Discrete Mathematics

1. Set theory

$A = \{1, 2, 3\}$
$A = \{a, b, c\}$ // Roaster form for infinite / large sets. $\cup$, ϕ

N = positive integers
Z → +ve, 0, −ve
Z+   Z−

Q − Rational nos.
C − complex no
R − Real no.

→ $C \supseteq R \supseteq Q \supseteq Z \supseteq N$

Universal sets are different.
ϕ are all same.

ϕ is a subset of every set.

1. $\phi \subseteq \phi$ T → Every set is a subset of itself.
2. $\phi \in \phi$ F
3. $\phi \subseteq \{\phi\}$ T → ϕ is a subset of every set.
4. $\phi \in \{\phi\}$ T
5. $\{\phi\} \in \phi$ F
6. $\{\phi\} \subseteq \{\phi\}$ T
7. $\{\phi\} \in \{\phi\}$ F

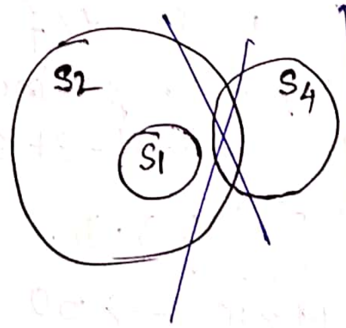
$E = \{x : x^2 - 3x + 2 = 0\}$
$F = \{2, 1\}$
$G = \{1, 2, 2, 1, 6/3\}$.

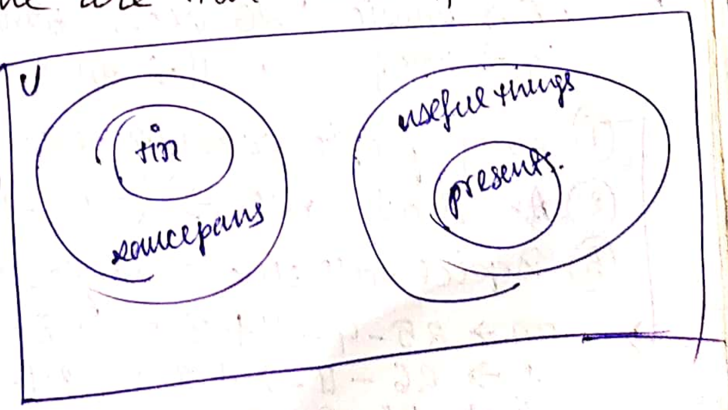
Venn diagram

1st set contains all the students of NITA [S_1]
S_2 → All the 3rd sem std., CSE (2018 batch)
Make a V.D representing these sets.

page-107

Q/ ① is my saucepans are the only things I have that are made of tin. (S_1, S_2)
② is I find all my presents to be useful (S_3, S_4)
③ None of my saucepans are of slightest use.
Conclusion → Your presents to me are not made of tin





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$S = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$

- Cardinality of $S = 3$
- power set of S.
  $\rightarrow \{\{\{\phi\}, \{\phi, \{\phi\}\}\}$,
  $\{\phi\}$, $\{\phi, \{\phi\}\}$,
  $\{\phi, \{\phi, \{\phi\}\}\}$,
  $\{\{\phi\}, \{\phi, \{\phi\}\}\}$,
  $\phi\}$

Q find the no. of mathematics students at the college taking atleast 1 of the languages → French, German & Russian, given the following data.

65 study French
45 study German
42 study Russian.
20 study F and G.
25 study F and R
15 study G and R
8 study all 3 languages

Principle of exclusion
$A \cup B \cup C = A + B + C - A \cap B - A \cap C - B \cap C + A \cap B \cap C$

→ ~~Principle~~

$65 + 45 + 42 - 20 - 25 - 15 + 8$
$= 100$

Q/ Consider the following data for 120 math students at the college studying the languages F, G, R,

$65 \rightarrow F$, $45 \rightarrow G$, $42 \rightarrow R$
$20 \rightarrow F, G$
$25 \rightarrow F, R$
$15 \rightarrow G, R$
$8 \rightarrow$ all 3

Find the no of students who study atleast 1 of the 3 languages.
& no of std. who study only F, only G, only R.

→ $N(F) = N(F) - N(F \cap G) - N(F \cap R) + N(F \cap G \cap R)$
$= 65 - 20 - 25 + 8$
$= 65 - 45 + 8$ ~~= 65 - 53 =~~ 28

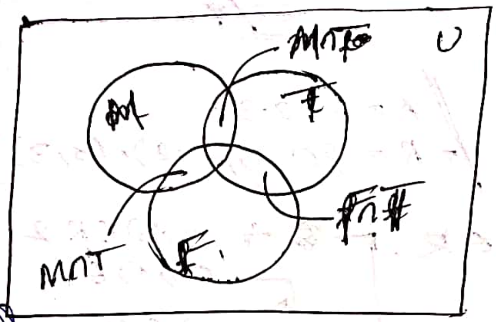
$N(G) = 45 - 15 - 20 + 8 = 18$

$N(R) = 42 - 25 - 15 + 8 = 42 - 40 + 8 = 10$

Q/ In a survey of 60 people it was found that
$25 \rightarrow$ music 26 read time 26 read fortune
$9 \rightarrow m \& f$, $11 \rightarrow m \& t$, $8 \rightarrow t \& f$, $3 \rightarrow$ all 3.

(1) No of people reading exactly 1 magazine
(2) At least one → $25 + 26 + 26 - 9 - 11 - 8 + 3 = 80 - 28 = 52$
(3) Depict all the 8 regions of the V.D.

$m \rightarrow 25 - 9 - 11 + 3 = 28 - 20 = 8$
$t \rightarrow 26 - 11 - 8 + 3 = 29 - 19 = 10$ → 30
$f \rightarrow 26 - 9 - 8 + 3 = 29 - 17 = 12$



Q/ Among 50 students in a class, 26 got an A in the 1st exam 21 got an A in the 2nd exam.
If 17 std. didn't get an A in either exam,
How many std. got an A in both exams?

$U \to 50 \quad E1 \to 26 \quad E2 \to 21$

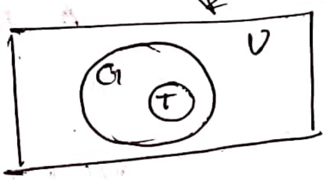
$26+21+17-50$
$64-50$
14

47
17
64

$A \cup B = A + B - A \cap B$
$A \cap B = A + B - A \cup B$
$= 26 + 21 -$
$= 4$

G

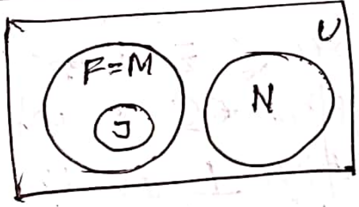
1. All guilty people will be arrested
2. All thieves are guilty people.
3. All the thieves are arrested (T)



~~Keep in your~~ mind The validity of follow..

1. All my friends are musicians
2. John = frnd.
3. None of my neighbours are musician.

C → John is my neighbour (F)



A
26 → E1
21 → E2
17 → A

23/7

Relation

Cartesian product. (A×B)

A {1, 2}
B {a, b, c}

{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)}

A = {1, 2, 3}
R = {(1,1), (2,2), (3,3)}

(1) Reflexive → ∀a ∈ A, (a,a) ∈ R.
(2) Symmetric → ∀(a,b) ∈ R : (b,a) ∈ R
(3) Transitive → ∀ (a,b) ∈ R & (b,c) ∈ R → (a,c) ∈ R

Q/ R_1 = {(1,1), (1,2), (2,3), (1,3), (4,4)} → transitive   A = {1, 2, 3, 4}
R_2 = {(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)} → (reflexive, symmetric transitive)
R_3 = {(1,3), (2,1)}
R_4 = ∅ → symmetric, transitive
R_5 = (A×A) → cross pdt on universal rel^n.
R_5 = {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)}

all three

(1,3) ✗ (3,1) → ∴ not sym.

