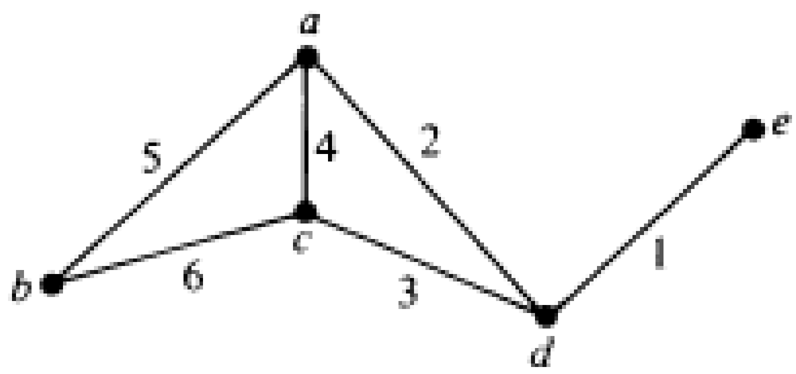
DAY 3

Isomorphism

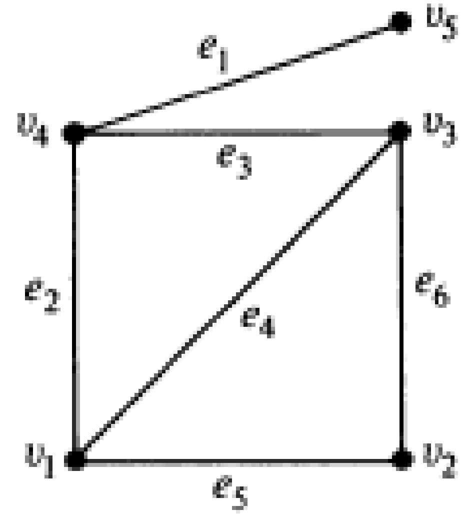
Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if there is a

bijection (an one-to-one correspondence) function f

from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if f(a) and f(b) are adjacent in G_2, for all a and b in V_1. Such a function f is called an isomorphism.



(a)



(b)

The vertices a, b, c, d, e in graph (a) correspond to $V_{1,}$ $V_{2,}$ $V_{3,}$ V_4 and V_5 in graph (b)

The edges 1,2,3,4,5,6 in graph (a) correspond to $e_{1,}$ $e_{2,}$ …… e_6 in graph (b) respectively.

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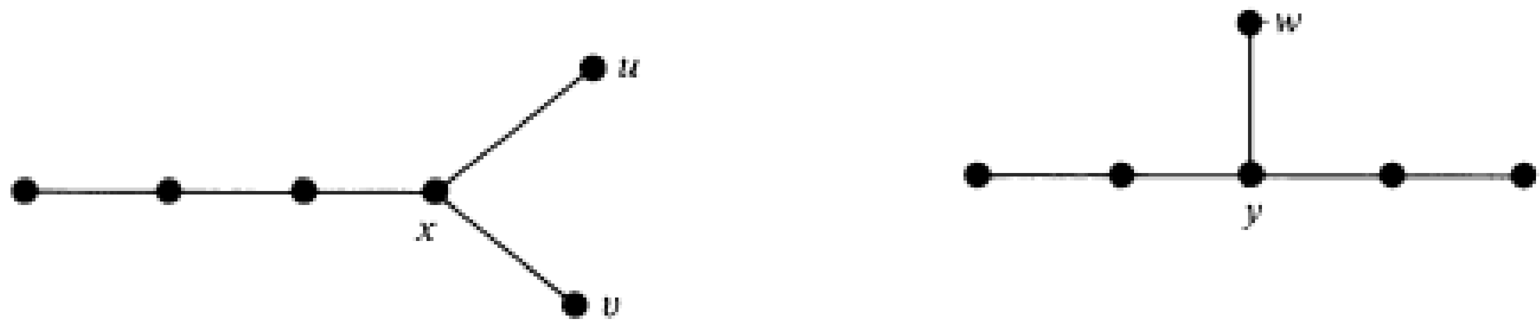
The two Isomorphic Graphs must have:

- the same number of vertices,
- the same number of edges and
- an equal number of vertices with a given degree.

