Examination Scheduling

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May 25, 2016

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Outline

PROBLEM

Problem

Find a good examination schedule for the exam period of the TUM.

Criteria for a good examination schedule

- Each exam is planned in exactly one period

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- Rooms for an exam are minimized

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- Each exam is planned in exactly one period
- In each room there is only one exam at a time
- There are enough seats for each exam
- No student has to write two exams at the same time
- Rooms for an exam are minimized
- Time between exams is maximized

MODELING THE PROBLEM

Variables

$$x_{i,k,l} := \begin{cases} 1, & \text{if exam i is written in period l in room k} \\ 0, & \text{otherwise} \end{cases}$$

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$$y_{i,l} := \begin{cases} 1, & \text{if exam i is written in period l} \\ 0, & \text{otherwise} \end{cases}$$

Constraints

- Connecting the variables x and y:

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$$\text{(1)} \sum_{\text{rooms k}} \mathrm{x}_{\mathrm{i,k,l}} \quad \leq \mathrm{y}_{\mathrm{i,l}} \cdot \mathrm{M} \quad \forall \text{ exams i}, \forall \text{ periods l}$$

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$$(2) \quad \sum_{\text{rooms } k} \mathrm{x}_{\mathrm{i,k,l}} \geq \mathrm{y}_{\mathrm{i,l}} \qquad \forall \text{ exams i}, \forall \text{ periods l}$$

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$$\text{c.f. } (2) \quad \sum_{\text{rooms k}} x_{i,k,l} \quad \geq y_{i,l} \qquad \forall \text{ exams i}, \forall \text{ periods l}$$

Constraints

- There are enough seats for the students in the exam rooms:

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$$(4) \quad \sum_{\substack{\text{periods } l, \\ \text{rooms } k}} c_k \cdot x_{i,k,l} \geq s_i \qquad \forall \text{ exams } i$$

$$s_i := \#\text{ students taking exam } i$$
$$c_k := \#\text{ seats in room } k$$

Constraints

- In every room there is only one exam at a given time:

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$$(5)\ \sum_{\text{exams } i} x_{i,k,l} \qquad \leq 1 \qquad \forall \text{ rooms } k, \forall \text{ periods } l$$

Constraints

- There are no conflicts for students taking multiple exams:

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$$(6) \quad \sum_{\substack{\text{exams j,} \\ \text{j conflicts with i}}} y_{j,l} \quad \leq (1 - y_{i,l}) \cdot M \quad \forall \text{ exams i}, \forall \text{ periods l}$$

Objective function

- Minimize the total number of rooms:

$$\text{(a)}\min \sum_{\substack{\text{exams } i,\\ \text{rooms } k,\\ \text{periods } l}} x_{i,k,l}$$

Objective function

- Maximize the time between two exams:

$$\text{(b)}\ \max \sum_{\substack{\text{exams i, j} \\ \text{i conflicts with j}}} \mathrm{d}_{\mathrm{i,j}}$$

$$\mathrm{d}_{\mathrm{i,j}} := \text{distance between exams i and j}$$

Objective function

- Combine the previous two objective functions using a weighting factor $\gamma > 0$:

$$(\mathrm{a} + \mathrm{b}) \min \sum_{\substack{\text{exams i,} \\ \text{rooms k,} \\ \text{periods l}}} \mathrm{x}_{\mathrm{i,k,l}} - \gamma \cdot \sum_{\substack{\text{exams i, j} \\ \text{i conflicts with j}}} \mathrm{d}_{\mathrm{i,j}}$$