Q/
1. Relation ≤ Z of ~~posi~~ integers. (Reflexive, transitive)   (1,2) (2,3) → (1,3)
2. Relation ⊆ C of sets. (Reflexive, transitive)
3. Relation ⊥ on the set L of lines in the plane (Symmetric)
4. Relation ∥ on the set L of lines in the plane (symmetric, reflexive, transitive)
5. Relation / on a set N of positive integers ⇒ a ÷ b → (Reflexive, ✗, Trans)

every no is div by itself

Important ✱

✱ Let R be a Relation defined on a set of positive integers such that (x,y) ∈ R if & only if (x−y) ÷ 3.
P.t : R is an equivalence relation.

→ Let (a,a) ∈ R
Now, (a−a) ÷ 3 = 0
∴ R is reflexive.

Let (a,b) ∈ R
Let $\frac{(a-b)}{3} = k$ ⇒ a − b = 3k
∴ b − a = −3k
∴ $\frac{b-a}{3} = -k$
∴ R is symmetric.

Antisymmetric imp

$\forall$ $(a,b) \in R$, $(b,a) \in R$ if and only if $a = b$

$R_1 = \{1,1\}$ → antisym.

$R_2 = \{(1,2),(2,1)\}$ → sym

$R_3 = \{(1,1),(1,2),(2,1)\}$ ↳ sym but not antisym

$R_4 = \{(1,1),(1,2)\}$ ↳ antisym but not sym.

* Partial Ordered set (POSET)

Let R be a relation defined of a set of ordered pair of +ve integers such that $(x,y),(u,v) \in Z^+ \times Z^+$ if and only if $\frac{u}{x} = \frac{v}{y}$, say whether R is equivalence or POSET?

→ (1) $\forall (a,b) \in R$, $\frac{a}{a} = \frac{b}{b}$ ∴ Reflexive.

~~(a,b) ∈ R~~ ∴ $(a,b) R (a,b)$.

(2) $\forall (a,b),(c,d) \in R$, $\left\{\frac{a}{c} = \frac{b}{d}\right\}$

Let $(a,b) R (c,d)$

∴ $\frac{c}{a} = \frac{d}{b} = m$ (say), where $m \in N$

∴ $c = ma$, $d = mb$.

Now, ∴ $\frac{a}{c} = \frac{a}{ma} = \frac{1}{m}$

and $\frac{b}{d} = \frac{b}{mb} = \frac{1}{m}$

∴ $\frac{a}{c} = \frac{b}{d}$ ∴ $(c,d) R (a,b)$

∴ symmetric.

(3) $(a,b) R (c,d)$ ~~(c,d) ∈ R~~

$(c,d) R (e,f)$ → $(a,b) R (e,f)$ ⇒ $\frac{e}{a} = \frac{f}{b}$

$\frac{e}{c} = \frac{f}{d}$

∴ $\frac{c}{a} = \frac{d}{b} = m$ (say), where $m \in N$ ∴ $c = ma$, $d = mb$.

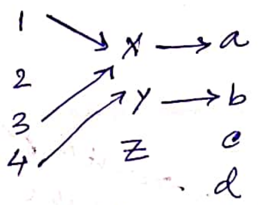
~~now, again~~ $\frac{e}{c} = \frac{f}{d} = n$ ⇒ $e = cn = man$, $f = dn = mbn$

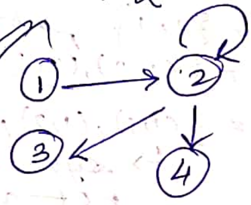
$\frac{e}{a} = \frac{man}{a} = mn$ $\quad$ $\frac{f}{b} = \frac{mbn}{b} = mn$

24/7/19

Composition of Relation

$R \circ S \rightarrow \{(1,a),(3,a),(4,b)\}$





consider the directed graph & find out whether it follows a equivalence relⁿ or not?

→ Not reflexive, not symmetric, not transitive

Q Relation $R = \{(a,b),(c,d),(d,b)\}$ (b,b)
Relation $S = \{(d,b),(b,e),(c,a),(a,c)\}$

find
① ROS
② (SOR)OR
③ RO(SOR)
④ ROR
⑤ SOS
⑥ ROROR

① $ROS = \{(a,e),(c,b),(b,e)\}$

② $SOR = \{(d,b),(c,b),(a,d)\}$

$(SOR)OR = \{(a,b), \ldots\}$

③ $RO(SOR) = \phi$

④ $ROR = \{(c,b)\}$ — $\{(a,b),(b,b)\}$

⑤ $SOS = \{(d,e),(c,e),(a,a)\}$

⑥ $(ROR)OR = \phi$ → $\{(a,b),(b,b)\}$

Q Let R and S be 2 relations on a set of +ve integers I such that $R = \{(a,3a) \mid a \in I\}$
$S = \{(a,a+1) \mid a \in I\}$

3a, 9a

① ROS → $ROS = \{(a, 3a+1) \mid a \in I\}$

② ROR → $ROR = \{(a, 9a) \mid a \in I\}$

③ (ROR)OR → $(ROR)OR = \{(a, 27a) \mid a \in I\}$

④ (ROS)OR → $(ROS)OR = \{(a, 9a+1) \mid a \in I\}$

Q. Consider a set of all animals, the relation is defined as: "is of same species as" → whether R is equivalence or not.

→ Yes. R S T

Q ~~a = b+mt~~ $a \equiv b \pmod m$ → [congruence relation]

↳ [Let m be a ~~fixed~~ +ve integers.
2 int a & b are said to be congruent modulo m
written as $a \equiv b \pmod m$]
whether the congruence relation is equivalence or not.
↳ $\boxed{\text{yes}}$

→

25/7/19

Equivalence class & Partition.

consider the following relation R on set $S = \{1, 2, 3\}$
$R = \{(1,1), (1,2), (2,1), (3,3)\}$ → R S T
1. show whether it is an equivalence relation or not.

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$

1 → 1 | 2 → 1 | 3 → 3
1 → 2 | 2 → 2

equal

equivalence classes (of 1 & 2 are equal)

$S = \{1, 2, 3\}$

$[1] = \{1, 2\}$ $[1] = [2]$
$[2] = \{1, 2\}$
$[3] = \{3\}$

S/R → poset set

Partition,
$P = [\{1, 2\}, \{3\}]$

Q// Let R be the following equivalence relation on the set $A = \{1, 2, 3, 4, 5, 6\}$, $R = \{(1,1), (1,5), (2,2), (2,3), (2,6), (3,2), (3,3), (3,6), (4,4), (5,1), (5,5), (6,2), (6,3), (6,6)\}$
Find the partion of A induced by R & find eq. class (A/R) of R.

$[1] = \{1, 5\}$
$[2] = \{2, 3, 6\}$
$[3] = \{2, 3, 6\}$
$[4] = \{4\}$
$[5] = \{1, 5\}$
$[6] = \{2, 3, 6\}$

$[2] = [3] = [6]$
$[1] = [5]$

$P = [\{1, 5\}, \{2, 3, 6\} \{4\}]$

$a \equiv b \bmod c$

Div. Rem. $c)a(k$ b (a-b) div c

~~Q. A congruence relation is equivalence.~~

→ Let R_5 be the relation on the set Z of integers defined by $\boxed{x \equiv y \bmod 5}$. Find equivalence classes & partition.

div. $\boxed{\text{Rem}}$ divisor

R → from 0 to 5.