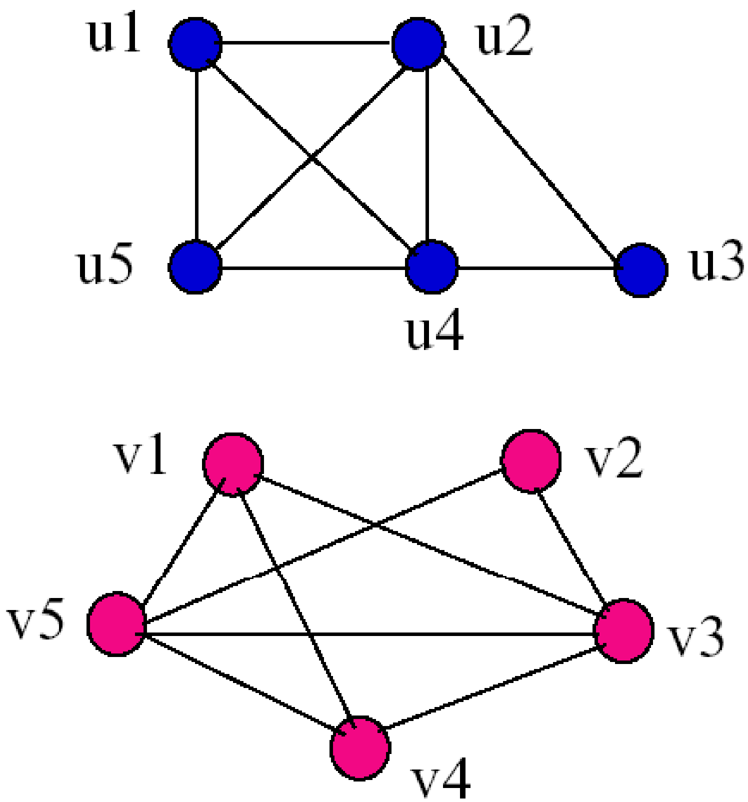
Further checking…



(a) (b)

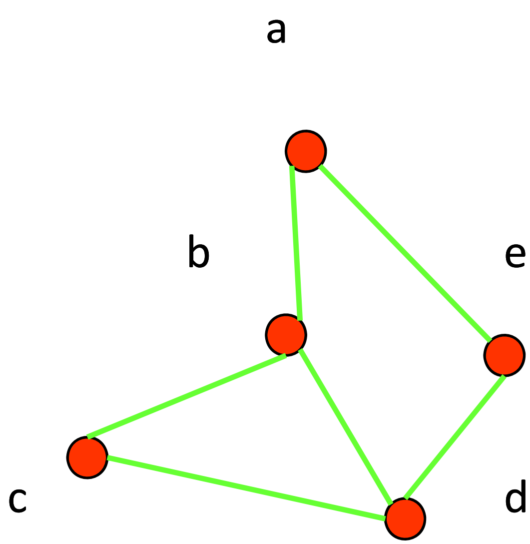
Are they isomorphic??

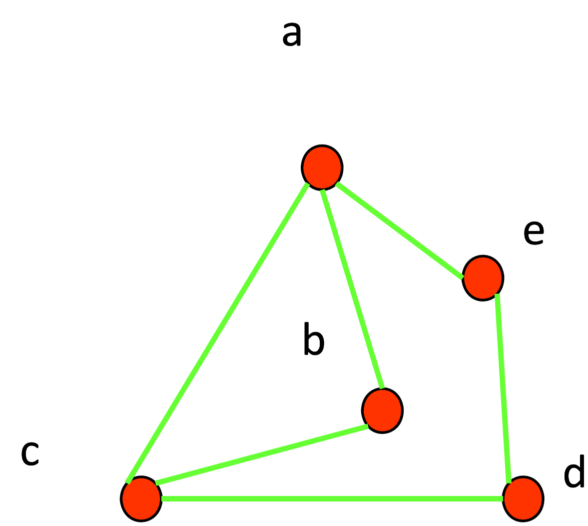
Question for Self Study



Are they isomorphic??

