CONTENTS AND SHORT BIBLIOGRAPHY
DISCRETE AND ALGORITHMIC GEOMETRY, UPC, 2019

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Due to my teaching (and grading) load this semester, I have not had time to prepare lecture notes for this class. But since I more or less directly copied the content from various sources, I hope to make your job of studying the material easier by explicitly listing the chapters I used. A big thanks to Moritz Otth for pushing me do compile this list!

The overarching theme are realizations of oriented matroids.

1. Examples and axiom systems

This material is directly copied from [10, Lecture 1].

1.1. **Matroids.** Examples:

- Vector and point configurations
- algebraic matroids
- transversal matroids
- graphical matroids

Axiom systems:

- Independent sets
- bases
- circuits
- cocircuits
- rank function
- flats
- hereditarily pure simplicial complexes
- universally shellable simplicial complexes
- greedily optimizable independent set systems

I didn't end up introducing Coxeter matroids, or talk much about matroid base polytopes.

1.2. **Oriented Matroids.** Axiom systems:

- Circuits
- Cocircuits

Chirotopes weren't introduced until later. I did not use [2] except for its Theorem 7.4.2, even though it's great as a reference.

The exercises in this section were taken from [10], [12], [3], [1], [15].

2. Constructions

After covering the direct sum, deletion and contraction following [10, Lecture 2], we switch sources.

3. Oriented matroid / Gale duality

The oriented matroid / Gale duality construction follows [15, Chapter 6], as does the discussion of Radon partitions and affine Gale diagrams. The example of a non-rational polytope is [15, Example 6.21]. The presentation of the Milnor-Thom-Oleinik-Petrovski theorem is from [8].

3.1. **Regular triangulations and the secondary polytope.** I initially tried to follow [6, Chapter 5], but found it to be too verbose for presentation in class. For a leisurely introduction it works great, though. In the end, I used [14, Chapters 7,8]. An additional source is [15, Chapter 9].

The exercises were taken from [6] and [15].

4. Gröbner bases and the Grassmannian

The primary sources here are [5, Chapter 2] and [14, Chapters 10–12]. A secondary source is [11]. The example on the key sniffing attack is [13, Section 3.1]. The Gröbner basis of the Plücker ideal is from [9, Chapter 14].

5. Realizations of oriented matroids

The relevant papers are [7] and [4].

References

[1] Federico Ardila and Sara Billey. Flag arrangements and triangulations of products of simplices. *Adv. Math.*, 214(2):495–524, 2007.

[2] Anders Björner, Michel Las Vergnas, Bernd Sturmfels, Neil White, and Günter Ziegler. *Oriented matroids. 2nd ed.*, volume 46 of *Encyclopedia of Mathematics and Its Applications*. Cambridge: Cambridge University Press, 2nd ed. edition, 1999.

[3] Jürgen G. Bokowski. *Computational oriented matroids. Equivalence classes of matrices within a natural framework.* Cambridge: Cambridge University Press, 2006.

[4] Madeline Brandt and Amy Wiebe. The slack realization space of a matroid. *Algebr. Comb.*, 2(4):663–681, 2019.

[5] David A. Cox, John Little, and Donal O'Shea. *Ideals, varieties, and algorithms. An introduction to computational algebraic geometry and commutative algebra. 4th revised ed.* Undergraduate Texts in Mathematics. Cham: Springer, 4th revised ed. edition, 2015.

[6] Jesús A. De Loera, Jörg Rambau, and Francisco Santos. *Triangulations. Structures for algorithms and applications.*, volume 25 of *Algorithms and Computation in Mathematics*. Berlin: Springer, 2010.

[7] João Gouveia, Antonio Macchia, Rekha R. Thomas, and Amy Wiebe. The slack realization space of a polytope. *SIAM J. Discrete Math.*, 33(3):1637–1653, 2019.

[8] Jiří Matoušek. *Lectures on discrete geometry.*, volume 212 of *Graduate Texts in Mathematics*. New York, NY: Springer, 2002.

[9] Ezra Miller and Bernd Sturmfels. *Combinatorial commutative algebra.*, volume 227 of *Graduate Texts in Mathematics*. New York, NY: Springer, 2005.

[10] Vic Reiner. Lecture notes for the ACE Summer School 2005 in Geometric Combinatorics. `http://www-users.math.umn.edu/~reiner/Talks/Vienna05/index.html`, 2005.

[11] Jürgen Richter-Gebert. *Perspectives on projective geometry. A guided tour through real and complex geometry.* Berlin: Springer, 2011.

[12] Alexander Schrijver. *Combinatorial optimization. Polyhedra and efficiency (3 volumes).*, volume 24 of *Algorithms and Combinatorics*. Berlin: Springer, 2003.

[13] A.J.M. Segers. Algebraic Attacks from a Gröbner Basis Perspective. Master's thesis, Technische Universiteit Eindhoven, 2004. `https://www.win.tue.nl/~henkvt/images/ReportSegersGB2-11-04.pdf`.

[14] Rekha R. Thomas. *Lectures in geometric combinatorics.*, volume 33 of *Student Mathematical Library*. Providence, RI: American Mathematical Society (AMS); Princeton, NJ: Institute for Advanced Studies, 2006.

[15] Günter M. Ziegler. *Lectures on polytopes.*, volume 152 of *Graduate Texts in Mathematics*. Berlin: Springer-Verlag, 1995.