$\frac{x-y}{5}$

$5)x(k$
y

$\boxed{x = 5k + y}$

Rem → $\boxed{0, 1, 2, 3, 4}$

~~$[0] = \{0, 10, 15, -5, 0, \ldots\}$~~

$x = 5k + y$

$[1] = \{1, 6, 11, 16\}$
$[2] = \{2, 7, 12, 17\}$
$[3] = \{3, 8, 13, 18\}$
$[4] = \{4, 9, 14, 19\}$
$[5] = \{5, 10, 15, 20\}$

Q. Consider the set of words $W = \{$sheet, last, sky, wash, wind, sit$\}$

Find W/R where

R is eq. relation of W defined by either:

(i) "has the same no. of letters as"

(ii) "begins with the same letter as"

$R_1 = \{$(last, wash), (last, wind), (wash, wind), (sky, sit), (wash, last), (wind, last), (wind, wash), (last, last), (wash, wash), (wind, wind), (sit, sky), (sit, sit), (sky, sky), (sheet, sheet)$\}$

$R_2 = \{$(sheet, sky), (sky, sheet), (sheet, sheet), (sky, sky), (wash, wind), (wind, wash), (wind, wind), (wash, wash), (last, last), (sheet, sit), (sit, sheet), (sit, sit), (sky, sit), (sit, sky)$\}$

$W/R_1 = [\{$wash, wind, last$\}, \{$sit, sky$\}, \{$sheet$\}]$

$W/R_2 = [\{$sheet, sky, sit$\}, \{$wash, wind$\}, \{$last$\}]$

[last] = {wash, wind, last}

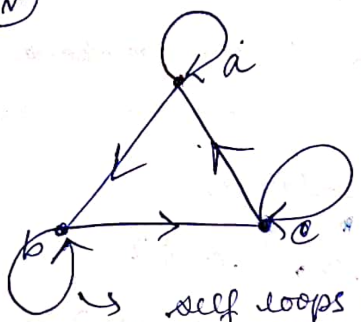
[wash] = {wind, last, wash}

[sky] = {sit, sky}

[sheet] = {sheet}

[wind] = {wind, wash, last}

sit = {sit, sky}



Directed graph
{ (a,a), (a,b), (b,b), (b,c), (c,c), (c,a) }

{ Symmetric ✗, Transitive ✗ }

self loops ✓ Reflexive

$$\begin{array}{c|c} & a \quad b \quad c \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

Not an equivalence relation because $a \not R a$
∴ not reflexive.

bRa, $a \not R b$ ✗, ✗

(Relation from row to column)

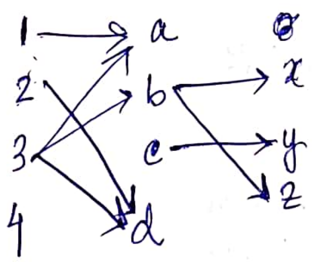
---

for symmetricity (upper & lower half of diagonals) must be similar.

For AoB, multiply [A] & [B] ⇝ non zero ⇒ Relation
zero ⇒ $\cancel{R}$

$$M_R = \begin{array}{c|c} & a \quad b \quad c \quad d \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad M_S = \begin{array}{c|c} & x \quad y \quad z \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

→ ROS, $M_{ROS} = M_R \times M_S = $
$$\begin{array}{c|c} & x \quad y \quad z \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$



Closure

- Same properties, all the elements satisfying the prop are closure.

Q Let $A = \{1,2,3,4\}$

$R = \{(1,3),(2,2),(1,4),(3,4)\}$

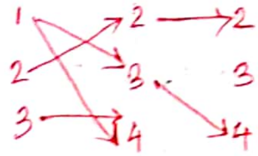
~~Rel~~ find the R, S, T closure of this relation.

→

$R_r = \{(1,3),(2,2),(1,4),(3,4),(1,1),(2,2),(3,3),(4,4)\}$

$R_s = \{(1,3),(2,2),(1,4),(3,4),(3,1),(4,1),(4,3)\}$

$R_T = \{(1,3),(2,2),(1,4),(3,4)\}$



(1, 4)
(2,2)

(*) To find Transitive closure find RoR

Q Let $X = \{1,2,3,4\}$

Let R be a relation such that $\frac{x-y}{3}$ find whether it is equivalence or not.

$R = \{(1,1),(2,2),(3,3),(4,4),(1,4),(4,1)\}$ R, S, T

Q Show that the relation $R = \{(a,b) : (a-b) = \text{even integer}\}$ is an equivalence relation.

→ ~~Reflle~~ Reflexive: Let $a \in R$

$\therefore (a-a) = 0$, which is an even integer

$\therefore$ R is reflexive

Symmetric: Let $(a,b) \in R$ such that $a-b = 2k$, which is an integer

$\therefore b-a = 2(-k)$, also an even integer.

$\therefore (b,a) \in R$

$\therefore$ R is symmetric.

Transitive: Let $(a,b) \in R$ and $(b,c) \in R$

$\therefore a-b = 2k$

and $b-c = 2m$

$\therefore a-b+b-c = 2k+2m$

~~$\therefore a-c-b+b = 2k+2m$~~

$\Rightarrow a-c = 2(k+m)$

$\therefore (a,c) \in R$

$\therefore$ R is transitive.

Q Let $A = \{1,2,3,4,5\}$, find the equivalence relation generated by the partition $R = \{\{1,3,5\},\{2,4\}\}$

→ $R = \{(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),(5,3),(5,5),(2,2),(2,4),(4,2),(4,4)\}$

$R = \{(1,3),(4,5)\}$ $A = \{1,2,3,4,5\}$

Domain $= \{1,4\}$ $R^{-1} = \{(3,1),(5,4)\}$

Range $= \{3,5\}$

Q Find whether the following Rel^n s over +ve ints N, is reflexive, symmetric, antisymmetric & transitive.

$R_1 \Rightarrow x > y$ ⟶ R X S X AS X T ✓ (1,2) (2,3) (1,3) ⇒ R ✓ S ✓ AS X T

$R_2 \Rightarrow xy =$ square of an integer.

$R_3 \Rightarrow x+y = 10$

$R_4 \Rightarrow x+4y = 10$

$R_5 \Rightarrow x+y = \cancel{6}\,5$

It will be AS because there is no such (1,4) (4,1) (a,b) or (b,a).

(b,a) (a,b)

	R	S	AS	T
1.	X	X	✓	✓
2.	✓	✓	X	✓
3.	X	✓		X
4.	X	X		
5.	X			

Q If R is a relation on the set of all people, determine whether the following rel^n s are equivalence or not?

