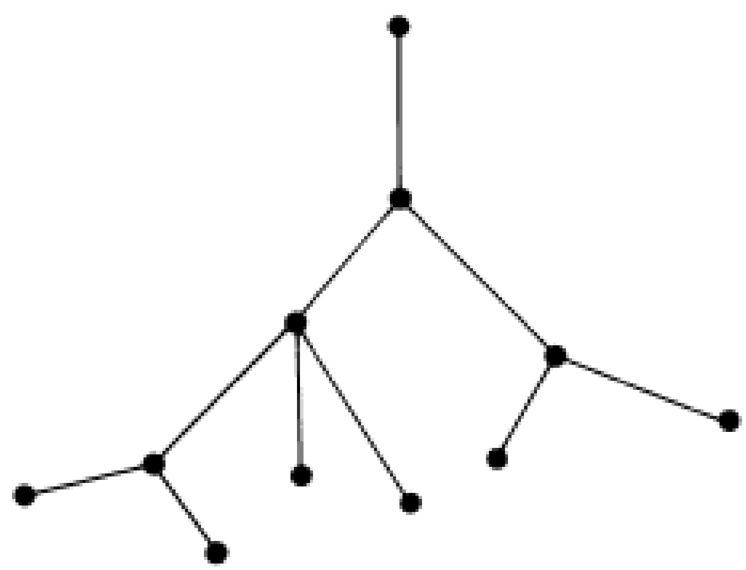
DAY 6

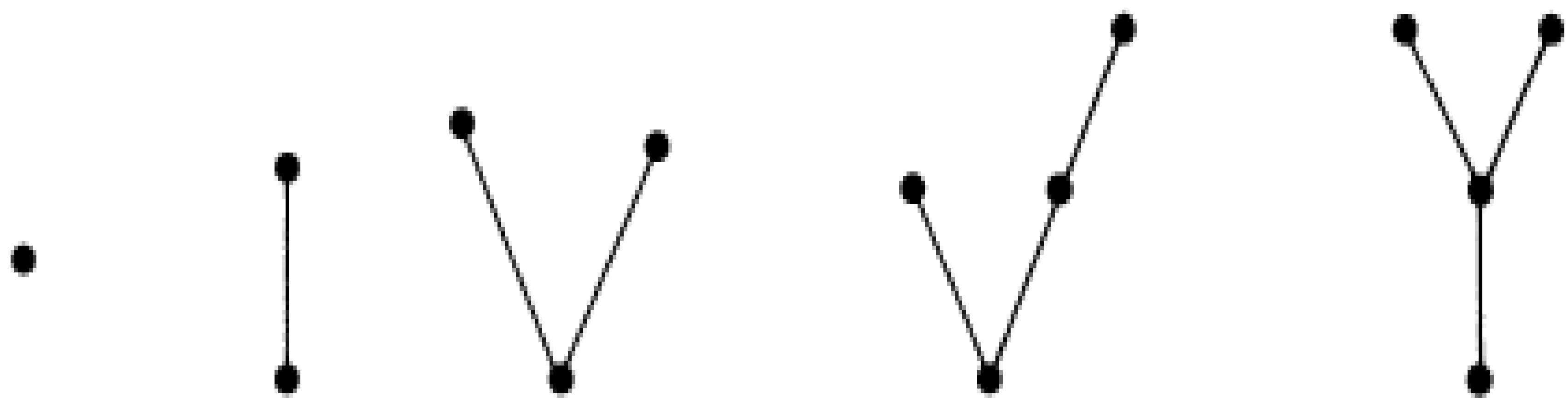
Tree

A tree is a connected graph that is circuit less.

The edges of a tree are known as **branches**. Elements of trees are called their nodes. The nodes without child nodes are called **leaf nodes**.

