جامعة الجلالة
GALALA UNIVERSITY

Artificial Intelligence

Chapter 4: Learning from Examples

Forms of Learning

- The technology of machine learning has become a standard part of software engineering.
- Any time you are building a software system, even if you don't think of it as an AI agent, components of the system can potentially be improved with machine learning.
  - For example, software to analyze images of galaxies with a machine-learned model.

Forms of Learning

- An agent is learning if it improves its performance after making observations about the world.
- When the agent is a computer,
  - we call it machine learning: a computer observes some data, builds a model based on the data, and uses it.
- Any component of an agent program can be improved by machine learning.

Types of learning

- In supervised learning the agent observes input-output pairs and learns a function that maps from input to output.
  - For example, the inputs could be camera images, each one is either “bus” or “pedestrian,” etc. An output like this is called a label.
- In unsupervised learning the agent learns patterns in the input without any explicit feedback.
  - The most common unsupervised learning task is clustering.
    - For example, when shown millions of images taken from the Internet, a computer vision system can identify a large cluster of similar images “cats.”

Types of learning

- In reinforcement learning the agent learns from a series of reinforcements: rewards and punishments.
    - For example, at the end of a chess game the agent is told that it has won (a reward) or lost (a punishment).

Supervised Learning

- Given a training set of example input–output pairs

$$(x_1, y_1), (x_2, y_2), \ldots (x_N, y_N) ,$$

- where each pair was generated by an unknown function $y = f(x)$ discover a function that approximates the true function f
- We can say y is the ground truth
- We can evaluate that with a second sample of (x_i, y_i) pairs called a test set.

Prediction Problems: Classification vs. Numeric Prediction

- Classification
  - predicts categorical class labels (discrete or nominal)
  - classifies data (constructs a model) based on the training set and the values (class labels) in a classifying attribute and uses it in classifying new data.
- Numeric Prediction
  - models continuous-valued functions, i.e., predicts unknown or missing values
- Typical applications
  - Credit/loan approval:
  - Medical diagnosis: if a tumor is cancerous or benign.
  - Web page categorization: which category it is.

Classification—A Two-Step Process

- Model construction: describing a set of predetermined classes
    - Each sample is assumed to belong to a predefined class, as determined by the class label attribute
    - The set of tuples used for model construction is training set
    - The model is represented as classification rules, decision trees, or mathematical formula
- Model usage: for classifying future or unknown objects
    - Estimate accuracy of the model
        - The known label of test sample is compared with the classified result from the model
        - Accuracy rate is the percentage of test set samples that are correctly classified by the model
        - Test set is independent of training set (otherwise overfitting)
    - If the accuracy is acceptable, use the model to classify new data
- Note: If *the test set* is used to select models, it is called validation (test) set

Process (1): Model Construction

Training Data

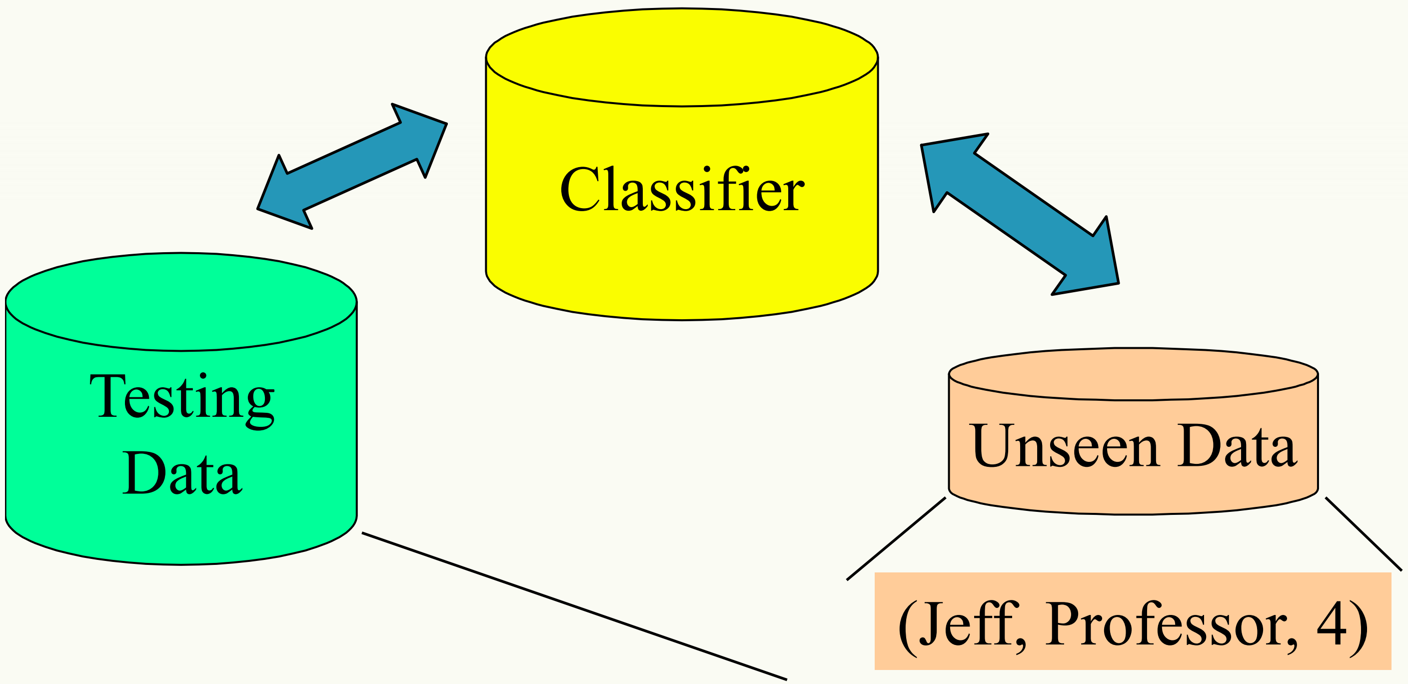
Classification Algorithms

Classifier (Model)

