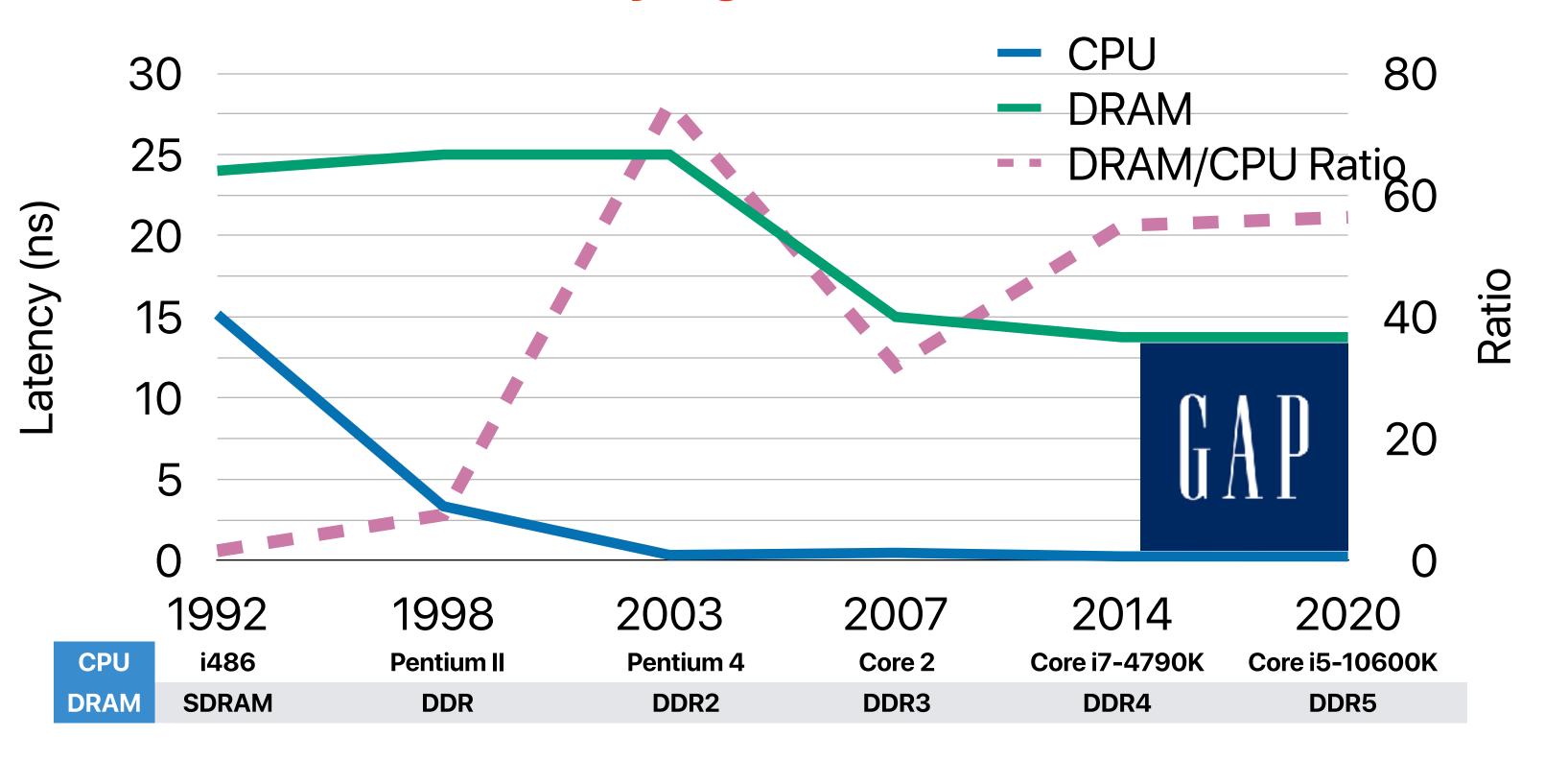
# Memory Hierarchy (4): Cache Misses and How to Address Them (cont.)

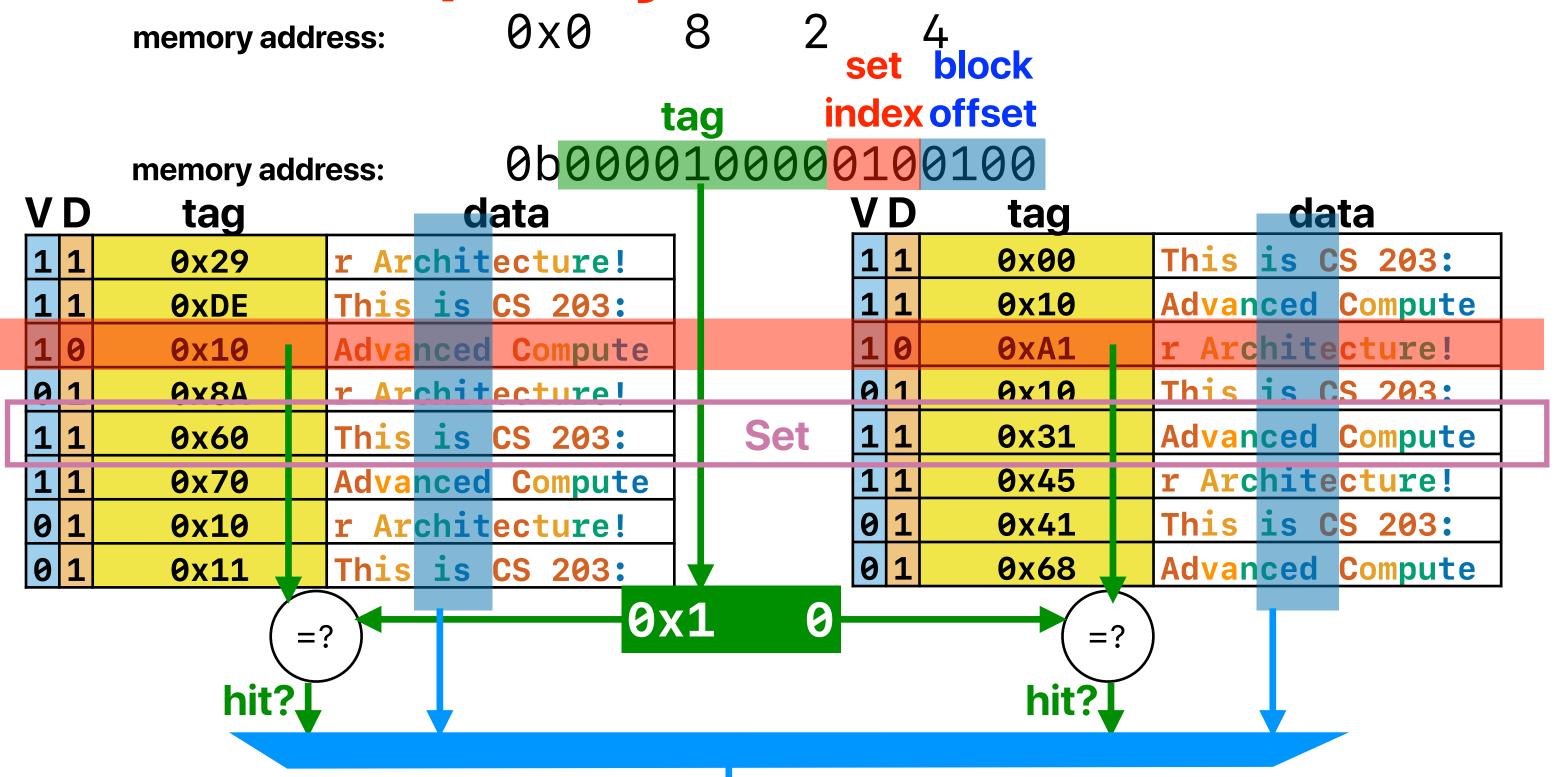
Hung-Wei Tseng

#### Recap: the "latency" gap between CPU and DRAM



Recap: Memory Hierarchy **Processor** fastest 1 **Processor** < 1ns Core fastest L1\$ Registers 32 or L2\$ SRAM\$ a few ns L3\$ larger GBs **DRAM** tens of ns TBs Storage us/ms larger

## Recap: Way-associative cache



#### Recap: Knowing where and why do I have cache misses

- Simulating code's cache behavior
  - Figure out where in the code will access memory
  - Generate the memory addresses that the code will access
  - Partition the addresses using C=ABS
  - Emulate the cache management
- Three types of misses
  - Compulsory misses
  - Conflict misses
  - Capacity misses

## Recap: NVIDIA Tegra X1

100% miss rate!