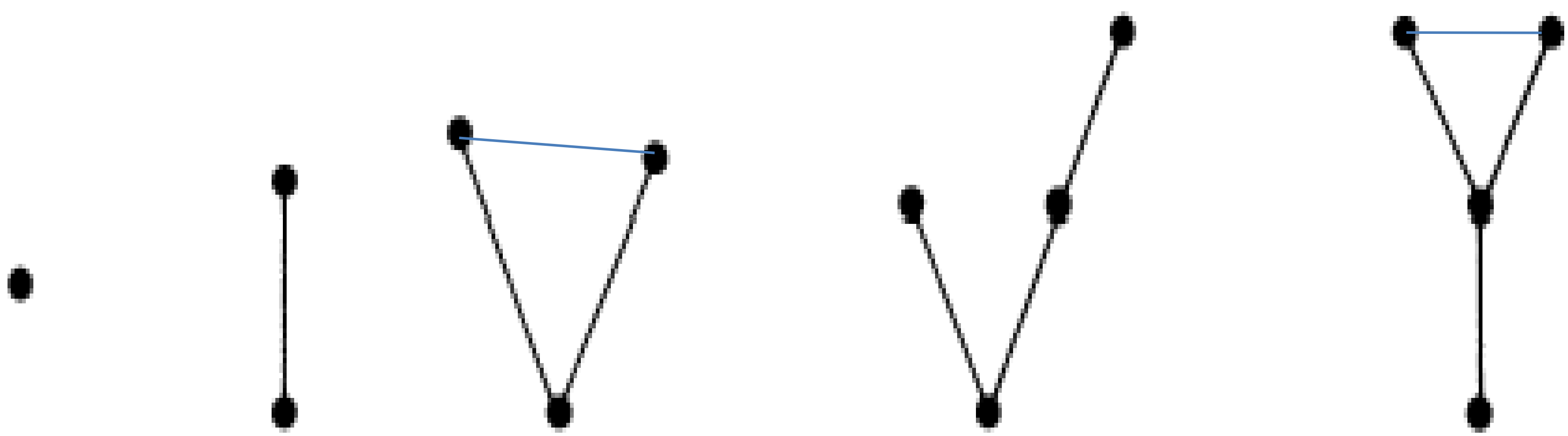


Contd..



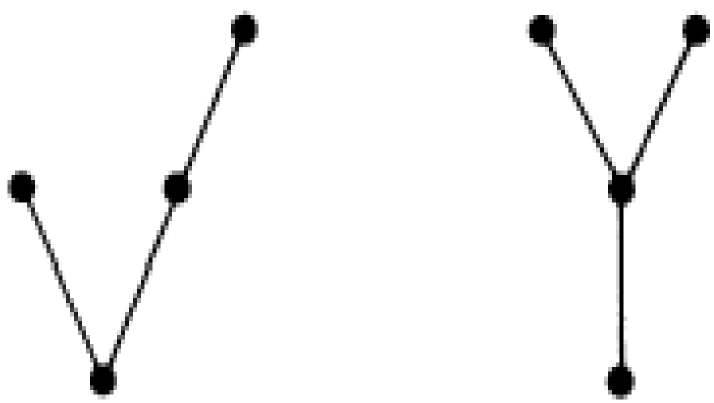
Trees with one, two, three, and four vertices.

Contd..

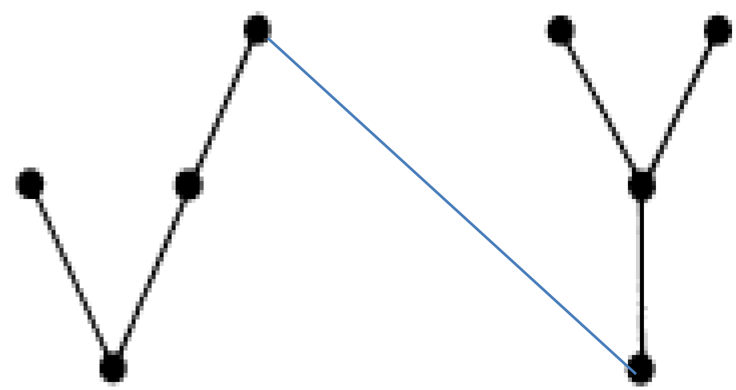


Forest

A disjoint collection of trees is called a forest.



(a)



(b)

Properties of Tree

There is one and only one path between every pair of vertices in a tree, T.

If in a graph G there is one and only one path between every pair of vertices, G is a tree.

A tree with n vertices has $n - 1$ edges.

Any connected graph with n vertices and $n - 1$ edges is a tree.

A graph is a tree if and only if it is minimally connected.

