DAY 8

Spanning Tree

A tree T is said to be a *spanning tree* of a connected graph G if T is a subgraph of G and T contains all vertices of G. For instance, the subgraph in heavy lines in Fig. 3-17 is a spanning tree of the graph shown.

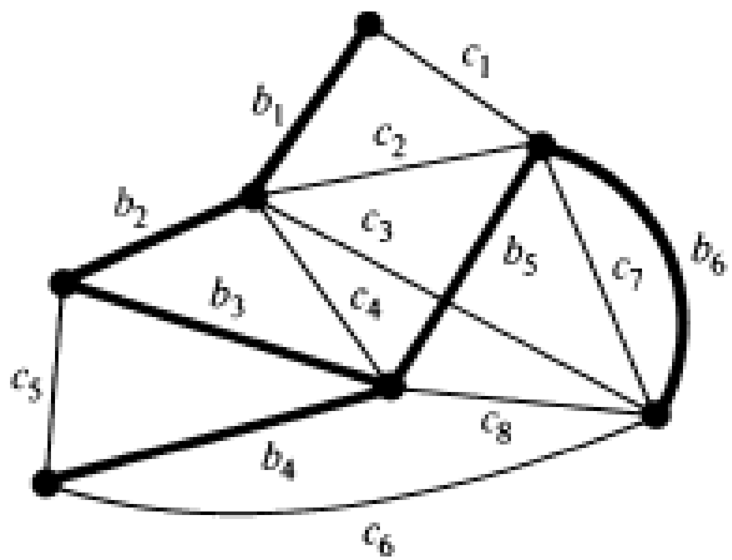
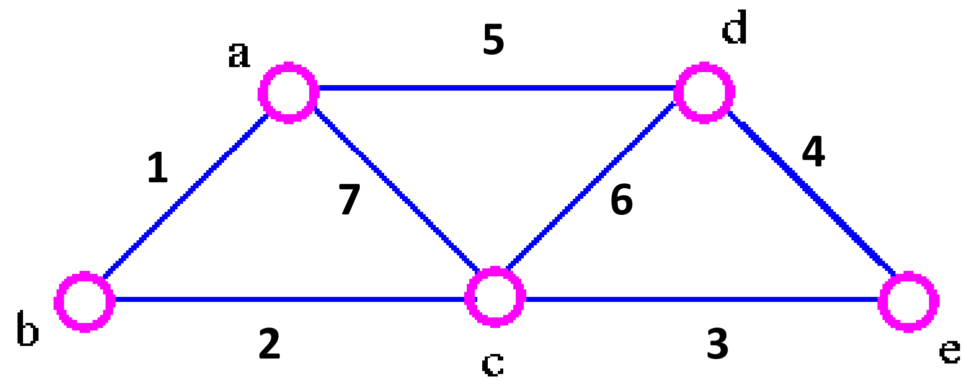


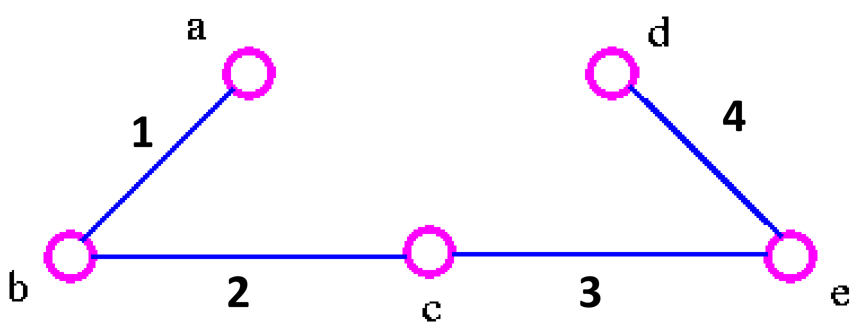
Fig. 3-17 Spanning tree.

Contd..

A spanning tree is a sub-graph of G that includes all the vertices of G. The edges of the graphs which are present in the tree are called **branches** and remaining edges of the graph are called **chords**.