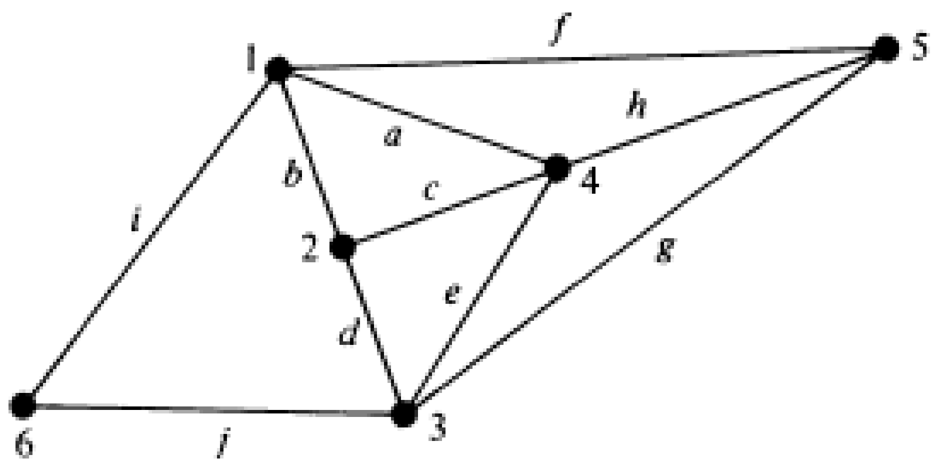
Question for Self Study



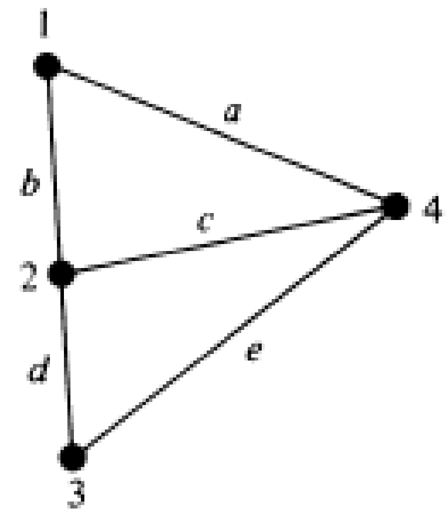


Are they isomorphic??

Sub Graph



(a)



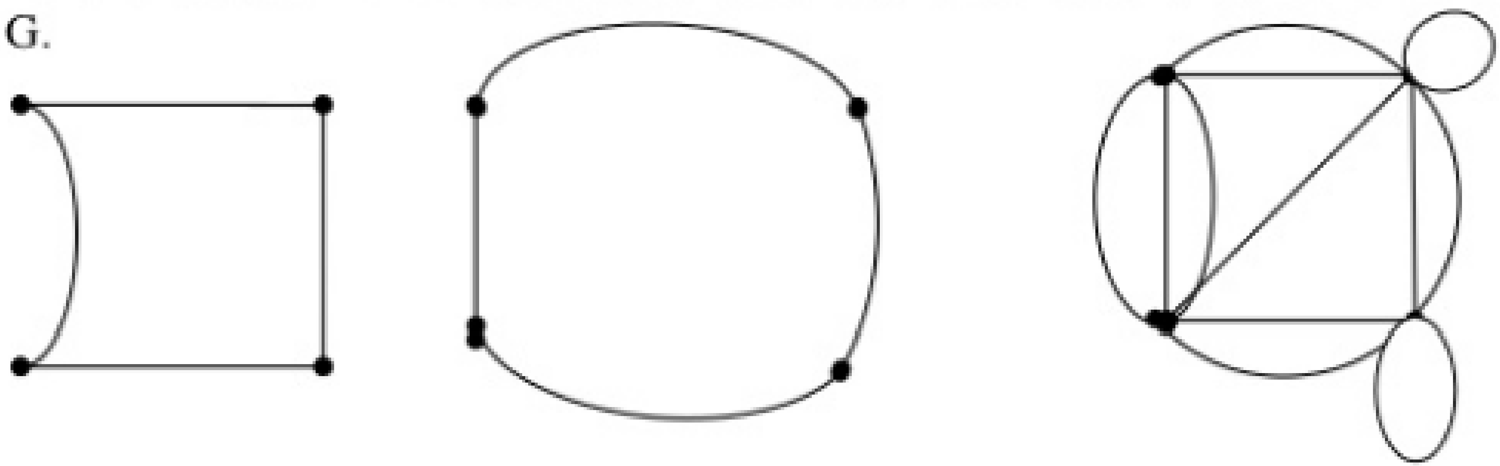
(b)

Observations

1. Every graph is its own subgraph.
2. A subgraph of a subgraph of G is a subgraph of G.
3. A single vertex in a graph G is a subgraph of G.
4. A single edge in G, together with its end vertices, is also a subgraph of G.

