Inclass 20: Information Theory

[SCS4049] Machine Learning and Data Science

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Logistic regression

Negative logarithm of the likelihood, which gives the cross-entropy error function

$$E(\mathbf{w}) = -\log p(\mathbf{t} \mid \mathbf{w}) = -\sum_{n=1}^{N} \{t_n \log y_n + (1 - t_n) \log(1 - y_n)\} \quad (1)$$

Taking the gradient of the error function, we obtain

$$\nabla E(\mathbf{w}) = \sum_{n=1}^{N} (y_n - t_n)\mathbf{x}_n \quad (2)$$

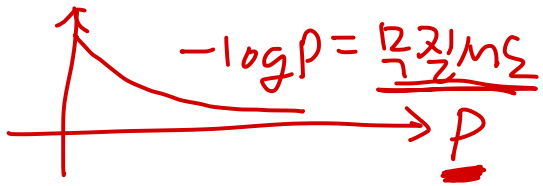
Entropy

Discrete random variable: X

Probability mass function: $p(x)$

Entropy = 평균 정보량 = 평균 무질서도

p↑ 무질서도↓, p↓ 무질서도↑

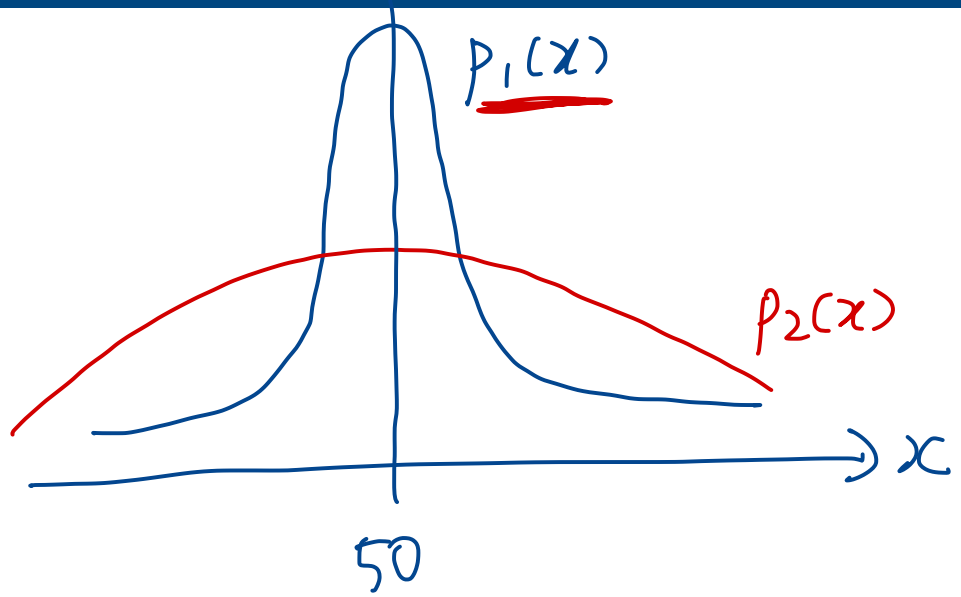


$$H(p) = \mathrm{E}[-\log p] = -\sum p(x) \log p(x) \tag{3}$$

Entropy $H(p)$ p: 확률분포.

