ is same as at $x = 0$

to put $x=0$ in (1)

$$f(0) = \frac{\pi}{4} - \frac{2}{\pi}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right)$$

$$\frac{\pi^2}{8} = \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right)$$

$$f(\pm\pi) = \frac{1}{2}\left[f(\pi^+) + f(\pi^-)\right]$$

$$= \frac{1}{2}(0 + \pi)$$

$$= \frac{\pi}{2}$$

Put $x = \pm\pi$ in

$$\frac{\pi}{2} - \frac{\pi}{4} = \frac{+2}{\pi}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right)$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

at $x = -\pi$ is same as at $x = \pi$

expand in a series of sine and cosine of multiple angles of x; the periodic function 'f' with period '2π' is defined as

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$$

also calculate sum of series at $x = 0, \frac{\pi}{2}, \pm\pi$

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Partial differentiate ~~equation~~ equation

$z = f(x,y)$

$\frac{\partial z}{\partial x} = p$, $\frac{\partial z}{\partial y} = q$ $\frac{\partial^2 z}{\partial x^2} = r$

$\frac{\partial^2 z}{\partial x \partial y} = s$, $\frac{\partial^2 z}{\partial y^2} = t$

Formation at partial differential equation -

1. by eleminating the arbitury constant

2. by eleminating arbitury function.

Find partial differential equation by eleminating a, b from equation

$z = ax + by + a^2 + b^2$ — (i)

~~$\frac{\partial z}{\partial x}$~~

differentiate eqn (I) both sides partially w.r.t. 'x' we get -

$\frac{\partial z}{\partial x} = a + 0 + 0$ — (ii)

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again differentiating equation 1 partially w.r.t. y we get -

$$\frac{\partial z}{\partial y} = b \quad \text{—— (iii)}$$

substituting the valus of 'a' and 'b' from equation (ii) and (iii) in equation (i) we get -

$$z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

Form the Partial differential equation by eleminating a, b from $z = (x^2+a)(y^2+b)$ —— (i)

differentiating eqn (i) partially w.r.t. x we get

$$\frac{\partial z}{\partial x} = (y^2+b)\,2x \quad \text{——}$$

$$\frac{\partial z}{\partial x} \times \frac{1}{2x} = (y^2+b) \quad \text{—— (ii)}$$

again differentiating eqn (i) partially w.r.t. y we get

$$\frac{\partial z}{\partial y} = (x^2+a)\,2y$$

$$(x^2+a) = \frac{1}{2y}\frac{\partial z}{\partial y} \quad \text{—— (iii)}$$

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to put values of (x^2+a) and (y^2+b)

from (iii) and eqn (ii) respectively in eqn (

$$z = \left(\frac{1}{2y}\frac{\partial z}{\partial y}\right)\left(\frac{1}{2x}\frac{\partial z}{\partial x}\right)$$

$$4xyz = \frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y}$$

eleminate the arbitary function 'x' from

$z = f(x^2-y^2)$ and form a partial

differential eqn

$$z = f(x^2-y^2) \quad \text{--- (i)}$$

differential eqn (i) partially w.r.t. y

$$\frac{\partial z}{\partial y} = -2y\,\frac{\partial f}{\partial y}(x^2-y^2) \quad \text{--- (ii)}$$

differentiate eqn (i) partially w.r.t x

$$\frac{\partial z}{\partial x} = 2x.\frac{\partial f}{\partial x}(x^2-y^2) \quad \text{--- (iii)}$$

~~(ii)~~ (iii) / (ii) $\Rightarrow$

$$\frac{\partial z/\partial x}{\partial z/\partial y} = \frac{-2x}{2y} = \frac{-x}{y}$$

$$y\frac{\partial z}{\partial x} = -x\frac{\partial z}{\partial y}$$

$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$$

Ex - $x+y+z = f(x^2+y^2+z^2)$

eliminate arbitrary ~~variable~~ function 'f'

differential w.r.t. x partially

$$1+\frac{\partial z}{\partial x} = f'(x^2+y^2+z^2) \times \left(2x+2z\frac{\partial z}{\partial x}\right) \quad (i)$$

$$1+\frac{\partial z}{\partial y} = f'(x^2+y^2+z^2) \times \left(2y+2z\frac{\partial z}{\partial y}\right) \quad (ii)$$

~~$\frac{\partial z}{\partial z} = f'(x^2+y^2+z^2) \times 2z$~~ (iii)

$$\frac{\frac{\partial z}{\partial x}}{\frac{\partial z}{\partial y}} = \frac{2x}{2y}$$

$$\boxed{y\frac{\partial z}{\partial x} = x\frac{\partial z}{\partial y}}$$

$y+yp+zq+zpq$

$= x+xq+zp+zpq$

~~$p(y+z)+y = 1$~~

$$y+y\frac{\partial z}{\partial x} = x+x\frac{\partial z}{\partial y}$$

$$y-x = x\frac{\partial z}{\partial y} - y\frac{\partial z}{\partial x}$$

$$\frac{1+p}{1+q} = \left(\frac{2x+2zp}{2y+2zq}\right)$$

$$\left(\frac{1+p}{1+q}\right) = \frac{x+zp}{y+zq}$$

$p(z-y)+q(x-z)$

$= y-x$

$\underline{Q_1}$:- form partial differential equation by eliminating arbitrary constants from

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad \text{——} (1)$$

z is a function of x and y

differentiating eqn (i) w.r.t x →

$$\frac{2x}{a^2} + \frac{2z}{c^2}\frac{\partial z}{\partial x} = 0$$

$$2xc^2 + 2za^2\frac{\partial z}{\partial x} = 0 \quad \text{——} (2)$$

differentiating eqn (i) w.r.t y

$$\frac{2y}{b^2} + \frac{2z}{c^2}\frac{\partial z}{\partial y} = 0$$

$$2yc^2 + 2zb^2\frac{dz}{\partial y} = 0 \quad \text{——} (3)$$

$$b^2 = -\left(\frac{yc^2}{zq}\right) \quad \text{——} (4)$$

~~to differentiate eqn (4) by y~~

$$\cancel{0 = -c^2\left(\frac{1}{zq} + \frac{y}{q} \times \frac{-1}{z^2}\frac{dz}{dy} + \frac{y}{z}\frac{-1}{q^2} \times \frac{\partial^2 z}{\partial y^2}\right)}$$

$$\cancel{0 = -c^2(}$$

take eqn (2)

$$\frac{2xc^2}{} + 2za^2\frac{\partial z}{\partial x} = 0$$

$$xc^2 + za^2\frac{\partial z}{\partial x} = 0$$

$$x\frac{c^2}{a^2} + z\frac{\partial z}{\partial x} = 0 \longrightarrow (5)$$

to differentiate eqn (5) with respect x

$$\frac{c^2}{a^2} + \frac{\partial z}{\partial x} \times \frac{\partial z}{\partial x} + z\frac{\partial^2 z}{\partial x^2} = 0 \longrightarrow (6)$$

to put value of $\frac{c^2}{a^2}$ ~~from~~ from eqn (5) into (6)

$$-\frac{z}{x}\frac{\partial z}{\partial x} + \left(\frac{\partial z}{\partial x}\right)^2 + z\frac{\partial^2 z}{\partial x^2} = 0 \longrightarrow (7)$$

take eqn (3)

$$y\frac{c^2}{b^2} + z\frac{\partial z}{\partial y} = 0 \longrightarrow (3)$$

todifferentiate eqn (3) ~~by~~ w.r.t y

$$\frac{c^2}{b^2} + \left(\frac{\partial z}{\partial y}\right)^2 + z\frac{\partial^2 z}{\partial y^2} = 0 \longrightarrow (8)$$

to put value of $\frac{c^2}{b^2}$ from eqn (3) to eqn (8)

$$-\frac{z}{y}\frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial y}\right)^2 + z\frac{\partial^2 z}{\partial y^2} = 0 \longrightarrow (9)$$

eqn (7) and eqn (9) are two differential eqns of eqn (1).

Question: Find the differential equation of all planes which are at a constant distance 'a' from the origin.

the equation of plane in normal form is $= lx + my + nz = a$ — (1)

where l, m, n are directional cosines from the origin to the plane,

then $l^2 + m^2 + n^2 = 1$ ⟶ (4)

diff. eqn (1) w.r.t. x

$$l + n\frac{\partial z}{\partial x} = 0 \longrightarrow (2)$$

differentiate eqn (1) w.r.t. y

$$m + n\frac{\partial z}{\partial y} = 0 \longrightarrow (3)$$

to put values of l, m in eqn (4)

$$n^2\left(\frac{\partial z}{\partial x}\right)^2 + n^2\left(\frac{\partial z}{\partial y}\right)^2 + n^2 = 1$$

$$n^2 = \frac{1}{(p^2+q^2+1)} \quad \text{———} \quad (6)$$

to put value of l, m in eqⁿ (1)

$$-\frac{n\,\partial z}{\partial x}\,x + \left(-\frac{n\,\partial z}{\partial y}\,y\right) + nz = a$$

$$nz - x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = \frac{a}{n} \qquad \text{——— (5)}$$

to put evalue of $\frac{1}{n}$ from eqn (6) into eqⁿ (1)

$$\boxed{\sqrt{p^2+q^2+1} \times a = nz - px - qy}$$

<u>Question</u> ⇒ Form P.D.E. by eleminating the arbitrary function from equation ⇒

$$z = e^{my}\,\phi(x-y) \qquad \text{——— (1)}$$

differentiate equation (1) w.r.t 'x'

$$\frac{\partial z}{\partial x} = e^{my}\,\phi'(x-y) \qquad \text{——— (2)}$$

differentiate equation (1) w.r.t 'y'

$$\frac{\partial z}{\partial y} = -m\,e^{my}\,\phi'(x-y) \qquad \text{——— (3)}$$

$$2/3 \Rightarrow \frac{\partial z}{\partial x}\Big/\frac{\partial z}{\partial y} = -\frac{1}{m}$$

$$\Rightarrow p = -q/m \quad \text{Ans}$$

Note ⇒ the differentiation of arbitrary function w.r.t. x and y will be same because ~~it is~~ first the operator is differentiated which will give same result in both cases after that the variable inside it is differentiated

Q1) form P.D.E. by eliminating arbitrary functions -

$$z = f(x+ay) + \phi(x-ay) \longrightarrow (1)$$

differentiate eqn (1) w.r.t. x

$$\frac{\partial z}{\partial x} = f'(x+ay) + \phi'(x-ay) \longrightarrow (2)$$

differentiate eqn (1) w.r.t. y

$$\frac{\partial z}{\partial y} = f'(x+ay) \times +a - a \times \phi'(x-ay) \longrightarrow (3)$$

differentiating equation (2) w.r.t. x

$$\frac{\partial^2 z}{\partial x^2} = f''(x+ay) + \phi''(x-ay) \longrightarrow (4)$$

differentiating equation (3) w.r.t. y

$$\frac{\partial^2 z}{\partial y^2} = a^2 f''(x+ay) + a^2 f''(x-ay) \longrightarrow (5)$$

