

## CHAPTER 3

# *Data and Signals*

One of the major functions of the physical layer is to move data in the form of electromagnetic signals across a transmission medium. Whether you are collecting numerical statistics from another computer, sending animated pictures from a design workstation, or causing a bell to ring at a distant control center, you are working with the transmission of **data** across network connections.

Generally, the data usable to a person or application are not in a form that can be transmitted over a network. For example, a photograph must first be changed to a form that transmission media can accept. Transmission media work by conducting energy along a physical path.

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**To be transmitted, data must be transformed to electromagnetic signals.**

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### 3.1 ANALOG AND DIGITAL

Both data and the signals that represent them can be either **analog** or **digital** in form.

#### **Analog and Digital Data**

Data can be analog or digital. The term **analog data** refers to information that is continuous; **digital data** refers to information that has discrete states. For example, an analog clock that has hour, minute, and second hands gives information in a continuous form; the movements of the hands are continuous. On the other hand, a digital clock that reports the hours and the minutes will change suddenly from 8:05 to 8:06.

Analog data, such as the sounds made by a human voice, take on continuous values. When someone speaks, an analog wave is created in the air. This can be captured by a microphone and converted to an analog signal or sampled and converted to a digital signal.

Digital data take on discrete values. For example, data are stored in computer memory in the form of 0s and 1s. They can be converted to a digital signal or modulated into an analog signal for transmission across a medium.

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Data can be analog or digital. Analog data are continuous and take continuous values.  
Digital data have discrete states and take discrete values.

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## Analog and Digital Signals

Like the data they represent, signals can be either analog or digital. An analog signal has infinitely many levels of intensity over a period of time. As the wave moves from value *A* to value *B*, it passes through and includes an infinite number of values along its path. A digital signal, on the other hand, can have only a limited number of defined values. Although each value can be any number, it is often as simple as 1 and 0.

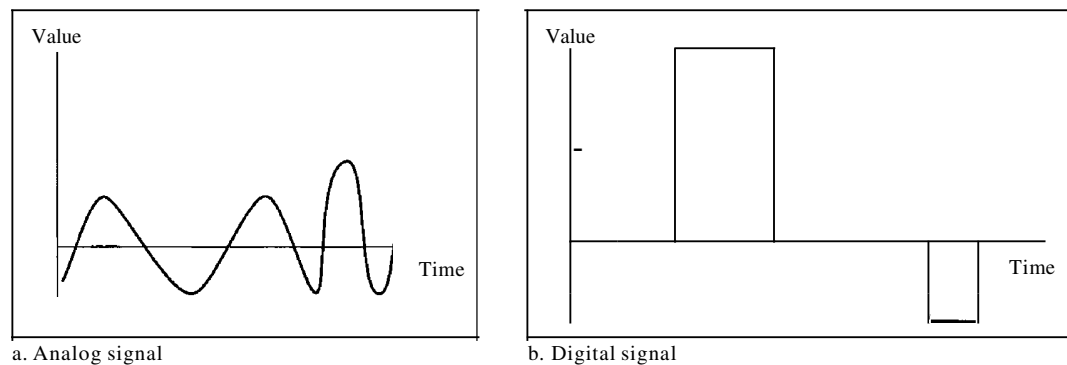
The simplest way to show signals is by plotting them on a pair of perpendicular axes. The vertical axis represents the value or strength of a signal. The horizontal axis represents time. Figure 3.1 illustrates an analog signal and a digital signal. The curve representing the analog signal passes through an infinite number of points. The vertical lines of the digital signal, however, demonstrate the sudden jump that the signal makes from value to value.

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Signals can be analog or digital. Analog signals can have an infinite number of values in a range; digital signals can have only a limited number of values.

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Figure 3.1 Comparison of analog and digital signals



## Periodic and Nonperiodic Signals

Both analog and digital signals can take one of two forms: *periodic* or *nonperiodic* (sometimes refer to as *aperiodic*, because the prefix *a* in Greek means "non").

A periodic signal completes a pattern within a measurable time frame, called a period, and repeats that pattern over subsequent identical periods. The completion of one full pattern is called a cycle. A nonperiodic signal changes without exhibiting a pattern or cycle that repeats over time.

Both analog and digital signals can be periodic or nonperiodic. In data communications, we commonly use periodic analog signals (because they need less bandwidth,

as we will see in Chapter 5) and nonperiodic digital signals (because they can represent variation in data, as we will see in Chapter 6).

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In data communications, we commonly use periodic  
analog signals and nonperiodic digital signals.

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## 3.2 PERIODIC ANALOG SIGNALS

Periodic analog signals can be classified as simple or composite. A simple periodic analog signal, a sine wave, cannot be decomposed into simpler signals. A composite periodic analog signal is composed of multiple sine waves.

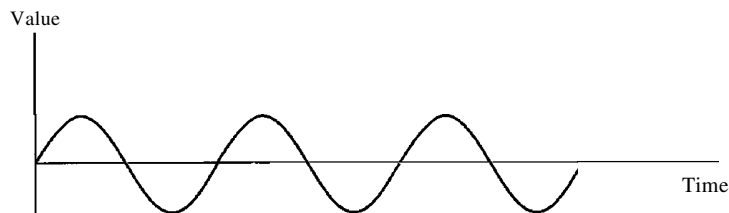
### Sine Wave

The sine wave is the most fundamental form of a periodic analog signal. When we visualize it as a simple oscillating curve, its change over the course of a cycle is smooth and consistent, a continuous, rolling flow. Figure 3.2 shows a sine wave. Each cycle consists of a single arc above the time axis followed by a single arc below it.

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Figure 3.2 A sine wave

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We discuss a mathematical approach to sine waves in Appendix C.

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A sine wave can be represented by three parameters: the *peak amplitude*, the *frequency*, and the *phase*. These three parameters fully describe a sine wave.

### Peak Amplitude

The peak amplitude of a signal is the absolute value of its highest intensity, proportional to the energy it carries. For electric signals, peak amplitude is normally measured in *volts*. Figure 3.3 shows two signals and their peak amplitudes.

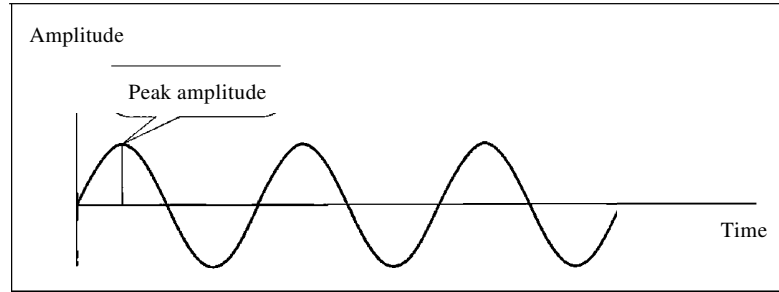
### Example 3.1

The power in your house can be represented by a sine wave with a peak amplitude of 155 to 170 **V**. However, it is common knowledge that the voltage of the power in U.S. homes is 110 to 120 **V**.

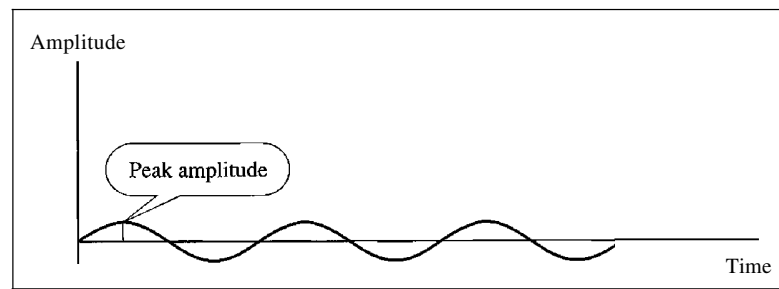
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**Figure 3.3** Two signals with the same phase and frequency, but different amplitudes
 

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a. A signal with high peak amplitude



b. A signal with low peak amplitude

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This discrepancy is due to the fact that these are root mean square (rms) values. The signal is squared and then the average amplitude is calculated. The peak value is equal to  $2^{1/2}$  x rms value.

### Example 3.2

The voltage of battery is a constant; this constant value can be considered a sine wave, as we will see later. For example, the peak value of an AA battery is normally 1.5 V.

### Period and Frequency

Period refers to the amount of time, in seconds, a signal needs to complete 1 cycle. Frequency refers to the number of periods in 1 s. Note that period and frequency are just one characteristic defined in two ways. Period is the inverse of frequency, and frequency is the inverse of period, as the following formulas show.

$$f = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{f}$$

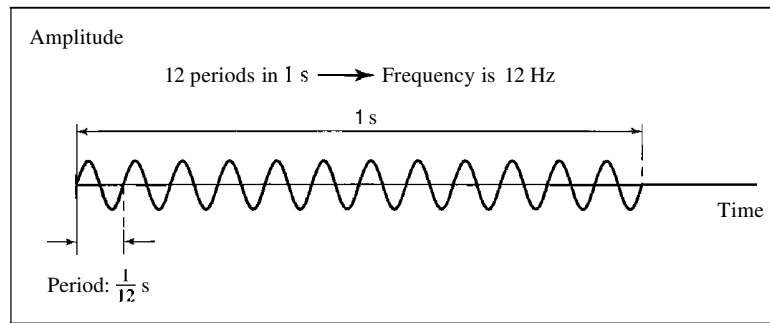
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Frequency and period are the inverse of each other.

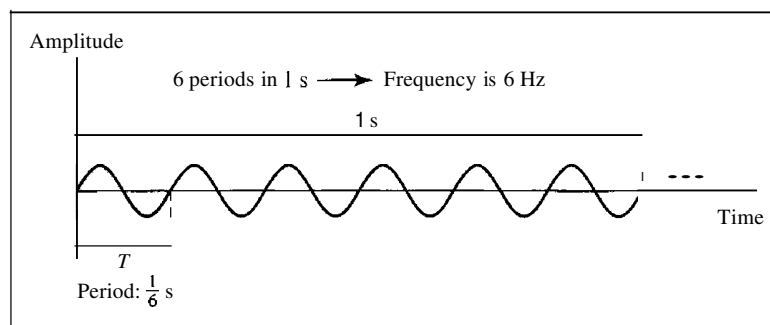
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Figure 3.4 shows two signals and their frequencies.

Figure 3.4 Two signals with the same amplitude and phase, but different frequencies



a. A signal with a frequency of 12 Hz



b. A signal with a frequency of 6 Hz

Period is formally expressed in seconds. Frequency is formally expressed in Hertz (Hz), which is cycle per second. Units of period and frequency are shown in Table 3.1.

Table 3.1 Units of period and frequency

Unit	Equivalent	Unit	Equivalent
Seconds (s)	1 s	Hertz (Hz)	1 Hz
Milliseconds (ms)	$10^{-3}$ s	Kilohertz (kHz)	$10^3$ Hz
Microseconds ( $\mu$ s)	$10^{-6}$ s	Megahertz (MHz)	$10^6$ Hz
Nanoseconds (ns)	$10^{-9}$ s	Gigahertz (GHz)	$10^9$ Hz
Picoseconds (ps)	$10^{-12}$ s	Terahertz (THz)	$10^{12}$ Hz

### Example 3.3

The power we use at home has a frequency of 60 Hz (50 Hz in Europe). The period of this sine wave can be determined as follows:

$$T = \frac{1}{f} = \frac{1}{60} = 0.0166 \text{ s} = 0.0166 \times 10^3 \text{ ms} = 16.6 \text{ ms}$$

This means that the period of the power for our lights at home is 0.0166 s, or 16.6 ms. Our eyes are not sensitive enough to distinguish these rapid changes in amplitude.

*Example 3.4*

Express a period of 100 ms in microseconds.

**Solution**

From Table 3.1 we find the equivalents of 1 ms (1 ms is  $10^{-3}$  s) and 1 s (1 s is  $10^6$   $\mu$ s). We make the following substitutions:

$$100 \text{ ms} = 100 \times 10^{-3} \text{ s} = 100 \times 10^{-3} \times 10^6 \mu\text{s} = 10^2 \times 10^{-3} \times 10^6 \mu\text{s} = 10^5 \mu\text{s}$$

*Example 3.5*

The period of a signal is 100 ms. What is its frequency in kilohertz?

**Solution**

First we change 100 ms to seconds, and then we calculate the frequency from the period (1 Hz =  $10^{-3}$  kHz).

$$100 \text{ ms} = 100 \times 10^{-3} \text{ s} = 10^{-1} \text{ s}$$

$$f = \frac{1}{T} = \frac{1}{10^{-1}} \text{ Hz} = 10 \text{ Hz} = 10 \times 10^{-3} \text{ kHz} = 10^{-2} \text{ kHz}$$

*More About Frequency*

We already know that frequency is the relationship of a signal to time and that the frequency of a wave is the number of cycles it completes in 1 s. But another way to look at frequency is as a measurement of the rate of change. Electromagnetic signals are oscillating waveforms; that is, they fluctuate continuously and predictably above and below a mean energy level. A 40-Hz signal has one-half the frequency of an 80-Hz signal; it completes 1 cycle in twice the time of the 80-Hz signal, so each cycle also takes twice as long to change from its lowest to its highest voltage levels. Frequency, therefore, though described in cycles per second (hertz), is a general measurement of the rate of change of a signal with respect to time.

