Unit 7

Association Rule

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What Is Frequent Pattern Analysis?

- Frequent pattern: a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set
- First proposed by Agrawal, Imielinski, and Swami [AIS93] in the context of frequent itemsets and association rule mining
- Motivation: Finding inherent regularities in data
  - What products were often purchased together?— Beer and diapers?!
  - What are the subsequent purchases after buying a PC?
  - What kinds of DNA are sensitive to this new drug?
  - Can we automatically classify web documents?
- Applications
  - Basket data analysis, cross-marketing, Web log (click stream) analysis, and DNA sequence analysis.

Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction.

Market-Basket transactions

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Example of Association Rules

{Diaper} $\rightarrow$ {Beer},
{Milk, Bread} $\rightarrow$ {Eggs,Coke},
{Beer, Bread} $\rightarrow$ {Milk},

Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count (σ)**
  - Frequency of occurrence of an itemset
  - E.g. $\sigma(\{\text{Milk, Bread,Diaper}\}) = 2$
- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. $s(\{\text{Milk, Bread, Diaper}\}) = 2/5$
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Definition: Association Rule

- Association Rule
  - An implication expression of the form X $\rightarrow$ Y, where X and Y are itemsets
  - Example:
    {Milk, Diaper} $\rightarrow$ {Beer}

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

- Rule Evaluation Metrics
  - **Support (s)**
    - Fraction of transactions that contain both X and Y
  - **Confidence (c)**
    - Measures how often items in Y appear in transactions that contain X

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
    - support ≥ *minsup* threshold
    - confidence ≥ *minconf* threshold

- Brute-force approach:
    - List all possible association rules
    - Compute the support and confidence for each rule
    - Prune rules that fail the *minsup* and *minconf* thresholds

    $\Rightarrow$ Computationally prohibitive!

Mining Association Rules

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Example of Rules:

{Milk,Diaper} $\rightarrow$ {Beer} (s=0.4, c=0.67)
{Milk,Beer} $\rightarrow$ {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} $\rightarrow$ {Milk} (s=0.4, c=0.67)
{Beer} $\rightarrow$ {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} $\rightarrow$ {Milk,Beer} (s=0.4, c=0.5)
{Milk} $\rightarrow$ {Diaper,Beer} (s=0.4, c=0.5)

Observations:

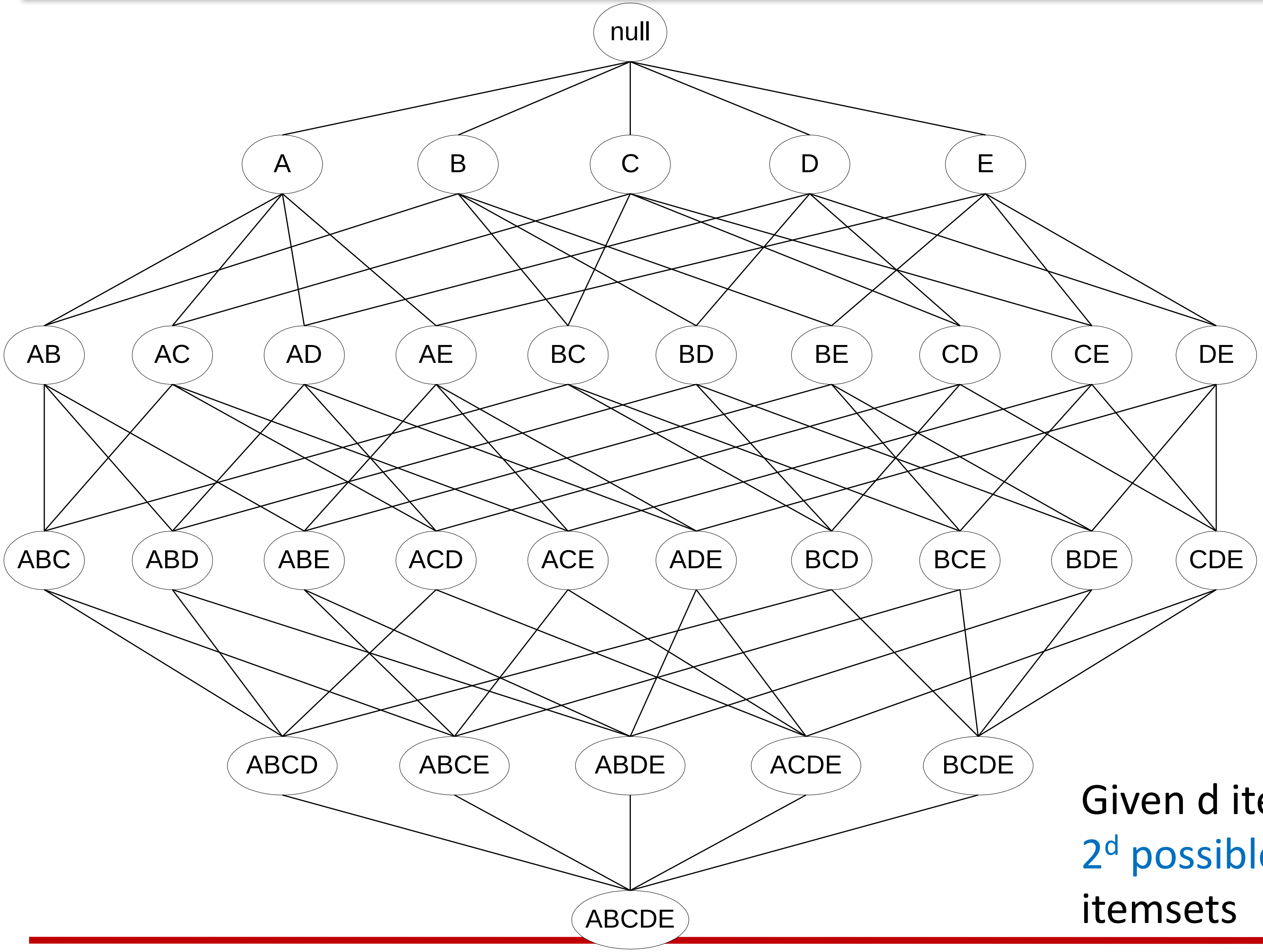
- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Mining Association Rules

