A General Introduction to Game Theory: A Dichotomy Approach*

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Abstract. Submissions to Problem Set 1 for COMPSCI/ECON 206 Computational Microeconomics, 2022 Spring Term (Seven Week - Second) instructed by Prof. Luyao Zhang at Duke Kunshan University.
[luyao] This is the version with general feedback.
Overleaf Hotkeys

Keywords: computational economics · game theory · innovative education.

1 Part I: Self-Introduction (2 points)

[luyao] **comments**: Most of you did great for this part. However, for a better production quality, you must consider the following revisions.

- Put all figures in a sub-folder for better files managements
- Simplify the name/label of the figure file and keep it consistent with photo number and title. (no space and special characters)
- Some of you used the comment function to change the default color, which is not conventional for general color changes
- Check out more latex color functions here: using the xcolor packages. You can see that I am playing with page and background colors

[luyao] **instructions**

- insert your professional photo with number, title, and labels
- insert your short bio (around 100 words)
- make your name in a color that is not the default black

[luyao] **more hints** Please try to avoid rasterized images for line-art diagrams and schemas. Whenever possible, use vector graphics instead.

* Supported by Duke Kunshan University

[luyao] **examples**

Fig. 1. Instructor: Luyao Zhang

Short bio: Luyao (Sunshine) Zhang is Assistant Professor of Economics and Senior Research Scientist at the Data Science Research Center at Duke Kunshan University (DKU). She has an abiding passion for *interdisciplinary collaborations*, especially those related to *Computational Economics* (Algorithmic Game Theory and Mechanism Design), *Artificial Intelligence* (Machine Learning, AI Trust, Human-Computer Interaction), *Cryptoeconomics* (Blockchain for social good, DeFi, and Consensus Algorithms), *Behavioral Science* (Bounded Rationality, Trust, and Cooperation), and *Interdisciplinary Big Data* (Social Media, Sustainability, and Global Health). Her publications appear in economic journals for general interest and beyond, including *American Economic Review: Papers and Proceedings*, *the Review of Economics and Statistics, the World Economy*, *Nature Scientific Data*, *ACM CCS*, *Remote Sensing*, *Journal of Digital Earth*, *Data and Information Management*, etc. (Source: Duke Scholar)

2 Part II: Reflections on Game Theory (5 points)

[luyao] **comments**: For this part, you should experiment the 4 citation styles functions listed in the instruction.
[luyao] **instructions**

- describe the major milestones of game theory by citing the original authors' seminal publications. (around 150 words)
- you must provide in-text citations by experimenting the following nabib package functions:
  - [1]
  - Neumann and Morgenstern [1]
  - Neumann and Morgenstern

- 1947

- you must have all citations in the end bibliography using the latex functions
- you must have a .bib file uploaded that follows the *IEEE Style* strictly.
- you must have all in-text citation in hyperlink that directs us to the original source online.

[luyao] **examples**

- 1928: Game theory did not exist as a unique field until Von Neumann [2] published the paper On the Theory of Games of Strategy in 1928.[2]
- 1944: The publication of the book "Theory of Games and Economic Behavior" by mathematician John von Neumann and economist Oskar Morgenstern by Princeton University Press, the groundbreaking text that crated the interdisciplinary research field of game theory.[3]
- 1951: John F. Nash, Nobel Prize Laureates in 1994, introduced the distinction between cooperative games, in which binding agreement can be made, and non-cooperative games, where binding agreements are not feasible. Nash developed an equilibrium concepts for non-cooperative games that later came to be called Nash Equilibrium.[4]
- 1965, Reinhard Selten: Nobel Prize Laureates in 1994, was the first to refine the Nash Equilibrium concept for analyzing dynamic strategic interaction. He has also applied these reined concepts to analyses of competition with only a few sellers.[5]
- 1967: John C. Harsanyi, Nobel Prize Laureates in 1994, showed how games of incomplete information can be analyzed, thereby providing a theoretical foundation for a lively field of research - the economics of information - which focuses on strategic situations where difference agents do not know each other's objectives.[6]
- 1985: Thomas C. Schelling Nobel Prize Laureates in 1985, Schelling's work prompted new developments in game theory and accelerated its use and application throughout the social sciences.[7]
- 1985: Robert J. Aumann, Nobel Prize Laureates in 1985, Robert Aumann was the first to conduct a full-fledged formal analysis of so-called infinitely repeated games. His research identified exactly what outcomes can be upheld over time in long-run relations.[7]