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Frequency is the rate of change with respect to time. Change in a short span of time means high frequency. Change over a long span of time means low frequency.

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If the value of a signal changes over a very short span of time, its frequency is high. If it changes over a long span of time, its frequency is low.

*Two Extremes*

What if a signal does not change at all? What if it maintains a constant voltage level for the entire time it is active? In such a case, its frequency is zero. Conceptually, this idea is a simple one. If a signal does not change at all, it never completes a cycle, so its frequency is 0 Hz.

But what if a signal changes instantaneously? What if it jumps from one level to another in no time? Then its frequency is infinite. In other words, when a signal changes instantaneously, its period is zero; since frequency is the inverse of period, in this case, the frequency is  $1/0$ , or infinite (unbounded).

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If a signal does not change at all, its frequency is zero.  
 If a signal changes instantaneously, its frequency is infinite.

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## Phase

The term phase describes the position of the waveform relative to time 0. If we think of the wave as something that can be shifted backward or forward along the time axis, phase describes the amount of that shift. It indicates the status of the first cycle.

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Phase describes the position of the waveform relative to time 0.

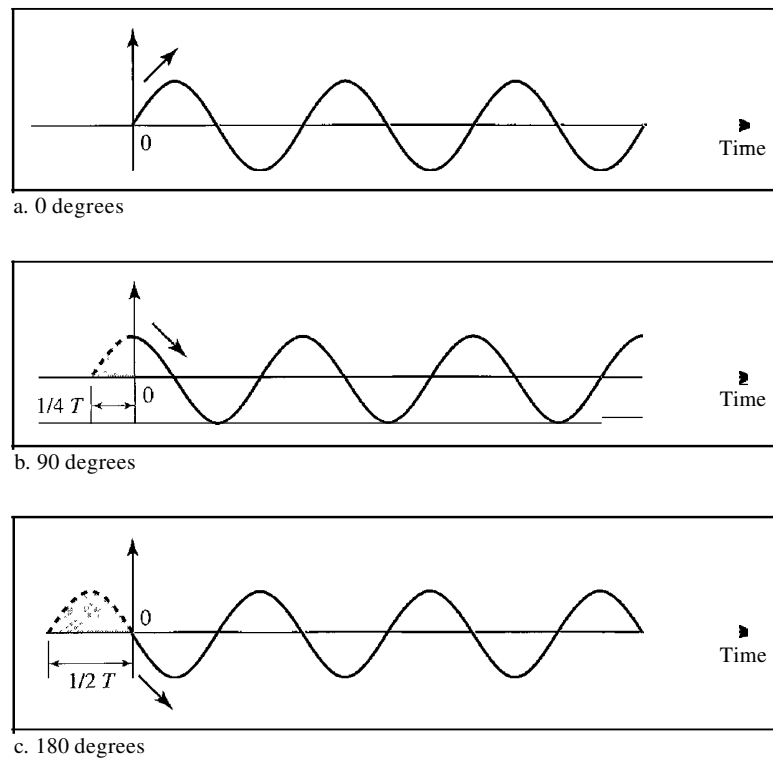
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Phase is measured in degrees or radians [ $360^\circ$  is  $2\pi$  rad;  $1^\circ$  is  $2\pi/360$  rad, and 1 rad is  $360/(2\pi)^\circ$ ]. A phase shift of  $360^\circ$  corresponds to a shift of a complete period; a phase shift of  $180^\circ$  corresponds to a shift of one-half of a period; and a phase shift of  $90^\circ$  corresponds to a shift of one-quarter of a period (see Figure 3.5).

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Figure 3.5 Three sine waves with the same amplitude and frequency, but different phases

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Looking at Figure 3.5, we can say that

1. A sine wave with a phase of  $0^\circ$  starts at time 0 with a zero amplitude. The amplitude is increasing.
2. A sine wave with a phase of  $90^\circ$  starts at time 0 with a peak amplitude. The amplitude is decreasing.

3. A sine wave with a phase of  $180^\circ$  starts at time 0 with a zero amplitude. The amplitude is decreasing.

Another way to look at the phase is in terms of shift or offset. We can say that

1. A sine wave with a phase of  $0^\circ$  is not shifted.
2. A sine wave with a phase of  $90^\circ$  is shifted to the left by  $\frac{1}{4}$  cycle. However, note that the signal does not really exist before time 0.
3. A sine wave with a phase of  $180^\circ$  is shifted to the left by  $\frac{1}{2}$  cycle. However, note that the signal does not really exist before time 0.

### Example 3.6

A sine wave is offset  $\frac{1}{6}$  cycle with respect to time 0. What is its phase in degrees and radians?

**Solution**

We know that 1 complete cycle is  $360^\circ$ . Therefore,  $\frac{1}{6}$  cycle is

$$\frac{1}{6} \times 360^\circ = 60^\circ = 60 \times \frac{2\pi}{360} \text{ rad} = \frac{\pi}{3} \text{ rad} \approx 1.046 \text{ rad}$$

## Wavelength

Wavelength is another characteristic of a signal traveling through a transmission medium. Wavelength binds the period or the frequency of a simple sine wave to the propagation speed of the medium (see Figure 3.6).

Figure 3.6 Wavelength and period



While the frequency of a signal is independent of the medium, the wavelength depends on both the frequency and the medium. Wavelength is a property of any type of signal. In data communications, we often use wavelength to describe the transmission of light in an optical fiber. The wavelength is the distance a simple signal can travel in one period.

Wavelength can be calculated if one is given the propagation speed (the speed of light) and the period of the signal. However, since period and frequency are related to each other, if we represent wavelength by  $\lambda$ , propagation speed by  $c$  (speed of light), and frequency by  $f$ , we get

$$\text{Wavelength} = \text{propagation speed} \times \text{period} = \frac{\text{propagation speed}}{\text{frequency}}$$



The propagation speed of electromagnetic signals depends on the medium and on the frequency of the signal. For example, in a vacuum, light is propagated with a speed of  $3 \times 10^8$  m/s. That speed is lower in air and even lower in cable.

The wavelength is normally measured in micrometers (microns) instead of meters. For example, the wavelength of red light (frequency  $= 4 \times 10^{14}$ ) in air is

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{4 \times 10^{14}} = 0.75 \times 10^{-6} \text{ m} = 0.75 \mu\text{m}$$

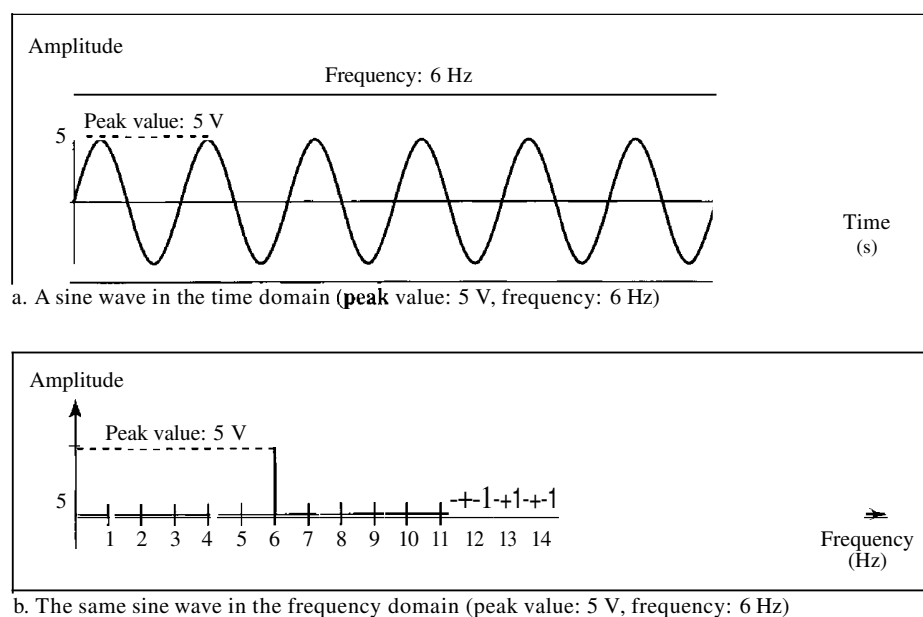
In a coaxial or fiber-optic cable, however, the wavelength is shorter ( $0.5 \mu\text{m}$ ) because the propagation speed in the cable is decreased.

## Time and Frequency Domains

A sine wave is comprehensively defined by its amplitude, frequency, and phase. We have been showing a sine wave by using what is called a time-domain plot. The time-domain plot shows changes in signal amplitude with respect to time (it is an amplitude-versus-time plot). Phase is not explicitly shown on a time-domain plot.

To show the relationship between amplitude and frequency, we can use what is called a frequency-domain plot. A frequency-domain plot is concerned with only the peak value and the frequency. Changes of amplitude during one period are not shown. Figure 3.7 shows a signal in both the time and frequency domains.

Figure 3.7 The time-domain and frequency-domain plots of a sine wave



It is obvious that the frequency domain is easy to plot and conveys the information that one can find in a time domain plot. The advantage of the frequency domain is that we can immediately see the values of the frequency and peak amplitude. A complete sine wave is represented by one spike. The position of the spike shows the frequency; its height shows the peak amplitude.

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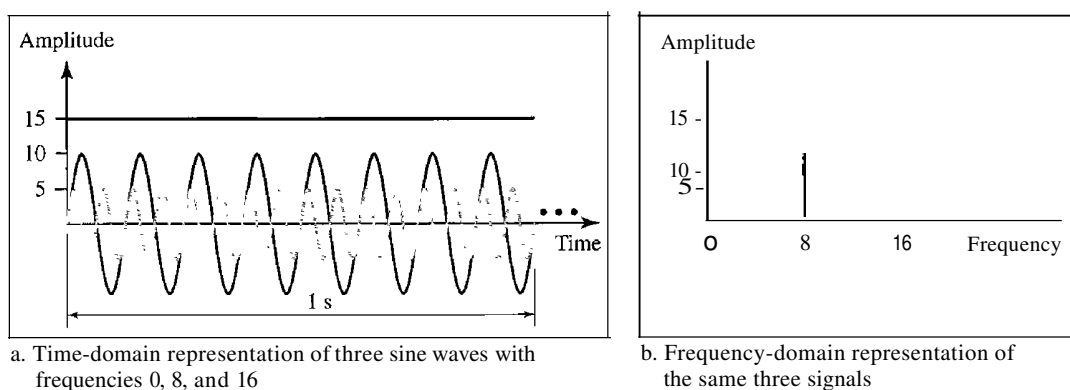
A complete sine wave in the time domain can be represented by one single spike in the frequency domain.

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### Example 3.7

The frequency domain is more compact and useful when we are dealing with more than one sine wave. For example, Figure 3.8 shows three sine waves, each with different amplitude and frequency. All can be represented by three spikes in the frequency domain.

Figure 3.8 The time domain and frequency domain of three sine waves



## Composite Signals

So far, we have focused on simple sine waves. Simple sine waves have many applications in daily life. We can send a single sine wave to carry electric energy from one place to another. For example, the power company sends a single sine wave with a frequency of 60 Hz to distribute electric energy to houses and businesses. As another example, we can use a single sine wave to send an alarm to a security center when a burglar opens a door or window in the house. In the first case, the sine wave is carrying energy; in the second, the sine wave is a signal of danger.

If we had only one single sine wave to convey a conversation over the phone, it would make no sense and carry no information. We would just hear a buzz. As we will see in Chapters 4 and 5, we need to send a composite signal to communicate data. A composite signal is made of many simple sine waves.

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A **single-frequency** sine wave is not useful in data communications; we need to send a composite signal, a signal made of many simple sine waves.

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In the early 1900s, the French mathematician Jean-Baptiste Fourier showed that any composite signal is actually a combination of simple sine waves with different frequencies, amplitudes, and phases. Fourier analysis is discussed in Appendix C; for our purposes, we just present the concept.

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According to Fourier analysis, any composite signal is a combination of simple sine waves with different frequencies, amplitudes, and phases. Fourier analysis is discussed in Appendix C.

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A composite signal can be periodic or nonperiodic. A periodic composite signal can be decomposed into a series of simple sine waves with discrete frequencies—frequencies that have integer values (1, 2, 3, and so on). A nonperiodic composite signal can be decomposed into a combination of an infinite number of simple sine waves with continuous frequencies, frequencies that have real values.

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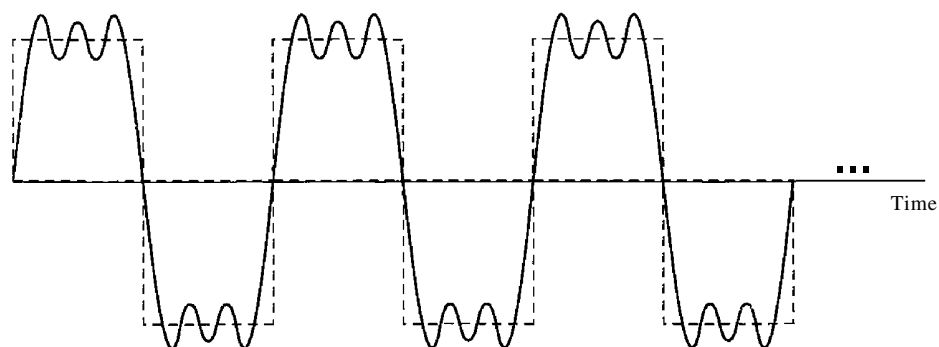
If the composite signal is periodic, the decomposition gives a series of signals with discrete frequencies; if the composite signal is nonperiodic, the decomposition gives a combination of sine waves with continuous frequencies.

