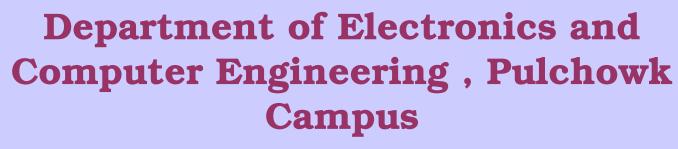
Digital Logic



Assistant Professor



Tribhuvan University



Sequential Logic Counters and Registers

Registers

- Introduction: Registers
 - Simple Registers
 - Registers with Parallel Load
- Using Registers to implement Sequential Circuits
- Shift Registers
 - Serial In/Serial Out Shift Registers
 - Serial In/Parallel Out Shift Registers
 - Parallel In/Serial Out Shift Registers
 - Parallel In/Parallel Out Shift Registers

Sequential Logic Counters and Registers

- Bidirectional Shift Registers
- An Application Serial Addition





Sequential Logic Counters and Registers

Counters

- Introduction: Counters
- Asynchronous (Ripple) Counters
- Asynchronous Counters with MOD number < 2ⁿ
- Asynchronous Down Counters
- Cascading Asynchronous Counters



- Synchronous (Parallel) Counters
- Up/Down Synchronous Counters
- Designing Synchronous Counters
- Decoding A Counter
- Counters with Parallel Load
- Shift Register Counters
 - Ring Counters
 - Johnson Counters
- Random-Access Memory (RAM)



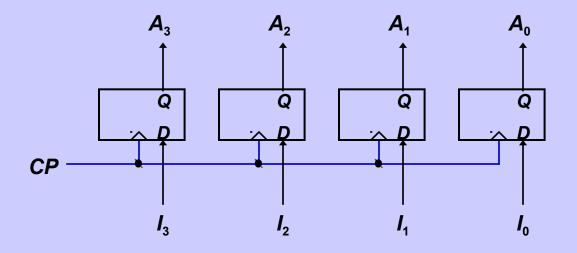
Introduction: Registers

- An n-bit register has a group of n flip-flops and some logic gates and is capable of storing n bits of information.
- The flip-flops store the information while the gates control when and how new information is transferred into the register.
- Some functions of register:
 - retrieve data from register
 - store/load new data into register (serial or parallel)
 - shift the data within register (left or right)



Simple Registers

- No external gates.
- Example: A 4-bit register. A new 4-bit data is loaded every clock cycle.

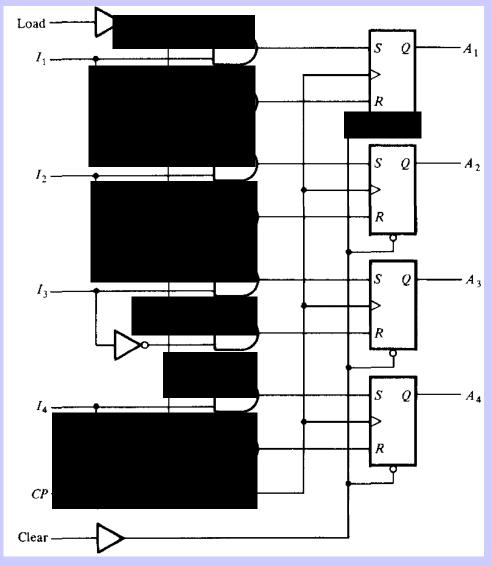




Registers With Parallel Load

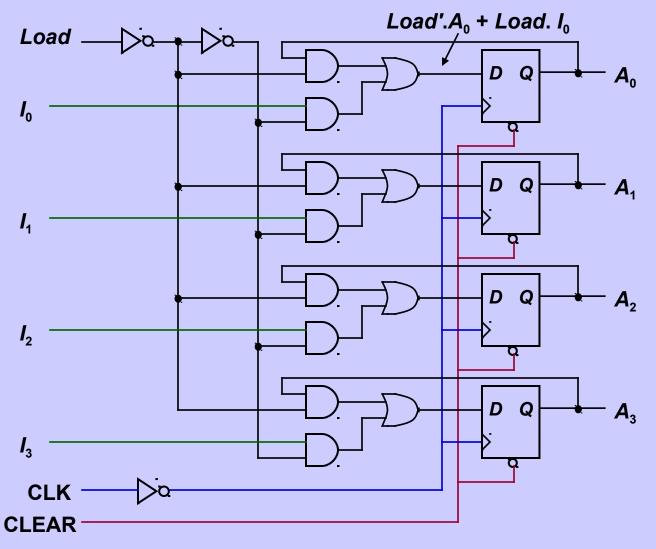
- Instead of loading the register at every clock pulse, we may want to control when to load.
- Loading a register: transfer new information into the register. Requires a load control input.
- Parallel loading: all bits are loaded simultaneously.

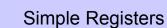
Registers With Parallel Load



4-bit Register with parallel load

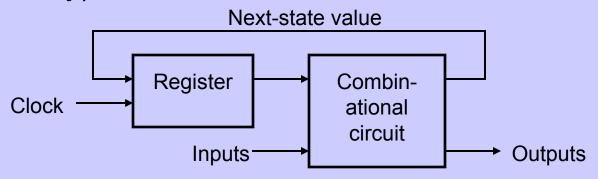
Registers With Parallel Load





Using Registers to implement Sequential Circuits

A sequential circuit may consist of a *register* (memory) and a *combinational circuit*.



- The external inputs and present states of the register determine the next states of the register and the external outputs, through the combinational circuit.
- The combinational circuit may be implemented by any of the methods covered in MSI components and Programmable Logic Devices.

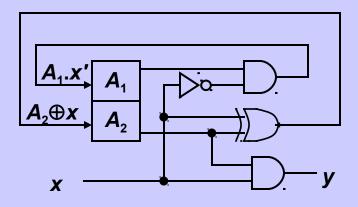
Using Registers to implement Sequential Circuits

Example 1:

$$A_{1}^{+} = \sum m(4,6) = A_{1}.x'$$

 $A_{2}^{+} = \sum m(1,2,5,6) = A_{2}.x' + A_{2}'.x = A_{2} \oplus x$
 $y = \sum m(3,7) = A_{2}.x$

Present state		Input	Output		
A_1	A_2	X	A_1^{\dagger}	A_2^+	У
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	0	0	1
1	0	0	1	0	0
1	0	1	0	1	0
1	1	0	1	1	0
1	1	1	0	0	1

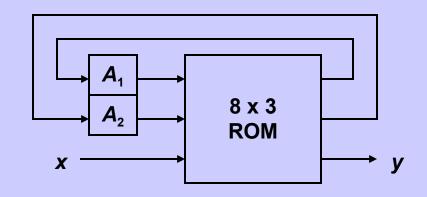




Using Registers to implement Sequential Circuits

Example 2: Repeat example 1, but use a ROM.

