Examination Scheduling

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Outline

Problem

Modeling The Problem

Next Steps

PROBLEM

Problem

Find a good examination schedule for the exam period of the TUM.

Criteria for a good examination schedule

- Each exam is planned in exactly one period

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Criteria for a good examination schedule

- Each exam is planned in exactly one period
- In each room there is only one exam at a time
- There are enough seats for each exam
- No student has to write two exams at the same time
- Rooms for an exam are minimized
- Time between exams is maximized

MODELING THE PROBLEM

Variables

$$x_{i,k,l} := \begin{cases} 1, & \text{if exam } i \text{ is written in period } l \text{ in room } k \\ 0, & \text{otherwise} \end{cases}$$

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$$y_{i,l} := \begin{cases} 1, & \text{if exam } i \text{ is written in period } l \\ 0, & \text{otherwise} \end{cases}$$

Constraints

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$$(1) \qquad \sum_{\text{rooms } k} x_{i,k,l} \qquad \leq y_{i,l} \cdot M \qquad \forall \text{ exams } i, \forall \text{ periods } l$$

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$$(1) \qquad \sum_{\text{rooms } k} x_{i,k,l} \leq y_{i,l} \cdot M \qquad \forall \text{ exams } i, \forall \text{ periods } l$$

$$(2) \qquad \sum_{\text{rooms } k} x_{i,k,l} \geq y_{i,l} \qquad \forall \text{ exams } i, \forall \text{ periods } l$$

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$$(\text{c.f. } 2) \qquad \sum_{\text{rooms } k} x_{i,k,l} \geq y_{i,l} \qquad \forall \text{ exams } i, \forall \text{ periods } l$$

Constraints

- There are enough seats for the students in the exam rooms:

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$$(4) \qquad \sum_{\substack{\text{periods } l,\\ \text{rooms } k}} c_k \cdot x_{i,k,l} \qquad \geq s_i \qquad \forall \text{ exams } i$$

$$s_i := \# \text{ students taking exam } i$$

$$c_k := \# \text{ seats in room } k$$

Constraints

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$$(5) \qquad \sum_{\text{exams } i} x_{i,k,l} \qquad \leq 1 \qquad \forall \text{ rooms } k, \forall \text{ periods } l$$

Constraints

- There are no conflicts for students taking multiple exams:

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$$(6) \qquad \sum_{\substack{\text{exams } j, \\ j \text{ conflicts with } i}} y_{j,l} \quad \leq (1 - y_{i,l}) \cdot M \quad \forall \text{exams } i, \forall \text{periods } l$$

Objective function

- Minimize the total number of rooms:

Objective function

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(a) $$\min \sum_{\substack{\text{exams } i,\\ \text{rooms } k,\\ \text{periods } l}} x_{i,k,l}$$

Objective function

- Maximize the time between two exams:

Objective function

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(b) $$\max \sum_{\text{exams } i} \min_{\substack{\text{exam } j \\ \text{conflicts with } i}} d_{i,j}$$

$$d_{i,j} := \text{distance between exams i and j}$$

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$$\text{(a,b)} \qquad \min \sum_{\substack{\text{exams } i,\\ \text{rooms } k,\\ \text{periods } l}} x_{i,k,l} \; - \; \gamma \cdot \sum_{\text{exams } i} \min_{\substack{\text{exam } j\\ \text{conflicts with } i}} d_{i,j}$$

