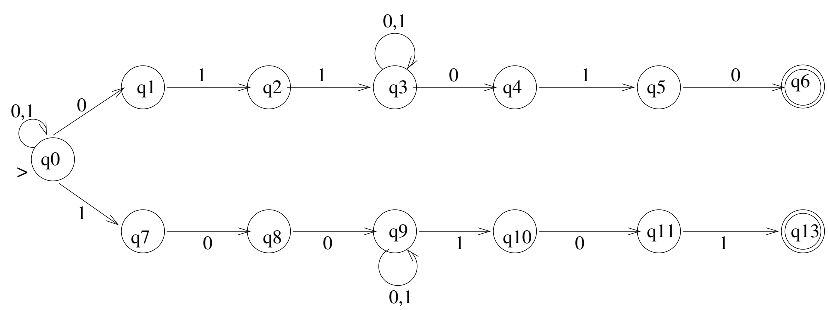
CSE303 Midterm 1 Solutions

1. (**20 Pts**) Design an NFA (non-deterministic finite automata) to accept the set of strings of 0's and 1's that either

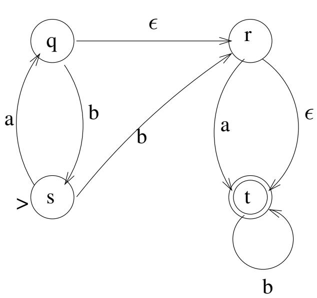
   (a) end in 010 and have 011 somewhere preceding, or

   (b) end in 101 and have 100 somewhere preceding

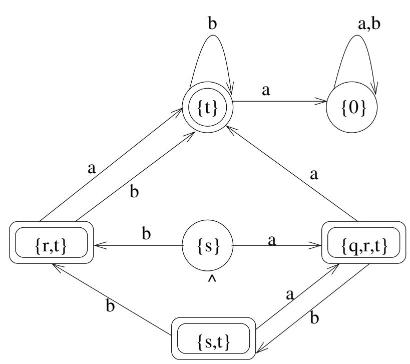
   Solution:



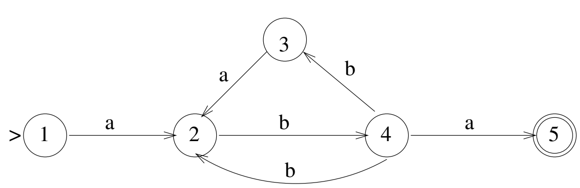
2. (**15 Pts**) Transform the following NFA (note that ϵ stands for null-string) into an equivalent DFA (deterministic finite automata)



   Solution:



3. (**10 Pts**) What is the language accepted by the following finite state automata?



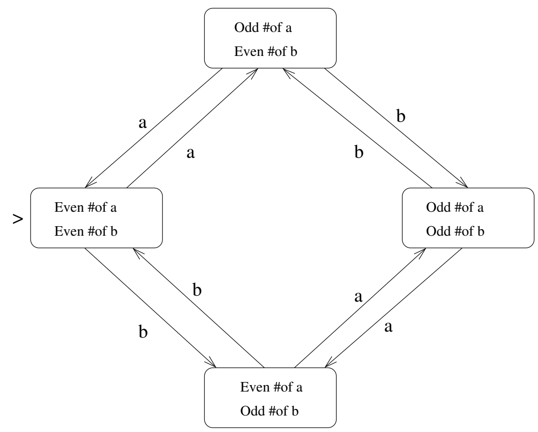
Solution: $a(bb\ +\ bba)^*ba$ or $ab(bb\ +\ bab)^*a$

4. (**15 Pts**) Let $\Sigma = \{a, b\}$. Write regular expression for the language L consisting of all strings in Σ^* with exactly one occurrence of the substring aaa.

   Solution: $(b + ab + aab)^*aaa(baa + ba + b)^*$

5. (**20 Pts**) Construct DFA to accept $L = \{w \in \{a, b\}^* \ \mid w$ has an odd number of a's and an even number of b's$\}$. Your construction must be direct, without using NFA.

   Solution:



6. (**20 Pts**)

   (a) Let Σ be a *finite* alphabet. Prove or disprove that the set Σ^* of all finite length words (using letters from Σ) is countable.

   Solution:

   If Σ is a finite alphabet, the set Σ^* of all words using letters from Σ is countably infinite. Note that Σ is nonempty by definition. Recall that $\Sigma^* = \bigcup_{k=0}^{\infty} \Sigma^k$ where each Σ^k is finite. Thus Σ^* is a countable union of countable sets, and hence Σ^* itself is countable since the union of countably many countable sets is countable (Homework 2, Question 3).

   (b) Let Σ be an *infinite* alphabet. Prove or disprove that the set Σ^* of all finite length words (using letters from Σ) is countable.

   Solution:

   For each $k \in \mathbb{P}$, the set Σ^k of all words of length k is in one-to-one correspondence with the product set $\Sigma^k = \Sigma \times \Sigma \times \ldots \times \Sigma$ (k times). In fact, the correspondence maps each word $a_1a_2\ldots a_k$ to the k-tuple $(a_1, a_2, \ldots, a_k)$. So each set Σ^k is countable by Homework 2, Question 4. The 1-element set $\Sigma^0 = \{\epsilon\}$ is countable too. Hence $\Sigma^* = \bigcup_{k=0}^{\infty} \Sigma^k$ is countable since the union of countably many countable sets is countable (Homework 2, Question 3).