✓ ① aRb ⇒ a, b same age. R ✓ S ✓ T ✓

✓ ② a, b have same parents. R ✓ S ✓ T ✓

X ③ a and b share a common parent. R ✓ S ✓ T X

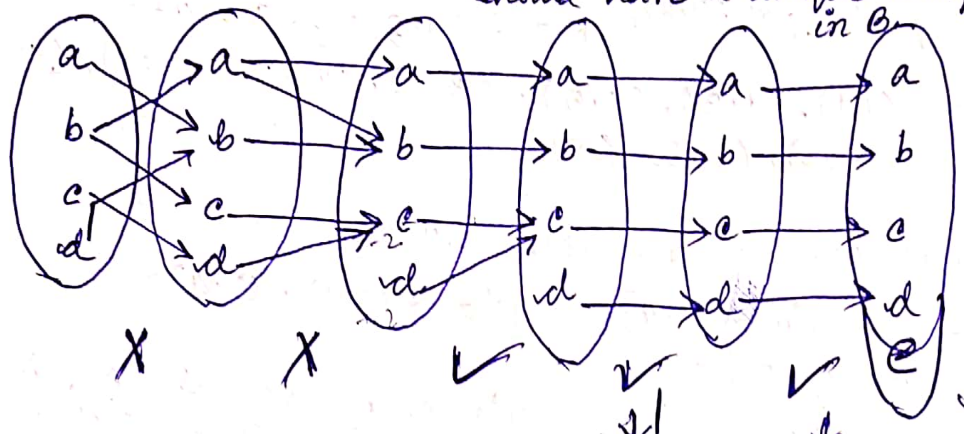
X ④ a & b have met. R X S ✓ T X

X ⑤ a is brother of b. R X S X T ✓

X ⑥ a & b speak a common language R ✓ S ✓ T X

Function : Every element of set A should have a unique image in B.

31/7



(one to one) — onto

(one to one) — not onto.

- one to one → If every element has a diff & one image.
- onto → If every element of B is an image

one to one correspondence.

- Injective (one to one)
- Surjective (onto)
- Bijective → $\overset{1-1}{\text{correspondence}}$ → Invertible.
  (onto & one to one)

Q1 Is a relation a function?

Q2 Is a function a relation?

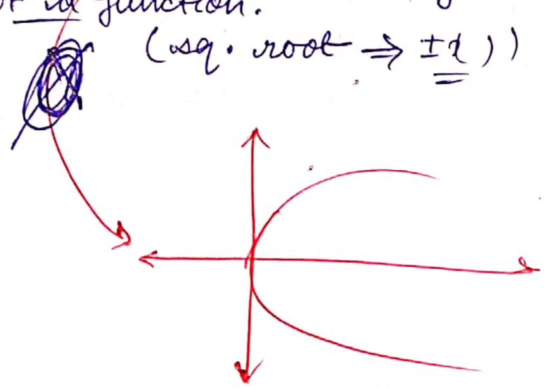
Is fn a subset of rel^n or is rel^n a subset of fn^n?

→ • All the functions are relations but all relations aren't functions.

• fn is a subset of rel^n.
  fn is not necessarily a proper subset of rel^n.

Q/ ① $f(x) = x^2$ ② $f(x) = \sqrt{x}$ consider the validity of ① & ② as a fn.

not a function. (sq. root ⇒ ±x)) [2 images]



Q ~~Let~~ A be a set of students in the school.
Determine which of the following assignments define a function
(a) on A.

1. To each student, assign his age. - Yes
2. To each student, assign his teacher X
3. To each student, " his gender Yes
4. To each student; his friends. X

Q/ ~~Verify~~ To each person on the earth assign the number which corresponds to ~~the~~ his/her age. X

Q/ To each country in the world assign the lat & long of the capital. ✓

Q/ To each book written ~~bye~~ only 1 author assign the author. X
→ multiple books by same author.

Q/ To each country in the world which has a PM assign the PM ✓

(#) Any declarative statement i.e. either true or false but not both.

~~1/8~~
20/8

↑

Logic & $\boxed{\text{proposition}}$al calculus

P : Today is Monday → false
Q : 2+2=4 → true.
R : Do your Homework (neither T nor false)

(And) $\wedge$ Conjunction
(or) $\vee$ Disjunction
(not) $\neg$ Negation.

p	q	p∨q
0	0	
0	1	
1	0	
0	0	

p	¬p
F	T
T.	F

p	q	p∧q
F	F	F
T	F	F
F	T	F
T	T	T

p	q	p∨q
F	F	F
T	F	T
F	T	T
T	T	T

Tautology: If the final value is ~~false~~ true for all the conditions it is called tautology.

Contradiction / Fallacy: If the final value is false for all the conditions.

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

→ whenever 2 truth tables' logical propositions are true, they are called logically equivalent to each other.

Implication: If p then q; $p \rightarrow q$

Biimplication: Iff p/q ; $p \leftrightarrow q$.

1. If p then q,
2. If p, q
3. p is sufficient for q. → A suff cond for q is p.
4. q if p.
5. q when p
6. A necessary condition for p is q. → q is necessary for p
7. q unless negation of p.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

only false when 1st is T & 2nd F.

p whenever q

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

X-NOR

$P_1 = p \rightarrow q$

$P_2 = q \rightarrow r$

$Q = p \rightarrow r$

only valid if $P_1 \wedge P_2 \rightarrow Q$ is a tautology.

~~$P_1 \wedge P_2 \rightarrow Q$~~

p	q	r	$P_1 = p \to q$	$P_2 = q \to r$	$Q = p \to r$	$P_1 \wedge P_2$	$P_1 \wedge P_2 \to Q$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	1	0	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	1	0	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Q Consider the following arguments :

P_1) If a man is bachelor he is unhappy.
P_2) If he is unhappy he dies young.
Q.) Q – bachelor dies young.

Law of syllogism

p : bachelor q : unhappy r : dies young.

p	q	r	P_1	P_2	Q	$P_1 \wedge P_2$	$P_1 \wedge P_2 \to Q$
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
T	F	T	F	T	T	F	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

$p \to q \to$ cond^n / Implication
$q \to p \to$ converse
$\neg p \to \neg q \to$ Inverse
$\neg q \to \neg p \to$ contrapositive

converse $\equiv$ inverse; implication $\equiv$ contrapositive

Determine the contrapositive of each statement.

1. If John is a poet then he is poor.
2. Only if Mark studies will he pass the test.
   (q = Mark studies, p = he pass the test)

$\neg q \to \neg p \to$ contrapositive.

1. If john is not ~~a poet~~ poor then he ~~is sick~~ is not a poet.
2. ~~He won't pass the test if Mark~~ If Mark doesn't study then he won't pass.

only if q p
$\to$ p only if q
$\to$ if p then q.

p	q	$p \to q$	$\neg p$	$\neg q$	$\neg p \to \neg q$	$q \to p$	$\neg q \to \neg p$
F	F	T	T	T	T	T	T
T	F	F	F	T	T	T	F
F	T	T	T	F	F	F	T
T	T	T	F	F	T	T	T

$\neg p \to \neg q \Leftrightarrow q \to p$: equivalent

$p \to q \Leftrightarrow \neg q \to \neg p$: equivalent

Determine the validity of following argument: if 2 sides of a Δ are equal then the opp angles are equal. $\to P_1$
(p = 2 sides of a Δ are equal, q = the opp angles are equal)

(2) 2 sides of a Δ aren't equal. $\to P_2$

$Q \to$ The opp angles aren't equal.

$P_1 \wedge P_2 \to Q$.

p	q	$p \to q$ (P_1)	$\neg p$ (P_2)	$\neg q$ (Q)	~~$\neg p \to \neg q$~~	$P_1 \wedge P_2$	$P_1 \wedge P_2 \to Q$
F	F	T	T	T	T	T	T
T	F	F	F	T	T	F	T
F	T	T	T	F	F	~~T~~ F	F
T	T	T	F	F	T	F	T

1. You can access the internet from campus only if you are a CS major or you're not a freshman.

convert it into a logical expression:

1. You can access the internet from campus p
2. you are a cs major q
3. are a Freshman r

$q \vee \neg r$

$\cancel{p \leftrightarrow (q \vee \neg r)}$

$p \leftrightarrow q \vee \neg r$

2. You can't ride the roller coaster if you're under 5 ft tall unless you are older than 16 yrs. → you can't ride the RC.

