

Examination Scheduling

Alexander Eckl, Maximilian Fiedler, Mickael Grima,
Roland Halbig

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Technische Universität München

Outline

Problem

Modeling The Problem

Next Steps

PROBLEM

Problem

Find a good examination schedule for the exam period of the TUM.

Criteria for a good examination schedule

- Each exam is planned in exactly one period

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- Each exam is planned in exactly one period
- In each room there is only one exam at a time
- There are enough seats for each exam
- No student has to write two exams at the same time
- Rooms for an exam are minimized
- Time between exams is maximized

MODELING THE PROBLEM

Variables

$$x_{i,k,l} := \begin{cases} 1, & \text{if exam } i \text{ is written in period } l \text{ in room } k \\ 0, & \text{otherwise} \end{cases}$$

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$$y_{i,l} := \begin{cases} 1, & \text{if exam } i \text{ is written in period } l \\ 0, & \text{otherwise} \end{cases}$$

Constraints

- Connecting the variables x and y :

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$$(1) \quad \sum_{\text{rooms } k} x_{i,k,l} \leq y_{i,l} \cdot M \quad \forall \text{ exams } i, \forall \text{ periods } l$$

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$$(2) \quad \sum_{\text{rooms } k} x_{i,k,l} \geq y_{i,l} \quad \forall \text{ exams } i, \forall \text{ periods } l$$

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Constraints

- There are enough seats for the students in the exam rooms:

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$$(4) \quad \sum_{\substack{\text{periods } l, \\ \text{rooms } k}} c_k \cdot x_{i,k,l} \geq s_i \quad \forall \text{ exams } i$$

$s_i := \#$ students taking exam i

$c_k := \#$ seats in room k

Constraints

- In every room there is only one exam at a given time:

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$$(5) \quad \sum_{\text{exams } i} x_{i,k,l} \leq 1 \quad \forall \text{ rooms } k, \forall \text{ periods } l$$

Constraints

- There are no conflicts for students taking multiple exams:

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$$(6) \quad \sum_{\substack{\text{exams } j, \\ j \text{ conflicts with } i}} y_{j,l} \leq (1 - y_{i,l}) \cdot M \quad \forall \text{exams } i, \forall \text{periods } l$$

Objective function

- Minimize the total number of rooms:

$$(a) \quad \min \sum_{\substack{\text{exams } i, \\ \text{rooms } k, \\ \text{periods } l}} x_{i,k,l}$$

Objective function

- Maximize the time between two exams:

$$(b) \quad \max \sum_{\substack{\text{exams } i, j \\ i \text{ conflicts with } j}} d_{i,j}$$

$d_{i,j} :=$ distance between exams i and j

Objective function

- Combine the previous two objective functions using a weighting factor $\gamma > 0$:

$$(a,b) \quad \min \sum_{\substack{\text{exams } i, \\ \text{rooms } k, \\ \text{periods } l}} x_{i,k,l} - \gamma \cdot \sum_{\substack{\text{exams } i,j \\ i \text{ conflicts with } j}} d_{i,j}$$

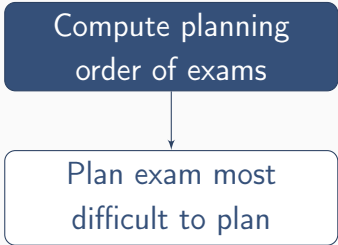
NEXT STEPS

Finding a feasible starting solution

Compute planning
order of exams

Finding a feasible starting solution

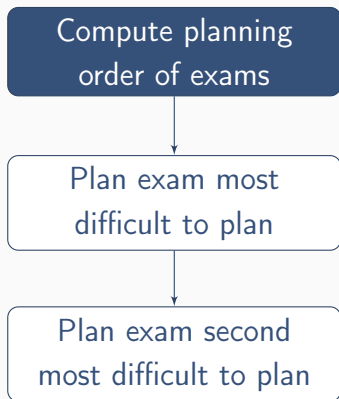
Compute planning
order of exams



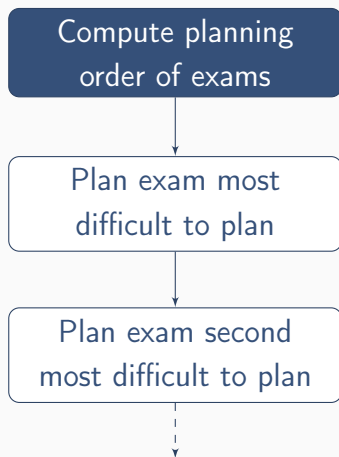
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graph TD; A[Compute planning order of exams] --> B[Plan exam most difficult to plan];
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Plan exam most
difficult to plan

