THEORY OF COMPUTATION

CSC-251

UNIT 1

DWIT

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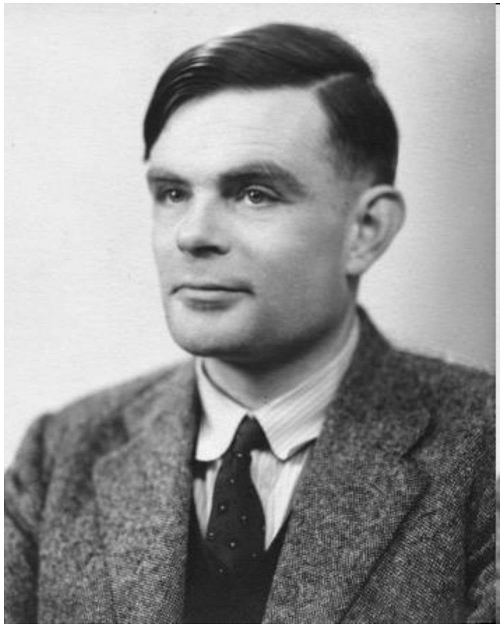
Outline

- Finite Automata: Deterministic and Non-deterministic Finite Automata
- Equivalence of Deterministic and Non-deterministic Finite Automata with Epsilon-Transition

Automata Theory

- Study of abstract computing devices or machines
- Automaton = an abstract computing device



ALAN TURING(1912-1954)

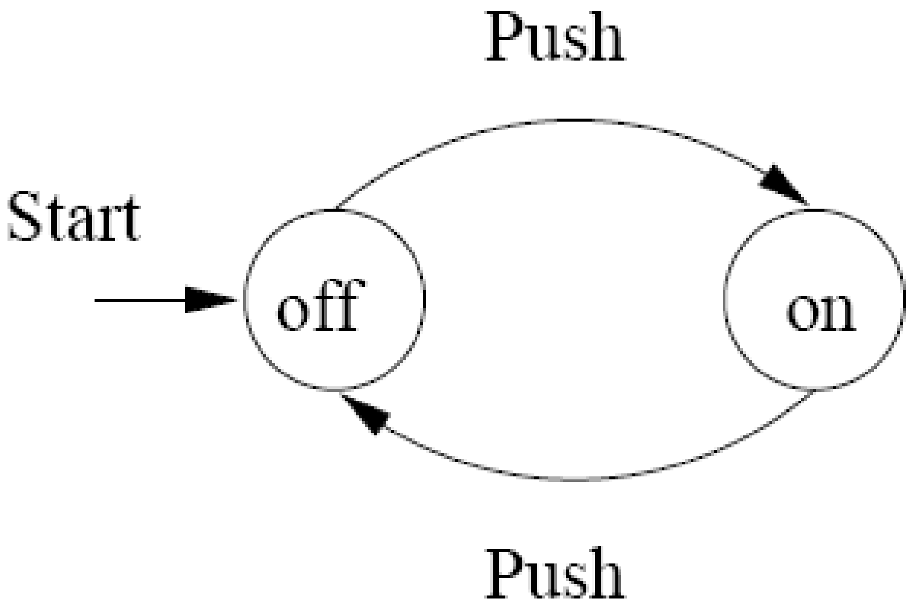
- Father of Modern Computer Science
- English mathematician
- Studied abstract machines called

Turing machines even before computers existed

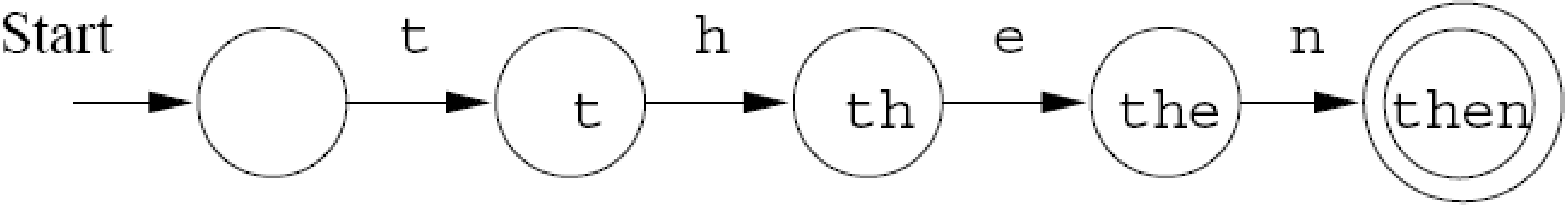
- Turing test?

Finite Automata is used as a model for

- Software for designing digital cicuits
- Lexical analyzer of a compiler(characters into tokens)
- Software for verifying finite state systems, such as communication protocols.
- Example: Finite Automaton modelling an on/off switch



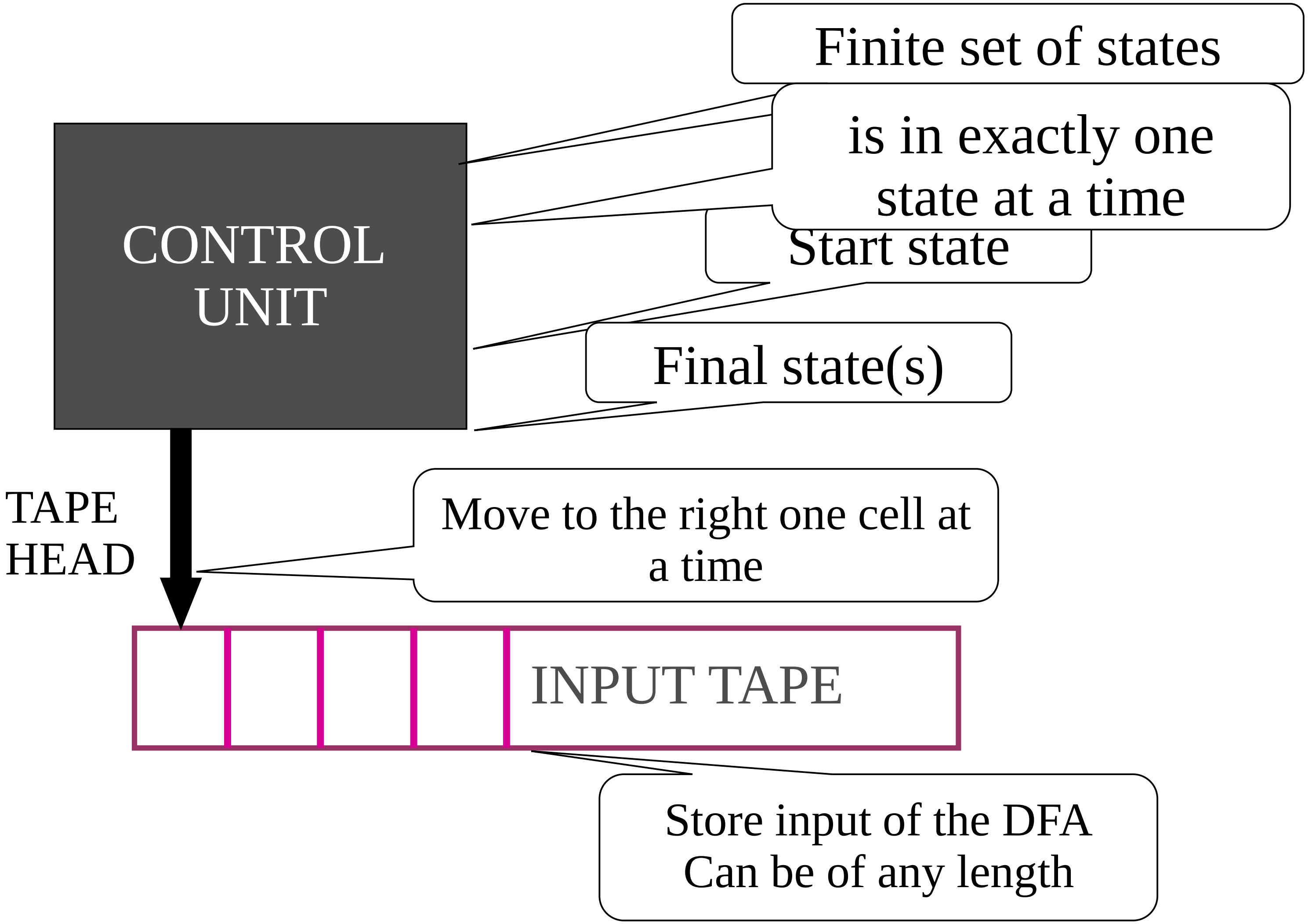
FA recognizing the string” then”



