An Experimental Look at the Stability of Graph Convolutional Networks against Topological Perturbations

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1. Research Question

- How do different properties of a graph impact the stability of its graph convolutional network(GCN) against topological perturbations?
- Which combinations of graph properties can act as reliable indicators of instability in GCNs?

2. Background

- **GCNs:** Models built on a special form of convolution that uses a **shift operator** [1]

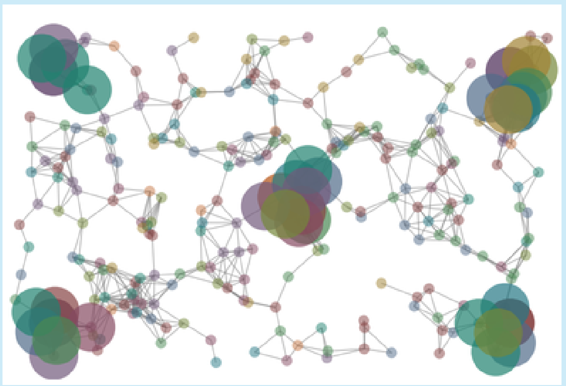

Figure 1: A visualization of the graph convolution operation, when the shift operator is applied twice []

- **Stability:** A GCN's ability to output accurate information when the graph's topology is different, i.e. perturbed
- Theoretical bounds have already been set on stability, but those bounds are often loose [2]
- Relationship between properties of graphs and real stability is still unknown

3. Methodology

- The task under investigation is semi-supervised node classification on **undirected** and **unweighted** graphs
- Topology Adaptive Graph Convolutional Networks (TAGConv) [3] are used as the GCN implementation

- **Measuring Stability:**
  - A TAGConv is trained on a graph **G**, the final layer's output **x** is saved
  - **G** is perturbed and the output of the trained TAGConv for perturbed **G** is stored (**y**)
  - The error is calculated using the relative euclidian distance between **x** and **y**:

$$y = \frac{\|g_\theta(\mathbf{S},\mathbf{x}) - g_\theta(\mathbf{S}_p,\mathbf{x})\|_2}{\|g_\theta(\mathbf{S},\mathbf{x})\|_2}$$

- **Challenge:** Existing graph datasets have a limited range in graph properties [4]
- **Solution:** Synthetically generate the datasets using:
  - Stochastic Block Model [5]
  - Lancichinetti–Fortunato–Radicchi Model [6]

- **Size and Types of Perturbations:**
  - The size of the perturbation is 10% of the number of edges in the graph
  - The perturbations should not result in self loops or create parallel edges
  - **Addition:** Add an edge between two random nodes
  - **Deletion:** Delete a random edge
  - **Rewiring:** Add and delete edges in the graph, while keeping the degree sequence untouched

4. Graph Properties Under Investigation

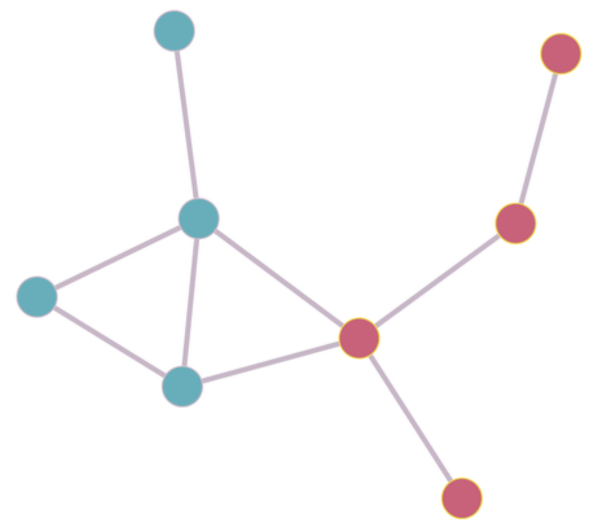

Figure 2: A graph with 8 nodes, generated using the Stochastic Block Model with a diameter of 4, connectivity of 1, clustering of 0.35, assortativity of 0.55 and centrality of 0.51

- **Diameter:** Maximum shortest path between any pair of vertices in a graph.
- **Edge Connectivity:** The minimum number of edges that need to be removed to make the graph disconnected
- **Clustering:** Measure of how much nodes tend to cluster together. Calculated by dividing the number of closed triplets by the total number of triplets (both open and closed)
- **Assortativity:** Specifically, nominal assortativity, a measure of how likely similar nodes are to be connected to each other.
- **Closeness Centrality:** The average distance of a node to every other node in the graph. In the context of this research, it is averaged over all nodes.

