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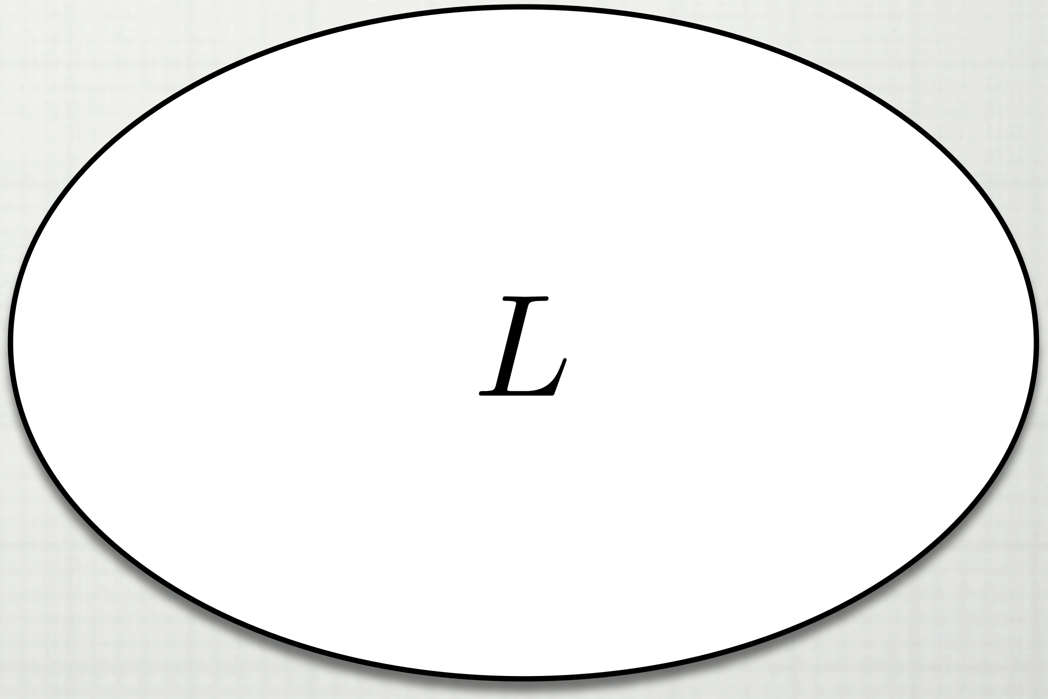

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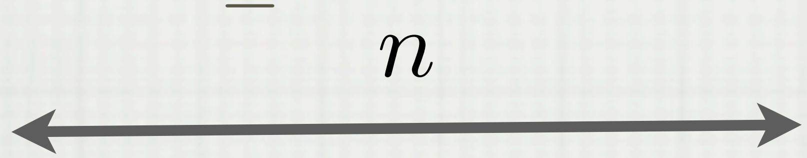