G



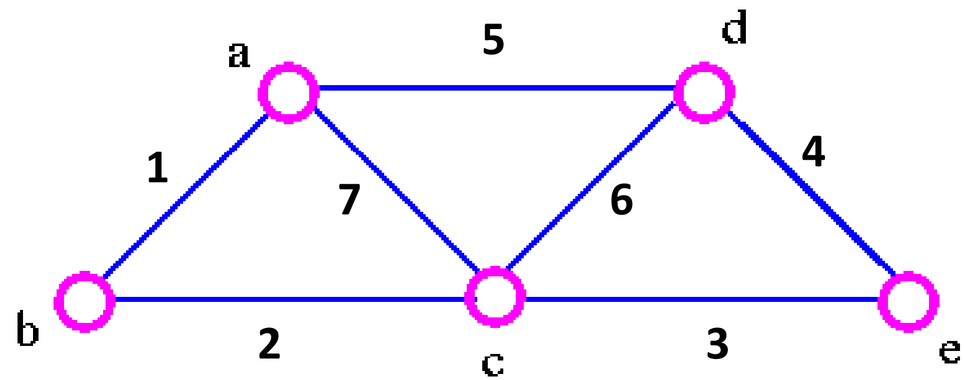
Branches = {1,2,3,4}

Chords = { 5,6,7}

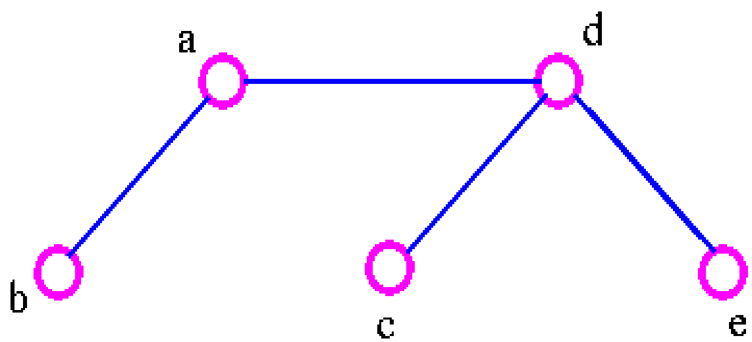
T1

Solve

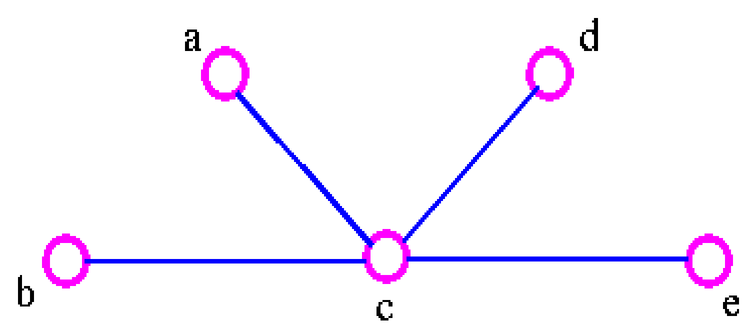
Identify the branches and chords from T2 and T3.



