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CS 180 HW2

CH 8 # 15

Solution:

To show Nearby Electromagnetic Observation Problem (NEOP) is NP-complete I need to show that the vertex cover problem is reducible to it, that is Vertex cover$\leq_p$NEOP. Consider a graph G with number of vertices V and number of edges E. Let every edge represent a frequency and let every vertex represent a geographic location at which a sensor can be placed. Furthermore, not every pair of vertices in G has an edge between them, so let each nonexistent edge be represented by an interference source pair, (edges not made, geographic location). Now finding a vertex cover of size k for a graph represented in this fashion is akin to finding the k sensor solution to the NEOP problem that represents it. This shows that NEOP is NP-complete as Vertex cover is known to be NP-complete.

CH 8 #22

Solution:

I will solve this by breaking the problem up into cases.

case 1: $k = 1$

In this case I will simply return any vertex in the graph G as all single vertex are trivially their own independent set.

case 2: $k > 1$

In this case we must consider the set V of size S which contains all nodes in our given graph. For each node $v \in V$ connect current node to every other node in V that it is not already connect to. Pass this new graph to the provided "black-box algorithm A". If A returns true leave the current edges and move onto the next node in the graph. If A returns false remove all the new edges which were created. Mark the current vertex v as visited and move onto the next unvisited vertex in V. Repeat this method until all vertices have been visited. Once all nodes have been visited simply return the set of nodes which have number of edges less than $S - 1$ where S was the number of vertices in the given graph. This solves the problem in polynomial time because A is given as running in polynomial time, and there will be S calls to V where S will be a constant number of nodes. Lastly the step of returning the set of independent nodes can also be completed in some constant time C. Thus running time for this method is $S \cdot A + C = O(A)$, as required.

CH 8 # 36

Solution:

To show that Daily Special Scheduling (DSS) is NP-complete I need to show that the Hamiltonian path problem is reducible to it. Consider a graph G with number of vertices V and number of edges E. Let every vertex be represented by some special in the DSS problem, and let every edge represent an ingredient needed by the specials(vertices) it is connected to. With these representations in place let each ingredient have a useable life of two days so that each ingredient used in today's special will have to be used in the next day's special. Similarly, let each ingredient come in bundles of size two. Now, finding a Hamiltonian path for any graph represented in this fashion is akin to finding the solution to the DSS problem that represents it. This shows that DSS is NP-complete as Hamiltonian path is know to be NP-complete.