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### Example 3.8

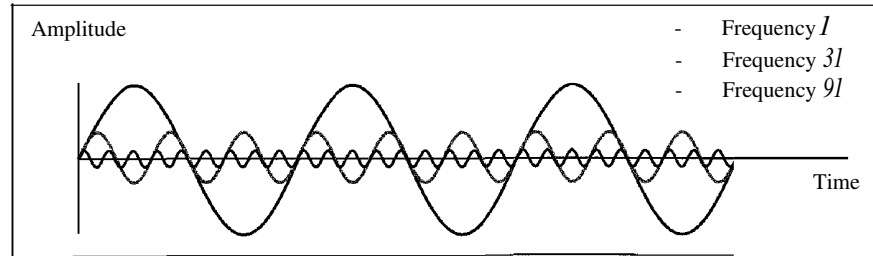
Figure 3.9 shows a periodic composite signal with frequency  $f$ . This type of signal is not typical of those found in data communications. We can consider it to be three alarm systems, each with a different frequency. The analysis of this signal can give us a good understanding of how to decompose signals.

Figure 3.9 A composite periodic signal

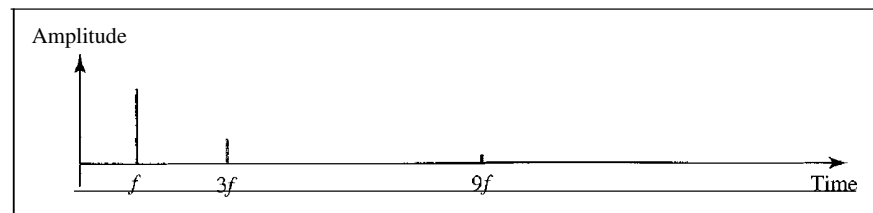


It is very difficult to manually decompose this signal into a series of simple sine waves. However, there are tools, both hardware and software, that can help us do the job. We are not concerned about how it is done; we are only interested in the result. Figure 3.10 shows the result of decomposing the above signal in both the time and frequency domains.

The amplitude of the sine wave with frequency  $f$  is almost the same as the peak amplitude of the composite signal. The amplitude of the sine wave with frequency  $3f$  is one-third of that of the first, and the amplitude of the sine wave with frequency  $9f$  is one-ninth of the first. The frequency

Figure 3.10 *Decomposition of a composite periodic signal in the time and frequency domains*

a. Time-domain decomposition of a composite signal



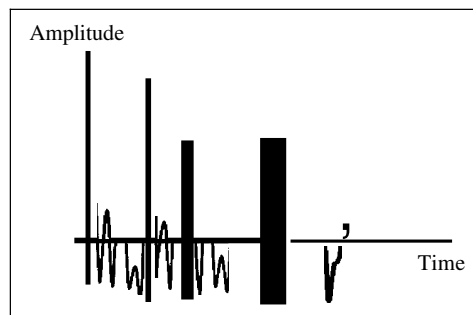
b. Frequency-domain decomposition of the composite signal

of the sine wave with frequency  $f$  is the same as the frequency of the composite signal; it is called the fundamental frequency, or first harmonic. The sine wave with frequency  $3f$  has a frequency of 3 times the fundamental frequency; it is called the third harmonic. The third sine wave with frequency  $9f$  has a frequency of 9 times the fundamental frequency; it is called the ninth harmonic.

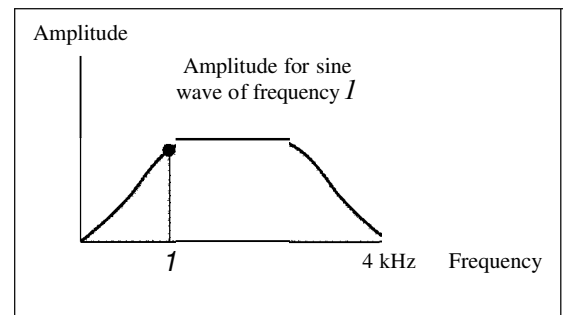
Note that the frequency decomposition of the signal is discrete; it has frequencies  $f$ ,  $3f$ , and  $9f$ . Because  $f$  is an integral number,  $3f$  and  $9f$  are also integral numbers. There are no frequencies such as  $1.2f$  or  $2.6f$ . The frequency domain of a periodic composite signal is always made of discrete spikes.

### Example 3.9

Figure 3.11 shows a nonperiodic composite signal. It can be the signal created by a microphone or a telephone set when a word or two is pronounced. In this case, the composite signal cannot be periodic, because that implies that we are repeating the same word or words with exactly the same tone.

Figure 3.11 *The time and frequency domains of a nonperiodic signal*

a. Time domain



b. Frequency domain

In a time-domain representation of this composite signal, there are an infinite number of simple sine frequencies. Although the number of frequencies in a human voice is infinite, the range is limited. A normal human being can create a continuous range of frequencies between 0 and 4 kHz.

Note that the frequency decomposition of the signal yields a continuous curve. There are an infinite number of frequencies between 0.0 and 4000.0 (real values). To find the amplitude related to frequency  $f$ , we draw a vertical line at  $f$  to intersect the envelope curve. The height of the vertical line is the amplitude of the corresponding frequency.

## Bandwidth

The range of frequencies contained in a composite signal is its bandwidth. The bandwidth is normally a difference between two numbers. For example, if a composite signal contains frequencies between 1000 and 5000, its bandwidth is  $5000 - 1000$ , or 4000.

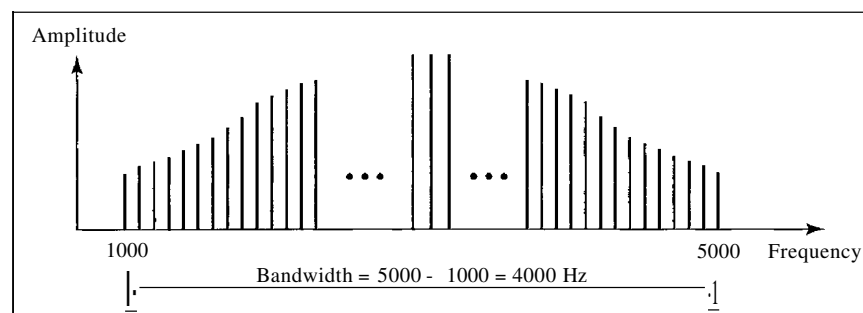
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The bandwidth of a composite signal is the difference between the highest and the lowest frequencies contained in that signal.

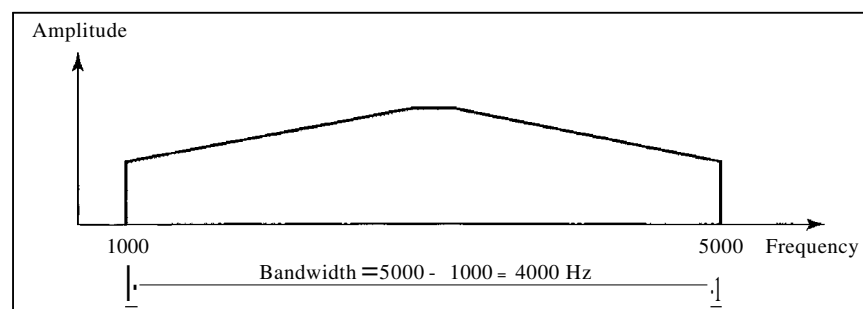
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Figure 3.12 shows the concept of bandwidth. The figure depicts two composite signals, one periodic and the other nonperiodic. The bandwidth of the periodic signal contains all integer frequencies between 1000 and 5000 (1000, 1001, 1002, ...). The bandwidth of the nonperiodic signal has the same range, but the frequencies are continuous.

Figure 3.12 The bandwidth of periodic and nonperiodic composite signals



a. Bandwidth of a periodic signal



b. Bandwidth of a nonperiodic signal

*Example 3.10*

If a periodic signal is decomposed into five sine waves with frequencies of 100, 300, 500, 700, and 900 Hz, what is its bandwidth? Draw the spectrum, assuming all components have a maximum amplitude of 10 V.

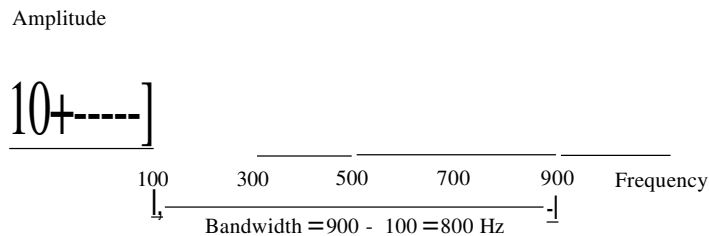
**Solution**

Let  $f_h$  be the highest frequency,  $f_l$  the lowest frequency, and  $B$  the bandwidth. Then

$$B = f_h - f_l = 900 - 100 = 800 \text{ Hz}$$

The spectrum has only five spikes, at 100, 300, 500, 700, and 900 Hz (see Figure 3.13).

Figure 3.13 The bandwidth for Example 3.10

*Example 3.11*

A periodic signal has a bandwidth of 20 Hz. The highest frequency is 60 Hz. What is the lowest frequency? Draw the spectrum if the signal contains all frequencies of the same amplitude.

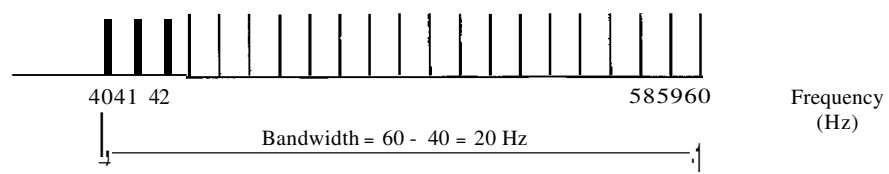
**Solution**

Let  $f_h$  be the highest frequency,  $f_l$  the lowest frequency, and  $B$  the bandwidth. Then

$$B = f_h - f_l \Rightarrow 20 = 60 - f_l \Rightarrow f_l = 60 - 20 = 40 \text{ Hz}$$

The spectrum contains all integer frequencies. We show this by a series of spikes (see Figure 3.14).

Figure 3.14 The bandwidth for Example 3.11

*Example 3.12*

A nonperiodic composite signal has a bandwidth of 200 kHz, with a middle frequency of 140 kHz and peak amplitude of 20 V. The two extreme frequencies have an amplitude of 0. Draw the frequency domain of the signal.

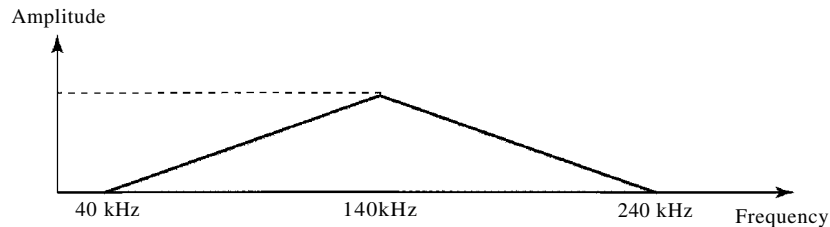
**Solution**

The lowest frequency must be at 40 kHz and the highest at 240 kHz. Figure 3.15 shows the frequency domain and the bandwidth.

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Figure 3.15 The bandwidth for Example 3.12

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**Example 3. B**

An example of a nonperiodic composite signal is the signal propagated by an AM radio station. In the United States, each AM radio station is assigned a 10-kHz bandwidth. The total bandwidth dedicated to AM radio ranges from 530 to 1700 kHz. We will show the rationale behind this 10-kHz bandwidth in Chapter 5.

**Example 3. J4**

Another example of a nonperiodic composite signal is the signal propagated by an FM radio station. In the United States, each FM radio station is assigned a 200-kHz bandwidth. The total bandwidth dedicated to FM radio ranges from 88 to 108 MHz. We will show the rationale behind this 200-kHz bandwidth in Chapter 5.

**Example 3. /5**

Another example of a nonperiodic composite signal is the signal received by an old-fashioned analog black-and-white TV. A TV screen is made up of pixels (picture elements) with each pixel being either white or black. The screen is scanned 30 times per second. (Scanning is actually 60 times per second, but odd lines are scanned in one round and even lines in the next and then interleaved.) If we assume a resolution of 525 x 700 (525 vertical lines and 700 horizontal lines), which is a ratio of 3:4, we have 367,500 pixels per screen. If we scan the screen 30 times per second, this is  $367,500 \times 30 = 11,025,000$  pixels per second. The worst-case scenario is alternating black and white pixels. In this case, we need to represent one color by the minimum amplitude and the other color by the maximum amplitude. We can send 2 pixels per cycle. Therefore, we need  $11,025,000/2 = 5,512,500$  cycles per second, or Hz. The bandwidth needed is 5.5124 MHz. This worst-case scenario has such a low probability of occurrence that the assumption is that we need only 70 percent of this bandwidth, which is 3.85 MHz. Since audio and synchronization signals are also needed, a 4-MHz bandwidth has been set aside for each black and white TV channel. An analog color TV channel has a 6-MHz bandwidth.

