# EXAMINATION SCHEDULING

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# OUTLINE

Problem

ILP Formulation

Improvements and Next Steps

# PROBLEM

Find a good examination schedule for the exam period of the TUM

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6. Time between exams is maximized

# ILP FORMULATION

$$x_{i,k,l} := \begin{cases} 1, & \text{if exam i is written in period l in room k} \\ 0, & \text{sonst} \end{cases}$$

$$y_{i,l} := \begin{cases} 1, & \text{if exam i is written in period l} \\ 0, & \text{sonst} \end{cases}$$

0. Connecting variables x and y

$$(1) \qquad \sum_{\forall \text{ rooms } k} x_{i,k,l} \leq y_{i,l} \cdot M \qquad \forall \text{ exams } i, \forall \text{ periods } l$$

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$$(2) \qquad \sum_{\forall \text{ rooms } k} x_{i,k,l} \geq y_{i,l} \qquad \forall \text{ exams } i, \forall \text{ periods } l$$

# EACH EXAM IS PLANNED

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2. There must be enough seats for the students in the chosen rooms

$$(4) \qquad \sum_{\substack{\forall \text{ periods } l, \\ \forall \text{ rooms } k}} c_k * x_{i,k,l} \geq s_i \qquad \forall \text{ exams } i$$

$$s_i := \# \text{ students taking exam i.}$$

$$c_k := \# \text{ seats in room k.}$$

3. In each room there is only one exam at a time

$$(5) \qquad \sum_{\forall \text{ exams } i} x_{i,k,l} \leq 1 \qquad \forall \text{ rooms } k, \forall \text{ periods } l$$

4. No student has to write two exams at the same time

$$(6) \qquad \sum_{\substack{\forall \text{ exams j:} \\ \text{i,j have a conflict}}} y_{j,l} \leq (1 - y_{i,l}) * M \qquad \forall \text{ exams i, } \forall \text{ periods l}$$

TODO

# IMPROVEMENTS AND NEXT STEPS

## HEURISTICS – FINDING A GOOD STARTING SOLUTION

- Plan exams that are difficult to plan first
- Caculate difficulty by:
  - number of students taking exam
  - Identifying cliques in conflict graph
- Use graph-coloring to identify exams that can be written in parallel

1. Clique Constraints

$$(7) \qquad \sum_{\text{j in clique}} \mathrm{y}_{\mathrm{i,l}} \leq 1 \quad \forall \text{cliques calculated from conflict graph}$$

# CONSTANTS

$$
\begin{aligned}
s_i &:= \text{Number of students signed up for exam i.} \\
c_k &:= \text{Number of available seats in the lecture room k.} \\
Q &:= \text{Kollisionsmatrix} \\
q_{i,j} &:= \begin{cases} 0, & \text{falls Prüfung i und j gleichzeitig stattfinden können} \\ 1, & \text{sonst} \end{cases} \\
T &:= \text{Sperrmatrix} \\
t_{i,j} &:= \begin{cases} 1, & \text{falls Raum k zum Zeitintervall l geöffnet ist} \\ 0, & \text{sonst} \end{cases} \\
h_l &:= \text{Anzahl der Stunden von Periode l nach Beginn des Prüfungszeitraume}
\end{aligned}
$$

# VARIABLEN

$$x_{i,k,l} := \begin{cases} 1, & \text{wenn Prüfung i zum Zeitpunkt l in Raum k stattfindet} \\ 0, & \text{sonst} \end{cases}$$

$$y_{i,l} := \begin{cases} 1, & \text{wenn Prüfung i im Zeitinterval l stattfindet} \\ 0, & \text{sonst} \end{cases}$$

Dimensionen:

$$n : AnzahlderPrfungen$$

$$r : AnzahlderRume$$

$$p : AnzahlderZeitintervalle$$

1. Verknüpfung der Variablen

$$\sum_{k=1}^{r} x_{i,k,l} \leq y_{i,l} \cdot r \quad \forall i \in [n] \forall l \in [p]$$

$$\sum_{k=1}^{r} x_{i,k,l} \geq y_{i,l} \quad \forall i \in [n] \forall l \in [p]$$

2. Jede Prüfung wird auf genau einem Zeitinterval eingeplant

$$\sum_{l=1}^{p} y_{i,l} = 1 \quad \forall i \in [n]$$

3. Konfliktvermeidung

$$\sum_{j=1, j>i}^{n} q_{i,j} y_{j,l} \leq (1 - y_{i,l}) \sum_{\nu=1}^{n} q_{i,\nu} \quad \forall i \in [n], \forall l \in [p]$$

5. Alle Studierenden bekommen einen Platz

$$\sum_{l=1}^{p}\sum_{k=1}^{r} c_k x_{i,k,l} \geq s_i \quad \forall i \in [n]$$

6. Jedem Raum wird je Zeit maximal eine Prüfung zugeteilt

$$\sum_{i=1}^{n} x_{i,k,l} \leq t_{k,l} \quad \forall k \in [r], \forall l \in [p]$$

7. Clique Constraints

$$\sum_{jinclique} y_{i,l} \leq 1 \quad \forall l \in [p]$$

$$\min \sum_{i=1}^{n} \sum_{k=1}^{r} \sum_{l=1}^{p} s_i x_{i,k,l} - \gamma \min_{j>i: q_{i,j}>0} |\Delta h_{i,j}|$$

where $\Delta h_{i,j} := \sum_{l=1}^{p} h_l (y_{i,l} - y_{j,l})$. Resolving abs:

$$\begin{aligned}
\min \sum_{i=1}^{n} \sum_{k=1}^{r} s_i x_{i,k} - \gamma w \\
\text{s.t.} \quad z_{i,j} &\leq \Delta h_{i,j} + \delta_{i,j}(h_p - h_1) \quad \forall i,j \in [n] \\
z_{i,j} &\leq -\Delta h_{i,j} + (1 - \delta_{i,j})(h_p - h_1) \quad \forall i,j \in [n] \\
z_{i,j} &\geq \Delta h_{i,j} \quad \forall i,j \in [n] \\
z_{i,j} &\geq -\Delta h_{i,j} \quad \forall i,j \in [n] \\
w &\geq z_{i,j} \forall i,j \in [n]
\end{aligned}$$

http://lpsolve.sourceforge.net/5.1/absolute.htm