Address			Outputs			
1	2	3	1	2	3	
0	0	0	0	0	0	
0	0	1	0	1	0	
0	1	0	0	1	0	
0	1	1	0	0	1	
1	0	0	1	0	0	
1	0	1	0	1	0	
1	1	0	1	1	0	
1	1	1	0	0	1	



ROM truth table

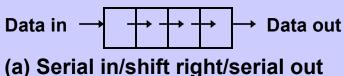


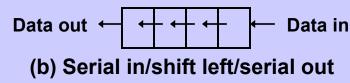
Shift Registers

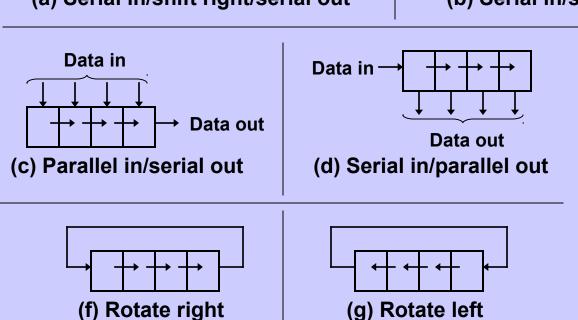
- Another function of a register, besides storage, is to provide for data movements.
- Each stage (flip-flop) in a shift register represents one bit of storage, and the shifting capability of a register permits the movement of data from stage to stage within the register, or into or out of the register upon application of clock pulses.

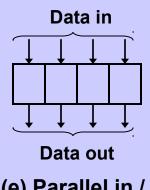
Shift Registers

 Basic data movement in shift registers (four bits are used for illustration).







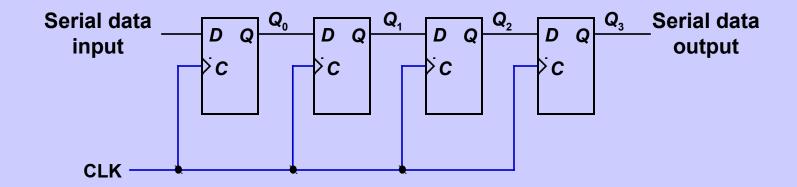






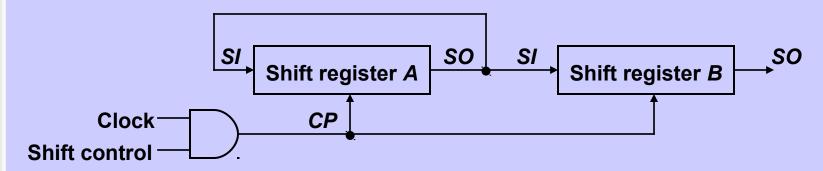
Serial In/Serial Out Shift Registers

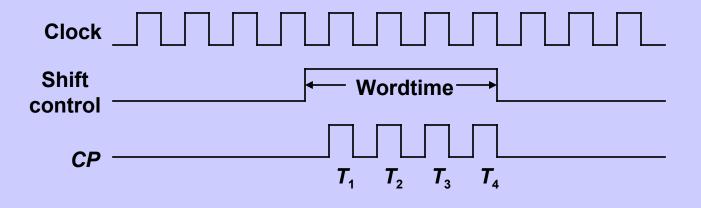
 Accepts data serially – one bit at a time – and also produces output serially.



Serial In/Serial Out Shift Registers

 Application: Serial transfer of data from one register to another.







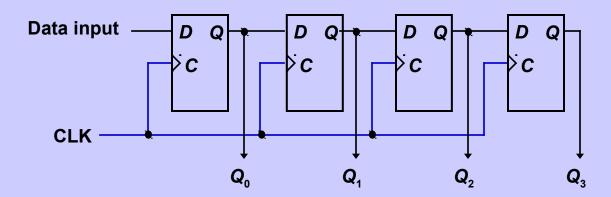
Serial-transfer example.

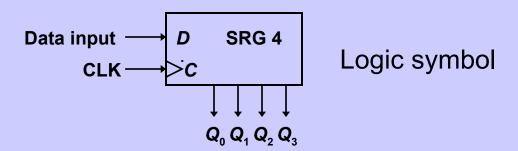
Timing Pulse	Shi	ft re	giste	er A	Shif	ft re	giste	er B	Serial output of <i>B</i>
Initial value	(1	0、	1、	1	0	0、	1、	0	0
After T ₁	1	1 1	0	1	1	0	0	1	1
After T ₂	1	1	1	0	1	1	0	0	0
After T ₃	0	1	1	1	0	1	1	0	0
After T ₄	1	0	1	1	1	0	1	1	1



Serial In/Parallel Out Shift Registers

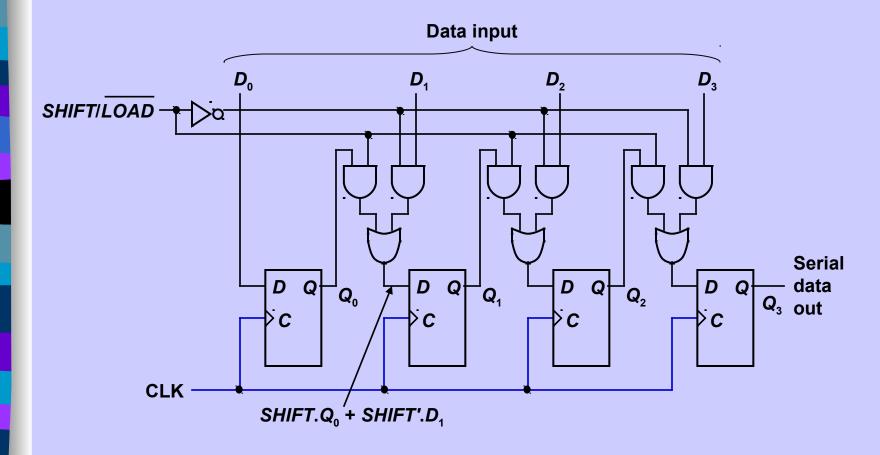
- Accepts data serially.
- Outputs of all stages are available simultaneously.





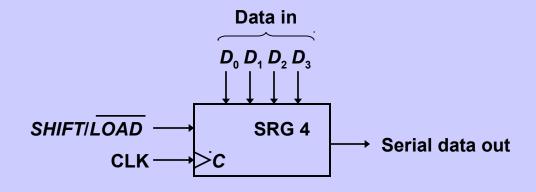
Parallel In/Serial Out Shift Registers

Bits are entered simultaneously, but output is serial.



Parallel In/Serial Out Shift Registers

Bits are entered simultaneously, but output is serial.