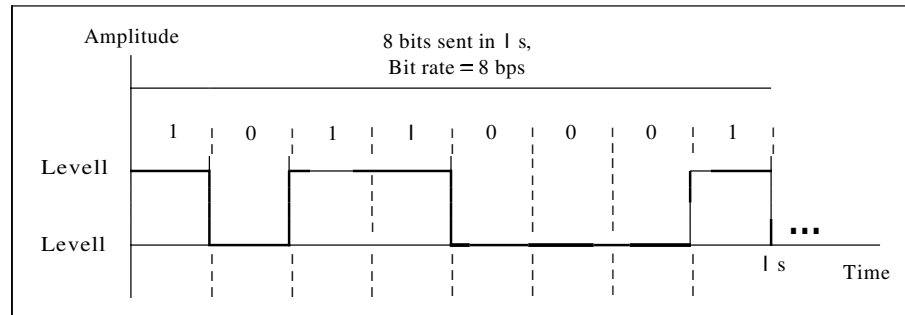
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## 3.3 DIGITAL SIGNALS

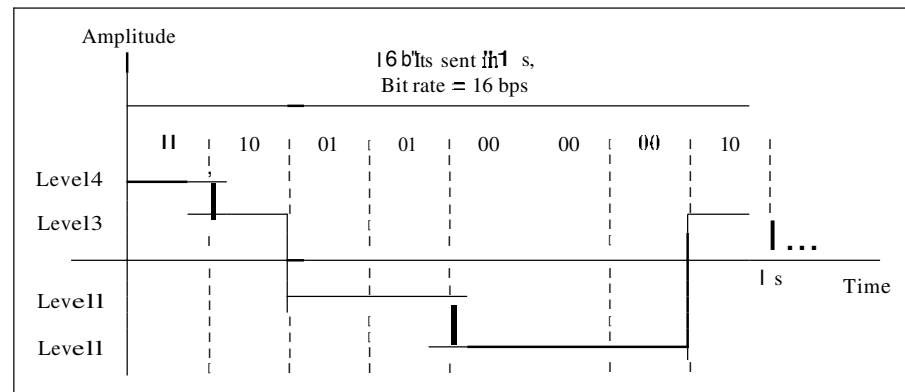
In addition to being represented by an analog signal, information can also be represented by a digital signal. For example, a 1 can be encoded as a positive voltage and a 0 as zero voltage. A digital signal can have more than two levels. In this case, we can

send more than 1 bit for each level. Figure 3.16 shows two signals, one with two levels and the other with four.

Figure 3.16 Two digital signals: one with two signal levels and the other with four signal levels



a. A digital signal with two levels



b. A digital signal with four levels

We send 1 bit per level in part a of the figure and 2 bits per level in part b of the figure. In general, if a signal has  $L$  levels, each level needs  $\log_2 L$  bits.

Appendix C reviews information about exponential and logarithmic functions.

### Example 3.16

A digital signal has eight levels. How many bits are needed per level? We calculate the number of bits from the formula

$$\text{Number of bits per level} = \log_2 8 = 3$$

Each signal level is represented by 3 bits.

### Example 3.17

A digital signal has nine levels. How many bits are needed per level? We calculate the number of bits by using the formula. Each signal level is represented by 3.17 bits. However, this answer is not realistic. The number of bits sent per level needs to be an integer as well as a power of 2. For this example, 4 bits can represent one level.



## Bit Rate

Most digital signals are nonperiodic, and thus period and frequency are not appropriate characteristics. Another *term-bit rate* (instead of *frequency*)-is used to describe digital signals. The bit rate is the number of bits sent in 1s, expressed in bits per second (bps). Figure 3.16 shows the bit rate for two signals.

### Example 3.18

Assume we need to download text documents at the rate of 100 pages per minute. What is the required bit rate of the channel?

#### Solution

A page is an average of 24 lines with 80 characters in each line. If we assume that one character requires 8 bits, the bit rate is

$$100 \times 24 \times 80 \times 8 = 1,636,000 \text{ bps} = 1.636 \text{ Mbps}$$

### Example 3.19

A digitized voice channel, as we will see in Chapter 4, is made by digitizing a 4-kHz bandwidth analog voice signal. We need to sample the signal at twice the highest frequency (two samples per hertz). We assume that each sample requires 8 bits. What is the required bit rate?

#### Solution

The bit rate can be calculated as

$$2 \times 4000 \times 8 = 64,000 \text{ bps} = 64 \text{ kbps}$$

### Example 3.20

What is the bit rate for high-definition TV (HDTV)?

#### Solution

HDTV uses digital signals to broadcast high quality video signals. The HDTV Screen is normally a ratio of 16 : 9 (in contrast to 4 : 3 for regular TV), which means the screen is wider. There are 1920 by 1080 pixels per screen, and the screen is renewed 30 times per second. Twenty-four bits represents one color pixel. We can calculate the bit rate as

$$1920 \times 1080 \times 30 \times 24 = 1,492,992,000 \text{ or } 1.5 \text{ Gbps}$$

The TV stations reduce this rate to 20 to 40 Mbps through compression.

## Bit Length

We discussed the concept of the wavelength for an analog signal: the distance one cycle occupies on the transmission medium. We can define something similar for a digital signal: the bit length. The bit length is the distance one bit occupies on the transmission medium.

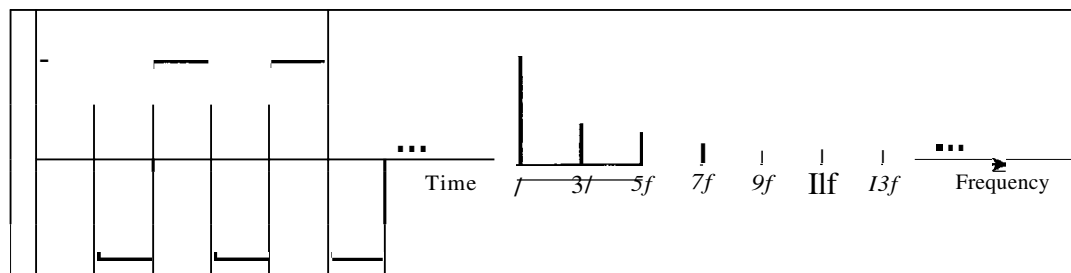
$$\text{Bit length} = \text{propagation speed} \times \text{bit duration}$$

## Digital Signal as a Composite Analog Signal

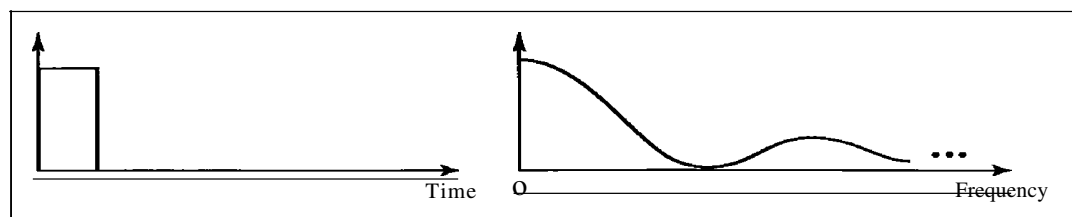
Based on Fourier analysis, a digital signal is a composite analog signal. The bandwidth is infinite, as you may have guessed. We can intuitively come up with this concept when we consider a digital signal. A digital signal, in the time domain, comprises connected vertical and horizontal line segments. A vertical line in the time domain means a frequency of infinity (sudden change in time); a horizontal line in the time domain means a frequency of zero (no change in time). Going from a frequency of zero to a frequency of infinity (and vice versa) implies all frequencies in between are part of the domain.

Fourier analysis can be used to decompose a digital signal. If the digital signal is periodic, which is rare in data communications, the decomposed signal has a frequency-domain representation with an infinite bandwidth and discrete frequencies. If the digital signal is nonperiodic, the decomposed signal still has an infinite bandwidth, but the frequencies are continuous. Figure 3.17 shows a periodic and a nonperiodic digital signal and their bandwidths.

Figure 3.17 The time and frequency domains of periodic and nonperiodic digital signals



a. Time and frequency domains of periodic digital signal



b. Time and frequency domains of nonperiodic digital signal

Note that both bandwidths are infinite, but the periodic signal has discrete frequencies while the nonperiodic signal has continuous frequencies.

## Transmission of Digital Signals

The previous discussion asserts that a digital signal, periodic or nonperiodic, is a composite analog signal with frequencies between zero and infinity. For the remainder of the discussion, let us consider the case of a nonperiodic digital signal, similar to the ones we encounter in data communications. The fundamental question is, How can we send a digital signal from point A to point B? We can transmit a digital signal by using one of two different approaches: baseband transmission or broadband transmission (using modulation).

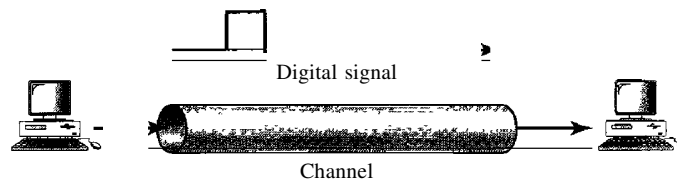
*Baseband Transmission*

Baseband transmission means sending a digital signal over a channel without changing the digital signal to an analog signal. Figure 3.18 shows baseband transmission.

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Figure 3.18 *Baseband transmission*

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A digital signal is a composite analog signal with an infinite bandwidth.

---

Baseband transmission requires that we have a low-pass channel, a channel with a bandwidth that starts from zero. This is the case if we have a dedicated medium with a bandwidth constituting only one channel. For example, the entire bandwidth of a cable connecting two computers is one single channel. As another example, we may connect several computers to a bus, but not allow more than two stations to communicate at a time. Again we have a low-pass channel, and we can use it for baseband communication. Figure 3.19 shows two low-pass channels: one with a narrow bandwidth and the other with a wide bandwidth. We need to remember that a low-pass channel with infinite bandwidth is ideal, but we cannot have such a channel in real life. However, we can get close.

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Figure 3.19 *Bandwidths of two low-pass channels*

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a. Low-pass channel, wide bandwidth



b. Low-pass channel, narrow bandwidth

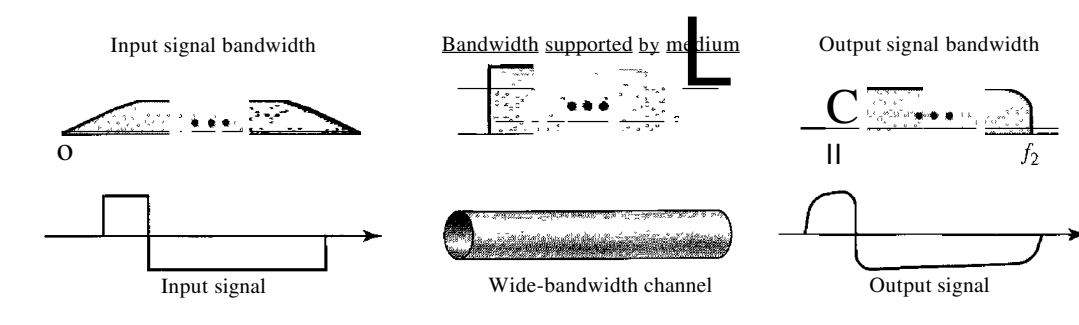
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Let us study two cases of a baseband communication: a low-pass channel with a wide bandwidth and one with a limited bandwidth.

### Case 1: Low-Pass Channel with Wide Bandwidth

If we want to preserve the exact form of a nonperiodic digital signal with vertical segments vertical and horizontal segments horizontal, we need to send the entire spectrum, the continuous range of frequencies between zero and infinity. This is possible if we have a dedicated medium with an infinite bandwidth between the sender and receiver that preserves the exact amplitude of each component of the composite signal. Although this may be possible inside a computer (e.g., between CPU and memory), it is not possible between two devices. Fortunately, the amplitudes of the frequencies at the border of the bandwidth are so small that they can be ignored. This means that if we have a medium, such as a coaxial cable or fiber optic, with a very wide bandwidth, two stations can communicate by using digital signals with very good accuracy, as shown in Figure 3.20. Note that  $f_1$  is close to zero, and  $f_2$  is very high.

Figure 3.20 Baseband transmission using a dedicated medium



Although the output signal is not an exact replica of the original signal, the data can still be deduced from the received signal. Note that although some of the frequencies are blocked by the medium, they are not critical.

Baseband transmission of a digital signal that preserves the shape of the digital signal is possible only if we have a low-pass channel with an infinite or very wide bandwidth.

### Example 3.21

An example of a dedicated channel where the entire bandwidth of the medium is used as one single channel is a LAN. Almost every wired LAN today uses a dedicated channel for two stations communicating with each other. In a bus topology LAN with multipoint connections, only two stations can communicate with each other at each moment in time (timesharing); the other stations need to refrain from sending data. In a star topology LAN, the entire channel between each station and the hub is used for communication between these two entities. We study LANs in Chapter 14.

