Examination Scheduling

Alexander Eckl, Maximilian Fiedler, Mickael Grima,
Roland Halbig

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Technische Universität München

Outline

PROBLEM

Problem

Find a good examination schedule for the exam period of the TUM.

Criteria for a good examination schedule

- Each exam is planned in exactly one period

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- In each room there is only one exam at a time
- There are enough seats for each exam
- No student has to write two exams at the same time
- Rooms for an exam are minimized
- Time between exams is maximized

MODELING THE PROBLEM

Variables

$$x_{i,k,l} := \begin{cases} 1, & \text{if exam i is written in period l in room k} \\ 0, & \text{otherwise} \end{cases}$$

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$$y_{\mathrm{i,l}} := \begin{cases} 1, & \text{if exam i is written in period l} \\ 0, & \text{otherwise} \end{cases}$$

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$$(1) \qquad \sum_{\text{rooms k}} \mathrm{x}_{\mathrm{i,k,l}} \qquad \leq \mathrm{y}_{\mathrm{i,l}} \cdot \mathrm{M} \qquad \forall \text{ exams i}, \forall \text{ periods l}$$

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$$(2) \qquad \sum_{\text{rooms k}} \mathrm{x}_{\mathrm{i,k,l}} \geq \mathrm{y}_{\mathrm{i,l}} \qquad \forall \text{ exams i}, \forall \text{ periods l}$$

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$$(3) \qquad \sum_{\text{periods l}} \mathrm{y_{i,l}} \qquad = 1 \qquad \forall \text{ exams i}$$

Constraints

- There are enough seats for the students in the exam rooms:

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$$(4) \qquad \sum_{\substack{\text{periods } l,\\ \text{rooms } k}} c_k \cdot x_{i,k,l} \geq s_i \qquad \forall \text{ exams } i$$

$$s_i := \# \text{ students taking exam } i$$
$$c_k := \# \text{ seats in room } k$$

Constraints

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$$(5) \qquad \sum_{\text{exams i}} \mathrm{x}_{\mathrm{i,k,l}} \qquad \leq 1 \qquad \forall \text{ rooms k}, \forall \text{ periods l}$$

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$$(6) \qquad \sum_{\substack{\text{exams j,}\\ \text{j conflicts i}}} \mathrm{y_{j,l}} \quad \leq (1 - \mathrm{y_{i,l}}) \cdot \mathrm{M} \quad \forall \text{exams i, } \forall \text{periods l}$$

Objective function

- Minimize the total number of rooms:

Objective function

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(a) $$\min \sum_{\substack{\text{exams } i,\\ \text{rooms } k,\\ \text{periods } l}} x_{i,k,l}$$

Objective function

- Maximize the time between two exams:

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$$\text{(b)} \qquad \max \sum_{\text{exams i}} \min_{\substack{\text{exam j} \\ \text{conflicts with i}}} \mathrm{d_{i,j}}$$

$$\mathrm{d_{i,j}} := \text{distance between exams i and j}$$

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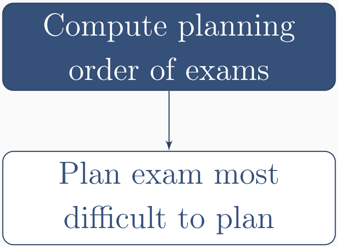
$$\text{(a,b)} \qquad \min \sum_{\substack{\text{exams i,}\\ \text{rooms k,}\\ \text{periods l}}} \mathrm{x}_{\mathrm{i,k,l}} - \gamma \cdot \sum_{\text{exams i}} \min_{\substack{\text{exam j}\\ \text{conflicts with i}}} \mathrm{d}_{\mathrm{i,j}}$$

