THEORY OF COMPUTATION

CSC-251

UNIT 1

DWIT

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Theory of Computation

- **theory of computation** is the branch that deals with how efficiently problems can be solved on a model of computation, using an algorithm

Unit 1: (14 Hrs)

- **Review of Mathematical Preliminaries**: Sets, Logic, Functions, Relations, Languages and proofs.
- Finite Automata: Deterministic and Non- deterministic Finite Automata, Equivalence of Deterministic and Non-deterministic Finite Automata with Epsilon-Transition.
- Regular Expression and languages, Equivalence of Regular Expressions and Finite Automata, Algebraic Laws for Regular Expressions, Properties of Regular Languages, Pumping Lemma for Regular Languages, Minimization of Finite State Machine.

Sets

- A set is a group of objects.
- The objects in a set are called the elements, or members, of the set.
- Eg; The set of positive integers less than 100 can be denoted as

$$\{1,2,3,\ldots\ldots,99\}$$

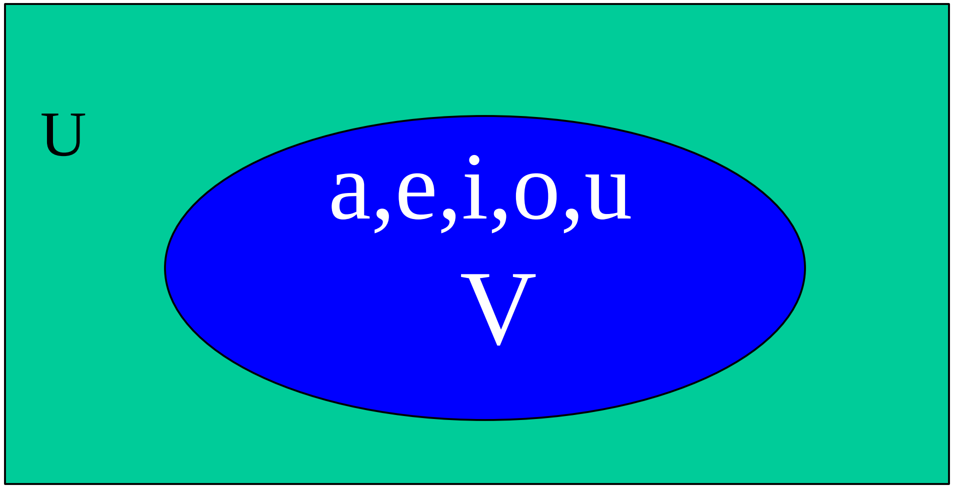
- Eg; A set can also consists of unrelated elements:

{a,1,Ram,Kathmandu}

- Equal Sets: $\{1,3,3,3,4,4,4\}$ and $\{1,3,4\}$

SET contd……

- A set can be described by using a Venn diagram



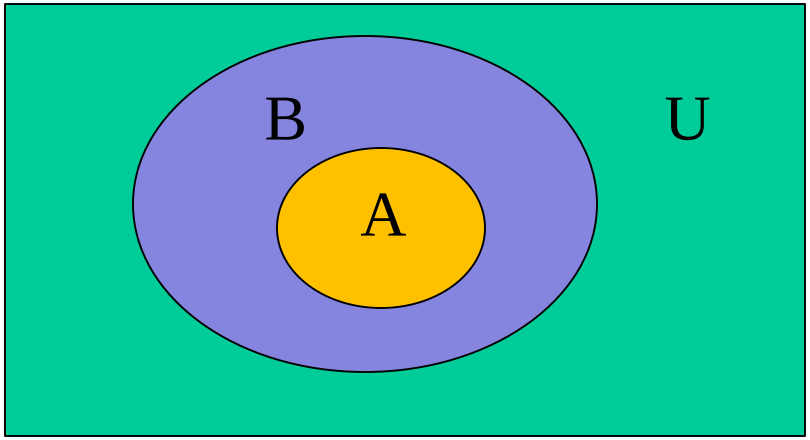
V, the set of vowels in English alphabet

- **empty set :** set that has no elements, denoted by ϕ

- **Subset**
  - The set *A* is said to be a subset of *B* if and only if every element of *A* is also an element of *B, denoted by* $A \subseteq B$

A={1,2,3} B={1,2,3,4,5,6,7}

- **Proper Subset**
  - **If a set A is a subset of set B but that $\mathrm{A} \neq \mathrm{B}$, A is called to be a proper subset of B denoted as $\mathrm{A} \subset \mathrm{B}$**



- **Power set** of a set S is the set of all subsets of S, denoted by P(S) or 2^S
  - Power Set of S={0,1,2}
  - $\{\phi,\{0\},\{1\},\{2\},\{0,1\},\{0,2\},\{1,2\},\{0,1,2\}\}$