Edge-disjoint Subgraph

Edge-Disjoint Subgraphs: Two (or more) subgraphs g1, and g2 of a graph G are said to be edge disjoint if g1, and g2 do not have any edges in common.
For example, the following two graphs are edge-disjoint sub-graphs of the graph G.

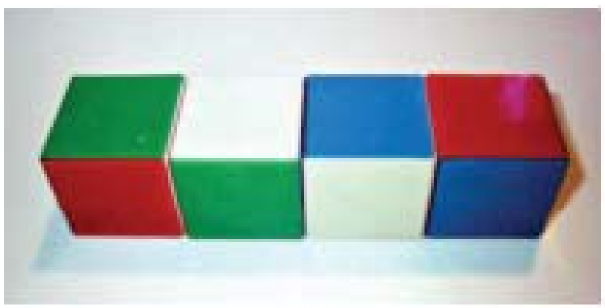


Note that although edge-disjoint graphs do not have any edge in common, they may have vertices in common. Sub-graphs that do not even have vertices in common are said to be vertex disjoint. (Obviously, graphs that have no vertices in common cannot possibly have edges in common.)

Question for Self Study

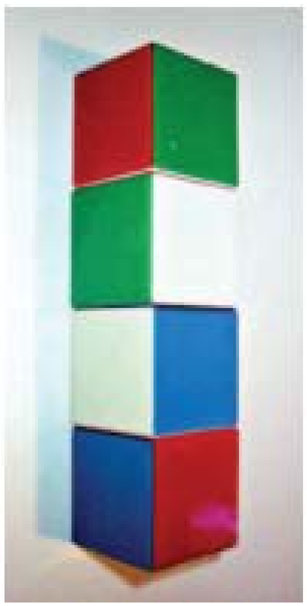
1. What is vertex-disjoint subgraph?