NAME	RANK	YEARS	TENURED
Mike	Assistant Prof	3	no
Mary	Assistant Prof	7	yes
Bill	Professor	2	yes
Jim	Associate Prof	7	yes
Dave	Assistant Prof	6	no
Anne	Associate Prof	3	no

IF rank = 'professor'
OR years > 6
THEN tenured = 'yes'

Process (2): Using the Model in Prediction



NAME	RANK	YEARS	TENURED
Tom	Assistant Prof	2	no
Merlisa	Associate Prof	7	no
George	Professor	5	yes
Joseph	Assistant Prof	7	yes

Tenured?

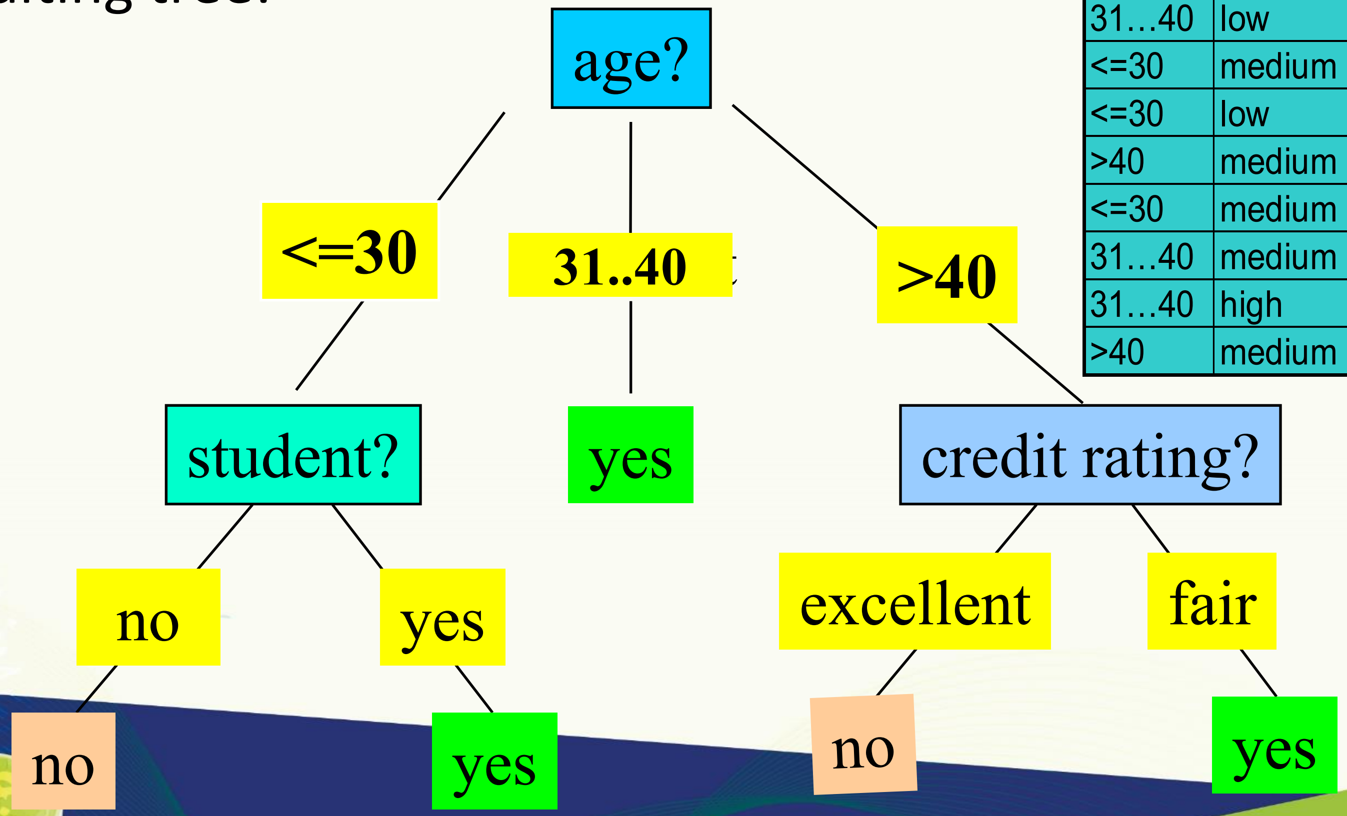
Yes

Learning Decision Trees

- A decision tree is a representation of a function that maps a vector of attribute values to a single output value—a "decision."
- A decision tree reaches its decision by performing a sequence of tests, starting at the root and following the appropriate branch until a leaf is reached.