G



T2

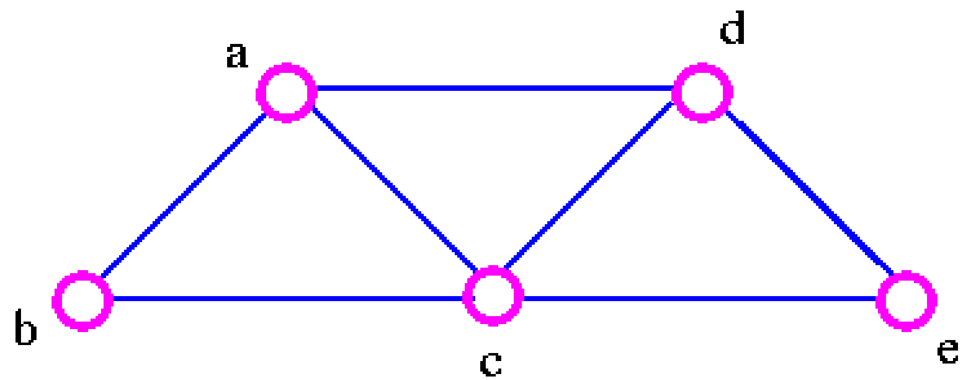


T3

Constructing a ST

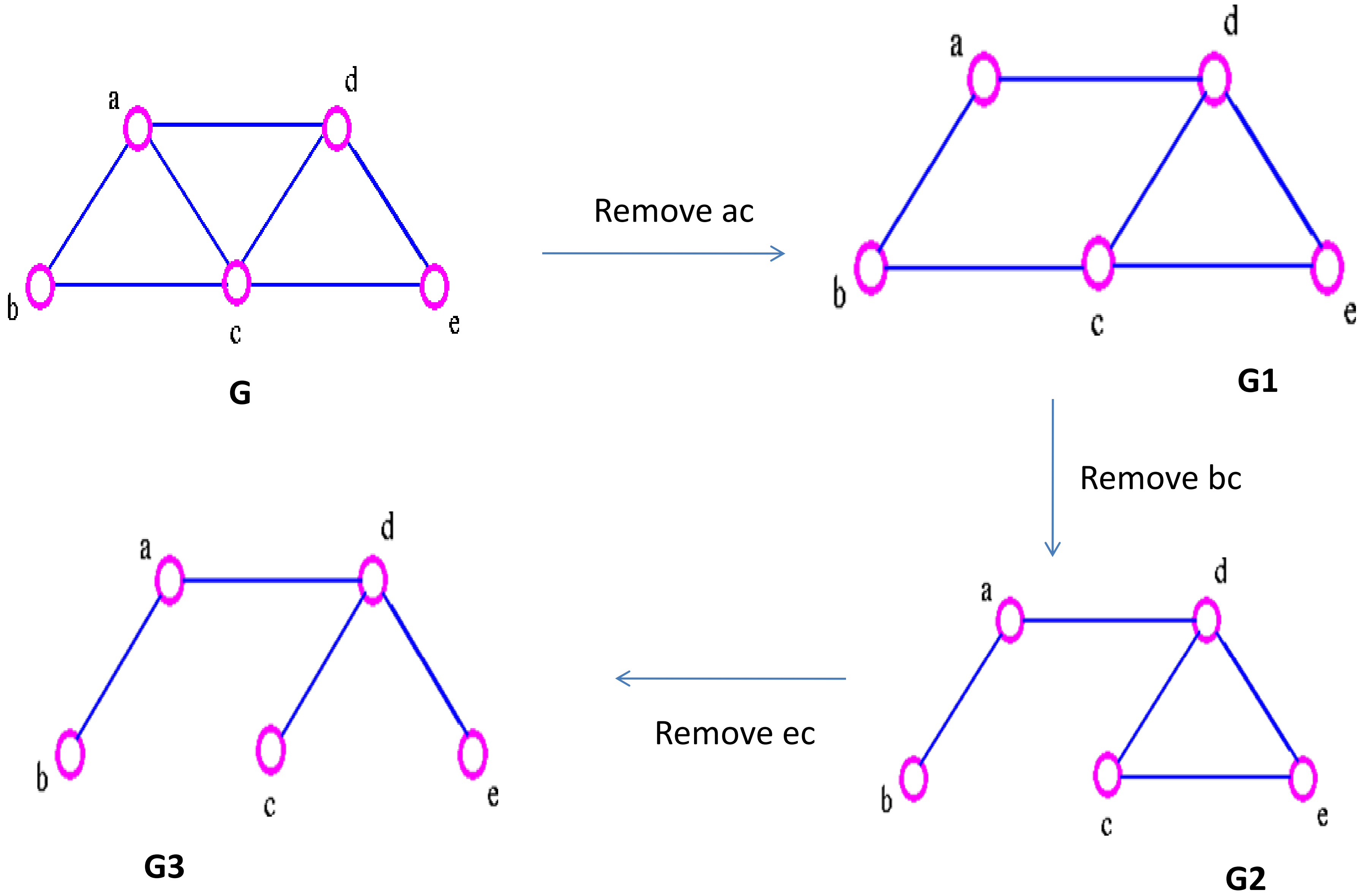
Cutting-down Method:

1. Start choosing any cycle in G.
2. Remove one of cycle's edges.
3. Repeat this procedure until there are no cycle left.



G

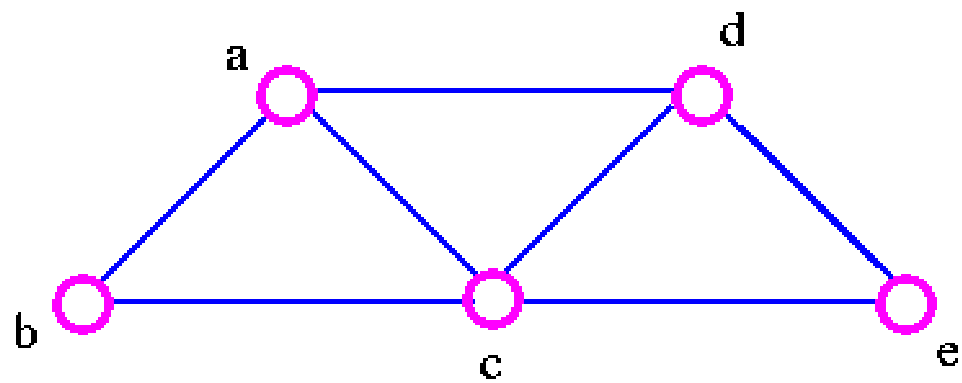
Contd..