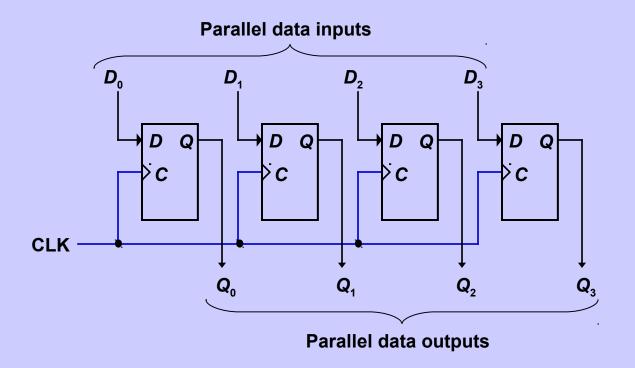


Logic symbol



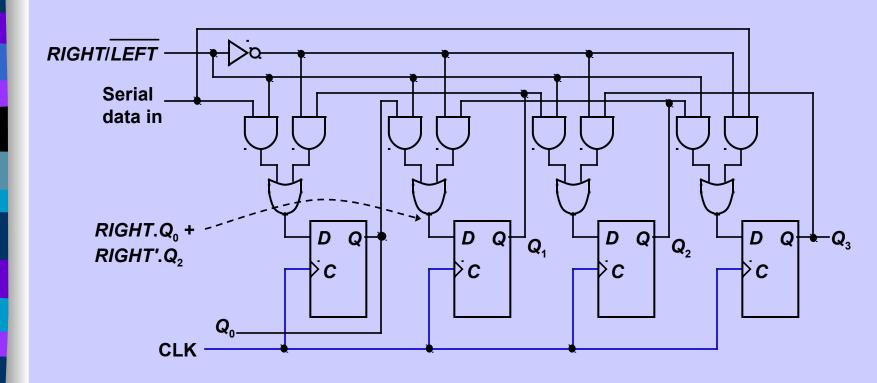
Parallel In/Parallel Out Shift Registers

Simultaneous input and output of all data bits.



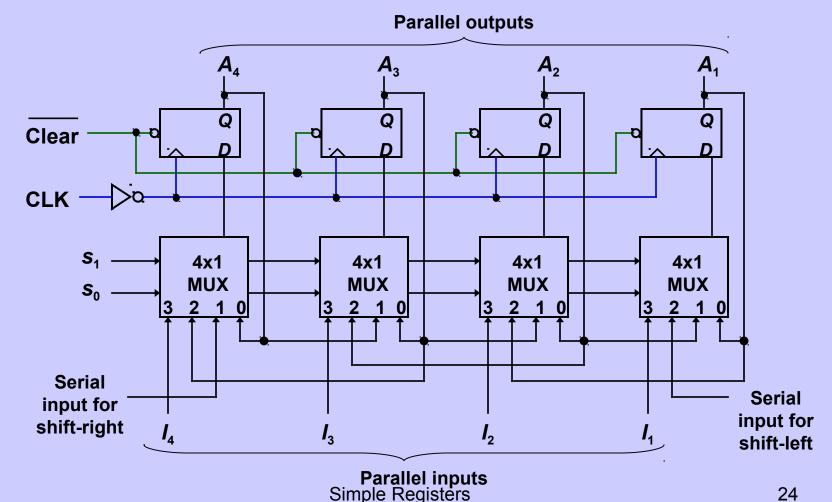
Bidirectional Shift Registers

 Data can be shifted either left or right, using a control line RIGHT/LEFT (or simply RIGHT) to indicate the direction.



Bidirectional Shift Registers

4-bit bidirectional shift register with parallel load.





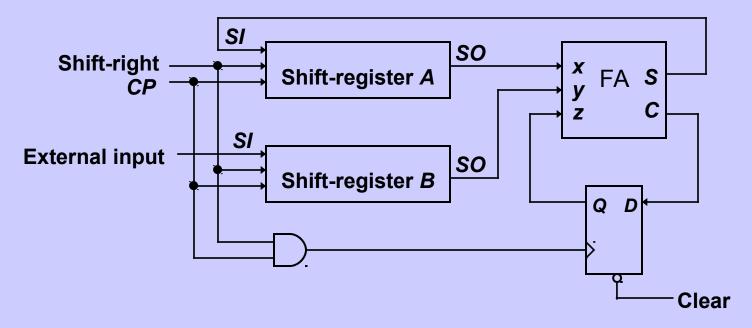
4-bit bidirectional shift register with parallel load.

Mode (Control	
S ₁	S ₀	Register Operation
0	0	No change
0	1	Shift right
1	0	Shift left
1	1	Parallel load



An Application – Serial Addition

- Most operations in digital computers are done in parallel. Serial operations are slower but require less equipment.
- A serial adder is shown below. A ← A + B.



Serial Adder

. These functions are specified in the excitation table and can be simp f maps:

$$JQ = xy$$

$$KQ = x'y' = (x + y)'$$

$$S = x \oplus y \oplus Q$$

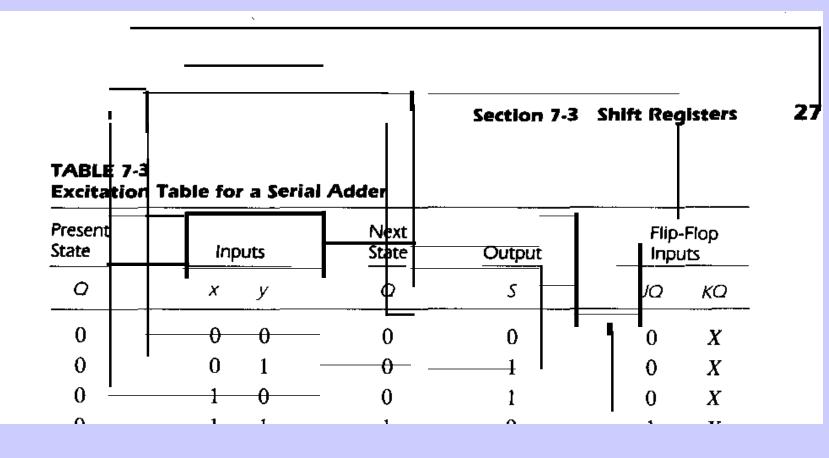
on in Fig. 7-11, the circuit consists of three gates and a JK flip-flop. isters are also included in the diagram to show the complete serial add out S is a function not only of x and y, but also of the present state of Q is a function of the present values of x and y that come out of the shift registers.

$$JQ = xy$$

$$KQ = x'y' = (x + y)'$$

$$S = x \oplus y \oplus Q$$

Serial Adder







An Application – Serial Addition

A = 0100; B = 0111. A + B = 1011 is stored in A after 4 clock pulses.

Initial:	A: 0 1 0 <u>0</u> B: 0 1 1 <u>1</u>	Q: <u>0</u>
Step 1: 0 + 1 + 0 S = 1, C = 0	A: 1 0 1 <u>0</u> B: x 0 1 <u>1</u>	Q : <u>0</u>
Step 2: 0 + 1 + 0 S = 1, C = 0	A: 1 1 0 <u>1</u> B: x x 0 <u>1</u>	Q : <u>0</u>
Step 3: 1 + 1 + 0 S = 0, C = 1	A: 0 1 1 <u>0</u> B: x x x <u>0</u>	Q: <u>1</u>
Step 4: 0 + 0 + 1 S = 1, C = 0	A: 1 0 1 1 B: x x x x	Q: <u>0</u>



Introduction: Counters

- Counters are circuits that cycle through a specified number of states.
- Two types of counters:
 - synchronous (parallel) counters
 - asynchronous (ripple) counters
- Ripple counters allow some flip-flop outputs to be used as a source of clock for other flip-flops.
- Synchronous counters apply the same clock to all flip-flops.

