Problem 7

1. (10 points) Calculate the critical depth and the corresponding specific energy for a discharge of 5.0 m^3/sec in the following channels.

   (a.) Rectangular channel, B = 2.0 m.
   (b.) Triangular channel, m = 0.5.
   (c.) Trapezoidal channel, B = 2.0 m, m = 1.5.

(a.) Rectangular channel $q = \frac{Q}{B} = \frac{5}{2} = 2.5\ m^3/s\cdot m$

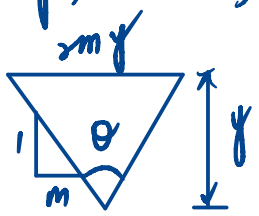
$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{2.5^2}{9.81}} = 0.86\ m$$

$$E_c = \frac{3}{2} y_c = 1.5 \cdot (0.86) = 1.29\ m$$

(b.) Triangular channel, m=0.5

$A = my^2$ $\quad \frac{TQ^2}{gA_c^3} = 1$ (critical condition)

$T = 2my$



$$* \frac{Q^2}{g} = \frac{A_c^3}{T_c} = \frac{m^3 y_c^6}{2m y_c} = \frac{m^2 y_c^5}{2}$$

$$y_c = \left(\frac{2Q^2}{g m^2}\right)^{\frac{1}{5}} \therefore y_c = \left(\frac{2\times 5^2}{(9.81)\times(0.5)^2}\right)^{\frac{1}{5}} = 1.83$$

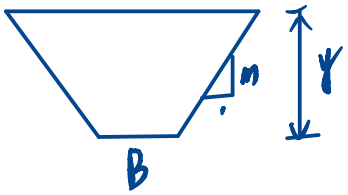
$$E_c = y_c + \frac{V_c^2}{2g} = y_c + \frac{Q^2}{2gA_c^2} = y_c + \frac{y_c^5 \cdot g m^2 \cdot \frac{1}{2}}{2g(my_c^2)^2} = y_c + \frac{y_c}{4} = 1.25 y_c$$

$$E_c = 1.25\, y_c = (1.25)\cdot(1.83) = 2.29\ m$$

(c.) Trapezoidal channel, B=2 m m=1.5

$A = (B+my)y$ $\quad T = (B+2my)$

$$* \frac{Q^2}{g} = \frac{A_c^3}{T} \Rightarrow \frac{Q^2}{g} = \frac{(B+my_c)^3 y_c^3}{(B+2my_c)}$$



$$\frac{5^2}{9.81} = \frac{(2+1.5y_c)^3 y_c^3}{(2+2\times 1.5 y_c)}, \quad y_c = 0.71\ m$$

$$A_c = (2+1.5\times 0.71)\times 0.71 = 2.18\ m^2$$

$$V_c = \frac{Q}{A_c} = \frac{5}{2.18} = 2.29\ m/s$$

$$E_c = y_c + \frac{V_c^2}{2g} = 0.71 + \frac{(2.29)^2}{19.62} = 0.98\ m$$