B.Tech. 3rd Semester End-Term Examination – 2021
Name of Subject: Engineering Mathematics-III/Mathematics - III
Subject Code: UME03C12/UEE03C13/C16/UCS03B02/C10/ UEI03C13/UPE03C14/UCH03C17/UBE03C15/UCE03C14/ UEC03B07/UCS03C01

Full Marks: 50 | **Submit the answer script PDF in this email: nita.ma.cse.a@gmail.com** | **Time: 2 hours**

Symbols used here have their usual meanings

Group – A

Choose the correct answer from the following: [10 × 2 = 20]

1. The particular integral of the differential equation $(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)z = e^{y+2x}$ is

(a) xe^{y+2x} (b) x^2e^{y+2x}
(c) $\frac{x}{2}e^{y+2x}$ (d) None of these

2. If $f(x) = x^2 - 2, -2 \le x \le 2$, then a_n is

(a) $\frac{16(-1)^{n+1}}{n^2\pi^2}$ (b) $\frac{16(-1)^n}{n^2\pi^2}$
(c) $\frac{32(-1)^n}{n^2\pi^2}$ (d) $\frac{8(-1)^n}{n^2\pi^2}$

3. Let $u(x,y) = f(xe^y) + g(y^2cosy)$, where f and g are infinitely differentiable functions. Then the partial differential of minimum order satisfied by u is

(a) $u_{xy} + xu_{xx} = u_x$ (b) $u_{xy} + xu_{xx} = xu_x$
(c) $u_{xy} - xu_{xx} = u_x$ (d) None of these

4. A rod of **30 cm** length has its ends **P** and **Q** kept **20°C** and **80°C** respectively until steady state condition has been prevailed. The temperature at each end point is suddenly reduced to **0°C** and kept so. The conditions for solving the equation will be

(a) $u(0,t) = 0 = u(30,t)$ and $u(x,0) = 20 + (\frac{60}{10})x$ (b) $u(0,t) = 0 = u(30,t)$ and $u(x,0) = 20 + (\frac{60}{30})x$
(c) $u_t(0,t) = 0 = u_t(30,t)$ and $u(x,0) = 20 + (\frac{60}{10})x$ (d) None of these

5. If $f(x) = \begin{cases} 1, & 0 < x < \frac{1}{2} \\ 0, & \frac{1}{2} < x < 1 \end{cases}$, then b_1 in half range sine series is equal to

(a) $1/\pi$ (b) $2/3$
(c) $3/\pi$ (d) $4/\pi$

6. If $X \sim N(30, 5^2)$, then the value of $P(26 \le X \le 40)$ is:

(a) 0.7653 (b) 0.5
(c) 0.9659 (d) None of these

7. The lines of regression in a bivariate distribution are: $X + 9Y = 7$ *and* $Y + 4X = \frac{49}{3}$. Then the correlation coefficient between X and Y is

(a) $1/6$ (b) $-1/6$
(c) $1/5$ (d) None of these

8. If X follows a Poisson distribution such that $P(X = 1) = 2P(X = 2)$, then the value of $P(X = 0)$ is

(a) e^{-1} (b) 1
(c) e^{-2} (d) 0

9. Let the probability density function of a random variable X is : $f(x) = A\sin\frac{\pi x}{5}, 0 \le x \le 5$. Then the median value of X is

(a) $\pi/10$ (b) $5/2$
(c) $\pi/20$ (d) $1/10$

10. A random variable X can assume only positive integral value n with a probability proportional to $\frac{1}{3^n}$, then $E(X)$ is

(a) $1/4$ (b) 0
(c) $3/4$ (d) 2

Group – B

Choose the correct answer from the following: [10 × 3 = 30]

11. The particular integral of the differential equation $(D^3 - 3DD'^2 - 2D'^3)z = \cos(x+2y) - e^y(3+2x)$ is

(a) $\frac{1}{27}\sin(x+2y) + xe^y$ (b) $\frac{1}{27}\text{cox}(x+2y) + xe^x$
(c) $\frac{1}{27}\sin(x+2y) - x^2e^x$ (d) None of these