		11101011				
	Address (Hex)	Address in binary	Tag	Index	Hit? Miss?	Replace?
a[0]	0x10000	0 <mark>b0001000</mark> 00000000000000	8x0	0x0	Compulsory Miss	
b[0]	0x20000	0 <mark>b0010000</mark> 00000000000000	0x10	0x0	Compulsory Miss	
c[0]	0x30000	0 <mark>b0011000</mark> 00000000000000	0x18	0x0	Compulsory Miss	
d[0]	0x40000	0 <mark>b0100000</mark> 000000000000000	0x20	0x0	Compulsory Miss	
e[0]	0x50000	0 <mark>b0101000</mark> 00000000000000	0x28	0x0	Compulsory Miss	a[0-7]
a[1]	0x10008	0 <mark>b0001000</mark> 00000000001000	0x8	0x0	<b>Conflict Miss</b>	b[0-7]
b[1]	0x20008	0b00100000000000001000	0x10	0x0	<b>Conflict Miss</b>	c[0-7]
c[1]	0x30008	0b00110000000000001000	0x18	0x0	<b>Conflict Miss</b>	d[0-7]
d[1]	0x40008	0b01000000000000001000	0x20	0x0	<b>Conflict Miss</b>	e[0-7]
e[1]	0x50008	0b01010000000000001000	0x28	0x0	<b>Conflict Miss</b>	a[0-7]
÷	:	<b>:</b>	:	:	<b>:</b>	:

## Recap: NVIDIA Tegra X1 (cont.)

		tag mack onset				
	Address (Hex)	Address in binary	Tag	Index	Hit? Miss?	Replace?
a[7]	0x10038	0 <mark>b0001000</mark> 0000000111000	8x0	0x0	<b>Conflict Miss</b>	
b[7]	0x20038	0 <mark>b0010000</mark> 00000000111000	0x10	0x0	<b>Conflict Miss</b>	
c[7]	0x30038	0 <mark>b0011000</mark> 0000000111000	0x18	0x0	<b>Conflict Miss</b>	
d[7]	0x40038	0 <mark>b0100000</mark> 0000000111000	0x20	0x0	<b>Conflict Miss</b>	
e[7]	0x50038	0 <mark>b0101000</mark> 0000000 <mark>111000</mark>	0x28	0x0	<b>Conflict Miss</b>	a[0-7]
a[8]	0x10040	0 <mark>b0001000</mark> 0000001000000	0x8	0x1	Compulsory Miss	
b[8]	0x20040	0b0010000000001000000	0x10	0x1	Compulsory Miss	
c[8]	0x30040	0b00110000000001000000	0x18	0x1	Compulsory Miss	
d[8]	0×40040	0b0100000000001000000	0x20	0x1	Compulsory Miss	
e[8]	0x50040	0b01010000000001000000	0x28	0x1	Compulsory Miss	a[8-15]
:	<u>:</u>	:	:	:	<u>:</u>	:

## Recap: NVIDIA Tegra X1 — Loop Fission

C = ABS

	Address (Hex)	Address in binary	Tag	Index	Hit? Miss?	Replace?
a[0]	0×10000	0 <mark>b0001000</mark> 00000000000000	8x0	0x0	Compulsory Miss	
b[0]	0x20000	0 <mark>b0010000</mark> 00000000000000	0x10	0x0	Compulsory Miss	
c[0]	0x30000	0 <mark>b0011000</mark> 00000000000000	0x18	0x0	Compulsory Miss	
e[0]	0x50000	0 <mark>b0101000</mark> 00000000000000	0x28	0x0	Compulsory Miss	
a[1]	0x10008	0 <mark>b0001000</mark> 000000000001000	0x8	0×0	Hit	
b[1]	0x20008	0 <mark>b0010000</mark> 00000000001000	0x10	0×0	Hit	
c[1]	0x30008	0b00110000000000001000	0x18	0×0	Hit	
e[1]	0x50008	0b01010000000000001000	0x28	0x0	Hit	

## Recap: NVIDIA Tegra X1 — Loop Fission

• Size 32KB, 4-way set associativity, 64B block, LRU policy, write-allocate, write-back, and assuming 64-bit address.

	Address (Hex)	Address in binary	Tag	Index	Hit? Miss?	Replace?
a[7]	0x10038	0 <mark>b0001000</mark> 0000000111000	8x0	0x0	Hit	
b[7]	0x20038	0 <mark>b0010000</mark> 0000000111000	0x10	0x0	Hit	
c[7]	0x30038	0b00110000000000111000	0x18	0x0	Hit	
e[7]	0x50038	01010000000000111000	0x28	0x0	Hit	
a[8]	0×10040	0 <mark>b0001000</mark> 0000001000000	0x8	0x1	Compulsory Miss	
b[8]	0x20040	0 <mark>b0010000</mark> 000001000000	0x10	0x1	Compulsory Miss	
c[8]	0x30040	0b0011000000001000000	0x18	0x1	Compulsory Miss	
e[8]	0x50040	0b01010000000001000000	0x28	0x1	Compulsory Miss	

$$\frac{512}{8} = 64$$

We need only 4 blocks in 64 sets to accommodate everything

## Recap: NVIDIA Tegra X1 — Loop Fission (cont.)

C = ABS

	Address (Hex)	Address in binary	Tag	Index	Hit? Miss?	Replace?
e[0]	0x50000	0 <mark>b0101000</mark> 00000000000000	0x28	0x0	Hit	
d[0]	0×40000	0 <mark>b0010000</mark> 000000000000000	0x20	0x0	Compulsory Miss	
e[0]	0x50000	0 <mark>b0101000</mark> 00000000000000	0x28	0×0	Hit	
e[1]	0x50008	0 <mark>b0101000</mark> 00000000001000	0x28	0x0	Hit	
d[1]	0x40008	0 <mark>b0001000</mark> 0000000001000	0x20	0x0	Hit	
e[1]	0x50008	0 <mark>b0101000</mark> 00000000001000	0x28	0x0	Hit	

	:	<u>-</u>	<u>-</u>	-	-			Ξ
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	•	<b>:</b>	<b>;</b>	:	:			:
	•	•	•	•	•	•		•
_								
		- $        -$	01-04-04-0000000004-00004	20	00 0	- 4	11 <u>-</u> ±	
е	[8] 0	x50040	0b0101000000000100000	90 0x	28 0×	(1	Hit	
е	[8] 0	x50040	0b0101000000000100000	90 0x	28 0×	(1	Hit	
		x50040 x40040	0b0101000000000100000 0b0100000000000100000				<b>Hit</b> ulsory Miss	
d	[8] 0	x40040	0b010000000000100000	90 0x	20 0x		ulsory Miss	
d	[8] 0	x40040	0b010000000000100000	90 0x	20 0x	(1 Compu	ulsory Miss	
d	[8] 0			90 0x	20 0x	(1 Compu		

## Details of Loop Fission on Tegra X1

```
double a[16384], b[16384], c[16384];
/* c = 0x10000, a = 0x20000, b = 0x30000 */
for(i = 0; i < 512; i++)
    e[i] = a[i] * b[i] + c[i];
//load a, b, c and then store to e
for(i = 0; i < 512; i++)
    e[i] /= d[i];
//load e, load d, and then store to e</pre>
```

Misses	Memory Accesses	Miss Rate
$4 \times \frac{512}{8}$	4 × 512	
<u>512</u> <u>8</u>	3 × 512	
$5 \times \frac{512}{9}$	7 × 512	0.0893

## **Loop optimizations**

```
for(i = 0; i < ARRAY_SIZE; i++)
{
  for(j = 0; j < ARRAY_SIZE; j++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}</pre>
```

## Loop interchange

```
for(j = 0; j < ARRAY_SIZE; j++)
{
  for(i = 0; i < ARRAY_SIZE; i++)
    {
    c[i][j] = a[i][j]+b[i][j];
  }
}</pre>
```

```
m
```

\_oop fission



## **Takeaways: Software Optimizations**

- Data layout capacity miss, conflict miss, compulsory miss
- Loop interchange conflict/capacity miss
- Loop fission conflict miss when \$ has limited way associativity

#### **Outline**

- Writing cache-friendly code
- Virtual memory
- Sample Midterm

# How can programmer improve memory performance? (cont.)

## Data structures



## What if we change the processor?

- If we have an intel processor with a 48KB, 12-way, 64Bblocked L1 cache, which version of code performs better?
  - A. Version A, because the code incurs fewer cache misses
  - B. Version B, because the code incurs fewer cache misses
  - C. Version A, because the code incurs fewer memory references
  - D. Version B, because the code incurs fewer memory references
  - E. They are about the same



## What if we change the processor?

- If we have an intel processor with a 32KB, 8-way, 64B-blocked L1 cache, which version of code performs better?
  - A. Version A, because the code incurs fewer cache misses
  - B. Version B, because the code incurs fewer cache misses
  - C. Version A, because the code incurs fewer memory references
  - D. Version B, because the code incurs fewer memory references
  - E. They are about the same

```
m
```

## **Loop optimizations**

```
for(i = 0; i < ARRAY_SIZE; i++)
{
  for(j = 0; j < ARRAY_SIZE; j++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}</pre>
```