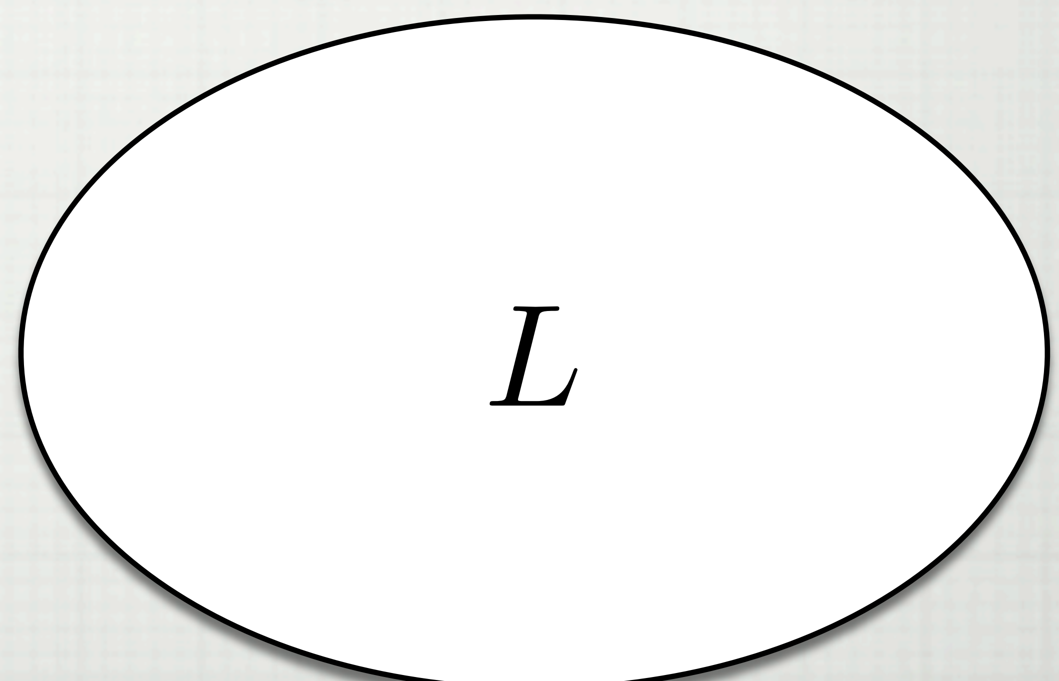

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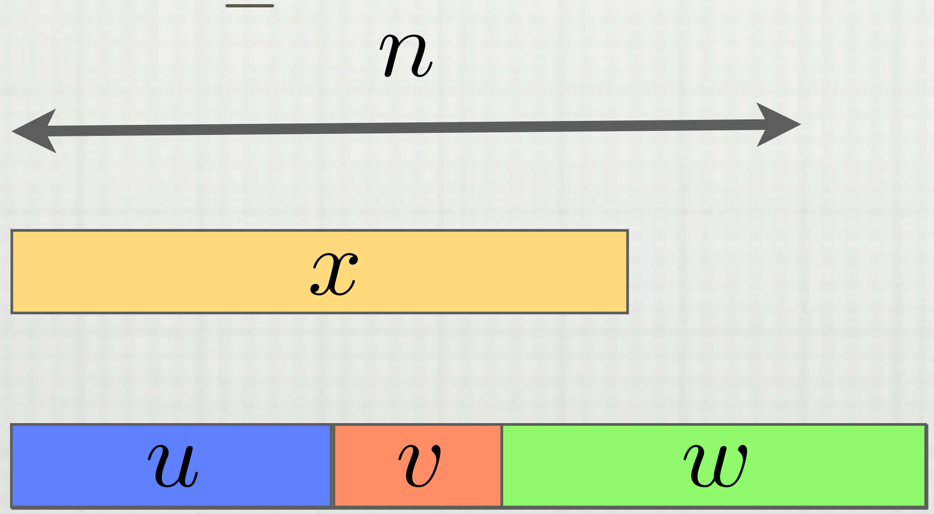