### Case 2: Low-Pass Channel with Limited Bandwidth

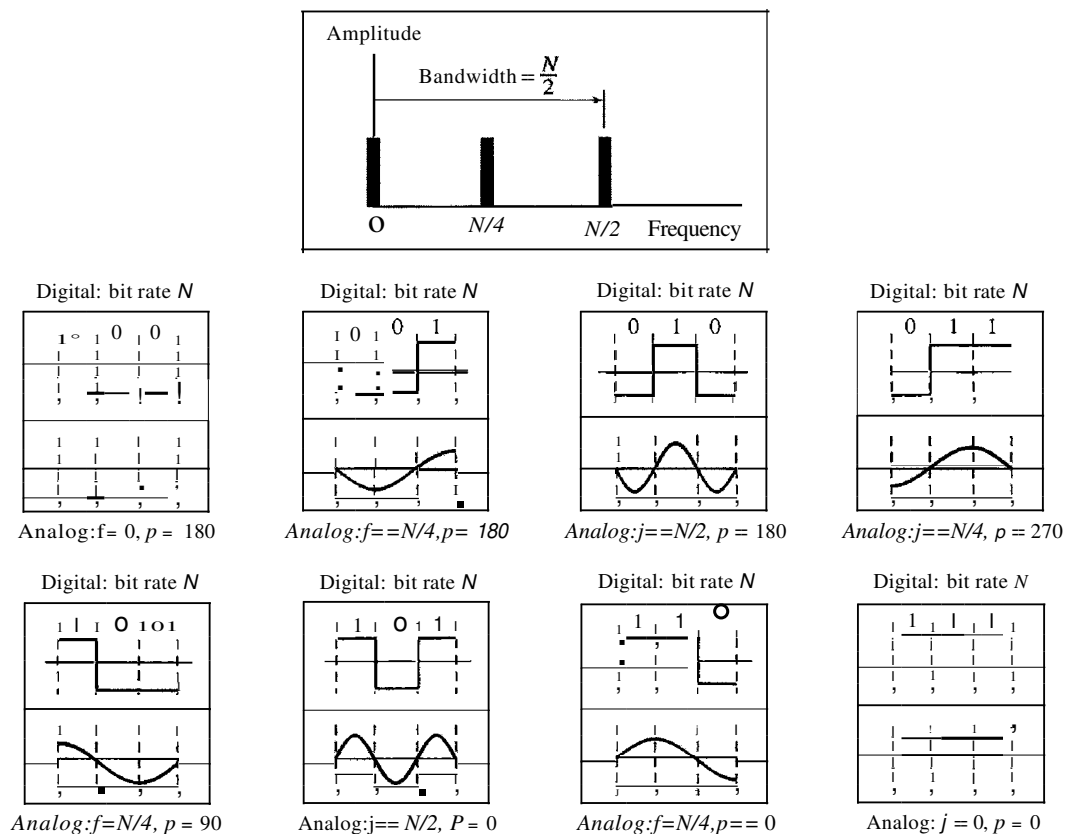
In a low-pass channel with limited bandwidth, we approximate the digital signal with an analog signal. The level of approximation depends on the bandwidth available.

**Rough Approximation** Let us assume that we have a digital signal of bit rate  $N$ . If we want to send analog signals to roughly simulate this signal, we need to consider the worst case, a maximum number of changes in the digital signal. This happens when the signal

carries the sequence 01010101 ... or the sequence 10101010 ... To simulate these two cases, we need an analog signal of frequency  $f = N/2$ . Let 1 be the positive peak value and 0 be the negative peak value. We send 2 bits in each cycle; the frequency of the analog signal is one-half of the bit rate, or  $N/2$ . However, just this one frequency cannot make all patterns; we need more components. The maximum frequency is  $N/2$ . As an example of this concept, let us see how a digital signal with a 3-bit pattern can be simulated by using analog signals. Figure 3.21 shows the idea. The two similar cases (000 and 111) are simulated with a signal with frequency  $f = 0$  and a phase of  $180^\circ$  for 000 and a phase of  $0^\circ$  for 111. The two worst cases (010 and 101) are simulated with an analog signal with frequency  $f = N/2$  and phases of  $180^\circ$  and  $0^\circ$ . The other four cases can only be simulated with an analog signal with  $f = N/4$  and phases of  $180^\circ$ ,  $270^\circ$ ,  $90^\circ$ , and  $0^\circ$ . In other words, we need a channel that can handle frequencies 0,  $N/4$ , and  $N/2$ . This rough approximation is referred to as using the first harmonic ( $N/2$ ) frequency. The required bandwidth is

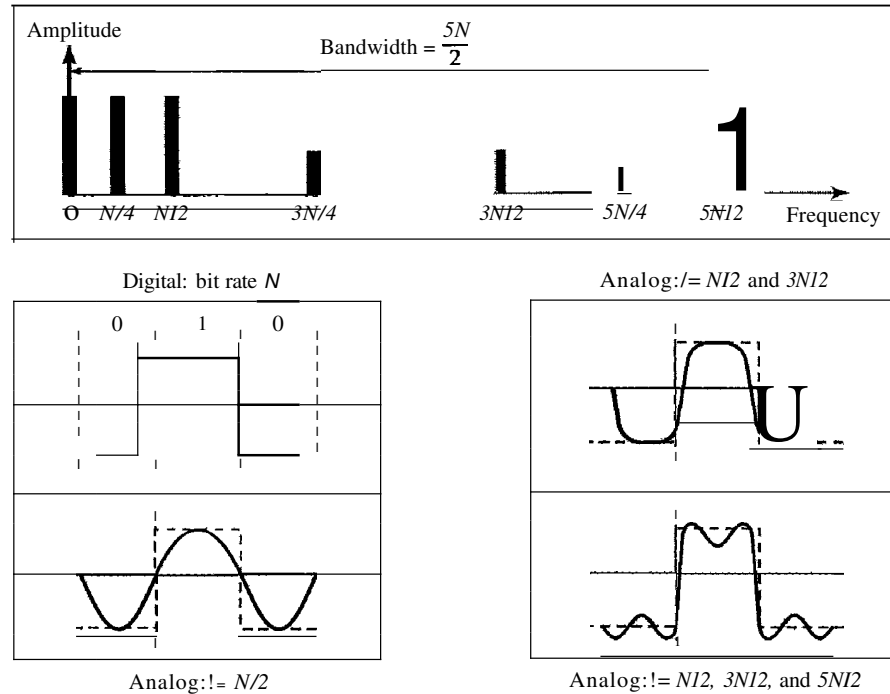
$$\text{Bandwidth} = \frac{N}{2} - 0 = \frac{N}{2}$$

**Figure 3.21** Rough approximation of a digital signal using the first harmonic for worst case



**Better Approximation** To make the shape of the analog signal look more like that of a digital signal, we need to add more harmonics of the frequencies. We need to increase the bandwidth. We can increase the bandwidth to  $3N/2$ ,  $5N/2$ ,  $7N/2$ , and so on. Figure 3.22 shows the effect of this increase for one of the worst cases, the pattern 010.

Figure 3.22 Simulating a digital signal with three first harmonics



Note that we have shown only the highest frequency for each harmonic. We use the first, third, and fifth harmonics. The required bandwidth is now  $5N/2$ , the difference between the lowest frequency  $0$  and the highest frequency  $5N/2$ . As we emphasized before, we need to remember that the required bandwidth is proportional to the bit rate.

**In** baseband transmission, the required bandwidth is proportional to the bit rate; if we need to send bits faster, we need more bandwidth.

By using this method, Table 3.2 shows how much bandwidth we need to send data at different rates.

Table 3.2 Bandwidth requirements

Bit Rate	Harmonic 1	Harmonics 1,3	Harmonics 1,3,5
$n = 1$ kbps	$B = 500\text{Hz}$	$B = 1.5$ kHz	$B = 2.5$ kHz
$n = 10$ kbps	$B = 5$ kHz	$B = 15$ kHz	$B = 25$ kHz
$n = 100$ kbps	$B = 50$ kHz	$B = 150$ kHz	$B = 250$ kHz

### Example 3.22

What is the required bandwidth of a low-pass channel if we need to send 1 Mbps by using baseband transmission?

**Solution**

The answer depends on the accuracy desired.

- The minimum bandwidth, a rough approximation, is  $B = \text{bit rate} / 2$ , or 500 kHz. We need a low-pass channel with frequencies between 0 and 500 kHz.
- A better result can be achieved by using the first and the third harmonics with the required bandwidth  $B = 3 \times 500 \text{ kHz} = 1.5 \text{ MHz}$ .
- Still a better result can be achieved by using the first, third, and fifth harmonics with  $B = 5 \times 500 \text{ kHz} = 2.5 \text{ MHz}$ .

**Example 3.23**

We have a low-pass channel with bandwidth 100 kHz. What is the maximum bit rate of this channel?

**Solution**

The maximum bit rate can be achieved if we use the first harmonic. The bit rate is 2 times the available bandwidth, or 200 kbps.

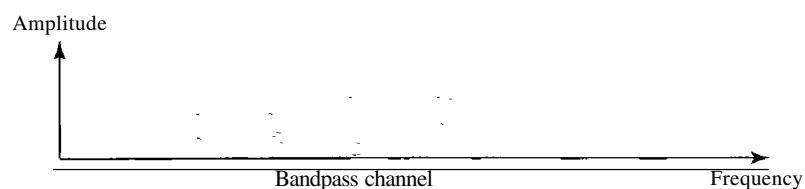
**Broadband Transmission (Using Modulation)**

Broadband transmission or modulation means changing the digital signal to an analog signal for transmission. Modulation allows us to use a bandpass channel—a channel with a bandwidth that does not start from zero. This type of channel is more available than a low-pass channel. Figure 3.23 shows a bandpass channel.

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Figure 3.23 Bandwidth of a bandpass channel

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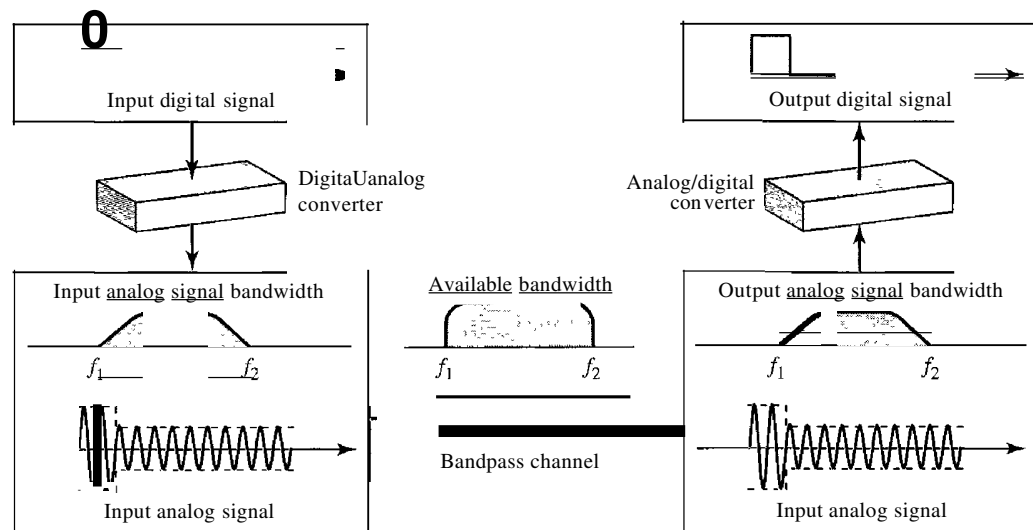
Note that a low-pass channel can be considered a bandpass channel with the lower frequency starting at zero.

Figure 3.24 shows the modulation of a digital signal. In the figure, a digital signal is converted to a composite analog signal. We have used a single-frequency analog signal (called a carrier); the amplitude of the carrier has been changed to look like the digital signal. The result, however, is not a single-frequency signal; it is a composite signal, as we will see in Chapter 5. At the receiver, the received analog signal is converted to digital, and the result is a replica of what has been sent.

---

**If the available channel is a bandpass channel, we cannot send the digital signal directly to the channel; we need to convert the digital signal to an analog signal before transmission.**

---

**Figure 3.24** Modulation of a digital signal for transmission on a bandpass channel**Example 3.24**

An example of broadband transmission using modulation is the sending of computer data through a telephone subscriber line, the line connecting a resident to the central telephone office. These lines, installed many years ago, are designed to carry voice (analog signal) with a limited bandwidth (frequencies between 0 and 4 kHz). Although this channel can be used as a low-pass channel, it is normally considered a bandpass channel. One reason is that the bandwidth is so narrow (4 kHz) that if we treat the channel as low-pass and use it for baseband transmission, the maximum bit rate can be only 8 kbps. The solution is to consider the channel a bandpass channel, convert the digital signal from the computer to an analog signal, and send the analog signal. We can install two converters to change the digital signal to analog and vice versa at the receiving end. The converter, in this case, is called a *modem* (modulator/demodulator), which we discuss in detail in Chapter 5.

**Example 3.25**

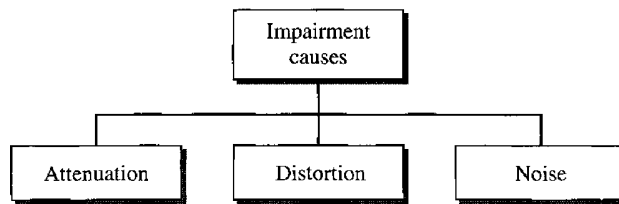
A second example is the digital cellular telephone. For better reception, digital cellular phones convert the analog voice signal to a digital signal (see Chapter 16). Although the bandwidth allocated to a company providing digital cellular phone service is very wide, we still cannot send the digital signal without conversion. The reason is that we only have a bandpass channel available between caller and callee. For example, if the available bandwidth is  $W$  and we allow 1000 couples to talk simultaneously, this means the available channel is  $W/1000$ , just part of the entire bandwidth. We need to convert the digitized voice to a composite analog signal before sending. The digital cellular phones convert the analog audio signal to digital and then convert it again to analog for transmission over a bandpass channel.