12. If $f(x) = \begin{cases} 1, & -1 \le x < 0 \\ -2, & 0 < x \le 1 \end{cases}$ and f is expanded in a Fourier series $\frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n\cos\left(\frac{n\pi x}{1}\right) + b_n\sin\left(\frac{n\pi x}{1}\right)]$, then a_0, a_3 and b_3 are (in this order)

(a) $(0, 0, -\frac{2}{\pi})$ (b) $(-1, 0, -\frac{2}{\pi})$
(c) $(1, 0, \frac{2}{\pi})$ (d) None of these

13. The general integral of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ is

(a) $x^2 + y^2 + z = \varphi(xz + y), \varphi$ being an arbitrary function (b) $x^2 - y^2 + z = \varphi(xz + y), \varphi$ being an arbitrary function
(c) $x^2 + y^2 - z = \varphi(xz - y), \varphi$ being an arbitrary function (d) $x^2 - y^2 - z = \varphi(xz - y), \varphi$ being an arbitrary function

14. The partial differential equation derived from the equation $F\left(x - y - z, \frac{x^2-y^2}{z^2}\right) = 0$ by eliminating the arbitrary function F, will be

(a) $(x^2 - y^2 - yz)p + (x^2 - y^2 - zx)q = z(x - y)$ (b) $(x^2 + y^2 + yz)p + (x^2 - y^2 - zx)q = z(x - y)$
(c) $(x^2 - y^2 + yz)p + (x^2 - y^2 + zx)q = z(x - y)$ (d) None of these

15. The complete integral of the equation $q^2 = z^2p^2(1 - p^2)$ will be

(a) $(a^2z^2 + 1)^{1/2} = ax + a^2y + b$ (b) $(a^2 - z^2)^{1/2} = ax + a^2y + b$
(c) $(a^2z^2 - 1)^{1/2} = ax + a^2y + b$ (d) None of these

16. The incidence of occupational disease in an industry is such that the workers have a 20% chance of suffering from it. Then the probability that out of six workers chosen at random, four or more will suffer from the disease is

(a) $52/3125$ (b) $51/3125$
(c) $53/3125$ (d) None of these

17. It is known that the probability that an item produced by a certain machine will be defective is 0.01. Then the probability that random sample of 100 items selected at random from the total output will contain not more than one defective item is:

(a) $1/e$ (b) $3/e$
(c) $2/e$ (d) None of these

18. The value of p in a binomial distribution with $n = 6$ and $9\,P(X = 4) = P(X = 2)$ is given by

(a) $p = -1/2$ (b) $p = 1/4$
(c) $p = 1$ (d) $p = 0$

19. Given the following values of X and Y:

X:	1	3	4	5	7	8	10
Y:	2	6	8	10	14	16	20

The coefficient of correlation between X and Y is

(a) 1 (b) 0
(c) 0.9 (d) None of these

20. If in a bivariate distribution $var(X) = 9$, Regression lines are $8X - 10Y = -66, 40X - 18Y = 214$, then the values of $E(X), E(Y), r(X,Y)$ are respectively

(a) 17, 14, -0.6 (b) 13, 17, 0.6
(c) 17, 14, 0.6 (d) None of these

* * * * * * * * * * * * * * * * * * *

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

National Institute of Technology, Agartala
Department of Mathematics

Name of the Examination : End Sem examination   Date : 06/12/2021

Subject Name : Engineering Mathematics III   Subject Code : UCS03C16

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Enrollment no : 20UCS119   Registration no : 2012709

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Answerscript :

Group A		Group B	
Question no.	Option	Question no.	Option
1	(A)	11	(A)
2	(B)	12	(B)
3	(C)	13	(A)
4	(B)	14	(A)
5	$\frac{2}{\pi}$ (B)	15	(D)
6	A)	16	(C)
7	(B)	17	(C)
8	(A)	18	(B)
9	(B)	19	(A)
10	(C)	20	(B)

Group A:

Q.1. $(D^3 - 4D^2D' + 5DD'^2 - 2D'^3)z = e^{y+2x}$.