$= E[-\log p]$

$= \sum_x -\log p(x) \cdot p(x)$ 같

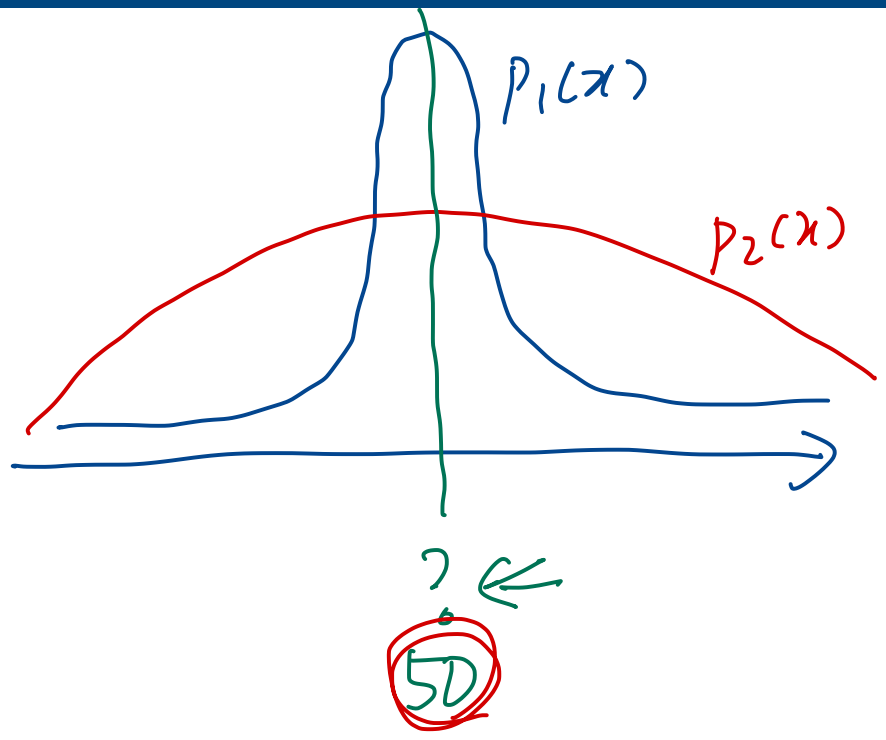


$p_1(x) \rightsquigarrow 49, 47, 51, 53, 48, 52, \cdots$

$p_2(x) \rightsquigarrow 31, 62, 53, 34, 47, \sim\sim$

'하나의 샘플이 가지고 있는 정보가 작다.' ← 뻔하다 = 무작위성 ↓ = 엔트로피 ↓ (평균적일 것)

하나하나의 샘플이 가진 정보가 많다. ← 덜 뻔하다 = 무작위성 ↑ = 엔트로피 ↑ (평균적일 것)



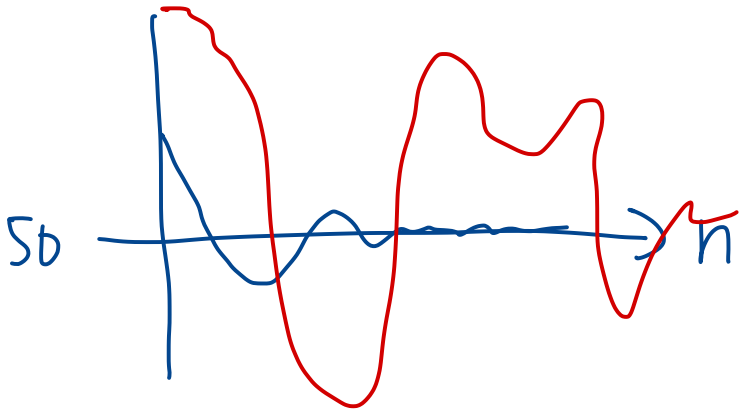
$p_1(x) \sim 51, 49, 48, 52, 53, \cdots$

$\bar{x} = \frac{1}{N}\sum x_n$

$p_2(x) \sim 31, 47, 59, 62, 38, \cdots$

$\bar{x} = \frac{1}{N}\sum x_n$

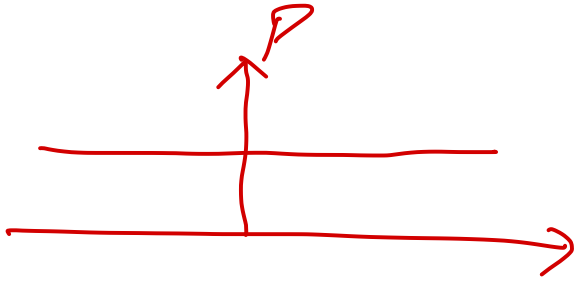
하나의 샘플이 측정된 데이터가 아니라
평균을 측정하는데
기여하는 정도 = 정보량.



Entropy와 불확실성, 그리고 정보량

Entropy가 최대인 확률 분포

- Discrete: uniform distribution
- Continuous: Gaussian distribution



Cross-entropy and relative entropy

Entropy $= E_p[-\log p] = \sum_x -p(x)\log p(x)$

Cross-entropy

p, q

$$H(p,q) = -\mathrm{E}_p[\log q] = -\sum p(x)\log q(x) \quad (4)$$

$\neq H(q,p)$

Relative entropy

KL divergence

p, q

$$\mathcal{D}_{\mathrm{KL}}(p \,||\, q) = \mathrm{E}_p\left[\log \frac{p}{q}\right] = -\sum p(x)\log\frac{p(x)}{q(x)} \quad (5)$$

$$= -\sum p(x)\{\log p(x) - \log q(x)\}$$

두 분포가 같을 때

$$\mathcal{D}_{\mathrm{KL}}(p \,||\, q) = 0 \iff p(x) = q(x) \;\; \forall x \quad (6)$$