Decision Tree Induction: An Example

- Training data set: Buys_computer
- The data set follows an example of Quinlan's ID3 (Playing Tennis)
- Resulting tree:



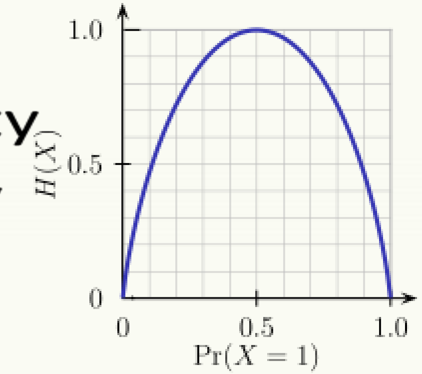
age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Algorithm for Decision Tree Induction

- Basic algorithm (a greedy algorithm)
  - Tree is constructed in a top-down recursive divide-and-conquer manner.
  - At start, all the training examples are at the root
  - Attributes are categorical (if continuous-valued, they are discretized in advance)
  - Examples are partitioned recursively based on selected attributes
  - Test attributes are selected on the basis of a heuristic or statistical measure (e.g., information gain)
- Conditions for stopping partitioning
  - All samples for a given node belong to the same class
  - There are no remaining attributes for further partitioning – majority voting is employed for classifying the leaf
  - There are no samples left

Brief Review of Entropy

- Entropy (Information Theory)
  - A measure of uncertainty associated with a random variable
  - Calculation: For a discrete random variable *Y* taking m distinct values $\{y_1, \ldots, y_m\}$,
    - $H(Y) = -\sum_{i=1}^{m} p_i \log(p_i)$, where $p_i = P(Y = y_i)$
  - Interpretation:
    - Higher entropy => higher uncertainty
    - Lower entropy => lower uncertainty



m = 2

- Conditional Entropy
  - $H(Y|X) = \sum_x p(x) H(Y|X = x)$

Attribute Selection Measure: Information Gain (ID3/C4.5)

- Select the attribute with the highest information gain
- Let p_i be the probability that an arbitrary sample in D belongs to class C_i, estimated by $|C_{i,D}|/|D|$.
- Expected information (entropy) needed to classify a tuple in D:

$$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$

- Information needed (after using A to split D into v partitions) to classify D:

$$Info_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \times Info(D_j)$$

- Information gained by branching on attribute A

$$Gain(A) = Info(D) - Info_A(D)$$

Attribute Selection: Information Gain

- Class P: buys_computer = "yes"
- Class N: buys_computer = "no"

$$Info(D) = I(9,5) = -\frac{9}{14}\log_2\left(\frac{9}{14}\right) - \frac{5}{14}\log_2\left(\frac{5}{14}\right) = 0.940$$

age	p_i	n_i	$I(p_i, n_i)$
<=30	2	3	0.971
31…40	4	0	0
>40	3	2	0.971

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

$$Info_{age}(D) = \frac{5}{14}I(2,3) + \frac{4}{14}I(4,0) + \frac{5}{14}I(3,2) = 0.694$$

$\frac{5}{14}I(2,3)$ means "age <=30" has 5 out of 14 samples, with 2 yes'es and 3 no's. Hence

$$Gain(age) = Info(D) - Info_{age}(D) = 0.246$$

Similarly,

$$Gain(income) = 0.029$$
$$Gain(student) = 0.151$$
$$Gain(credit\_rating) = 0.048$$

```
#Importing required libraries
 import pandas as pd
import numpy as np
from sklearn.datasets import load_iris
from sklearn.tree import DecisionTreeClassifier
from sklearn.model_selection import train_test_split
data = load_iris() print('Classes to predict: ', data.target_names)
#Extracting data attributes
X = data.data
### Extracting target/ class labels
y = data.target

print('Number of examples in the data:', X.shape[0])
#First four rows in the variable 'X'
X[:4]

#Output
Out: array([[5.1, 3.5, 1.4, 0.2],
       [4.9, 3. , 1.4, 0.2],
       [4.7, 3.2, 1.3, 0.2],
       [4.6, 3.1, 1.5, 0.2]])
```

```
#Using the train_test_split to create train and test sets.
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state = 47, test_size = 0.25)
#Importing the Decision tree classifier from the sklearn library.
from sklearn.tree import DecisionTreeClassifier
clf = DecisionTreeClassifier(criterion = 'entropy')

#Training the decision tree classifier.
clf.fit(X_train, y_train)

#Output:
Out:DecisionTreeClassifier(class_weight=None, criterion='entropy', max_depth=None,
            max_features=None, max_leaf_nodes=None,
            min_impurity_decrease=0.0, min_impurity_split=None,
            min_samples_leaf=1, min_samples_split=2,
            min_weight_fraction_leaf=0.0, presort=False, random_state=None,
            splitter='best')
#Predicting labels on the test set.
y_pred =  clf.predict(X_test)
```

```
#Importing the accuracy metric from sklearn.metrics library

from sklearn.metrics import accuracy_score
print('Accuracy Score on train data: ', accuracy_score(y_true=y_train, y_pred=clf.predict(X_train)))
print('Accuracy Score on test data: ', accuracy_score(y_true=y_test, y_pred=y_pred))

#Output:
Out: Accuracy Score on train data:  1.0
    Accuracy Score on test data:  0.9473684210526315
```

Gain Ratio for Attribute Selection (C4.5)

- Information gain measure is biased towards attributes with a large number of values.
- C4.5 (a successor of ID3) uses gain ratio to overcome the problem (normalization to information gain).

$$SplitInfo_A(D) = -\sum_{j=1}^{v} \frac{|D_j|}{|D|} \times \log_2\left(\frac{|D_j|}{|D|}\right)$$

  - GainRatio(A) = Gain(A)/SplitInfo(A)
- Ex.

$$SplitInfo_{income}(D) = -\frac{4}{14} \times \log_2\left(\frac{4}{14}\right) - \frac{6}{14} \times \log_2\left(\frac{6}{14}\right) - \frac{4}{14} \times \log_2\left(\frac{4}{14}\right) = 1.557$$