(4) $\times a^2$ − (5)

$$a^2 \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$$

$$\boxed{a^2 r - t = 0}$$

<u>Question</u> ⇒ Form P.D.E. by eliminating arbitrary function from

eqⁿ ⇒ $f(x+y+z,\ x^2+y^2-z^2) = 0$ — (1)

~~differentiate equation (1) w.r.t x~~

$$\cancel{f'(x+y+z,\ x^2+y^2-z^2)\,(1+2x)} = 0 \quad — (2)$$

$$\left(1+\partial z/\partial x,\ +2x-2z\frac{\partial z}{\partial x}\right)$$

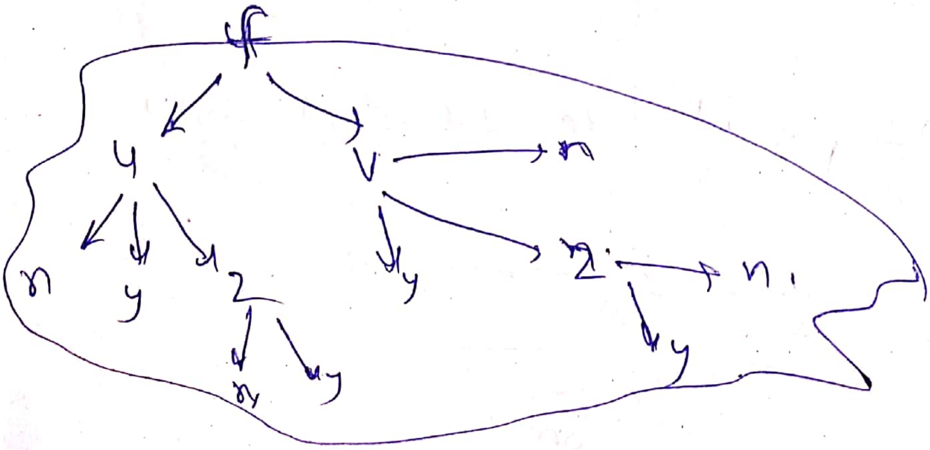
~~differentiate eqⁿ (1) w.r.t y~~ ⇒

$$\cancel{f'(x+y+z,\ x^2+y^2-z^2)\left(1+\frac{\partial z}{\partial y},\ 2y-2z\frac{\partial z}{\partial y}\right)} = 0$$

let $x+y+z = u$

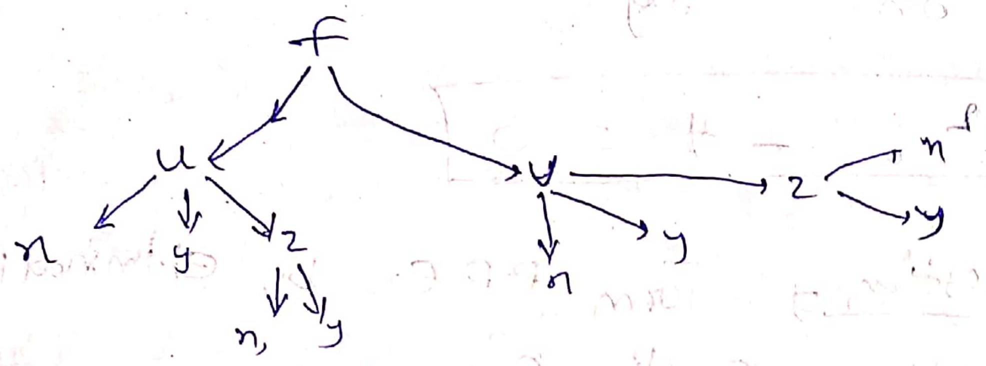
$x^2+y^2-z^2 = v$

$f(u,v) = 0$



$$f(u,v) = 0 \quad \text{——} \quad (1)$$

differentiate equation (1) w.r.t x



$$\cancel{\frac{\partial f}{\partial u}\left[\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z} \times \frac{\partial z}{\partial x}\right]} =$$

$$\frac{\partial f}{\partial u}\left[\frac{\partial u}{\partial x} + \cancel{\frac{\partial u}{\partial z} \times \frac{\partial z}{\partial x}}\right] + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial x} + \cancel{\frac{\partial v}{\partial z}\frac{\partial z}{\partial x}}\right) = 0$$

$u = x+y+z \qquad v = x^2+y^2-z^2$

$$\frac{\partial u}{\partial x} = 1 + \frac{\partial z}{\partial x} \qquad \frac{\partial v}{\partial x} = 2x - 2z\frac{\partial z}{\partial x}$$

$$\frac{\partial u}{\partial y} = 1 + \frac{\partial z}{\partial y} \qquad \frac{\partial v}{\partial y} = 2y - 2z\frac{\partial z}{\partial y}$$

$$\frac{\partial u}{\partial z} = 1$$

$$\frac{\partial f}{\partial u}\left(1+\frac{\partial z}{\partial x}\right) + \frac{\partial f}{\partial v}\left(2x - 2z\frac{\partial z}{\partial x}\right) = 0$$

$$\frac{\partial f}{\partial u} \Big/ \frac{\partial f}{\partial v} = \frac{\left(2z\frac{\partial z}{\partial x} - 2x\right)}{\left(1+\frac{\partial z}{\partial x}\right)}$$

differentiating eqn(1) w.r.t y

$$\cancel{\frac{\partial f}{\partial y} \times \frac{\partial y}{\partial y} + \frac{\partial f}{\partial y} \times \frac{\partial u}{\partial z} \times \frac{\partial z}{\partial y} = 0}$$

$$\frac{\partial f}{\partial u} \times \frac{\partial u}{\partial y} + \frac{\partial f}{\partial v} \times \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial f}{\partial u}\left(1 + \frac{\partial z}{\partial y}\right) + \frac{\partial f}{\partial v}\left(2y - 2z\frac{\partial z}{\partial y}\right) = 0$$

$$\frac{\partial f}{\partial u} \Big/ \frac{\partial f}{\partial v} = \frac{\left(2z\frac{\partial z}{\partial y} - 2y\right)}{\left(1 + \frac{\partial z}{\partial y}\right)} \quad \text{———} \quad (3)$$

$$\frac{\cancel{2}z\frac{\partial z}{\partial x} - \cancel{2}x}{1 + \frac{\partial z}{\partial x}} = \frac{\cancel{2}z\frac{\partial z}{\partial y} - \cancel{2}y}{\left(1 + \frac{\partial z}{\partial y}\right)}$$

$$\left(1 + \frac{\partial z}{\partial y}\right)\left(z\frac{\partial z}{\partial x} - x\right) = \left(1 + \frac{\partial z}{\partial x}\right)\left(z\frac{\partial z}{\partial y} - y\right)$$

$$z\frac{\partial z}{\partial x} - z\frac{\partial z}{\partial y} - x + y - x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x}(z + y) - \frac{\partial z}{\partial y}(z + x) - x + y = 0$$

Eliminate arbitrary function f & F from $y = f(x-at) +$
$F(x+at)$ —— (1)

diff. eqn (1) partially w.r.t x ⇒

$$\frac{\partial y}{\partial x} = f'(x-at) + F'(x+at) \text{ —— (2)}$$

again diff. ~~eqn (2)~~ w.r.t x we get

$$\frac{\partial^2 y}{\partial x^2} = f''(x-at) + F''(x+at) \rightarrow (2)$$

diff. eqn (1) partially w.r.t t

$$\frac{\partial y}{\partial t} = -af'(x-at) + aF'(x+at)$$

again diff w.r.t t

$$\frac{\partial^2 y}{\partial t^2} = +a^2 f''(x-at) + a^2 F''(x+at)$$

—— (3)

to put value of $f''(x-at) + F''(x+at)$ from eqn (2) into eqn (3)

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

Ans

form a P.D.E. by eliminating the arbitrary functions f & g from

$$z = f(x^2-y) + g(x^2+y) \longrightarrow (1)$$

diff eq^n (1) w.r.t x

$$\frac{\partial z}{\partial x} = 2x\, f'(x^2-y) + 2x\, g'(x^2+y)$$

again diff eqn w.r.t x

$$\frac{\partial^2 z}{\partial x^2} = 4x^2 f''(x^2-y) + 2f'(x^2-y) + 4x^2 g''(x^2+y) + 2g'(x^2+y) \longrightarrow (2)$$

diff eqn (1) w.r.t y

$$\frac{\partial z}{\partial y} = -f'(x^2-y) + g'(x^2+y)$$

again diff w.r.t y

$$\frac{\partial^2 z}{\partial y^2} = f''(x^2-y) + g''(x^2+y) \longrightarrow (3)$$

$$\frac{\partial^2 z}{\partial x^2} = 4x^2 \frac{\partial^2 z}{\partial y^2} + \frac{1}{x}\frac{\partial z}{\partial x}$$

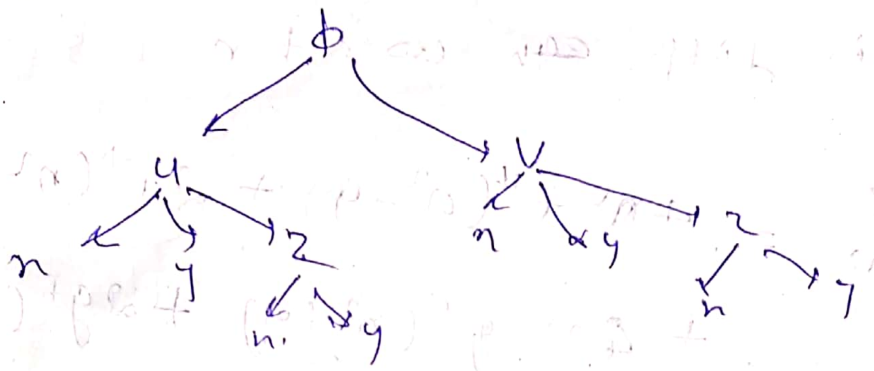
Question→ Form a P.D.E. by elimination

$$\phi(x^2+y^2+z^2,\ z^2-2xy)=0 \quad \text{—— (1)}$$

$$u = x^2+y^2+z^2 \quad \text{——}$$

$$v = z^2-2xy$$

$$\phi(u,v)=0 \longrightarrow (2)$$



diff. eqn (2) w.r.t. x

$$\frac{\partial \phi}{\partial x} \Rightarrow \frac{\partial \phi}{\partial u} \times \frac{\partial u}{\partial x} + \frac{\partial \phi}{\partial v} \cdot \frac{\partial v}{\partial x} = 0$$

$$u = x^2+y^2+z^2$$

$$\frac{\partial u}{\partial x} = 2x + 2z\frac{\partial z}{\partial x}$$

$$\frac{\partial u}{\partial y} = 2y + 2z\frac{\partial z}{\partial y}$$

$$v = z^2 - 2xy \qquad \frac{\partial v}{\partial y} =$$

$$\frac{\partial v}{\partial x} = 2z\frac{\partial z}{\partial x} - 2y \qquad 2z\frac{\partial z}{\partial y} - 2x$$

again diff. eqn (2) Partially w.r.t. y $\Rightarrow$

$$\frac{\partial \phi}{\partial u} \times \frac{\partial u}{\partial y} + \frac{\partial \phi}{\partial v} \times \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial \phi}{\partial u}\left(2y + 2z\frac{\partial z}{\partial y}\right) + \frac{\partial \phi}{\partial v}\left(2z\frac{\partial z}{\partial y} - 2x\right) = 0$$

