Examination Scheduling

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Outline

Problem Formulation

Modeling The Problem

Next Steps

PROBLEM FORMULATION

Problem

Find a good examination schedule for the exam period of the TUM.

Criteria for a good examination schedule

- Each exam is planned in exactly one period

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- In each room there is only one exam at a time
- There are enough seats for each exam
- No student has to write two exams at the same time
- Rooms for an exam are minimized
- Time between exams is maximized

MODELING THE PROBLEM

Variables

$$x_{i,k,l} := \begin{cases} 1, & \text{if exam } i \text{ is written in period } l \text{ in room } k \\ 0, & \text{otherwise} \end{cases}$$

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$$y_{i,l} := \begin{cases} 1, & \text{if exam } i \text{ is written in period } l \\ 0, & \text{otherwise} \end{cases}$$

Constraints

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$$(1) \quad \sum_{\text{rooms } k} x_{i,k,l} \qquad \leq y_{i,l} \cdot M \qquad \forall \text{ exams } i, \forall \text{ periods } l$$

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$$(1) \quad \sum_{\text{rooms } k} x_{i,k,l} \leq y_{i,l} \cdot M \qquad \forall \text{ exams } i, \forall \text{ periods } l$$

$$(2) \quad \sum_{\text{rooms } k} x_{i,k,l} \geq y_{i,l} \qquad \forall \text{ exams } i, \forall \text{ periods } l$$

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Constraints

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$$(4) \quad \sum_{\substack{\text{periods } l,\\ \text{rooms } k}} c_k \cdot x_{i,k,l} \qquad \geq s_i \qquad \forall \text{ exams } i$$

$$s_i := \# \text{ students taking exam } i$$

$$c_k := \# \text{ seats in room } k$$

Constraints

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$$(5) \sum_{\text{exams } i} x_{i,k,l} \qquad \leq 1 \qquad \forall \text{ rooms } k, \forall \text{ periods } l$$

Constraints

- There are no conflicts for students taking multiple exams:

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$$(6) \quad \sum_{\substack{\text{exams } j, \\ j \text{ conflicts with } i}} y_{j,l} \quad \leq (1 - y_{i,l}) \cdot M \quad \forall \text{ exams } i, \forall \text{ periods } l$$

Objective function

- Minimize the total number of rooms:

$$\min \sum_{\substack{\text{exams } i,\\ \text{rooms } k,\\ \text{periods } l}} x_{i,k,l}$$

Objective function

- Maximize the time between two exams:

$$\max \sum_{\substack{\text{exams } i,j \\ i \text{ conflicts with } j}} d_{i,j}$$

$$d_{i,j} := \text{distance between exams i and j}$$

Objective function

- Combine the previous two objective functions using a weighting factor $\gamma > 0$:

$$\min \sum_{\substack{\text{exams } i,\\ \text{rooms } k,\\ \text{periods } l}} x_{i,k,l} - \gamma \cdot \sum_{\substack{\text{exams } i,j\\ i \text{ conflicts with } j}} d_{i,j}$$

NEXT STEPS

Finding a feasible starting point

- Use graph-coloring to schedule exams without conflicts.

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  - Identifying cliques in conflict graph

Improvements to the model

- Add Clique Constraints:

$$(7) \sum_{i \text{ in clique}} y_{i,l} \quad \leq 1 \quad \forall \text{ cliques calculated from the conflict graph}$$

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- Add Clique Constraints:

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- Remove absolute value in the objective function.
- Improve running time using heuristics, pre-solving, etc.

Data acquisition

- -dummy-
- -From exam coordinator for Mathematics-
- -From central exam coordinator-
- -Direct export from TUMonline-