# for(j = 0; j < ARRAY\_SIZE; j++ LOOP interchange

```
for(j = 0; j < ARRAY_SIZE; j++)
{
  for(i = 0; i < ARRAY_SIZE; i++)
    {
     c[i][j] = a[i][j]+b[i][j];
  }
}</pre>
```

m

Loop fission



4

Loop fusion

## **Takeaways: Software Optimizations**

- Data layout capacity miss, conflict miss, compulsory miss
- Loop interchange conflict/capacity miss
- Loop fission conflict miss when \$ has limited way associativity
- Loop fusion capacity miss when \$ has enough way associativity

# Tiling/Blocking Algorithm

## What is an M by N "2-D" array in C?

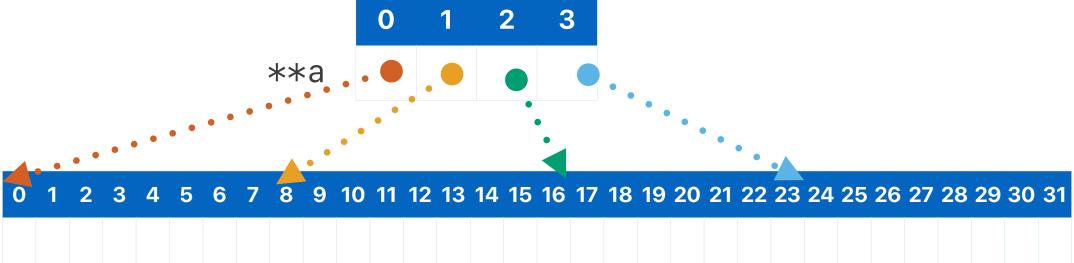
```
a = (double **)malloc(M*sizeof(double *));
for(i = 0; i < N; i++)
{
   a[i] = (double *)malloc(N*sizeof(double));
}</pre>
```

## a[i][j] is essentially a[i\*N+j]

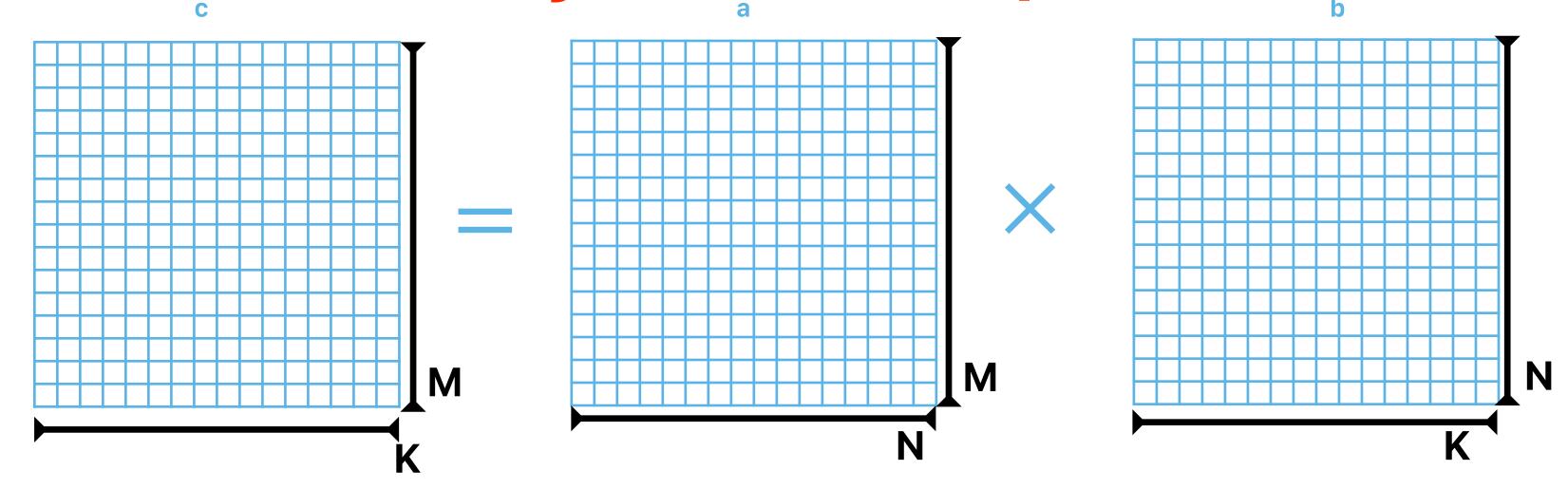
#### abstraction



#### physical implementation



## Case Study: Matrix Multiplications



```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
}</pre>
```

Algorithm class tells you it's O(n<sup>3</sup>)

If M=N=K=1024, it takes about 2 sec

How long is it take when M=N=K=2048?



#### What kind(s) of misses are there in Matrix Multiplications

• Considering the case where M=N=K=2048, what do you think the majority type(s) of cache misses are we seeing?

```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
}</pre>
```

- A. Compulsory miss
- B. Capacity miss
- C. Conflict miss
- D. Capacity & conflict miss
- E. Compulsory & conflict miss



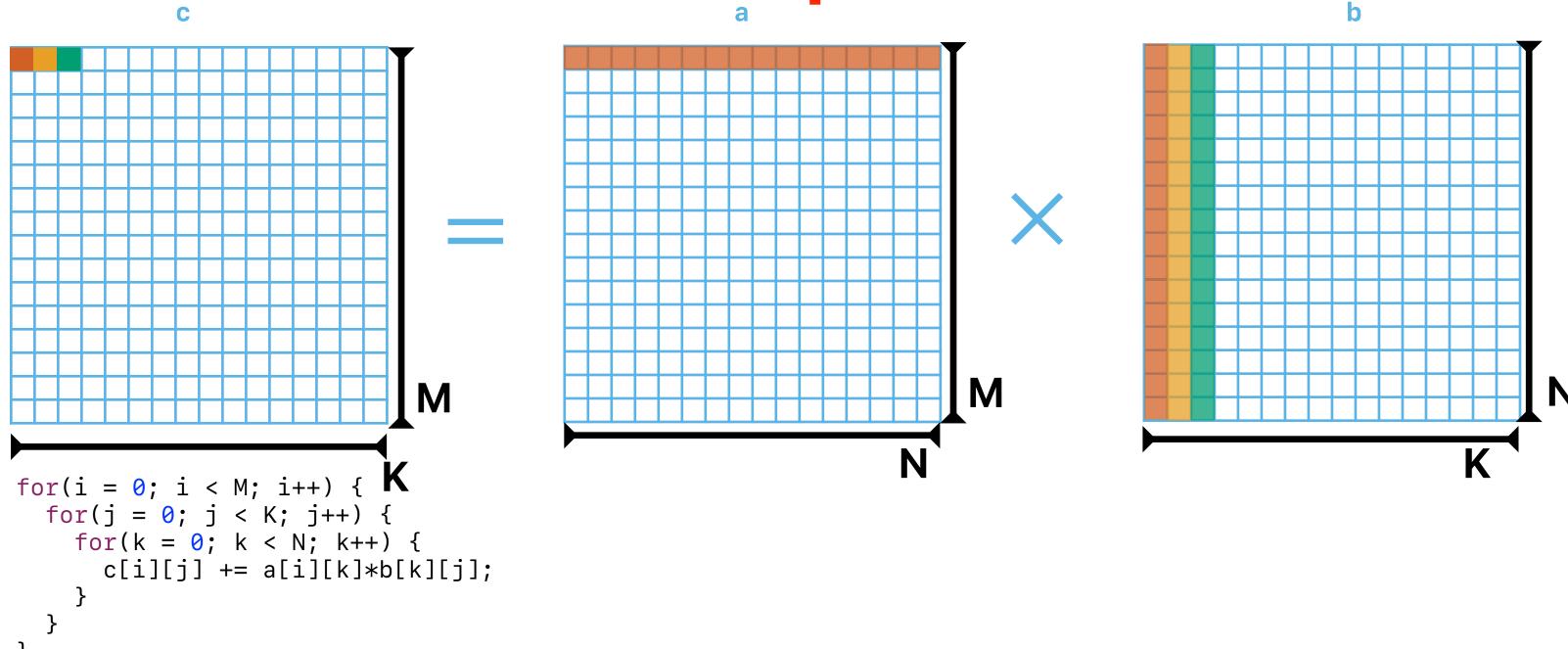
#### What kind(s) of misses are there in Matrix Multiplications

• Considering the case where M=N=K=2048, what do you think the majority type(s) of cache misses are we seeing?

```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
}</pre>
```

- A. Compulsory miss
- B. Capacity miss
- C. Conflict miss
- D. Capacity & conflict miss
- E. Compulsory & conflict miss

## **Matrix Multiplications**



## Matrix Multiplication — let's consider "b"

```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
}</pre>
```

	Address	Tag	Index
b[0][0]	0x20000	0x20	0x0
b[1][0]	0x24000	0x24	0x0
b[2][0]	0x28000	0x28	0x0
b[3][0]	0x2C000	0x2C	0x0
b[4][0]	0x30000	0x30	0x0
b[5][0]	0x34000	0x34	0x0
b[6][0]	0x38000	0x38	0x0
b[7][0]	0x3C000	0x3C	0x0
b[8][0]	0x40000	0x40	0x0
b[9][0]	0x44000	0x44	0x0
b[10][0]	0x48000	0x48	0x0
b[11][0]	0x4C000	0x4C	0x0
b[12][0]	0x50000	0x50	0x0
b[13][0]	0x54000	0x54	0x0
b[14][0]	0x58000	0x58	0x0
b[15][0]	0x5C000	0x5C	0x0
b[16][0]	0x60000	0x60	0×0

 If the row dimension (N) of your matrix is 2048, each row element with the same column index is

$$2048 \times 8 = 16384 = 0x4000$$
 away from each other

Each set can store only 12 blocks! So we will start to kick out b[0][0-7], b[1][0-7] ...