  - gain_ratio(income) = 0.029/1.557 = 0.019
- The attribute with the maximum gain ratio is selected as the splitting attribute

$$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i) \qquad Info_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \times Info(D_j)$$

$$SplitInfo_A(D) = -\sum_{j=1}^{v} \frac{|D_j|}{|D|} \times \log_2\left(\frac{|D_j|}{|D|}\right) \qquad Gain(A) = Info(D) - Info_A(D)$$

$$GainRatio(A) = \frac{Gain(A)}{SplitInfo(A)}$$

- Calculate ***Entropy*** of Class attribute:

buys_computer	
yes	no
9	5

$$Info(D) = I(9,5) = -\frac{9}{14}log_2\left(\frac{9}{14}\right) - \frac{5}{14}log_2\left(\frac{5}{14}\right) = \mathbf{0.9403}$$

- Calculate ***Gain Ratio*** of all other attributes:

$$-\frac{2}{5}log_2\left(\frac{2}{5}\right) - \frac{3}{5}log_2\left(\frac{3}{5}\right)$$

		Class		
		yes	no	
age	youth	2	3	5
	middle_aged	4	0	4
	senior	3	2	5
				14

$$Info_{age}(D) = \frac{5}{14}I(2,3) + \frac{4}{14}I(4,0) + \frac{5}{14}I(3,2)$$

$$= \frac{5}{14} * 0.971 + \frac{4}{14} * 0 + \frac{5}{14} * 0.971 = 0.3467 + 0 + 0.3467 = \mathbf{0.6934}$$

$$Gain(age) = Info(D) - Infoage_{(D)} = 0.9403 - 0.6934 = \mathbf{0.2469}$$

$$SplitInf0_{age}(D) = -\frac{5}{14} * \log2\left(\frac{5}{14}\right) - \frac{4}{14} * \log_2\left(\frac{4}{14}\right) - \frac{5}{14} * \log2\left(\frac{5}{14}\right) = \mathbf{1.5774}$$

$$\mathbf{GainRatio(age)} = \frac{Gain(A)}{SplitInfo(A)} = \frac{0.246}{1.5774} = \mathbf{0.1559}$$

		Class		
		yes	no	
income	low	3	1	4
	medium	4	2	6
	high	2	2	4
				14

$$Info_{income}(D) = \frac{4}{14}I(3,1) + \frac{6}{14}I(4,2) + \frac{4}{14}I(2,2)$$

$$= \frac{4}{14} * 0.8113 + \frac{6}{14} * 0.9183 + \frac{4}{14} * 1 = 0.2318 + 0.3935 + 0.2857 = \mathbf{0.911}$$

$$Gain(income) = 0.9403 - 0.911 = \mathbf{0.0293}$$

$$SplitInf0_{income}(D) = -\frac{4}{14} * \log2\left(\frac{4}{14}\right) - \frac{6}{14} * \log_2\left(\frac{6}{14}\right) - \frac{4}{14} * \log2\left(\frac{4}{14}\right) = \mathbf{1.5566}$$

$$\mathbf{GainRatio(income)} = \frac{0.0293}{1.5566} = \mathbf{0.0188}$$

age	income	student	credit_rating	buys_computer
youth	high	no	fair	no
youth	high	no	excellent	no
middle_aged	high	no	fair	yes
senior	medium	no	fair	yes
senior	low	yes	fair	yes
senior	low	yes	excellent	no
middle_aged	low	yes	excellent	yes
youth	medium	no	fair	no
youth	low	yes	fair	yes
senior	medium	yes	fair	yes
youth	medium	yes	excellent	yes
middle_aged	medium	no	excellent	yes
middle_aged	high	yes	fair	yes
senior	medium	no	excellent	no

$$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$

$$Info_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \times Info(D_j)$$

$$SplitInfo_A(D) = -\sum_{j=1}^{v} \frac{|D_j|}{|D|} \times \log_2\left(\frac{|D_j|}{|D|}\right)$$

$$Gain(A) = Info(D) - Info_A(D)$$

$$GainRatio(A) = \frac{Gain(A)}{SplitInfo(A)}$$

- Calculate ***Entropy*** of Class attribute:

buys_computer	
yes	no
9	5

$$Info(D) = I(9,8) = -\frac{9}{14}log_2\left(\frac{9}{14}\right) - \frac{5}{14}log_2\left(\frac{5}{14}\right) = \mathbf{0.9403}$$

age	income	student	credit_rating	buys_computer
youth	high	no	fair	no
youth	high	no	excellent	no
middle_aged	high	no	fair	yes
senior	medium	no	fair	yes
senior	low	yes	fair	yes
senior	low	yes	excellent	no
middle_aged	low	yes	excellent	yes
youth	medium	no	fair	no
youth	low	yes	fair	yes
senior	medium	yes	fair	yes
youth	medium	yes	excellent	yes
middle_aged	medium	no	excellent	yes
middle_aged	high	yes	fair	yes
senior	medium	no	excellent	no

- Calculate ***Gain Ratio*** of all other attributes:

		Class		
		yes	no	
student	yes	6	1	7
	no	3	4	7
				14

$$Info_{student}(D) = \frac{7}{14}I(6,1) + \frac{7}{14}I(3,4)$$

$$= \frac{7}{14} * 0.5917 + \frac{7}{14} * 0.9852 = 0.2958 + 0.4926 = \mathbf{0.7884}$$

$$Gain(student) = 0.9403 - 0.7884 = \mathbf{0.1519}$$

$$SplitInf0_{student}(D) = -\frac{7}{14} * \log2\left(\frac{7}{14}\right) - \frac{7}{14} * \log_2\left(\frac{7}{14}\right) = \mathbf{1}$$

$$\mathbf{GainRatio(student)} = \frac{0.1519}{1} = \mathbf{0.1519}$$

		Class		
		yes	no	
credit_rating	fair	6	2	8
	excellent	3	3	6
				14

$$Info_{credit\_rating}(D) = \frac{8}{14}I(6,2) + \frac{6}{14}I(3,3)$$

$$= \frac{8}{14} * 0.8113 + \frac{6}{14} * 1 = 0.4636 + 0.4286 = \mathbf{0.8922}$$

$$Gain(credit-rating) = 0.9403 - 0.8922 = \mathbf{0.0481}$$

$$SplitInf0_{credit-rating}(D) = -\frac{8}{14} * \log2\left(\frac{8}{14}\right) - \frac{6}{14} * \log_2\left(\frac{6}{14}\right) = \mathbf{0.9852}$$

$$\mathbf{GainRatio(credit-rating)} = \frac{0.0481}{0.9852} = \mathbf{0.0488}$$

$$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$

$$Info_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \times Info(D_j)$$

$$SplitInfo_A(D) = -\sum_{j=1}^{v} \frac{|D_j|}{|D|} \times \log_2\left(\frac{|D_j|}{|D|}\right)$$

$$Gain(A) = Info(D) - Info_A(D)$$

$$GainRatio(A) = \frac{Gain(A)}{SplitInfo(A)}$$

- Calculate ***Entropy*** of Class attribute:

buys_computer	
yes	no
9	5

$$Info(D) = I(9,8) = -\frac{9}{14} log_2\left(\frac{9}{14}\right) - \frac{5}{14} log_2\left(\frac{5}{14}\right) = \mathbf{0.9403}$$

age	income	student	credit_rating	buys_computer
youth	high	no	fair	no
youth	high	no	excellent	no
middle_aged	high	no	fair	yes
senior	medium	no	fair	yes
senior	low	yes	fair	yes
senior	low	yes	excellent	no
middle_aged	low	yes	excellent	yes
youth	medium	no	fair	no
youth	low	yes	fair	yes
senior	medium	yes	fair	yes
youth	medium	yes	excellent	yes
middle_aged	medium	no	excellent	yes
middle_aged	high	yes	fair	yes
senior	medium	no	excellent	no

- Calculate ***Gain Ratio*** of all other attributes:

		Class		
		yes	no	
student	yes	6	1	7
	no	3	4	7
				14

$$Info_{student}(D) = \frac{7}{14} I(6,1) + \frac{7}{14} I(3,4)$$
$$= \frac{7}{14} * 0.5917 + \frac{7}{14} * 0.9852 = 0.2958 + 0.4926 = \mathbf{0.7884}$$

$$Gain(student) = 0.9403 - 0.7884 = \mathbf{0.1519}$$

$$SplitInf0_{student}(D) = -\frac{7}{14} * \log2\left(\frac{7}{14}\right) - \frac{7}{14} * \log_2\left(\frac{7}{14}\right) = 1$$

$$\boldsymbol{GainRatio(student)} = \frac{0.1519}{1} = \mathbf{0.1519}$$

		Class		
		yes	no	
credit_rating	fair	6	2	8
	excellent	3	3	6
				14

$$Info_{credit\_rating}(D) = \frac{8}{14} I(6,2) + \frac{6}{14} I(3,3)$$
$$= \frac{8}{14} * 0.8113 + \frac{6}{14} * 1 = 0.4636 + 0.4286 = \mathbf{0.8922}$$

$$Gain(credit - rating) = 0.9403 - 0.8922 = \mathbf{0.0481}$$

$$SplitInf0_{credit-rating}(D) = -\frac{8}{14} * \log2\left(\frac{8}{14}\right) - \frac{6}{14} * \log_2\left(\frac{6}{14}\right) = \mathbf{0.9852}$$

$$\boldsymbol{GainRatio(credit - rating)} = \frac{0.0481}{0.9852} = \mathbf{0.0488}$$

- As, the Gain Ratio of "age" is highest,
- So "**age**" is the best attribute & becomes the root node of the decision tree.

Age?

youth

income	student	credit_rating	class
high	no	fair	no
high	no	excellent	no
medium	no	fair	no
low	yes	fair	yes
medium	yes	excellent	yes

middle_aged

income	student	credit_rating	class
high	no	fair	yes
low	yes	excellent	yes
medium	no	excellent	yes
high	yes	fair	yes

senior

income	student	credit_rating	class
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$

$Info_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \times Info(D_j)$

$SplitInfo_A(D) = -\sum_{j=1}^{v} \frac{|D_j|}{|D|} \times \log_2(\frac{|D_j|}{|D|})$

$Gain(A) = Info(D) - Info_A(D)$

$GainRatio(A) = \frac{Gain(A)}{SplitInfo(A)}$

- For Left subtree: Calculate ***Entropy*** of Class attribute:

buys_computer	
yes	no
2	3

$$Info(D) = I(2,3) = -\frac{2}{5}log_2\left(\frac{2}{5}\right) - \frac{3}{5}log_2\left(\frac{3}{5}\right) = 0.971$$

income	student	credit_rating	class
high	no	fair	no
high	no	excellent	no
medium	no	fair	no
low	yes	fair	yes
medium	yes	excellent	yes