## 3.4 TRANSMISSION IMPAIRMENT

Signals travel through transmission media, which are not perfect. The imperfection causes signal impairment. This means that the signal at the beginning of the medium is not the same as the signal at the end of the medium. What is sent is not what is received. Three causes of impairment are attenuation, distortion, and noise (see Figure 3.25).



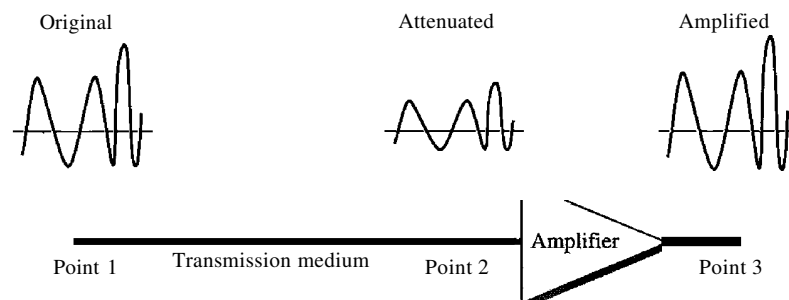
Figure 3.25 Causes of impairment



## Attenuation

Attenuation means a loss of energy. When a signal, simple or composite, travels through a medium, it loses some of its energy in overcoming the resistance of the medium. That is why a wire carrying electric signals gets warm, if not hot, after a while. Some of the electrical energy in the signal is converted to heat. To compensate for this loss, amplifiers are used to amplify the signal. Figure 3.26 shows the effect of attenuation and amplification.

Figure 3.26 Attenuation



## Decibel

To show that a signal has lost or gained strength, engineers use the unit of the decibel. The decibel (dB) measures the relative strengths of two signals or one signal at two different points. Note that the decibel is negative if a signal is attenuated and positive if a signal is amplified.

$$\text{dB} = 10 \log_{10} \frac{P_2}{P_1}$$

Variables  $P_1$  and  $P_2$  are the powers of a signal at points 1 and 2, respectively. Note that some engineering books define the decibel in terms of voltage instead of power. In this case, because power is proportional to the square of the voltage, the formula is  $\text{dB} = 20 \log_{10} (V_2/V_1)$ . In this text, we express dB in terms of power.

*Example 3.26*

Suppose a signal travels through a transmission medium and its power is reduced to one-half. This means that  $P_2 = \frac{1}{2} P_1$ . In this case, the attenuation (loss of power) can be calculated as

$$10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{0.5 P_1}{P_1} = 10 \log_{10} 0.5 = 10(-0.3) = -3 \text{ dB}$$

A loss of 3 dB (-3 dB) is equivalent to losing one-half the power.

*Example 3.27*

A signal travels through an amplifier, and its power is increased 10 times. This means that  $P_2 = 10 P_1$ . In this case, the amplification (gain of power) can be calculated as

$$10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{10 P_1}{P_1} = 10 \log_{10} 10 = 10(1) = 10 \text{ dB}$$

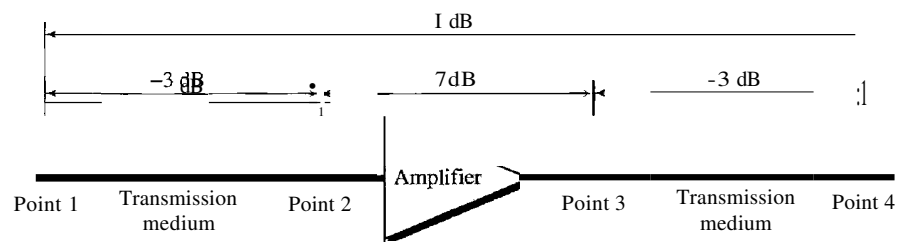
*Example 3.28*

One reason that engineers use the decibel to measure the changes in the strength of a signal is that decibel numbers can be added (or subtracted) when we are measuring several points (cascading) instead of just two. In Figure 3.27 a signal travels from point 1 to point 4. The signal is attenuated by the time it reaches point 2. Between points 2 and 3, the signal is amplified. Again, between points 3 and 4, the signal is attenuated. We can find the resultant decibel value for the signal just by adding the decibel measurements between each set of points.

---

Figure 3.27 Decibels for Example 3.28

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In this case, the decibel value can be calculated as

$$\text{dB} = -3 + 7 - 3 = +1$$

The signal has gained in power.

*Example 3.29*

Sometimes the decibel is used to measure signal power in milliwatts. In this case, it is referred to as  $\text{dB}_m$  and is calculated as  $\text{dB}_m = 10 \log_{10} P_m$ , where  $P_m$  is the power in milliwatts. Calculate the power of a signal if its  $\text{dB}_m = -30$ .

**Solution**

We can calculate the power in the signal as

$$\begin{aligned} \text{dB}_m &= 10 \log_{10} P_m = -30 \\ \log_{10} P_m &= -3 \quad P_m = 10^{-3} \text{ mW} \end{aligned}$$

**Example 3.30**

The loss in a cable is usually defined in decibels per kilometer (dB/km). If the signal at the beginning of a cable with -0.3 dB/km has a power of 2 mW, what is the power of the signal at 5 km?

**Solution**

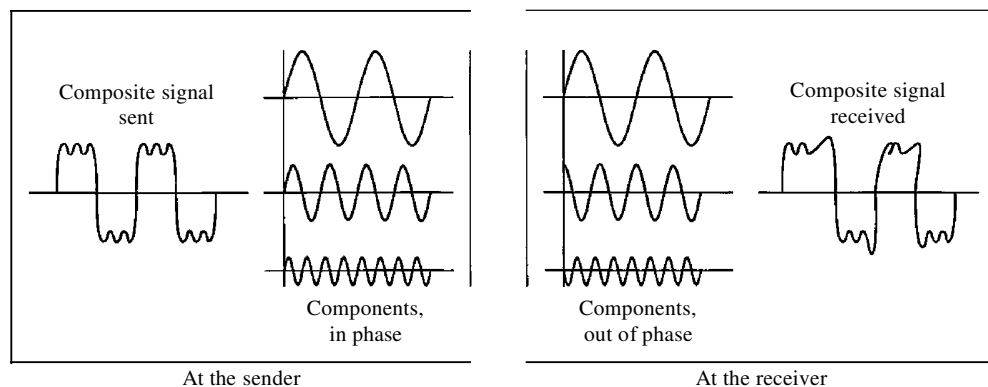
The loss in the cable in decibels is  $5 \times (-0.3) = -1.5$  dB. We can calculate the power as

$$\begin{aligned} \text{dB} &= 10 \log_{10} \frac{P_2}{P_1} = -1.5 \\ \frac{P_2}{P_1} &= 10^{-0.15} = 0.71 \\ P_2 &= 0.71 P_1 = 0.7 \times 2 = 1.4 \text{ mW} \end{aligned}$$

**Distortion**

**Distortion** means that the signal changes its form or shape. Distortion can occur in a composite signal made of different frequencies. Each signal component has its own propagation speed (see the next section) through a medium and, therefore, its own delay in arriving at the final destination. Differences in delay may create a difference in phase if the delay is not exactly the same as the period duration. In other words, signal components at the receiver have phases different from what they had at the sender. The shape of the composite signal is therefore not the same. Figure 3.28 shows the effect of distortion on a composite signal.

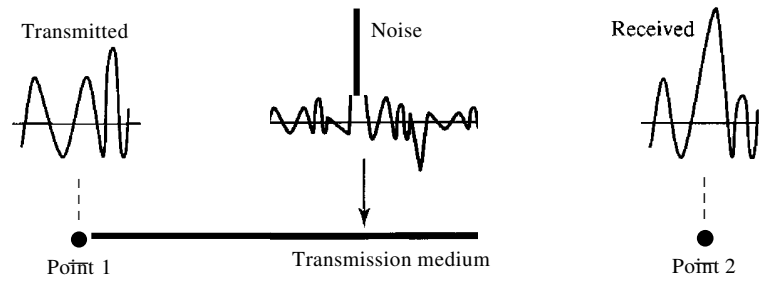
Figure 3.28 Distortion



## Noise

Noise is another cause of impairment. Several types of noise, such as thermal noise, induced noise, crosstalk, and impulse noise, may corrupt the signal. Thermal noise is the random motion of electrons in a wire which creates an extra signal not originally sent by the transmitter. Induced noise comes from sources such as motors and appliances. These devices act as a sending antenna, and the transmission medium acts as the receiving antenna. Crosstalk is the effect of one wire on the other. One wire acts as a sending antenna and the other as the receiving antenna. Impulse noise is a spike (a signal with high energy in a very short time) that comes from power lines, lightning, and so on. Figure 3.29 shows the effect of noise on a signal. We discuss error in Chapter 10.

Figure 3.29 Noise



### Signal-to-Noise Ratio (SNR)

As we will see later, to find the theoretical bit rate limit, we need to know the ratio of the signal power to the noise power. The signal-to-noise ratio is defined as

$$\text{SNR} = \frac{\text{average signal power}}{\text{average noise power}}$$

We need to consider the average signal power and the average noise power because these may change with time. Figure 3.30 shows the idea of SNR.

SNR is actually the ratio of what is wanted (signal) to what is not wanted (noise). A high SNR means the signal is less corrupted by noise; a low SNR means the signal is more corrupted by noise.

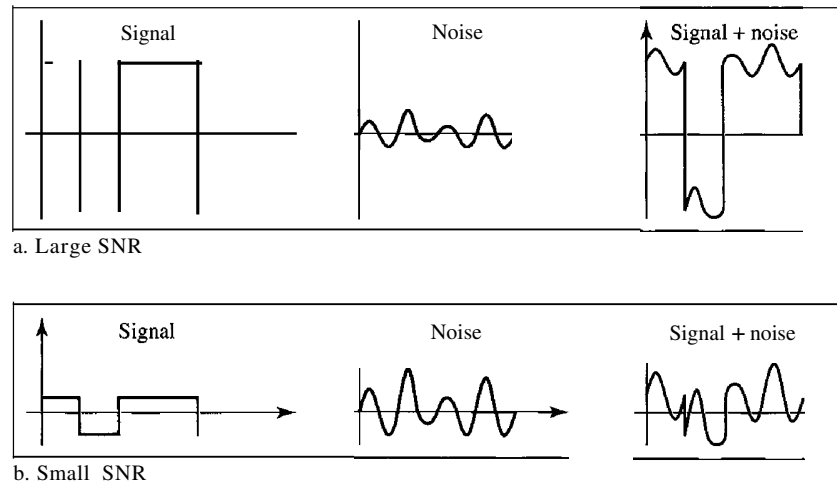
Because SNR is the ratio of two powers, it is often described in decibel units,  $\text{SNR}_{\text{dB}}$ , defined as

$$\text{SNR}_{\text{dB}} = 10 \log_{10} \text{SNR}$$

### Example 3.31

The power of a signal is 10 mW and the power of the noise is 1  $\mu\text{W}$ ; what are the values of SNR and  $\text{SNR}_{\text{dB}}$ ?

Figure 3.30 Two cases of SNR: a high SNR and a low SNR

**Solution**

The values of SNR and  $\text{SNR}_{\text{dB}}$  can be calculated as follows:

$$\text{SNR} = \frac{10.000 \mu\text{W}}{1 \text{ mW}} = 10,000$$

$$\text{SNR}_{\text{dB}} = 10 \log_{10} 10,000 = 10 \log_{10} 10^4 = 40$$

**Example 3.32**

The values of SNR and  $\text{SNR}_{\text{dB}}$  for a noiseless channel are

$$\text{SNR} = \frac{\text{signal power}}{0} = \infty$$

$$\text{SNR}_{\text{dB}} = 10 \log_{10} \infty = \infty$$

We can never achieve this ratio in real life; it is an ideal.

## 3.5 DATA RATE LIMITS

A very important consideration in data communications is how fast we can send data, in bits per second, over a channel. Data rate depends on three factors:

1. The bandwidth available
2. The level of the signals we use
3. The quality of the channel (the level of noise)

Two theoretical formulas were developed to calculate the data rate: one by Nyquist for a noiseless channel, another by Shannon for a noisy channel.

### Noiseless Channel: Nyquist Bit Rate

For a noiseless channel, the Nyquist bit rate formula defines the theoretical maximum bit rate

$$\text{BitRate} = 2 \times \text{bandwidth} \times \log_2 L$$

In this formula, bandwidth is the bandwidth of the channel,  $L$  is the number of signal levels used to represent data, and BitRate is the bit rate in bits per second.

According to the formula, we might think that, given a specific bandwidth, we can have any bit rate we want by increasing the number of signal levels. Although the idea is theoretically correct, practically there is a limit. When we increase the number of signal levels, we impose a burden on the receiver. If the number of levels in a signal is just 2, the receiver can easily distinguish between a 0 and a 1. If the level of a signal is 64, the receiver must be very sophisticated to distinguish between 64 different levels. In other words, increasing the levels of a signal reduces the reliability of the system.

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Increasing the levels of a signal may reduce the reliability of the system.

---

#### Example 3.33

Does the Nyquist theorem bit rate agree with the intuitive bit rate described in baseband transmission?

