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CSM146, Winter 2018
Problem Set 0: Math prerequisites

Problem 1

Solution: $y' = sin(z)e^{-x} \cdot -xsin(z)e^{-x}$

Problem 2

(a) Solution: $y^t z = 11$

(b) Solution: $Xy = \begin{pmatrix} 14 \\ 10 \end{pmatrix}$

(c) Solution: $det\Big(X\Big) = 6 - 4 = 2$ thus X is invertable and $X^{-1} = \frac{1}{2}\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}$

(d) Solution: $rank\Big(X\Big) = 2$

Problem 3

(a) Solution: sample mean (expectation) $= \dfrac{3}{5}$

(b) Solution: variance $= \Big[\mathrm{E(X)}^2 - \mathrm{E(X^2)}\Big] = \left(\dfrac{3}{5}\right)^2 - \left(\dfrac{3}{5}\right) = .24$

(c) Solution: Probaility $= .5^5 = .03125$

(d) Solution: Need to maximize $(x)^3\,(1-x)^2$where $x < 1\,x > 0$. The maximum value of this function is .6. That is $P(Xi = 1) = 0.6$

Due to the way I used the function to find this value, and the fact that said function is convex between the specified bounds proves this probability to be the correct maximum.

(e) Solution: 0.1

Problem 4

(a) Solution: false

(b) Solution: true

(c) Solution: false

(d) Solution: false

(e) Solution: true

Problem 5

Solution:

(a) v

(b) iv

(c) ii

(d) i

(e) iii

Problem 6

(a) Solution: mean $= p$, variance $= p(1-p)$

(b) Solution: variance$(2X) = 4 \cdot \sigma^2$, and variance$(X+2) = \sigma^2$

Problem 7

(a) Solution:
(i)ln(n)=O(lg(n))
(ii)f(n)=O(g(n))
(iii)f(n)=O(g(n))
NOTE: these answers are considering the functions as $n \to \infty$

(b) Solution: As the title of this function says a divide and conquer algorithm will solve this index finding problem. Specifically a binary search algorithm. That is, first divide our array in half and test the $\frac{n}{2}$th element. If the element is 1 we know the interval $[0, \frac{n}{2}]$ has the index we are looking for, else $[\frac{n}{2}, n]$ has it. This is becasue we know all 0's must occur before any 1's. After we get our new interval do the same thing. That is, take the midpoint, test its value and throw out the half that cant have the first 1/last 0. Once our interval is only 2 elements big we will have our solution. This runs in time O(log(n)) because of the way we are repeatedly dividing our input array in half.

Problem 8

(a) Solution: By the independence of X and Y $f(x,y) = f(x) \cdot f(y)$ so,

$$\begin{aligned} E(XY) &= \int_{-\infty}^{\infty} f_{x,y}(x,y) \cdot xy \cdot dxdy \\ &= \int_{-\infty}^{\infty} f_x(x)xdx \cdot \int_{-\infty}^{\infty} f_y(y)ydy \\ &= E(X) \cdot E(Y) \end{aligned}$$

As to be proved.

(b) Solution:

(i) On a fair die the chance of rolling a 3 is $\frac{1}{6}$. $\frac{1}{6} \cdot 6000 = 1000$.

(ii) Central Limit Theorem

Problem 9

(a) Solution:

(b) Solution:

(i) Eigenvalue: Scalar solutions to the characteristic equation of a matrix. Eigenvector: A non-zero vector which when multiplied by a specific linear transfowmation is only scaled by an eigenvalueλ.

(ii) Eigenvalues: 1,3

Eigenvectors: $\lambda = 1, v = \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix}$ & $\lambda = 3, v = \begin{bmatrix} x_1 \\ x_1 \end{bmatrix}$

(iii)Given a matrix A, and eigenvalue $\lambda$and an eigenvector x then $Ax = \lambda x$ and

$$\begin{aligned} AAx &= A\lambda x \\ &= \lambda Ax \\ &= \lambda^2 x \end{aligned}$$

Since A^kcan be represented as k left multiplications of A then $A^k x = \lambda^k x$ as the equations shown above would suggest. Since x doesnt change x is still an eigenvector of A and the eigenvalues of A will become λ^k.

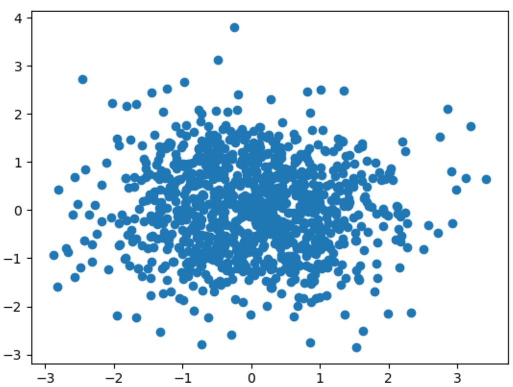
(c) Solution:

(i) $(a^T x)\frac{d}{dx} = (a^T)$

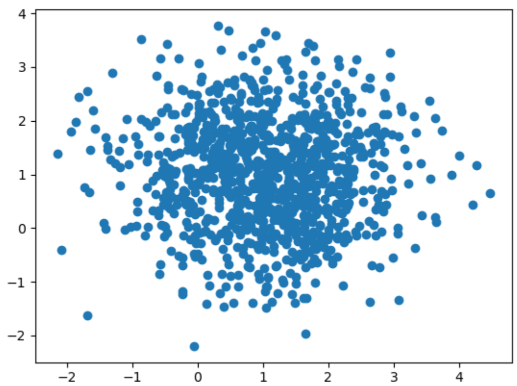
(ii)$(x^T Ax)\frac{d}{dx} = (Ax)$ & $(x^T Ax)\frac{d^2}{dx^2} = A$

Problem 10

(a) Solution:



(b) Solution:



(c) Solution:

(d) Solution:

(e) Solution:

Problem 11

```
import numpy as np
import scipy.linalg as lg
A = np.mat("1 0; 1 3")
evals, evecs = lg.eig(A)
print("eigenvalues: ",evals, '/n')
print("eigenvectors: ",evecs)
```

The Output maximum eigenvalue can be seen to be 3 with corresponding eigenvector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Problem 12

Solution:

(a) Titanic Dataset

(b) http://www.cs.toronto.edu/~delve/data/titanic/desc.html

(c) The data set is the 2201 people who were onboard the Titanic when it sank. The features include social class, age, sex, and survival. Survival of passengers with a certian sets of features is what is being predicted, however the real interest is in interpretation and how survival realtes to the other attributes.

(d) 2201 examples in this dataset

(e) 4 attributes to each example, as stated above; class, age, sex, and survival.