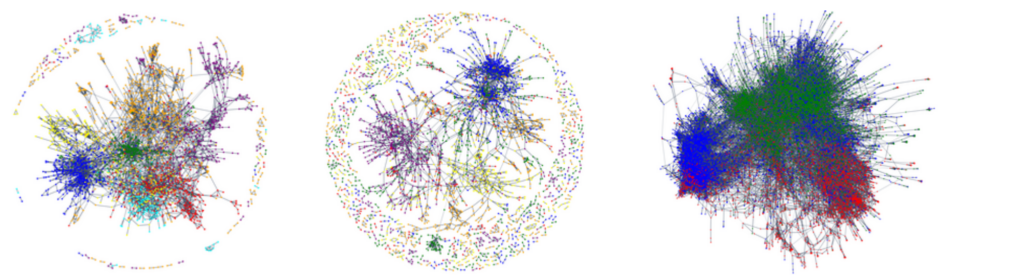

Figure 3: Real world graph datasets that were tested. From left to right: Cora, CiteSeer, PubMed.

	LFR			SBM			Citeseer	Cora	Pubmed
	Max	Mean	Min	Max	Mean	Min	-	-	-
Assortativity	0.8855	0.8715	0.8453	0.9221	0.7515	0.0029	0.6707	0.7711	0.686
Centrality	0.4724	0.388	0.3359	0.5227	0.3916	0.3223	0.3516	0.2189	0.1603
Connectivity	0	0	0	21	8.5068	0	0	0	2
Density	0.0606	0.0234	0.0096	0.1129	0.0125	0.0254	0.0016	0.0029	0.0005
Diameter	17	11.068	7	8	4.1356	3	28	19	18

Figure 4: Properties of the graphs that were tested. Both synthetic and real.

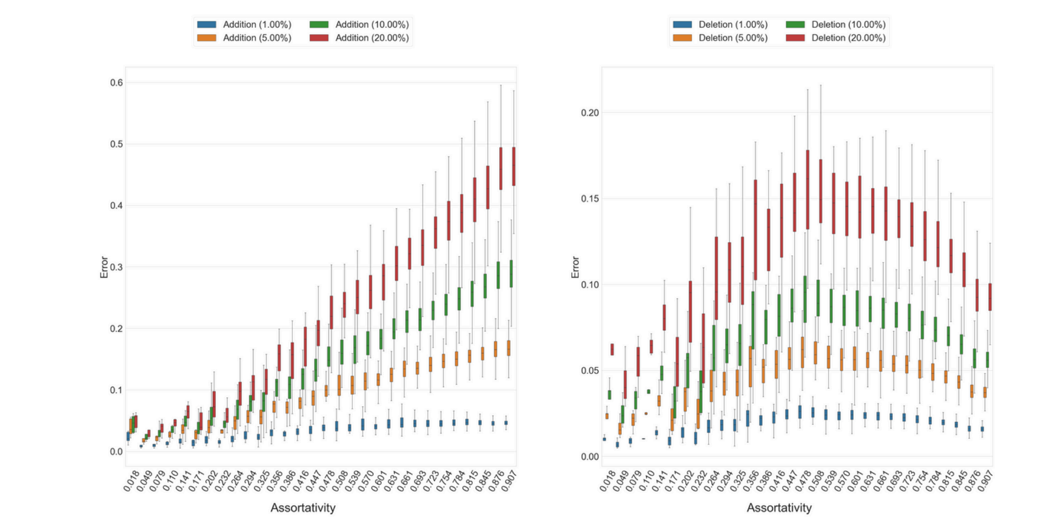

Figure 4: Box plot displaying errors of graphs with different assortativity, for both addition and deletion