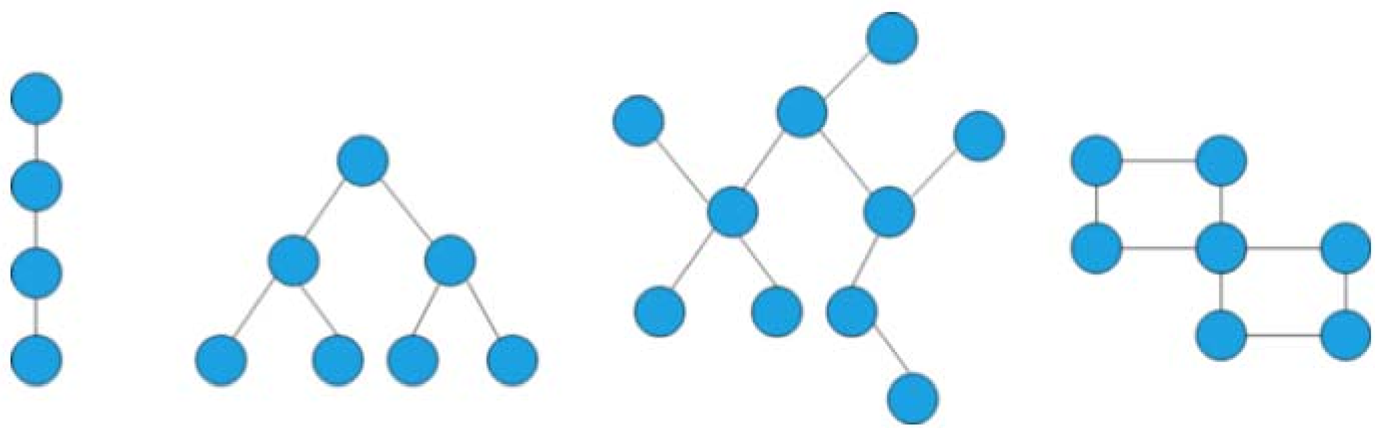
A graph G with n vertices, $n - 1$ edges, and no circuits is connected.

Contd..

The results of the preceding six theorems can be summarized by saying that the following are five different but equivalent definitions of a tree. That is, a graph G with n vertices is called a tree if

1. G is *connected* and is *circuitless*, or
2. G is *connected* and has $n - 1$ *edges*, or
3. G is *circuitless* and has $n - 1$ *edges*, or
4. There is *exactly one path* between every pair of vertices in G, or
5. G is a *minimally connected* graph.