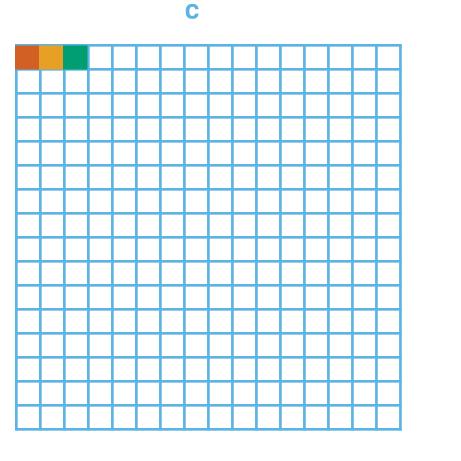
## Now, when we work on c[0][1]

	Address	Tag	Index
b[0][0]	0×20000	0x20	0x0
b[1][0]	0x24000	0x24	0x0
b[2][0]	0x28000	0x28	0x0
b[3][0]	0x2C000	0x2C	0x0
b[4][0]	0x30000	0x30	0x0
b[5][0]	0x34000	0x34	0x0
b[6][0]	0x38000	0x38	0x0
b[7][0]	0x3C000	0x3C	0x0
b[8][0]	0x40000	0x40	0x0
b[9][0]	0x44000	0x44	0x0
b[10][0]	0x48000	0x48	0x0
b[11][0]	0x4C000	0x4C	0x0
b[12][0]	0x50000	0x50	0x0
b[13][0]	0x54000	0x54	0x0
b[14][0]	0x58000	0x58	0x0
b[15][0]	0x5C000	0x5C	0x0
b[16][0]	0x60000	0x60	0x0

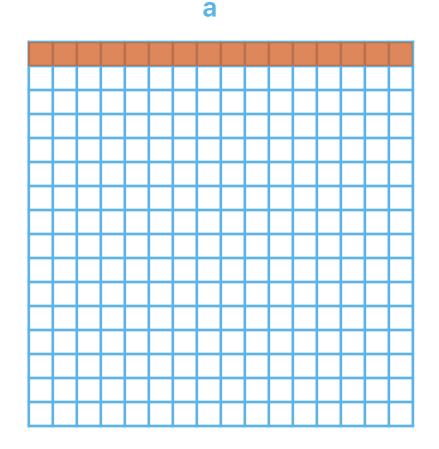
	Address	Tag	Index	
b[0][1]	0x20008	0x20	0x0	Conflict Miss
b[1][1]	0x24008	0x24	0x0	Conflict Miss
b[2][1]	0x28008	0x28	0x0	Conflict Miss
b[3][1]	0x2C008	0x2C	0x0	Conflict Miss
b[4][1]	0x30008	0x30	0×0	Conflict Miss
b[5][1]	0x34008	0x34	0x0	Conflict Miss
b[6][1]	0x38008	0x38	0×0	Conflict Miss
b[7][1]	0x3C008	0x3C	0x0	Conflict Miss
b[8][1]	0x40008	0×40	0×0	Conflict Miss
b[9][1]	0x44008	0x44	0x0	Conflict Miss
b[10][1]	0x48008	0x48	0×0	Conflict Miss
b[11][1]	0x4C008	0x4C	0x0	Conflict Miss
b[12][1]	0x50008	0x50	0×0	Conflict Miss
b[13][1]	0x54008	0x54	0x0	Conflict Miss
b[14][1]	0x58008	0x58	0×0	Conflict Miss
b[15][1]	0x5C008	0x5C	0x0	Conflict Miss
b[16][1]	0x60008	0x60	0x0	Conflict Miss

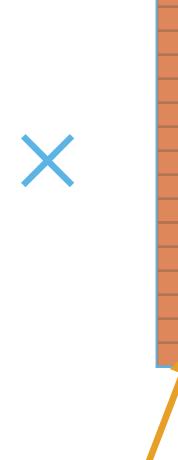
Each set can store only 12 blocks! So we will start to kick out b[0][0-7], b[1][0-7] ...

## **Matrix Multiplications**



```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
}</pre>
```





These are conflict misses as we have cached them before

These are These are conflict misses as we have as we have before

b

## **Matrix Multiplications**

C



Unlikely to be kept in the cache

b

- If each dimension of your matrix is 2048
  - Each row or column takes  $2048 \times 8$  Bytes = 16 KB
  - The L1-\$ of intel Core i7 is 48 KB, 12-way, 64-byte blocked
  - You can only hold at most 3 rows or columns of each matrix!
  - You need the more columns when j increase!
- We will have capacity misses when we work on a new rapacity miss!

#### What kind(s) of misses are there in Matrix Multiplications

• Considering the case where M=N=K=2048, what do you think the majority type(s) of cache misses are we seeing?

```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
}</pre>
```

- A. Compulsory miss
- B. Capacity miss
- C. Conflict miss
- D. Capacity & conflict miss
- E. Compulsory & conflict miss

#### Ideas regarding reducing misses in matrix multiplications

- Reducing conflict misses we need to reduce the length of a column that we visit within a period of time
- Reducing capacity misses we need to reduce the length of a row that we visit within a period of time

#### Mathematical view of MM

$$c_{i,j} = \sum_{k=0}^{k=N-1} a_{i,k} \times b_{k,j} = \sum_{k=0}^{k=\frac{N}{2}-1} a_{i,k} \times b_{k,j} + \sum_{k=\frac{N}{2}}^{k=N-1} a_{i,k} \times b_{k,j}$$

$$= \sum_{k=0}^{k=\frac{N}{4}-1} a_{i,k} \times b_{k,j} + \sum_{k=\frac{N}{4}}^{k=\frac{N}{2}-1} a_{i,k} \times b_{k,j} + \sum_{k=\frac{N}{2}}^{k=\frac{N}{4}-1} a_{i,k} \times b_{k,j} + \sum_{k=3N4-1}^{k=N-1} a_{i,k} \times b_{k,j}$$

$$\vdots$$

Let's break up the multiplications and accumulations into something fits in the cache well

## Matrix Multiplication — let's consider "b"

```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
```

0x34000

0x38000

0x3C000

0x40000

0x44000

0x48000

0x4C000

0x50000

0x54000

0x58000

0x5C000

0x60000

b[5][0]

b[6][0]

b[7][0]

b[8][0]

b[9][0]

b[10][0]

b[11][0]

b[12][0]

b[13][0]

b[14][0]

b[15][0]

b[16][0]

	Address	Tag	Index
b[0][0]	0x20000	0x20	0x0
b[1][0]	0x24000	0x24	0x0
b[2][0]	0x28000	0x28	0x0
b[3][0]	0x2C000	0x2C	0x0
b[4][0]	0x30000	0x30	0x0_

0x34

0x38

0x3C

0x40

0x44

0x48

0x4C

0x50

0x54

0x58

0x5C

0x60

0x0

 If the row dimension of your matrix is 2048, each row element with the same column index is

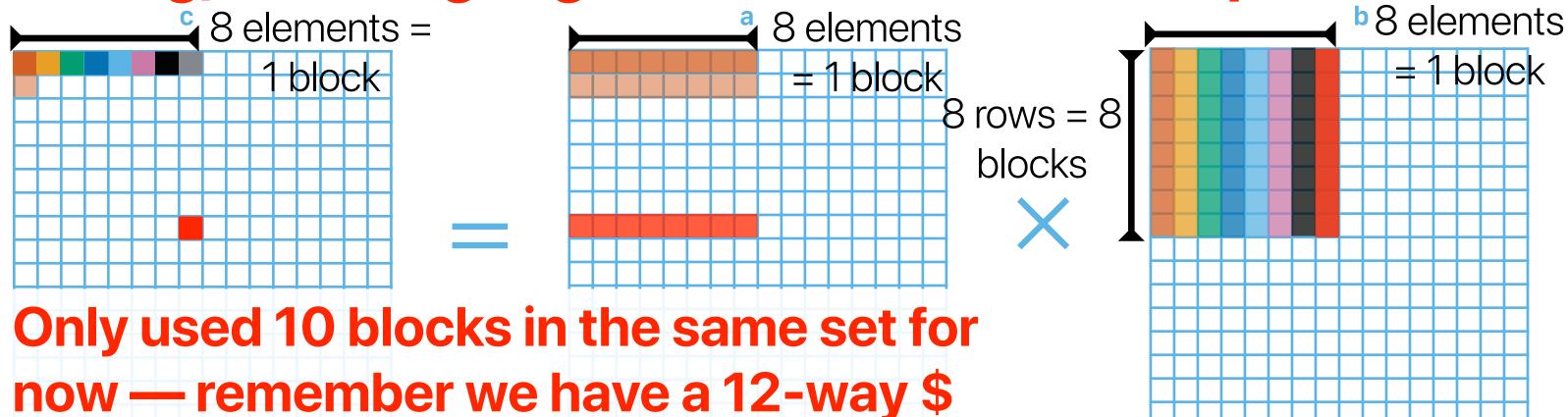
$$2048 \times 8 = 16384 = 0x4000$$

If we stop at somewhere before 12 blocks, we should be fine!

