Practice Assignment PA06

Sets

Exercise 6–1

Determine whether each of these pairs of sets is equal.

(1) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}, \{5, 3, 1\}$ (2) $\{\{1\}\}, \{1, \{1\}\}$ (3) $\emptyset, \{\emptyset\}$

Exercise 6–2

Determine whether each of these statements is True or False. If an item is partially incorrect or mathematically not well-formed, the entire item should be considered False.

(a) $0 \in \emptyset$
(b) $\emptyset \in \{0\}$
(c) $\{0\} \subset \emptyset$
(d) $\emptyset \subset \{0\}$
(e) $\{0\} \in \{0\}$
(f) $\{0\} \subset \{0\}$
(g) $\emptyset \subseteq \emptyset$
(h) $\emptyset \in \{\emptyset\}$
(i) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
(j) $\{\emptyset\} \in \{\emptyset\}$
(k) $\{\emptyset\} \in \{\{\emptyset\}\}$
(l) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
(m) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
(n) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$
(o) $x \in \{x\}$
(p) $\{x\} \subseteq \{x\}$
(q) $\{x\} \in \{x\}$
(r) $\{x\} \in \{\{x\}\}$
(s) $\emptyset \subseteq \{x\}$
(t) $\emptyset \in \{x\}$
(u) $|A \times B| \leq |B| \times |A|$
(v) $|\{A \times B\}| \in \{|B| \times |A|\}$

Exercise 6–3

Suppose $A = \{a, b, c\}$. Mark each statement True or False.

(a) $\{b, c\} \in \mathcal{P}(A)$
(b) $\{\{a\}\} \subseteq \mathcal{P}(A)$
(c) $\emptyset \subseteq A$
(d) $\{\emptyset\} \subseteq \mathcal{P}(A)$
(e) $\emptyset \subseteq A \times A$
(f) $\{a, c\} \in A$
(g) $\{a, b\} \in A \times A$
(h) $(c, c) \in A \times A$

Exercise 6–4

Find two sets A and B such that $A \in B$ and $A \subseteq B$.

Exercise 6–5

What is the cardinality of each of these sets?

(1) $\emptyset$
(2) $\{\emptyset\}$
(3) $\{\{\emptyset\}\}$
(4) $\{\emptyset, \{\emptyset\}\}$

(5) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

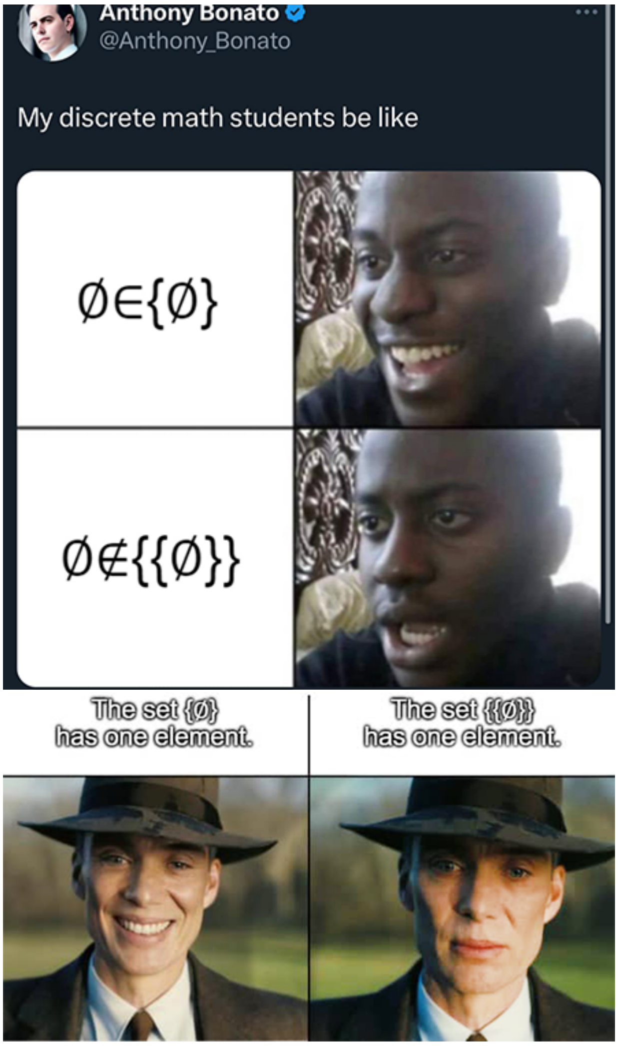
Exercise 6–6

How many elements does each of these sets have where a and b are distinct elements?