- Two-step approach:
  - Frequent Itemset Generation
    - Generate all itemsets whose support $\geq$ minsup
  - Rule Generation
    - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

- Frequent itemset generation is still computationally expensive

Frequent Itemset Generation



Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

Brute-force approach:

- Each itemset in the lattice is a candidate frequent itemset
- Count the support of each candidate by scanning the database

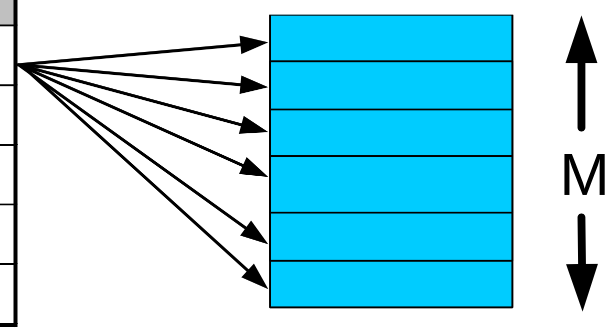
Transactions

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

N

w

List of Candidates



- Match each transaction against every candidate
- Complexity ~ O(NMw) => Expensive since $M = 2^d$!!!

Reducing Number of Candidates

Apriori principle:

If an itemset is frequent, then all of its subsets must also be frequent

Apriori principle holds due to the following property of the support measure:

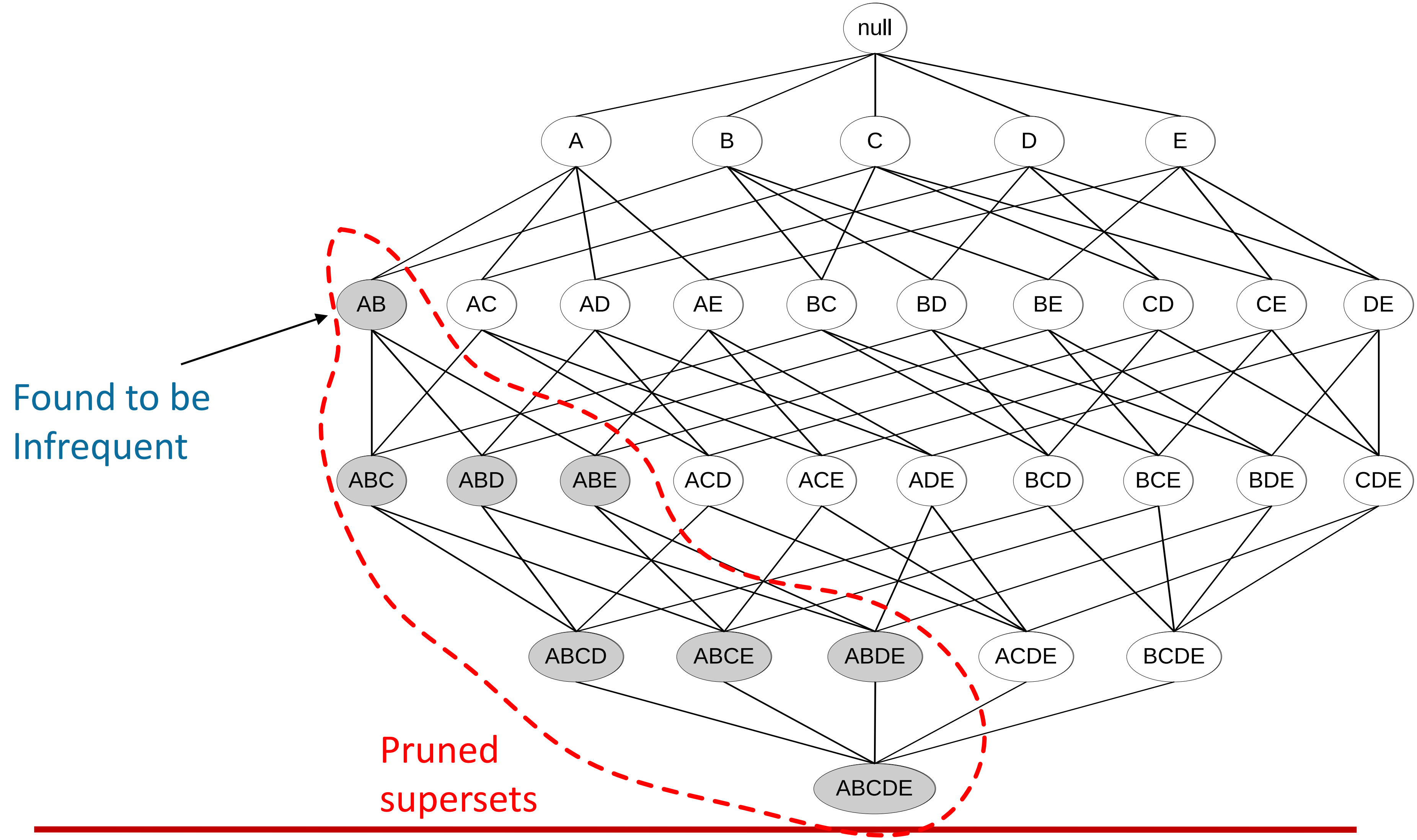
$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

Support of an itemset never exceeds the support of its subsets

This is known as the anti-monotone property of support

Anti-monotone: if a set can’t pass a test, all of its superset will fail the same test as well

Illustrating Apriori Principle



Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Items (1-itemsets)

Itemset	Count
{Bread,Milk}	**3**
{Bread,Beer}	**2**
{Bread,Diaper}	**3**
{Milk,Beer}	**2**
{Milk,Diaper}	**3**
{Beer,Diaper}	**3**

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	**3**

If every subset is considered,

$^6C_1 + {^6C_2} + {^6C_3} = 41$

With support-based pruning,

$6 + 6 + 1 = 13$

Q: Total number of possible frequent itemsets ???

Apriori Algorithm

Method:

- Let k=1
- Generate frequent itemsets of length 1
- Repeat until no new frequent itemsets are identified
  - Generate length (k+1) candidate itemsets from length k frequent itemsets
  - Prune candidate itemsets containing subsets of length k that are infrequent
  - Count the support of each candidate by scanning the DB
  - Eliminate(prune) candidates that are infrequent, leaving only those that are frequent

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D

C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{5}	3

C_2

itemset
{1 2}
{1 3}
{1 5}
{2 3}
{2 5}
{3 5}

Scan D

C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

L_2

itemset	sup
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

itemset
{1,2, 3}
{1,3 ,5}
{2,3, 5}

C_3

itemset
{2 3 5}

Scan D

L_3

itemset	sup
{2 3 5}	2

Why {1 2 3}, {1 2 5}, {1 3 5} are not listed in C3???