Finite Automata (FA)

- Finite collections of states with transition rules that take you from one state to another
- Recognizer for “Regular Languages”
- *Deterministic Finite Automata (DFA)*
  - The machine can exist in only one state at any given time
- *Non-deterministic Finite Automata (NFA)*
  - The machine can exist in multiple states at the same time

Finite Automata



Finite set of states

is in exactly one state at a time

Start state

Final state(s)

Move to the right one cell at a time

Store input of the DFA
Can be of any length

Alphabets

- finite set of symbols.
- {0,1}
- {a,b,c}

Strings

- The set of strings over an alphabet Σ is the set of lists, each element of which is a member of Σ
- Example: abc
- Σ* denotes this set of strings
- {0,1}* = {ε, 0, 1, 00, 01, 10, 11, 000,001, . . . }

Languages

- A language is a subset of Σ^* for some alphabet Σ
- The set of strings of 0's and 1's with no two consecutive 1's.
- L = {ε, 0, 1, 00, 01, 10, 000, 001, 010,100, 101, 0000, 0001, 0010, 0100,0101, 1000, 1001, 1010, . . . }
- Let L(A) be a language *recognized* by a DFA A.
  - Then L(A) is called a "*Regular Language*".

Deterministic Finite Automata(DFA)

- States ---->Determined
- **Quintuple {Q, Σ, δ, qo, F}**
  - A finite set of states (Q, typically).
  - An input alphabet (Σ, typically).
  - A transition function (δ, typically).
    - δ is a function $Q \times \Sigma \rightarrow Q$
  - A start state (q0, in Q, typically).
  - A set of final states (F ⊆ Q, typically).

Graph Representation of DFA’s

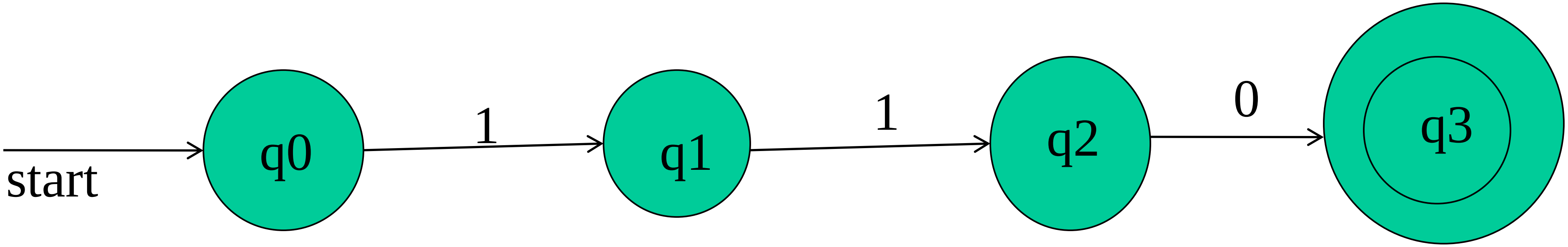
- Nodes = states.
- Arcs represent transition function.
- Arrow labeled “Start” to the start state.
- Final states indicated by **double circles**

The Transition Function

- Takes two arguments:
  - a state and
  - an input symbol.
- δ(q, a) = the state that the DFA goes to when it is in state q and input a is received.