Since each block has 8 elements, let's break up in 8 for now

- -8 elements from a[i]
- **—8 columns each covers 8 rows**

## Tiling/Blocking Algorithm for Matrix Multiplications

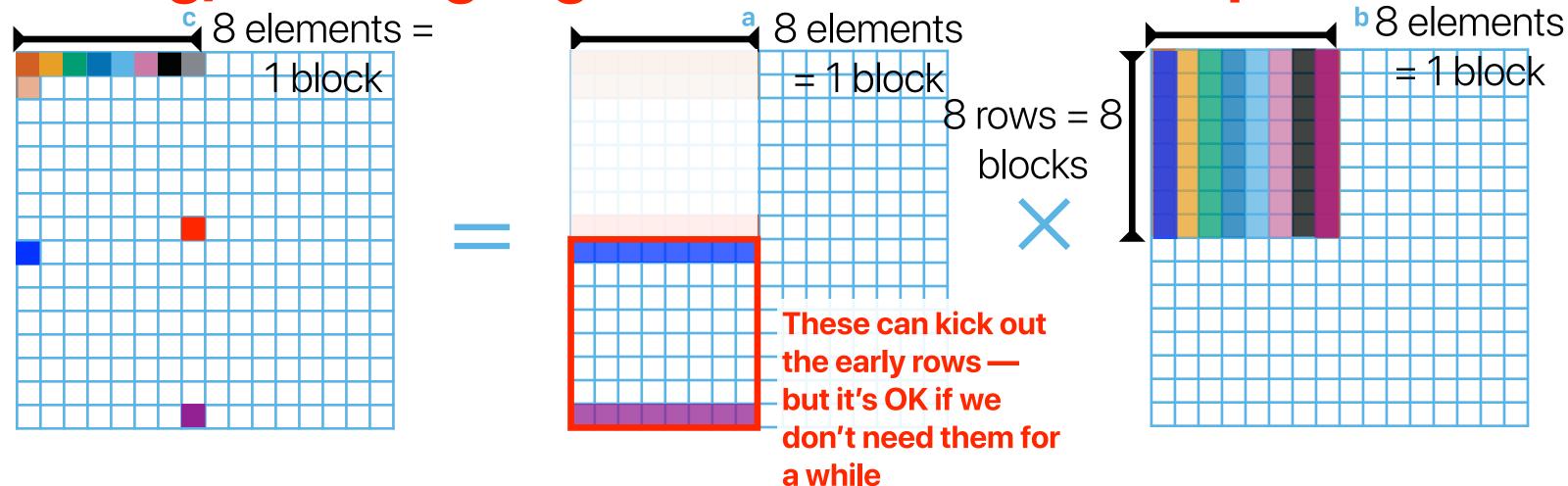


#### Only compulsory misses —

$$miss\_rate = \frac{total\ misses}{total\ accesses} = \frac{8+8}{3\times8\times8} = 0.083$$

These are still around when we move to the next row in the "tile"

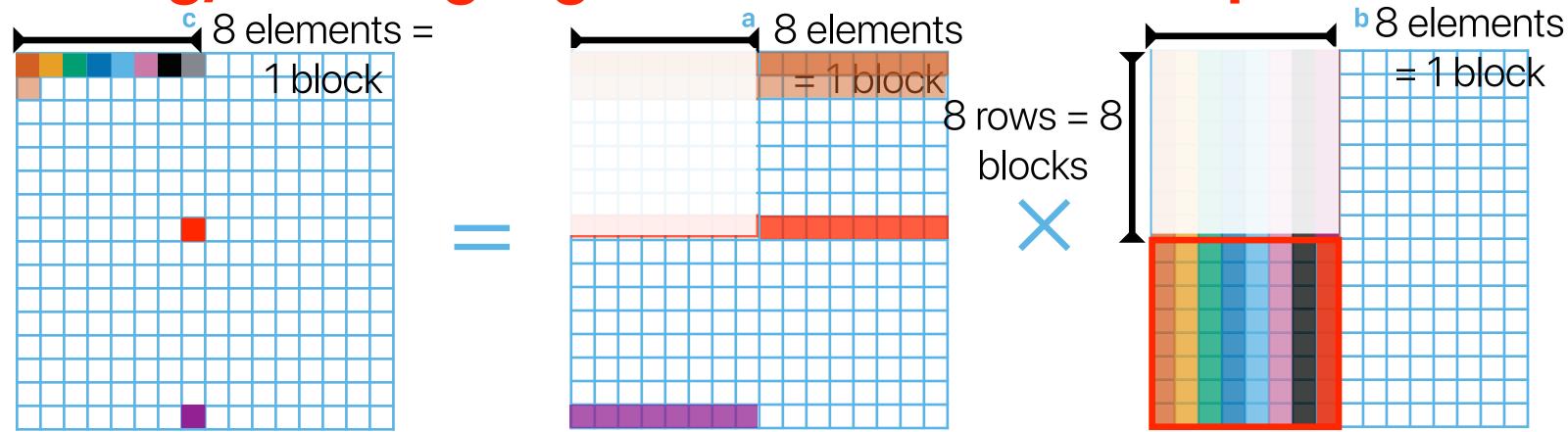
### Tiling/Blocking Algorithm for Matrix Multiplications



# Bringing miss rate even further lower now —

$$miss\_rate = \frac{total\ misses}{total\ accesses} = \frac{8+8+8}{3\times8\times8+3\times8\times8} = 0.042$$

#### Tiling/Blocking Algorithm for Matrix Multiplications



```
for(i = 0; i < M; i+=tile\_size) \\ for(j = 0; j < K; j+=tile\_size) \\ for(k = 0; k < N; k+=tile\_size) \\ for(ii = i; ii < i+tile\_size; ii++) \\ for(jj = j; jj < j+tile\_size; jj++) \\ for(kk = k; kk < k+tile\_size; kk++) \\ c[ii][jj] += a[ii][kk]*b[kk][jj];
```

# Why is "8" not the best performing?

size	tile_size	IC	Cycles	СРІ	CT_ns	ET_s	DL1_miss_rat e	DL1_accesses
2048	4	97765686275	24149510064	0.247014	0.193189	4.665430	0.015102	43641501149
2048	8	80996985555	21043742544	0.259809	0.193444	4.070776	0.010135	38128531445
2048	16	74473114435	19369857501	0.260092	0.193204	3.742332	0.071790	36105122733
2048	32	71543334296	27812871208	0.388756	0.193112	5.371009	0.217011	35214370198

More instructions due to more loop control overhead!

"8" indeed has the best miss rate



#### What kind(s) of misses can tiling algorithm remove?

 Comparing the naive algorithm and tiling algorithm on matrix multiplication, what kind of misses does tiling algorithm help to remove? (assuming an intel Core i7)

```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
}</pre>
```

- A. Compulsory miss
- B. Capacity miss
- C. Conflict miss
- D. Capacity & conflict miss
- E. Compulsory & conflict miss

# Block/tile

```
for(i = 0; i < M; i+=tile_size) {
  for(j = 0; j < K; j+=tile_size) {
    for(k = 0; k < N; k+=tile_size) {
     for(ii = i; ii < i+tile_size; ii++)
        for(jj = j; jj < j+tile_size; jj++)
        for(kk = k; kk < k+tile_size; kk++)
        c[ii][jj] += a[ii][kk]*b[kk][jj];
  }
}</pre>
```



#### What kind(s) of misses can tiling algorithm remove?

 Comparing the naive algorithm and tiling algorithm on matrix multiplication, what kind of misses does tiling algorithm help to remove? (assuming an intel Core i7)

```
for(i = 0; i < M; i++) {
  for(j = 0; j < K; j++) {
    for(k = 0; k < N; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
}</pre>
```

- A. Compulsory miss
- B. Capacity miss
- C. Conflict miss
- D. Capacity & conflict miss
- E. Compulsory & conflict miss

# Block/tile

```
for(i = 0; i < M; i+=tile_size) {
  for(j = 0; j < K; j+=tile_size) {
    for(k = 0; k < N; k+=tile_size) {
     for(ii = i; ii < i+tile_size; ii++)
        for(jj = j; jj < j+tile_size; jj++)
        for(kk = k; kk < k+tile_size; kk++)
        c[ii][jj] += a[ii][kk]*b[kk][jj];
  }
}</pre>
```