Frequent Pattern Tree

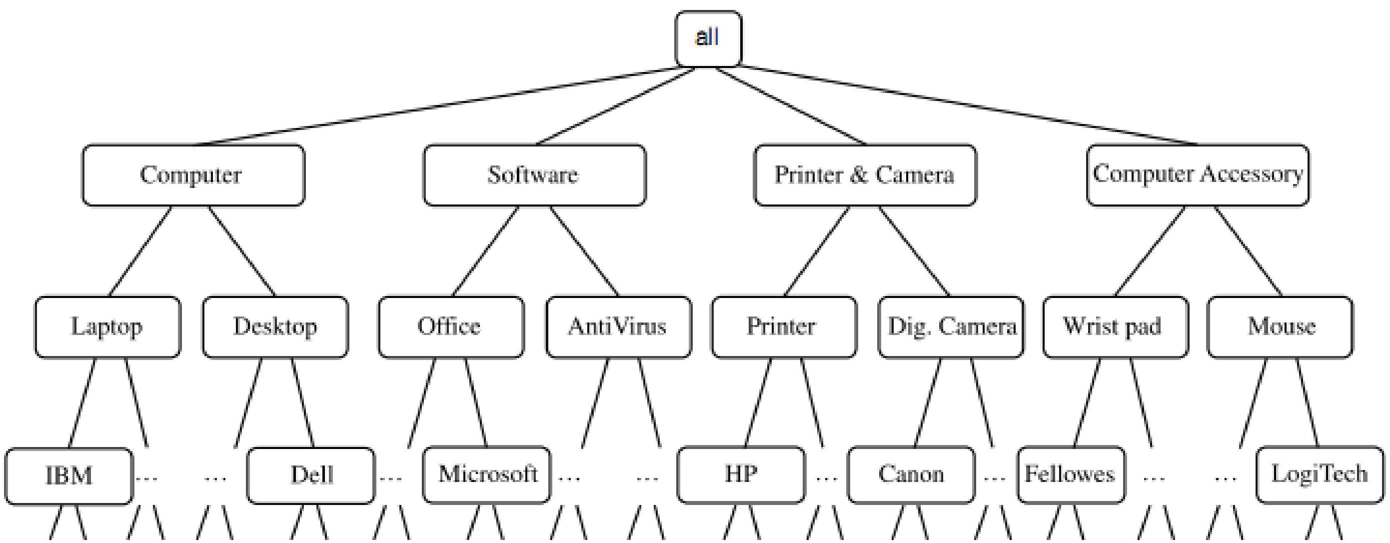
Generating Association Rule (Example)

- **Given a frequent itemset L**
  - Find all non-empty subsets F in L, such that the association rule $F \Rightarrow \{L-F\}$ satisfies the minimum confidence
  - Create the rule $F \Rightarrow \{L-F\}$

- **If L={A,B,C}**
  - The candidate itemsets are: $AB \Rightarrow C$, $AC \Rightarrow B$, $BC \Rightarrow A$, $A \Rightarrow BC$, $B \Rightarrow AC$, $C \Rightarrow AB$
  - In general, there are 2^K-2 candidate solutions, where k is the length of the itemset L

Recap : A Concept Hierarchy

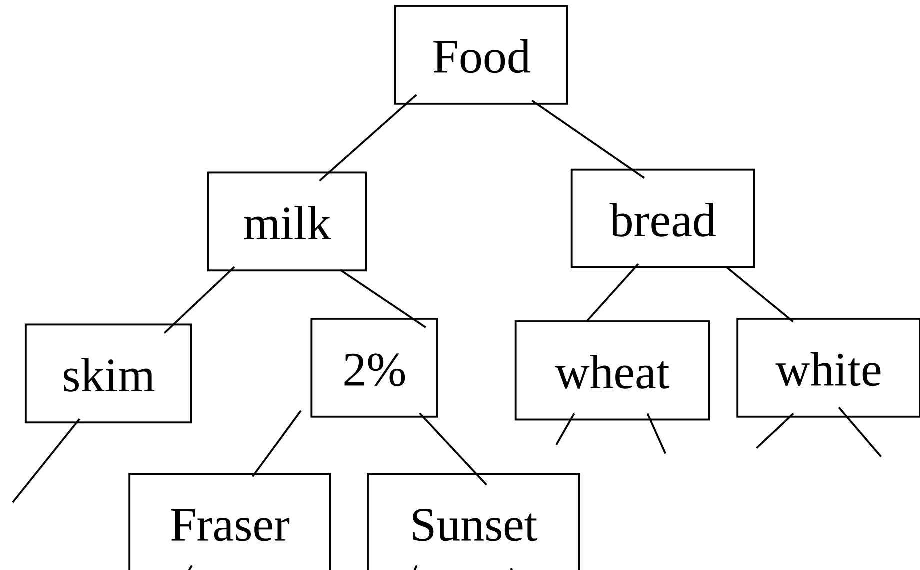
TID	*Items Purchased*
T100	IBM-ThinkPad-T40/2373, HP-Photosmart-7660
T200	Microsoft-Office-Professional-2003, Microsoft-Plus!-Digital-Media
T300	Logitech-MX700-Cordless-Mouse, Fellowes-Wrist-Rest
T400	Dell-Dimension-XPS, Canon-PowerShot-S400
T500	IBM-ThinkPad-R40/P4M, Symantec-Norton-Antivirus-2003
...	...