Construct a DFA that accepts **110**

Construct a DFA that accepts 110



Construct a DFA that accepts 001101

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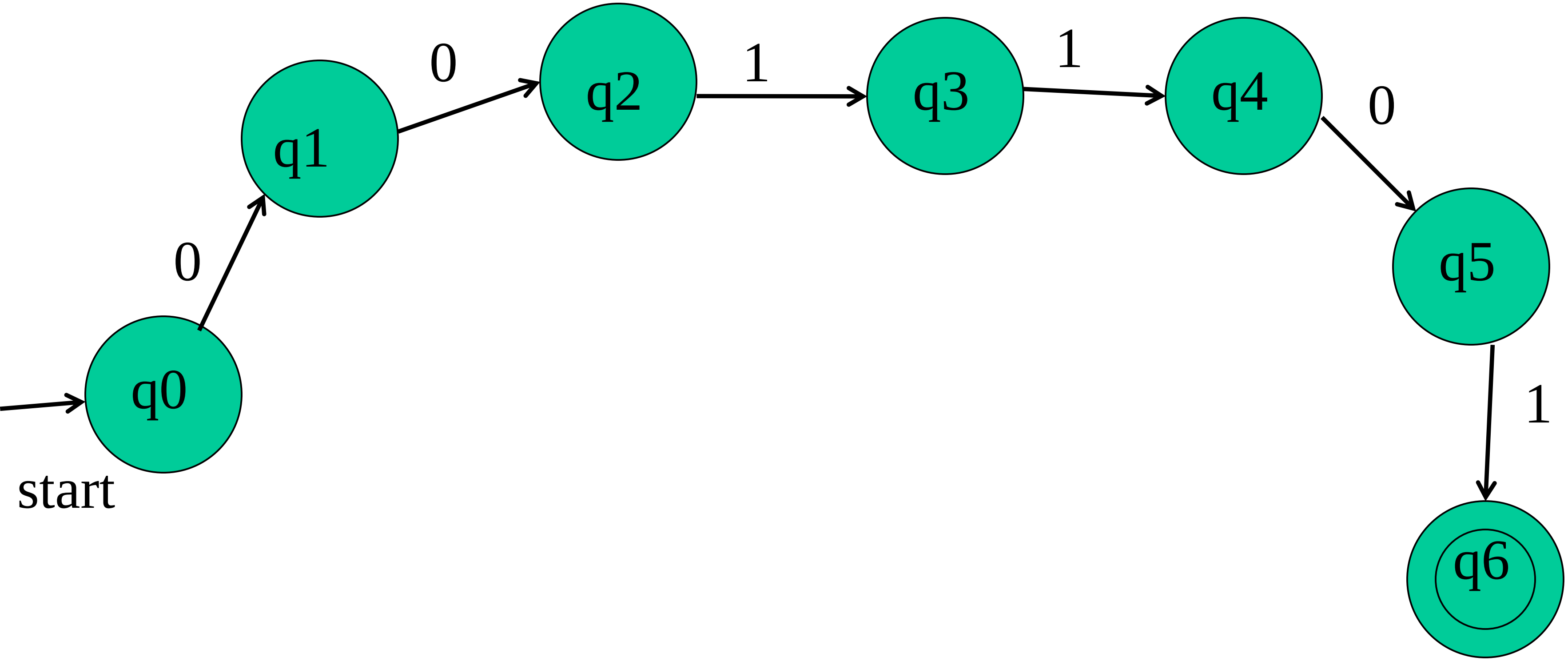
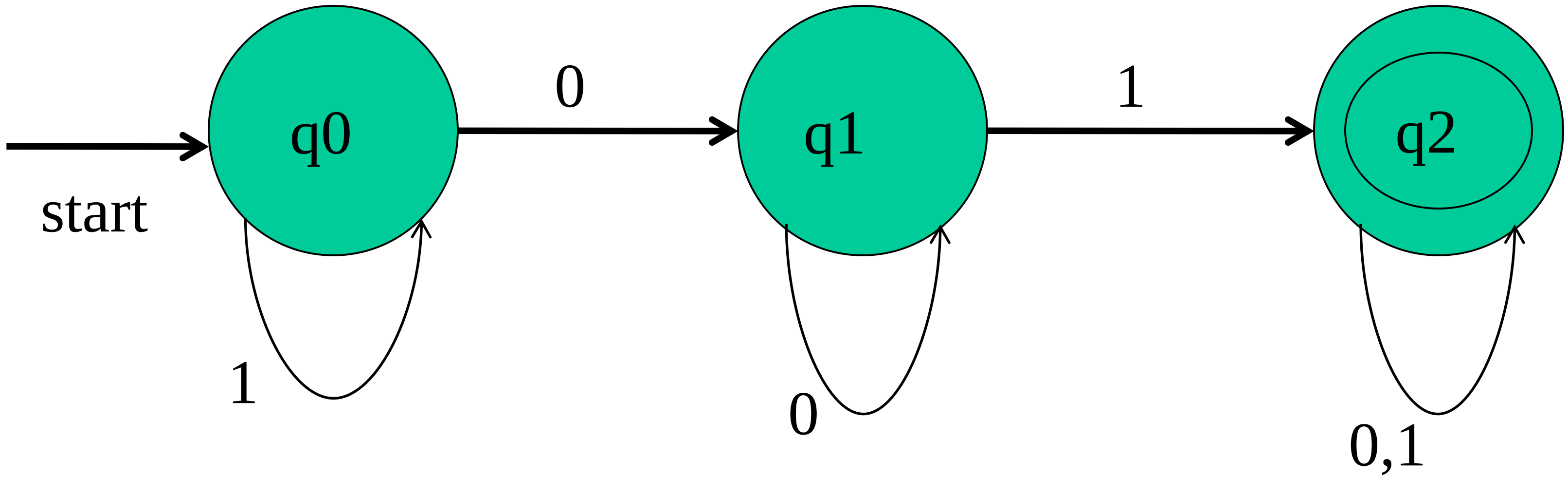