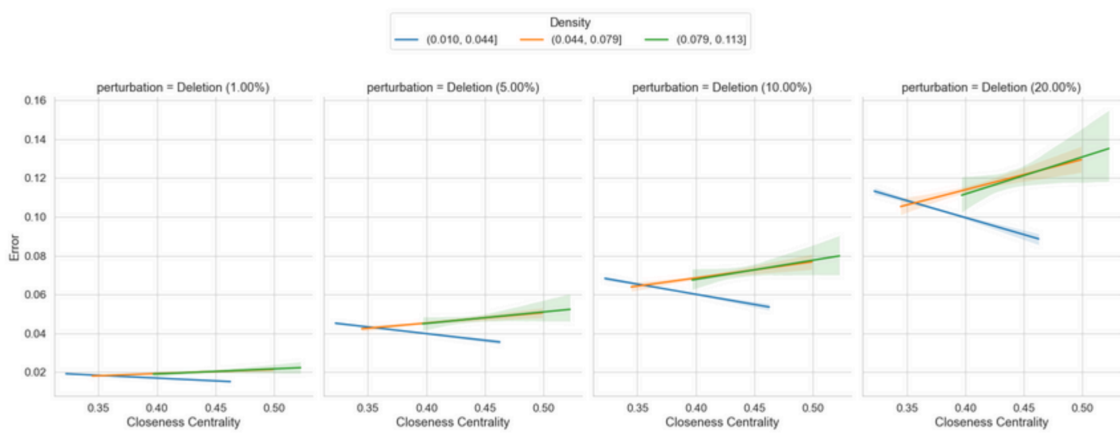

Figure 5: Plots showcasing how the mean error of each graph from deletion perturbation responds to closeness centrality for graphs with different density and perturbation sizes. The lines are fit to the data using linear regression, with the light colour hues around them representing the confidence intervals of 95%.

5. Results

- Expected error from dataset addition: Cora > PubMed > CiteSeer, based on assortativity and centrality.
- **Addition results:** Graphs with more connected components may experience greater impact, explaining result mismatch.
- **Deletion results:** PubMed is most robust, CiteSeer performs worse than Cora despite lower density and higher centrality, suggesting other factors. Area for future research.

Type	Size	Cora	CiteSeer	PubMed
Addition	**1.00%**	0.1493	0.1645	0.1088
	5.00%	0.3286	0.3483	0.2403
	10.00%	0.4397	0.4567	0.3154
	20.00%	0.5656	0.566	0.3965
Deletion	**1.00%**	0.0272	0.0267	0.0221
	5.00%	0.0689	0.079	0.0549
	10.00%	0.1129	0.1354	0.0871
	20.00%	0.2052	0.2553	0.1531
Rewire	**1.00%**	0.0659	0.0775	0.0405
	5.00%	0.1534	0.1732	0.0982
	10.00%	0.2251	0.2445	0.1491
	20.00%	0.3338	0.3458	0.2287

- **Note on addition:** The error gap between core and CiteSeer decreasing for higher perturbations are supportive of the results on Figure 4, since assortativity is more detrimental for larger perturbations.