lecture 2

Multiple-Level Association Rules

- Items often form hierarchy.
- Items at the lower level are expected to have lower support.
- Rules regarding itemsets at appropriate levels could be quite useful.
- We can explore shared multi-level mining



TID	Items
T1	{111, 121, 211, 221}
T2	{111, 211, 222, 323}
T3	{112, 122, 221, 411}
T4	{111, 121}
T5	{111, 122, 211, 221, 413}

Mining Multi-Level Associations

- A top_down, progressive deepening approach:
  - First find high-level strong rules:

    milk $\rightarrow$ bread [20%, 60%].
  - Then find their lower-level "weaker" rules:

    2% milk $\rightarrow$ wheat bread [6%, 50%].
- Variations at mining multiple-level association rules.
  - Association rules with multiple, alternative hierarchies:

    2% milk $\rightarrow$ *Wonder bread*

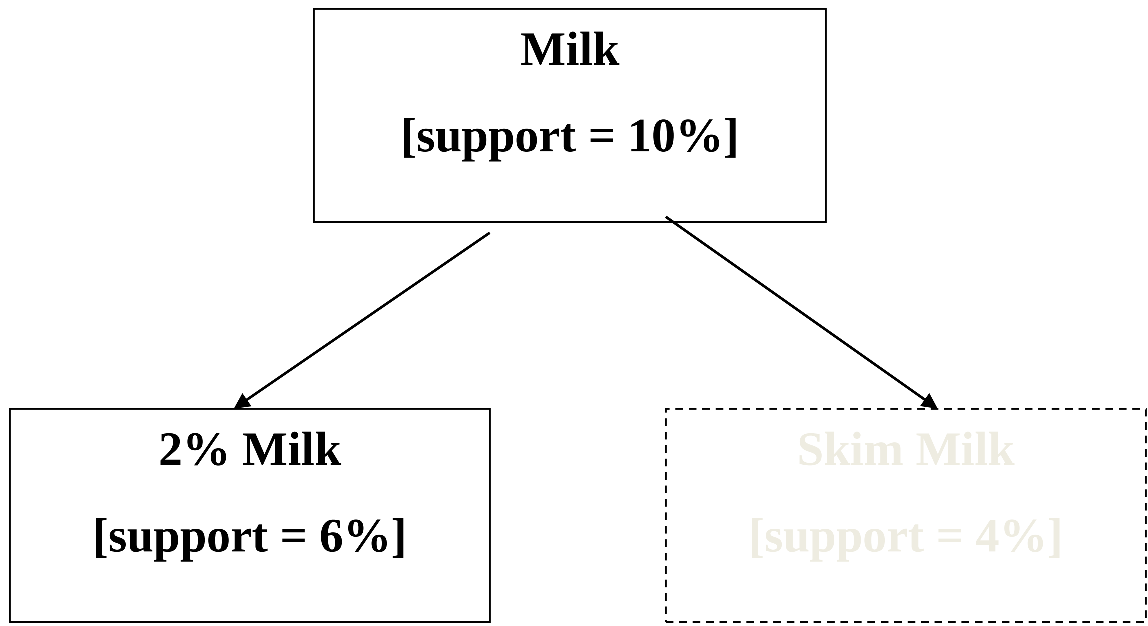
Multi-level Association: Uniform Support vs. Reduced Support