a
d
b
c
e
G
Remove ac
a
d
b
c
e
G1
Remove bc
a
d
b
c
e
G3
Remove ec
a
d
b
c
e
G2

Constructing a ST

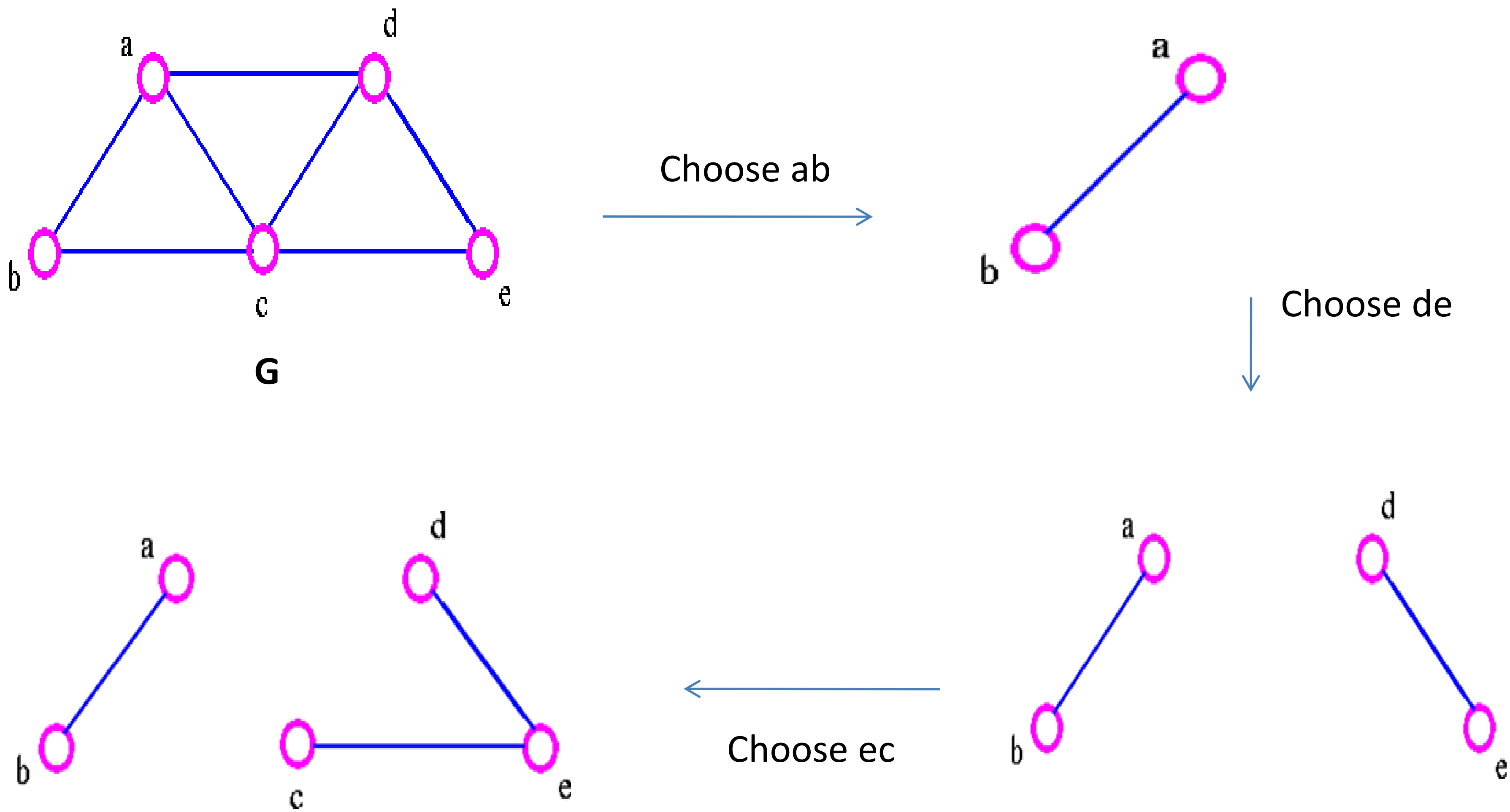
Building-up Method

1. Select edges of G one at a time. in such a way that no cycles are created.
2. Repeat this procedure until all vertices are included.