$$\frac{\partial \phi}{\partial u} \Big/ \frac{\partial \phi}{\partial v} = \frac{\left(2x - 2z\frac{\partial z}{\partial y}\right)}{\left(2y + 2z\frac{\partial z}{\partial y}\right)} \quad \text{--- (3)}$$

$$\frac{\partial \phi}{\partial u}\left(2x + 2z\frac{\partial z}{\partial x}\right) + \frac{\partial \phi}{\partial v}\left(2z\frac{\partial z}{\partial x} - 2y\right) = 0$$

$$\frac{\partial \phi}{\partial u} \Big/ \frac{\partial \phi}{\partial v} = \frac{\left(2y - 2z\frac{\partial z}{\partial x}\right)}{\left(2x + 2z\frac{\partial z}{\partial x}\right)} \quad \text{--- (4)}$$

$$\left(x - z\frac{\partial z}{\partial y}\right)\left(x + z\frac{\partial z}{\partial x}\right) = \left(z\frac{\partial z}{\partial y} + y\right)\left(y - z\frac{\partial z}{\partial x}\right)$$

$$(x - zq)(x + zp) = (zq + y)(y - zp)$$

Q:- eleminate arbitrary function ϕ from eqn

$$\phi\left(x + y + z,\ x^2 + y^2 - z^2\right) = 0$$

⇒ Legrange's method of solving Ist order 1st degree partial differential eqn

$$\boxed{Pp + Qq = R}$$

$p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

$P, Q, R \ni f(x, y, z)$ or constant

lagrange's auxiliary equation are -

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

general solution →

$$\phi(f_1(x, y, z), f_2(x, y, z)) = 0$$

Type - 1

Q: solve $\frac{y^2 z}{x} p + xz\,q = y^2$

Leegranges auxiliary eqn are

$$\frac{dx}{\frac{y^2 z}{x}} = \frac{dy}{xz} = \frac{dz}{y^2} \quad \leftarrow (2)$$

$$\frac{x\,dx}{y^2 z} = \frac{dy}{x z}$$

$$x^2\,dx = y^2\,dy$$

$$x^3 = y^3 + c_1$$

$$x^3 - y^3 = c_1 \longrightarrow (3)$$

$$\frac{x\,dx}{y^2 z} = \frac{dz}{y^2}$$

$$x\,dx = z\,dz$$

$$x^2 - z^2 = c_2 \longrightarrow (4)$$

from eqn (3) & (4)

general soln is

$$\phi(x^3 - y^3,\ x^2 - z^2) = 0 \quad \text{Ans}$$

Type-2

solve $p + 3q = 5z + \tan(y - 3x)$

$$\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{5z + \tan(y-3x)}$$

$$3\,dx = dy$$

$$3x - y = c_1 \longrightarrow (1)$$

$$dz = (5z + \tan(y - 3x))\,dx$$

$$dz = (5z + \tan c_1)\,dx$$

$$\frac{dz}{(5z + \tan c_1)} = dx$$

$$\frac{\log(5z + \tan c_1)}{5} = x + c_2$$

$$\log(5z + \tan c_1) = 5x + c_2$$

$$5x - \log(5z + \tan c_1) = c_2$$

$$\phi(3x - y,\ 5x - \log(5z + \tan(3x - y))) = 0$$

<u>type III</u>

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

take a_1, a_2, a_3 as multiplier.

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{a_1 dx + a_2 dy + a_3 dz}{a_1 P + a_2 Q + a_3 R}$$

if $a_1 P + a_2 Q + a_3 R = 0$ then

$$a_1 dx + a_2 dy + a_3 dz = 0$$

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solve $\Rightarrow$

$$(z^2-2yz-y^2)p+(xy+zx)q = xy-zx \quad \text{--- (1)}$$

lagrange's auxilliary eqn are

$$\frac{dx}{(z^2-2yz-y^2)} = \frac{dy}{(xy+zx)} = \frac{dz}{xy-zx} \quad \text{--- (2)}$$

taking (2) nd 3rd fraction

$$dy\,(x(y-z)) = dz\,(x(y+z))$$

$$dy\,(y-z) = dz\,(y+z)$$

$$y\,dy - z\,dy = y\,dz + z\,dz$$

$$\cancel{y(dy-dz) = z(dy+dz)}$$

$$y\,dy - z\,dz = z\,dy + y\,dz$$

$$\int y\,dy - \int z\,dz = \int d(zy)$$

$$\frac{y^2}{2} - \frac{z^2}{2} = zy + c$$

$$y^2 - z^2 = 2zy + c_1 \quad \text{--- (3)}$$

$$y^2 - z^2 - 2yz = c_1 \quad \text{--- (3)}$$

$$x(z^2-2yz-y^2) + y(xy+zx) + z(xy-zx)$$

$$= 0$$

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taking x, y, z as multiplier then

each fraction of eq^n (2) is ——

$$\frac{x\,dx + y\,dy + z\,dz}{xz^2 - 2x^2z - xy^2 + \cancel{xy^2} + \cancel{xyz} + \cancel{xyz} - xy^2}$$

$x\,dx + y\,dy + z\,dz = 0$ —— (4)

$x^2 + y^2 + z^2 = c_2$ —— (5)

$\phi(y^2 - z^2 - 2yz, x^2 + y^2 + z^2) = 0$

—— Ans

~~for~~ $(x+2z)p + (4zx - y)q = 2x^2 + y$

langrange auxiliary eq

$$\frac{dx}{x+2z} = \frac{dy}{(4zx-y)} = \frac{dz}{(2x^2+y)}$$

taking $(-2x, 1, 1)$ as multiplier

$\cancel{-2x^2} - \cancel{4zx} + \cancel{4zx} - \cancel{y} + \cancel{2x^2} + \cancel{y} = 0$

$(-2x, 1, 1)$

$$-2x\,dx + dy + dz = 0$$

$$-\frac{\cancel{2}x^2}{\cancel{2}} + y + z = c_1 \quad \text{—— (1)}$$

$$-x^2 + y + z = c_1 \quad \text{—— (2)}$$

~~$x^2y + 2zyx - 8z^2x + 2yz$~~

~~$xyz + 2yz^2 + 8z^2x + 2yz$~~

taking $y, x, -2z$ as multipl.

$$\cancel{xy} + \cancel{2}yz + \cancel{4}zx^2 - \cancel{yx} - \cancel{4}zx^2 - 2\cancel{yz}$$
$$\Rightarrow$$

$$y\,dx + x\,dy - 2z\,dz = 0$$

$$xy - z^2 = c_2 \quad \text{—— (2)}$$

Ans

$$\phi(-x^2 + y + z,\ xy - z^2) = 0$$

type-4

solve $(x^2 - y^2 - yz)p + (x^2 - y^2 - zx)q = z(x - y)$ —— (1)

~~Lagrange~~

lagrange's auxiliary eqⁿ are ⇒

$$\frac{dx}{(x^2-y^2-yz)} = \frac{dy}{(x^2-y^2-zx)} = \frac{dz}{z(x-y)} \quad \text{—②}$$

taking $(1,-1,0)$ as multiplier

then each fraction of eqⁿ (2) will be

$$\frac{dx-dy}{x^2-y^2-yz-x^2+y^2+zx} = \frac{dx-dy}{z(x-y)} =$$

$$\frac{d(x-y)}{z(x-y)} \quad \text{—③}$$

taking $x, -y, 0$ as multiplier

$$\frac{x\,dx - y\,dy}{x(x^2-y^2-yz) - y(x^2-y^2-zx)}$$

$$= \frac{x\,dx - y\,dz}{x^3 - xy^2 - xyz - x^2y + y^3 + yzx}$$

$$= \frac{x\,dx - y\,dz}{x^2(x-y) + y^2(y-x)}$$

$$= \frac{x\,dx - y\,dz}{(x^2-y^2)(x-y)}$$

from eqn (2), (3) & (4) we get

$$\frac{dz}{z(x-y)} = \frac{x\,dx - y\,dy}{(x-y)(x^2-y^2)} = \frac{d(x-y)}{z(x-y)}$$

taking 1st and 3rd fraction of eqn (5)

$$\frac{dz}{z(x-y)} = \frac{d(x-y)}{z(x-y)}$$

$$z = x-y+c$$

$$\underline{\underline{z+y-x = c_1}} \quad \text{A}$$

taking 1st and 2nd fraction of eqn (5)

$$\frac{dz}{z(x-y)} = \frac{x\,dx - y\,dz}{(x^2-y^2)(x-y)}$$

$$\frac{dz}{z} = \frac{d(x^2-y^2)}{2(x^2-y^2)}$$

$$2\log z = \log(x^2-y^2) + c_2$$

$$2\log z = \log(x^2-y^2) + \log c_2$$

$$\underline{\underline{\frac{z^2}{(x^2-y^2)} = c_2}} \quad \text{--- (7)}$$

$$\phi\left(z+y-x,\ \frac{z^2}{x^2-y^2}\right) = 0$$

Type-I

Q:- $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

lagrange's auxilary eqn are

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$

taking 2nd and 3rd fraction ② we get

$$\frac{dy}{2xy} = \frac{dz}{2xz}$$

$$\frac{dy}{y} = \frac{dz}{z}$$

$$\log y = \log z + \log c_1 \Rightarrow$$

$$\frac{y}{z} = c_1 \longrightarrow ③$$

taking x, y, z as multiplier then each fraction will be equal to

$$= \frac{x\,dx + y\,dy + z\,dz}{x^3 - xy^2 - xz^2 + 2xy^2 + 2xz^2}$$

$$= \frac{x\,dx + y\,dy + z\,dz}{x^3 + xy^2 + xz^2}$$

$$= \frac{x\,dx + y\,dy + z\,dz}{x(x^2+y^2+z^2)}$$

$$= \frac{d(x^2+y^2+z^2)}{2x(x^2+y^2+z^2)} \quad \text{——— (4)}$$

From eqn (2) & (4) we get →

$$\frac{dz}{2xz} = \frac{d(x^2+y^2+z^2)}{2x(x^2+y^2+z^2)}$$

$$\frac{dz}{z} = \frac{d(x^2+y^2+z^2)}{(x^2+y^2+z^2)}$$

$$\log z = \log(x^2+y^2+z^2) + \log c_2$$

$$\frac{z}{(x^2+y^2+z^2)} = c_2 \quad \text{——— (5)}$$

$$\phi\left(\frac{y}{z}, \frac{z}{x^2+y^2+z^2}\right) = 0 \quad \text{Ans}$$

Ques → (1) $xyp + y^2q = zxy - zx^2$

(2) $py + qx = xyz^2(x^2-y^2)$

(3) $(x^2-y^2-z^2)p + 2xyq = 2xz$

$z(p-q) = z^2 + (x+y)^2$

$p + 2q = 3x^2 \sin(y+2x)$

(III) type

1. $z(x+y)p + z(x-y)q = x^2+y^2$

2. $(mz-ny)p + (nx-lz)q = ly-mx$

3. $x(y^2-z^2)p + y(z^2-x^2)q = z(x^2-y^2)$

4. $(x+2z)p + q(4zx-y) = 2x^2+y$

5. $(z^2-2yz-y^2)p + (xy+zx)q$
$= xy - zx$

6. $(x-y)p + (x+y)q = 2xz$

7. $(3x+y-z)p + (x+y-z)q = 2(z-y)$

8. $x(x^2+3y^2)p - y(3x^2+y^2)q =$
$2z(y^2-x^2)$

9. $(y+zx)p - (x+yz)q + y^2 - x^2 = 0$

type-2

1. $y^2(x-y)p + x^2(y-x)q = z(x^2+y^2)$

2. $(x^2-y^2-z^2)p + 2xyq = 2xz$

3. $(x^2-y^2-yz)p + (x^2-y^2-zx)q = z(x+y)$

5. $p+q = x+y+z$

6. $(1+y)p + (1+x)q = z$

type-1

(i) $zp = -x$

(ii) $y^2p - xyq = x(z-2y)$

(iii) $x^2p + y^2q + z^2 = 0$

(i) $zp = -x$ $\quad p = \frac{\partial z}{\partial x}$

$z\frac{\partial z}{\partial x} = -x$

$-\int x\,\partial x = \int z\,\partial z + c$

$-\frac{x^2}{2} = \frac{z^2}{2} + c$

$z^2 + x^2 = c$ Ans

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② $\underbrace{y^2}_{P} p \; \underbrace{-xy}_{Q} q = \underbrace{x(z-2y)}_{R}$