$a = 2, b = 1$

$$P.I. = \frac{1}{f(a,b)} e^{y+2x}$$

$$\Rightarrow \frac{1}{8-16+10-2} e^{y+2x} \quad \because f(a,b) = 0$$

$$\Rightarrow \frac{x}{f'(a,b)} e^{y+2x} = \frac{x}{12-16+5} e^{y+2x}$$

$$= x e^{y+2x} \quad \text{(Ans)} \quad \text{Option (a)}$$

Q.2. $f(x) = x^2 - 2, \; -2 \leq x \leq 2$

$$a_n = \frac{2}{4}\int_{-2}^{2}(x^2-2)\cos\left(\frac{2\pi n x}{4}\right)dx$$

$$= \frac{1}{2}\left[\int_{-2}^{2}(x^2-2)\cos\left(\frac{\pi n x}{2}\right)dx\right]$$

$$= \frac{1}{2}\left[\int_{-2}^{2}x^2\cos\left(\frac{n\pi x}{2}\right)dx - 2\int_{-2}^{2}\cos\left(\frac{n\pi x}{2}\right)dx\right]$$

$$= \frac{1}{2}\left[\frac{x^2 2}{\pi n}\sin\left(\frac{\pi n x}{2}\right) - \int\frac{2x\,2}{n\pi}\sin\left(\frac{\pi n x}{2}\right)dx\right]_{-2}^{2} - \frac{2\times 2}{n\pi}\left|\sin\left(\frac{n\pi x}{2}\right)\right|_{-2}^{2}\Bigg]$$

$$= \frac{1}{2}\left\{-2\times\frac{2}{n\pi}\{0\}\right\} + \frac{1}{2}\left[\frac{x^2 2}{\pi n}\sin\left(\frac{n\pi x}{2}\right) - \frac{4}{n\pi}\left(x\left(\frac{-2}{n\pi}\right)\cos\left(\frac{\pi n x}{2}\right)\right.\right.$$

$$\left.\left. + \frac{1}{n\pi}\frac{2}{\pi n}\sin\left(\frac{n\pi x}{2}\right)\right)\right]_{-2}^{2}$$

$$= 0 + \frac{1}{2}\left[\frac{2}{\pi n}x^2\sin\left(\frac{n\pi x}{2}\right) + \frac{8x}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right) + \frac{8}{n^2\pi^2}\sin\left(\frac{n\pi x}{2}\right)\right]_{-2}^{2}$$

$$= \frac{1}{2}\left[\frac{8\times 2}{n^2\pi^2}(1) + \frac{8\times 2}{n^2\pi^2}\cos(n\pi)\right]$$

$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}\times 8\times 2}{n^2\pi^2}\right) = \frac{16}{n^2\pi^2}(-1)^n \quad \text{Option (b)}$$

Q.3. $U(x,y) = f(e^y) + g(y^2 \cos y)$

$U_x = \frac{\partial U}{\partial x} = e^y . f'(xe^y)$

$U_{xy} = \frac{\partial^2 U}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial U}{\partial x}\right) = \frac{\partial}{\partial y}\left[e^y f'(xe^y)\right]$

$= e^y . f''(xe^y) . xe^y + e^y f'(xe^y)$

$U_{xx} = \frac{\partial^2 U}{\partial x^2} = e^y f'(xe^y) . e^y$

$U_{xy} - xU_{xx} = U_x$ Option (C)

Q.5. $f(x) = \begin{cases} 1 & 0 < x < \frac{1}{2} \\ 0 & \frac{1}{2} < x < 2 \end{cases}$

$b_n = 2\int_0^e f(x) \sin(n\pi x) dx = 2\int_0^{1/2} f(x) \sin(n\pi x) dx$

$= 2\int_0^{1/2} 1 . \sin(n\pi x) dx$

$= 2\left|-\frac{\cos n\pi x}{n\pi}\right|_0^{1/2} = \frac{2}{n\pi}\left[-\sin n\pi + 1\right]$

$\rightarrow = \frac{2}{\pi}[0+1] = \frac{2}{\pi}$ Option (B)

Q.4. $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ In steady state, $\frac{\partial u}{\partial t} = 0$.