Construct a DFA to accept string containing a 0 followed by a 1

Construct a DFA to accept string containing a 0 followed by a 1



Construct a DFA to accept string containing even no. of 0’s and any no. of 1’s

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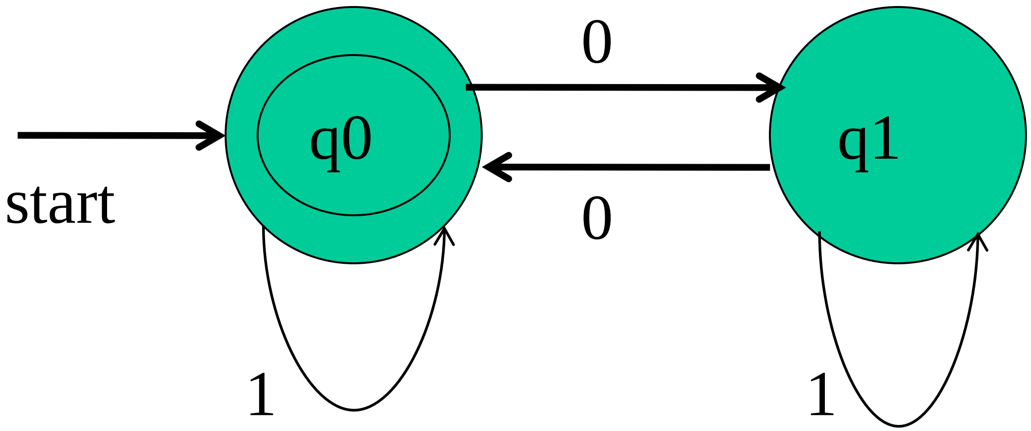
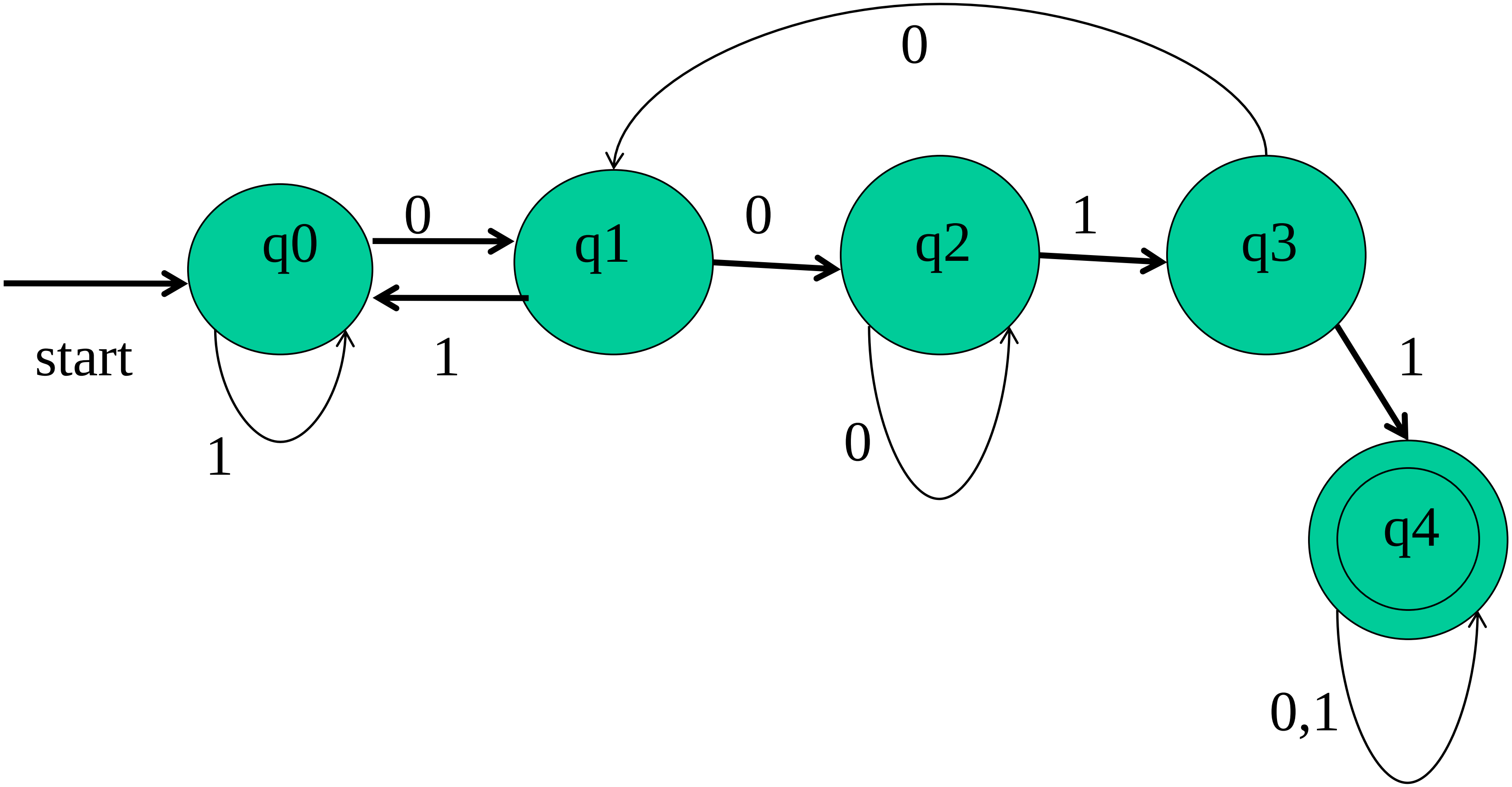


Construct a DFA to accept string containing two consecutive 0’s followed by two consecutive 1’s

Construct a DFA to accept string containing two consecutive 0's followed by two consecutive 1's



Construct a DFA to accept string that ends in 1101

An automaton A that accepts

$$L = \{x01y : x, y \in \{0,1\}^*\}$$

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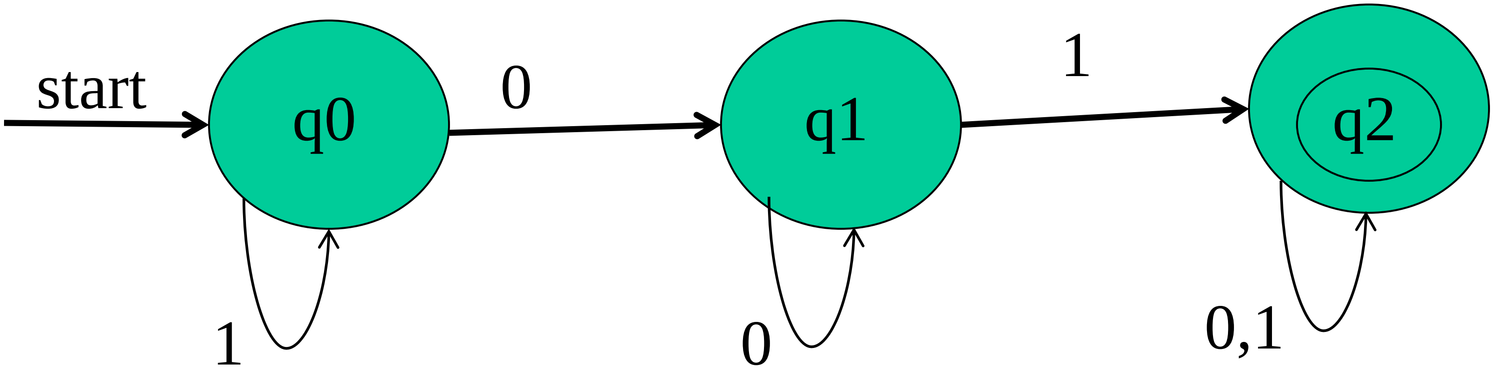

01,11010,1100011

111000 not accepted..why

DFA accepting all and only strings with an even number of 0's and an even number of 1's