- Calculate ***Gain Ratio*** of all other attributes:

		Class		
		yes	no	
income	low	1	0	1
	medium	1	1	2
	high	0	2	2
				5

$$Info_{income}(D) = \frac{1}{5}I(1,0) + \frac{2}{5}I(1,1) + \frac{2}{5}I(0,2)$$
$$= \frac{1}{5} * 0 + \frac{2}{5} * 1 + \frac{2}{5} * 0 = 0 + 0.4 + 0 = \mathbf{0.4}$$

$$Gain(income) = 0.971 - 0.4 = \mathbf{0.571}$$

$$SplitInf0_{income}(D) = -\frac{1}{5} * \log2\left(\frac{1}{5}\right) - \frac{2}{5} * \log_2\left(\frac{2}{5}\right) - \frac{2}{5} * \log_2\left(\frac{2}{5}\right) = \mathbf{1.5219}$$

$$\mathbf{GainRatio(income)} = \frac{0.571}{1.5219} = \mathbf{0.3751}$$

		Class		
		yes	no	
student	yes	2	0	2
	no	0	3	3
				5

$$Info_{student}(D) = \frac{2}{5}I(2,0) + \frac{3}{5}I(0,3) = \frac{2}{5} * 0 + \frac{3}{5} * 0 = \mathbf{0}$$

$$Gain(age) = 0.971 - 0 = \mathbf{0.971}$$

$$SplitInf0_{student}(D) = -\frac{2}{5} * \log_2\left(\frac{2}{5}\right) - \frac{3}{5} * \log2\left(\frac{3}{5}\right) = \mathbf{0.9709}$$

$$\mathbf{GainRatio(student)} = \frac{0.971}{0.9709} = \mathbf{1}$$

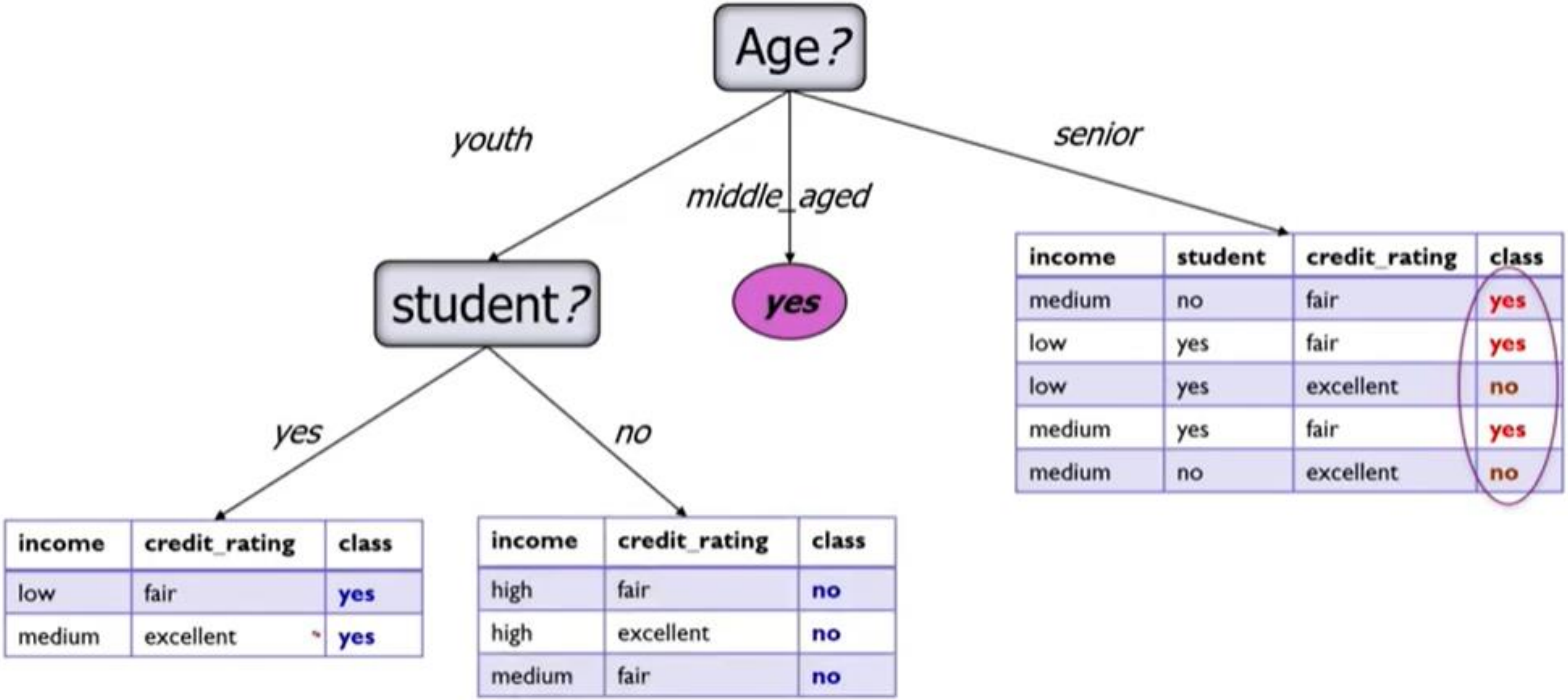
		Class		
		yes	no	
credit_ rating	fair	1	2	3
	excellent	1	1	2
				5

$$Info_{credit\_rating}(D) = \frac{3}{5}I(1,2) + \frac{2}{5}I(1,1)$$
$$= \frac{3}{5} * 0.9183 + \frac{2}{5} * 1 = 0.3443 + 0.4 = \mathbf{0.7443}$$

$$Gain(credit\_rating) = 0.971 - 0.7443 = \mathbf{0.2267}$$

$$SplitInf0_{credit\_rating}(D) = -\frac{3}{5} * \log2\left(\frac{3}{5}\right) - \frac{2}{5} * \log_2\left(\frac{2}{5}\right) = \mathbf{0.9709}$$