(1) $\mathcal{P}(\{a, b, \{a, b\}\})$

(2) $\mathcal{P}(\{\emptyset, a, \{a\}, \{\{a\}\}\})$

(3) $\mathcal{P}(\mathcal{P}(\emptyset))$



Exercise 6–7

Determine whether each of the given sets is the power set of some set. If the given set is a power set, give the set of which it is a power set. Consider a and b as distinct elements.

(1) $\emptyset$

(2) $\{\emptyset, \{a\}\}$

(3) $\{\emptyset, \{a, \emptyset\}\}$

(4) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$

(5) $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

(6) $\{\emptyset,\ \{\emptyset\},\ \{a\},\ \{\{a\}\},\ \{\{\{a\}\}\},\ \{\emptyset, a\},\ \{\emptyset, \{a\}\},\ \{\emptyset, \{\{a\}\}\},\ \{a, \{a\}\},\ \{a, \{\{a\}\}\},\ \{\{a\}, \{\{a\}\}\},\ \{\emptyset, a, \{a\}\},\ \{\emptyset, a, \{\{a\}\}\},\ \{\emptyset, \{a\}, \{\{a\}\}\},\ \{a, \{a\}, \{\{a\}\}\},\ \{\emptyset, a, \{a\}, \{\{a\}\}\}\}$

Exercise 6–8

Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by giving a containment proof (that is, prove that the left side is a subset of the right side and that the right side is a subset of the left side).

Exercise 6–9

Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by giving a proof using logical equivalence.

Exercise 6–10

Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ if and only if $A \subseteq B$.

Exercise 6–11

Prove or disprove each of the following:

(1) $A - (B \cap C) = (A - B) \cup (A - C)$.

(2) If A, B, and C are sets, then $A - (B \cap C) = (A - B) \cap (A - C)$.

(3) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$.

(4) If A and B are two sets with the same power set, then we can conclude that $A = B$.

Exercise 6–12

Show that if $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$

Exercise 6–13

What is the Cartesian Product $A \times B$, where A is the set of courses offered by the mathematics department at a university and B is the set of mathematics professors at this university? Give an example of how this Cartesian product can be used.

Exercise 6–14

Suppose that $A \times B = \emptyset$, where A and B are sets. What can you conclude?

Exercise 6–15

Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

(1) $A \times B \times C$.

(2) $C \times B \times A$.

(3) $C \times A \times B$.

(4) $B \times B \times B$.

Exercise 6–16

Find A^3 if

(1) $A = \{a\}$

(2) $A = \{0, a\}$

Exercise 6–17

How many different elements does $A \times B \times C$ have if A has m elements, B has n elements, and C has p elements?

Exercise 6–18

Explain why $(A \times B) \times (C \times D)$ and $A \times (B \times C) \times D$ are not the same.

Exercise 6–19

Translate each of these quantifications into English and determine its truth value.

(1) $\exists x \in \mathbb{R}(x^3 = -1)$

(2) $\exists x \in \mathbb{Z}(x + 1 > x)$

(3) $\forall x \in \mathbb{Z}(x - 1 \in \mathbb{Z})$

(4) $\forall x \in \mathbb{Z}(x^2 \in \mathbb{Z})$

Exercise 6–20

Find the truth set of each of these predicates where the domain is the set of integers.

(1) $p(x) : x^3 \geq 1$

(2) $Q(x) : x^2 = 2$

(3) $R(x) : x < x^2$

Exercise 6–21

Let A and B be sets. Show that

(1) $(A \cap B) \subseteq A$.

(2) $A \subseteq (A \cup B)$.

(3) $A - B \subseteq A$.

(4) $A \cap (B - A) = \emptyset$.

(5) $A \cup (B - A) = A \cup B$

Exercise 6–22

Let A, B, and C be sets. Show that

(1) $(A \cup B) \subseteq (A \cup B \cup C)$.

(2) $(A \cap B \cap C) \subseteq (A \cap B)$.

(3) $(A - B) - C \subseteq A - C$.

(4) $(A - C) \cap (C - B) = \emptyset$.

(5) $(B - A) \cup (C - A) = (B \cup C) - A$.

Exercise 6–23

Show that if A and B are sets, then

(1) $A - B = A \cap \bar{B}$

(2) $(A \cap B) \cup (A \cap \bar{B}) = A$