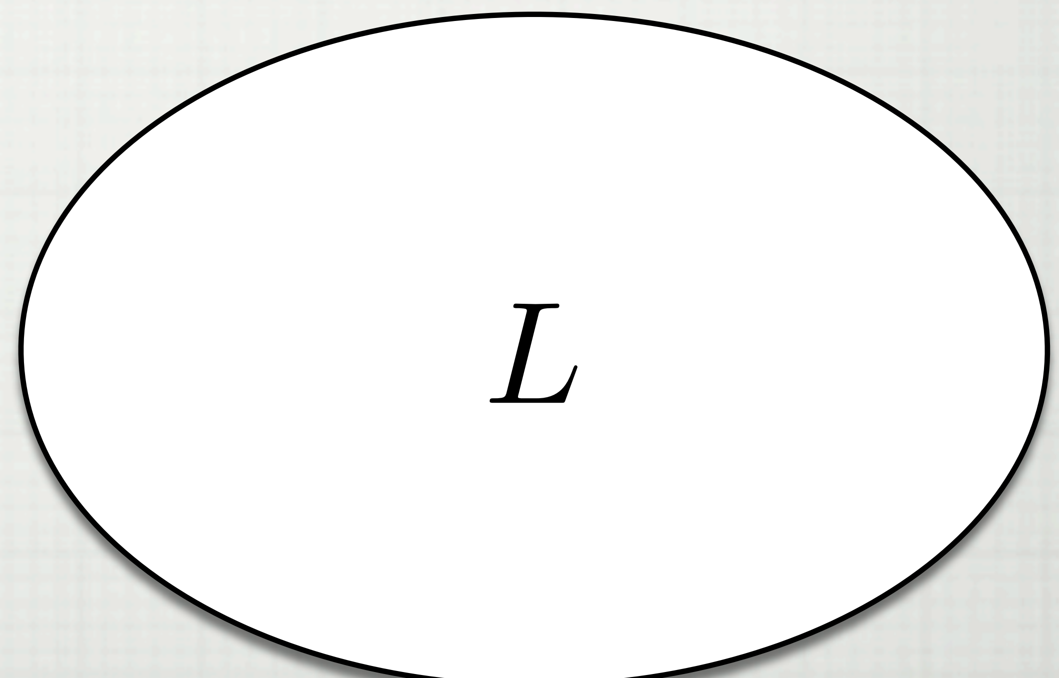

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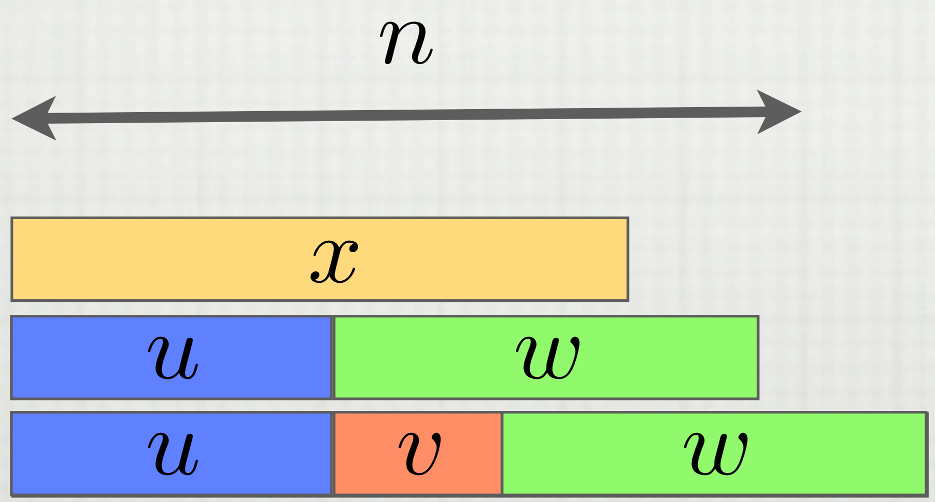