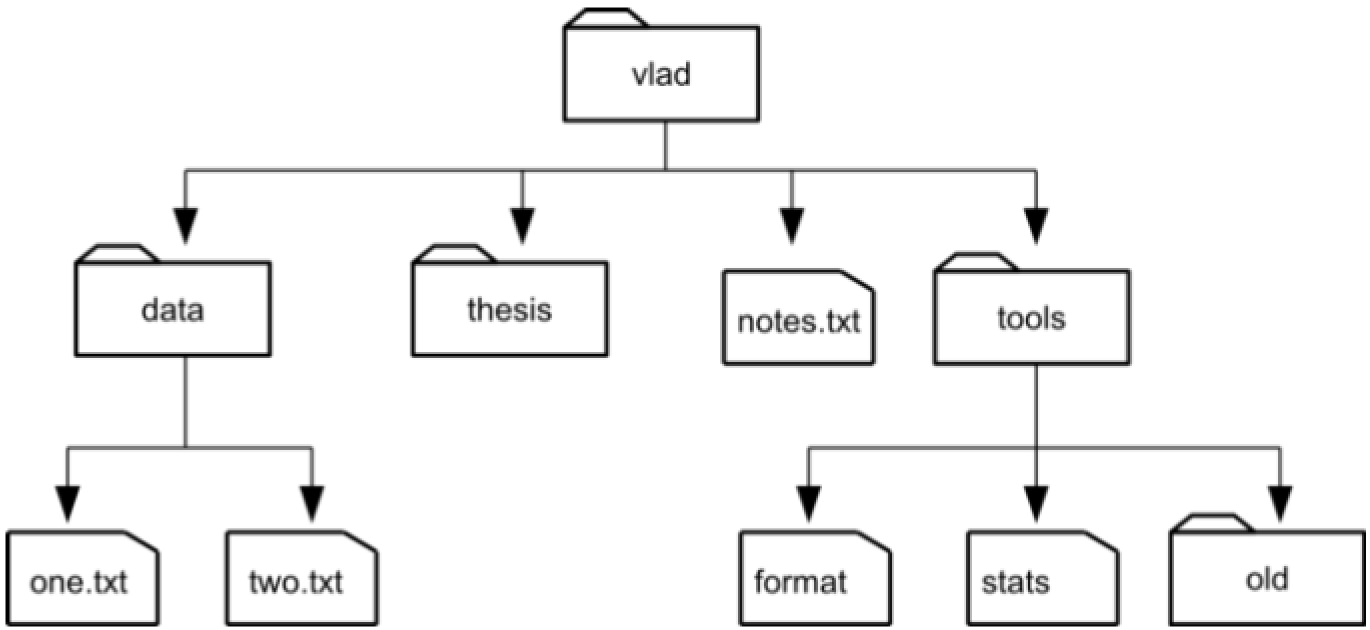
Solve



As per the properties, which one is not a tree?

Uses of Tree

File Structure



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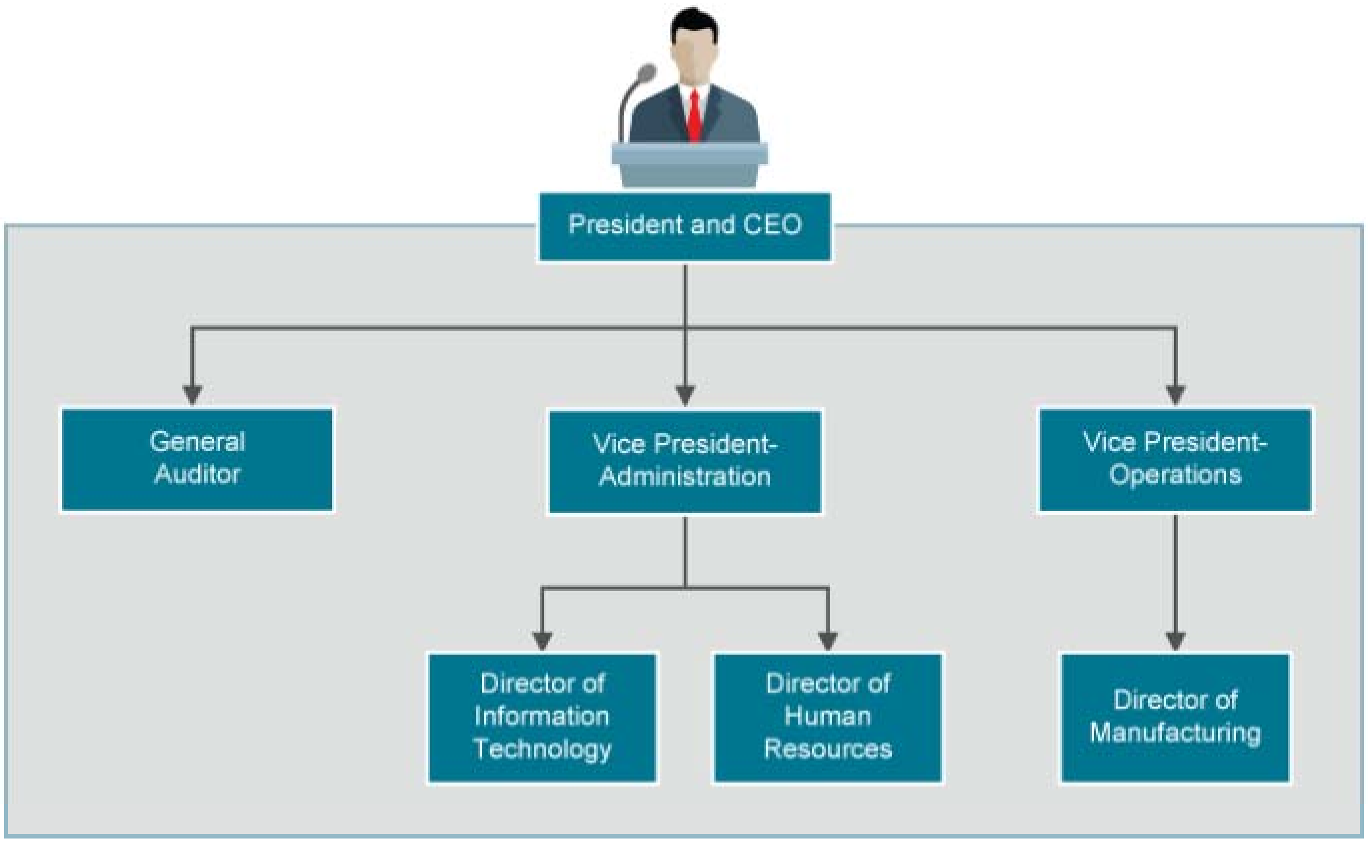
Contd..