NEXT STEPS

Finding a feasible starting solution

Compute planning order of exams

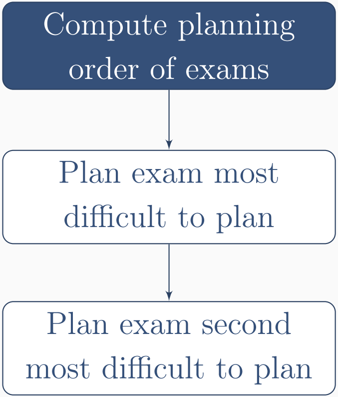
Finding a feasible starting solution

Compute planning order of exams

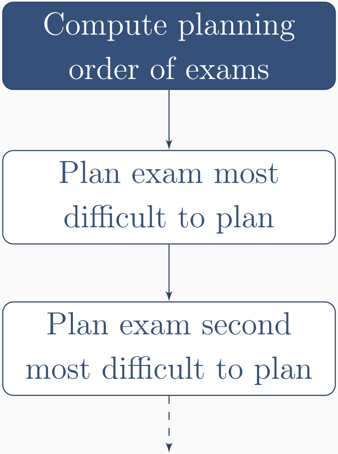
↓

Plan exam most difficult to plan

Finding a feasible starting solution



Finding a feasible starting solution



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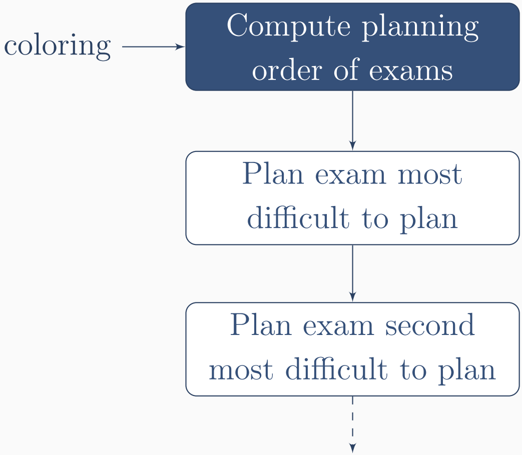

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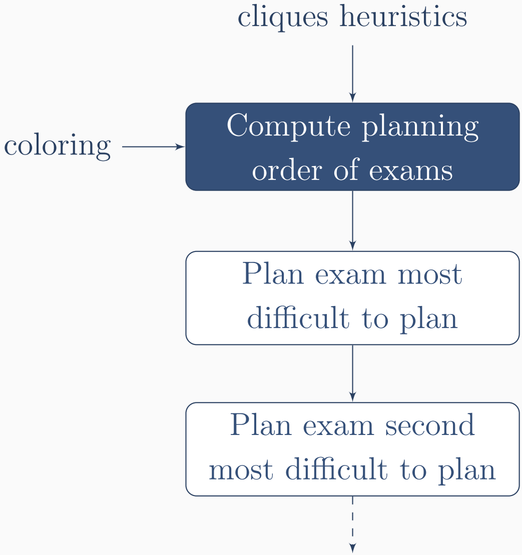

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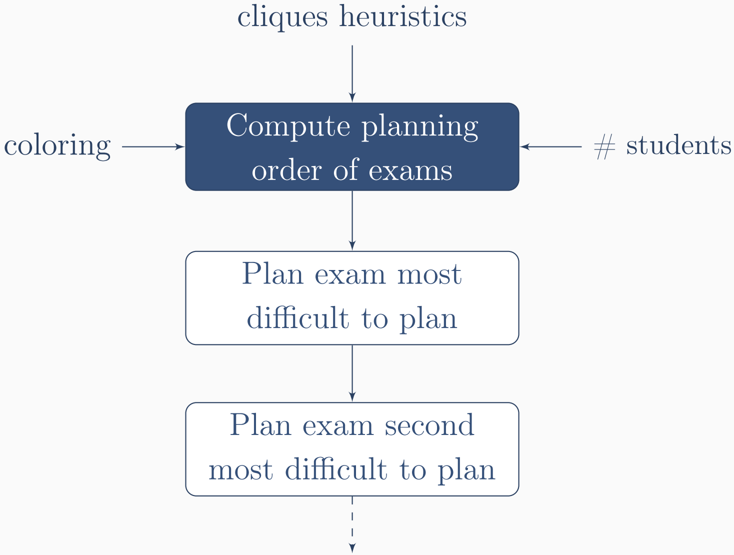

Improvements to the model

- Add Clique Constraints:

$$(7) \sum_{\text{i in clique}} y_{i,l} \qquad \leq 1 \qquad \forall \text{cliques}$$

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- Remove absolute value in the objective function.

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- Remove absolute value in the objective function.
- Improve running time using heuristics, pre-solving, etc.
- Using a path based model and column generation

Data acquisition

Figure 1: Server, hopefully somewhere in Germany