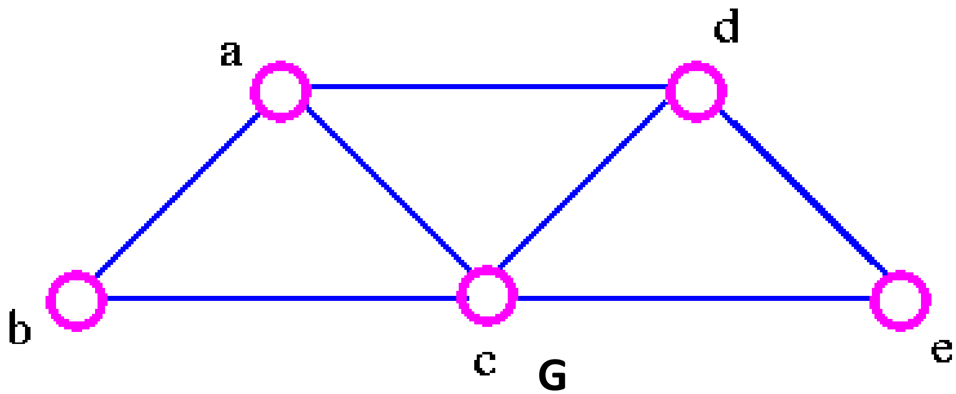


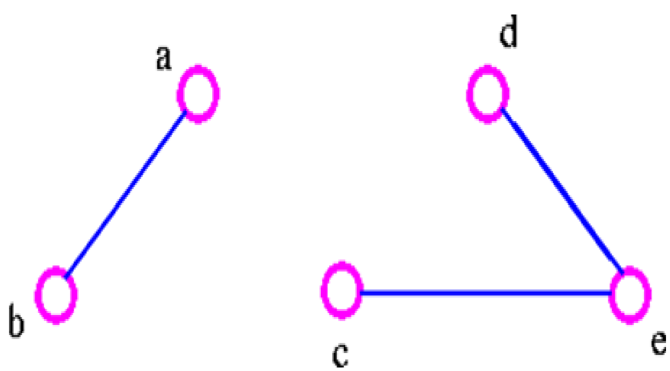
G

Contd..



Contd..





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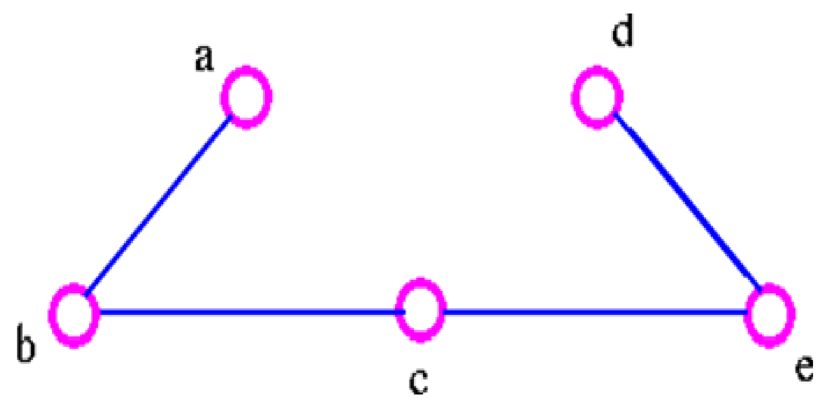


T1

General Properties of Spanning Tree

- A connected graph G can have more than one spanning tree.
- All possible spanning trees of graph G, have the same number of edges and vertices.
- The spanning tree does not have any cycle (loops).
- Removing one edge from the spanning tree will make the graph disconnected, i.e. the spanning tree is **minimally connected**.
- Adding one edge to the spanning tree will create a circuit or loop, i.e. the spanning tree is **maximally acyclic**.

Application of Spanning Tree

- **Network connectivity with minimum wiring**
- **Computer Network Routing Protocol**
- **Cluster Analysis**
- **Electronic circuit board design**
- **Biomedical image analysis**

Solve

Define a Spanning Forest with appropriate diagram.

Relation between Branch and Chord

THEOREM 3-12

With respect to any of its spanning trees, a connected graph of n vertices and e edges has $n - 1$ tree branches and $e - n + 1$ chords.

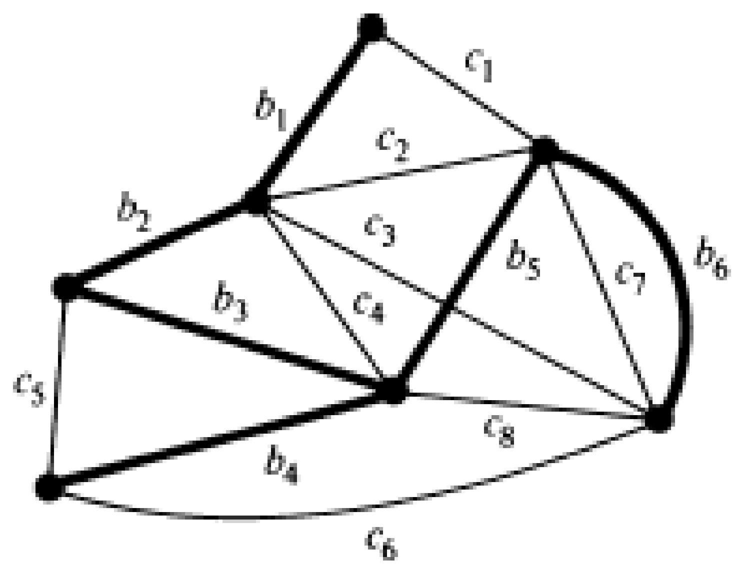


Fig. 3-17 Spanning tree.

n = 7, e = 14

Branch = (n-1) = 6

Chord = (e-n+1) = 8

Contd..