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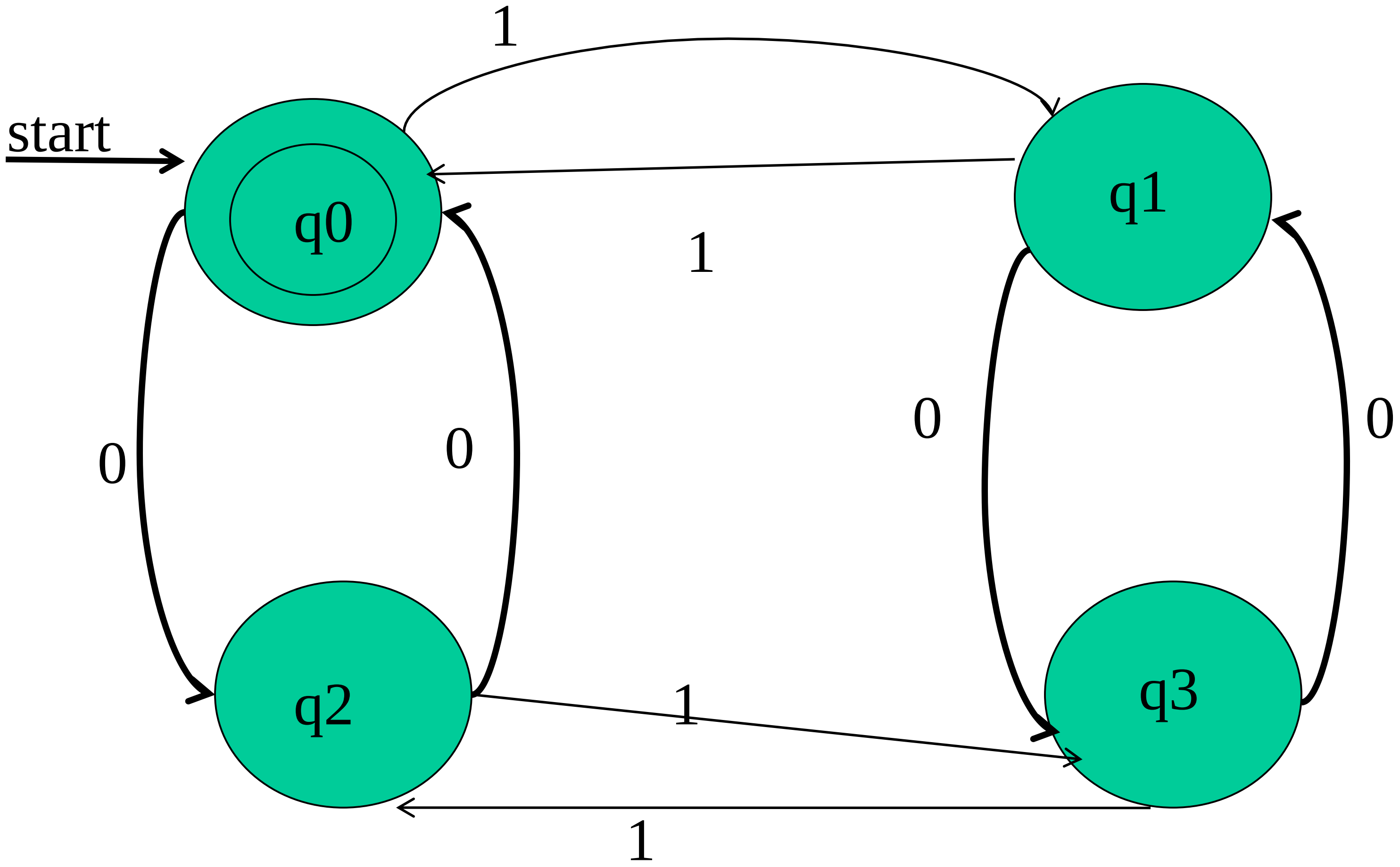
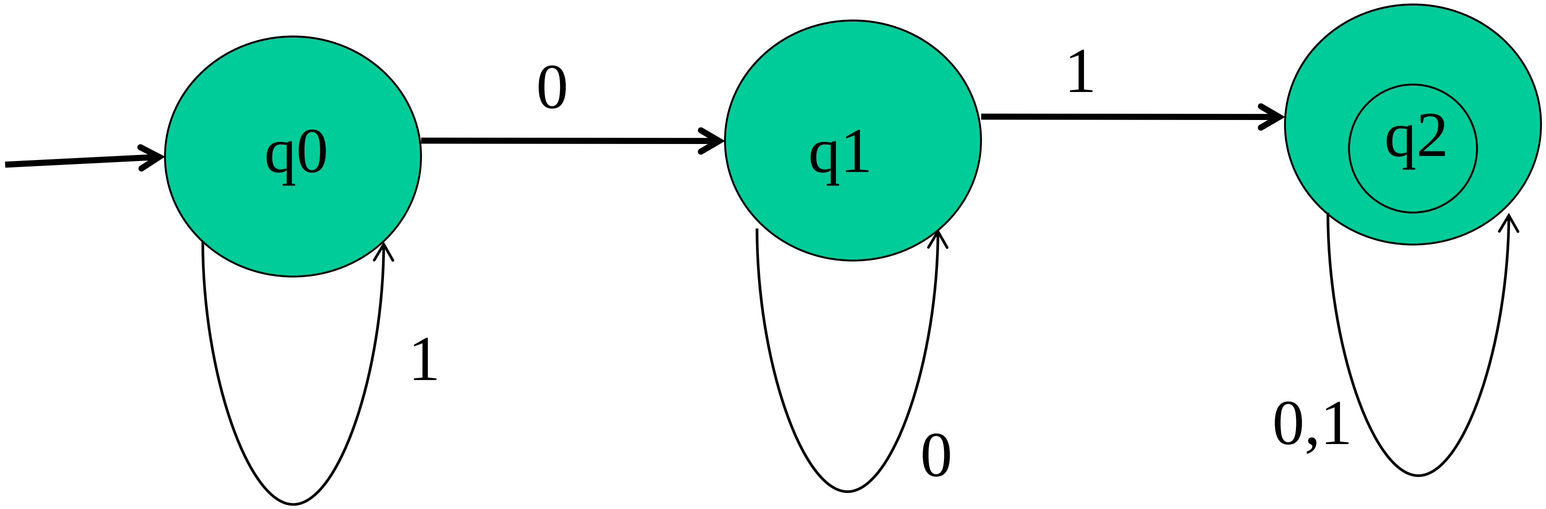


L={w|w has both an even no of 0’s and even no of 1’s

DFA accepting all strings with a substring 01



Construct a DFA

- Set of all strings ending in 00
- Set of all strings with three consecutive 0’s
- Set of strings with 011 as substring

Transition Function

$\delta(q1,101) = \delta(\,\delta(q1,1)\,,01)$

$= \delta(q1,01)$

$= \delta(\,\delta(q1,0)\,,1)$

$= \delta(q2,1)$

$= \delta(\,\delta(q2,0)\,,\varepsilon)$

$= \delta(q2,\varepsilon)$

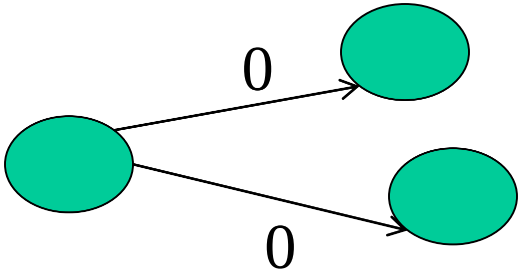
$= q2$

Nondeterministic Finite Automata

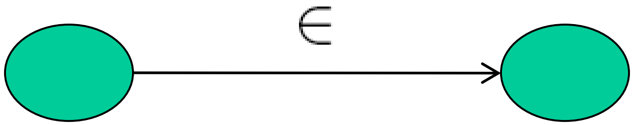
- The machine can exist in multiple states at the same time
- Transitions could be non-deterministic
- Power to be in several states at once
- **Guess** which state to go to next
- Easier to design than DFA
- Each transition function therefore maps to a set of states
- Differs with DFA in the transition function
    - Multiples states

Nondeterministic Finite Automata

- Nondeterministic move
  - On reading an input symbol, the automaton can choose to make a transition to one of selected states.