1. can't ride the roller coaster → p
2. you're under 5 → q
3. unless you're >16 yrs. → r

$(q \wedge \neg r) \leftrightarrow \neg p$

$\boxed{\text{unless} \rightarrow \text{not , and}}$ → or

q	r	r'	qr'	$q+r$
0	0	1	0	0
0	1	0	0	1
1	0	1	1	1
1	1	0	0	1

can ride the R.C ⇒ p
under 5 feet tall ⇒ q
older 16 years ⇒ r

$(q \vee \neg r) \rightarrow \neg p$

$$\begin{array}{l} p \rightarrow q \\ \neg p \\ \hline \neg q \end{array} \qquad \begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \neg p \end{array}$$

$p \rightarrow q, \neg q \vdash \neg p$

A unless B
A, if not B
B or A

22/8

Propositional function
Quantifiers

A propositional function defined on A is an expression $P(x)$, which has the property that $P(x)$ is true or false for each value of $x \in A$.

$\forall x, P(x)$
for all the values of x, $P(x)$ is true. — Universal quantifier

$\exists x, P(x)$
There exists some values of x for $P(x)$ is true. — existential Quantifier

Q Use quantifiers and propositional function to state the Relationship betⁿ the following elements.
1. Students and clever.
2. Integer and positive.
3. Integer & Real number
4. Tigers, white

Ans: 1. Relationship : The students are clever.
$P(x) \rightarrow$ $\boxed{\forall x, P(x)}$ ×
$\forall x$, $P(x)$ is true.

2. $\exists x, P(x)$
3. $\forall x, P(x)$
4. $\exists x, P(x)$

$P(x,y) \rightarrow x+y=5$

$\forall x \; \forall y \; P(x,y)$
$\forall x \; \exists y \; P(x,y)$
$\exists x \; \forall y \; P(x,y)$
$\exists x \; \exists y \; P(x,y)$

T

Write the following in terms of quantifiers :—

1. Every student has atleast 1 course where the lecturer is a teaching assistant. $\forall x, \exists y \; P(x, y)$

$\forall x \; \exists y \; \exists z \; \cancel{C}(x, y, z)$

28/8/19

① $\forall x \forall y \; P(x, y)$
② $\forall y \forall x \; P(x, y)$
③ $\forall x \; \exists y \; P(x, y)$
④ $\forall y \; \exists x \; P(x, y)$
⑤ $\exists x \forall y \; P(x, y)$
⑥ $\exists y \; \forall x \; P(x, y)$
⑦ $\exists x \exists y \; P(x, y)$
⑧ $\exists y \; \exists x \; P(x, y)$

1. Which of them are equal.
2. Which of them implies another one.

$\boxed{x + y = 5}$

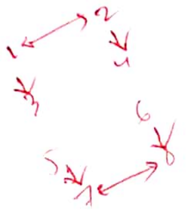
$$\boxed{\begin{array}{l} (1) \; \forall x \forall y \; P(x, y) \; \times \\ (2) \; \forall y \forall y \; P(x, y) \; \times \end{array}} \; \Big] \text{ same}$$

(3) $\forall x \exists y \; P(x, y)$
(4) $\forall y \; \exists x \; P(x, y)$

(7) $\exists x \; \exists y \; P(x, y)$ ✓
(8) $\exists y \; \exists x \; P(x, y)$ ✓] same.

$$\left\{ \begin{array}{l} 3 - 4 \; ✓ \\ 3 - 5 \; \times \\ 4 - 6 \; \times \end{array} \right.$$

$$\begin{tikzcd} 1 \arrow[r, leftrightarrow] \arrow[d] & 2 \arrow[d] \\ 3 \arrow[d, "\times"'] & 4 \arrow[d, "\times"] \\ 5 \arrow[d] & 6 \arrow[d] \\ 7 \arrow[r, leftrightarrow] & 8 \end{tikzcd}$$



Q. Write the following ~~using quantifiers as~~ propositional quantifiers : $\forall x\, p(x)$

① Everyone enrolled in the university has lived in a dormitory. → ~~$\forall x\, p(x,y)$~~ $\forall x$

② All students in this class understand logic. → $\forall x\, p(x)$

③ All movies directed by Quinton Tarrantino are wonderful $\forall x\, p(x)$

④ Someone in Mumbai has never seen Amitabh Bachhan $\exists x\, \neg p(x)$

⑤ Every student who has taken a course in DMS can take a course in algorithms.

$\forall x\, p(x)$

$(A \rightarrow B)$
$\Downarrow$
is equivalent to $A' + B$.

$(\forall(x) \wedge \forall(y)) \rightarrow p(x) \vee q(x)$

① All tigers and lions hunt when threatened or hungry.

$(\forall(x) \wedge \forall(y)) \rightarrow P(z) \vee P(a)$

// Permutation Function

Q Let the universe of discourse of x is the set $A = \{1,2,3,4\}$ and for y, is the set $B = \{5,6,7,8\}$ and the predicate $P(x,y)$ is defined as follows $P(x,y)$ is such that $x < y$, find the truth values of the 8 propositions involving existential & universal quantifiers.

$\forall x \forall y \; P(x,y)$ $\quad P(x,y) = \{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),(3,8),(4,5),(4,6),(4,7),(4,8)\}$

$\forall x \exists y \; P(x,y)$

$\exists x \forall y \; P(x,y)$

$\exists x \exists y$

$\forall y \forall x \; P(x,y)$

$\forall y \exists x$

$\exists y \forall x$

$\exists y \exists x$

3/9/19

Group Theory

$A = \{0, 1\}$ over the operation $+$ / Ex-OR.

Algebraic System (A, OP) → consists of a set & an operatn.

		+ / Ex-OR
0	0	0
0	1	1
1	0	1
1	1	0

(i) closure → perform the operation, if all the elements belong to the set, then its said to be closed.

(ii) Associative → $A+(B+C) = (A+B)+C$ — take a, b, c. Prove it using truth table.

(iii) Identity → $A + I = A$ → $\{+ \text{ op}: 0;\ \times \text{ op}: 1\}$

(iv) Inverse →
$$\boxed{\begin{aligned} a + a^{-1} &= I \\ 0 + 0 &= 0 \\ 1 + 1 &= 0 \end{aligned}}$$
→ Inverses should exist for all elements.

$(0)^{-1} = 0 \quad (1)^{-1} = 1$

(v) Commutative $(a+b = b+a)$

Closure → Groupoid (Gr)

Associative → Semigroup (SG)

Identity → Monoid (M) → Gr, SG.

Inverse → Group (G) → M, SG, Gr

Commutative → Abelian (A) → Gr, SG, M, G.

• A SG is a gr.

Q/ Consider a set N of +ve integers. Find whether N is closed under addition ✓ multiplication ✓ subtractⁿ & division.

subtraction, division → not closed.