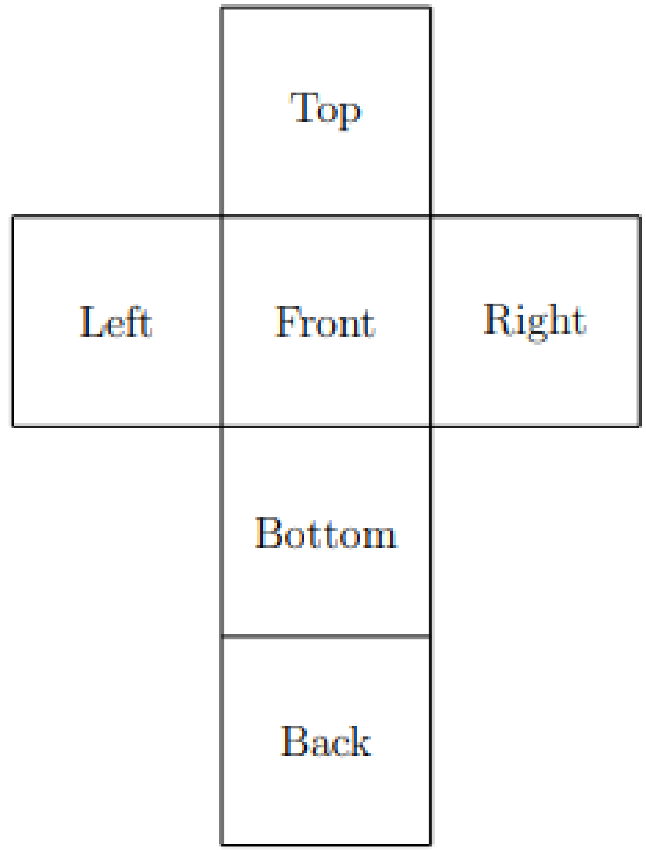
Multicolor cube problem



Instant Insanity

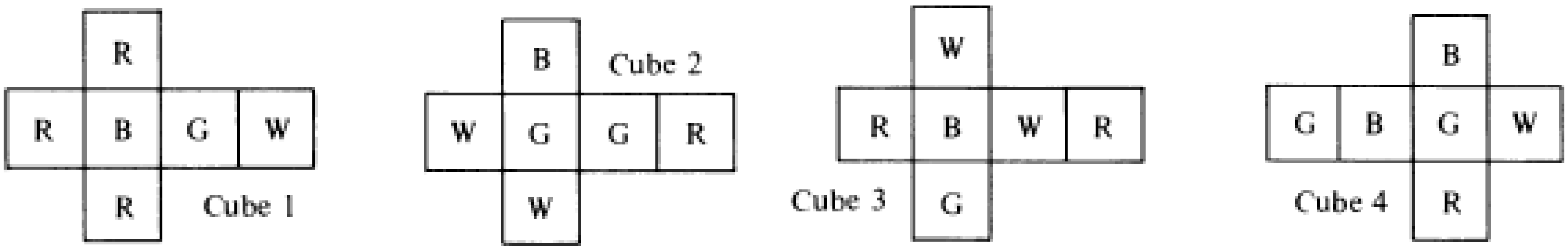
Multicolor cube problem

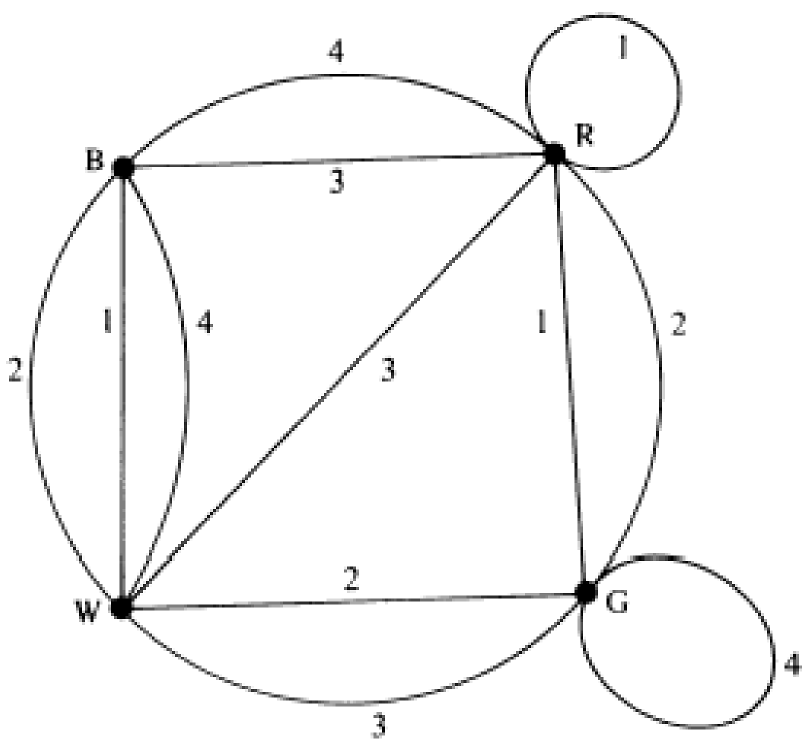




A net of the cube

Multicolor cube problem

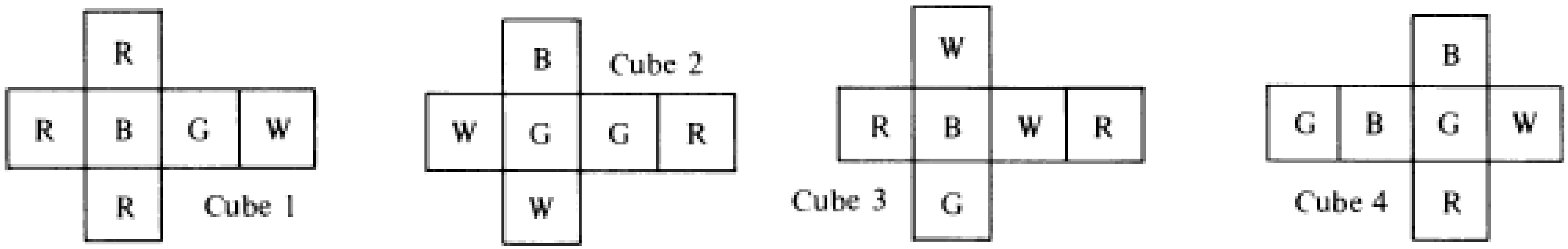


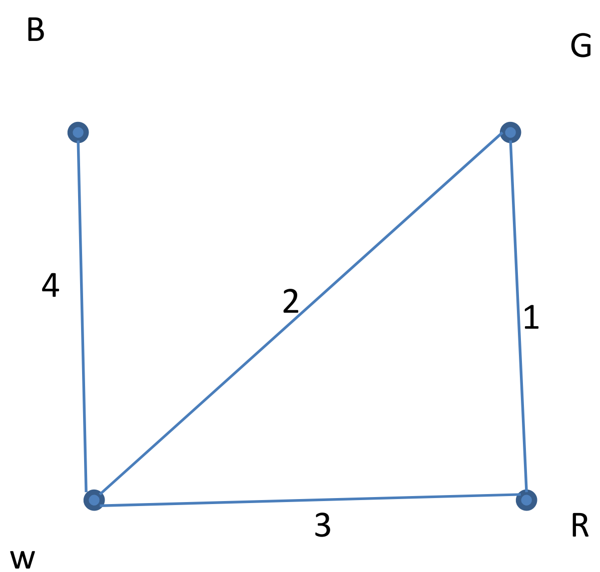


Steps

1. Put every color as a separate vertex.
2. Join the vertices with edges in such a way that each edge represents the opposite faces of a cube and the name of the edge is represented by the number of the cube.
3. Check for the degree of a vertex which must be equal to the total occurrences of a color in all four cubes.