Corporate Hierarchy



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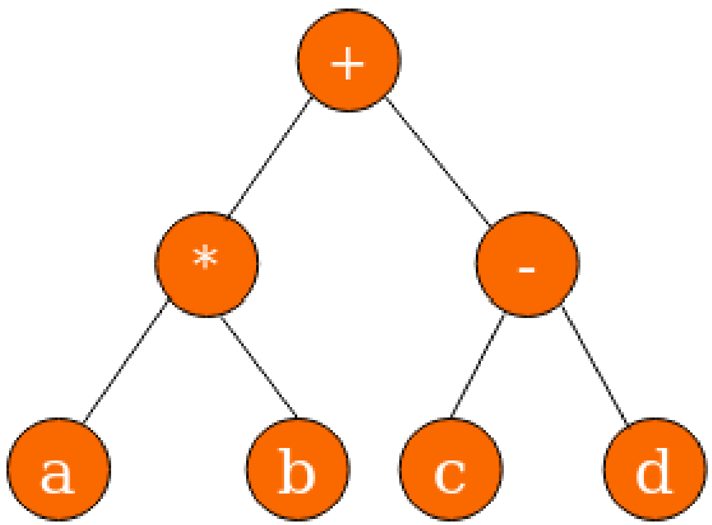
Evaluating Expressions



$(a * b) + (c - d)$

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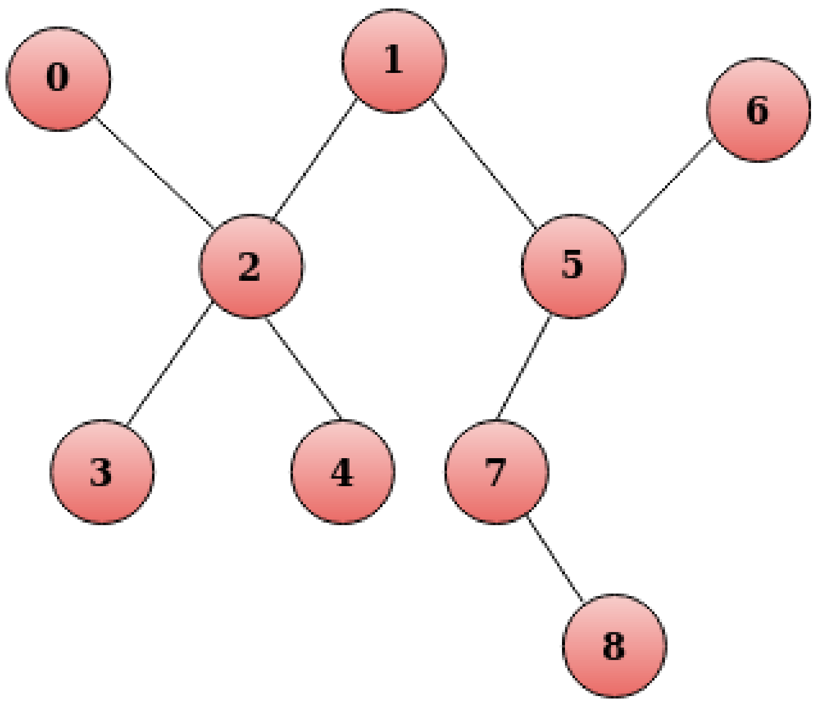
Contd..

