PROBABILITY.

Sample Space - The collection of all possible outcome of an experiment.
eg. tossing a coin one time $S = \{H, T\}$.
Rolling a dice one time $S = \{1, 2, 3, 4, 5, 6\}$.

Random Experiment - Experiments whose outcomes are unpredictable is known as random experiment.
Eg. Tossing an unbiased coin.
Draw a card from pack of 52 cards.

Event - The outcome of an experiment is known as events. mathematically event is subset of sample space.
eg. getting head while tossing a coin one time.
getting even no while rolling a dice.

Types of Event :-

1. Complementary Event : $\overset{\text{event}}{E}$, E^c or $\bar{E}$ it contains all the outcomes of sample space that is not in E.
$S = \{1, 2, 3, 4, 5, 6\}$ $E = [\text{odd no}] = \{1, 3, 5\}$

$$E^c = \{2, 4, 6\}$$

2. Equally Likely Events - E and F are said to be equally likely events if their probability are equal.

$$P(E) = P(F)$$

3. Mutually Exclusive Events - If no common outcome b/w 2 events.

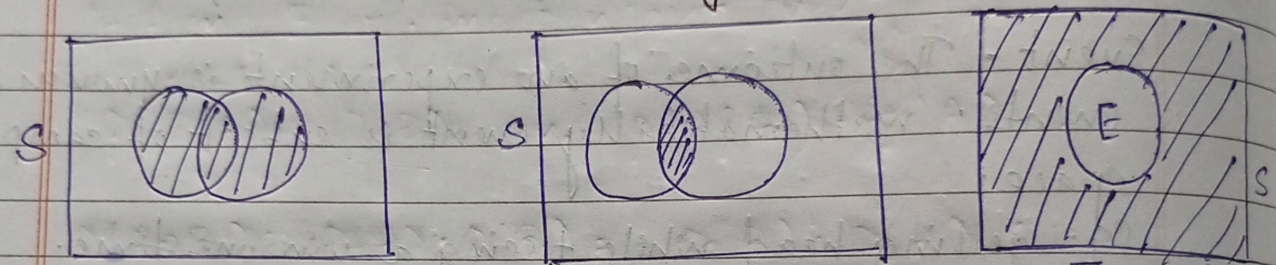
$$E \cap F = \phi \text{ (null set)}$$

Collective Exhaustive Events : E and F if both events gives complete sample space.

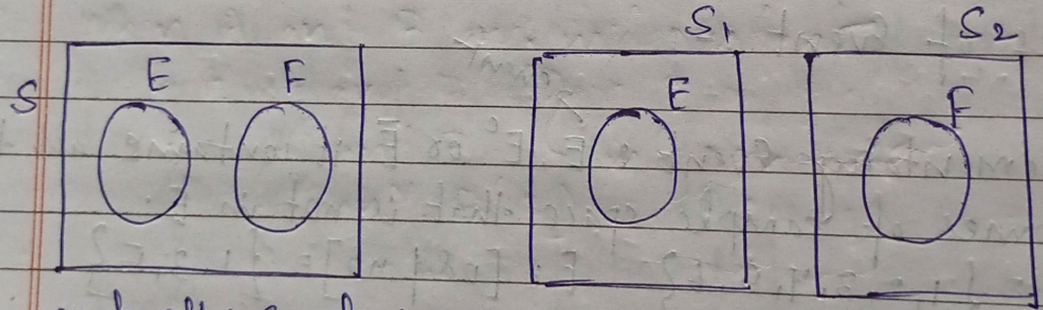
$$E \cup F = S.$$

Independent Events : Two events are independent if p(E) doesn't affect F has happened or not and vice versa.

Venn Diagram.



$E \cup F$ $\quad$ $E \cap F$ $\quad$ $\bar{E}$ or E^c



mutually Exclusive $\quad$ Independent events.

<u>probability</u> :- The ratio of no of ways an event can happen to the no of ways sample space can happen.

$$S = \{1, 2, 3, 4, 5, 6\} \quad n(S) = 6.$$

$$E(\text{odd no}) = \{1, 3, 5\} \quad n(S) = 3$$

$$P = \frac{n(E)}{n(S)} = 3/6 = 1/2.$$

Conditional probability :-

If their is an event E, $p(E) > 0$.
then probability of happening F when E is already happened

$$p(F/E) = \frac{p(F \cap E)}{p(E)}.$$

$$p(F/E) = \frac{\text{no of outcomes in } (F \cap E)}{\text{no of outcomes in } E}.$$

Rules of probability:

① $0 \leq p(E) \leq 1$

② $p(S) = 1$ (Definite event)

③ $p(\phi) = 0$ (Impossible event).