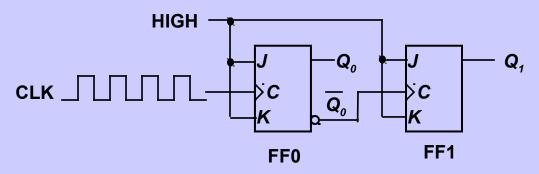


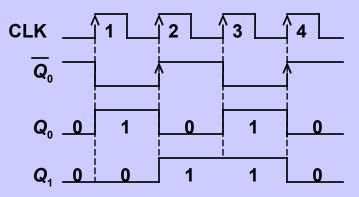
- Asynchronous counters: the flip-flops do not change states at exactly the same time as they do not have a common clock pulse.
- Also known as ripple counters, as the input clock pulse "ripples" through the counter – cumulative delay is a drawback.
- n flip-flops \rightarrow a MOD (modulus) 2^n counter. (Note: A MOD-x counter cycles through x states.)
- Output of the last flip-flop (MSB) divides the input clock frequency by the MOD number of the counter, hence a counter is also a frequency divider.

As

Asynchronous (Ripple) Counters

- Example: 2-bit ripple binary counter.
- Output of one flip-flop is connected to the clock input of the next more-significant flip-flop.

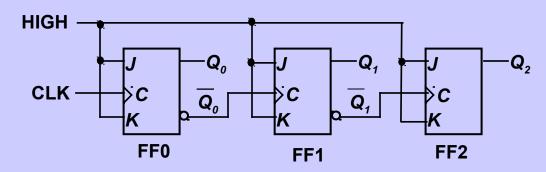


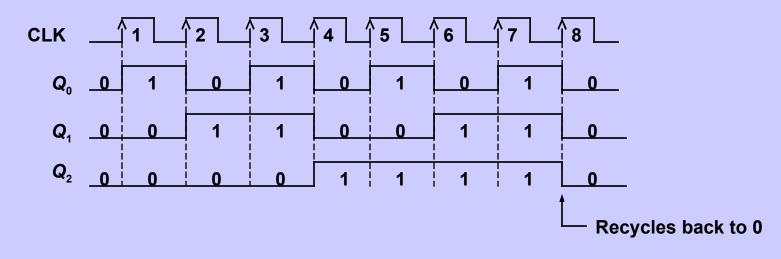


Timing diagram

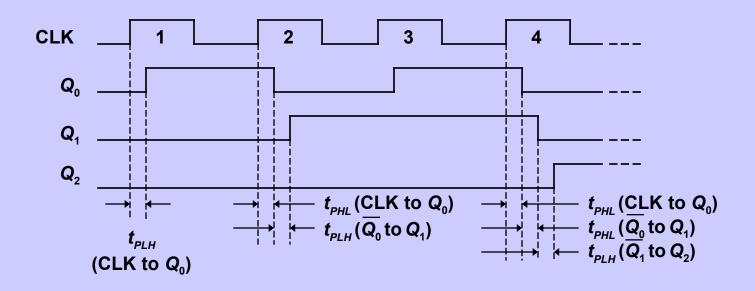
$$00 \rightarrow 01 \rightarrow 10 \rightarrow 11 \rightarrow 00 \dots$$

Example: 3-bit ripple binary counter.

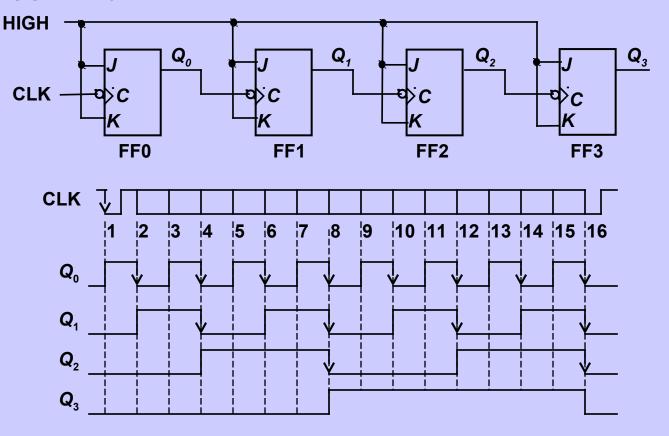




- Propagation delays in an asynchronous (ripple-clocked) binary counter.
- If the accumulated delay is greater than the clock pulse, some counter states may be misrepresented!



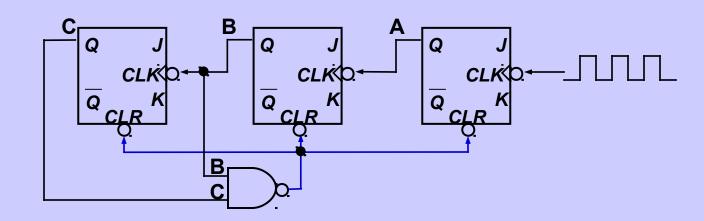
Example: 4-bit ripple binary counter (negative-edge triggered).



Asyn. Counters with MOD no. < 2ⁿ

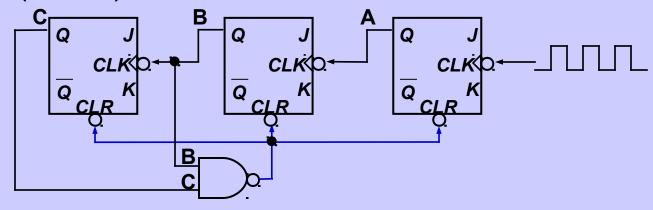
- States may be skipped resulting in a truncated sequence.
- Technique: force counter to recycle before going through all of the states in the binary sequence.
- Example: Given the following circuit, determine the counting sequence (and hence the modulus no.)

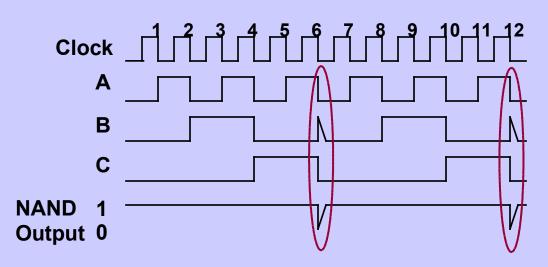
All *J*, *K* inputs are 1 (HIGH).



Example (cont'd):

All *J*, *K* inputs are 1 (HIGH).

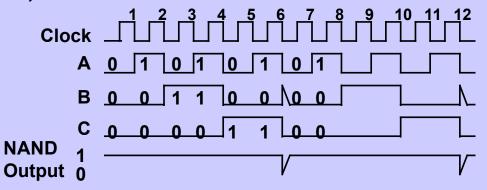


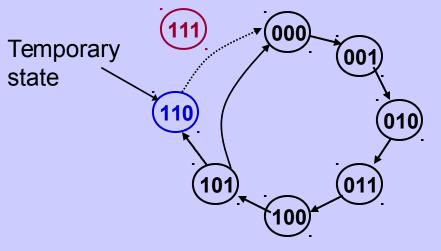


MOD-6 counter

produced by clearing (a MOD-8 binary counter) when count of six (110) occurs.

 Example (cont'd): Counting sequence of circuit (in CBA order).