~~general solution is~~ → Lagrange's eqns are:

$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{\partial z}{x(z-2y)} \quad \text{--- (1)}$$

to solve Ist and IInd fraction of eqn ①

$$\frac{dx}{y^2} = \frac{-dy}{xy}$$

$$x\,dx = -y\,dy$$

$$x^2 + y^2 \pm C_1 \text{ ——— } (2')$$

to solve IInd and IIIrd fraction of eqn ①

$$\frac{-dy}{xy} = \frac{dz}{x(z-2y)}$$

$$-\frac{(1-y)}{y}\,dy = \frac{dz}{z}$$

$$dy - \frac{dy}{y} - \frac{dz}{z} = 0$$

integrating both side

$$y - \log y - \log z = C_2 \text{ --- } (3)$$

general solution of eqn is

$$\phi\left((x^2+y^2),\ (y - \log y - \log z)\right) = 0$$

(3) $\underbrace{x^2}_{P} p + \underbrace{y^2}_{Q} q = \underbrace{-z^2}_{R}$

~~general solution of this eqn is~~
lagrange's eqns are-

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{-z^2} \quad \text{———} \quad (1)$$

to solve Ist and IInd fraction of eqn (1)

$$\frac{dx}{x^2} - \frac{dy}{y^2} = 0$$

integrate both sides

$$-\frac{1}{x} + \frac{1}{y} = c_1 \quad \text{———} \quad (2)$$

to solve Ist and IIIrd fraction of eqn (1)

$$\frac{dx}{x^2} + \frac{dz}{z^2} = 0$$

Integrating both sides

$$-\frac{1}{x} - \frac{1}{z} = c_2$$

$$\frac{1}{x} + \frac{1}{z} = c_2 \quad \text{———} \quad (3)$$

general solution is

$$\phi\left(\frac{-1}{x} + \frac{1}{y}, \frac{1}{x} + \frac{1}{z}\right) = 0$$

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(9) $xyp + y^2 q = zxy - 2x^2$

Lagrange's equation are -

$$\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{zxy - 2x^2} \quad \text{--- (1)}$$

take Ist and IInd fraction of eqn (1)

$$\frac{dx}{x} = \frac{dy}{y}$$

$$\log x - \log y = c_1$$

$$\frac{x}{y} = c_1 \quad \text{--- (2)}$$

take Ist and IIIrd fractio

$$\frac{dx}{y} = \frac{dz}{zy - 2x}$$

$$dx = \frac{dz}{z - 2c_1}$$

$$x = \log(z - 2c_1) + c_2 = 0$$

$$x - \log(z - 2\, x/y) = c_2$$

general soln is

$$\phi\left(x/y ,\ x - \log\left(z - 2x/y\right)\right) = 0 \quad \text{Ans}$$

(5) $py \, \frac{\partial z}{\partial x}$ ... by eqn $= xyz^2(x^2-y^2)$

lagrange's equations are —

$$\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xyz^2(x^2-y^2)} \quad \text{—— (1)}$$

to take Ist and IInd fraction of eqn (1)

$$\frac{dx}{y} = \frac{dy}{x} \quad -$$

$$x^2 - y^2 = C_1 \quad \text{—— (2)}$$

to take Ist and IIIrd fraction of eqn (1)

$$\cancel{C_1 \, dx = \frac{dz}{}}$$

$$x C_1 \, dx = \frac{dz}{z^2}$$

$$\frac{x^2}{2} + \frac{1}{z} = C_2$$

$$\frac{x^2}{2} + \frac{1}{z} = C_2 \quad \text{—— (3)}$$

General solution is

$$\phi\left((x^2-y^2), \left(\frac{x^2}{2} + \frac{1}{z}\right)\right) = 0 \quad \text{Ans}$$

(6)

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(2) $z(x+y)p + z(x-y)q = x^2+y^2$

lagrange's equations are −

$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{(x^2+y^2)} \quad \text{——} \ (1)$$

$\cancel{2x+2y \quad 2x-2y \quad x^2+y^2}$

to solve I^st and II^nd fraction

$$(x-y)\,dx = (x+y)\,dy$$

$$xdx - ydx = xdy + ydy$$

$$xdx = xdy + ydx + ydy$$

$$\frac{x^2}{2} = xy + \frac{y^2}{2} + c_1$$

$\cancel{x^2}$

$$x^2 - y^2 - 2xy = c_1 \quad \text{——} \ (2)$$

Qu Given P;

~~p = India~~

p = 7004

(7) $z(p-q) = z^2 + (x+y)^2$

$zp - zq = z^2 + (x+y)^2$

auxiliary eqn is

$$\frac{dx}{z} = \frac{dy}{-z} = \frac{dz}{z^2 + (x+y)^2} \longrightarrow (1)$$

to solve Ist and IInd fraction

$-dx = dy$

$x+y = c_1 \longrightarrow (1)$

to solve Ist and IIIrd fractio

$$\frac{dx}{z} = \frac{dz}{z^2 + c_1^2}$$

$$\int dx = \int \frac{z\,dz}{z^2 + c_1^2}$$

$$x = \frac{1}{2} \log(z^2 + c_1^2) + c_2$$

$2x - \log(z^2 + c_1^2) = c_2$

General soln is

$\phi\left((x+y), \left(2x - \log(z^2 + (x+y)^2)\right)\right) = 0$

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Charpit's method for solving Ist order higher degree partial differential equation.

$$f(x, y, z, p, q) = 0$$

charpit's auxiliary equations are-

$$\frac{dp}{f_x + p f_z} = \frac{dq}{f_y + q f_z} = \frac{dz}{-p f_p - q f_q} = \frac{dx}{-f_p}$$

$$= \frac{dy}{-f_q}$$

to find value of p or q

or relation b/w p and q

$$dz = p dx + q dy$$

$$z = f(x, y)$$

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

$$dz = p dx + q dy$$

5

Example → Find the complete integral of

$$px + qy = pq \quad (1)$$

Here, the given equation is

$$px + qy - pq = 0 = f(x, y, z, p, q)$$

charpit's auxiliary equations are

$$\frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dz}{-pf_p - qf_q} = \frac{dx}{-f_p} = \frac{dy}{-f_q}$$

here,

$f(x) = p$, $f_y = q$, $f_z = 0$

$f_p = x - q$, $f_q = y - p$

now subtituting the values of f_x, f_y, f_z, f_p, f_q in eq^n 2 we get

$$\frac{dp}{p} = \frac{dq}{q} = \frac{dz}{-p(x-q) - q(y-p)} = \frac{dx}{-(x-q)} = \frac{dy}{-(y-p)} \quad (3)$$

taking Ist and IInd fraction of eq^n 3

$$\frac{dp}{p} = \frac{dq}{q}$$

$\frac{p}{q} = c_1$

$p = aq$ —— (4)

putting $p = aq$ in eqn (1), we get

$aqx + qy - aq^2 = 0$

$q(ax + y - aq) = 0$

$\therefore ax + y - aq = 0$ $\{q \neq 0\}$

$q = \frac{xa + y}{a}$

$p = ax + y$

$dz = p\,dx + q\,dy$

$dz = (ax+y)\,dx + \left(\frac{ax+y}{a}\right) dy$

$a\,dz = (a^2x + ay)\,dx + (ax+y)\,dy$

$\int a\,dz = \int a^2x\,dx + y\,dy + a(y\,dx + x\,dy)$

$az = \frac{a^2x^2}{2} + \frac{y^2}{2} + axy + b$

$2az = a^2x^2 + y^2 + 2axy + b$

$2az = (ax+y)^2 + b$

Q. Find the complete integral of (2)

$$p^2 - y^2 q = y^2 - x^2 \quad \text{--- (I)}$$

The Charpit's auxiliary eqn of (I) is

$$\frac{dp}{f_x + p f_z} = \frac{dq}{f_y + q f_z} = \frac{dz}{-p f_p - q f_q} =$$

$$\frac{dx}{-f_p} = \frac{dy}{-f_q} \quad \text{--- (2)}$$

$$f(x) = 2x$$

$$f_y = -2y(1+q)$$

$$f_z = 0$$

$$f_p = 2p$$

$$f_q = -y^2$$

$$\frac{dp}{2x + 0} = \frac{dq}{-2y(1+q)} = \frac{dz}{-2p^2 + qy^2} = \frac{dx}{-2p} = \frac{dy}{+y^2} \quad (3)$$

taking Ist and IVth fraction become

$$\frac{dp}{2x} = \frac{dx}{-2p}$$

$$\cancel{-2p^2\, dp = 2x\, dx}$$

$$~~-\frac{2}{3}p^3 - \frac{2n^2}{2} = a~~$$

$$~~2p^3 + 3n^2 = a~~$$

$$~~p = \frac{(a-3n^2)}{2}~~$$

$$-2p\,dp = 2n\,dn$$

$$p^2 = a^2 - n^2$$

$$p = \sqrt{a^2 - n^2}$$

subtituting value of p in eqn (I)

$$a^2 - n^2 - y^2 q = y^2 - n^2$$

$$a^2 - y^2 q - y^2 = 0$$

$$-y^2 q = y^2 - a^2$$

$$~~q = a^2 y^2~~$$

$$q = -1 + \frac{a^2}{y^2}$$

$$p = \sqrt{a^2 - n^2}$$

(I) we know

$$dz = p\,dn + q\,dy$$

$$\int dz = \int \sqrt{a^2 - n^2}\,dn + \int (-1 + a^2 y^{-2})\,dy + b$$

$$z = \frac{n}{2}\sqrt{a^2 - n^2} + \frac{a^2}{2}\sin^{-1}\frac{n}{a} - \frac{a^2}{y} - y + b$$