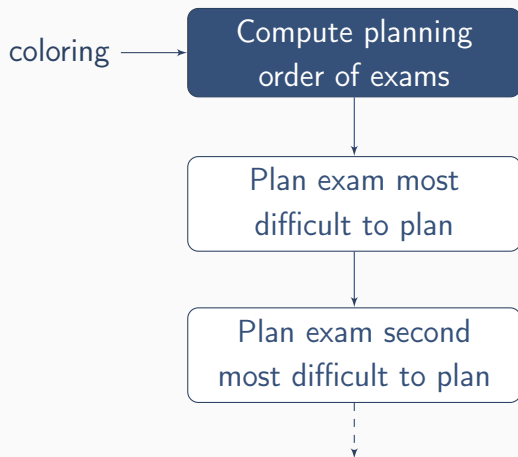
Finding a feasible starting solution



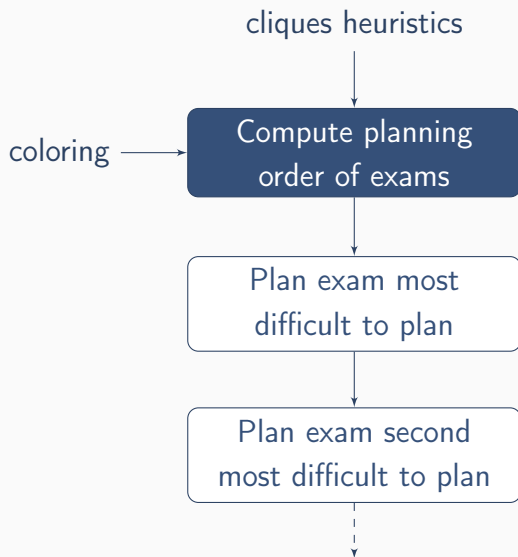
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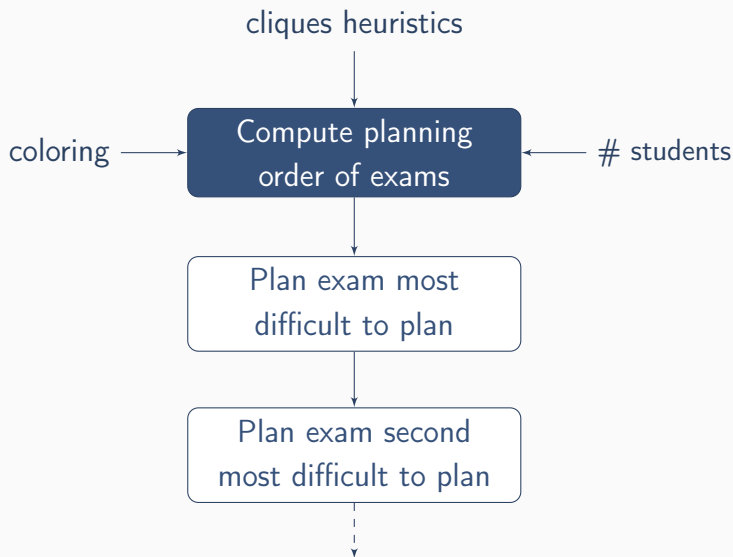
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Improvements to the model

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$$(7) \quad \sum_{i \text{ in clique}} y_{i,l} \leq 1 \quad \forall \text{ cliques}$$

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- Improve running time using heuristics, pre-solving, etc.
- Use a path based model and column generation

Data acquisition



Figure 1: Server, hopefully somewhere in Germany