NEXT STEPS

Finding a feasible starting solution

Compute planning order of exams

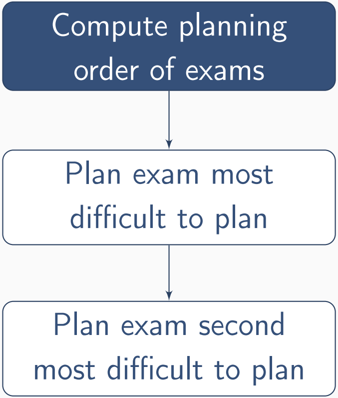
Finding a feasible starting solution

Compute planning order of exams

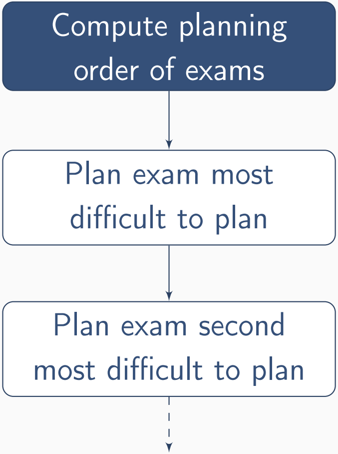
↓

Plan exam most difficult to plan

Finding a feasible starting solution



Finding a feasible starting solution



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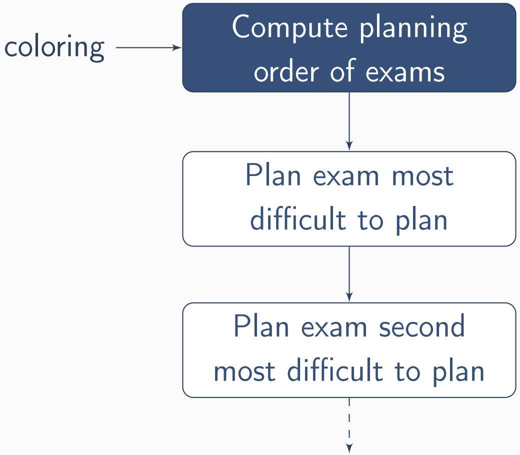

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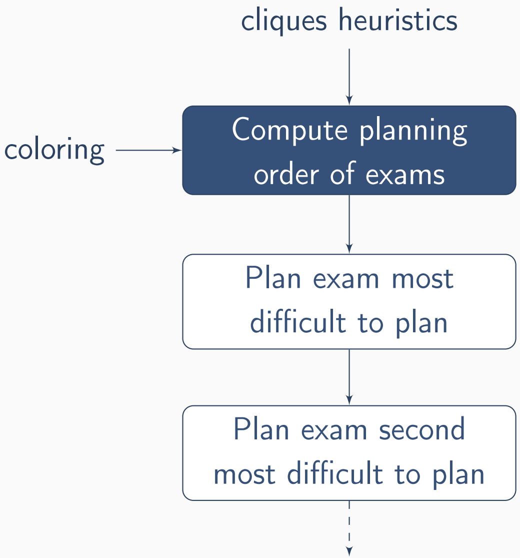

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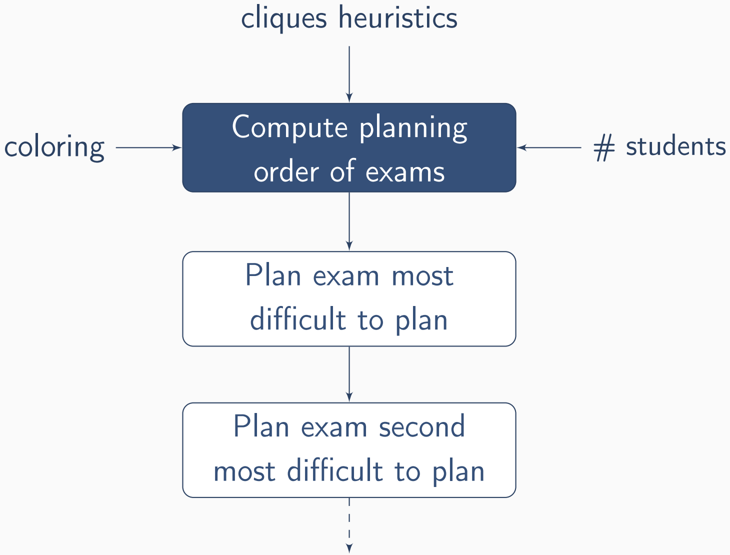

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- Use a path-based model and column generation.

Data acquisition

Figure 1: Server, hopefully somewhere in Germany