Q:- find the complete integral of $z^2(p^2z^2+q^2) = 1$ — (1)

$$p^2z^4+q^2z^2-1 = 0$$

the charpit's auxiliary eqn is

$$\frac{dp}{f_x+pf_z} = \frac{dq}{f_y+qf_z} = \frac{dz}{-pf_p-qf_q}$$

$$= \frac{dx}{-f_p} = \frac{dy}{-f_q} \quad \text{— (2)}$$

$f_x = 0$, $f_y = 0$, ~~$dz = 4z^3p^2+q$~~

$f_z = 4p^2z^3+2q^2z$

$f_p = 2pz^4$

$f_q = 2qz^2$

to put all values in eqn (2)

$$\frac{dp}{p(4p^2z^3+2q^2z)} = \frac{dq}{q(4p^2z^3+2q^2z)} = \frac{dz}{-2p^2z^4-2q^2z^2}$$

$$= \frac{dx}{-2pz^4} = \frac{dy}{-2qz^2}$$

taking Ist and IInd fraction

$$\frac{p}{q} = a,$$

$$p = aq$$

$$z^2(a^2q^2z^2+q^2)=1$$

$$q=\sqrt{\frac{1}{z^2(a^2z^2+1)}}$$

$$q=\frac{1}{z}(a^2z^2+1)^{-1/2}$$

$$p=\frac{a}{z}(a^2z^2+1)^{-1/2}$$

$$dz=p\,dx+q\,dy$$

$$dz=\frac{a}{z}(a^2z^2+1)^{-1/2}dx+$$

$$\frac{1}{z}(a^2z^2+1)^{-1/2}dy$$

$$\cancel{zdz=a(a^2z^2+1)^{-1/2}}$$

$$\cancel{(a^2z^2+1)^{-1/2}}$$

$$\frac{z\,dz}{(a^2z^2+1)^{-1/2}}=a\,dx+dy$$

$$a^2z^2+1=t$$

$$2a^2z\,dz=dt$$

$$\frac{dt\times\sqrt{t}}{2a^2}=a\,dx+dy$$

$$\frac{1}{2a^2}\times\frac{2}{3}(t)^{3/2}=ax+y+b$$

$$(a^2z^2+1)^{3/2}=3a^3x+3a^2y+b$$

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Question

Find the complete integral of ⑧

$(z+px)^2 = q$

$z^2 + p^2x^2 + 2zpx = q$ — ①

Charpit's auxiliary eqn is

$$\frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dz}{-pf_p - qf_q} = \frac{dx}{-f_p} = \frac{dy}{-f_q}$$

~~f(x)~~

$f_x = 2xp^2 + 2zp$

$f_y = 0$

$f_z = 2z + 2px$

$f_p = x^2 + 2zx = 2x(z+px)$

$f_q = -1$

$$\frac{dp}{2xp^2+2zp + 2zp + 2p^2x} = \frac{dq}{2zq + 2pqx}$$

$$= \frac{dz}{-px^2 - 2pzx - q} = \frac{dx}{-x^2 - 2zx} = \frac{dy}{+1}$$

$$\frac{dp}{4xp^2 + 4zp} = \frac{dq}{2zq + 2pqx} = \frac{dz}{-px^2 - 2}$$

taking 2nd and 4th

$$\frac{dq}{2q(z+px)} = \frac{dx}{\cancel{x^2 - 2zx}\; -2x(z+px)}$$

$$qx = a$$

$$q = \frac{a}{x}$$

$$(z+px)^2 = \frac{a}{x}$$

$$px = \left(\frac{a}{x}\right)^{1/2} - z$$

$$p = \left(\frac{a}{x}\right)^{1/2} \times \frac{1}{x} - \frac{z}{x}$$

$$dz = p\,dx + q\,dy$$

$$dz = \frac{(a)^{1/2}}{x^{3/2}}\,dx + \frac{a}{x} \times dy$$

$$dz = \left(\frac{\sqrt{a}}{x^{3/2}} - \frac{z}{x}\right)dx - \frac{a}{x}\,dy$$

$$x\,dz = \left(\frac{\sqrt{a}}{\sqrt{x}} - z\right)dx - a\,dy$$

$$x\,dz + z\,dx = \frac{\sqrt{a}}{\sqrt{x}}\,dx - a\,dy$$

$$d(xz) = \int \frac{\sqrt{a}}{\sqrt{x}}\,dx - a\int dy + c$$

$$\boxed{xz = 2\sqrt{ax} - ay + c}$$

Az

Q. $(p^2+q^2)x = pz$

$p^2x + q^2x - pz = 0$

$$\frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dz}{-pf_p - qf_q}$$

$$= \frac{dx}{-f_p} = \frac{dy}{-f_q}$$

$f_x = p^2+q^2$

$f_y = 0$

$f_z = -p$

$f_p = 2px - z$

$f_q = 2qx$

$$\frac{dp}{\cancel{p^2}+q^2-\cancel{p^2}} = \frac{dq}{-pq} = \frac{dz}{pz-2p^2x-2q^2x} = \frac{dx}{-f_p}$$

taking Ist and IInd fraction

$-pdp = q\,dq$

$-\frac{p^2}{2} - \frac{q^2}{2} = a$

$p^2+q^2 = a^2$ —— (1)

$ax - pz = 0$

$$p = \frac{a^2x}{z}$$

$$q = \sqrt{a^2 - \left(\frac{a^2x}{z}\right)^2}$$

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$$dz = p\,dx + q\,dy$$

$$dz = \frac{a^2 x}{z}\,dx + \frac{1}{z}\sqrt{a^2z^2 - (a^2x)^2}\,dy$$

~~$z\,dz = ax\,dx + \sqrt{az^2 - (ax)^2}\,dy$~~

$$z\,dz = a^2 x\,dx + a\sqrt{z^2 - a^2x^2}\,dy$$

$$\frac{z\,dz - a^2 x\,dx}{a\sqrt{z^2 - a^2x^2}} = dy$$

$$z^2 - a^2x^2 = t$$

$$2(z\,dz - a^2 x\,dx) = dt$$

$$\frac{dt}{2a\sqrt{t}} = dy$$

$$\sqrt{t} = dy + b$$

$$\sqrt{z^2 - a^2x^2} = dy + b$$

$$a\sqrt{z^2 - a^2x^2} = y + b$$

Ans

Qu. $2xz - px^2 - 2qxy + pq = 0$

~~$f_x = 2z - 2px - 2qy$~~

f

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$2xz - px^2 - 2qxy + pq = 0$

auxiliary eqn

$$\frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dz}{-pf_p - qf_q}$$

$$= \frac{dx}{-f_p} = \frac{dy}{-f_q}$$

$f_x = 2z - 2px - 2qy$

$f_y = 2qx$

$f_z = 2x$

$f_p = -x^2 + q$

$f_q = p - 2xy$

$$\frac{dp}{2z-2qy} = \frac{dq}{2qx - 2qx} = \frac{dz}{px^2 - pq - pq + 2x}$$

$$= \frac{dx}{+x^2 - q} = \frac{dy}{-p + 2xy}$$

from second fraction $\underline{\underline{q = a}}$

$\underline{2xz - px^2 = 0}$

$\cancel{p = \frac{2z}{x}}$

$p(a - x^2) = 2axy - 2xz$

$p =$

$$dz = \frac{2z}{x}dx + 0$$

$$\cancel{dz = 2}$$

$$\frac{dz}{z} = \frac{2}{x}dx$$

$$\log z = 2\log x + \cancel{c}$$

$$\log\frac{z}{x^2} = a$$

$$\frac{z}{x^2} = b$$

$$p = \cancel{2a}\ \frac{2x(ay - z)}{(a - x^2)}$$

$$q = a$$

$$dz = \frac{2x(ay - z)}{a - x^2}dx + a\,dy$$

$$\frac{dz}{ay - z} - \frac{a\,dy}{ay - z} = \frac{2x\,\cancel{(ay-z)}}{(a - x^2)}dx$$

$$-\log(ay - z) = -\log(a - x^2) + b$$

$$\frac{ay - z}{a - x^2} = b$$

$$(ay - z) = b(a - x^2)$$

Qu ⇒ find complete Integral of

$yzp^2 - q = 0$

$$\frac{dp}{f_x + p f_z} = \frac{dq}{f_y + q f_z} = \frac{dz}{-p f_p - q f_q} = \frac{dx}{-f_p} = \frac{dy}{-f_q}$$

$f_x = 0$

$f_y = zp^2$ $\quad f_z = yp^2$

$f_p = 2pyz$, $f_q = -1$

$$\frac{dp}{yp^3} = \frac{dq}{zp^2 + yp^2 q} = \frac{dz}{-2p^2yz + q} =$$

$$\frac{dx}{-2pyz} = \frac{dy}{1}$$

$$\frac{dp}{}$$

$$\frac{-2}{p^2} = \frac{y^2}{2} + a$$

$p^2(y+a) = -2$

$$p = \sqrt{\frac{-2}{y+a}}$$

$$q = \frac{yz \times -2}{y+a}$$

$$\frac{dp}{yp^3} = \frac{dy}{1}$$

$$\frac{dp}{p^3} = y\,dy$$

$$\frac{-1}{\cancel{2}p^2} \cancel{dp} = \frac{y^2}{\cancel{2}} + \frac{a^2}{\cancel{2}}$$

$$\frac{1}{p^2} = -(y^2+a)$$

$$\cancel{p = \sqrt{\frac{1}{-(y^2+a)}}}$$

$$\cancel{q = \frac{-yz}{(y^2+a)}}$$

$$\cancel{dz = \sqrt{\frac{-1}{y^2+a}}\,dx + \frac{yz\,dy}{(y^2+a)}}$$

$$\cancel{\frac{\left(dz + \frac{yz\,dy}{y^2+a}\right)}{\sqrt{\frac{-1}{y^2+a}}} = dx}$$

$$\frac{1}{\cancel{p}} \quad \frac{1}{p^2} = a^2 - y^2$$

$$p = \sqrt{\frac{1}{a^2-y^2}}$$

$$q = yz\sqrt{\frac{1}{a^2-y^2}}$$

$$dz = \frac{1}{\sqrt{a^2-y^2}}\,dx + \frac{yz}{\sqrt{a^2-y^2}}\,dy$$

$$\cancel{dx = (dz - yz\,dy)}$$

$dx = \sqrt{a^2 - y^2}$ $dz = \dfrac{-yz\,dy}{\sqrt{a^2-y^2}}$

$$pxy + pq + qy = yz$$

$$\frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dz}{-pf_p - qf_q} =$$

$$\frac{dx}{-f_p} = \frac{dy}{-f_q}$$

$$f_x = py, \quad f_y = px + q - z$$

$$f_z = -y, \quad f_p = xy + q, \quad f_q = p + y$$

$$\frac{dp}{py - py} = \frac{dq}{px + q - z - qy}$$

$$\underline{p = a}$$

$$axy + aq + qy = yz$$

$$q(a+y) = yz - axy$$

$$q = \frac{yz - axy}{(a+y)}$$

$$dz = a\,dx + \frac{yz - axy}{(a+y)}\,dy$$

$$\cancel{dz + \frac{axy - yz}{(a+y)}\,dy = a\,dx}$$

$$\cancel{(a+y)\,dz = a(a+y)\,dx + (yz - axy)\,dy}$$

$$\cancel{a\,dz + y\,dz - yz\,dy = a^2 dx + ay\,dx - axy\,dy}$$

$$dz - a\,dx = \frac{yz - axy}{a+y}\,dy$$

$$d(z - ax) = \frac{y(z - ax)}{(a+y)}\,dy$$

$$\frac{d(z - ax)}{(z - ax)} = \frac{y}{(a+y)}\,dy$$

$$\log(z - ax) = y - k\log(y + a) + b$$

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$D^n + A_1 D^{n-1} D' + A_2 D^{n-2} D'^2 + \cdots A_n D'^n) z$

$= f(x,y)$

$D = \frac{\partial}{\partial x}, \quad D' = \frac{\partial}{\partial y}$ ①

$(D^2 + 1) = e^x$

A.E $\Rightarrow m^2 + 1 = 0$ {replacing $D = m$, $D' = 1$}

$m = \pm 1$

$C.F = c_1 \cos x + c_2 \sin x$

auxiliary eqn

$A_0 m^n + A_1 m^{n-1} + \cdots A_n = 0$

$= m_1, m_2, m_3 \cdots m_n$ (distined roots)

$C.F = \phi_1(y + m_1 x) + \phi_2(m_2 x) + \cdots \phi_n(y + m_n x)$

where $\phi_1, \phi_2 \cdots \phi_n$ are arbitary functions.