$\frac{\partial^2 u}{\partial x^2} = 0$ . . . . (i) Integrating,

$u = ax + b$ . .

$u = 20$, when $x = 0$ $\rightarrow$ $b = 20$

$u = 80$, $x = 30$. $\rightarrow$ $80 = 30a + b$

$a = 2$

$\therefore$ equation reduces to $u = 2x + 20$

Hence boundary condition are.

(i) $u(0,t) = 0$, For $t > 0$

(ii) $u(30,t) = 0$, For $t > 0$ Option (B)

(iii) $u(x,0) = 2x + 20$, For $0 < x < 30$

$u(0,t) = 0$, $u(30,t)$ & $u(x,0) = 20 + \frac{60}{30}x$

Q.6. For $X = 26$, $Z = \frac{26-30}{5} = -\frac{4}{5} = -0.8$

For, $X = 40 \Rightarrow Z = \frac{40-30}{5} = 2$

$P(26 \le X \le 40) = P(-0.8 \le Z \le 2)$

$\Rightarrow P(-0.8 \le Z \le 0) + P(0 \le Z \le 2)$

$\Rightarrow P(0 \le Z \le 0.8) + P(0 \le Z < 2)$

$\Rightarrow 0.2881 + 0.4772 \Rightarrow 0.7653$. Option (A)

Q.7. X on Y, $Y + 4X = \frac{49}{3}$

$\Rightarrow 4X = \frac{49}{3} - Y$

$\Rightarrow X = \frac{49}{12} - \frac{Y}{4}$ $\quad b_{xy} = -\frac{1}{4}$

Y on X, $X + 9Y = 7$

$9Y = 7 - X$

$Y = \frac{7}{9} - \frac{X}{9}$ $\quad b_{yx} = -\frac{1}{9}$

$r = \sqrt{\left(-\frac{1}{9}\right)\left(-\frac{1}{4}\right)} = \frac{1}{6} < 1$

Correlation coefficient between X and Y is $-\frac{1}{6}$ ~~Option (A)~~

Option (B)

Q.8. $P(x ; \mu) = \dfrac{e^{-\mu}\mu^{x}}{x!}$

Given, $P(X=0) = P(X=1) = k$

$\dfrac{e^{-\mu}\mu^0}{0!} = \dfrac{e^{-\mu}\mu^1}{1!} \Rightarrow e^{-\mu} = e^{-\mu}\mu$

$\Rightarrow e^{-\mu}(1-\mu) = 0$

Since, $e^{-\mu} \neq 0$, $\Rightarrow \mu = 1$

$P(X=0) = \dfrac{e^{-\mu}\mu^0}{0!} = e^{-\mu} = e^{-1} = \dfrac{1}{e}$ Option (A)

Q.9. $A\displaystyle\int_0^5 \sin\left(\frac{\pi x}{5}\right) dx = 1$

$\rightarrow \dfrac{5A}{\pi}\left[-\cos\left(\dfrac{\pi x}{5}\right)\right]_0^5 = 1$

$\rightarrow A[\cos\pi - \cos 0] = -\dfrac{\pi}{5}$

$\rightarrow A(-2) = -\dfrac{\pi}{10}$ $\quad A = \dfrac{\pi}{10}$

Let m be the median,

$\displaystyle\int_0^m A\sin\left(\frac{\pi x}{5}\right) dx = \frac{1}{2}$

$\rightarrow -\dfrac{\pi}{10} \times \dfrac{5}{\pi}\left[\cos\left(\dfrac{\pi x}{5}\right)\right]_0^m = \dfrac{1}{2}$

$\rightarrow \cos\dfrac{\pi m}{5} - 1 = -1$

$\rightarrow \cos\left(\dfrac{\pi m}{5}\right) = 0 \rightarrow \dfrac{\pi m}{5} = \dfrac{\pi}{2}$

$\rightarrow m = \dfrac{5}{2}$ Option (B)

10.