3 Part III: Nash Equilibrium: Definition, Theorem, and Proof (3 points)

[luyao] **comments**: For future endeavors, I suggest the following:

1. try to make the definitions concise and self-content
2. add formal in-text citations of textbook using the nabib package
3. add page numbers for any textbook or book citations (must mention chapter, pages, and the original definition/theorem number)
4. For more instructors, refer to https://en.wikibooks.org/wiki/LaTeX and LaTeX/Mathematics. For example, for most math functions to work, you need to import a package $\backslash usepackage\{amsmath\}$, $\backslash usepackage\{amsfonts\}$, and $\backslash usepackage\{amssymb\}$
5. For how to stylize the real number symbol, refer to: *https://www.physicsread.com/latex-real-number/*

[luyao] **instructions**

- provide the definition, theorem, and proof from the two text books (source must be cited) on Nash Equilibrium
- you must utilize the headings of subsection, subsubsection, and paragraph to structure the section
- you must use the definition, theorem, and proof environment
- you must provide basic discussions to compare the definition, theorem, and proof
- you can refer to the instructions for more styling options: *https://www.overleaf.com/learn/latex/Theorems_and_proofs*

[luyao] **examples**

3.1 Nash Equilibrium: The definitions

3.1.1. The Economist Perspectives

Refer to Textbook: Osborne, Martin J. and Ariel Rubinstein. 1994. A Course in Game Theory (Chapter 2, Page 14, DEFINITION 14.1)

Definition 1 (Nash Equilibrium). *A* ***Nash Equilibrium*** *of a strategic game $\langle N, A_i, (\succeq_i)\rangle$ is a profile $a^* \in A$ of actions with the property that for every player $i \in N$, we have:*

$$(a^*_{-i}, a^*_i) \succeq_i (a^*_{-i}, a_i), \forall \in A_i.$$

And, a strategic game $\langle N, A_i, (\succeq_i)\rangle$ consist of:

- *a finite set N as the set of players*
- *for each player $i \in N$, a nonempty set A_i as the set of actions available to player i*
- *for each player $i \in N$, a preference relation $\succeq_i$ on $A = \times_{j\in N} A_j$*

3.1.2. The Computer Scientist Perspectives

Refer to Textbook: Shoham, Yoav, and Kevin Leyton-Brown. 2008. Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations. Cambridge: Cambridge University Press. (Chapter 3, Page 62, Definition 3.3.4)

Definition 2 (Nash Equilibrium). *A strategy profile $s^* = (s^*_1, ..., s^*_n) \in S$ is a* ***Nash Equilibrium*** *of a normal for game (N, A, μ) if, $\forall$ agents i, s^*_i is a best response to s^*_{-i}:*

$$\mu_i(s^*_i, s^*_{-i}) \geq \mu_i(s_i, s^*_{-i}), \forall - i.$$

And a normal game (N, A, μ) consist of:

- *N, a finite set of n players, indexed by i*
- *$A = A_1 \times ...A_n$, where A_i is a finite set of actions available to player i. Each vector $a = (a_1, ..., a_n) \in A$ is called an action profile; the set of mixed strategy for player i is $S_i = \prod(A_i)$, where for any set X, $\prod(X)$ denotes the set of all probability distributions over X*
- *$\mu = (\mu_1, ..., \mu_n)$ where $\mu_i : A \mapsto \mathbb{R}$*

3.2 Nash Equilibrium: The thereom

3.2.1. The Economist Perspectives

Refer to Textbook: Osborne, Martin J. and Ariel Rubinstein. 1994. A Course in Game Theory (Chapter 3, Page: 33, Proposition 33.1)

Proposition 1. *Every finite strategic game has a mixed strategy Nash Equilibrium.*

Proof. Let $G = \langle N, A_i, (\succeq_i) \rangle$ be a strategic game, and for each player i let m_i be the number of members of the set A_i. Then we can identify the set $\delta(A_i)$ of player i's mixed strategy with the set of vectors (p_1, p_{m_i}) for which $p_k \geq 0$ for all k and $\sum_{k=1}^{m_i} p_k = 1$ (p_k being the probability with which player i uses his kth pure strategy). This set is nonempty, convex, and compact. Since expected payoff is linear in the probabilities, each player's payoff function in the mixed extension of G is both quasi-concave in his own strategy and continuous. Thus the mixed extension of G satisfies all the requirements of Proposition 20.3.

