

EXAMINATION SCHEDULING

Alexander Eckl, Maximilian Fiedler, Mickael Grima, Roland Halbig

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Technische Universität München

Problem

ILP Formulation

Improvements and Next Steps

PROBLEM

Find a good examination schedule for the exam period of the TUM

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6. Time between exams is maximized

ILP FORMULATION

$$x_{i,k,l} := \begin{cases} 1, & \text{if exam } i \text{ is written in period } l \text{ in room } k \\ 0, & \text{sonst} \end{cases}$$

$$y_{i,l} := \begin{cases} 1, & \text{if exam } i \text{ is written in period } l \\ 0, & \text{sonst} \end{cases}$$

0. Connecting variables x and y

$$(1) \quad \sum_{\forall \text{ rooms } k} x_{i,k,l} \leq y_{i,l} \cdot M \quad \forall \text{ exams } i, \forall \text{ periods } l$$

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$$(1) \quad \sum_{\forall \text{ rooms } k} x_{i,k,l} \leq y_{i,l} \cdot M \quad \forall \text{ exams } i, \forall \text{ periods } l$$

$$(2) \quad \sum_{\forall \text{ rooms } k} x_{i,k,l} \geq y_{i,l} \quad \forall \text{ exams } i, \forall \text{ periods } l$$

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$$(3) \quad \sum_{\forall \text{ periods } l} y_{i,l} = 1 \quad \forall \text{ exams } i$$

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2. There must be enough seats for the students in the chosen rooms

$$\sum_{\substack{\forall \text{ periods } l, \\ \forall \text{ rooms } k}} c_k * x_{i,k,l} \geq s_i \quad \forall \text{ exams } i$$

$s_i := \#$ students taking exam i .

$c_k := \#$ seats in room k .

3. In each room there is only one exam at a time

$$\sum_{\forall \text{ exams } i} x_{i,k,l} \leq 1 \quad \forall \text{ rooms } k, \forall \text{ periods } l$$

4. No student has to write two exams at the same time

$$\sum_{\substack{\forall \text{ exams } j: \\ i, j \text{ have a conflict}}} y_{j,l} \leq (1 - y_{i,l}) * M \quad \forall \text{ exams } i, \forall \text{ periods } l$$

IMPROVEMENTS AND NEXT STEPS

s_i := Number of students signed up for exam i .

c_k := Number of available seats in the lecture room k .

Q := Kollisionsmatrix

$$q_{i,j} := \begin{cases} 0, & \text{falls Prüfung } i \text{ und } j \text{ gleichzeitig stattfinden können} \\ 1, & \text{sonst} \end{cases}$$

T := Sperrmatrix

$$t_{i,j} := \begin{cases} 1, & \text{falls Raum } k \text{ zum Zeitintervall } l \text{ geöffnet ist} \\ 0, & \text{sonst} \end{cases}$$

h_l := Anzahl der Stunden von Periode l nach Beginn des Prüfungszeitraumes

$$x_{i,k,l} := \begin{cases} 1, & \text{wenn Prüfung } i \text{ zum Zeitpunkt } l \text{ in Raum } k \text{ stattfindet} \\ 0, & \text{sonst} \end{cases}$$

$$y_{i,l} := \begin{cases} 1, & \text{wenn Prüfung } i \text{ im Zeitintervall } l \text{ stattfindet} \\ 0, & \text{sonst} \end{cases}$$

Dimensionen:

n : Anzahl der Prüfungen

r : Anzahl der Räume

p : Anzahl der Zeitintervalle

1. Verknüpfung der Variablen

$$\sum_{k=1}^r x_{i,k,l} \leq y_{i,l} \cdot r \quad \forall i \in [n] \forall l \in [p]$$

$$\sum_{k=1}^r x_{i,k,l} \geq y_{i,l} \quad \forall i \in [n] \forall l \in [p]$$

2. Jede Prüfung wird auf genau einem Zeitintervall eingeplant

$$\sum_{l=1}^p y_{i,l} = 1 \quad \forall i \in [n]$$

3. Konfliktvermeidung

$$\sum_{j=1, j>i}^n q_{i,j} y_{j,l} \leq (1 - y_{i,l}) \sum_{\nu=1}^n q_{i,\nu} \quad \forall i \in [n], \forall l \in [p]$$

5. Alle Studierenden bekommen einen Platz

$$\sum_{l=1}^p \sum_{k=1}^r c_{k,i,k,l} x_{i,k,l} \geq s_i \quad \forall i \in [n]$$

6. Jedem Raum wird je Zeit maximal eine Prüfung zugeteilt

$$\sum_{i=1}^n x_{i,k,l} \leq t_{k,l} \quad \forall k \in [r], \forall l \in [p]$$

7. Clique Constraints

$$\sum_{j \text{ in clique}} y_{i,l} \leq 1 \quad \forall l \in [p]$$

$$\min \sum_{i=1}^n \sum_{k=1}^r \sum_{l=1}^p s_i x_{i,k,l} - \gamma \min_{j>i: q_{i,j}>0} |\Delta h_{i,j}|$$

where $\Delta h_{i,j} := \sum_{l=1}^p h_l(y_{i,l} - y_{j,l})$. Resolving abs:

$$\min \sum_{i=1}^n \sum_{k=1}^r s_i x_{i,k} - \gamma w$$

$$\text{s.t. } z_{i,j} \leq \Delta h_{i,j} + \delta_{i,j}(h_p - h_1) \quad \forall i, j \in [n]$$

$$z_{i,j} \leq -\Delta h_{i,j} + (1 - \delta_{i,j})(h_p - h_1) \quad \forall i, j \in [n]$$

$$z_{i,j} \geq \Delta h_{i,j} \quad \forall i, j \in [n]$$

$$z_{i,j} \geq -\Delta h_{i,j} \quad \forall i, j \in [n]$$

$$w \geq z_{i,j} \quad \forall i, j \in [n]$$

<http://lpsolve.sourceforge.net/5.1/absolute.htm>