X	1	2	3	4	5
$P(X=x)$	$\frac{1}{3}$	$\frac{1}{3^2}$	$\frac{1}{3^3}$		

$$E(X) = \Sigma x\, p(x)$$

$$S = 1 \cdot \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} \cdots$$

$$\frac{S}{3} = 1 \cdot \frac{1}{3^2} + \frac{2}{3^3} \cdots$$

$$\frac{2}{3}S = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} \cdots$$

$$\frac{2}{3}S = \frac{\frac{1}{3}}{1-\frac{1}{3}} \Rightarrow \frac{2}{3}S = \frac{1}{2} \Rightarrow S = \frac{3}{4}$$

$$E(X) = \frac{3}{4}$$

Option (c)

Q.11. $(D^3 - 3DD'^2 - 2D'^3) Z = \cos(x+2y) - e^y(3+2x)$

$\Rightarrow$ PI : $\dfrac{1}{(D^3 - 3DD'^2 - 2D'^3)} \left(\cos(x+2y) - e^y(3+2x)\right)$

$= \dfrac{1}{(D+D')^2 (D-2D')} \left\{ \cos(x+2y) - e^y(3+2x) \right\}$

$= \dfrac{1}{(D+D')^2 (D-2D')} \{\cos(x+2y)\} - \dfrac{1}{(D+D')^2 (D-2D')} e^y (3+2x)$

$= -\dfrac{1}{27} \iiint \cos v \, dv - e^y \dfrac{1}{(D+D'+1)^2 (D\cancel{x} - 2D' - 2)} (3+2x$

$= \dfrac{\sin(x+2y)}{27} - e^y \left[\dfrac{1}{(D-2D'-2)} (3\cancel{x}+2x) + 2(-2)) \right]$

$= \dfrac{\sin(x+2y)}{27} + \dfrac{e^y}{2} \left[\dfrac{1}{1 - \frac{(D-2D')}{2}} \right] \cdot \cancel{2}x - 1)]$

$= \dfrac{\sin(x+2y)}{27} + \dfrac{x e^y}{2}$ Option (A)

Q.12. $F(x) = \begin{cases} 1 & -1 \le x < 0 \\ -2 & 0 < x \le 1 \end{cases}$

$\dfrac{a_0}{2} = \dfrac{1}{2} \displaystyle\int_{-1}^{1} f(x)\,dx = \dfrac{1}{2} \left[\int_{-1}^{0} dx + \int_{0}^{1} -2\,dx \right]$

$= \dfrac{1}{2} \left[\cancel{2x} [x]_{-1}^{0} \cancel{2} - 2[x]_{1}^{0} \right] = \dfrac{1}{2} [1 - 2]$

$\dfrac{a_0}{2} = -\dfrac{1}{2} \Rightarrow \boxed{a_0 = 1}$

$a_3 = \dfrac{1}{1} \displaystyle\int f(x) \cos(3\pi x)\,dx$

$= \displaystyle\int_{-1}^{0} \cos(3\pi x)\,dx + \int_{0}^{1} (-2) \cos(3\pi x)\,dx$

$= \left[\dfrac{\sin 3\pi x}{3\pi} \right]_{-1}^{0} - 2 \left[\dfrac{\sin(3\pi \cancel{x}}{3\pi} \right]_{0}^{1}$ $\boxed{a_3 = 0}$

$$b_3 = \frac{1}{1}\int_{-1}^{1} f(x)\sin(3\pi x)\,dx$$

$$\Rightarrow \int_{-1}^{0}\sin(3\pi x)\,dx + \int_{0}^{1}(-2)\sin(3\pi x)\,dx$$

$$= -\left[\frac{\cos(3\pi x)}{3\pi}\right]_{-1}^{0} - 2\left[\frac{-\cos(3\pi x)}{3\pi}\right]_{0}^{1}$$

$$= \left[-\frac{1}{3\pi} - \left(\frac{1}{3\pi}\right)\right] - 2\left[\frac{1}{3\pi} + \frac{1}{3\pi}\right]$$

$$b_3 = -\frac{2}{3\pi} - \frac{4}{3\pi} = -\frac{2}{\pi}$$

$\cancel{b_3 = -2}$ $\quad (a_0, a_3, b_3) = \left(-1, 0, -\frac{2}{\pi}\right)$ Option (B)