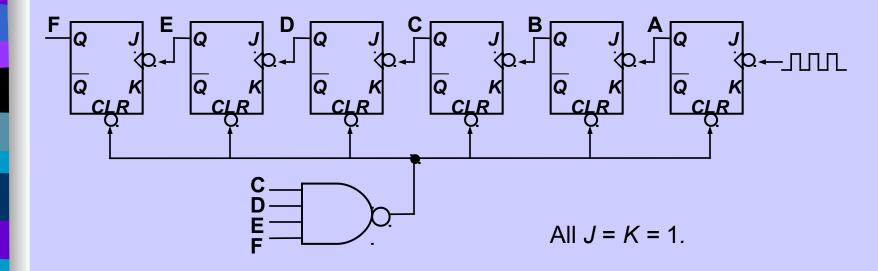




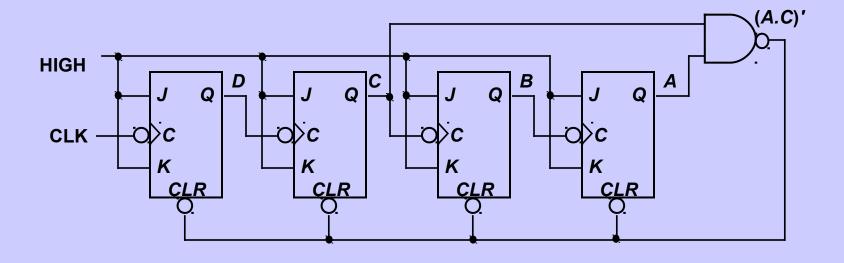
Counter is a MOD-6 counter.



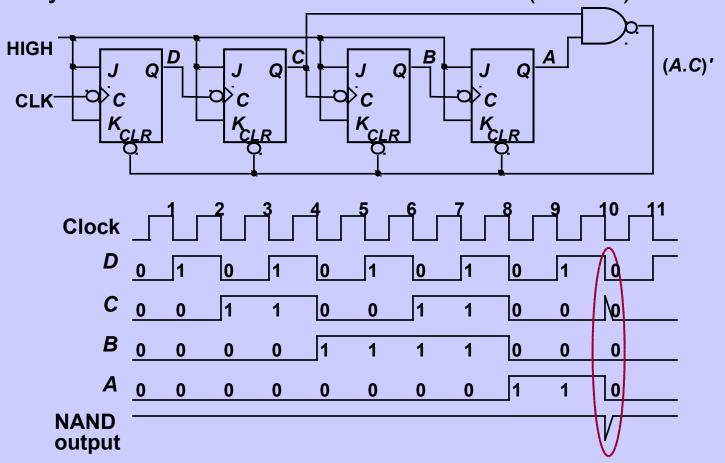
- Exercise: How to construct an asynchronous MOD-5 counter? MOD-7 counter? MOD-12 counter?
- Question: The following is a MOD-? counter?



- Decade counters (or BCD counters) are counters with 10 states (modulus-10) in their sequence. They are commonly used in daily life (e.g.: utility meters, odometers, etc.).
- Design an asynchronous decade counter.

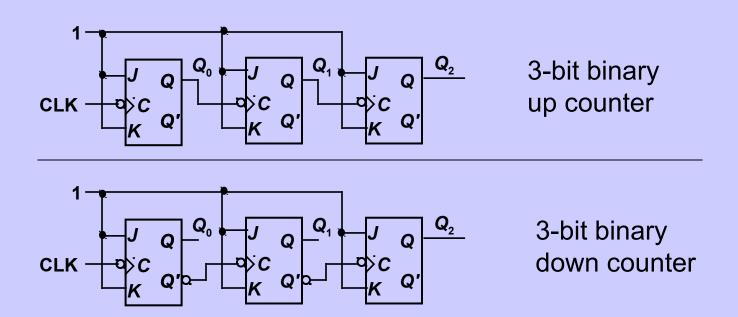


Asynchronous decade/BCD counter (cont'd).



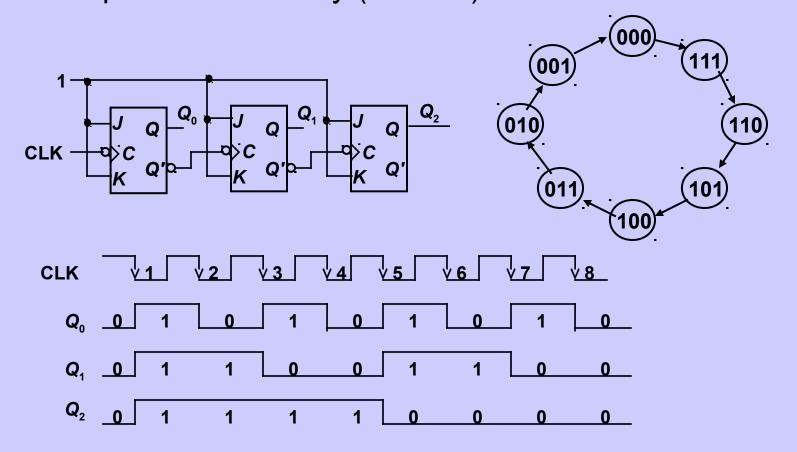
Asynchronous Down Counters

- So far we are dealing with up counters. Down counters, on the other hand, count downward from a maximum value to zero, and repeat.
- Example: A 3-bit binary (MOD-2³) down counter.



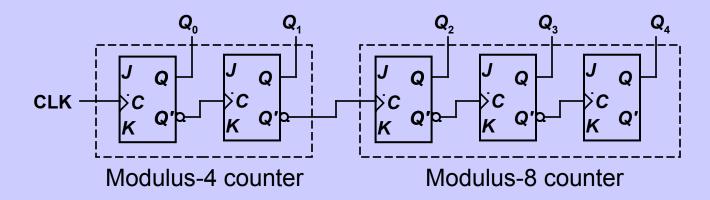
Asynchronous Down Counters

Example: A 3-bit binary (MOD-8) down counter.



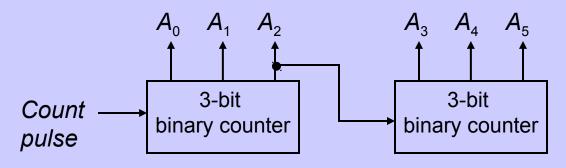
Cascading Asynchronous Counters

- Larger asynchronous (ripple) counter can be constructed by cascading smaller ripple counters.
- Connect last-stage output of one counter to the clock input of next counter so as to achieve higher-modulus operation.
- Example: A modulus-32 ripple counter constructed from a modulus-4 counter and a modulus-8 counter.



Cascading Asynchronous Counters

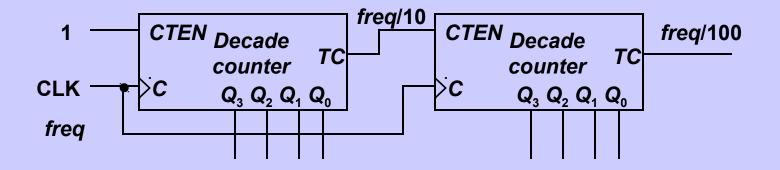
 Example: A 6-bit binary counter (counts from 0 to 63) constructed from two 3-bit counters.