# **Takeaways: Software Optimizations**

- Data layout capacity miss, conflict miss, compulsory miss
- Loop interchange conflict/capacity miss
- Loop fission conflict miss when \$ has limited way associativity
- Loop fusion capacity miss when \$ has enough way associativity
- Blocking/tiling capacity miss, conflict miss

# **Matrix Transpose**

```
for(i = 0; i < M; i+=tile_size) {
  for(j = 0; j < K; j+=tile_size) {
    for(k = 0; k < N; k+=tile_size) {
     for(ii = i; ii < i+tile_size; ii++)
        for(jj = j; jj < j+tile_size; jj++)
        for(kk = k; kk < k+tile_size; kk++)
        c[ii][jj] += a[ii][kk]*b[kk][jj];
    }
}</pre>
```

```
// Transpose matrix b into b_t
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {</pre>
  for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {</pre>
      b_t[i][j] += b[j][i];
  for(i = 0; i < M; i+=tile_size) {</pre>
     for(j = 0; j < K; j+=tile_size) {</pre>
       for(k = 0; k < N; k+=tile_size) {</pre>
         for(ii = i; ii < i+tile_size; ii++)</pre>
           for(jj = j; jj < j+tile_size; jj++)</pre>
              for(kk = k; kk < k+tile_size; kk++)</pre>
                // Compute on b_t
                c[ii][jj] += a[ii][kk]*b_t[jj][kk];
```

# The effect of transposition

size	tile_size	IC	Cycles	СРІ	CT_ns	ET_s	DL1_miss_rat e	DL1_accesses
2048	4	97765686275	24149510064	0.247014	0.193189	4.665430	0.015102	43641501149
2048	8	80996985555	21043742544	0.259809	0.193444	4.070776	0.010135	38128531445
2048	16	74473114435	19369857501	0.260092	0.193204	3.742332	0.071790	36105122733
2048	32	71543334296	27812871208	0.388756	0.193112	5.371009	0.217011	35214370198
2048	16	64810073062	15221864848	0.234869	0.193117	2.939604	0.024597	27045326530

Transpose improves the miss rate!



#### What kind(s) of misses can matrix transpose remove?

• By transposing a matrix, the performance of matrix multiplication can be further improved. What kind(s) of cache misses does matrix transpose help to remove?

```
for(i = 0; i < M; i+=tile_size) {
  for(j = 0; j < K; j+=tile_size) {
    for(k = 0; k < N; k+=tile_size) {
     for(ii = i; ii < i+tile_size; ii++)
        for(jj = j; jj < j+tile_size; jj++)
        for(kk = k; kk < k+tile_size; kk++)
        c[ii][jj] += a[ii][kk]*b[kk][jj];
  }
}</pre>
```

- A. Compulsory miss
- B. Capacity miss
- C. Conflict miss
- D. Capacity & conflict miss
- E. Compulsory & conflict miss

```
// Transpose matrix b into b_t
for(i = 0; i < ARRAY_SIZE; i++) {</pre>
  for(j = 0; j < ARRAY_SIZE; j++) {</pre>
       b_t[i][j] += b[j][i];
  for(i = 0; i < M; i+=tile_size) {</pre>
    for(j = 0; j < K; j+=tile_size) {</pre>
       for(k = 0; k < N; k+=tile_size) {</pre>
         for(ii = i; ii < i+tile_size; ii++)</pre>
           for(jj = j; jj < j+tile_size; jj++)</pre>
             for(kk = k; kk < k+tile_size; kk++)</pre>
                // Compute on b_t
                c[ii][jj] += a[ii][kk]*b_t[jj][kk];
```

#### What kind(s) of misses can matrix transpose remove?

• By transposing a matrix, the performance of matrix multiplication can be further improved. What kind(s) of cache misses does matrix transpose help to remove?

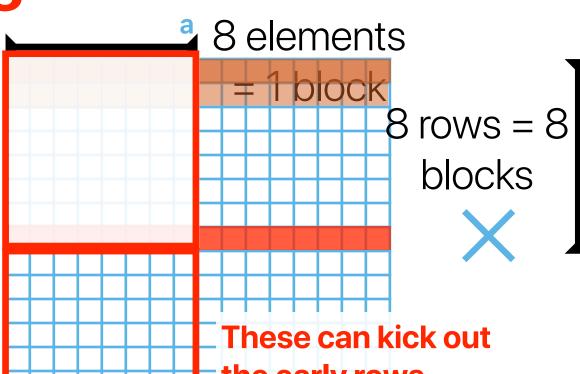
```
for(i = 0; i < M; i+=tile_size) {
  for(j = 0; j < K; j+=tile_size) {
    for(k = 0; k < N; k+=tile_size) {
     for(ii = i; ii < i+tile_size; ii++)
        for(jj = j; jj < j+tile_size; jj++)
        for(kk = k; kk < k+tile_size; kk++)
        c[ii][jj] += a[ii][kk]*b[kk][jj];
  }
}</pre>
```

- A. Compulsory miss
- B. Capacity miss
- C. Conflict miss
- D. Capacity & conflict miss
- E. Compulsory & conflict miss

#### // Transpose matrix b into b\_t for(i = 0; i < ARRAY\_SIZE; i++) {</pre> for(j = 0; j < ARRAY\_SIZE; j++) {</pre> $b_t[i][j] += b[j][i];$ for(i = 0; i < M; i+=tile\_size) {</pre> for(j = 0; j < K; j+=tile\_size) {</pre> for(k = 0; k < N; k+=tile\_size) {</pre> for(ii = i; ii < i+tile\_size; ii++)</pre> for(jj = j; jj < j+tile\_size; jj++)</pre> Block for(kk = k; kk < k+tile\_size; kk++)</pre> // Compute on b\_t c[ii][jj] += a[ii][kk]\*b\_t[jj][kk];

#### Tiling/Blocking Algorithm for Matrix Multiplications



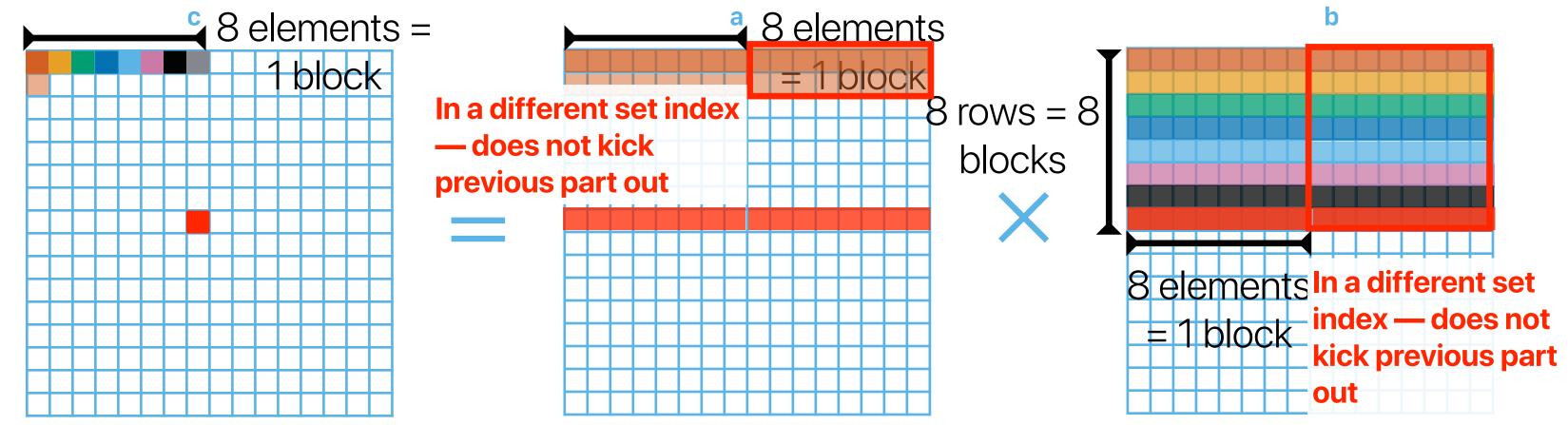


These can kick out
the early rows —
but it's OK if we
don't need them for
a while



**b**8 elements

#### Tiling/Blocking Algorithm for Transposed Matrix Multiplications



We can make the "tile\_size" larger without interfacing

#### What kind(s) of misses can matrix transpose remove?

• By transposing a matrix, the performance of matrix multiplication can be further improved. What kind(s) of cache misses does matrix transpose help to remove?

```
for(i = 0; i < M; i+=tile_size) {
  for(j = 0; j < K; j+=tile_size) {
    for(k = 0; k < N; k+=tile_size) {
     for(ii = i; ii < i+tile_size; ii++)
        for(jj = j; jj < j+tile_size; jj++)
        for(kk = k; kk < k+tile_size; kk++)
        c[ii][jj] += a[ii][kk]*b[kk][jj];
  }
}</pre>
```

- A. Compulsory miss
- B. Capacity miss
- C. Conflict miss
- D. Capacity & conflict miss
- E. Compulsory & conflict miss