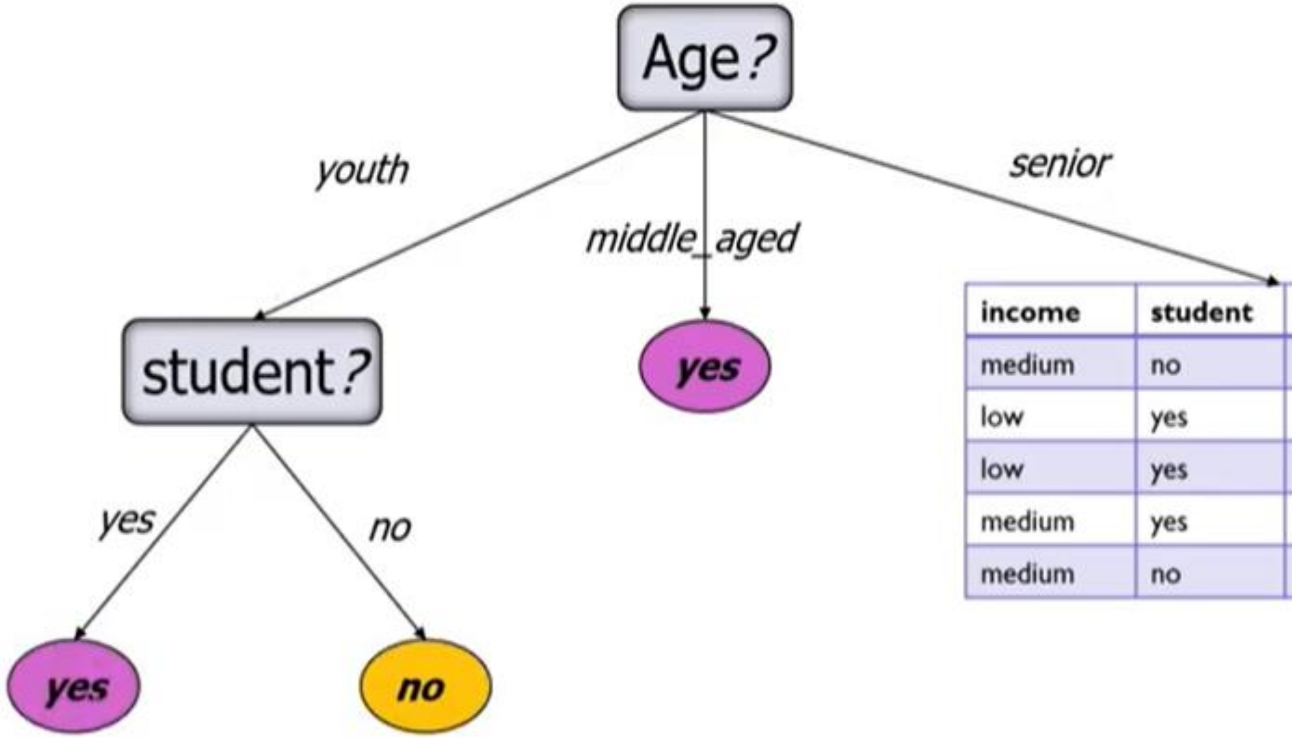
$$\mathbf{GainRatio(credit\_rating)} = \frac{0.2267}{0.9709} = \mathbf{0.2335}$$



income	student	credit_rating	class
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

income	credit_rating	class
low	fair	yes
medium	excellent	yes

income	credit_rating	class
high	fair	no
high	excellent	no
medium	fair	no



income	student	credit_rating	class
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

Gain Ratio [C4.5] - Example

$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$

$Info_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \times Info(D_j)$

$Gain(A) = Info(D) - Info_A(D)$

- For Right subtree: Calculate ***Entropy*** of Class attribute:

buys_computer	
yes	no
3	2

$$Info(D) = I(3,2) = -\frac{3}{5}log_2\left(\frac{3}{5}\right) - \frac{2}{5}log_2\left(\frac{2}{5}\right) = \mathbf{0.971} ✓$$

income	student	credit_rating	class
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

- Calculate ***Gain Ratio*** of all other attributes:

		Class		
		yes	no	
income	low	1	1	2
	medium	2	1	3
	high	0	0	0
				5

$$Info_{income}(D) = \frac{2}{5}I(1,1) + \frac{3}{5}I(2,1)$$
$$= \frac{2}{5} * 1 + \frac{3}{5} * 0.9183 = 0.4 + 0.551 = \mathbf{0.951}$$

$$Gain(income) = 0.971 - 0.951 = \mathbf{0.02}$$

$$SplitInfo_{income}(D) = -\frac{2}{5} * \log 2\left(\frac{2}{5}\right) - \frac{3}{5} * \log_2\left(\frac{3}{5}\right) = \mathbf{0.9709}$$