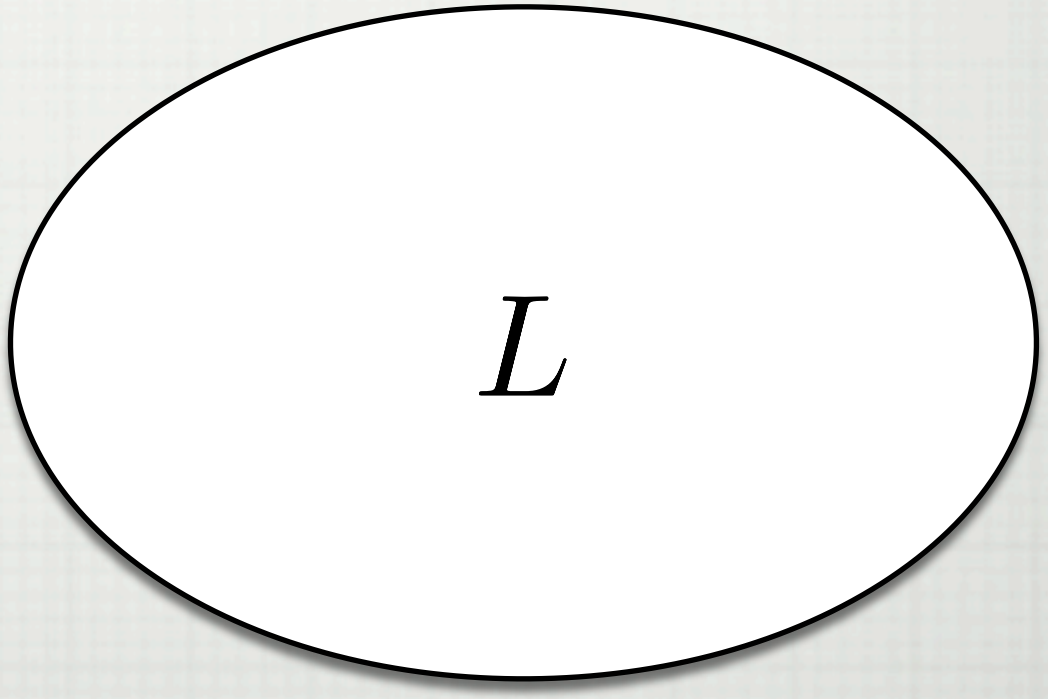

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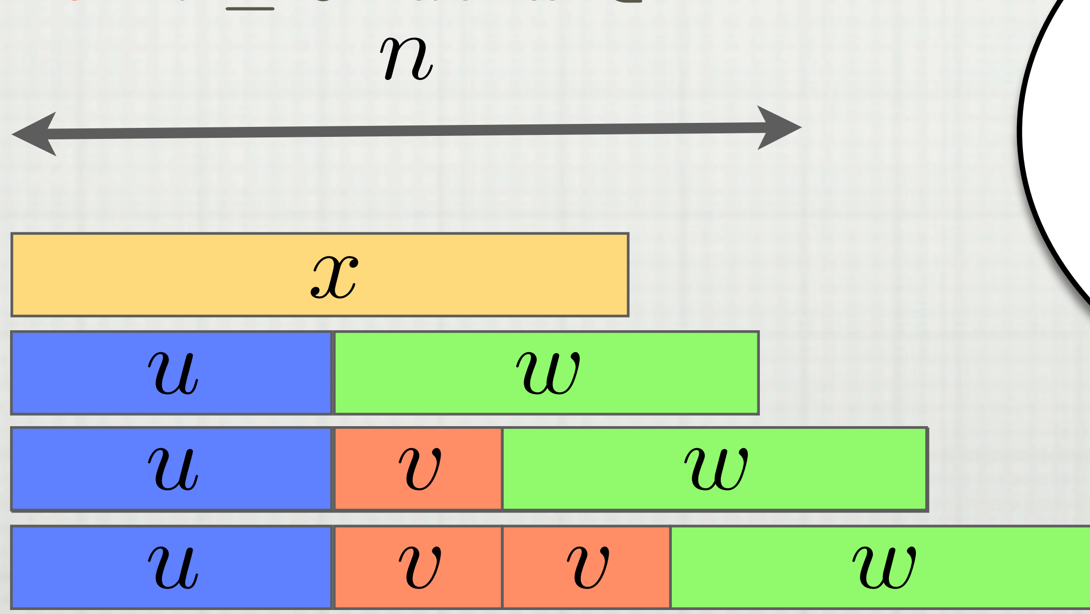

