Assignment 8 - Math 448

Assignment 8: Due 3/10 by email

A planet follows an elliptical orbit, which can be represented in a Cartesian (x, y) coordinate system by the equation $ay^2 + bxy + cx + dy + e = x^2$.

1. Using Linear Least Squares, determine the orbital parameters a, b,c,d,e given the following observations (as (x, y)-coordinates). (1.02,.39), (.95,.32), (.87,.27), (.77,.22), (.67,.18), (.56,.15), (.44,.13), (.30,.12), (.16,.13), (.01,.15)
Plot the resulting orbit for the given data points.

2. This least squares problem is nearly rank deficient. To see what effect this has on the solution, perturb the input data slightly by adding to each coordinate of each data point a random number drawn uniformly from the interval [-.005,.005]. and solve the least squares problem with the perturbed data. Compare the new values for the parameters with those previously computed. What effect does this difference have on the plot of the orbit? Can you explain this behavior?

3. Find the singular value decomposition for the 10×5 matrix in the original linear least squares problem. Use the singular value decomposition to compute the linear least squares problem using the first k singular values only, for $k = 1, \ldots 5$. For each of the 5 solutions plot the corresponding orbit along with the given data points.

4. Perturb the input data again, as in part 2. Compute the singular value decomposition of the new least squares matrix, and solve the least squares problem with the perturbed data, as in part 3, using k singular values. Compare the new values for the parameters with those previously computed in part 1 for each value of k. What effect des this difference have on the plots of the orbits? Can you explain this behavior? Which solution would you regard as better: one that fits the data more closely or one than is less sensitive to small perturbations in the data? Why?