4. From the completed graph find two edge-disjoint sub-graphs such that: one is facing north-south and the other is facing east-west.
5. Since every edge is the connection between two opposite faces, thus, the four edges in the N-S sub-graph will represent eight (8) faces out of which 4 will be North faces and 4 will be south faces for all the four cubes.
6. Similarly, , the four edges in the E-W sub-graph will represent eight (8) faces out of which 4 will be East faces and 4 will be West faces for all the four cubes.
7. Each of the 4 edges in a sub-graph has a label 1,2,3 or 4 and no color will appear in any of the four sides if and only if every edge in the sub graph has a degree 2.

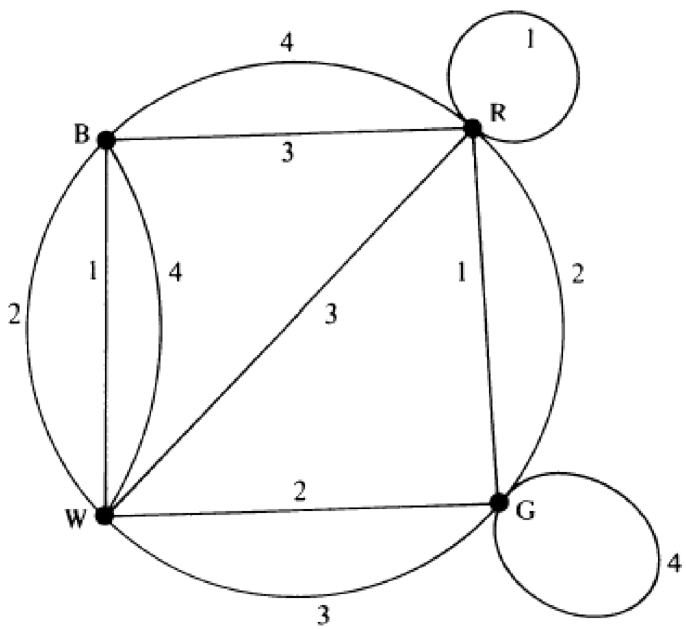
Contd..