Storing Undirected Trees



$[(0, 2), (2, 3), (2, 4), (2, 1), (1, 5), (5, 6), (5, 7), (7, 8)]$

$[0] \rightarrow [2]$

$[1] \rightarrow [2, 5]$

$[2] \rightarrow [0, 1, 3, 4]$

$[3] \rightarrow [2]$

$[4] \rightarrow [2]$

$[5] \rightarrow [1, 6, 7]$

$[6] \rightarrow [5]$

$[7] \rightarrow [5, 8]$

$[8] \rightarrow [7]$

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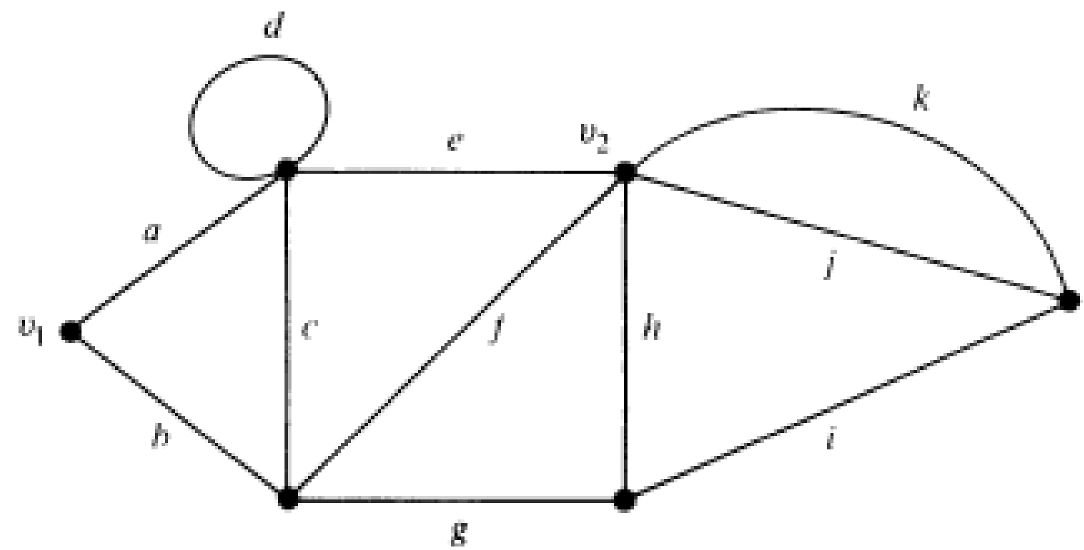
Distance

In a connected graph G, the *distance* $d(v_i, v_j)$ between two of its vertices v_i and v_j is the length of the shortest path (i.e., the number of edges in the shortest path) between them.

Path1: (a, e) length = 2
Path2: (a, c, f) length = 3
Path3 : (a, c, g, h) length = 4

Thus, d(v1,v2) = 2



Contd..

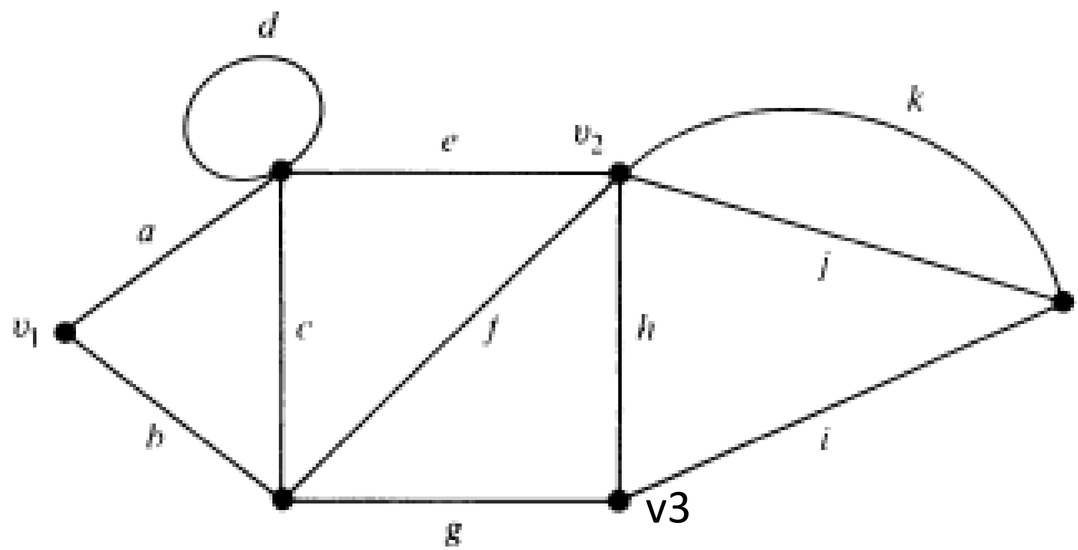
THEOREM 3-8

The distance between vertices of a connected graph is a metric.