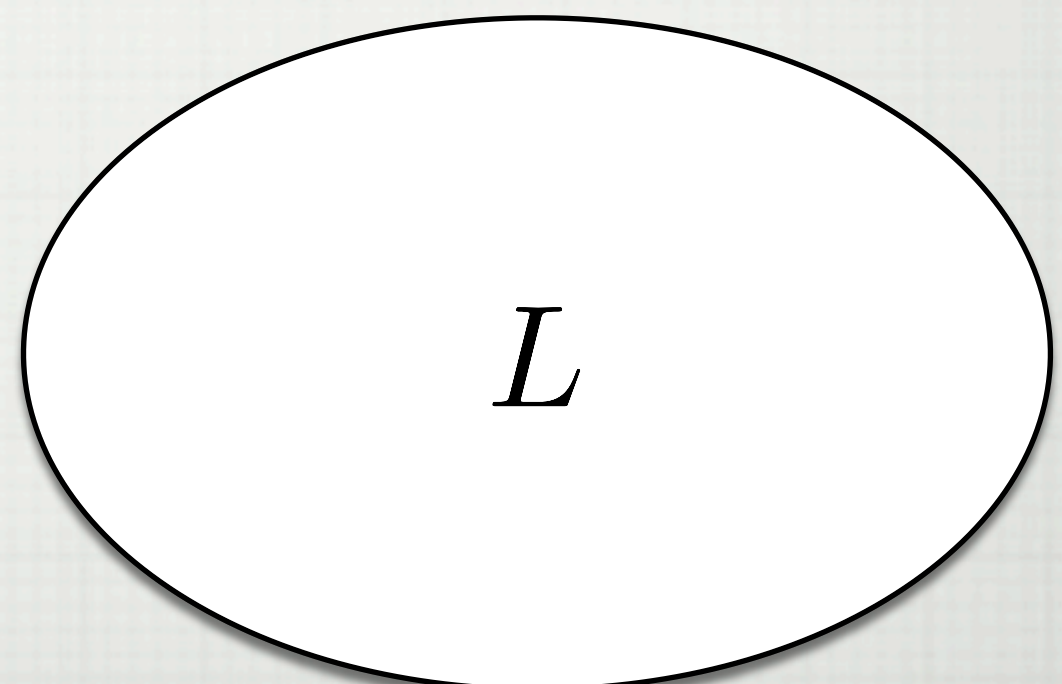

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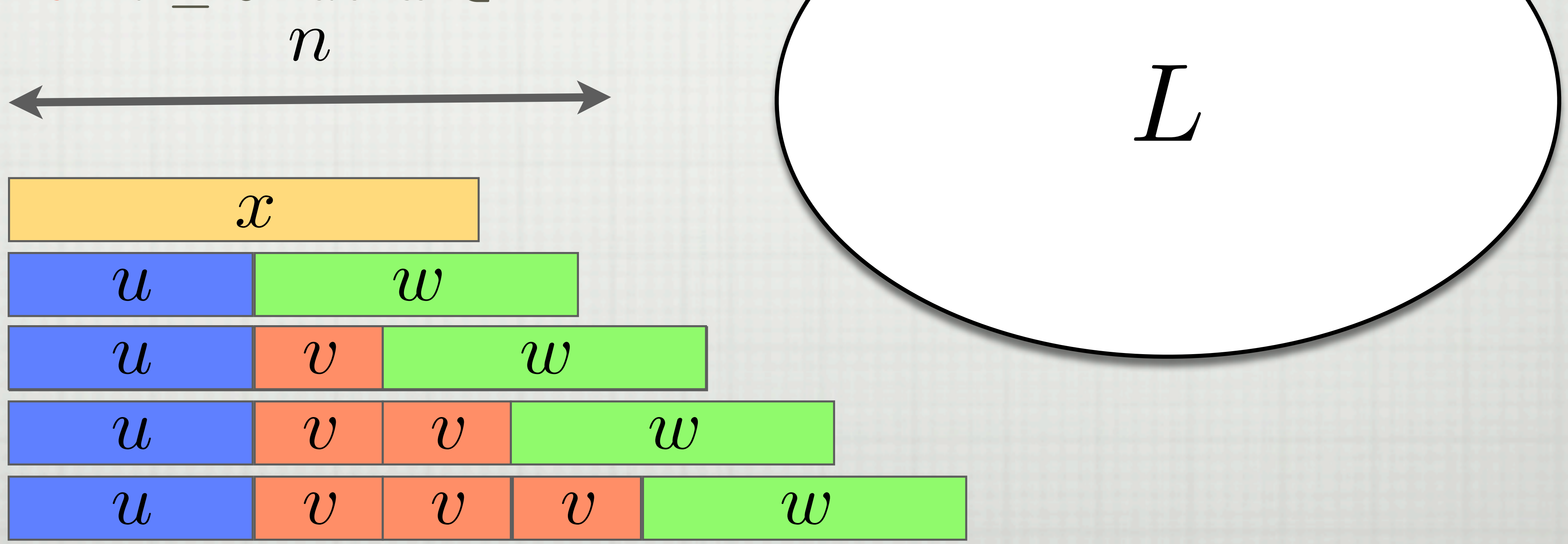