Cartesian Product of Two Sets

- set whose all members are all possible ordered pairs(a,b)
- The **Cartesian product** of two sets is defined as:

$$A\times B = \{(a, b) \mid a\in A \wedge b\in B\}$$

- **Example:** $A = \{x, y\}$, $B = \{a, b, c\}$

  $A\times B = \{(x, a), (x, b), (x, c), (y, a), (y, b), (y, c)\}$

- **Disjoint Sets**
  - Two sets are called **disjoint** if their intersection is empty, that is, they share no elements:

$$A\cap B = \varnothing$$

SET OPERATIONS

- **Union**: $A \cup B = \{x \mid x \in A \vee x \in B\}$

  Example**:** $A = \{a, b\}$, $B = \{b, c, d\}$

  $A \cup B = \{a, b, c, d\}$

- **Intersection**: $A \cap B = \{x \mid x \in A \wedge x \in B\}$

  Example: $A = \{a, b\}$, $B = \{b, c, d\}$

  $A \cap B = \{b\}$

• The **difference** between two sets A and B contains exactly those elements of A that are not in B:

$$A-B = \{x \mid x \in A \wedge x \notin B\}$$

Example**:** A = {a, b}, B = {b, c, d},

•The **complement** of a set A contains exactly those elements under consideration that are not in A:

$$-A = U-A$$

Example**:** U = **N**, B = {10,11,12, …}

$$-B = \{0, 1, 2, \ldots, 8, 9\}$$

Properties of SET

Identity	*Name*
$A \cup \phi = A$ $A \cap U - A$	Identity laws
$A \cup U = U$ $A \cap \phi = \phi$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation laws
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive laws
$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$	*De Morgan's* law

Properties of SET contd….

Logic

- use and study of valid reasoning
  - Propositional Logic
  - Predicate Logic

Propositional Logic

- A proposition – a sentence that can be either true or false.
- Syntax and Semantics
- logical connectives
- branch of logic that studies ways of joining and/or modifying entire propositions, statements or sentences to form more complicated propositions, statements or sentences,
- Example
  1. Paris is the capital of France and Paris has a population of over two million.
     Therefore, Paris has a population of over two million.
  2. Ram is a man. All men are mortal

     So, Ram is Mortal

Propositional Logic connectives

- A *propositional connective is a way of combining propositions so that the* truth or falsity of the compound proposition depends only on the truth or falsity of the components.

connective	*symbol*
not (negation)	$\neg$
and (conjunction)	$\wedge$
or (disjunction)	$\vee$
implies (implication)	$\Rightarrow$
iff (equivalence)	$\Leftrightarrow$

connectives using *truth tables*

p	q	$p \wedge q$	$p \vee q$	$p \Rightarrow q$	$p \Leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Propositional Logic Examples

- P means "It is hot."
- Q means "It is humid."
- R means "It is raining."
- $(P \wedge Q) \rightarrow R$

  "If it is hot and humid, then it is raining"
- $Q \rightarrow P$

  "If it is humid, then it is hot"

Note: Make Table and compare

Propositional Logic Examples

- The statements "The cafeteria either has no pasta, or it has no sauce" and "The cafeteria doesn't have both pasta and sauce" are two ways to say the same thing.
- If *p = "The cafeteria has pasta" and*
- *q = "The* cafeteria has sauce",
- then this means that

$$(\neg p) \vee (\neg q) \quad \text{and} \quad \neg(p \wedge q)$$

are equivalent(*De Morgan's Laws.)*

Law of the double negative

- $\neg\neg P$ and P are equivalent.
- If something is not false, it is true, and vice versa.

Contrapositive Statement

- “If it rained this morning, the grass is wet" and “If the grass is dry, it didn't rain this morning" are two more ways to say the
- same thing.
- Now if p = *“It rained this morning" and*
- q = *“The grass* is wet",
- then this means that

$$(p \Rightarrow q) \quad \text{and} \quad \underline{(\neg q) \Rightarrow (\neg p)}$$

Contrapositive

- Either it did not rain this morning, or the grass is wet.
- How Do you represent???

- **Implication** $P \rightarrow Q \rightarrow \neg P \vee Q$

"If I win the lottery, then I will give you half the money" is true exactly when I either don't win the lottery, or I give you half the money.

- **Equivalence** $P \leftrightarrow Q \rightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$

"Anna is healthy if and only if she is happy" is equivalent to "If Anna is healthy, then she is happy, and if Anna is happy, then she is healthy".

- **Absorption** $P \vee (P \wedge Q) \rightarrow P$

"Kate is happy, or Kate is happy and healthy" is true if and only if "Kate is happy" is true.

More on Propositional Equivalence

- **Tautology**
  - compound proposition that is always true
  - $P \vee \neg P$
- **Contradiction**
  - compound proposition that is always false
  - $P \wedge \neg P$.
- **Contingency**
  - compound proposition that is not a tautology or a contradiction

Well Formed Formula(WFF)

- A sentence (well formed formula) is defined as follows:
  - A symbol is a sentence
  - If S is a sentence, then $\neg$S is a sentence
  - If S is a sentence, then (S) is a sentence
  - If S and T are sentences, then (S $\vee$ T), (S $\wedge$ T), (S $\rightarrow$ T), and (S $\leftrightarrow$ T) are sentences

  A sentence results from a finite number of applications of the above rules

Inference Rules in Propositional logic

The process of deriving new sentences from old one is called **inference**.