#### Solution

They match when we have only two levels. We said, in baseband transmission, the bit rate is 2 times the bandwidth if we use only the first harmonic in the worst case. However, the Nyquist formula is more general than what we derived intuitively; it can be applied to baseband transmission and modulation. Also, it can be applied when we have two or more levels of signals.

#### Example 3.34

Consider a noiseless channel with a bandwidth of 3000 Hz transmitting a signal with two signal levels. The maximum bit rate can be calculated as

$$\text{BitRate} = 2 \times 3000 \times \log_2 2 = 6000 \text{ bps}$$

#### Example 3.35

Consider the same noiseless channel transmitting a signal with four signal levels (for each level, we send 2 bits). The maximum bit rate can be calculated as

$$\text{BitRate} = 2 \times 3000 \times \log_2 4 = 12,000 \text{ bps}$$

#### Example 3.36

We need to send 265 kbps over a noiseless channel with a bandwidth of 20 kHz. How many signal levels do we need?

**Solution**

We can use the Nyquist formula as shown:

$$\begin{aligned} 265,000 &= 2 \times 20,000 \times \log_2 L \\ \log_2 L &= 6.625 \quad L = 2^{6.625} = 98.7 \text{ levels} \end{aligned}$$

Since this result is not a power of 2, we need to either increase the number of levels or reduce the bit rate. If we have 128 levels, the bit rate is 280 kbps. If we have 64 levels, the bit rate is 240 kbps.

**Noisy Channel: Shannon Capacity**

In reality, we cannot have a noiseless channel; the channel is always noisy. In 1944, Claude Shannon introduced a formula, called the Shannon capacity, to determine the theoretical highest data rate for a noisy channel:

$$\text{Capacity} = \text{bandwidth} \times \log_2 (1 + \text{SNR})$$

In this formula, bandwidth is the bandwidth of the channel, SNR is the signal-to-noise ratio, and capacity is the capacity of the channel in bits per second. Note that in the Shannon formula there is no indication of the signal level, which means that no matter how many levels we have, we cannot achieve a data rate higher than the capacity of the channel. In other words, the formula defines a characteristic of the channel, not the method of transmission.

*Example 3.37*

Consider an extremely noisy channel in which the value of the signal-to-noise ratio is almost zero. In other words, the noise is so strong that the signal is faint. For this channel the capacity  $C$  is calculated as

$$C = B \log_2 (1 + \text{SNR}) = B \log_2 (1 + 0) = B \log_2 1 = B \times 0 = 0$$

This means that the capacity of this channel is zero regardless of the bandwidth. In other words, we cannot receive any data through this channel.

*Example 3.38*

We can calculate the theoretical highest bit rate of a regular telephone line. A telephone line normally has a bandwidth of 3000 Hz (300 to 3300 Hz) assigned for data communications. The signal-to-noise ratio is usually 3162. For this channel the capacity is calculated as

$$\begin{aligned} C &= B \log_2 (1 + \text{SNR}) = 3000 \log_2 (1 + 3162) = 3000 \log_2 3163 \\ &= 3000 \times 11.62 = 34,860 \text{ bps} \end{aligned}$$

This means that the highest bit rate for a telephone line is 34.860 kbps. If we want to send data faster than this, we can either increase the bandwidth of the line or improve the signal-to-noise ratio.

*Example 3.39*

The signal-to-noise ratio is often given in decibels. Assume that  $\text{SNR}_{\text{dB}} = 36$  and the channel bandwidth is 2 MHz. The theoretical channel capacity can be calculated as

$$\begin{aligned}\text{SNR}_{\text{dB}} = 10 \log_{10} \text{SNR} &\Rightarrow \text{SNR} = 10^{\text{SNR}_{\text{dB}}/10} \Rightarrow \text{SNR} = 10^{3.6} = 3981 \\ C = B \log_2 (1 + \text{SNR}) &= 2 \times 10^6 \times \log_2 3982 = 24 \text{ Mbps}\end{aligned}$$

*Example 3.40*

For practical purposes, when the SNR is very high, we can assume that  $\text{SNR} + 1$  is almost the same as SNR. In these cases, the theoretical channel capacity can be simplified to

$$C = B \times \frac{\text{SNR}_{\text{dB}}}{3}$$

For example, we can calculate the theoretical capacity of the previous example as

$$C = 2 \text{ MHz} \times \frac{36}{3} = 24 \text{ Mbps}$$

**Using Both Limits**

In practice, we need to use both methods to find the limits and signal levels. Let us show this with an example.

*Example 3.41*

We have a channel with a 1-MHz bandwidth. The SNR for this channel is 63. What are the appropriate bit rate and signal level?

**Solution**

First, we use the Shannon formula to find the upper limit.

$$C = B \log_2 (1 + \text{SNR}) = 10^6 \log_2 (1 + 63) = 10^6 \log_2 64 = 6 \text{ Mbps}$$

The Shannon formula gives us 6 Mbps, the upper limit. For better performance we choose something lower, 4 Mbps, for example. Then we use the Nyquist formula to find the number of signal levels.

$$4 \text{ Mbps} = 2 \times 1 \text{ MHz} \times \log_2 L \Rightarrow L = 4$$

---

The Shannon capacity gives us the upper limit;  
the Nyquist formula tells us how many signal levels we need.

---



## 3.6 PERFORMANCE

Up to now, we have discussed the tools of transmitting data (signals) over a network and how the data behave. One important issue in networking is the performance of the network-how good is it? We discuss quality of service, an overall measurement of network performance, in greater detail in Chapter 24. In this section, we introduce terms that we need for future chapters.

### Bandwidth

One characteristic that measures network performance is bandwidth. However, the term can be used in two different contexts with two different measuring values: bandwidth in hertz and bandwidth in bits per second.

#### *Bandwidth in Hertz*

We have discussed this concept. Bandwidth in hertz is the range of frequencies contained in a composite signal or the range of frequencies a channel can pass. For example, we can say the bandwidth of a subscriber telephone line is 4 kHz.

#### *Bandwidth in Bits per Seconds*

The term *bandwidth* can also refer to the number of bits per second that a channel, a link, or even a network can transmit. For example, one can say the bandwidth of a Fast Ethernet network (or the links in this network) is a maximum of 100 Mbps. This means that this network can send 100 Mbps.

#### *Relationship*

There is an explicit relationship between the bandwidth in hertz and bandwidth in bits per seconds. Basically, an increase in bandwidth in hertz means an increase in bandwidth in bits per second. The relationship depends on whether we have baseband transmission or transmission with modulation. We discuss this relationship in Chapters 4 and 5.

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In networking, we use the term *bandwidth* in two contexts.

- The first, *bandwidth in hertz*, refers to the range of frequencies in a composite signal or the range of frequencies that a channel can pass.
  - The second, *bandwidth in bits per second*, refers to the speed of bit transmission in a channel or link.
- 

#### *Example 3.42*

The bandwidth of a subscriber line is 4 kHz for voice or data. The bandwidth of this line for data transmission can be up to 56,000 bps using a sophisticated modem to change the digital signal to analog.

#### *Example 3.43*

If the telephone company improves the quality of the line and increases the bandwidth to 8 kHz, we can send 112,000 bps by using the same technology as mentioned in Example 3.42.

## Throughput

The throughput is a measure of how fast we can actually send data through a network. Although, at first glance, bandwidth in bits per second and throughput seem the same, they are different. A link may have a bandwidth of  $B$  bps, but we can only send  $T$  bps through this link with  $T$  always less than  $B$ . In other words, the bandwidth is a potential measurement of a link; the throughput is an actual measurement of how fast we can send data. For example, we may have a link with a bandwidth of 1 Mbps, but the devices connected to the end of the link may handle only 200 kbps. This means that we cannot send more than 200 kbps through this link.

Imagine a highway designed to transmit 1000 cars per minute from one point to another. However, if there is congestion on the road, this figure may be reduced to 100 cars per minute. The bandwidth is 1000 cars per minute; the throughput is 100 cars per minute.

### Example 3.44

A network with bandwidth of 10 Mbps can pass only an average of 12,000 frames per minute with each frame carrying an average of 10,000 bits. What is the throughput of this network?

#### Solution

We can calculate the throughput as

$$\text{Throughput} = \frac{12,000 \times 10,000}{60} = 2 \text{ Mbps}$$

The throughput is almost one-fifth of the bandwidth in this case.

## Latency (Delay)

The latency or delay defines how long it takes for an entire message to completely arrive at the destination from the time the first bit is sent out from the source. We can say that latency is made of four components: propagation time, transmission time, queuing time and processing delay.

$$\text{Latency} = \text{propagation time} + \text{transmission time} + \text{queuing time} + \text{processing delay}$$

### Propagation Time

Propagation time measures the time required for a bit to travel from the source to the destination. The propagation time is calculated by dividing the distance by the propagation speed.

$$\text{Propagation time} = \frac{\text{Distance}}{\text{Propagation speed}}$$

The propagation speed of electromagnetic signals depends on the medium and on the frequency of the signal. For example, in a vacuum, light is propagated with a speed of  $3 \times 10^8$  mfs. It is lower in air; it is much lower in cable.

*Example 3.45*

What is the propagation time if the distance between the two points is 12,000 km? Assume the propagation speed to be  $2.4 \times 10^8$  mls in cable.

**Solution**

We can calculate the propagation time as

$$\text{Propagation time} = \frac{12,000 \times 1,000}{2.4 \times 10^8} = 50 \text{ ms}$$

The example shows that a bit can go over the Atlantic Ocean in only 50 ms if there is a direct cable between the source and the destination.

*Transmission Time*

In data communications we don't send just 1 bit, we send a message. The first bit may take a time equal to the propagation time to reach its destination; the last bit also may take the same amount of time. However, there is a time between the first bit leaving the sender and the last bit arriving at the receiver. The first bit leaves earlier and arrives earlier; the last bit leaves later and arrives later. The time required for transmission of a message depends on the size of the message and the bandwidth of the channel.

$$\text{Transmission time} = \frac{\text{Message size}}{\text{Bandwidth}}$$

*Example 3.46*

What are the propagation time and the transmission time for a 2.5-kbyte message (an e-mail) if the bandwidth of the network is 1 Gbps? Assume that the distance between the sender and the receiver is 12,000 km and that light travels at  $2.4 \times 10^8$  mls.

**Solution**

We can calculate the propagation and transmission time as

$$\text{Propagation time} = \frac{12,000 \times 1,000}{2.4 \times 10^8} = 50 \text{ ms}$$

$$\text{Transmission time} = \frac{2,500 \times 8}{10} = 0.020 \text{ ms}$$

Note that in this case, because the message is short and the bandwidth is high, the dominant factor is the propagation time, not the transmission time. The transmission time can be ignored.

*Example 3.47*

What are the propagation time and the transmission time for a 5-Mbyte message (an image) if the bandwidth of the network is 1 Mbps? Assume that the distance between the sender and the receiver is 12,000 km and that light travels at  $2.4 \times 10^8$  mls.

**Solution**

We can calculate the propagation and transmission times as

$$\text{Propagation Time} = \frac{12,000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

$$\text{Transmission time} = \frac{5,000,000 \times 8}{10^6} = 40 \text{ s}$$

Note that in this case, because the message is very long and the bandwidth is not very high, the dominant factor is the transmission time, not the propagation time. The propagation time can be ignored.

*Queuing Time*

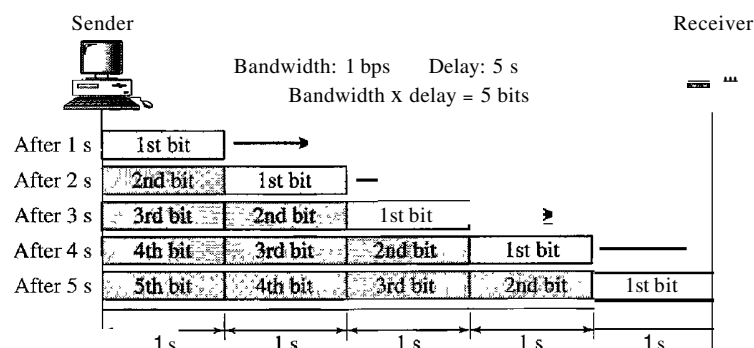
The third component in latency is the queuing time, the time needed for each intermediate or end device to hold the message before it can be processed. The queuing time is not a fixed factor; it changes with the load imposed on the network. When there is heavy traffic on the network, the queuing time increases. An intermediate device, such as a router, queues the arrived messages and processes them one by one. If there are many messages, each message will have to wait.

**Bandwidth-Delay Product**

Bandwidth and delay are two performance metrics of a link. However, as we will see in this chapter and future chapters, what is very important in data communications is the product of the two, the bandwidth-delay product. Let us elaborate on this issue, using two hypothetical cases as examples.