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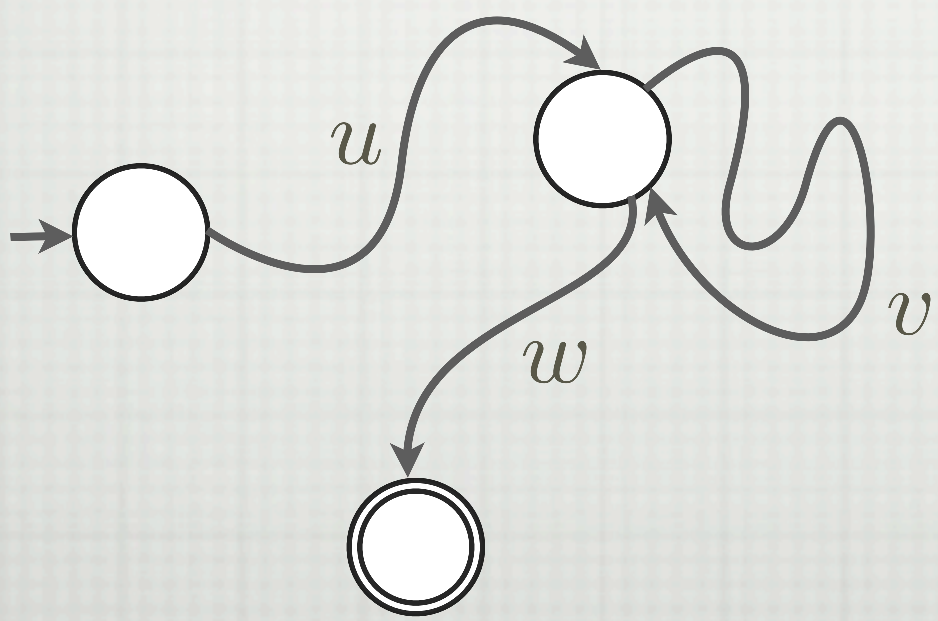

