#### examination scheduling

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#### outline

Problem

Formulation of Constraints

Objective Function

Improvements and Next Steps

# problem

## problem

Find a good examination schedule for the exam period of the  $\mathsf{TUM}$ 

• Each exam is planed in exactly one period

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- In each room there is only one exam at a time

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- Rooms for an exam are minimized
- Time between exams is maximized

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formulation of constraints

#### model - variables

$$x_{i,k,l} := \begin{cases} 1, & \text{if exam } i \text{ is written in period } l \text{ in room } k \\ 0, & \text{otherwise} \end{cases}$$

$$y_{i,l} := \begin{cases} 1, & \text{if exam } i \text{ is written in period } l \\ 0, & \text{otherwise} \end{cases}$$

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## connecting variables

Connecting variables x and y

(1) 
$$\sum_{\forall \text{ rooms } k} x_{i,k,l} \leq y_{i,l} \cdot M \qquad \forall \text{ exams } i, \ \forall \text{ periods } l$$

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(1) 
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(2) 
$$\sum_{\forall \text{ rooms } k} x_{i,k,l} \ge y_{i,l} \qquad \forall \text{ exams } i, \ \forall \text{ periods } l$$

## each exam is planned

Each exam is planned in exactly one period

(3) 
$$\sum_{\forall \text{ periods } I} y_{i,I} = 1 \qquad \forall \text{ exams } i$$

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c.f. (2) 
$$\sum_{\forall \text{ rooms } k} x_{i,k,l} \ge y_{i,l} \qquad \forall \text{ exams } i, \forall \text{ periods } l$$

#### there are enough seats

There must be enough seats for the students in the chosen rooms

$$(4) \sum_{\substack{\forall \text{ periods } I, \\ \forall \text{ rooms } k}} c_k \cdot x_{i,k,l} \geq s_i \qquad \forall \text{ exams } i$$

 $s_i := \#$  students taking exam i.  $c_k := \#$  seats in room k.

#### one exam at a time

In each room there is only one exam at a time

(5) 
$$\sum_{\forall \text{ exams } i} x_{i,k,l} \leq 1 \qquad \forall \text{ rooms } k, \forall \text{ periods } l$$

#### no conflicts

No student has to write two exams at the same time

(6) 
$$\sum_{\substack{\forall \text{ exams } j:\\ i,j \text{ have a conflict}}} y_{j,l} \leq (1-y_{i,l}) \cdot M \qquad \forall \text{ exams } i, \ \forall \text{ period}$$

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# objective function

## objective function

Minimize the total number of rooms

minimize 
$$\sum_{\text{exams } i \text{ rooms } k} \sum_{\text{periods } l} x_{i,k,l}$$

#### maximize time between exams

Maximize the time between two exams

$$\max \sum_{j>i:q_{i,j}>0} d_{i,j}$$

 $d_{i,j} := Distance between exams i and j$ 

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#### objective functions combined

Combine objective functions using weighting factor  $\gamma>0$ 

$$\min \sum_{i=1}^{n} \sum_{k=1}^{r} \sum_{l=1}^{p} x_{i,k,l} - \gamma \sum_{j>i:q_{i,j}>0} d_{i,j}$$

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improvements and next steps

## finding a feasible starting point

- Use graph-coloring to schedule exams without conflicts
- Plan difficult exams first
- Calculate difficulty by:
  - number of students taking exam
  - Identifying cliques in conflict graph

#### model improvements

#### Clique Constraints

(7) 
$$\sum_{\text{j in clique}} y_{i,l} \leq 1 \quad \forall \text{cliques calculated from conflict graph}$$

## data