**O** Case 1. Figure 3.31 shows case 1.

Figure 3.31 Filling the link with bits for case 1

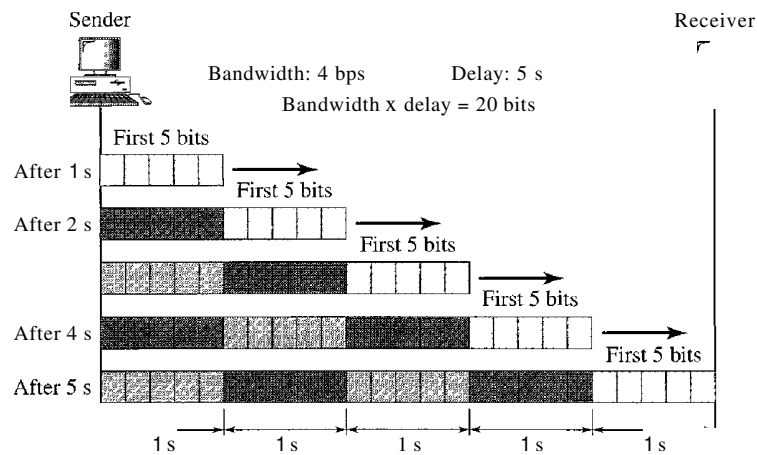


Let us assume that we have a link with a bandwidth of 1 bps (unrealistic, but good for demonstration purposes). We also assume that the delay of the link is 5 s (also unrealistic). We want to see what the bandwidth-delay product means in this case.

Looking at figure, we can say that this product  $1 \times 5$  is the maximum number of bits that can fill the link. There can be no more than 5 bits at any time on the link.

- O Case 2. Now assume we have a bandwidth of 4 bps. Figure 3.32 shows that there can be maximum  $4 \times 5 = 20$  bits on the line. The reason is that, at each second, there are 4 bits on the line; the duration of each bit is 0.25 s.

Figure 3.32 Filling the link with bits in case 2



The above two cases show that the product of bandwidth and delay is the number of bits that can fill the link. This measurement is important if we need to send data in bursts and wait for the acknowledgment of each burst before sending the next one. To use the maximum capability of the link, we need to make the size of our burst 2 times the product of bandwidth and delay; we need to fill up the full-duplex channel (two directions). The sender should send a burst of data of  $(2 \times \text{bandwidth} \times \text{delay})$  bits. The sender then waits for receiver acknowledgment for part of the burst before sending another burst. The amount  $2 \times \text{bandwidth} \times \text{delay}$  is the number of bits that can be in transition at any time.

The **bandwidth-delay** product defines the number of bits that can **fill** the link.

#### Example 3.48

We can think about the link between two points as a pipe. The cross section of the pipe represents the bandwidth, and the length of the pipe represents the delay. We can say the volume of the pipe defines the bandwidth-delay product, as shown in Figure 3.33.

Figure 3.33 Concept of bandwidth-delay product



## Jitter

Another performance issue that is related to delay is **jitter**. We can roughly say that jitter is a problem if different packets of data encounter different delays and the application using the data at the receiver site is time-sensitive (audio and video data, for example). **If** the delay for the first packet is 20 ms, for the second is 45 ms, and for the third is 40 ms, then the real-time application that uses the packets endures jitter. We discuss jitter in greater detail in Chapter 29.

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## 3.7 RECOMMENDED READING

For more details about subjects discussed in this chapter, we recommend the following books. The items in brackets [...] refer to the reference list at the end of the text.

### Books

Data and signals are elegantly discussed in Chapters 1 to 6 of [Pea92]. [Cou01] gives an excellent coverage about signals in Chapter 2. More advanced materials can be found in [Ber96]. [Hsu03] gives a good mathematical approach to signaling. Complete coverage of Fourier Analysis can be found in [Spi74]. Data and signals are discussed in Chapter 3 of [Sta04] and Section 2.1 of [Tan03].

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## 3.8 KEY TERMS

analog	Fourier analysis
analog data	frequency
analog signal	frequency-domain
attenuation	fundamental frequency
bandpass channel	harmonic
bandwidth	Hertz (Hz)
baseband transmission	jitter
bit rate	low-pass channel
bits per second (bps)	noise
broadband transmission	nonperiodic signal
composite signal	Nyquist bit rate
cycle	peak amplitude
decibel (dB)	period
digital	periodic signal
digital data	phase
digital signal	processing delay
distortion	propagation speed

propagation time	sine wave
queuing time	throughput
Shannon capacity	time-domain
signal	transmission time
signal-to-noise ratio (SNR)	wavelength

### 3.9 SUMMARY

- Data must be transformed to electromagnetic signals to be transmitted.
- Data can be analog or digital. Analog data are continuous and take continuous values. Digital data have discrete states and take discrete values.
- Signals can be analog or digital. Analog signals can have an infinite number of values in a range; digital signals can have only a limited number of values.
- In data communications, we commonly use periodic analog signals and nonperiodic digital signals.
- Frequency and period are the inverse of each other.
- Frequency is the rate of change with respect to time.
- Phase describes the position of the waveform relative to time 0.
- A complete sine wave in the time domain can be represented by one single spike in the frequency domain.
- A single-frequency sine wave is not useful in data communications; we need to send a composite signal, a signal made of many simple sine waves.
- According to Fourier analysis, any composite signal is a combination of simple sine waves with different frequencies, amplitudes, and phases.
- The bandwidth of a composite signal is the difference between the highest and the lowest frequencies contained in that signal.
- A digital signal is a composite analog signal with an infinite bandwidth.
- Baseband transmission of a digital signal that preserves the shape of the digital signal is possible only if we have a low-pass channel with an infinite or very wide bandwidth.
- If the available channel is a bandpass channel, we cannot send a digital signal directly to the channel; we need to convert the digital signal to an analog signal before transmission.
- For a noiseless channel, the Nyquist bit rate formula defines the theoretical maximum bit rate. For a noisy channel, we need to use the Shannon capacity to find the maximum bit rate.
- Attenuation, distortion, and noise can impair a signal.
- Attenuation is the loss of a signal's energy due to the resistance of the medium.
- Distortion is the alteration of a signal due to the differing propagation speeds of each of the frequencies that make up a signal.
- Noise is the external energy that corrupts a signal.
- The bandwidth-delay product defines the number of bits that can fill the link.

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## 3.10 PRACTICE SET

### Review Questions

1. What is the relationship between period and frequency?
2. What does the amplitude of a signal measure? What does the frequency of a signal measure? What does the phase of a signal measure?
3. How can a composite signal be decomposed into its individual frequencies?
4. Name three types of transmission impairment.
5. Distinguish between baseband transmission and broadband transmission.
6. Distinguish between a low-pass channel and a band-pass channel.
7. What does the Nyquist theorem have to do with communications?
8. What does the Shannon capacity have to do with communications?
9. Why do optical signals used in fiber optic cables have a very short wave length?
10. Can we say if a signal is periodic or nonperiodic by just looking at its frequency domain plot? How?
11. Is the frequency domain plot of a voice signal discrete or continuous?
12. Is the frequency domain plot of an alarm system discrete or continuous?
13. We send a voice signal from a microphone to a recorder. Is this baseband or broadband transmission?
14. We send a digital signal from one station on a LAN to another station. Is this baseband or broadband transmission?
15. We modulate several voice signals and send them through the air. Is this baseband or broadband transmission?

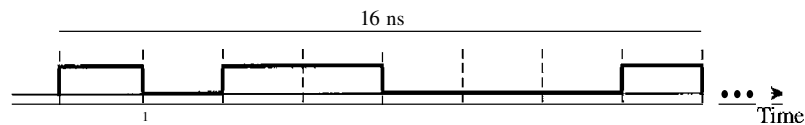
### Exercises

16. Given the frequencies listed below, calculate the corresponding periods.
  - a. 24Hz
  - b. 8 MHz
  - c. 140 KHz
17. Given the following periods, calculate the corresponding frequencies.
  - a. 5 s
  - b. 12  $\mu$ s
  - c. 220 ns
18. What is the phase shift for the following?
  - a. A sine wave with the maximum amplitude at time zero
  - b. A sine wave with maximum amplitude after 1/4 cycle
  - c. A sine wave with zero amplitude after 3/4 cycle and increasing
19. What is the bandwidth of a signal that can be decomposed into five sine waves with frequencies at 0, 20, 50, 100, and 200 Hz? All peak amplitudes are the same. Draw the bandwidth.



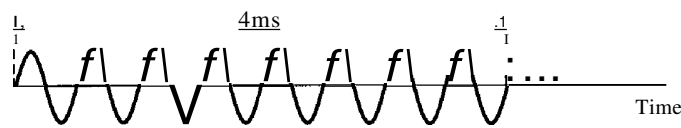
20. A periodic composite signal with a bandwidth of 2000 Hz is composed of two sine waves. The first one has a frequency of 100 Hz with a maximum amplitude of 20 V; the second one has a maximum amplitude of 5 V. Draw the bandwidth.
21. Which signal has a wider bandwidth, a sine wave with a frequency of 100 Hz or a sine wave with a frequency of 200 Hz?
22. What is the bit rate for each of the following signals?
- A signal in which 1 bit lasts 0.001 s
  - A signal in which 1 bit lasts 2 ms
  - A signal in which 10 bits last 20  $\mu$ s
23. A device is sending out data at the rate of 1000 bps.
- How long does it take to send out 10 bits?
  - How long does it take to send out a single character (8 bits)?
  - How long does it take to send a file of 100,000 characters?
24. What is the bit rate for the signal in Figure 3.34?

Figure 3.34 Exercise 24



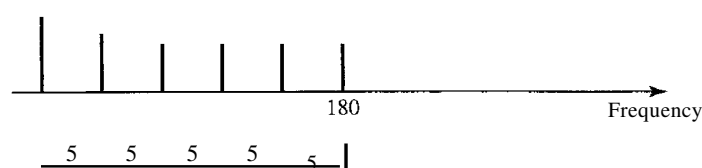
25. What is the frequency of the signal in Figure 3.35?

Figure 3.35 Exercise 25



26. What is the bandwidth of the composite signal shown in Figure 3.36.

Figure 3.36 Exercise 26



27. A periodic composite signal contains frequencies from 10 to 30 KHz, each with an amplitude of 10 V. Draw the frequency spectrum.
28. A non-periodic composite signal contains frequencies from 10 to 30 KHz. The peak amplitude is 10 V for the lowest and the highest signals and is 30 V for the 20-KHz signal. Assuming that the amplitudes change gradually from the minimum to the maximum, draw the frequency spectrum.
29. A TV channel has a bandwidth of 6 MHz. If we send a digital signal using one channel, what are the data rates if we use one harmonic, three harmonics, and five harmonics?
30. A signal travels from point A to point B. At point A, the signal power is 100 W. At point B, the power is 90 W. What is the attenuation in decibels?
31. The attenuation of a signal is -10 dB. What is the final signal power if it was originally 5 W?
32. A signal has passed through three cascaded amplifiers, each with a 4 dB gain. What is the total gain? How much is the signal amplified?
33. If the bandwidth of the channel is 5 Kbps, how long does it take to send a frame of 100,000 bits out of this device?
34. The light of the sun takes approximately eight minutes to reach the earth. What is the distance between the sun and the earth?
35. A signal has a wavelength of  $1\text{ }\mu\text{m}$  in air. How far can the front of the wave travel during 1000 periods?
36. A line has a signal-to-noise ratio of 1000 and a bandwidth of 4000 KHz. What is the maximum data rate supported by this line?
37. We measure the performance of a telephone line (4 KHz of bandwidth). When the signal is 10 V, the noise is 5 mV. What is the maximum data rate supported by this telephone line?
38. A file contains 2 million bytes. How long does it take to download this file using a 56-Kbps channel? 1-Mbps channel?
39. A computer monitor has a resolution of 1200 by 1000 pixels. If each pixel uses 1024 colors, how many bits are needed to send the complete contents of a screen?
40. A signal with 200 milliwatts power passes through 10 devices, each with an average noise of 2 microwatts. What is the SNR? What is the  $\text{SNR}_{\text{dB}}$ ?
41. If the peak voltage value of a signal is 20 times the peak voltage value of the noise, what is the SNR? What is the  $\text{SNR}_{\text{dB}}$ ?
42. What is the theoretical capacity of a channel in each of the following cases:
  - a. Bandwidth: 20 KHz       $\text{SNR}_{\text{dB}} = 40$
  - b. Bandwidth: 200 KHz     $\text{SNR}_{\text{dB}} = 4$
  - c. Bandwidth: 1 MHz       $\text{SNR}_{\text{dB}} = 20$
43. We need to upgrade a channel to a higher bandwidth. Answer the following questions:
  - a. How is the rate improved if we double the bandwidth?
  - b. How is the rate improved if we double the SNR?

44. We have a channel with 4 KHz bandwidth. If we want to send data at 100 Kbps, what is the minimum  $\text{SNR}_{\text{dB}}$ ? What is SNR?
45. What is the transmission time of a packet sent by a station if the length of the packet is 1 million bytes and the bandwidth of the channel is 200 Kbps?
46. What is the length of a bit in a channel with a propagation speed of  $2 \times 10^8$  m/s if the channel bandwidth is
  - a. 1 Mbps?
  - b. 10 Mbps?
  - c. 100 Mbps?
47. How many bits can fit on a link with a 2 ms delay if the bandwidth of the link is
  - a. 1 Mbps?
  - b. 10 Mbps?
  - c. 100 Mbps?
48. What is the total delay (latency) for a frame of size 5 million bits that is being sent on a link with 10 routers each having a queuing time of  $2 \mu\text{s}$  and a processing time of  $1 \mu\text{s}$ . The length of the link is 2000 Km. The speed of light inside the link is  $2 \times 10^8$  m/s. The link has a bandwidth of 5 Mbps. Which component of the total delay is dominant? Which one is negligible?