$$\mathbf{GainRatio(income)} = \frac{0.02}{0.9709} = \mathbf{0.0205} ✓$$

		Class		
		yes	no	
credit_ rating	fair	3	0	3
	excellent	0	2	2
				5

$$Info_{credit\_rating}(D) = \frac{3}{5}I(3,0) + \frac{2}{5}I(0,2) = \frac{3}{5} * 0 + \frac{2}{5} * 0 = \mathbf{0}$$

$$Gain(credit\_rating) = 0.971 - 0 = \mathbf{0.971}$$

$$SplitInfo_{credit\_rating}(D) = -\frac{3}{5} * \log 2\left(\frac{3}{5}\right) - \frac{2}{5} * \log_2\left(\frac{2}{5}\right) = \mathbf{0.9709}$$

$$\mathbf{GainRatio(credit\_rating)} = \frac{0.971}{0.9709} = \mathbf{1}$$

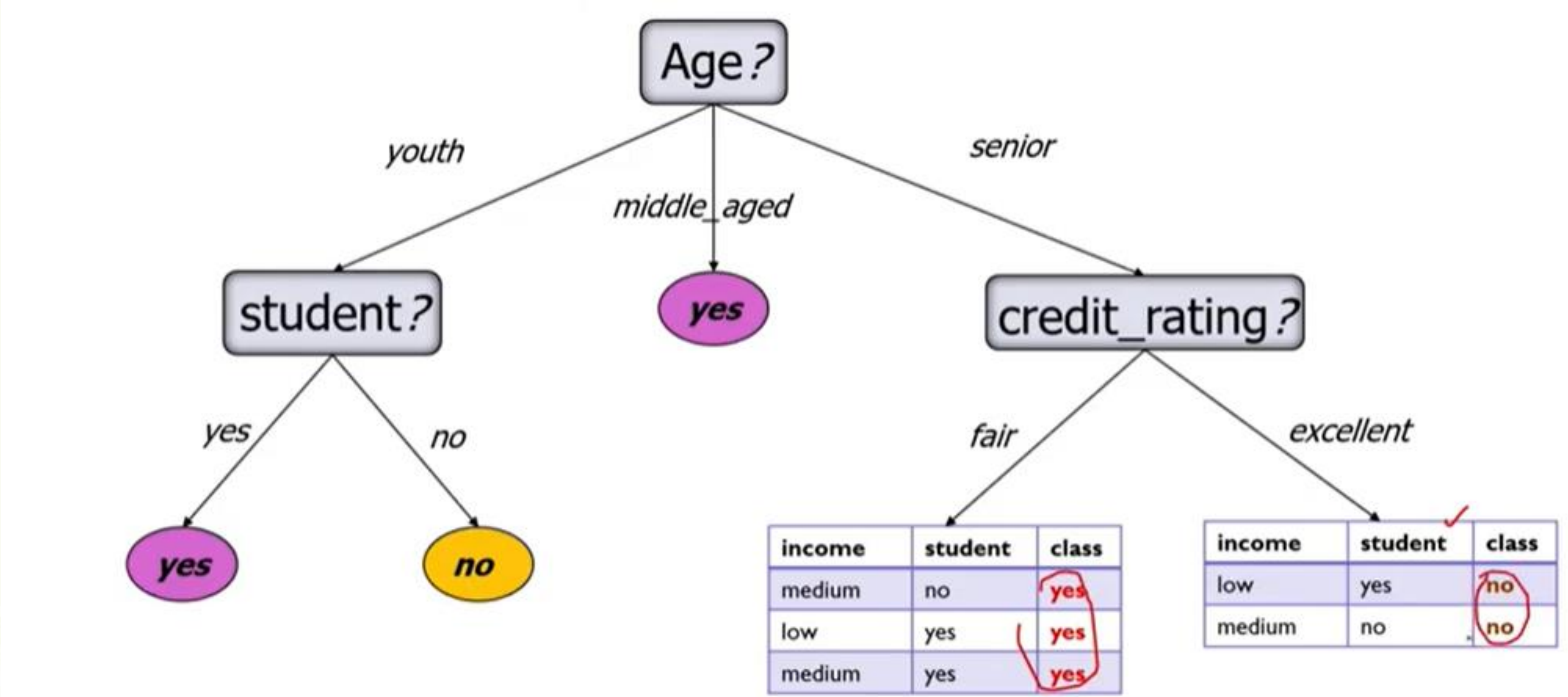
		Class		
		yes	no	
student	yes	2	1	3
	no	1	1	2
				5

$$Info_{student}(D) = \frac{3}{5}I(2,1) + \frac{2}{5}I(1,1)$$
$$= \frac{3}{5} * 0.9183 + \frac{2}{5} * 1 = 0.551 + 0.4 = \mathbf{0.951}$$

$$Gain(age) = 0.971 - .951 = \mathbf{0.02}$$

$$SplitInfo_{student}(D) = -\frac{3}{5} * \log_2\left(\frac{3}{5}\right) - \frac{2}{5} * \log 2\left(\frac{2}{5}\right) = \mathbf{0.9709}$$

$$\mathbf{GainRatio(student)} = \frac{0.02}{0.9709} = \mathbf{0.0205}$$



income	student	class
medium	no	yes
low	yes	yes
medium	yes	yes

income	student	class
low	yes	no
medium	no	no

What is the decision for

- X=[age, income, student, cradit]=[15,low,no,excellent]
- X=[age, income, student, cradit]=[40,low,no,excellent]

Comparing Attribute Selection Measures

- The two measures, in general, return good results but
  - **Information gain**:
    - biased towards multivalued attributes
  - **Gain ratio**:
    - tends to prefer unbalanced splits in which one partition is much smaller than the others