A_5	A_4	A_3	A_2	A_1	A_0
0	0	0	0	0	0
0	0	0	0	0	1
0	0	0	:	•	•
0	0	0	1	1	1
0	0	1	0	0	0
0	0	1	0	0	1
:	:	:	:	•	•

Cascading Asynchronous Counters

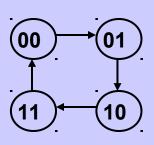
- If counter is a not a binary counter, requires additional output.
- Example: A modulus-100 counter using two decade counters.



TC = 1 when counter recycles to 0000



- Synchronous (parallel) counters: the flip-flops are clocked at the same time by a common clock pulse.
- We can design these counters using the sequential logic design process (covered in Lecture #12).
- Example: 2-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J,K inputs).



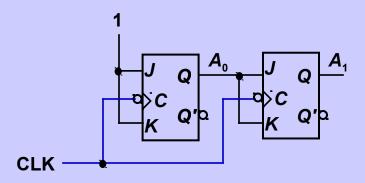
Present state			ext ate	Flip-flop inputs		
A ₁	A_0	A_1^+	$A_1^+ A_0^+$		TA ₀	
0	0	0	1	0	1	
0	1	1	0	1	1	
1	0	1	1	0	1	
1	1	0	0	1	1	



Example: 2-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J,K inputs).

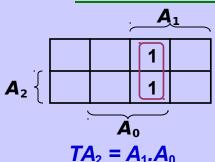
Pres sta			ext ate	-	Flip-flop inputs	
A ₁	A_0	A_1^{\dagger}	A_0^+	TA ₁	TA ₀	
0	0	0	1	0	1	
0	1	1	0	1	1	
1	0	1	1	0	1	
1	1	0	0	1	1	

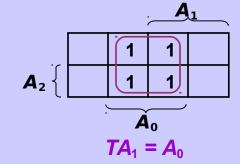
$$TA_0 = 1$$

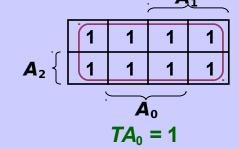


 Example: 3-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J, K inputs).

	rese state			Next state						lip-flo nputs	_
A_2	A_1	A_0	A_2^+	A_1^{\dagger}	A_0^{\dagger}	TA ₂	TA ₁	TA_0			
0	0	0	0	0	1	0	0	1			
0	0	1	0	1	0	0	1	1			
0	1	0	0	1	1	0	0	1			
0	1	1	1	0	0	1	1	1			
1	0	0	1	0	1	0	0	1			
1	0	1	1	1	0	0	1	1			
1	1	0	1	1	1	0	0	1			
1	1	1	0	0	0	1	1	1			





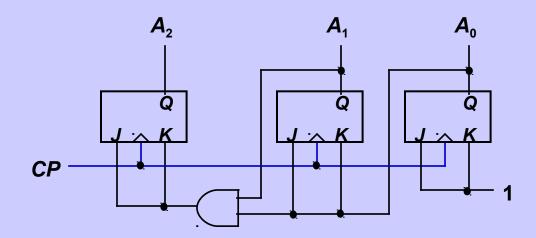




Example: 3-bit synchronous binary counter (cont'd).

$$TA_2 = A_1.A_0$$
 $TA_1 = A_0$ $TA_0 = 1$

$$TA_0 = 1$$





 Note that in a binary counter, the nth bit (shown underlined) is always complemented whenever

$$011...11 \rightarrow 100...00$$

or $111...11 \rightarrow 000...00$

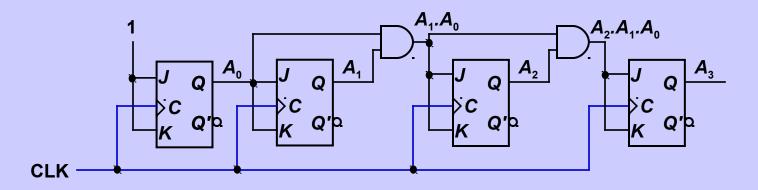
- Hence, X_n is complemented whenever $X_{n-1}X_{n-2} \dots X_1X_0 = 11...11$.
- As a result, if T flip-flops are used, then $TX_n = X_{n-1} \cdot X_{n-2} \cdot \dots \cdot X_1 \cdot X_0$



Example: 4-bit synchronous binary counter.

$$TA_3 = A_2 \cdot A_1 \cdot A_0$$

 $TA_2 = A_1 \cdot A_0$
 $TA_1 = A_0$
 $TA_0 = 1$



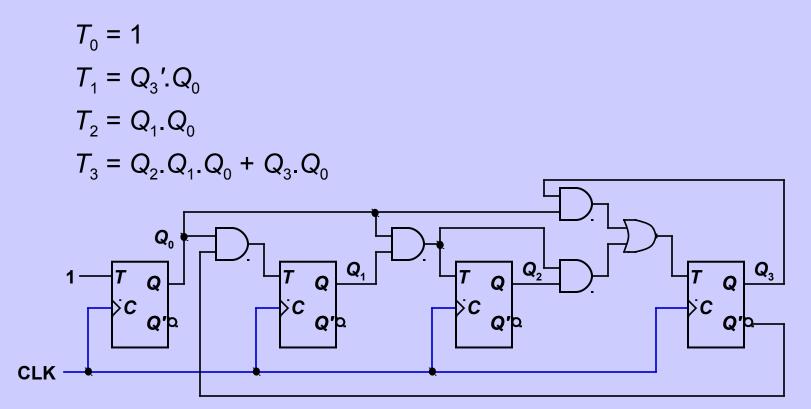


Example: Synchronous decade/BCD counter.

Clock pulse	Q_3	Q_2	Q_1	Q_0
Initially	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10 (recycle)	0	0	0	0

$$T_0 = 1$$
 $T_1 = Q_3'.Q_0$
 $T_2 = Q_1.Q_0$
 $T_3 = Q_2.Q_1.Q_0 + Q_3.Q_0$

 Example: Synchronous decade/BCD counter (cont'd).





Up/Down Synchronous Counters

- Up/down synchronous counter: a bidirectional counter that is capable of counting either up or down.
- An input (control) line *Up/Down* (or simply *Up*) specifies the direction of counting.
 - ❖ Up/Down = 1 → Count upward
 - ***** *Up/Down* = 0 \rightarrow Count downward

Up/Down Synchronous Counters

Example: A 3-bit up/down synchronous binary counter.

Clock pulse	Up	Q_2	Q_1	Q_0	Down
0		0	0	0	₹7
1		0	0	1	- ≼
2		0	1	0	₹
3	<u> </u>	0	1	1	₹
4	<u> </u>	1	0	0	₹
5	<u> </u>	1	0	1	—
6	<u> </u>	1	1	0	_
7		1	1	1	→

$$TQ_0 = 1$$

 $TQ_1 = (Q_0.Up) + (Q_0'.Up')$
 $TQ_2 = (Q_0.Q_1.Up) + (Q_0'.Q_1'.Up')$

Up counter	Down counter
$TQ_0 = 1$	$TQ_0 = 1$
$TQ_1 = Q_0$	$TQ_1 = Q_0'$
$TQ_2 = Q_0.Q_1$	$TQ_2 = Q_0'.Q_1'$

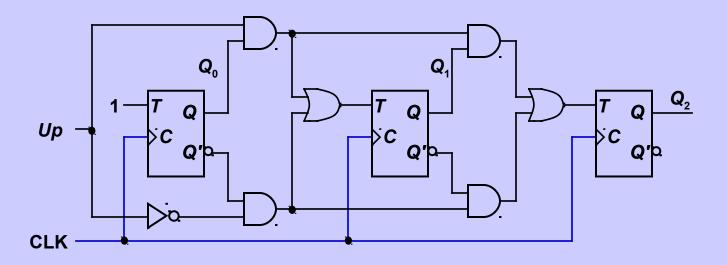


Up/Down Synchronous Counters

Example: A 3-bit up/down synchronous binary counter (cont'd).