- Uniform Support: the same minimum support for all levels
  - **+** One minimum support threshold. No need to examine itemsets containing any item whose ancestors do not have minimum support.
  - **–** Lower level items do not occur as frequently. If support threshold
    - too high $\Rightarrow$ miss low level associations
    - too low $\Rightarrow$ generate too many high level associations
- Reduced Support: reduced minimum support at lower levels

Uniform Support

Multi-level mining with uniform support

Level 1
min_sup = 5%

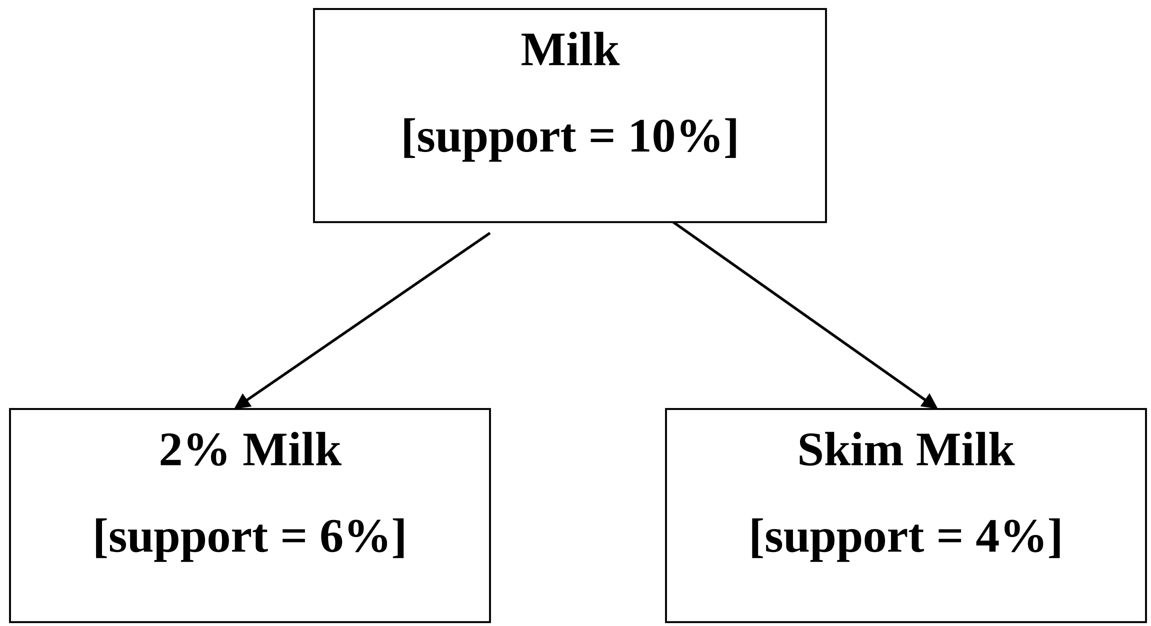


Level 2
min_sup = 5%

Reduced Support

Multi-level mining with reduced support

Level 1
min_sup = 5%



Level 2
min_sup = 3%

Multi-level Association: Redundancy Filtering

- Some rules may be redundant due to "ancestor" relationships between items.
- Example
  - milk $\Rightarrow$ wheat bread [support = 8%, confidence = 70%]
  - 2% milk $\Rightarrow$ wheat bread [support = 2%, confidence = 72%]
- We say the first rule is an ancestor of the second rule.

Multi-Dimensional Association: Concepts

- Single-dimensional rules:

  buys(X, “milk”) $\Rightarrow$ buys(X, “bread”)

- Multi-dimensional rules: ❍ 2 dimensions or predicates
  - Inter-dimension association rules (*no repeated predicates*)

    age(X,”19-25”) $\wedge$ occupation(X,“student”) $\Rightarrow$ buys(X,“coke”)
  - hybrid-dimension association rules (*repeated predicates*)

    age(X,”19-25”) $\wedge$ buys(X, “popcorn”) $\Rightarrow$ buys(X, “coke”)

Interestingness Measurements

- Objective measures

  Two popular measurements:

  ☆ *support;* and

  ⏱ *confidence*

- Subjective measures

  A rule (pattern) is interesting if

  ☆ it is *unexpected* (surprising to the user); and/or

  ⏱ *actionable* (the user can do something with it)