Q/ Let A and B denote the set of even and odd +ve integers. Check whether both of them are closed under addⁿ, sub, multiplicatⁿ & divⁿ.

even → A → addⁿ ✓
sub ✗
mult ✓
div ✗

odd → B → addⁿ ✗
sub ✗
mult ✓
div ✗

---

*	a	b	c	d
a	a	a	a	a
b	b	b	b	b
c	c	c	c	c
d	d	d	d	d

⟹ The final elements are all a part of the set
∴ Closure ✓

→ A ✓

$(a*b)*c = a*(b*c)$

$a*b \to a$, $a*c \to a$

$b*c \to b$, $a*b \to a$

same

identity (wrt a)

$a \times a = a$
$b \times a = b$
$c \times a = c$
$d \times a = d$

& should be unique.

	a	b	c	d
a	a	b	c	d
b	b	a	a	b
c	c	b	a	a
d	d	a	a	a

$(b+c)+d = b+(c+d)$

$a+d = b+a$

$d \quad b \quad a$

a

~~not asso~~

not a group

---

	a	b	c	d
a	a	d	c	b
b	b	a	d	c
c	c	b	a	d
d	d	c	b	a

$a \times a = a$

$b \times a = b$

$c \times a = c$

$d \times a = d$

$(a * b) * d \neq a * (b * d)$

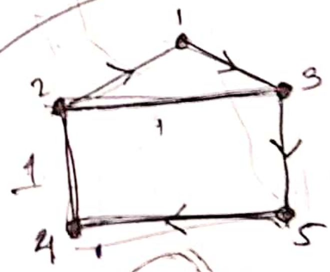
$d * d \qquad a * c$

$a \qquad c$

not associative

	a	b	c	d
a	a	b	c	d
b	d	a	b	c
c	c	d	a	b
d	b	c	d	a

$\Downarrow$



ham path → 4.

5

Q/ Show that the cube roots of unity form an abelian ~~group~~ under the multiplication of complex nos.

→

*	1	ω	ω²
1	1	ω	ω²
ω	ω	ω²	ω³
ω²	ω²	ω³	ω⁴

$$\left.\begin{array}{l} 1*1 = 1 \\ \omega*1 = \omega \\ \omega^2*1 = \omega^2 \end{array}\right\} \text{Identity}$$

$(1*\omega)*\omega^2$ $1*(\omega*\omega^2)$

$\omega*\omega^2$ $1*\omega^3$

ω^3

associative
$$\left\{\begin{array}{ll} (\omega*1)*\omega^2 & \omega*(1*\omega^2) \\ \omega*\omega^2 & \omega*\omega^2 \\ \omega^3 & \omega^3 \end{array}\right.$$

Inverse
$$\left\{\begin{array}{l} 1^{-1} = 1 \\ \omega^{-1} = \omega^2 \\ (\omega^2)^{-1} = \omega \end{array}\right.$$
⟹ Abelian

Q/ Show that 1, -1, i, -i forms an abelian group under multiplication.

	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

Associative :

$1*(-1*i*(-i))$

$1*(-i*(-i))$

$1*(-1)$

-1

$(1*(-1)*i)*(-i)$

$(-1*i)*(-i)$

$-i*(-i)$

-1

commutative

$1*(-1)*i*(-i)$

-1

$(-i)*i*(-1)*1$

-1

Identity = 1 .

• closure (All o/p's are in the same set)

Q. Let G be the set of all non-zero real nos. & let $A * B = \frac{AB}{2}$, prove that $(G, *)$ is an abelian group.

set of all integers ↳ ✗ closure (mult, div)

→ ~~G = R~~

- closure → follows.
- Associative →
- Identity → $\frac{a.b}{2}$ → 2 ... $\frac{a}{2}$ ... 2 = Identity

$a * I = a$

$\frac{a * I}{2} = a \quad \therefore I = 2.$

- Inverse ↳ $\frac{a \cdot a^{-1}}{2} = \frac{1}{2}$

---

*	a	b	c	d
a	a	b	c	d
b	d	a	b	c
c	c	d	a	b
d	b	c	d	a

$(a*b)*c$	$a*(b*c)$
$b*c$	$a*b$
b	b

$((b*c)*d$	$b*(c*d)$
$b*d$	$b*b$
c	a

not associative.

$A * A \to A$

- closure ✓
- associative ✗
- Identity $a * e = a$
- Inverse $a * a^{-1} = e$
- Commutative.

Groupoid
semigroup
monoid
group
abelian

$(A, *)$ → Algebraic structure.

	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

a c d

d b

a

(a, b, d)

a

closure ✓
associative ✓
identity ✓
Inverse

(a)

$a * a = a$	$a^{-1} = a$
$b * d = a$	$b^{-1} = d$
$c * c = a$	$c^{-1} = c$
$d * b = a$	$d^{-1} = b$

$A = 0, 1, 2, 3, 4$

Q A (A + mod 5) is a group.

$1+1$

*	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

- closure ✓
- associative

$(0*1)*2 \quad | \quad 0*(1*2)$

$1*2 \quad | \quad 0*3$

$3 \quad | \quad 3$

- Identity

$0*0=0$
$1*0=1$
$2*0=2$
$3*0=3$
$4*0=4$

$\Big\}\ 0$

- Inverse

$0*0=0 \rightarrow 0^{-1}=0$
$1*4=0 \rightarrow 1^{-1}=4$
$2*3=0 \rightarrow 2^{-1}=3$
$3*2=0 \rightarrow 3^{-1}=2$
$4*1=0 \rightarrow 4^{-1}=1$

$A = 1, 2, 3, 4$

Q2

	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

- closure ✓
- associative ✓

$(1*2)*3 \quad | \quad 1*(2*3)$

$2*3 \quad | \quad 1*1$

$1 \quad | \quad 1$

$(1*3)*4 \quad | \quad 1*(3*4) \quad | \quad (2*3)*4 \quad | \quad 2*(3*4)$

$3*4 \quad | \quad 1*2 \quad | \quad 1*4 \quad | \quad 2*2$

$2 \quad | \quad 2 \quad | \quad 4 \quad | \quad 4$

Identity 1 | $1^{-1}=1$
$2^{-1}=3$
$3^{-1}=2$
$4^{-1}=4$

$A\ (0 \ldots p-1)$
$A +$ mod $p \rightarrow$ group.

$A\ (0, 1, \ldots p-1) \to$ $A +$ mod $p \to$ group

$(1, \ldots p-1) \to$ ~~A mod~~ multiplication mod 5 $\to$ group

*	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

- closure ✓
- associative ✓

$(1 * -1) * i$ | $1 * (-1 * i)$

$-1 * i$ | $1 * -i$

$-i$ | $-i$

- identity $\to$ 1

$1^{-1} = 1$

$-1^{-1} = -1$

$i^{-1} = -i$

$-i^{-1} = i$

Q/ A = +ve int, $*$ = LCM } identity = 1, inverse $\to$ ✗

$3 \mid 3, 6$
$1, 2$

Q/ set of positive int $\to$ op is $a * b = a + b + 2$ — identity ✗

Q/ set of +ve, -ve int. $\to$ identity $\to$ -2

$\hookrightarrow$ inverse $\to$

$a * b = -2$

$a + b + 2 = -2$

$a = -4 - b$

$a^{-1} = b$ $\hookrightarrow b = -4 - a$

$a^{-1} = -4 - a$

Q/ $a * b = 2ab$ $\to$ identity $\to \frac{1}{2}$

$\boxed{a * b = a}$

$2ab = a$

$2b = 1$

$b = \frac{1}{2}$

$a * b = 1/2$

$2ab = 1/2$

$a^{-1} = b$

$b = \frac{1}{4a}$

$a * b = a + b - 2ab$

identity $\to 0$

$a + b - 2ab = 0$

$a + b = 2ab$

$b = a(2b - 1)$

17/10/19

① state whether or not each of the following subsets over the +ve int n is closed under the ~~oper~~ multiplication operation → all closed.

① set A = {0, 1}
② B = {1, 2}
③ C = {x : x is prime}
④ D = {2, 4, 6, 8, . . ∞} → x is even
⑤ E = {1, 3, 5 . . .} → x is odd.
⑥ F = {2, 4, 8, 16 . . .} → closed. → $x = 2^n$.

①

*	0	1
0	0	0
1	0	1

closed

②

*	1	2
1	1	2
2	2	4

not closed

③ x is prime . .
2 × 3 = 6 not closed .

④ 2, 4, 6, 8, 10, 12, 14 closed .