The distance between two variables can be represented by a function f(x, y) and this function is called a metric if it satisfies the following conditions:

1. Nonnegativity: $f(x, y) \geq 0$, and $f(x, y) = 0$ if and only if $x = y$.
2. Symmetry: $f(x, y) = f(y, x)$.
3. Triangle inequality: $f(x, y) \leq f(x, z) + f(z, y)$ for any z.

Contd..



1. d(v1,v2) = 2 which is >=0
2. d(v1,v2) = d(v2,v1) = 2
3. d(v1,v2) <= d(v1,v3) + d(v3,v2)

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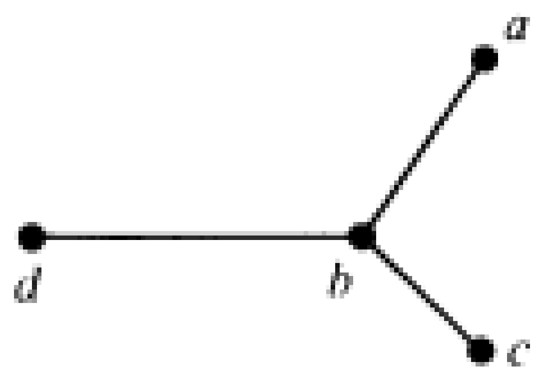
Distance in a tree

Distance between two vertices in a tree is the length of the only path available between them.

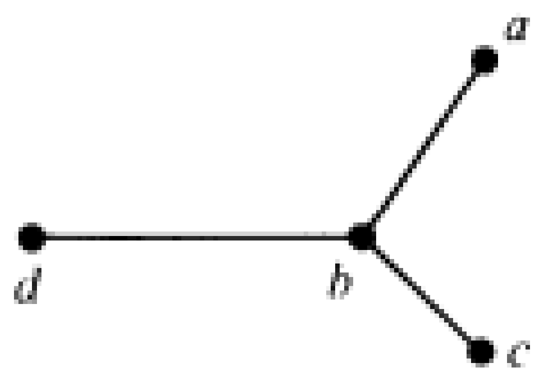


d(a, b) = 1
d(a, c) = 2
d(a, d) = 2
d(c, d) = 2

Eccentricity

The eccentricity $E(v)$ of a vertex v in a graph G is the distance from v to the vertex farthest from v in G; that is,

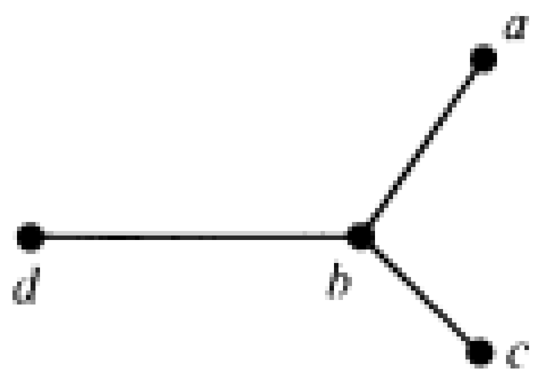
$$E(v) = \max_{v_i \in G} d(v, v_i).$$



E(a) = 2
E(b) = 1
E(c) = 2
E(d) = 2

Center

The node with minimum eccentricity is called the center of the tree.