RULE	PREMISE	CONCLUSION
Modus Ponens	$A, A \rightarrow B$	B
And Introduction	A, B	$A \wedge B$
And Elimination	$A \wedge B$	A
Double Negation	$\neg\neg A$	A
Unit Resolution	$A \vee B, \neg B$	A
Resolution	$A \vee B, \neg B \vee C$	$A \vee C$

Why Predicate Logic

- Propositional logic is a weak language
- Hard to identify “individuals” (e.g., Mary, 3)
- Can’t directly talk about properties of individuals or relations between individuals (e.g., “Bill is tall”)
- Generalizations, patterns, regularities can’t easily be represented (e.g., “all triangles have 3 sides”)
- First-Order Logic (abbreviated FOL or FOPC) is expressive enough to concisely represent this kind of information

FOL adds relations, variables, and quantifiers, e.g.,

“Every elephant is gray”: $\forall$ x (elephant(x) $\rightarrow$ gray(x))

“There is a white alligator”: $\exists$ x (alligator(X) ^ white(X))

Predicate Logic Provides

- **Variable symbols**
  - E.g., x, y
- **Connectives**
  - Same as in PL: not ($\neg$), and ($\wedge$), or ($\vee$), implies ($\rightarrow$), if and only if (biconditional $\leftrightarrow$)
- **Quantifiers**
  - Universal $\forall$**x**
  - Existential $\exists$**x**

Universal Quantification

- For all x in the universe of discourse, P(x) is true.
- Using the universal quantifier $\forall$:

  $\forall x\, P(x)$ “for all x P(x)” or “for every x P(x)”
- Example:
- Every DWIT students are good.

Existential Quantification

- There exists an x in the universe of discourse for which P(x) is true.
- Using the existential quantifier ∃:

$\exists x\, P(x)$ "There is an x such that P(x)."

"There is at least one x such that P(x)."

"There is an x such that x is a Maths professor and x is a genius." Or "At least one Maths professor is a genius."

$\exists x\, (P(x) \wedge G(x))$

Examples

- **A person's mother is that person's parent.**
  - $\forall X$ person (X)$\rightarrow$ parent(mother-of(X),X)
- **There are people who think this class is cool.**
  - $\exists X$ person (X) $\wedge$ T (X) or $\exists X$ person (X) $\rightarrow$ T (X)
- **John likes to eat every food.**
  - $\forall X$ food(X) $\rightarrow$ likes (john,X)
- **John likes at least one dish Jane likes.**
  - $\exists F$ food(F) $\wedge$ likes (jane, F) $\wedge$ likes (john, F)

- **Everybody likes some food.**
- **There is a food that everyone likes.**
- **Whenever someone eats a spicy dish, they’re happy.**

- **Everybody likes some food.**

  $\forall X \, \exists F \; food(F) \wedge likes(X,F)$

- **There is a food that everyone likes.**

  $\exists F \, \forall X \; food(F) \wedge likes(X,F)$

- **Whenever someone eats a spicy dish, they're happy.**

  $\forall X \, \exists F \; food(F) \wedge spicy(F) \wedge eats(X,F) \rightarrow happy(X)$

- John’s meals are spicy.
- Every city has a dogcatcher who has been bitten by every dog in town.

- All students are smart.
- Everyone in the world is a student and is smart.
- There is a student who is smart.
- Everyone likes someone(Someone is liked by everyone.)

FOPL Inference Rules

- **Universal instantiation(universal elimination)**
  - $\forall x\ P(x) : P(C)$
  - If $(\forall x)\ P(x)$ is true, then $P(C)$ is true, where C is *any* constant in the domain of x
  - Example:

  $(\forall x)$ eats(Cat, x) $\Rightarrow$ eats(Cat, IceCream)

- **Universal generalization(Universal Introduction)**
  - $P(A) \wedge P(B) \ldots : \forall x\ P(x)$

Inference Rules Contd…

- **Existential instantiation(existential elimination)**
  - From $(\exists x)\ P(x)$ infer $P(c)$
  - Example:
  - $(\exists x)$ eats(Ziggy, x) $\rightarrow$ eats(Ziggy, Stuff)

- **Existential generalization(existential introduction)**
  - If $P(c)$ is true, then $(\exists x)\ P(x)$ is inferred.
  - Example

  eats(Ziggy, IceCream) $\Rightarrow$ $(\exists x)$ eats(Ziggy, x)

Assignment

- Every gardener likes the sun.
- You can fool some of the people all of the time.
- You can fool all of the people some of the time.
- All purple mushrooms are poisonous.
- No purple mushroom is poisonous.
- There are exactly two purple mushrooms.
- Clinton is not tall.

Assignment

- **Differentiate Between Propositional Logic and Predicate Logic**