$D_{KL}(p||q) \geq 0$

$\simeq$ 차이의 정도 p, q

Cross-entropy and relative entropy

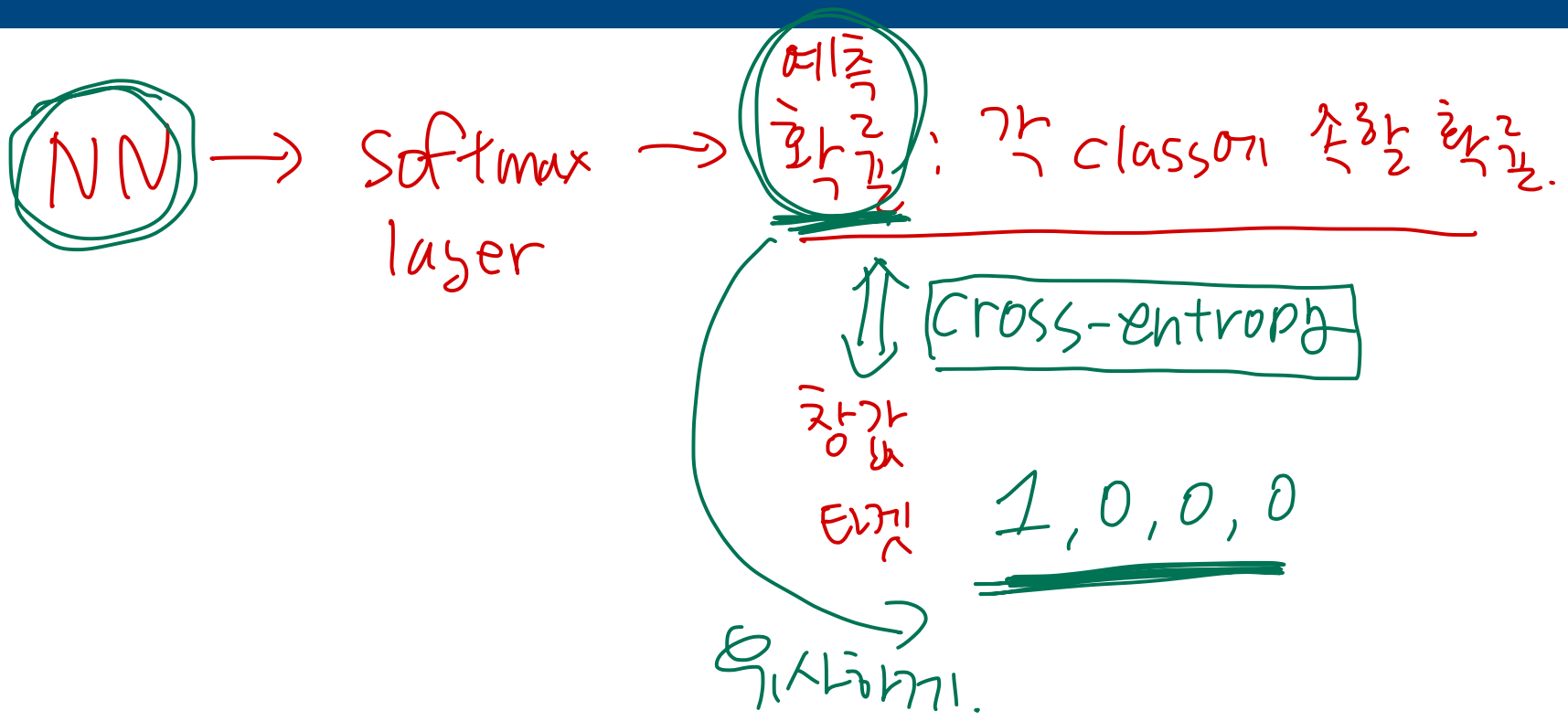
Cross-entropy = entropy + relative entropy

교차 엔트로피 상대 엔트로피.

$$H(p, q) = H(p) + \mathcal{D}_{\mathrm{KL}}(p \,||\, q) \tag{7}$$

$$-\Sigma p \log q = -\Sigma p \log p + \Sigma p \log p - p \log q$$

min cross-entropy $\Longleftrightarrow$ min relative entropy


NN
Softmax layer
예측 확률: 각 class의 소속될 확률.
cross-entropy
참값 태깅
1,0,0,0
역전파하기.