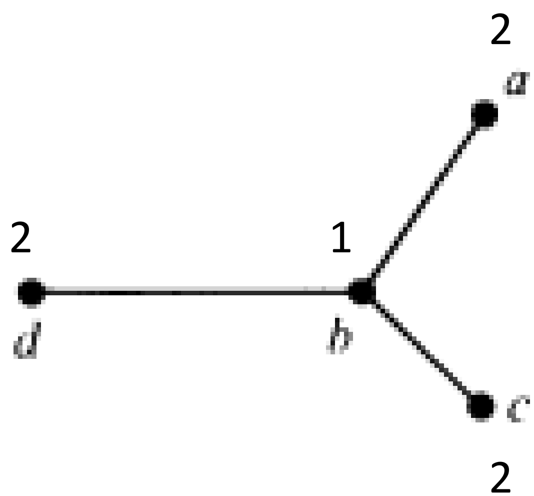


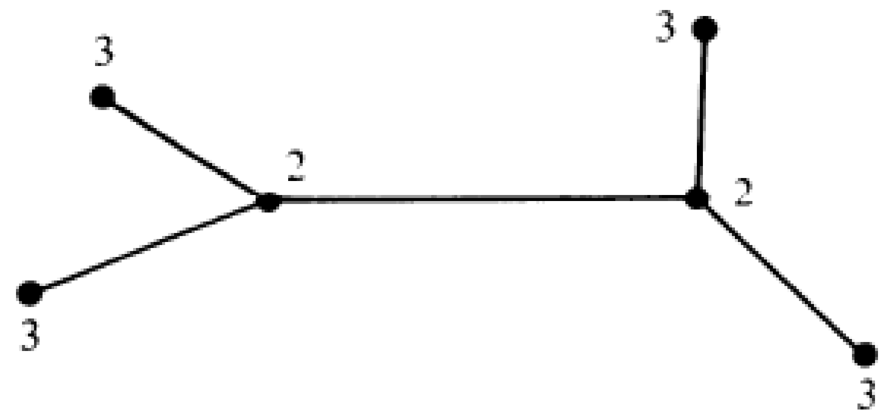
E(a) = 2
E(b) = 1
E(c) = 2
E(d) = 2

Therefore, b is the center of the tree.

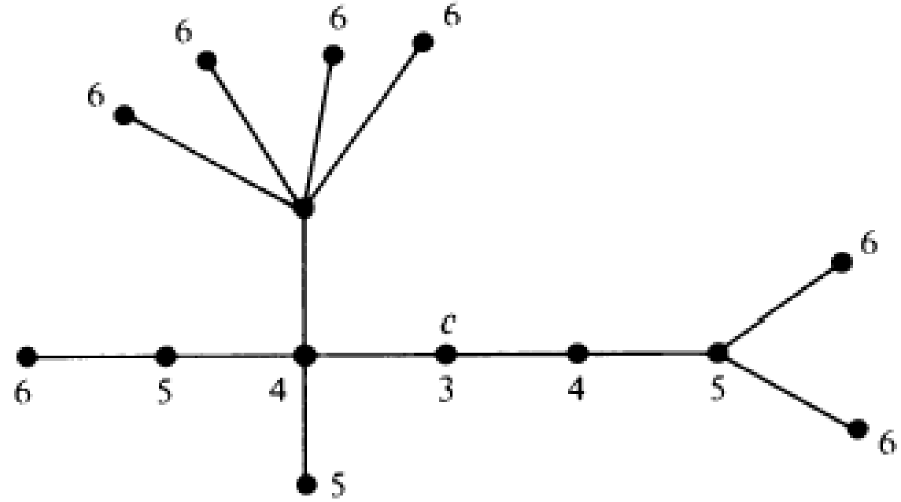
THEOREM 3-9

Every tree has either one or two centers.

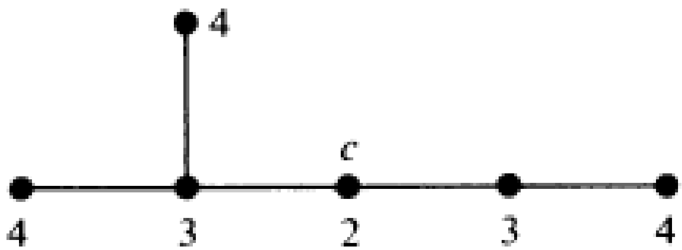




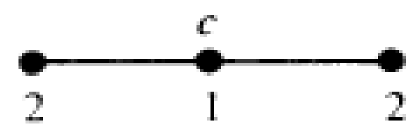
Center Identification



(a)

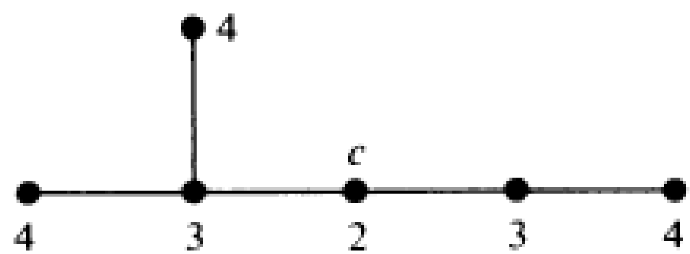


(b)



(c)

Radius and Diameter



(b)

Eccentricity of the center is called the radius of the tree.

Diameter is the length of the longest path in the tree.