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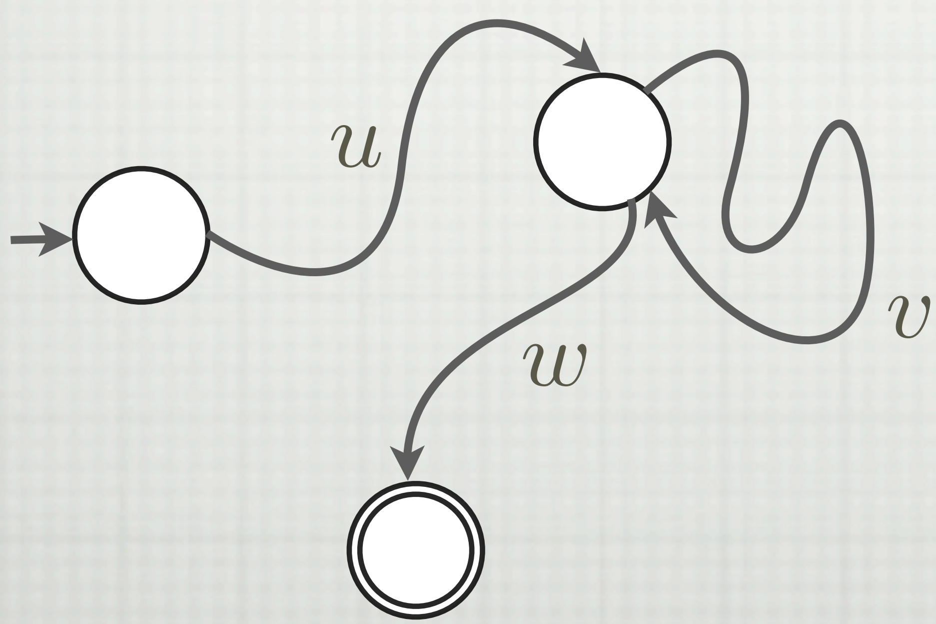

$$
\begin{aligned}
& uw \in L \\
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& uvvw \in L \\
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& \ldots
\end{aligned}
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USING PUMPING LEMMA TO PROVE NON-REGULARITY

$$\begin{array}{rcl} L \text{ regular} & \Longrightarrow & L \text{ satisfies P.L.} \\ L \text{ non-regular} & \Longrightarrow & ? \\ L \text{ non-regular} & \Longleftarrow & L \text{ doesn't satisfy P.L.} \end{array}$$

Negation:

$\exists\ n \in \mathbb{N}\ \forall\ x \in L$ with $|x| \geq n$
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(1)

(2)

(3)

(4)

Equivalently:
$(1) \wedge (2) \wedge (3) \Rightarrow$ **not**(4)
where **not**(4) is:
$\exists\, i : uv^i w \notin L$

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$$uv^0 w = uw = 0^s 0^p 1^n = 0^{s+p} 1^n \notin L, \quad \text{since } s + p \neq n$$

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If (1), (2), (3) hold then (4) fails: it is not the case that for all i, $uv^i w$ is in L.

In particular, let $i = 0$. $uw \notin L$.

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Both factors ≥ 2

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Can $0^n 0^n$ be written as $0^n 0^n = uvw$ such that $|uv| \leq n$ $|v| \geq 1$ and that for all i: $uv^i w \in L$?

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YES! Let $u = \epsilon$, $v = 00$, and $w = 0^{2n-2}$.

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Prove that $L = \{yy : y \in \{0,1\}^*\}$ is NOT regular.

Again we try to show that P.L. doesn't hold.
If L is regular, then by P.L. $\exists\, n$ such that ...
Let us consider $x = 0^n 0^n \in L$. Obviously $|x| \geq n$.

Can $0^n 0^n$ be written as $0^n 0^n = uvw$ such that $|uv| \leq n$ $|v| \geq 1$ and that for all i: $uv^i w \in L$?

YES! Let $u = \epsilon$, $v = 00$, and $w = 0^{2n-2}$.
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NO. We have chosen a bad string x. To show that L fails the P.L., we only need to exhibit some x that cannot be "pumped" (and $|x| \geq n$).

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A: Yes! Take $u = \epsilon$, $v = 0101$, $w = (01)^{2n-2}$.

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Another bad choice of x!

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Prove that $L = \{yy : y \in \{0,1\}^*\}$ is NOT regular.

Given n from the P.L., let $x = 0^n 1 0^n 1$. Again $x \in L$ and $|x| \geq n$.

$\forall u, v, w$ such that $0^n 1 0^n 1 = uvw$ and $|uv| \leq n$ and $|v| \geq 1$:

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Given n from the P.L., let $x = 0^n10^n1$. Again $x \in L$ and $|x| \geq n$.

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Since $|v|$ is at least 1, this is clearly not of the form yy.