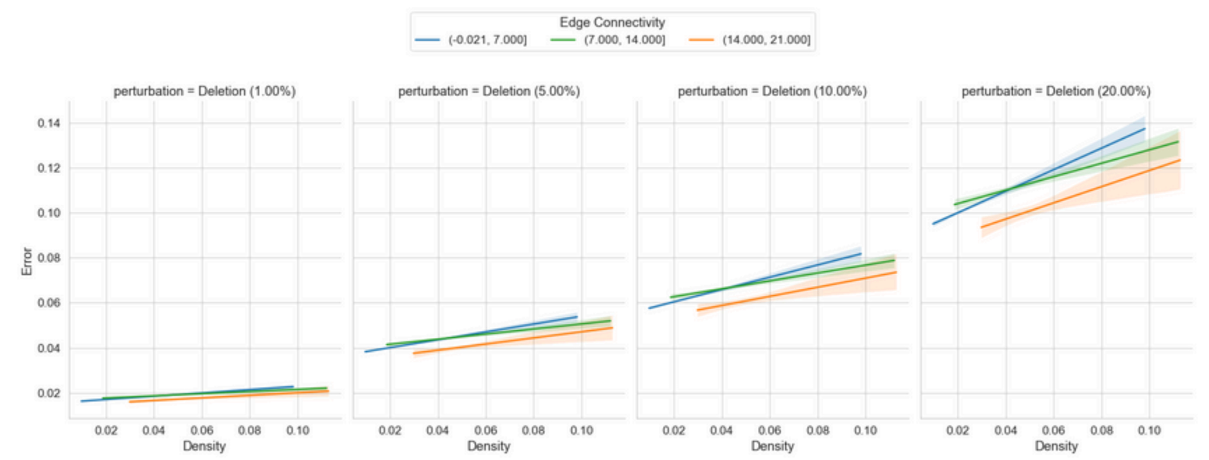

Figure 6: Plots showcasing how the mean error of each graph, due to deletion perturbation, responds to density for graphs with different edge connectivity and perturbation sizes.

- **Key Takeaways:**
  - The range of the errors of a GCN obtained from different perturbations is correlated with its vulnerability to **adversarial** perturbations, which are small perturbations introduced explicitly in order to cause a large deviation in output
  - Stability against addition is inversely correlated with assortativity while deletion is somewhat directly correlated. After a certain threshold of assortativity is achieved, the relationship changes.
  - High density is detrimental to graph stability, as it leads to more nodes being disconnected when an edge is removed
  - Similarly, the interaction of closeness centrality and density is highly dependent on the graph's density.
    - For low density graphs, the information loss from deletion is not very high and high closeness centrality has a stabilizing effect
    - As density gets higher, higher centrality means more nodes are reachable via a shorter path, which possibly acts as a catalyst to the destabilizing effects of high density

6. Conclusion

- Analyzed relationship between graph properties and GNN stability
- Focused on TAGConv, a GCN model
- Measured stability against three perturbation types: addition, deletion, rewiring
- **Synthetic Dataset Analysis:**
  - Used SBM and LFR graph generation models
  - More assortative graphs:
    - Vulnerable to addition perturbations
    - Resilient to deletion perturbations
  - Density and centrality showed opposite behavior
  - Rewiring effects combined traits of addition and deletion
- **Real-World Dataset Analysis:**
  - Inconsistencies likely due to untested features like number of connected components
  - Synthetic conclusions mostly applicable, especially for deletion perturbations
- **Conclusions:**
  - Preliminary analysis on impact of graph properties on GNN stability
  - Serves as a baseline for estimating GNN stability with heuristics

7. Future Work

- Exploration of additional graph properties beyond those analyzed.
- Investigation into the number of connected components and their impact on stability against perturbations.
- Analysis of graph properties that were not possible to sample, such as high density and high diameter.
- Extension of research to include different GNN architectures like Graph Attention Networks (GAT) and ChebNet.
- Examination of how various graph topologies affect the stability of different GNN tasks.
- Preliminary studies by team members:
  - Alex Brown: Impact of different perturbation types and strategies on TAGConv GNN stability.
  - Vladimir Rullens: Stability properties of GNNs trained for different tasks
  - Khoa Nguyen: Stability properties of different GNN architectures

8. References

[1] F. Gama, E. Isufi, G. Leus, and A. Ribeiro, 'Graphs, Convolutions, and Neural Networks', CoRR, vol. abs/2003.03777, 2020.

[2] H. Kenlay, D. Thanou, and X. Dong, 'Interpretable Stability Bounds for Spectral Graph Filters', CoRR, vol. abs/2102.09587, 2021.

[3] J. Du, S. Zhang, G. Wu, J. M. F. Moura, and S. Kar, 'Topology adaptive graph convolutional networks', CoRR, vol. abs/1710.10370, 2017.

[4] J. Palowitch, A. Tsitsulin, B. Mayer, and B. Perozzi, 'GraphWorld: Fake Graphs Bring Real Insights for GNNs', in Proceedings of the 28th ACM SIGKDD Conference on Knowledge Discovery and Data Mining, 2022.

[5] B. Karrer and M. E. J. Newman, 'Stochastic blockmodels and community structure in networks', Phys. Rev. E, vol. 83, p. 016107, Jan. 2011.

[6] A. Lancichinetti, S. Fortunato, and F. Radicchi, 'Benchmark graphs for testing community detection algorithms', Physical Review E, vol. 78, no. 4, Oct. 2008.