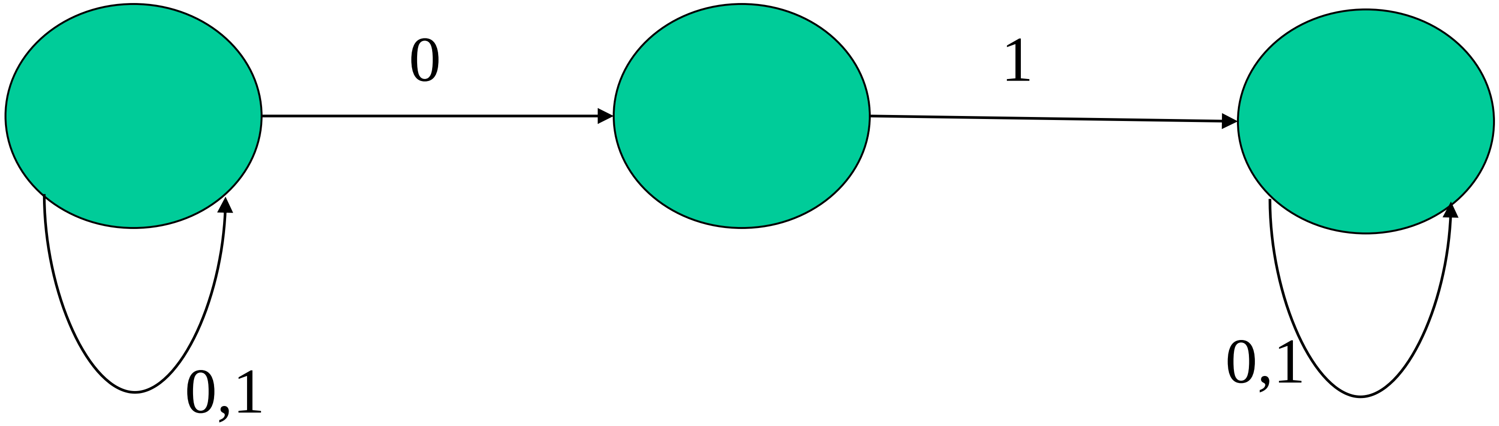
  - Without reading any symbol, the automaton can choose to make a transition to one of selected states or not.



Nondeterministic Finite Automata

- NFA is a quintuple
  - **{Q, Σ, δ, qo, F}**
- Q is a finite set of states
- **Σ** is a finite alphabet
- **δ** is a transition function from QX **Σ** to the powerset of Q
- A start state (q0, in Q, typically).
- A set of final states (F ⊆ Q, typically).

NFA for strings containing 01 or Regular expression: (0+1)*01(0+1)*



Transition function ?????

NFA: An automaton that accepts all and only strings ending in 01

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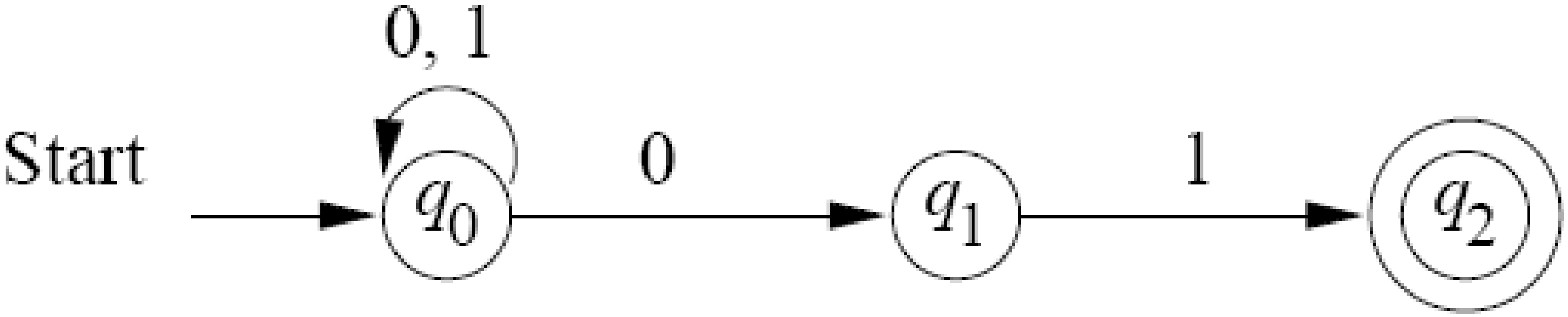
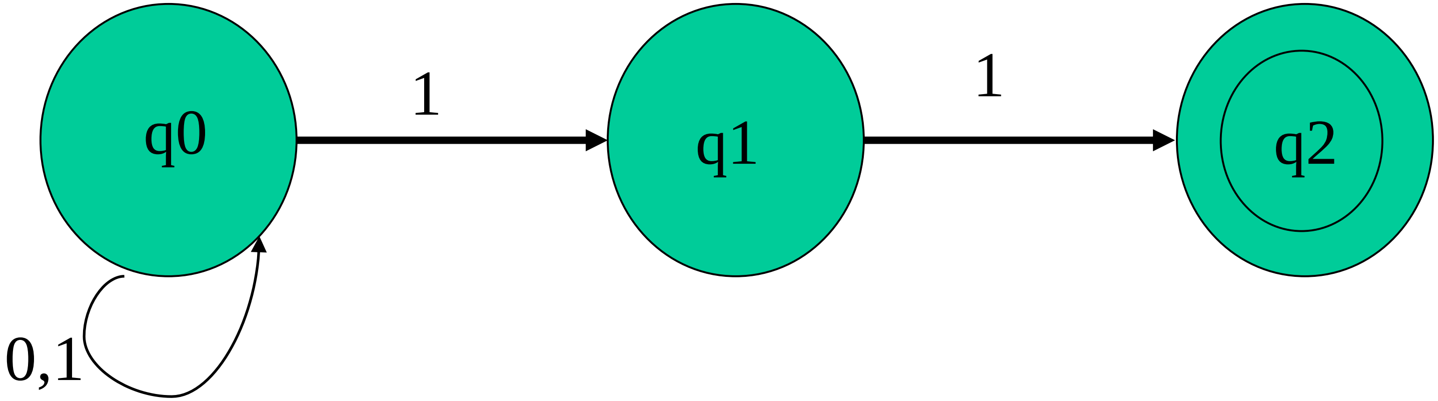


