Examination Scheduling

Alexander Eckl, Maximilian Fiedler, Mickael Grima,
Roland Halbig

May 25, 2016

Technische Universität München

Outline

PROBLEM

Problem

Find a good examination schedule for the exam period of the TUM.

Criteria for a good examination schedule

- Each exam is planned in exactly one period

Criteria for a good examination schedule

- Each exam is planned in exactly one period
- In each room there is only one exam at a time

Criteria for a good examination schedule

- Each exam is planned in exactly one period
- In each room there is only one exam at a time
- There are enough seats for each exam

Criteria for a good examination schedule

- Each exam is planned in exactly one period
- In each room there is only one exam at a time
- There are enough seats for each exam
- No student has to write two exams at the same time

Criteria for a good examination schedule

- Each exam is planned in exactly one period
- In each room there is only one exam at a time
- There are enough seats for each exam
- No student has to write two exams at the same time
- Rooms for an exam are minimized

Criteria for a good examination schedule

- Each exam is planned in exactly one period
- In each room there is only one exam at a time
- There are enough seats for each exam
- No student has to write two exams at the same time
- Rooms for an exam are minimized
- Time between exams is maximized

MODELING THE PROBLEM

Variables

$$x_{i,k,l} := \begin{cases} 1, & \text{if exam i is written in period l in room k} \\ 0, & \text{otherwise} \end{cases}$$

Variables

$$x_{i,k,l} := \begin{cases} 1, & \text{if exam i is written in period l in room k} \\ 0, & \text{otherwise} \end{cases}$$

$$y_{i,l} := \begin{cases} 1, & \text{if exam i is written in period l} \\ 0, & \text{otherwise} \end{cases}$$

Constraints

- Connecting the variables x and y:

Constraints

- Connecting the variables x and y:

$$(1) \sum_{\text{rooms k}} x_{i,k,l} \quad \leq y_{i,l} \cdot M \quad \forall \text{ exams i}, \forall \text{ periods l}$$

Constraints

- Connecting the variables x and y:

$$(1) \quad \sum_{\text{rooms k}} \mathrm{x}_{\mathrm{i,k,l}} \leq \mathrm{y}_{\mathrm{i,l}} \cdot \mathrm{M} \qquad \forall \text{ exams i, } \forall \text{ periods l}$$

$$(2) \quad \sum_{\text{rooms k}} \mathrm{x}_{\mathrm{i,k,l}} \geq \mathrm{y}_{\mathrm{i,l}} \qquad \forall \text{ exams i, } \forall \text{ periods l}$$

Constraints

- Each exam is planned in exactly one period:

Constraints

- Each exam is planned in exactly one period:

$$(3) \sum_{\text{periods l}} \mathrm{y}_{\mathrm{i,l}} \qquad = 1 \qquad \forall \text{ exams i}$$

- Each exam is planned in exactly one period:

$$(3) \quad \sum_{\text{periods l}} \mathrm{y_{i,l}} \qquad = 1 \qquad \forall \text{ exams i}$$

$$\text{c.f.} \quad (2) \quad \sum_{\text{rooms k}} \mathrm{x_{i,k,l}} \qquad \geq \mathrm{y_{i,l}} \qquad \forall \text{ exams i}, \forall \text{ periods l}$$

Constraints

- There are enough seats for the students in the exam rooms:

Constraints

- There are enough seats for the students in the exam rooms:

$$(4) \quad \sum_{\substack{\text{periods } l,\\ \text{rooms } k}} c_k \cdot x_{i,k,l} \geq s_i \qquad \forall \text{ exams } i$$

$$s_i := \ \# \text{ students taking exam } i$$
$$c_k := \ \# \text{ seats in room } k$$

Constraints

- In every room there is only one exam at a given time:

Constraints

- In every room there is only one exam at a given time:

$$(5) \quad \sum_{\text{exams i}} \mathrm{x}_{\mathrm{i,k,l}} \qquad \leq 1 \qquad \forall \text{ rooms k}, \forall \text{ periods l}$$

Constraints

- There are no conflicts for students taking multiple exams:

Constraints

- There are no conflicts for students taking multiple exams:

$$(6) \quad \sum_{\substack{\text{exams j,}\\ \text{j conflicts with i}}} \mathrm{y}_{\mathrm{j,l}} \quad \leq (1 - \mathrm{y}_{\mathrm{i,l}}) \cdot \mathrm{M} \quad \forall \text{ exams i}, \forall \text{ periods l}$$

Objective function

- Minimize the total number of rooms:

$$\text{(a)} \min \sum_{\substack{\text{exams } i,\\ \text{rooms } k,\\ \text{periods } l}} x_{i,k,l}$$

Objective function

- Maximize the time between two exams:

$$\text{(b)}\max \sum_{\substack{\text{exams } i,j \\ i \text{ conflicts with } j}} d_{i,j}$$

$$d_{i,j} := \text{distance between exams } i \text{ and } j$$

Objective function

- Combine the previous two objective functions using a weighting factor $\gamma > 0$:

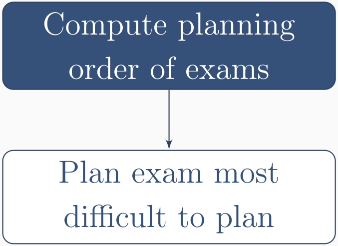
$$(a+b)\min \sum_{\substack{\text{exams } i,\\ \text{rooms } k,\\ \text{periods } l}} x_{i,k,l} - \gamma \cdot \sum_{\substack{\text{exams } i,j\\ i \text{ conflicts with } j}} d_{i,j}$$