#### // Transpose matrix b into b\_t for(i = 0; i < ARRAY\_SIZE; i++) {</pre> for(j = 0; j < ARRAY\_SIZE; j++) {</pre> b\_t[i][j] += b[j][i]; for(i = 0; i < M; i+=tile\_size) {</pre> for(j = 0; j < K; j+=tile\_size) {</pre> for(k = 0; k < N; k+=tile\_size) {</pre> for(ii = i; ii < i+tile\_size; ii++)</pre> for(jj = j; jj < j+tile\_size; jj++)</pre> Block for(kk = k; kk < k+tile\_size; kk++)</pre> // Compute on b\_t c[ii][jj] += a[ii][kk]\*b\_t[jj][kk];

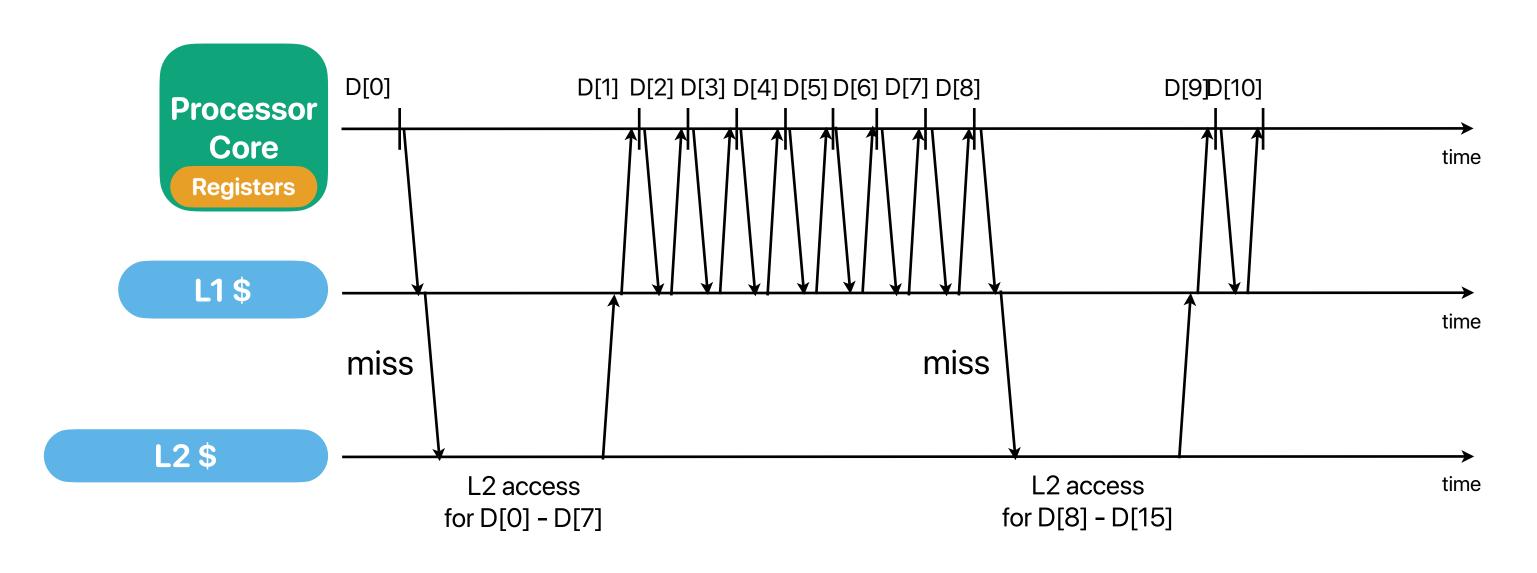
# **Takeaways: Software Optimizations**

- Data layout capacity miss, conflict miss, compulsory miss
- Loop interchange conflict/capacity miss
- Loop fission conflict miss when \$ has limited way associativity
- Loop fusion capacity miss when \$ has enough way associativity
- Blocking/tiling capacity miss, conflict miss
- Matrix transpose (a technique changes layout) conflict misses

# Prefetching

### Characteristic of memory accesses

```
for(i = 0;i < 1000000; i++) {
    D[i] = rand();
}</pre>
```



# **Prefetching**

```
for(i = 0; i < 1000000; i++) {
             D[i] = rand();
             // prefetch D[i+8] if i % 8 == 0
            D[0]
                            D[1] D[2] D[3] D[4] D[5] D[6] D[7] D[8] D[9] D[10] [11] D[12] D[13] D[14] D[15] D[16]
Processor
  Core
                                                                                         time
 Registers
                                                           prefetc
                   prefetch
                                                                                     prefetch
 L1$
                                                                                         time
            miss
                                 miss
                                                            miss
                                 L2 access
                                                              L2 access
                    L2 access
                                                                                         time
```

for D[16] - D[23]

for D[8] - D[15]

for D[0] - D[7]

L2\$

# Prefetching

- Identify the access pattern and proactively fetch data/ instruction before the application asks for the data/instruction
  - Trigger the cache miss earlier to eliminate the miss when the application needs the data/instruction
- Hardware prefetch
  - The processor can keep track the distance between misses. If there
    is a pattern, fetch miss\_data\_address+distance for a miss
- Software prefetch
  - Load data into some register
  - Using prefetch instructions

# Demo: make matrix transpose faster!

- x86 provide prefetch instructions
- As a programmer, you may insert \_mm\_prefetch in x86 programs to perform software prefetch for your code
- gcc also has a flag "-fprefetch-loop-arrays" to automatically insert software prefetch instructions

# **Takeaways: Software Optimizations**

- Data layout capacity miss, conflict miss, compulsory miss
- Loop interchange conflict/capacity miss
- Loop fission conflict miss when \$ has limited way associativity
- Loop fusion capacity miss when \$ has enough way associativity
- Blocking/tiling capacity miss, conflict miss
- Matrix transpose (a technique changes layout) conflict misses
- Prefetching compulsory misses

# Sample Midterm

#### **Format**

- 15 Multiple Choice Questions 30%
- 3 Assignment Style Free Answer Questions 70%
- Closed note, closed book, no outside materials

# Programmer's impact

• By adding the "sort" in the following code snippet, what the programmer changes in the performance equation to achieve **better** performance?

```
std::sort(data, data + arraySize);
       for (unsigned c = 0; c < arraySize*1000; ++c) {
               if (data[c%arraySize] >= INT_MAX/2)
                    sum ++;
B. IC
C. CT
D. IC & CPI
E. CPI & CT
```

#### How programming languages affect performance

- Performance equation consists of the following three factors
  - ① IC
  - 2 CPI
  - **3** CT

How many can the **programming language** affect?

- A. 0
- B. 1
- C. 2
- D. 3

# Demo — programmer & performance

```
for(i = 0; i < ARRAY_SIZE; i++)
{
  for(j = 0; j < ARRAY_SIZE; j++)
    {
    c[i][j] = a[i][j]+b[i][j];
  }
}</pre>
```

```
for(j = 0; j < ARRAY_SIZE; j++)
{
  for(i = 0; i < ARRAY_SIZE; i++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}</pre>
```

- How many of the following make(s) the performance of A better than B?
  - ① IC
  - ② CPI
  - **3** CT
  - A. 0
  - B. 1
  - C. 2
  - D. 3

# How compilers affect performance

- If we apply compiler optimizations for both code snippets A and B, how many of the following can we expect?
  - ① Compiler optimizations can reduce IC for both
  - ② Compiler optimizations can make the CPI lower for both
  - ③ Compiler optimizations can make the ET lower for both

  - A. 0
  - B. 1
  - C. 2
  - D. 3
  - E. 4

```
for(i = 0; i < ARRAY_SIZE; i++)
{
  for(j = 0; j < ARRAY_SIZE; j++)
    {
    c[i][j] = a[i][j]+b[i][j];
  }
}</pre>
```

```
for(j = 0; j < ARRAY_SIZE; j++)
{
  for(i = 0; i < ARRAY_SIZE; i++)
  {
    c[i][j] = a[i][j]+b[i][j];
  }
}</pre>
```

 $\mathbf{m}$ 

#### Amdahl's Law on Multicore Architectures

- Regarding Amdahl's Law on multicore architectures, how many of the following statements is/are correct?
  - ① If we have unlimited parallelism, the performance of each parallel piece does not matter as long as the performance slowdown in each piece is bounded
  - ② With unlimited amount of parallel hardware units, single-core performance does not matter anymore
  - ③ With unlimited amount of parallel hardware units, the maximum speedup will be bounded by the fraction of parallel parts
  - With unlimited amount of parallel hardware units, the effect of scheduling and data exchange overhead is minor
  - A. 0
  - B. 1
  - C. 2
  - D. 3
  - E. 4

#### How reflective is FLOPS?

- If you're given the FLOPS of an underlying GPU, how many situations below can the FLOPS be representative to the real performance?
  - ① The FLOPS remains the same on the same GPU even if we change the data size
  - ② The FLOPS remains the same on the same GPU even if we change the data type to double
  - ③ The FLOPS remains the same on the same GPU if we change the algorithm implementation
  - The ratio of FLOPS on two different GPUs reflects the ratio execution on these two GPUs when executing floating point applications
  - A. 0
  - B. 1
  - C. 2
  - D. 3
  - E. 4

### How programmer affects performance?

- Performance equation consists of the following three factors
  - ① IC
  - 2 CPI
  - **3** CT

How many can a **programmer** affect?