NEXT STEPS

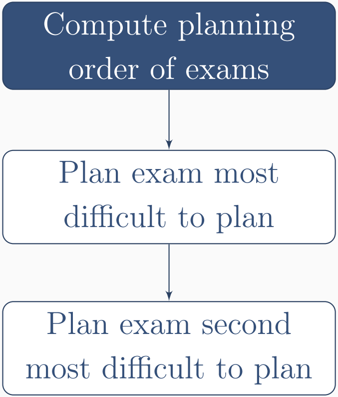
Finding a feasible starting solution

Compute planning order of exams

Finding a feasible starting solution



Finding a feasible starting solution



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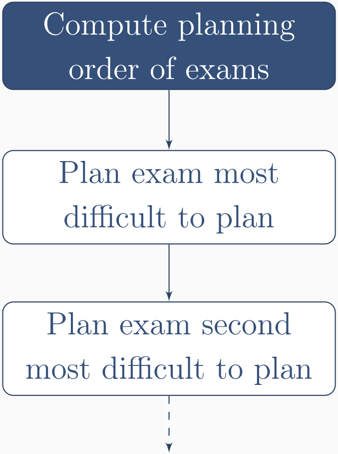

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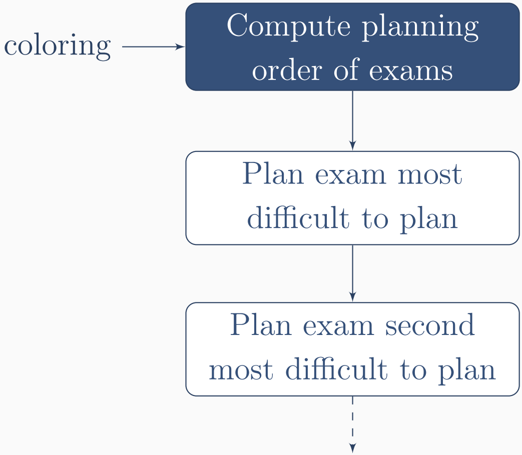

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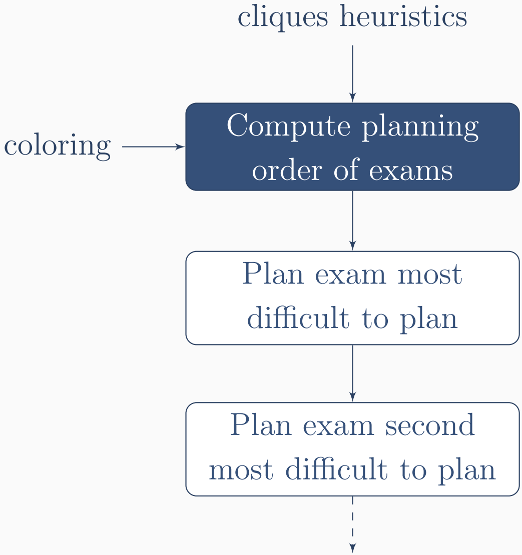

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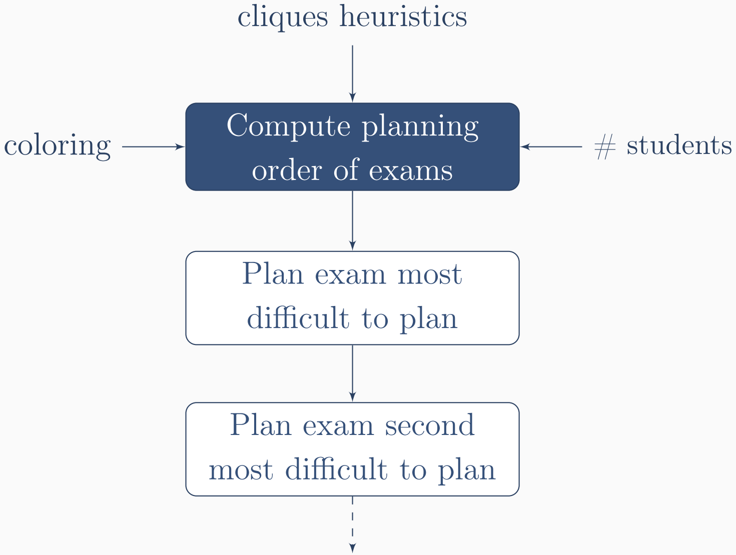

Improvements to the model

- Add clique constraints:

$$(7) \qquad \sum_{\text{i in clique}} \mathrm{y_{i,l}} \qquad \leq 1 \qquad \forall\, \text{cliques}, \forall \text{periods l}$$

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- Use a path-based model and column generation

Data acquisition

Figure 1: Server, hopefully somewhere in Germany