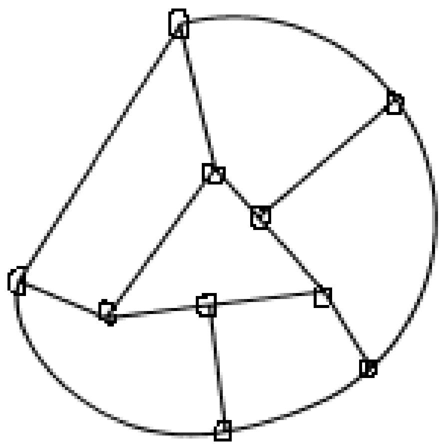


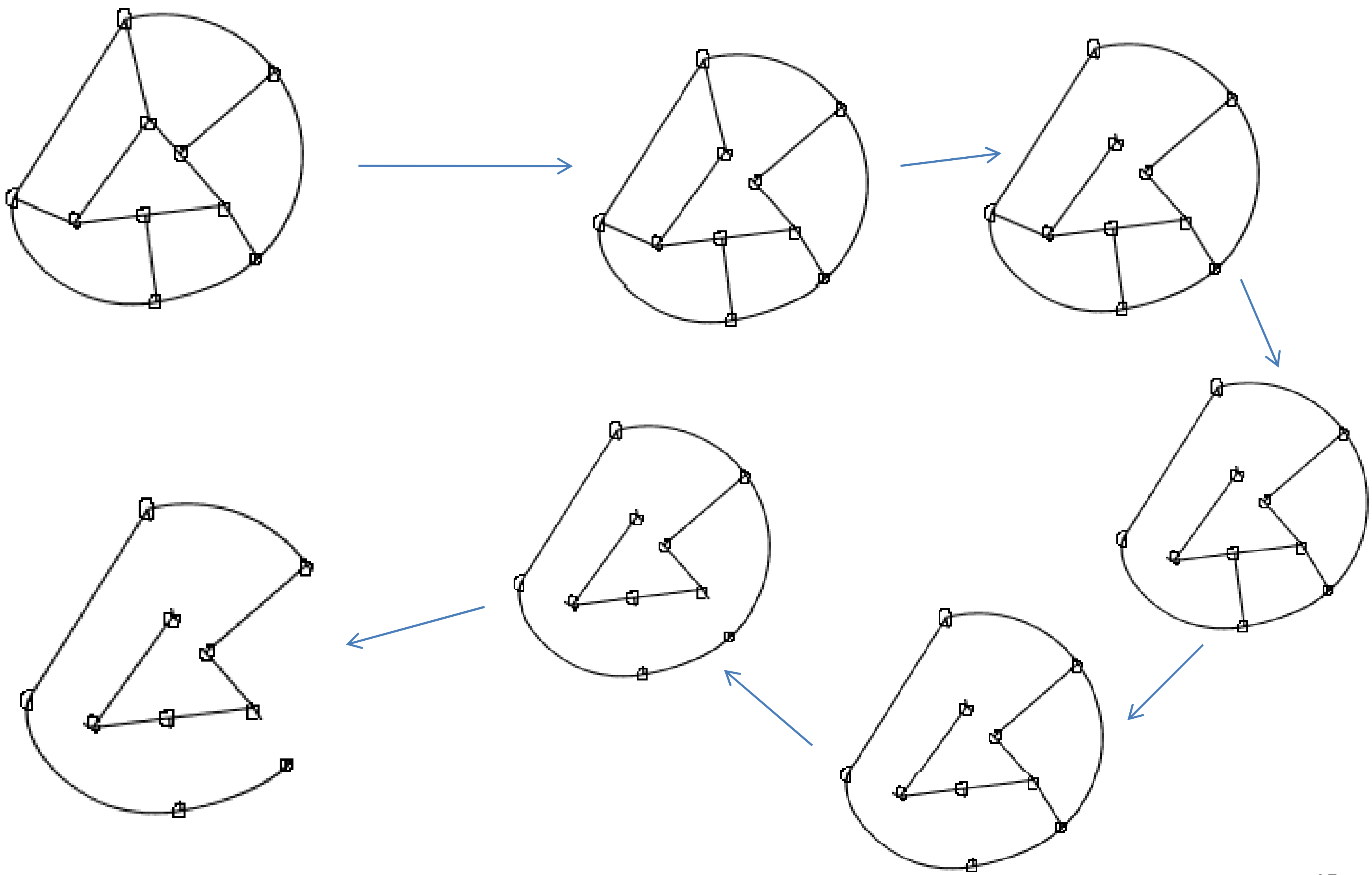
Fig. 3-18 Farm with walled plots of land.

n = 10, e = 15

Branch = (n-1) = 9

Chord = (e-n+1) = 6

Contd..

