Discrete and Algorithmic Geometry

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Sheet 0

due on Tuesday, November 12, 2019

Reading

(-4) Learn about the versioning software `git`, and practice until you become comfortable using it. Then check out the repository of this course using

`git clone git@gitlab.mat-apl.upc.edu:julian.pfeifle/2019-dag-upc` .

(-3) Create a new branch `your-name-cv` in the repository, and edit the file `participants.tex` to include a short cv and some information about your mathematical interests. Then `commmit` and `push` your changes, and create a pull request at `gitlab.mat-apl.upc.edu` so that all the different stories may be merged.

(-2) Learn about public key cryptography and the use, advantages and disadvantages of the software `gpg`.

(-1) Organize into teams of 2–3 people to work on the exercises, and edit `participants.tex` to reflect these changes. As always, `commit` and `push` your changes, and create a pull request at `gitlab.mat-apl.upc.edu`.

(0) Read up on two programming languages of your choice that you and your team will use in this course. One of these should be a scripting language for rapid iteration, the other a compiled language for efficiency. If you have never programmed before, a good choice for a scripted language is `python/sage`, and a good choice for a compiled language is `julia`. If you already know some languages, take the opportunity to learn a new one! Some suggestions are `c++`, `perl/raku`, `rust`, `haskell`.

Sheet 1

due on Tuesday, November 19, 2019

Writing

To submit your solutions to these exercises,
- ▷ create a new branch `your-team-sheet-1`,
- ▷ create a subdirectory `exercises/sheet1/your-awesome-team-name/`,
- ▷ and put your solutions to the exercises into a `.pdf` file into that directory.
- ▷ Now encrypt this `.pdf` using `julian.pfeifle@upc.edu.public.gpg.key`, and
- ▷ `add`, `commit` and `push` **only this encrypted pdf, not the original** `.tex`
- ▷ and create a pull request.

You will be graded collectively on these exercises, and individually in the final exam.

Exercises not submitted via this mechanism will not be graded.

Let $([n], \mathcal{I})$ be a matroid on the ground set $[n] = \{1, 2, \dots, n\}$ and whose independent sets are $\{I : I \in \mathcal{I}\}$. Recall the following definitions:
- ▷ For any proper subset $S \subset [n]$, the deletion $M \setminus S$ is the matroid on the ground set $[n] \setminus S$ whose independent sets are $\{I \subset [n] \setminus S : I \in \mathcal{I}\}$.
- ▷ The dual matroid $M^\star$ of M is the matroid on $[n]$ where I is a basis iff $[n] \setminus I$ is a basis of M.
- ▷ If $S \subset [n]$, then the contraction of M with respect to S is $M/S = (M^\star \setminus S)^\star$.

(1) Why does this notion of contraction agree with the notion of contraction in graph theory?

(2) Prove that if a matroid M is realizable over a ground field $\Bbbk$, then the dual matroid $M^\star$ is also realizable over $\Bbbk$.

> *Hint.* Suppose that M has rank d and n elements. After a change of basis, M can be represented by the $d \times n$ matrix $A = [I|B]$, where I is the $d \times d$ identity matrix, and B has size $d \times (n - d)$. Now find a matrix that represents $M^\star$.

(3) Consider the matroid given by the columns of

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix},$$

and compute its dual, and the contraction along some subsets of your choice.

Coding