3.2.2. The Computer Scientist Perspectives

Refer to Textbook: Shoham, Yoav, and Kevin Leyton-Brown. 2008. Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations. Cambridge: Cambridge University Press. (Chapter 3, Page 72, Theorem 3.3.22 (Nash 1951))

Theorem 1. *Every game with a finite number of players and action profiles has at least one Nash equilibrium.*

Proof. Given a strategy profile $s \in S$, for all $i \in N$ and $a_i \in A_i$ we define

$$\varphi_{i,a_i} = max(0, u_i(a_i, s_{-i}) - u_i(s)).$$

We then define the function $f : S \mapsto S$ by $f(s) = s'$, where

$$s'_i(a_i) = \frac{s_i(a_i) + \varphi_{i,a_i}(s)}{\sum_{b_i \in A_i} s_i(b_i) + \varphi_{i,b_i}(s)}$$

$$= \frac{s_i(a_i) + \varphi_{i,a_i}(s)}{1 + \sum_{b_i \in A_i} + \varphi_{i,b_i}(s)}$$

Intuitively, this function maps a strategy profile s to a new strategy profile s' in which each agent's actions that are better responses to s receive increased probability mass.

The function f is continuous since each φ_{i,a_i} is continuous. Since S is convex and compact and $f : S \mapsto S$, f must have at least one fixed point. We must now show that the fixed points of f are the Nash equilibria.

First, if s is a Nash equilibrium then all φ's are 0, making s a fixed point of f.Conversely, consider an arbitrary fixed point of f, s. By the linearity of expectation there must exist at least one action in the support of s, say a'_i, for which $u_{i,a'_i(s)} \leq u_i(s)$. From the definition of φ, $\varphi_{i,a'_i(s)} = 0$. Since s is a fixed point of f, $s'_i(a'_i) = s_i(a'_i)$. Consider Equation (3.5), the expression defining

$s_i'(a_i')$. The numerator simplifies to $s_i(a_i')$, and is positive since a_i' is in i's support. Hence the denominator must be 1. Thus for any i and $b_i \in A_i$, $\varphi_{i,b_i}(s)$ must equal 0. From the definition of φ, this can occur only when no player can improve his expected payoff by moving to a pure strategy. Therefore, s is a Nash equilibrium.

[luyao] **more hints**

3.3 A Subsection Sample

Please note that the first paragraph of a section or subsection is not indented. The first paragraph that follows a table, figure, equation etc. does not need an indent, either.

Subsequent paragraphs, however, are indented.

Sample Heading (Third Level) Only two levels of headings should be numbered. Lower level headings remain unnumbered; they are formatted as run-in headings.

Sample Heading (Fourth Level) The contribution should contain no more than four levels of headings. Table 1 gives a summary of all heading levels.

Definition 3 (Nash Equilibrium). *type definition here*

Theorem 2. *This is a sample theorem. The run-in heading is set in bold, while the following text appears in italics. Definitions, lemmas, propositions, and corollaries are styled the same way.*

Proof. Proofs, examples, and remarks have the initial word in italics, while the following text appears in normal font.

4 Part IV: Game Theory Glossary Tables (5 points)

[luyao] **comments**:
[luyao] **instructions**

- create a glossary table for the basic game theory glossaries by citing the original article
- you must cite the original publication that defines the glossaries (not the text books)
- you must at least include the following glossaries: game theory, non-cooperative game theory, cooperative game theory, normal form game, extensive form game, Nash Equilibrium, Bayesian Nash Equlibrium, sub-game Perfect Nash Equlibrium, Perfect Nash Equlibrium

[luyao] **more hints**

Table 1. Table captions should be placed above the tables.