If $m = m'$ (repeated n times)

$C.F = \phi_1(y + m'x) + x\phi_2(y + m'x) + \cdots$

$\cdots + x^{n-1}\phi_n(y + m'x)$

If non-repeated factor D' the left hand side of (1), part of C.F. will be

$C.F. = \phi(x)$

Corresponding to repeated factor $(D')^m$ in L.H.S. the part of C.F. will be

$C.F. = \phi_1(x) + y\,\phi_2(x) + \dots y^{m-1}\phi_m$

<u>Question</u> ⇒ solve $2r + 5s + 2t = 0$

~~$2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial y^2} + 2\,\partial$~~

$2\frac{\partial^2 z}{\partial x^2} + 5\times\frac{\partial^2 z}{\partial x\,\partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$

$z\left(2(D)^2 + 5\times DD' + 2\times(D')^2\right) = 0$ → (1)

auxilliary eqn of (1) is

$2m^2 + 5m + 2 = 0$ —— (2)

$2m^2 + 4m + m + 2 = 0$

$2m(m+2) + 1(m+2) = 0$

$m = -2, \frac{-1}{2}$

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p, q, r, s, t

$\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}$

therefore C.F =

$$C.F = \phi_1(y-2x) + \phi_2(y - \tfrac{1}{2}x)$$

$$\text{or } C.F. = \phi_1(y-2x) + \phi_2(2y-x)$$

general solution is

$$z = \phi_1(y-2x) + \phi_2(2y-x)$$

Ans

Question ⇒ solve $(D^3(D')^2 + D^2(D')^3)\,z = 0$ —— (1)

auxiliary eqn of (1) is

~~m^3~~

$$(D')^2(D^3 + D^2 D')\,z = 0$$

$$(D')^2 D^2 (D + D')\,z = 0$$

$$m^2(m+1) = 0$$

$$m = 0, 0, -1$$

$$CF = \phi_1(y+0x) + x\phi_2(y+0x) + \phi_3(y-x)$$

$$CF = \phi_1(y) + x\phi_2(y) + \phi_3(y-x) + \phi_4(x) + y\phi_5(x)$$

solve $(D^3 D' - 4D^2 D'^2 + 4D D'^3) z = 0$

$D' D (D^2 - 4DD' + 4D'^2) z = 0$

~~the~~ auxilliary eqn is →

$m(m^2 - 4m + 4) = 0$

$m(m^2 - 2m - 2m + 4) = 0$

$m((m-2)(m-2)) = 0$

$m = 0, 2, 2$

$C.F = \phi_1(y+2x) + x\,\phi_2(y+2x) + \phi_3(y+0x)$
$+ \phi_4(x)$

$C.F = \phi_1(y+2x) + x\,\phi_2(y+2x) + \phi_3(y)$
$+ \phi_4(x)$

general eqn is

$z = \phi_1(y+2x) + x\,\phi_2(y+2x) + \phi_3(y)$
$+ \phi_4(x)$

Question ⇒ Solve $(D^4 + D'^4)z = 0$

$m^4 + 1 = 0$

~~m = 1, i, -i, -1~~

$m = \frac{1 \pm i}{\sqrt{2}}, \frac{-1 \pm i}{\sqrt{2}}$

$(m^2 + 1)^2 - 2m^2 = 0$

$z_1 = \frac{-1+i}{\sqrt{2}}, \quad z_2 = \frac{-1-i}{\sqrt{2}}, \quad z_3 = \frac{1+i}{\sqrt{2}}, z_4 = \frac{1-i}{\sqrt{2}}$

$C.F. = \phi_1(y + z_1 x) + \phi_2(y + z_2 x) + \phi_3(y + z_3 x) + \phi_4(y + z_4 x)$

general eqn is

$z = C.F.$

Working rules for finding P.I. when R.H.S. function $f(x,y)$ is of the form $\phi(ax+by)$

$$P.I. = \frac{1}{f(D,D')}\phi(ax+by)$$

$$= \frac{1}{f(a,b)} \iint \cdots \iint \phi(v)\, dv\, dv\, dv \ldots dv$$

where $v = (ax+by)$

provided $f(a,b) \neq 0$

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If $f(a,b) = 0$

$$\frac{1}{(bD - aD')^n}\phi(ax+by) = \frac{x^n}{b^n \lfloor n}\phi(ax+by)$$

Question ⇒ solve $(D^2 + 3DD' + 2D'^2)z = x+y$

auxilliary eqn is

$$(m^2 + 3m + 2) = 0$$

$$m = -1,\ m - 2$$

$$C.F = \phi_1(y-x) + \phi_2(y-2x)$$

$$P.I. = \frac{1}{D^2 + 3DD' + 2D'^2}(x+y)$$

$$a = 1,\ b = 1$$

$$= \frac{1}{1^2 + 3\times1\times1 + 2\times1^2}\iint v\,dv\,dv$$

$$v = (x+y)$$

$$= \frac{1}{6}\times\frac{v^3}{6}$$

$$= \frac{1}{36}(x+y)^3$$

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general solution

$$z = \phi_1(y-x) + \phi_2(y-2x) + \frac{1}{36}(x+y)^3$$

Ques ⇒ $r + s - 2t = (2x+y)^{1/2}$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = (2x+y)^{1/2}$$

$$D^2 + DD' - 2D'^2 = (2x+y)^{1/2}$$

auxiliary eqn is

~~$m^2 + 1 - 2 = 0$~~

$m^2 + m - 2 = 0$

$m^2 + 2m - m - 2 = 0$

$m = -2, \; m = 1$

$$C.F = \phi_1(y-2x) + \phi_2(y+x)$$

$$P.I. = \frac{1}{D^2 + DD' - 2D'^2}\sqrt{(2x+y)}$$

$$\phi(ax+by) = \sqrt{2x+y}$$

$a = 2, \; b = 1$

$$P.I = \frac{1}{4+2-2}\iint v^{1/2}\,dv\,dv$$

~~$P.I = \frac{1}{4} \times \frac{v^3}{6}$~~

~~$P.I = \frac{1}{24} \times$~~

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$$P.I = \frac{\cancel{6}}{\cancel{3}} \times V^{5/2} \qquad \times \frac{1}{4} \times \frac{2}{3} \times \frac{1}{5}$$

$$P.I = \frac{1}{15} V^{5/2}$$

$$P.I = \frac{1}{15}(2x+y)^{5/2}$$

$$G.S = C.F + P.I$$

Solve →

$$(D^3 - 6D^2D' + 11DD'^2 - 6D'^3)z = e^{5x+6y}$$

auxiliary eqn is

$$m^3 - 6m^2 + 11m - 6 = 0$$

$$(m-1)\overline{)m^3 - 6m^2 + 11m - 6}(m^2 - 5m + 6$$

$$\cancel{m^3} - m^2$$

$$-\quad +$$

$$-\cancel{5m^2} + 11m$$

$$-5m^2 + 5m$$

$$+\quad -$$

$$6m - 6$$

$$6m - 6$$

$$(m-1)(m^2 - 3m - 2m + 6) = 0$$

$$(m-1)(m-3)(m-2) = 0$$

$$m = 1, 2, 3$$

$C.F = \phi_1(y+x) + \phi_2(y+2x) + \phi_3(y+3x)$

$$P.I = \frac{1}{D^3 - 6D^2D' + 11DD'^2 - 6D'^3}(e^{5x+6y})$$

$a = 5, \; b = 6$

$$\frac{1}{125 - 150\times 6 + 11\times 180 - 6\times 216}$$

$$\frac{1}{125 + 1980 - 900 - 1296}$$

$$\frac{1}{2105 - 2196}$$

$$-\frac{1}{91}$$

$$P.I = -\frac{1}{91}\iiint e^v \, dv$$

$$P.I = -\frac{1}{91}e^{(5x+6y)}$$

general solution $= C.F + P.I.$

Question $\Rightarrow$ solve $r - t = x - y$

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$$

$$D^2 - D'^2 = x - y$$

auxiliary eqn $\Rightarrow$

$m^2 - 1 = 0$

$m = \pm 1$

$C.F = \phi_1(y+x) + \phi_2(y-x)$

$$P.I = \frac{1}{D^2 - D'^2}(x-y)$$

$a = 1, b = -1$

$$\Rightarrow \cancel{\frac{x^4}{(-1)^2 \lfloor 2} \phi(x-y)}$$

$$\Rightarrow \cancel{\frac{x^4}{2} \phi(x-y)}$$

$$\Rightarrow \frac{1}{(D+D')(D-D')}$$

$\rightarrow$ to put $D = 1, D' = -1$ then $D+D' = 0$

$$\Rightarrow \frac{1}{(D+D')}\left[\frac{1}{1 - (-1)} \int v\, dv\right]$$

$$\Rightarrow \frac{1}{(D+D')}\left[\frac{1}{2} \times \frac{(x-y)^2}{2}\right]$$

$$= \frac{1}{4(D+D')}(x-y)^2$$

$$\Rightarrow \frac{1}{-4(-D-D')}(x-y)^2$$

<u>$n = 1$</u>

$$\Rightarrow \frac{-1}{4}\left(\frac{x^1}{(-1)^1 \lfloor 1} \cancel{\phi^1} (x-y)\right)$$

$$\Rightarrow \frac{-1}{4}\left(\frac{x}{-1} (x-y)^2\right)$$

$$\Rightarrow \frac{x}{4} (x-y)^2 \Rightarrow P.I.$$

general solution = CF + PI.