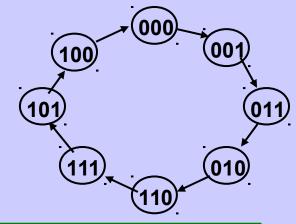
$$TQ_0 = 1$$

 $TQ_1 = (Q_0.Up) + (Q_0'.Up')$
 $TQ_2 = (Q_0.Q_1.Up) + (Q_0'.Q_1'.Up')$



Designing Synchronous Counters

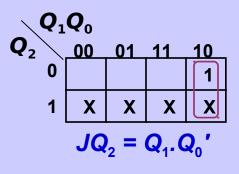
- Covered in Lecture #12.
- Example: A 3-bit Gray code counter (using JK flip-flops).

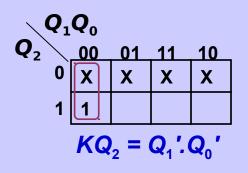


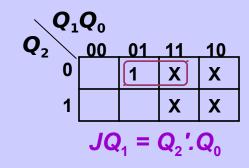
	Present state			Next <u>state</u>			Flip-flop inputs				
Q_2	Q_1	Q_0	Q_2^+	Q_1^{\dagger}	Q_0^{\dagger}	JQ ₂	KQ ₂	JQ ₁	KQ ₁	JQ_0	KQ_0
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	1	0	X	1	X	X	0
0	1	0	1	1	0	1	X	X	0	0	X
0	1	1	0	1	0	0	X	X	0	X	1
1	0	0	0	0	0	X	1	0	X	0	X
1	0	1	1	0	0	X	0	0	X	X	1
1	1	0	1	1	1	X	0	X	0	1	X
1	1	1	1	0	1	X	0	X	1	X	0

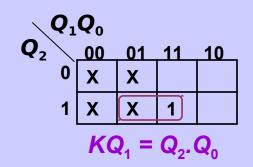
Designing Synchronous Counters

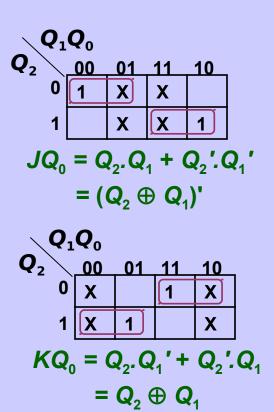
3-bit Gray code counter: flip-flop inputs.









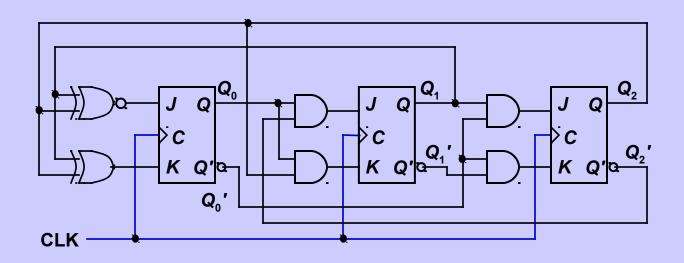




Designing Synchronous Counters

3-bit Gray code counter: logic diagram.

$$JQ_2 = Q_1.Q_0'$$
 $JQ_1 = Q_2'.Q_0$ $JQ_0 = (Q_2 \oplus Q_1)'$
 $KQ_2 = Q_1'.Q_0'$ $KQ_1 = Q_2.Q_0$ $KQ_0 = Q_2 \oplus Q_1$



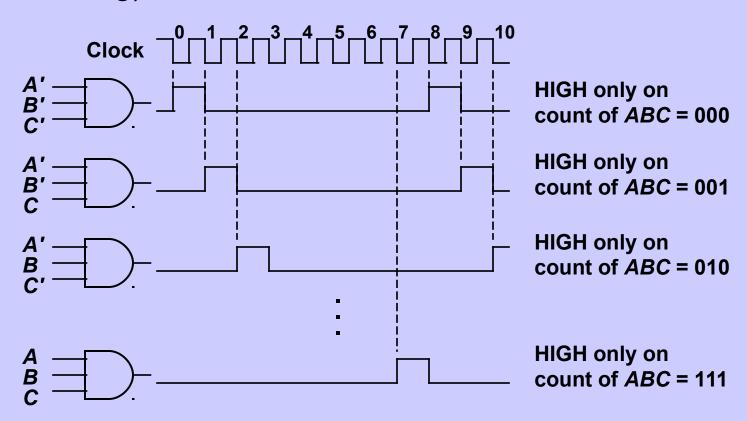


Decoding A Counter

- Decoding a counter involves determining which state in the sequence the counter is in.
- Differentiate between active-HIGH and active-LOW decoding.
- Active-HIGH decoding: output HIGH if the counter is in the state concerned.
- Active-LOW decoding: output LOW if the counter is in the state concerned.

Decoding A Counter

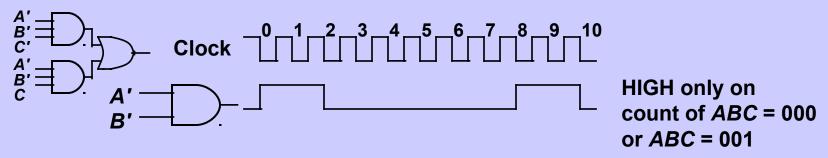
Example: MOD-8 ripple counter (active-HIGH decoding).



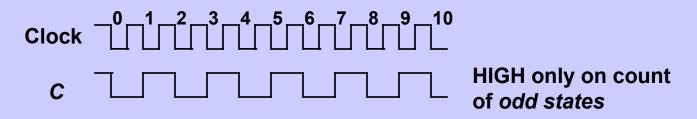


Decoding A Counter

 Example: To detect that a MOD-8 counter is in state 0 (000) or state 1 (001).



 Example: To detect that a MOD-8 counter is in the odd states (states 1, 3, 5 or 7), simply use C.



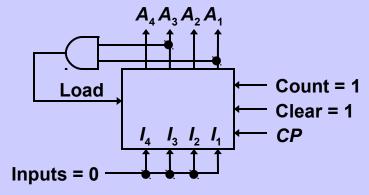


Counters with Parallel Load

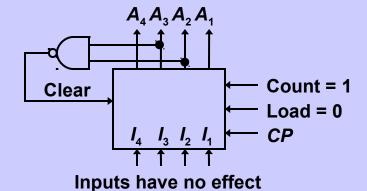
- Counters could be augmented with parallel load capability for the following purposes:
 - To start at a different state
 - To count a different sequence
 - As more sophisticated register with increment/decrement functionality.