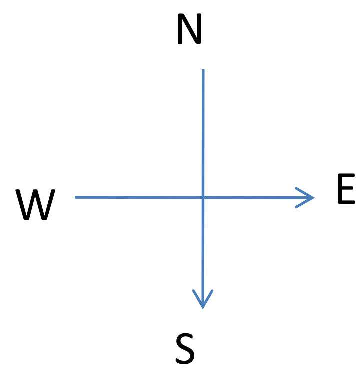


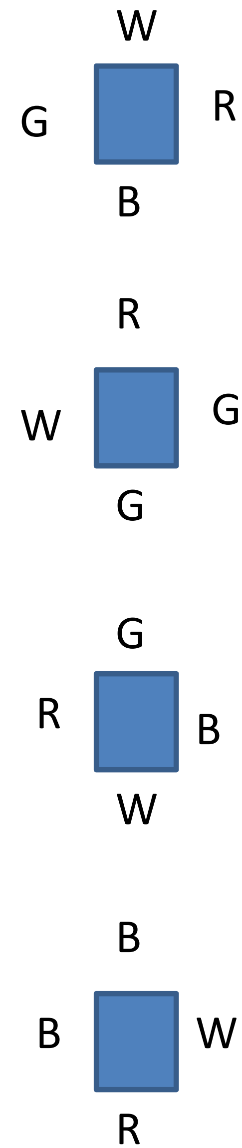


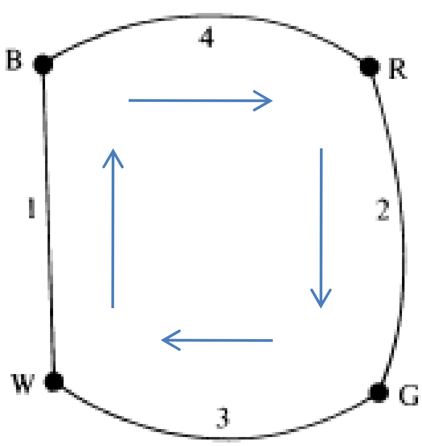
Please complete the graph and compare with the previous graph

Contd..

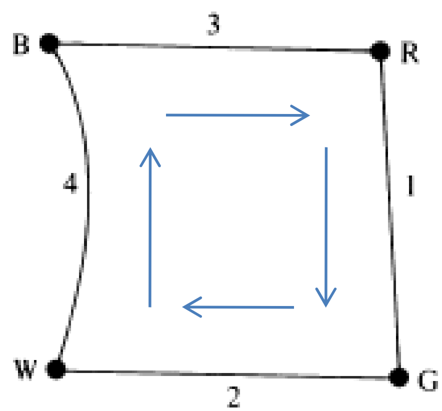






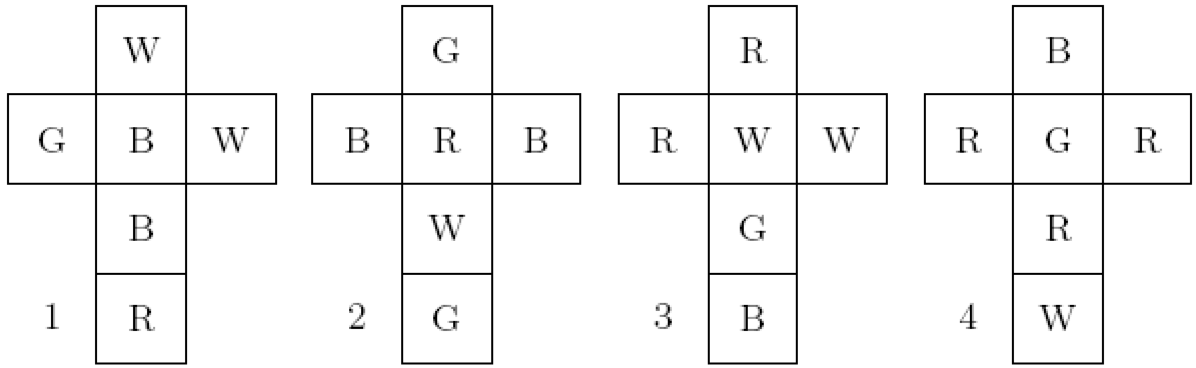


(a) North-South Subgraph



(b) East-West Subgraph

Solve by yourself



Given four cubes having the six faces variously colored with the following four colors. Is it possible to stack the cubes one on top of another such that no color appears twice on any of the four sides of this column?

Thanks for your patience!