Q.13. $(2xy-1)p + (z-2x^2)q = 2(x-yz)$

$$\frac{dx}{2xy-1} = \frac{dy}{z-2x^2} = \frac{dz}{2(x-yz)}$$

$$\rightarrow x\,dx + y\,dy + \frac{1}{2}dz = 0$$

$$\Rightarrow x^2 + y^2 + z = c_1^2 \quad \ldots$$

$$\rightarrow z\,dx + x\,dz + dy = 0$$

$$\rightarrow xz + y = c_2$$

$$x^2 + y^2 + z = \phi(xz+y)$$

Option (A)

16. Probability of catching disease $= \frac{20}{100} = \frac{1}{5}$

Let, probability of success $= p = \frac{1}{5}$

〃 of failure $(q) = 1 - \frac{1}{5} = \frac{4}{6}$.

Given, $n = 6$.

$\therefore \quad P(X \geq 4) = P(X=4) + P(X=5) + P(X=6)$

$$= {}^6C_4 \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^2 + {}^6C_5 \left(\frac{1}{5}\right)^5 \left(\frac{4}{5}\right)^1 + {}^6C_6 \left(\frac{1}{5}\right)^6 \left(\frac{4}{5}\right)^0$$

$$= \frac{1}{5^6} \left\{ 4^2 \times {}^6C_4 + 4 \times {}^6C_5 + {}^6C_6 \right\}$$

$$= \frac{1}{5^6} \left\{ 4^2 \times \frac{6}{4! \times 2!} + 4 \cdot 6 + 1 \right\}$$

$$= \frac{1}{5^6} \times (265) = \frac{53}{3125}$$ Option (c)

Q. 14. $F\left(x - y + z, \frac{x^2 - y^2}{z^2}\right) = 0$, Let $u = x - y - z$

$v = \frac{x^2 - y^2}{z^2}$

$$P = \begin{vmatrix} \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \end{vmatrix}$$

$\frac{\partial u}{\partial x} = 1$; $\frac{\partial u}{\partial y} = -1$; $\frac{\partial v}{\partial z} = -1$

$\frac{\partial v}{\partial x} = \frac{2x}{z^2}$; $\frac{\partial v}{\partial y} = \frac{-2y}{z^2}$

$\frac{\partial v}{\partial z} = -\frac{2}{z^3}(x^2 - y^2)$

$$= \begin{vmatrix} -1 & -1 \\ \frac{-2y}{z^2} & -\frac{2}{z^3}(x^2 - y^2) \end{vmatrix}$$

$$= \frac{2}{z^3}(x^2 - y^2) - \frac{2y}{z^2}$$

$$Q = \begin{vmatrix} \frac{\partial u}{\partial z} & \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial z} & \frac{\partial v}{\partial x} \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ -\frac{2(x^2 - y^2)}{z^3} & \frac{2x}{z^2} \end{vmatrix} = \frac{-2x}{z^2} + \frac{2(x^2 - y^2)}{z^3}$$

$$R = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ \frac{2x}{z^2} & \frac{-2y}{z^2} \end{vmatrix} = \frac{-2y}{z^2} + \frac{2x}{z^2}$$

The sol^n is $Pp + Qq = R$

$$= \left\{\frac{2}{z^3}(x^2-y^2) - \frac{2y}{z^2}\right\} p + \left\{\frac{-2x}{z^2} + \frac{2(x^2-y^2)}{z^3}\right\} q = \frac{-2y}{z^2} + \frac{2x}{z^2}$$

$$\rightarrow \quad (x^2 - y^2 - yz)p + (x^2 - y^2 - zx) q = z(x-y) \quad \text{Option (A)}$$

