Examination Scheduling

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Outline

PROBLEM FORMULATION

Problem

Find a good examination schedule for the exam period of the TUM.

Criteria for a good examination schedule

- Each exam is planned in exactly one period

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- In each room there is only one exam at a time
- There are enough seats for each exam
- No student has to write two exams at the same time
- Rooms for an exam are minimized
- Time between exams is maximized

MODELING THE PROBLEM

Variables

$$x_{i,k,l} := \begin{cases} 1, & \text{if exam } i \text{ is written in period } l \text{ in room } k \\ 0, & \text{otherwise} \end{cases}$$

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$$y_{i,l} := \begin{cases} 1, & \text{if exam } i \text{ is written in period } l \\ 0, & \text{otherwise} \end{cases}$$

Constraints

- Connecting the variables x and y:

$$(1) \quad \sum_{\text{rooms } k} x_{i,k,l} \quad \leq y_{i,l} \cdot M \quad \forall \text{ exams } i, \forall \text{ periods } l$$

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$$(1) \quad \sum_{\text{rooms } k} x_{i,k,l} \leq y_{i,l} \cdot M \qquad \forall \text{ exams } i, \forall \text{ periods } l$$

$$(2) \quad \sum_{\text{rooms } k} x_{i,k,l} \geq y_{i,l} \qquad \forall \text{ exams } i, \forall \text{ periods } l$$

Constraints

- Each exam is planned in exactly one period:

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$$\text{c.f. } (2) \quad \sum_{\text{rooms } k} x_{i,k,l} \quad \geq y_{i,l} \quad \forall \text{ exams } i, \forall \text{ periods } l$$

Constraints

- There are enough seats for the students in the exam rooms:

$$(4) \quad \sum_{\substack{\text{periods } l, \\ \text{rooms } k}} c_k \cdot x_{i,k,l} \qquad \geq s_i \qquad \forall \text{ exams } i$$

$$s_i := \ \# \text{ students taking exam } i$$

$$c_k := \ \# \text{ seats in room } k$$

Constraints

- In every room there is only one exam at a given time:

$$(5) \quad \sum_{\text{exams } i} x_{i,k,l} \qquad \leq 1 \qquad \forall \text{ rooms } k, \forall \text{ periods } l$$

Constraints

- There are no conflicts for students taking multiple exams:

$$(6) \qquad \sum_{\substack{\text{exams } j, \\ i \text{ conflicts with } j}} y_{j,l} \quad \leq (1 - y_{i,l}) \cdot M \quad \forall \text{ exams } i, \forall \text{ periods } l$$

Objective function

- Minimize the total number of rooms:

$$\min \sum_{\substack{\text{exams } i,\\ \text{rooms } k,\\ \text{periods } l}} x_{i,k,l}$$

Objective function

- Maximize the time between two exams:

$$\max \sum_{\substack{\text{exams } i,j \\ i > j}} d_{i,j}$$

$d_{i,j}$:= distance between exams i and j

Objective function

- Combine the previous two objective functions using a weighting factor $\gamma > 0$:

$$\min \sum_{\substack{\text{exams } i, \\ \text{rooms } k, \\ \text{periods } l}} x_{i,k,l} \; - \; \gamma \cdot \sum_{\substack{\text{exams } i,j \\ i > j}} d_{i,j}$$

NEXT STEPS

Finding a feasible starting point

- Use graph-coloring to schedule exams without conflicts.
- Plan difficult exams first.
- Calculate difficulty by:
  - Number of students taking exam
  - Identifying cliques in conflict graph

Improvements to the model

- Add Clique Constraints:

$$\sum_{\text{j in clique}} y_{i,I} \leq 1 \quad \forall \text{ cliques calculated from conflict graph} \tag{7}$$

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- Add Clique Constraints:

$$(7) \qquad \sum_{\text{j in clique}} y_{i,l} \leq 1 \quad \forall \text{ cliques calculated from conflict graph}$$

- Remove absolute value in the objective function.
- Improve running time using heuristics, pre-solving, etc.

Data acquisition

- -dummy-
- -From exam coordinator for Mathematics-
- -From central exam coordinator-
- -Direct export from TUMonline-