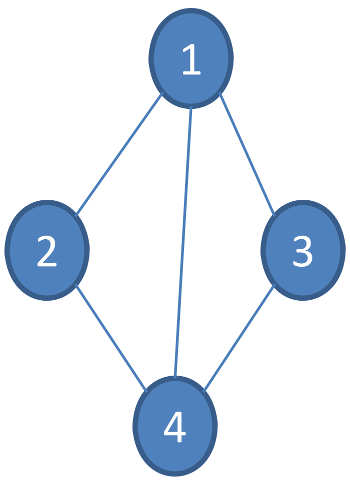


Rank and Nullity

$$\text{rank} \qquad r = n - k,$$
$$\text{nullity} \qquad \mu = e - n + k.$$

$$\begin{aligned}
\text{rank of } G &= \text{number of branches in any spanning} \\
&\quad \text{tree (or forest) of } G, \\
\text{nullity of } G &= \text{number of chords in } G, \\
\text{rank} + \text{nullity} &= \text{number of edges in } G.
\end{aligned}$$

Kirchhoff's Matrix-tree Theorem



G

Steps:

1. Construct the Laplacian Matrix Q for G which can be done as:

   a) for i != j

      if vertex i and j are adjacent in G, then $q_{ij} = -1$.

      else, $q_{ij} = 0$

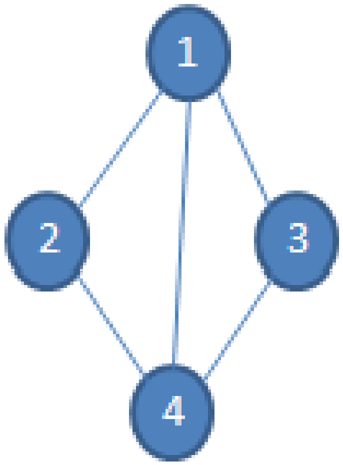
   b) for i = = j

   q_{ij} = degree of vertex i in G.

2. From Q, construct matrix Q' by deleting any one row and any one column from Q.

3. Calculate determinant of Q' to get total no. of ST in G.

Contd..



G

Q =

3	**-1**	**-1**	**-1**
-1	2	0	-1
-1	0	2	-1
-1	-1	-1	3

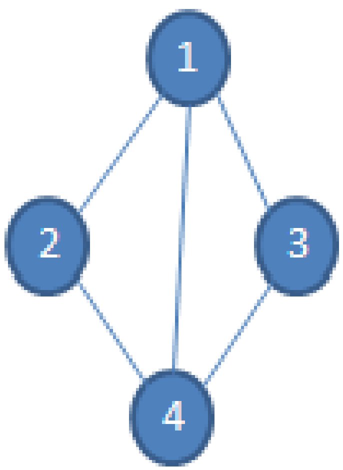
By deleting row 1 and column 1 we get:

Q' =

+	-	+
2	**0**	**-1**
0	2	-1
-1	-1	3

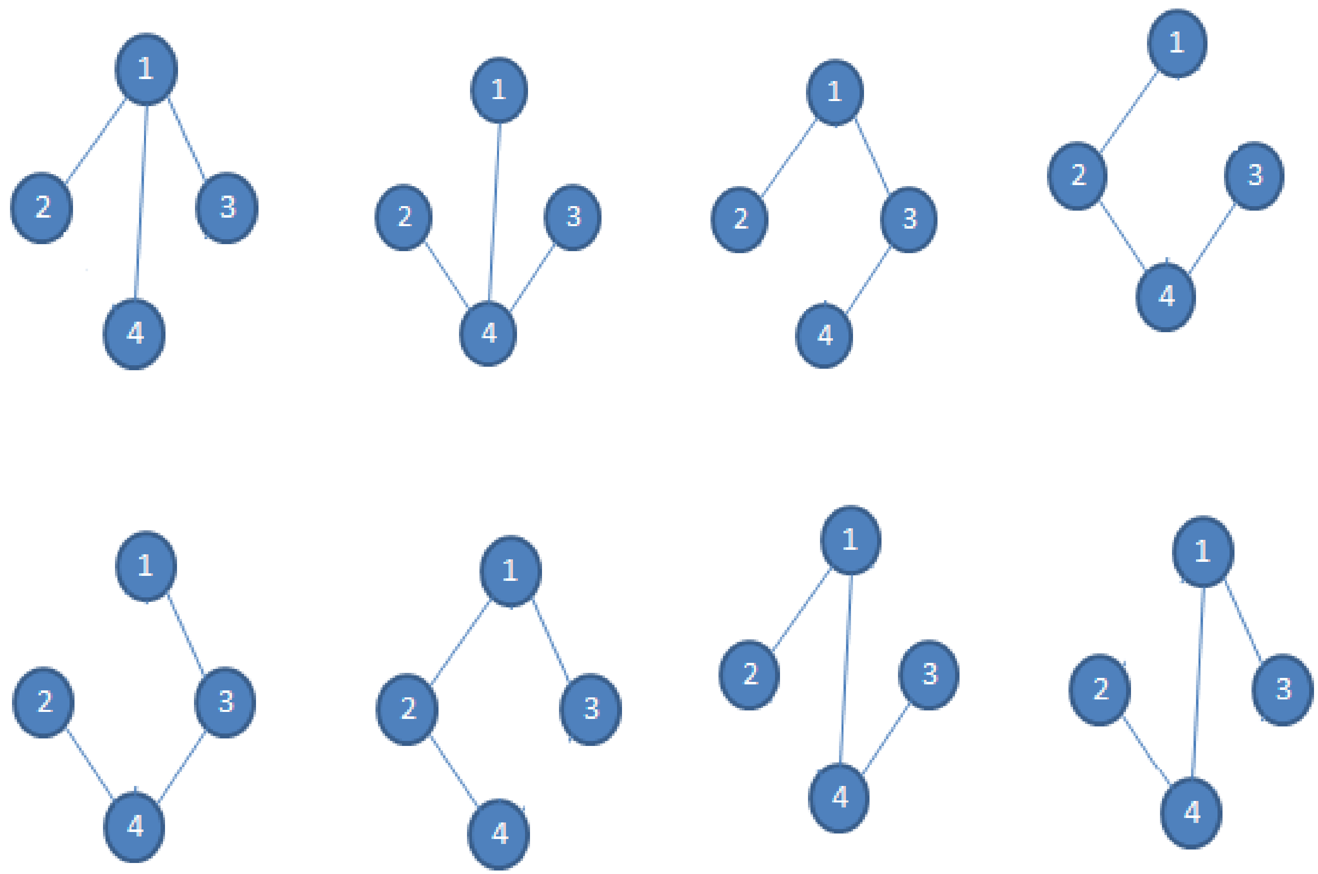
Det(Q') = 2 X [(2X3) – (-1)X(-1)] – 0 + (-1) x [0X(-1) – (2)x(-1)] = 8

Contd..

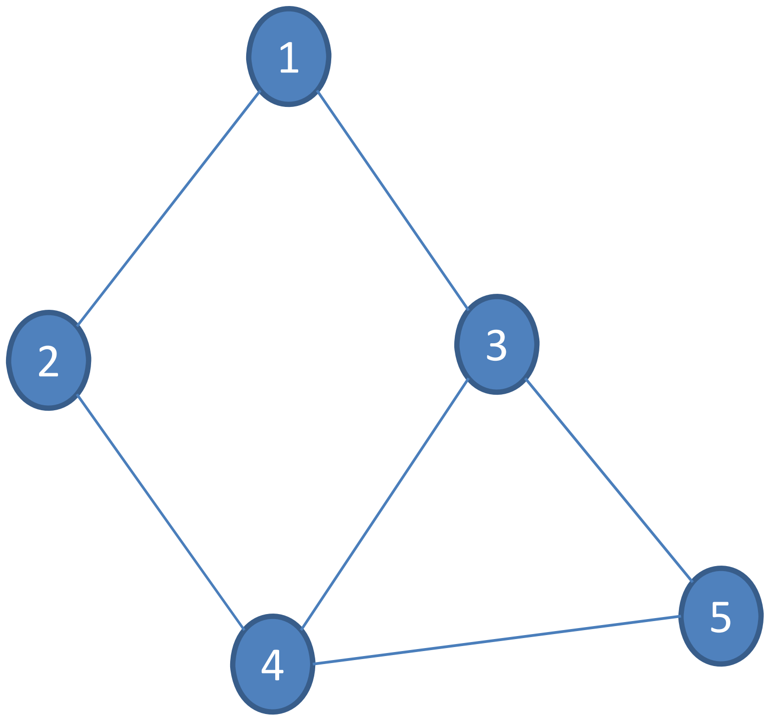


G

Therefore, total no. of Spanning Tree in G is = 8



Solve



Calculate the number of Spanning Tree in the above graph