A General Introduction to Game Theory: A Dichotomy Approach*

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Abstract. Submissions to Problem Set 1 for COMPSCI/ECON 206 Computational Microeconomics, 2022 Spring Term (Seven Week - Second) instructed by Prof. Luyao Zhang at Duke Kunshan University.

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1 Part I: Self-Introduction

Bio: Ray Zhu is a graduating student from the inaugural undergraduate class of Duke Kunshan University and Duke University. He is an economist and a young global leader living in a community of over 70 countries and regions. His academic interest lies in investment, behavioral finance, and FinTech and blockchain. He loves movies, jogging, cooking, traveling, and seafood.

Fig. 1. Ray Zhu's Professional Photo

* Supported by Duke Kunshan University

2 Part II: Reflections on Game Theory

- 1928, Game theory did not exist as a unique field until John von Neumann published the paper On the Theory of Games of Strategy in 1928.[1]
- 1944, The publication of the book "Theory of Games and Economic Behavior" by mathematician John von Neumann and economist Oskar Morgenstern by Princeton University Press, the groundbreaking text that crated the interdisciplinary research field of game theory.[2]
- 1951, John F. Nash, Nobel Prize Laureates in 1994, introduced the distinction between cooperative games, in which binding agreement can be made, and non-cooperative games, where binding agreements are not feasible. Nash developed an equilibrium concepts for non-cooperative games that later came to be called Nash Equilibrium.[3]
- 1965, Reinhard Selten, Nobel Prize Laureates in 1994, was the first to refine the Nash Equilibrium concept for analyzing dynamic strategic interaction. He has also applied these reined concepts to analyses of competition with only a few sellers.[4]
- 1967, John C. Harsanyi, Nobel Prize Laureates in 1994, showed how games of incomplete information can be analyzed, thereby providing a theoretical foundation for a lively field of research - the economics of information - which focuses on strategic situations where difference agents do not know each other's objectives.[5]
- Robert J. Aumann, Nobel Prize Laureates in 2005,Schelling's work prompted new developments in game theory and accelerated its use and application throughout the social sciences. [6]
- Thomas C. Schelling,Nobel Prize Laureates in 2005,Robert Aumann was the first to conduct a full-fledged formal analysis of so-called infinitely repeated games. His research identified exactly what outcomes can be upheld over time in long-run relations.[6]

3 Part III: Nash Equilibrium: Definition, Theorem, and Proof

3.1 Understand Nash Equilibrium

Definition, Theorem, and Proof We will go over the definition, theorem, and proof of Nash Equilibrium from our textbooks to help understand the theory and relevant concepts.

Reference Textbook: *Shoham, Yoav, and Kevin Leyton-Brown.* 2009. Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations. Cambridge: Cambridge University Press.

Definition 1 (Nash Equilibrium). *A strategy profile $s = (s_1, ..., s_n)$ is a Nash equilibrium if, for all agents i, s_i is a best response to s_{-i} Nash equilibrium.*

Theorem 1 (Nash, 1951). *Every game with a finite number of players and action profiles has at least one Nash equilibrium.*

Proof.

$$s_i^{'} = \frac{s_i\left(a_i\right) + \phi_{i,\alpha_i}\left(s\right)}{\Sigma_{b_i \in A_i} s_i\left(b_i\right) + \phi_{i,b_i}\left(s\right)} \tag{1}$$

Intuitively, this function maps a strategy profile s to a new strategy profile $s^{'}$ in which each agent's actions that are better responses to s receive increased probability mass.

The function f is continuous since each ϕ_{i,a_i} is continuous. Since S is convex and compact and $f : S \to S$, by Corollary 3.3.21 f must have at least one fixed point. We must now show that the fixed points of f are the Nash equilibria.

First, if s is a Nash equilibrium then all ϕ's are 0, making s a fixed point of f.

Conversely, consider an arbitrary fixed point of f , s. By the linearity of expectation there must exist at least one action in the support of s, say $a_i^{'}$, for which $u_{i,a_i^{'}} \leq u_i\left(s\right)$. From the definition of $\phi, \phi_{i,a_i^{'}}\left(s\right) = 0$. Since s is a fixed point of f, $s_i^{'}\left(a_i^{'}\right) = s_i\left(a_i^{'}\right)$. Consider Equation (3.5), the expression defining $s_i^{'}\left(a_i^{'}\right)$. The numerator simplifies to $s_i\left(a_i^{'}\right)$, and is positive since $a_i^{'}$ is in i's support. Hence the denominator must be 1. Thus for any i and $b_i \in A_i$, $\phi_{i,b_i}\left(s\right)$ must equal 0. From the definition of ϕ, this can occur only when no player can improve his expected payoff by moving to a pure strategy. Therefore, s is a Nash equilibrium.

4 Part IV: Game Theory Glossary Table

Table 1. Game Theory Glossary Table

Glossary	Definition	Sources
game theory	Game theory is the study of mathematical models of strategic interactions among rational agents	(Von Neumann 1928)[1]
non-cooperative game theory	"Our theory, in contradistinction, is based on the absence of coalitions in that it is assumed that each participant acts independently, without collaboration or communication with any of the other"	(Nash 1951)[3]
cooperative game theory	"A theory of n-person cooperative games... This theory is based on an analysis of the interrelationships of the various coalitions which can be formed by the players of the game"	(von Neumann and Morgenstern 1944)[2]
normal form game	The normal-form representation of a game includes all perceptible and conceivable strategies, and their corresponding payoffs, for each player	(von Neumann and Morgenstern 1944)[2]
extensive form game	An extensive-form game is a specification of a game in game theory, allowing (as the name suggests) for the explicit representation of a number of key aspects	(von Neumann and Morgenstern 1944)[2]
Nash Equilibrium	A solution (an equilibrium set) to a non-cooperative game where players, knowing the playing strategies of their opponents, have no incentive to change their strategy	(Nash 1951)[3]
Bayesian Nash Equlibrium	Bayesian game means a strategic game with incomplete information	(Harsanyi 1967)[5]
sub-game Perfect Nash Equlibrium	A strategy profile is a subgame perfect equilibrium if it represents a Nash equilibrium of every subgame of the original game	(Selten 1965)[4]

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