⑤ 1, 3, 5, 7, 9, 11, 13, 15, 17 .
→ closed

addition

①

+	0	1
0	0	1
1	1	2

not closed

②

	1	2
1	2	3
2	3	4

not closed .

③ → 2, 3, 5, 7 → 12 not closed .
④ → 2, 4, 6, 8, closed
⑤ → 1, 3, 5, 7 not closed
⑥ → 2, 4, 8, 16 → 12 not closed

Q/ Let G be reduced residue system modulo 15
G = 1, 2, 4, 7, 8, 11, 13, 14.

The set of integers betⁿ 1 & 15 which are co-prime to 15.

Prove whether G is a group under multiplication modulo 15.

	1	2	4	7	8	11	13	14
1	1	2	4	7	8	11	13	14
2	2	4	8	14	1	7	11	13
4	4	8	1	13	2	14	7	11
7	7	14	13	4	11	2	1	8
8	8	1	2	11	4	13	14	7
11	11	7	14	2	13	1	8	4
13	13	11	7	1	14	8	4	2
14	14	13	11	8	7	4	2	1

- closed
- Identity → 1

Inverse :

$1^{-1} = 1$

$2^{-1} = 8$

$4^{-1} = 4$

$7^{-1} = 13$

$8^{-1} = 2$

$11^{-1} = 11$

$13^{-1} = 7$

$14^{-1} = 14$

exponential → not associative

$2^{3^2} = (2\times2)\times(2\times2)\times(2\times2)$

$2^{3^2} = ~~(8\times8)\times(8\times8)~~$

(1, 2)

POSET $\Rightarrow$ [Reflexive, Antisymmetric, transitive]

e.g.

① $\boxed{R \leq}$

② (Z) over divisibility

~~(4,2)~~

Ref ✓

sym ✗

trans ✓

Antisym ✓

✗ because :

~~(4,-2)~~ (2,-2) (-2,2)

③ Z^+ over / $\rightarrow$ $\boxed{POSET}$

ref ✓

sym ✗

trans ✓

Antisym ✓

④ Let S be any collectn of sets the reln '$\subseteq$' of set inclusion over S

S = { s_1, s_2, s_3 ... } $\rightarrow$ • Reflexive ✓

[Any set is a subset of itself]

• Symmetric ✗ $a \subseteq b$, $b \not\subseteq a$

antisymmetric ✓ $a \subseteq b$, $b \subseteq a$ / $a = b$.

• transitive ✓ $a \subseteq b$, $b \subseteq c$ $\Rightarrow a \subseteq c$

S is a POSET.

Lattice

Consider a relation defined on the divisors of 12 over the divisibility operation.

{1, 2, 3, 4, 6, 12}

(1,1) (2,1) (3,1) (4,1) (6,1) (12,1)
(2,2) (4,2) (6,2) (12,2)
(3,3) (6,3) (12,3)
(4,4) (12,4) (12,6)
(6,6) (12,12)

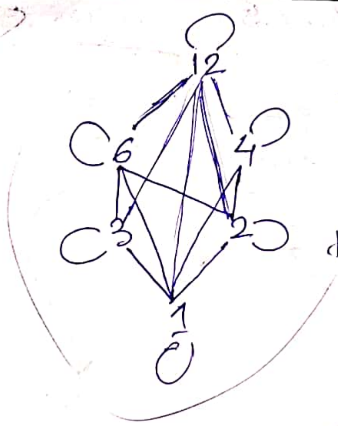
$\rightarrow$ reflexive ✓

$\rightarrow$ symmetric ✗

$\rightarrow$ antisymmetric ✓

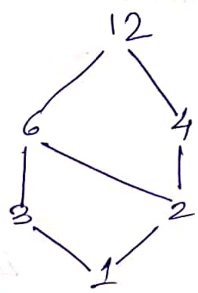
$\rightarrow$ transitive ✓

Hasse diagram

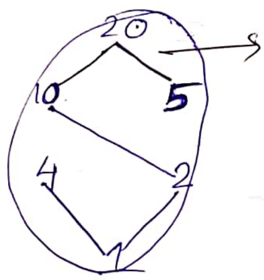


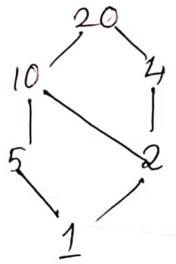
remove self loops →

remove → trans. elements.



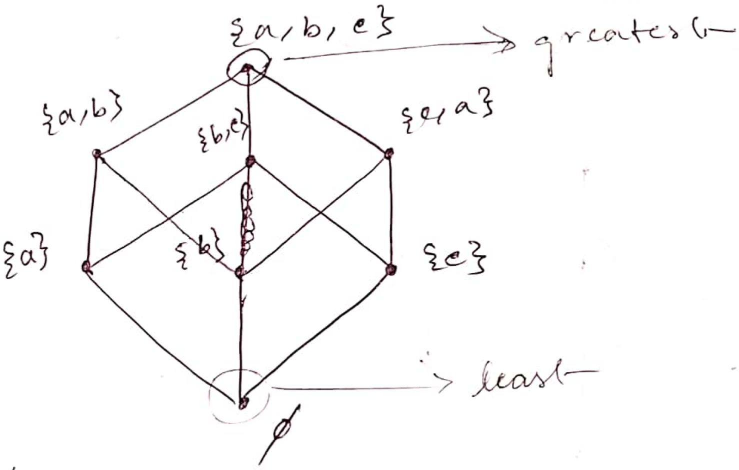
factors of 20 → 1, 2, 4, 5, 10, 20



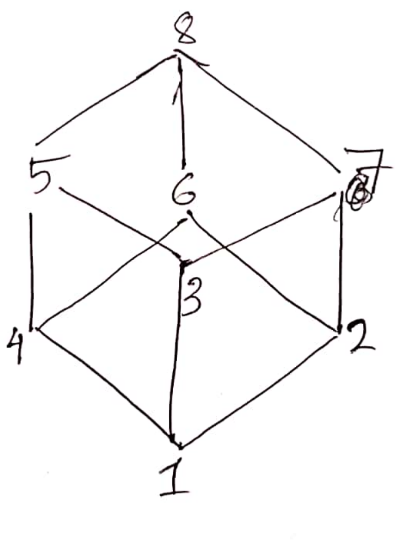


s = {a, b, c}

= { {a, b, c}, {a, b}, {b, c}, {a, c}, {a}, {b}, {c}, ∅ }



greatest
least
upperbound
lowerbound
greatest lower bound
least upper bound
maximal (2 max^{m})
minimal. (2 min^{m})



An element $m \in S$ is called UB if $x \leq m$ for all ~~$x \in I$~~ $x \in T$

// Upper bound

// lower bound.

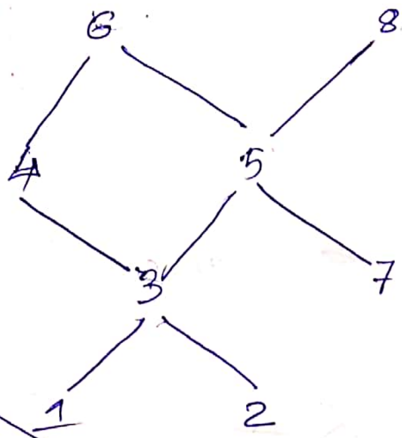
T = (3, 4, 5)

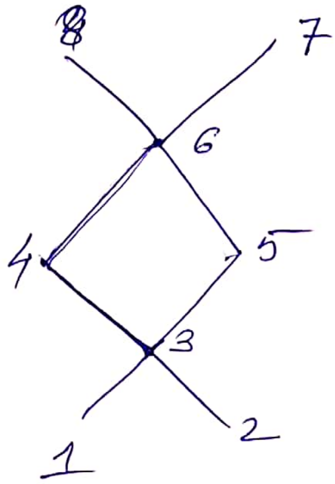
upper (T)

Lower (T)

6

1, 2, 3.





