Carlos Segarra Thursday, September 26th

Discrete and Algorithmic Geometry: Problems 4 and 5

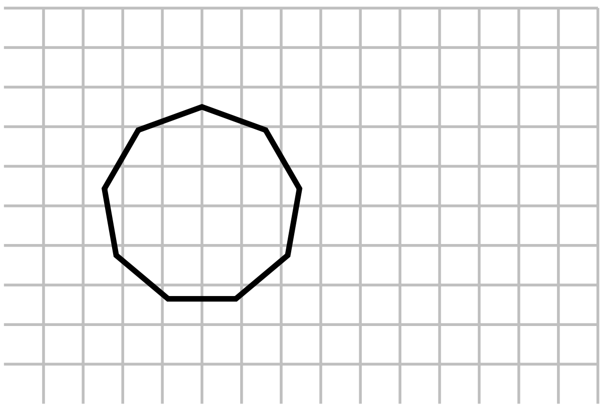
4. Propose an algorithm that, given a point p external to a convex polygon $\mathcal{P}$, finds the point of $\mathcal{P}$ closest to p. What happens if instead of finding the closest point we look for the farthest? What if we restrict the search to the vertices of $\mathcal{P}$?

Let $\{q_1, \ldots, q_n\}$ be the set of vertices of $\mathcal{P}$ in clock-wise order. And let f be the function s.t. for all $q \in \mathcal{P}$, sends p to $d(q, p)$. Before starting with the proof, let us make two observations:

Observation 1. *f is an uni-modal application. Alternatively, its derivative is a linear funcion that has only one zero.*

Observation 2. *Let q_{min} and q_{min+1} be the closest and second to closest vertices from $\mathcal{P}$ to p. Then, the closest point in $\mathcal{P}$ to p is either (i) q_{min} or (ii) contained in the $q_{min}q_{min+1}$ segment.*

Note that the second observation applies symetrically to the farthest point in $\mathcal{P}$. Further, once we find the two closest points, checking wether we are in case (i) or case (ii) from Obs 2 can be done in constant time testing if p belongs to the triangle drawn by the line throguh p parallel to $q_{min}q_{min+1}$ and the two lines through p_{min} orthogonal to the segments incident to p_{min}.



Below, we include the Algorithm for computing the closest point to a polygon, note that, in order to find the farthest we could use the same exact algorithm

Algorithm 1 Unknown TX T-DAG Generation from its root.

$start \leftarrow 1$
$end \leftarrow \frac{n}{2}$
while $start \leq end$ **do**
  $\Delta_A \leftarrow d(q_{start+1}, p) - d(q_{start}, p)$
  $\Delta_B \leftarrow d(q_{end+1}, p) - d(q_{end}, p)$
  switch sign(Δ_A), sign(Δ_B) **do**
    case $++$
    case $+-$
      $start \leftarrow end$
      $end \leftarrow end + \frac{end-start}{2}$
    case $-+$
      $end \leftarrow end - \frac{end-start}{2}$
    case $--$
      $end \leftarrow \frac{end}{2}$
if $q \in \Delta(q_{min}, \bar{p_1}, \bar{p_2})$ **then**
  $q_{min} = \underset{q \in \{q_{start}, q_{end}\}}{\text{argmin}} (d(q,p))$
else
  $q_{min} \leftarrow \text{line}(\perp q_{start}q_{end}, p) \cap q_{start}q_{end}$
$d_{min} \leftarrow d(q_{min}, p)$
return d_{min}

Running Time

5. Propose an algorithm that, given two disjoitn convex polygons, $\mathcal{P}$ and $\mathcal{Q}$, finds the closest pair of points $p \in \mathcal{P}$ and $q \in \mathcal{Q}$.