- A. 0
- B. 1
- C. 2
- D. 3

# **Data locality**

 Which description about locality of arrays matrix and vector in the following code is the most accurate?

```
for(uint64_t i = 0; i < m; i++) {
    result = 0;
    for(uint64_t j = 0; j < n; j++) {
        result += matrix[i][j]*vector[j];
    }
    output[i] = result;
}</pre>
```

- A. Access of matrix has temporal locality, vector has spatial locality
- B. Both matrix and vector have temporal locality, and vector also has spatial locality
- C. Access of matrix has spatial locality, vector has temporal locality
- D. Both matrix and vector have spatial locality and temporal locality
- E. Both matrix and vector have spatial locality, and vector also has temporal locality

#### intel Core i7

- L1 data (D-L1) cache configuration of Core i7
  - Size 32KB, 8-way set associativity, 64B block
  - Assume 64-bit memory address
  - Which of the following is NOT correct?
    - A. Tag is 52 bits
    - B. Index is 6 bits
    - C. Offset is 6 bits
    - D. The cache has 128 sets

#### intel Core i7

- D-L1 Cache configuration of intel Core i7
  - Size 48KB, 12-way set associativity, 64B block, LRU policy, writeallocate, write-back, and assuming 64-bit address.

```
double a[16384], b[16384], c[16384], d[16384], e[16384];
/* a = 0x10000, b = 0x20000, c = 0x30000, d = 0x40000, e = 0x50000 */
for(i = 0; i < 512; i++) {
    e[i] = (a[i] * b[i] + c[i])/d[i];
    //load a[i], b[i], c[i], d[i] and then store to e[i]
}</pre>
```

How many of the cache misses are **compulsory** misses?

- A. 12.5%
- B. 66.67%
- C. 68.75%
- D. 87.5%
- E. 100%

# What if we change the processor?

- If we have an intel processor with a 32KB, 8-way, 64B-blocked L1 cache, which version of code performs better?
  - A. Version A, because the code incurs fewer cache misses
  - B. Version B, because the code incurs fewer cache misses
  - C. Version A, because the code incurs fewer memory references
  - D. Version B, because the code incurs fewer memory references
  - E. They are about the same

#### What kind(s) of misses can tiling algorithm remove?

 Comparing the naive algorithm and tiling algorithm on matrix multiplication, what kind of misses does tiling algorithm help to remove? (assuming an intel Core i7)

```
for(i = 0; i < ARRAY_SIZE; i++) {
  for(j = 0; j < ARRAY_SIZE; j++) {
    for(k = 0; k < ARRAY_SIZE; k++) {
      c[i][j] += a[i][k]*b[k][j];
    }
  }
}</pre>
```

- A. Compulsory miss
- B. Capacity miss
- C. Conflict miss
- D. Capacity & conflict miss
- E. Compulsory & conflict miss

# Block

```
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
   for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
     for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {
      for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)
          for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
          for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
          c[ii][jj] += a[ii][kk]*b[kk][jj];
    }
}</pre>
```

#### What kind(s) of misses can matrix transpose remove?

• By transposing a matrix, the performance of matrix multiplication can be further improved. What kind(s) of cache misses does matrix transpose help to remove?

```
for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {
   for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {
     for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {
      for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)
          for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
          for(kk = k; kk < k+(ARRAY_SIZE/n); kk++)
          c[ii][jj] += a[ii][kk]*b[kk][jj];
   }
}</pre>
```

- A. Compulsory miss
- B. Capacity miss
- C. Conflict miss
- D. Capacity & conflict miss
- E. Compulsory & conflict miss

```
// Transpose matrix b into b_t
   for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {</pre>
      for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {</pre>
          b_t[i][j] += b[j][i];
    for(i = 0; i < ARRAY_SIZE; i+=(ARRAY_SIZE/n)) {</pre>
      for(j = 0; j < ARRAY_SIZE; j+=(ARRAY_SIZE/n)) {</pre>
        for(k = 0; k < ARRAY_SIZE; k+=(ARRAY_SIZE/n)) {</pre>
             for(ii = i; ii < i+(ARRAY_SIZE/n); ii++)</pre>
               for(jj = j; jj < j+(ARRAY_SIZE/n); jj++)
Block
                 for(kk = k; kk < k+(ARRAY_SIZE/n); kk++</pre>
                   // Compute on b_t
                   c[ii][jj] += a[ii][kk]*b_t[jj][kk];
```

# Tag in the cache

- Regarding the "tag" in cache, please identify the correct statement
  - A. The tag helps us to identify if the requesting address residing in the cache.
  - B. The tag comes from the least significant bits of the memory address.
  - C. Each memory address has a unique tag.
  - D. The tag gives the location of a word within a block

#### **Conflict misses**

- Which of the following statement best describes conflict misses
  - A. The cache is large enough to hold the referenced data objects, but because they map to the same cache block, the cache keeps missing.
  - B. When the size of the working set exceeds the size of the cache, the cache is just too small to handle this particular working set.
  - C. When the code access a data structure for the first time, the cache has a miss of the object.

# Remind yourself

- What are the limitations of compiler optimizations? Can you list two?
- Please define Amdahl's Law and explain each term in it
- Please define the CPU performance equation and explain each term.
- Can you list two things affecting each term in the performance equation?
- What's the difference between latency and throughput? When should you use latency or throughput to judge performance?
- What's "benchmark" suite? Why is it important?
- Why TFLOPS or inferences per second is not a good metrics?

# Performance equation

Consider the following c code snippet and x86 instructions implement the code snippet

If (1) count is set to 1,000,000,000, (2) a memory instruction takes 4 cycles, (3) a branch/jump instruction takes 3 cycles, (4) other instructions takes 1 cycle on average, and (5) the processor runs at 4 GHz, how much time is it take to finish executing the code snippet?

# Speedup of Y over X

 Consider the same program on the following two machines, X and Y. By how much Y is faster than X?

	Clock Rate	Instructions	Percentage of Type-A		Percentage of Type-B	CPI of Type-B	Percentage of Type-C	CPI of Type-C
Machine X	3 GHz	500000000	20%	8	20%	4	60%	1
Machine Y	5 GHz	500000000	20%	13	20%	4	60%	1

#### Q5 and Q8: cache performance on the following code

- D-L1 Cache configuration of intel Core i7 processor
  - Size 48KB, 12-way set associativity, 64B block, LRU policy, writeallocate, write-back, and assuming 64-bit address.

```
double data[16384]; /* &data[0] = 0x20000 */
uint64_t sum = 0;
for(uint i = 0; i < arg1; i++) {
    for(uint x = 0; x < size; x+=arg1) {
        sum += data[x];
    }
}
return sum;</pre>
```

What's the data cache miss rate for this code when arg1=1 and size = 16384?

#### Q5 and Q8: cache performance on the following code

- D-L1 Cache configuration of intel Core i7 processor
  - Size 48KB, 12-way set associativity, 64B block, LRU policy, writeallocate, write-back, and assuming 64-bit address.

```
double data[16384]; /* &data[0] = 0x20000 */
uint64_t sum = 0;
for(uint i = 0; i < arg1; i++) {
    for(uint x = 0; x < size; x+=arg1) {
        sum += data[x];
    }
}
return sum;</pre>
```

What's the data cache miss rate for this code when arg1=16 and size = 16384?

# Practicing Amdahl's Law

 Final Fantasy XV spends lots of time loading a map — within which period that 95% of the time on the accessing the H.D.D., the rest in the operating system, file system and the I/O protocol. If we replace the H.D.D. with a flash drive, which provides 100x faster access time. By how much can we speed up the map loading process?

# Practicing Amdahl's Law (2)

• Final Fantasy XV spends lots of time loading a map — within which period that 95% of the time on the accessing the H.D.D., the rest in the operating system, file system and the I/O protocol. If we replace the H.D.D. with a flash drive, which provides 100x faster access time and a better processor to accelerate the software overhead by 2x. By how much can we speed up the map loading process?

#### Announcement

- Midterm next Monday in person during the normal lecture time
  - 80 minute
  - In-person, closed book, closed note
  - You can bring a calculator, but not the mobile calculator app
  - Multiple choices and the assignment-style free answer questions
  - Will release a sample midterm tomorrow
  - Hung-Wei will host office hours today 3:30p-6p
- Update your Assignment #2!
  - Execute the script in the notebook's Section 2.4 and refresh the "browser", not your notebook
  - Q8's stride should be "8"
  - Due this Saturday
  - You should run the performance measurement yourself and calculate results based on that everyone should have a different answer
  - All questions this time require correct estimations in cache performance to help you better prepare the examines
- Reading Quiz #5 due next Wednesday before the lecture

# Computer Science & Engineering

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