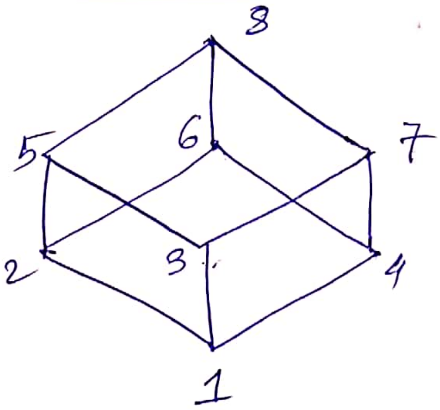
(4, 5)

6, 7, 8 → UB.

(6, 4, 3) → $\overset{UB}{6, 7, 8}$

(4, 8) → $\overset{UB}{8}$

(6, 4, 5) → $\overset{UB}{(6, 7, 8)}$



(3, 5)

UB → 5, 8

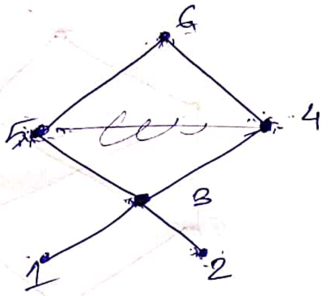
5, 6

(2, 1) LB.

Scanned by CamScanner

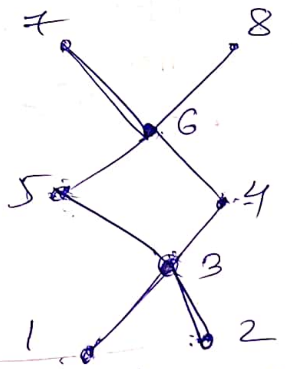
Q find out the upperbound & lower bound of following sets:

① (3,4,5)
② (2,3)
③ (6,1)
④ (4,5,6)
⑤ (1,2)
⑥ (5,4,1,2)



	UB	LB
①	6	1,2,3
②	3,4,5,6	2
③	6	1
④	6	1,2,3
⑤	3 4 5 6	∅
? ⑥	∅	∅

(7,8)

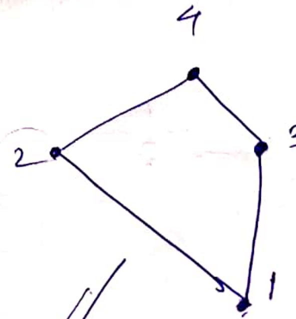


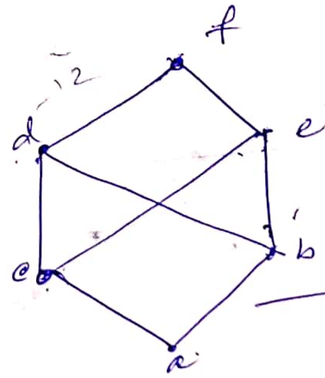
UB	LB
∅	1,2,3,4,5,6

UB → 6,7,8 :

	GLB	LUB	
(3,4,5)	3	6	
(2,3)	2	3	UB: 3,4,5,6,7,8 LB: 2
(6,1)	1	6	UB: 6,7,8 LB: 1
(4,5,6)	3	6 .	UB: 6,7,8 . LB: 1,2,3 .
(1,2)	does not exist	3	UB: 3,4,5,6,7,8 LB: ∅

* If in a hasse diagram, GLB & LUB exists for all subsets, then it is called a <u>lattice</u> .





→ not a lattice.

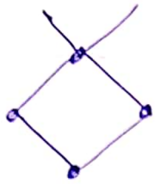
~~{1}{2}~~ lattic

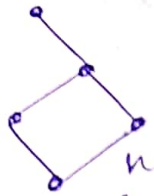
~~{1}{2}{3}{4}~~ (d, e)

UB : f
LB : a, b, c
no GLB

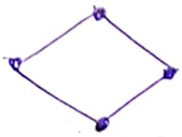
(b, c)
UB : d, e, f.
NO LUB

lattice





not a lattice



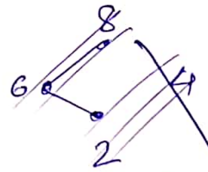
lattice ??

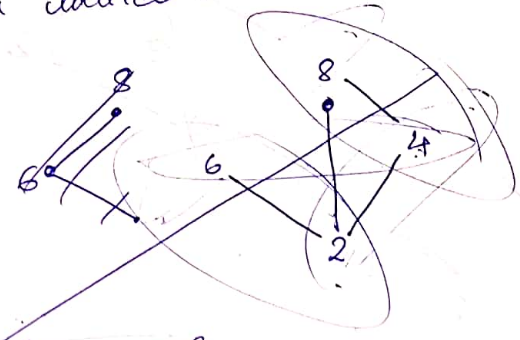
(7) Assume a POSET $A = \{2, 4, 6, 8\}$ over '/'.
Investigate:

① Every pair of element has a GLB

② has a LUB.

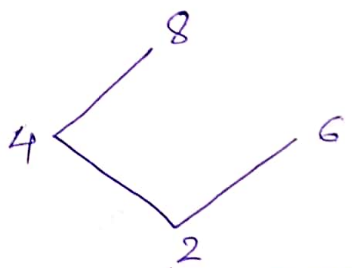
③ The given ~~cons~~ POSET is a lattice.





	UB	LB
8, 4	8	2
8, 2	8	2
6, 2	6	2
4, 2	8	2

8 — 6



$UB(4,6) = \phi$

LUB does not exist.

meet of a & b

GLB $a \wedge b$

LUB $a \vee b$

join of b.

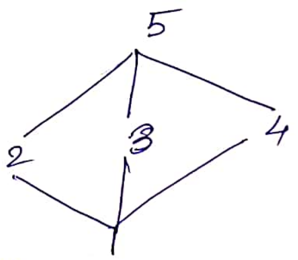
$(L, \wedge, \vee) \rightarrow$ lattice.

$$\begin{cases} a \vee a = a \\ a \wedge a = a \end{cases} \Bigg/ \begin{cases} a \wedge b = b \wedge a \\ a \vee b = b \vee a \end{cases}$$ law of duality.

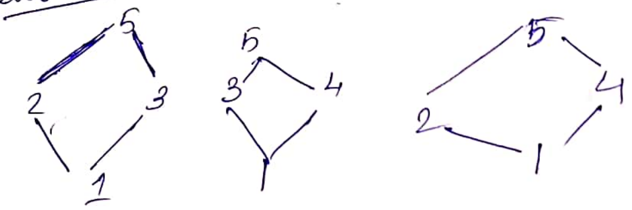
- Associative law.
- Absorption law

Q Let $X = \{1,2,3,4,5\}$

find all the sublattices of the given lattice.



Sublattices:



$(\mathbb{Z}, +, \times)$ → group (defined) G

Ring

Properties

① G should be associative over '$\times$'.

② should allow distributive law

$x(y+z) = xy + xz$

$(y+z)x = yx + zx$.

- socratica → for field theory.

field = Ring

i.e field should have all properties of ring.

① Commutative over '$\times$'.

② field that has identity element over '$\times$' is a ring with unity.

③ Inverse over '$\times$'

~~If ring is~~ called division ring.

$\boxed{\text{Commutative division ring is field.}}$