Question $\Rightarrow$ solve $(2D^2 - 5DD' + 2D'^2)z = \cancel{5}$ $5\sin(2x+y)$

auxiliary eqn $\Rightarrow$

$2m^2 - 5m + 2 = 0$

$2m^2 - 4m - m + 2 = 0$

$m = 2,\; m = \frac{1}{2}$

$C.F = \phi_1(y+2x) + \phi_2(2y+x)$

$$P.I = \frac{1}{2D^2 - 5DD' + 2D'^2} 5\sin(2x+y)$$

$$= \frac{1}{\cancel{(m-2)(}}$$

$a = 2$

$b = 1$

$$P.I = \frac{1}{(2D-D')(D-2D')} 5\sin(2x+y)$$

$$P.I = \frac{5 \times 1}{(D-2D')}\left[\frac{1}{(2D-D')}\int \sin v \, dv\right]$$

$$P.I = \frac{5 \times 1}{(D-2D')} \times \frac{1}{(2D-D')} \times -\cos(2x+y)$$

$$P.I. = \frac{5 \times}{D-2D'} \times \frac{1}{3} \times (-\cos(2x+y))$$

$$P.I = \frac{5}{3}\left[\frac{1}{D-2D'}(-\cos(2x+y))\right]$$

$$P.I = \frac{5}{3} \times \frac{x^1}{(1)^1 \, \underline{|1}}(-\cos(2x+y)) \qquad \begin{matrix} b=1 \\ a=2 \end{matrix}$$

$$P.I = \frac{-5}{3} x \cos(2x+y)$$

general solution = C.F + P.I.

\#

if R.H.S function $f(x,y)$ is of the form $x^m y^n$,

Question: $(D^2 - a^2 D'^2) z = x$

a.e is

$m^2 - a^2 = 0$

~~m = a f~~ $m = a, -a$

~~C.F = f_1 y(a + x)~~

$C.F = \phi_1(y+ax) + \phi_2(y-ax)$

$$P.I. = \frac{1}{(D^2 - a^2 D'^2)} x$$

$$P.I. = \frac{1}{D^2}\left[\frac{1}{\left(1 - a^2 \frac{D'^2}{D^2}\right)}\right] x$$

$$P.I. = \frac{1}{D^2}\left[1 - a^2 \frac{D'^2}{D^2}\right]^{-1} x$$

$$P.I. = \frac{1}{D^2}\left[1 + a^2\frac{D'^2}{D^2} + \left(a^2 \frac{D'^2}{D^2}\right)^2 + \left(\frac{a^2 D'^2}{D^2}\right)^3 + \cdots\right] x$$

$a=1$, $b=0$

$$P.I = \frac{1}{D^2} x \Rightarrow \frac{x^3}{6}$$

general solut $= \phi_1(y+ax) + \phi_2(y-ax) + \frac{x^3}{6}$

another approach for P.I.

$$P.I = \frac{1}{(D^2 - a^2 D'^2)} x$$

$$= \frac{1}{-a^2 D'^2 \left(1 - \frac{D^2}{a^2 D'^2}\right)} x$$

$$= \frac{-1}{a^2 D'^2}\left[1 + \frac{D^2}{a^2 D'^2} + \frac{D^4}{a^4 D'^4} + \cdots\right] x$$

$$= \frac{-1}{a^2 D'^2}(x + 0)$$

$$= \frac{-1}{a^2} \times x \times \frac{y^2}{2} = -\frac{1}{2a^2} xy^2$$

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now general solut. $\Rightarrow$ C.F + P.I

$$g.s. = C.F + \frac{1}{-2a^2} xy^2$$

Question $\Rightarrow$ Solve $\frac{\partial^3 z}{\partial x^3} - \frac{\partial^3 z}{\partial y^3} = x^3 y^3$

auxiliary eqn $\Rightarrow$

$m^3 - 1 = 0$

$m = 1, \omega, \omega^2$

~~$\phi_1\, y($~~

$\phi_1(y+x) + \phi_2(y+\omega x) + \phi_3(y+\omega^2 x)$

$$\frac{1}{D^3 - D'^3}(x^3y^3)$$

general

#

$$\frac{}{\left(\frac{D'^3}{D^3}\right)}(x^3y^3)$$

if

form x^m $\left(1 - \frac{D'^3}{D^3}\right)^{-1}(x^3y^3)$

Question $\Rightarrow$ $\left(1 + \frac{D'^3}{D^3} + \frac{D'^6}{D^6} + \frac{D'^9}{D^9} + \cdots\right)x^3y^3$

g.e. is $\frac{D'^3}{D^3}(x^3y^3) + 0\Big)$

$m^2 - a^2$

~~$m = a +$~~ $\frac{1}{D^6}\, x^6\Big)$

~~$C.F = \phi_1\, y($~~

$$P.I = \frac{1}{D^3}\left[x^3y^3 + \frac{x^6}{30}\right]$$

$$P.I = y^3 \times \frac{1}{120}x^6 + \frac{x^9}{6\times7\times8\times30}$$

$$= \frac{x^6y^3}{120} + \frac{x^9}{10080}$$

$$G.S. = C.F + P.I.$$

Qu ⇒ $(D^2 - 2DD' + D'^2)z = (e^{x+2y} + x^3)$

A.E. ⇒

$$m^2 - 2m + 1 = 0$$

$$m = 1, 1$$

$$C.F = \phi_1(x+y) + x\,\phi_2(y+x)$$

$$P.I = \frac{1}{(D^2 - 2DD' + D'^2)}(e^{x+2y} + x^3)$$

$$P.I = \frac{1}{(D^2 - 2DD' + D'^2)}e^{x+2y} + \frac{1}{D^2 - 2DD' + D'^2}x^3$$

$$= \frac{1}{1}\iint(e^v\,du)\,du + \frac{1}{(D-D')^2}x^3$$

$$= e^{x+2y} + \frac{1}{D^2}\left(1 - \frac{D'}{D}\right)^{-2}x^3$$

$$= e^{x+2y} + \frac{1}{D^2}\left(1 + \frac{2D'}{D} + \frac{3(D')^2}{D^2} + \frac{4(D')^3}{D^3} + \dots\right)$$

$$= e^{x+2y} + \frac{1}{D^2}\left(x^3 + 0\right)$$

$$P.I. = e^{x+2y} + \frac{x^5}{4 \times 5} = e^{x+2y} + \frac{x^5}{20}$$

$q.s = C.F + P.I$

~~Questions~~

General method for finding Particular Integral (P.I.)

In this case we shall use the following Formulas

Formula -1

$$\frac{1}{D - mD'} f(x,y) = \int f(x, c - mx)\, dx$$

where $c = y + mx$

Formula -2

$$\frac{1}{D + mD'} f(x,y) = \int f(x, c + mx)\, dx$$

where $c = y - mx$

Que) solve $(D^2 - DD' - 2D'^2)z = (y-1)e^x$

A.E.

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$m = 2, \; m = -1$

$C.F = \phi_1(y+2x) + \phi_2(y-x)$

$$P.I. = \frac{1}{(D^2 - DD' - 2D'^2)}(y-1)e^x$$

$$= \frac{1}{(D-2D')(D+D')}(y-1)e^x$$

$$= \frac{1}{(D+D')}\left[\frac{1}{(D-2D')}(y-1)e^x\right]$$

$$= \frac{1}{D+D'}\left[\int (c-2x-1)e^x\right] \quad c = y+2x$$

$$= \frac{1}{D+D'}\left[(c-2x-1)e^x + 2e^x\right]$$

$$= \frac{1}{(D+D')}\left[(c-2x+1)e^x\right]$$

$$= \frac{1}{(D+D')}\left[(y+2x-2x+1)e^x\right]$$

$$= \frac{1}{(D+D')}(y+1)e^x$$

$$\Rightarrow \frac{1}{(c'+x+1)}$$

$$= \int (c'+x+1)\, e^x\, dx \quad \text{where } c' = y.$$

$$= \int c' e^x\, dx + \int x e^x\, dx + \int e^x\, dx$$

$$= \int (y-x)\, e^x\, dx + \int x e^x\, dx + \int e^x\, dx$$

$$P.I. = y e^x + e^x$$

$$G.S = \phi_1(y+2x) + \phi_2(y-x) + y e^x + e^x$$

Question $\Rightarrow$ $r + s - 6t = y \cos x$

$$D^2 + DD' - 6(D')^2 = y \cos x$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial z}{\partial x\, \partial y} - 6\left(\frac{\partial z}{\partial y}\right)^2 = y \cos x$$

$$D^2 + DD' - 6(D')^2 = 0$$

A.E

$$m^2 + m - 6 = 0$$

$$m^2 + 3m - 2m - 6 = 0$$

$$m = -3,\ 2.$$

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$C.F = \phi_1(y-3x) + \phi_2(y+2x)$

$$P.I = \frac{1}{D^2 + DD' - 6(D')^2} y\cos x$$

$$P.I = \frac{1}{(D+3D')(D-2D')} y\cos x$$

$$P.I = \frac{1}{(D-2D')}\left[\frac{y\cos x}{(D+3D')}\right]$$

$$P.I = \frac{1}{D-2D'}\left[\int (c+3x)\cos x\, dx\right]$$

$c = y - 3x$

$$P.I = \frac{1}{(D-2D')}\left[(c+3x)\sin x + 3\cos x\right]$$

$$P.I = \frac{1}{(D-2D')}\left[(y - 3x + 3x)\sin x + 3\cos x\right]$$

$$= \frac{1}{D-2D'}\left[y\sin x + 3\cos x\right]$$

$$= \int ((c' - 2x)\sin x + 3\cos x)\, dx$$

$y = c' - 2x$

$$= \int (c' - 2x) - \cos x + 2\sin x + 3\sin x$$

$$= \left[(c' - 2x) - \cos x + 5\sin x\right]$$

$P.I. = -y\cos x + \sin x$

$G.S. = C.F. + (\sin x - y\cos x)$

<u>Ques</u>: Solve $(D^2 + DD' + D'^2)z = 2\cos y - x\sin y$

A.E.