NEXT STEPS

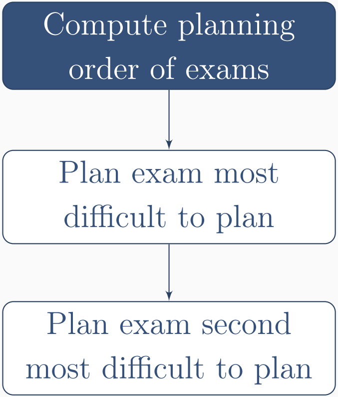
Finding a feasible starting solution

Compute planning order of exams

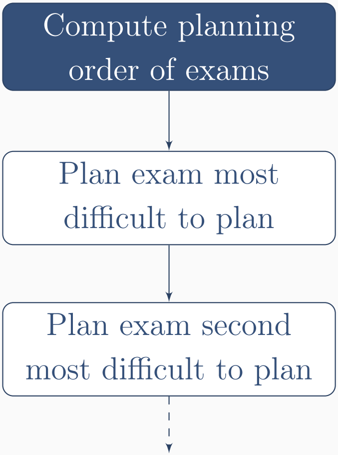
Finding a feasible starting solution



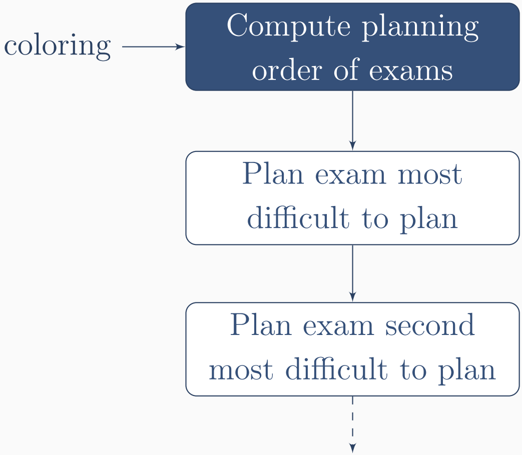
Finding a feasible starting solution



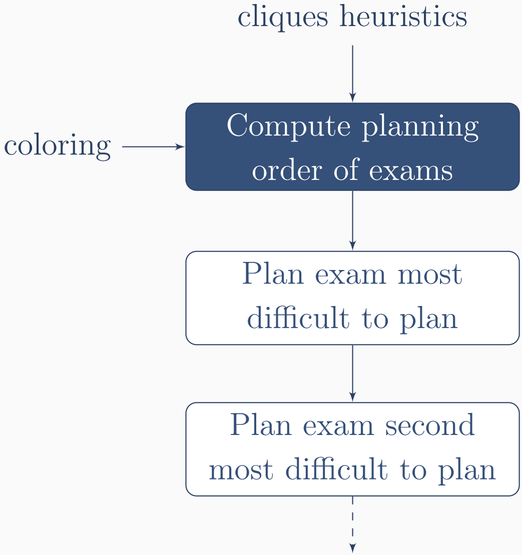
Finding a feasible starting solution



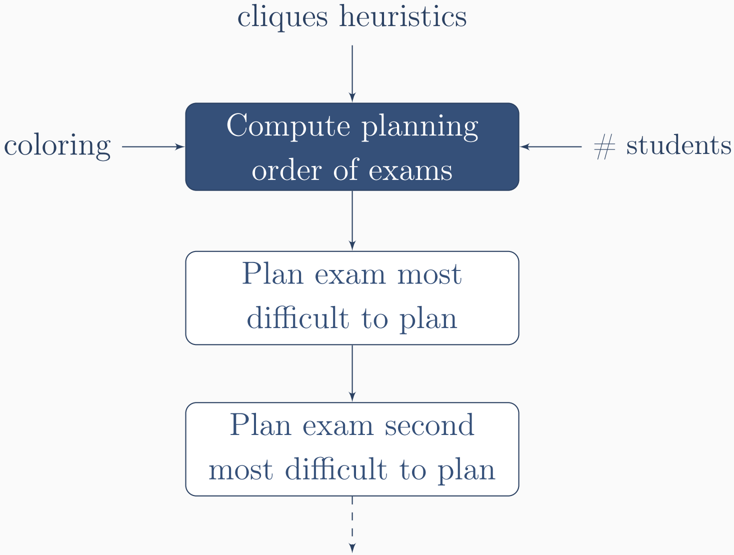
Finding a feasible starting solution



Finding a feasible starting solution



Finding a feasible starting solution



Improvements to the model

- Add Clique Constraints:

$$(7) \sum_{\text{i in clique}} \mathrm{y}_{\mathrm{i,l}} \qquad \leq 1 \qquad \forall\, \text{cliques}$$

- Add Clique Constraints:

$$(7)\ \sum_{\text{i in clique}} \mathrm{y_{i,l}} \qquad \leq 1 \qquad \forall\,\text{cliques}$$

- Remove absolute value in the objective function.

Improvements to the model

- Add Clique Constraints:

$$(7) \sum_{\text{i in clique}} y_{i,l} \qquad \leq 1 \qquad \forall \text{cliques}$$

- Remove absolute value in the objective function.
- Improve running time using heuristics, pre-solving, etc.

Improvements to the model

- Add Clique Constraints:

$$(7) \sum_{\text{i in clique}} y_{i,l} \qquad \leq 1 \qquad \forall \text{cliques}$$

- Remove absolute value in the objective function.
- Improve running time using heuristics, pre-solving, etc.
- Use a path based model and column generation

Data acquisition

Figure 1: Server, hopefully somewhere in Germany