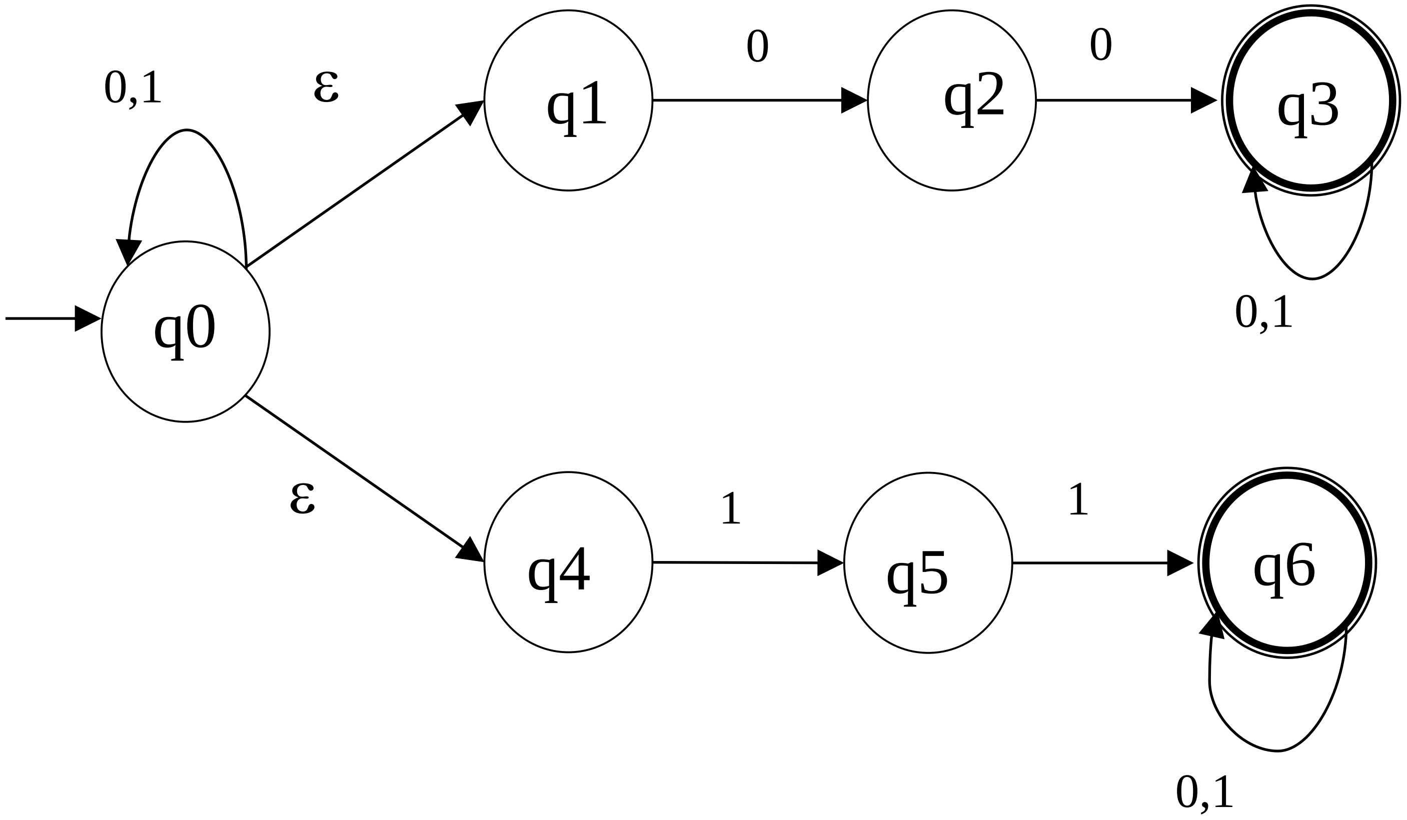
δ	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\emptyset$	$\{q_2\}$
$\star q_2$	$\emptyset$	$\emptyset$

An NFA accepting {$w \in \{0,1\}^*$ |w ends with 11}

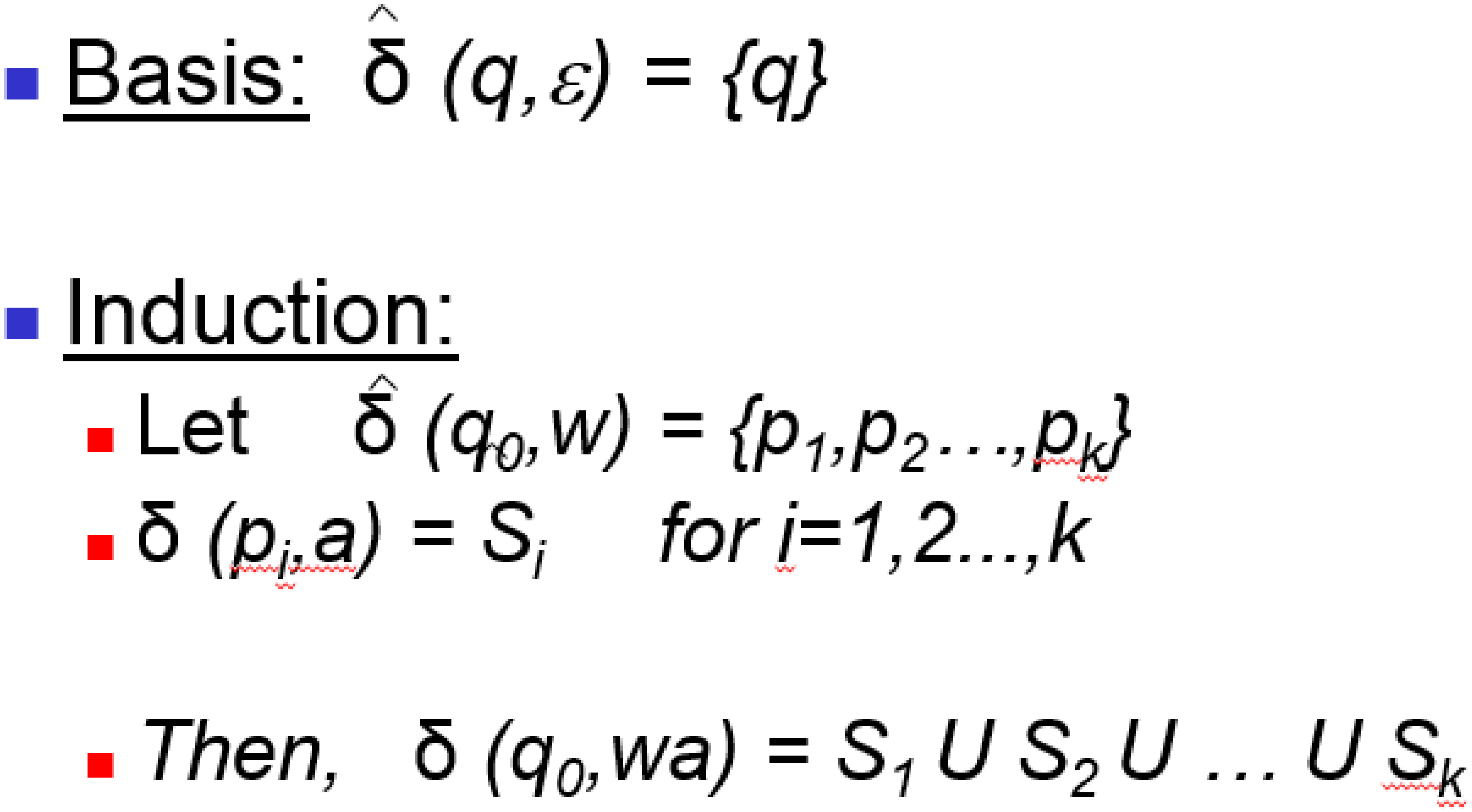


$\{w \in \{0,1\}^* \mid w$ has either 00 or 11 as substring$\}$

{*w* ∈ {0,1}* | *w* has either 00 or 11 as substring}



Extended Transition Function i.e. Extension of δ to NFA Paths



- <u>Basis:</u> $\hat{\delta}(q,\varepsilon) = \{q\}$
- <u>Induction:</u>
  - Let $\hat{\delta}(q_0,w) = \{p_1,p_2\ldots,p_k\}$
  - $\delta(p_i,a) = S_i$ *for i=1,2...,k*
  - *Then,* $\delta(q_0,wa) = S_1 \cup S_2 \cup \ldots \cup S_k$