Q.15.

$$q^2 = z^2 p^2 (1 - p^2)$$

$$f(p, q, z)$$

put $= q = pa$

$$(pa)^2 = z^2 p^2 (1-p^2)$$

$$\rightarrow \quad p^2 a^2 = z^2 p^2 (1-p^2)$$

$$\rightarrow \quad p^2 [a^2 - 1 + p^2] = 0$$

$$p^2 \neq 0, \quad p^2 = 1 - a^2 \quad \rightarrow \quad p = \pm \sqrt{1 - az}$$

$$dz = p\,(dx + a\,dy)$$

$$\rightarrow \quad dz = \pm\sqrt{1 - az}\,(dx + a\,dy)$$

$$\rightarrow \quad z^2 = (1-a^2)(x + ay + b)^2$$

(D) None of these

Q.17. By using poison distribution,

$$P(x;\mu) = \frac{e^{-\mu}\mu^x}{x!}$$

μ: The mean no. of successes that occur in a specified region

x: The actual no. of successes that occur.

$\mu = 100 \times 0.01 = 1$ $\quad P(x;\mu) = P(x=0;1) + P(x=1;1)$

$$\Rightarrow P(x;1) = \frac{e^{-1}1^0}{0!} + \frac{e^{-1} \times 1^1}{1!}$$

$$= e^{-1} + e^{-1} = \frac{2}{e} \quad \text{Option (C)}$$

Q.18. $n=6$, $9 \times P(X=4) = P(X=2)$

$\Rightarrow P(X=4) = {}^6C_4\, p^4 (1-p)^2$

$\Rightarrow P(X=2) = {}^6C_2\, p^2 (1-p)^4$

$\Rightarrow 9\ {}^6C_4\, p^4 (1-p)^2 = {}^6C_2\, p^2 (1-p)^4$

$\Rightarrow 3p^2 (1-p) = p(1-p)^2$

$\Rightarrow 3p = 1-p \quad \Rightarrow p = \frac{1}{4}$ Option (B)

Group B:

Q.19.

X	Y	X^2	Y^2	XY
1	2	1	4	2
3	6	9	36	18
4	8	16	64	32
5	10	25	100	50
7	14	49	196	98
8	16	64	256	128
10	20	100	400	200
$\Sigma X = 38$	$\Sigma Y = 76$	$\Sigma X^2 = 264$	$\Sigma Y^2 = 1056$	$\Sigma XY = 528$

$$r = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)\left(\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right)}}$$

$$= \frac{528 - \frac{38 \times 76}{7}}{\sqrt{\left(264 - \frac{(38)^2}{7}\right)\left(1056 - \frac{(76)^2}{7}\right)}}$$

$$= \frac{528 - 412.57}{\sqrt{(264 - 206.28)(1056 - 825.14)}}$$

$$= \frac{115.43}{\sqrt{57.72 \times 230.86}}$$

$$= \frac{115.43}{7.59 \times 15.1940} = 1.009 \approx 1$$

Option (a) 1.

Q.20. $Var(X) = 9$

$$8X - 10Y = -66, \quad 40X - 18Y = 214$$

$$X = \frac{10Y - 66}{8} \qquad X = \frac{18Y + 214}{40}$$

$$\frac{10Y - 66}{8} = \frac{18Y + 214}{40}$$

$$\Rightarrow 50Y - 330 = 18Y + 214$$

$$Y = 17 \qquad X = \frac{170 - 66}{8} = 13$$

$$E(X) = 13, \quad E(Y) = 17$$

$$X = \cancel{\frac{10}{8}}\, Y - \frac{66}{8}, \quad Y = \frac{40}{18}X - \frac{214}{18}$$

$$\frac{1}{r} = \sqrt{\frac{10}{8} \times \frac{40}{18}}$$

$$\frac{1}{r} = \frac{5}{3} \Rightarrow r = \frac{3}{5} = 0.6$$

(b) 13, 13, 0.6