④ If event E has probability → $p(E)$

$$p(\bar{E}) = p(E^c) = 1 - p(E).$$

⑤ $p(A \cup B) = p(A) + p(B) - p(A \cap B)$.

For mutually exclusive events $p(A \cap B) = 0$

$$p(A \cup B) = p(A) + p(B).$$

⑥ $p(A \cup B \cup C) = p(A) + p(B) + p(C) - p(A \cap B) - p(B \cap C) - p(C \cap A) + p(A \cap B \cap C)$

~~Gate 2017~~ :

If A, B, C are mutually exclusive

$p(A \cup B \cup C) = p(A) + p(B) + p(C)$:

(6) If $A \subset B$ (If A subset of B)
$p(A) < p(B)$.

(7) $p(A \cap B) = p(A) \cdot p(B|A) = p(B) \cdot p(A|B)$

If A and B are independant event

$p(B|A) = p(B)$ and $p(A|B) = p(A)$.

$p(A \cap B) = p(A) \cdot p(B) = p(B) \cdot p(A)$:

(8) When A and B are independant
then $\left.\begin{array}{l} A \text{ and } B^c \\ A^c \text{ and } B \\ A^c \text{ and } B^c \end{array}\right\}$ independant.

(9) A, B and C are independant then
$\left.\begin{array}{l} A \text{ and } B \\ B \text{ and } C \\ C \text{ and } A \end{array}\right\}$ independant.

Gate Question:-

1. An exam consists of 2 papers, paper 1 and 2. The probability of failing in paper 1 is 0.3 and in 2 is 0.2. A student has failed in paper 2, the

prob of failing in paper 1 is 0.6. The probability of a student failing in both papers are.

→ A – Event of failing in paper 1 $p(A) = 0.3$
B – failing in paper 2. $p(B) = 0.2$

$p(A/B) = 0.6$.

then $p(A \cap B) = ?$

$p(A \cap B) = p(B) \cdot p(A/B)$
$= 0.2 \times 0.6 = 0.12$.

2. A fair dice is rolled twice. The probability that an odd no will follow an even no is

(a) 1/2 (b) 1/6
(c) 1/3 (d) 1/4

$S = \{1,2,3,4,5,6\}$

$p(o) = \frac{n(o)}{n(s)} = 3/6 = 1/2$

$p(e) = \frac{n(e)}{n(s)} = 3/6 = 1/2$

$p(o \cdot e) = p(one)$
$p(o) \cdot p(e)$
$= 1/2 \times 1/2 = 4$.

3. Suppose a fair 6-sided die is rolled once. If the value on the die is 1,2, or 3 the die is rolled for second time. What is the probability that the sum total of values that turn up is atleast 6?

$p(sum \geq 6) = p(6 \text{ in first time}) + p(1,5) + p(1,6)$
$+ p(2,4) + p(2,5) + p(2,6) + p(3,3)$

$+ p(3,4) + p(3,5) + p(3,6)$

$= \frac{1}{6} + 9 \times \frac{1}{36} = \frac{15}{36}$.

$p(1,5) = p(1) \cdot p(5) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Q. A single die is thrown twice. What is the probability that the sum is neither 8 nor 9.

$S = \{(1,1), (1,2), (1,3) \ldots (1,6)$
$(2,1) (2,2) \ldots (2,6)$
$\vdots$
$(6,1), (6,2) \ldots (6,6)\}$

$n(S) = 36$.

A = getting sum is either 8 or 9.

$A = \{(2,6), (3,5) (3,6) (4,4), (4,5) (5,3), (5,4), (6,2), (6,3)\}$.

$n(A) = 9$

$p(A) = \frac{n(A)}{n(S)} = \frac{9}{36} = 1/4$

For neigther 8 or 9 $\quad 1 - \frac{1}{4} = 3/4$.

Q. A fair coin is tossed till a head appears for the first time. The probability that the no of required tosses is odd is

1/3 $\qquad$ 1/2 $\qquad$ 2/3 $\qquad$ 3/4

→ p (no of tosses required should be odd)

→ = p (no of tosses 1, 3, 5, 7)

= p(1) + p(3) + p(5) + ...

p(1) = p (head in 1st toss) = $1/2$.

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p(3) = p(tail) × p(tail) × p(head) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

p(5) = p(tail) × p(tail) × p(tail) × p(tail) × p(tail) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \left(\frac{1}{2}\right)^5$.

$p(A) = p(1) + p(3) + p(5) + p(7) + \ldots\ldots$

$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \ldots$

$= \frac{1}{2} + \frac{1}{8} + \frac{1}{32}$

Geometric Series $a = 1/2$, $r = \frac{1/8}{1/2} = \frac{1}{4}$.

$= \frac{a}{1-r} = \frac{1/2}{1-1/4} = \frac{2}{3}$ (Ans).