Language of an NFA

- An NFA accepts *w* if *there exists at least one* path from the start state to an accepting (or final) state that is labeled by *w*
- $L(N) = \{ w \mid \hat{\delta}(q_0,w) \cap F \neq \Phi \}$

Equivalence of DFA & NFA: Subset construction

- Subset construction starts from and NFA $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$
- Describe a DFA

  $$D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$$

  such that L(D)=L(N)
- If-part: A language L is accepted by a DFA if it is accepted by an NFA
- Given any NFA N, we can construct a DFA D such that L(N)=L(D)
- How to convert an NFA into a DFA?
  - Observation: In an NFA, each transition maps to a *subset* of states
  - Idea: Represent:

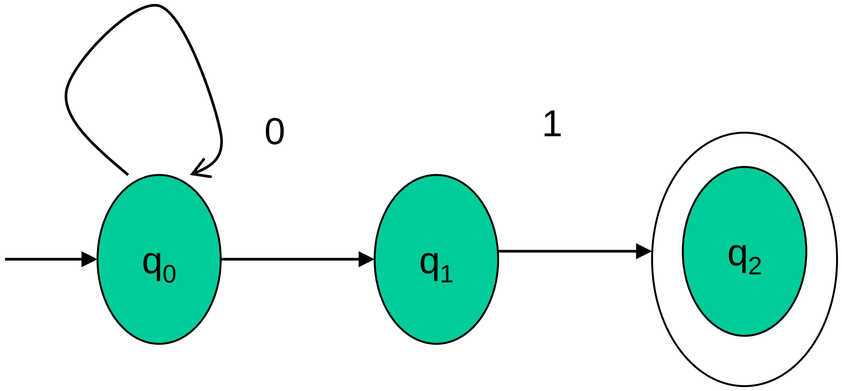
    each “subset of NFA_states” ➔ a single “DFA_state”

NFA to DFA by subset construction

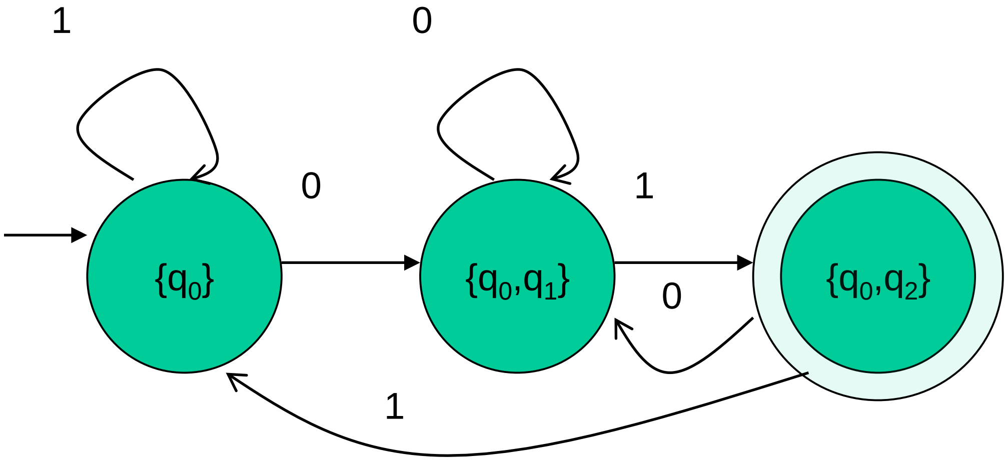
- Let N = $\{Q_N, \sum, \delta_N, q_0, F_N\}$
- Goal: Build D=$\{Q_D, \sum, \delta_D, \{q_0\}, F_D\}$ s.t. L(D)=L(N)
- Construction:

1. Q_D= all subsets of Q_N (i.e., power set)
2. F_D=set of subsets S of Q_N s.t. $S \cap F_N \neq \Phi$
3. δ_D: for each subset S of Q_N and for each input symbol a in $\sum$:
   - $\delta_D(S,a) = \cup\ \delta_N(p,a)$

$L = \{w \mid w \text{ ends in } 01\}$



NFA



DFA

FA with ε-Transitions

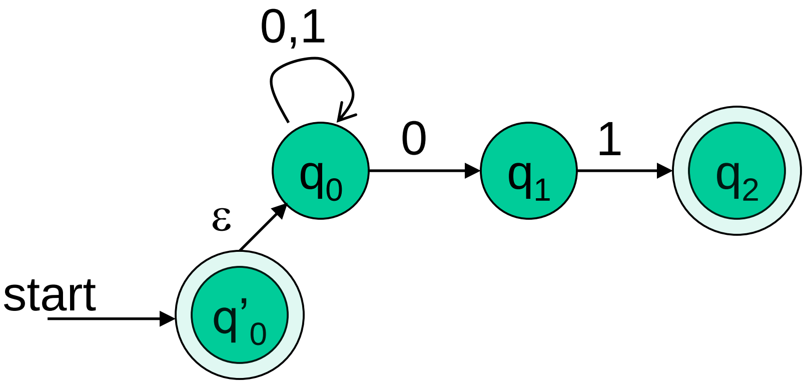
- We can allow explicit ε-transitions in finite automata
  - i.e., a transition from one state to another state without consuming any additional input symbol
  - Makes it easier sometimes to construct NFAs

***Definition:* ε -NFAs are those NFAs with at least one explicit ε-transition defined.**

- ε -NFAs have one more column in their transition table

Example of an ε-NFA

L = {w | w is empty, or if non-empty will end in 01}



δ_E	0	1	ε
→ *q'$_0$	Ø	Ø	{q'$_0$,q$_0$}
q$_0$	{q$_0$,q$_1$}	{q$_0$}	{q$_0$}
q$_1$	Ø	{q$_2$}	{q$_1$}
*q$_2$	Ø	Ø	{q$_2$}

ECLOSE(q'$_0$)

ECLOSE(q$_0$)

ECLOSE(q$_1$)

ECLOSE(q$_2$)

- ε-closure of a state q, ***ECLOSE(q)***,
- If state p is in ECLOSE(q) and there is a transition from state p to state r labelled ε, then r is in ECLOSE(q).