Glossary	Definition	Sources
Title (centered) second line	**Lecture Notes**	14 point, bold
1st-level heading	**1 Introduction**	12 point, bold
2nd-level heading	**2.1 Printing Area**	10 point, bold
3rd-level heading	**Run-in Heading in Bold.** Text follows	10 point, bold
4th-level heading	*Lowest Level Heading.* Text follows	10 point, italic

[luyao] **examples**

Table 2. Game Theory Glossary Tables

Glossary	Definition	Sources
Game Theory	Game theory is the study of mathematical models of strategic interactions among rational agents.	Myerson [10]
Non-comparative Game Theory	Non-comparative Game Theory is a mixed-strategy Nash equilibrium for any game with a finite set of actions and prove that atleast one (mixed-strategy) Nash equilibrium must exist in such a game.	Nash [11]
cooperative Game Theory	"A theory of n-person cooperative games. . . This theory is based on an analysis of the interrelationships of the various coalitions which can be formed bythe players of the game"	Neumann and Morgenstern [1]
Normal form game	Normal form game is a much more simple special one, which was nevertheless shown to be fully equivalent to the former (extensive form).	Neumann and Morgenstern [1]
Nash Equilibrium	A steady state of the play of a strategic game in which each player holds the correctexpectation about the other players' behavior and acts rationally	Nash [4]
Bayesian Nash Equilibrium	A strategy profile that maximizes the expected payoff for each player given their beliefs and given the strategies played by the other players	Harsanyi [6]
Sub-game Perfect Nash Equilibrium	A strategy profile is a subgame perfectequilibrium if it represents a Nash equilibrium of every subgame of the original game	Selten [5]
Perfect Bayesian Equilibrium	In a PBE, (P) the strategies form a Bayesian equilibrium for each continuation game, given the specified beliefs, and (B) beliefs are updated from period to period in accordance with Bayes rule whenever possible, and satisfy a "no-signaling-what-you-don't-know" condition.	Fudenberg and Tirole [12]
Evolutionary Bayesian Equilibrium	studies players who adjust their strategies over time according to rules that are not necessarily rational or farsighted.	Newton [13]

Bibliography

[1] V. Neumann and O. Morgenstern, *Theory of Games and Economic Behavior. (Second edition.)*. Princeton University Press, 1947. [Online]. Available: https://www.amazon.com/Theory-Games-Economic-Behavior-Commemorative/dp/0691130612

[2] J. Von Neumann, "Zur theorie der gesellschaftsspiele," *Mathematische Annalen*, vol. 100, pp. 295–320, 12 1928. [Online]. Available: https://cs.uwaterloo.ca/~y328yu/classics/vonNeumann.pdf

[3] J. Von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*. Princeton University Press, 1944. [Online]. Available: https://press.princeton.edu/books/paperback/9780691130613/theory-of-games-and-economic-behavior

[4] J. Nash, "Non-cooperative games," *The Annals of Mathematics*, vol. 54, p. 286, 09 1951. [Online]. Available: https://www.cs.vu.nl/~eliens/download/paper-Nash51.pdf

[5] R. Selten, "Spieltheoretische behandlung eines oligopolmodells mit nachfrageträgheit," *Zeitschrift für die gesamte Staatswissenschaft : ZgS*, vol. 121, 1965. [Online]. Available: https://www.econbiz.de/Record/spieltheoretische-behandlung-eines-oligopolmodells-mit-nachfragetr%C3%A4gheit-selten-reinhard/10002792917

[6] J. C. Harsanyi, "Games with incomplete information played by "bayesian" players," *Management Science*, vol. 14, pp. 159–182, 11 1967. [Online]. Available: http://www.dklevine.com/archive/refs41175.pdf

[7] A. R.J., "What is game theory trying to accomplish?" *Frontiers of Economics*, vol. Basil Blackwell, Oxford, 5, 1985. [Online]. Available: https://www.scirp.org/(S(351jmbntvnsjt1aadkozje))/reference/referencespapers.aspx?referenceid=2126572

[8] M. J. Osborne and A. Rubinstein, *A course in game theory*. MIT press, 1994. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0899825699907236

[9] Y. Shoham and K. Leyton-Brown, *Multiagent systems: Algorithmic, game-theoretic, and logical foundations*. Cambridge University Press, 2008. [Online]. Available: https://dl.acm.org/doi/pdf/10.1145/1753171.1753181

[10] R. B. Myerson, *Game Theory: Analysis of Conflict.* Harvard University Press, 1991.

[11] F. Nash, J, *Non-Cooperative Games.* Princeton University Press, 1950.

[12] D. Fudenberg and J. Tirole, "Perfect bayesian equilibrium and sequential equilibrium," *journal of Economic Theory*, vol. 53, no. 2, pp. 236–260, 1991.

[13] J. Newton, "Evolutionary game theory: A renaissance," *Games*, vol. 9, no. 2, p. 31, 2018.