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 =$

$(m+1)^2 =$

$m = -1, -1$

$C.F. = \phi_1(y - x) + x\,\phi_2(y - x)$

$$P.I. = \frac{1}{D^2 + DD' + D'^2}(2\cos y - x\sin y)$$

$$P.I. = \frac{1}{(D+D')(D+D')}(2\cos y - x\sin y)$$

$$= \frac{1}{(D+D')}\left[\int 2\cos(c+x) - x\sin(c+x)\,dx\right.$$

$y = c + x$

$$= \frac{1}{(D+D')}\left[2\sin(c+x) - [-x\cos(c+x) + \sin(c+x)]\right.$$

$= \frac{1}{D+D'}\left[\sin(c+x) + x\cos(c+x)\right]$

$= \frac{1}{D+D'}\left[\sin y + x\cos y\right]$

$= \int \underline{\sin(c+x) + x\cos(c+x)}\, dx$

$= \cancel{-\cos(c+x)} + x\sin(c+x) + \cancel{\cos(c+x)}$

$= x\sin(c+x) \cancel{-2\cos(c+x)}$

$= x\sin y \cancel{-2\cos y}$

general solutn $\Rightarrow$ $C.F + x\sin y.$

Qus $\Rightarrow$ solve $(D^2 - DD' - 2D'^2)z =$ ~~$(2x^2+xy-y^2)$~~

$= (2x^2 + xy - y^2)\sin xy - \cos xy$

A.E $\Rightarrow$ $m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$m(m-2) + 1(m-2) = 0$

$m = -1,\ m = 2$

$C.F = \phi_1(y-x) + \phi_2(y+2x)$

$$P.I = \frac{(2x^2+xy-y^2)(\sin xy) - \cos xy)}{(D+D')(D-2D')}$$

$$= \frac{1}{(D-2D')}\left[\int (2x^2 + x(c+x) - (c+x)^2)\sin x(c+x) - \cos x(c+x)\, dx\right]$$

$y = c+x$

$$= \cancel{\frac{1}{(D-2D')}\left[\frac{2x^3}{3}\right.}$$

$$= \frac{1}{(D-2D')}\left[\int (2x^2 - cx - c^2)\sin x(c+x) - \cos x(c+x)\, dx\right]$$

$$= \cancel{\frac{1}{(D-2D')}\left[(2x^2 - cx - c^2)\right.}$$

$$= \frac{1}{(D-2D')}\left[\int (c+2x)(x-c)\sin(x^2+cx) - \cos(cx+x^2)\, dx\right]$$

$$= \frac{1}{D-2D'}\left[(x-c)\int (c+2x)\sin(x^2+cx) - \iint (c+2x)\sin(x^2+cx) - \int \cos(cx+xy)\,dx\right]$$

$x^2 + cx = t$

$(2x+c)\,dx = dt$

$\int \sin t \, dt$

$-\cos t$

$$= \frac{1}{D-2D'}\left[(x-c)\times -\cos(x^2+cx) + \int \cos(\cancel{x^2+cx})\,dx - \int \cos(\cancel{cx+xy})\,dx\right]$$

$$= \frac{1}{D-2D'}\left[- \cos(x^2 + c\, x)\,(x-c) \right]$$

$$= \int \cancel{x-c)(c-x)\cos(x^2+c\,x)}\, dx$$

$$= \frac{1}{D-2D'}\left[(y+\cancel{2x}) \left(\cos(x^2+4x-x^2)\right)\right]$$

$$= \frac{1}{D-2D'}\left[y \cos 4x \right]$$

$$= \int \underline{(c-2x)\cos(c-2x)x\, dx}$$

$$= \int (c-2x)\cos(cx-2x^2)\, dx$$

$$cx - 2x^2 = t$$

$$c - 4x = dt$$

$$(c-2x)\,dx = dt + 2x$$

$$= \int \cos t \times (dt + 2x)$$

$$= \int \cos t\, dt + \cos t\, 2x$$

Que - solve $(D^2 + DD' - 6D'^2)z = x^2 \sin(x+y)$

Ques-11 : $(D^3 + 2D^2D' - DD'^2 - 2D'^3)z = (y+2)e^x$

Q. $(D^3 + 3DD'^2 - 2D'^3)z = \cos(x+2y) - e^y(3+2x)$

dimensional

Heat flow equation

the one dimensional heat flow equation is

$$\frac{\partial y}{\partial t} = c^2 \frac{\partial^2 y}{\partial x^2} \longleftarrow (1)$$

Let, $y(x,t) = X(x)\, T(t)$

be a solution of eqn (1)

$$\frac{\partial^2 y}{\partial x^2} = X''(x)\, T(t)$$

$$\frac{\partial y}{\partial t} = X(x)\, T'(t)$$

substituting in eqn (1)

$$X(x)\, T'(t) = c^2 \left(X''(x)\, T(t) \right)$$

$$\frac{X''(x)}{X(x)} = \frac{1}{c^2} \frac{T'(t)}{T(t)} = K \text{ ——— say } (2)$$

considering $\frac{X''(x)}{X(x)} = K$, we get

$$X''(x) - KX(x) = 0 \text{ ——— } (3)$$

Again, considering

$$\frac{1}{c^2} \frac{T'(t)}{T(t)} = K$$

$$T'(t) = c^2 K T(t) \quad \text{--- (4)}$$

from eq^n (3)

$$\frac{\partial^2 X}{\partial x^2} - KX = 0$$

The following cases arising according to the value of 'K'

Case 1 ⇒ Let 'K' be +ve say, m^2

from eq^n (4) we get

$$\frac{d^2 X}{dx^2} - KX = 0$$

A.E.
$$D^2 - m^2 = 0$$

$$D = \pm m$$

$$C.F = C_1 e^{mx} + C_2 . e^{-mx} \quad \text{--- (5)}$$

from eqn (4)

$$T'(t) = c^2 m^2 \, T(t)$$

$$\frac{\partial T}{\partial t} = c^2 m^2 T$$

$$\int \frac{\partial T}{T} = \int c^2 m^2 \, \partial t$$

$$\log T = c^2 m^2 t + \log c_3$$

$$\frac{T}{c_3} = e^{c^2 m^2 t}$$

$$T(t) = c_3 \, e^{c^2 m^2 t} \longrightarrow (6)$$

i.e. $u(x,t) = (c_1 e^{mx} + c_2 e^{-mx}) \, c_3 e^{c^2 m^2 t}$

$\longrightarrow (7)$

Case - 2

If k be $-ve$, say $-km^2$

from eqn (3) we get

$$\frac{\partial^2 X}{\partial x^2} - kX = 0$$

$$\frac{\partial^2 X}{\partial x^2} + m^2 X = 0$$

$$D^2 + m^2 = 0$$

$$D = \pm m i$$

$$X(x) = C_4 \cos mx + C_5 \sin mx \quad \text{(8)}$$

from eqn (4) 0

$$T' = -c^2 m^2 t$$

$$\int \frac{dT}{T} = \int c^2 m^2 \, dt$$

$$\log T = -c^2 m^2 t + \log C_6$$

$$T(t) = C_6 \, e^{-c^2 m^2 t} \quad \text{(9)}$$

$$u(x,t) = (C_4 \cos mx + C_5 \sin mx) \, C_6 \, e^{-c^2 m^2 t}$$

<u>case-3</u> <u>$K = 0$</u>

from eqn 3 $\frac{d^2 X}{dx^2} - K X(x) = 0$

$$\frac{d^2 X}{dx^2} = 0$$

$$X(x) = (C_7 + C_8 x)$$

from eqn (4) we get

$T = C_9$

$\frac{\partial T}{\partial t} = 0$

$u(x,t) = (C_7 + C_8 x) C_9$ —— (10)

~~How~~ out of these 3 solutions, a solution is chosen which is consistent ~~now~~ with the ~~Pro~~ physible nature of problem

Since we are dealing with the problem of heat conduction the temperature must decrease in increase of time.

Hence the solution of heat flow eqn is

~~$u(x,t) = C_4$ (const)~~

$u(x,t) = (C_4 \cos mx + C_5 \sin mx) C_6 e^{-m^2 c^2 t}$

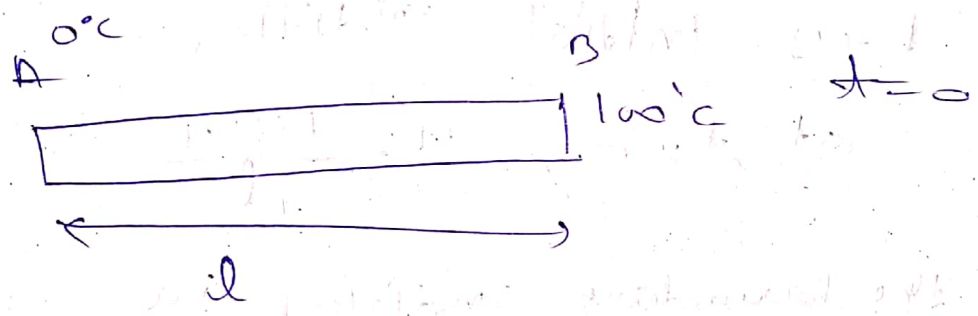
$u(x,t) = (A \cos mx + B \sin mx) e^{-c^2 m^2 t}$

$A = C_4 C_6, \quad B = C_5 C_6$

are arbitrary constants,

A condition is known as "steady state" if the depending variables are independent of time (t)

Problem: A literally equivalent bar of length 'l' has its ends A, B maintain at 0°C and 100°C respectively until steady state condition prevail. If temperature of B is suddenly reduces to 0°C and kept so while that of A is maintain at 0°C. Find the temperature at distance 'x' from A at any time 't'.



the eqn of heat conduction is

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{(1)}$$

at steady state condition at t=0

u is independent of t

i.e. t=0, $\frac{\partial u}{\partial t} = 0$

eqn (1) reduces to

$$\frac{\partial^2 u}{\partial x^2} = 0 \quad \text{—— (2)}$$

solving eqn (2)

$$u = ~~c~~ ax + b \quad \text{—— (3)}$$

$$u = ax + b$$

at $x = 0$, $u = 0$, $b = 0$

at $x = ~~l~~ l$, $u = 100$

$$a = \frac{100}{l}, \; b = 0$$

$$u = \frac{100}{l} x \quad \text{—— (4)}$$

they initial condition

at $t = 0$, $u = \frac{100x}{l}$

the boundary conditions are

(i) at $x = 0$, $u = 0$ for all t

(ii) at $x = l$, $u = 0$, for all t,

i.e. $u(l, t) = 0$

since 'u' decreases as 't' increases

the solution of eqn (1) will be

of form

$$u = (c_1 \cos mx + c_2 \sin mx) e^{-c^2 m^2 t} \quad \text{—— (8)}$$

now applying the condition (2) in eqn (8)

$$0 = c_1 e^{-c^2 m^2 t}$$

therefor $c_1 = 0$

$$u = (c_2 \sin mx) e^{-c^2 m^2 t} \quad \text{—— (6)}$$

now applying condition (3) in eqn (6)

we get,

$$0 = c_2 \sin ml \, e^{-c^2 m^2 t}$$

$$c_2 \sin ml = 0$$

c_2 can't be zero

$$\sin ml = 0$$

$$\underline{ml = n\pi}$$

$$m = \frac{n\pi}{l} \quad \text{to put in eqn (6)}$$

$$u = c_2 \sin \frac{n\pi}{l} \, e^{-c^2 \left(\frac{n\pi}{l}\right)^2 t}$$

putting $n = 1, 2, 3 \, ------$

and adding all these solutions by principal of superposition general solution of eqn (1) will be

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{l} x \; e^{-c^2 \frac{n^2 \pi^2}{l^2} t} \quad \text{—— (7)}$$

∴ applying condition (I) in eqn (7) we get

$$\frac{100x}{l} = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$b_n = \frac{2}{l}\int_0^l \frac{100x}{l}\sin\frac{n\pi x}{l}\,dx$$

$$b_n = \frac{200}{l^2}\int_0^l x\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{200}{l^2}\left[x\left(-\cos\frac{n\pi x}{l}\right)\left(\frac{l}{n\pi}\right) + \sin\frac{n\pi x}{l}\left(\frac{l}{n\pi}\right)^2\right]_0^l$$

$$= \frac{200}{l^2} \times l \times \frac{l}{n\pi}(-\cos n\pi)$$

$$= \frac{200}{n\pi}(-1)^{n+1}$$

Ans ⇒

$$\sum_{n=1}^{100} \frac{200}{n\pi}(-1)^{n+1}\sin\frac{n\pi x}{l}\, e^{-\frac{c^2 n^2\pi^2}{l^2}t}$$

clairaut eqn

general form is

$$z = px + qy + f(p,q)$$

in this case g.s. will be

$$z = ax + by + f(a,b)$$