Counters with Parallel Load

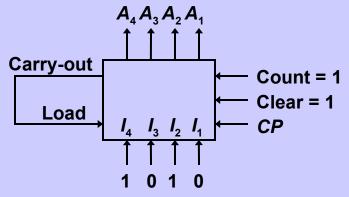
Different ways of getting a MOD-6 counter:



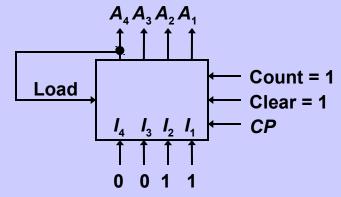
(a) Binary states 0,1,2,3,4,5.



(b) Binary states 0,1,2,3,4,5.



(c) Binary states 10,11,12,13,14,15.

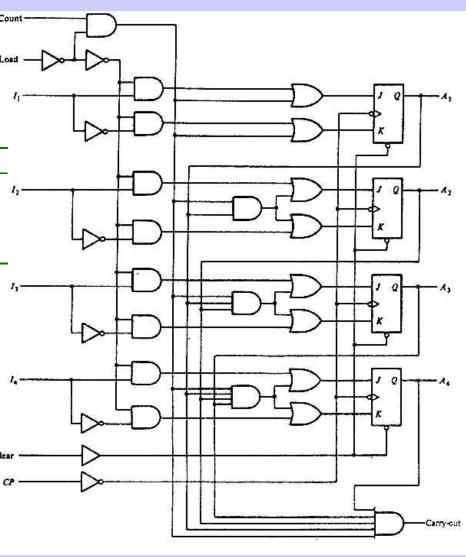


(d) Binary states 3,4,5,6,7,8.

Counters with Parallel Load

4-bit counter with parallel load.

Clear	СР	Load	Count	Function
0	X	X	X	Clear to 0
1	X	0	0	No change
1	1	1	X	Load inputs
1	1	0	1	Next state





Shift Register Counters

- Shift register counter: a shift register with the serial output connected back to the serial input.
- They are classified as counters because they give a specified sequence of states.
- Two common types: the Johnson counter and the Ring counter.

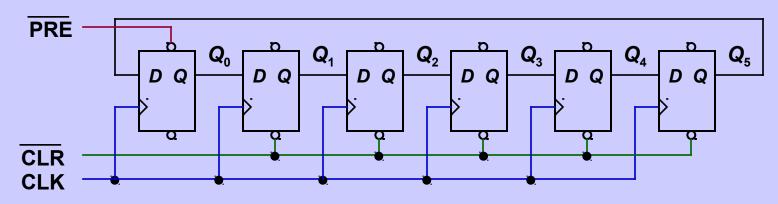


Ring Counters

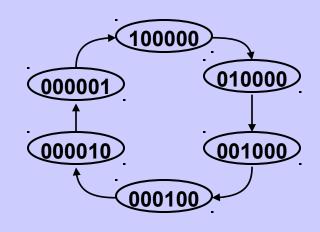
- One flip-flop (stage) for each state in the sequence.
- The output of the last stage is connected to the D input of the first stage.
- An n-bit ring counter cycles through n states.
- No decoding gates are required, as there is an output that corresponds to every state the counter is in.

Ring Counters

Example: A 6-bit (MOD-6) ring counter.



Clock	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5
→ 0	1	0	0	0	0	0
1	0	1	0	0	0	0
2	0	0	1	0	0	0
3	0	0	0	1	0	0
4	0	0	0	0	1	0
└ 5	0	0	0	0	0	1_



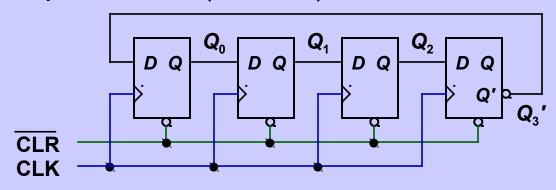


Johnson Counters

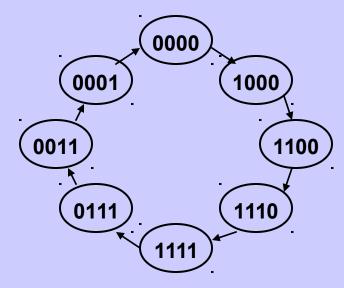
- The complement of the output of the last stage is connected back to the D input of the first stage.
- Also called the twisted-ring counter.
- Require fewer flip-flops than ring counters but more flip-flops than binary counters.
- An n-bit Johnson counter cycles through 2n states.
- Require more decoding circuitry than ring counter but less than binary counters.

Johnson Counters

Example: A 4-bit (MOD-8) Johnson counter.



Clock	Q_0	Q_1	Q_2	Q_3
→ 0	0	0	0	0
1	1	0	0	0
2	1	1	0	0
3	1	1	1	0
4	1	1	1	1
5	0	1	1	1
6	0	0	1	1
<u></u>	0	0	0	1

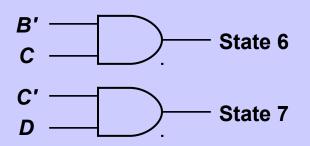


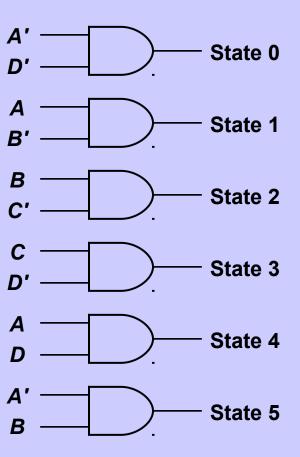


Johnson Counters

Decoding logic for a 4-bit Johnson counter.

Clock	Α	В	С	D	Decoding
→ 0	0	0	0	0	A'.D'
1	1	0	0	0	A.B'
2	1	1	0	0	B.C'
3	1	1	1	0	C.D'
4	1	1	1	1	A.D
5	0	1	1	1	A'.B
6	0	0	1	1	B'.C
└ 7	0	0	0	1	C'.D

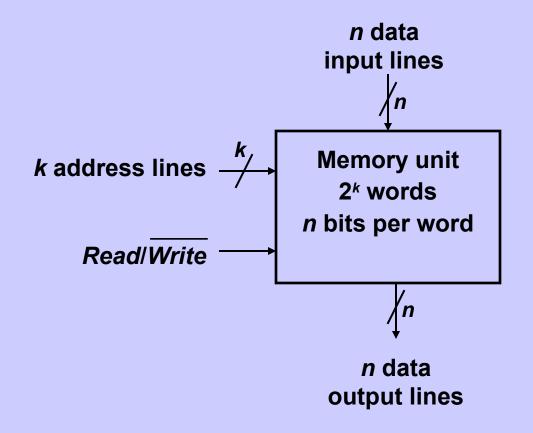






- A memory unit stores binary information in groups of bits called words.
- The data consists of n lines (for n-bit words). Data input lines provide the information to be stored (written) into the memory, while data output lines carry the information out (read) from the memory.
- The address consists of k lines which specify which word (among the 2k words available) to be selected for reading or writing.
- The control lines Read and Write (usually combined into a single control line Read/Write) specifies the direction of transfer of the data.

Block diagram of a memory unit:





Content of a 1024 x 16-bit memory:

Memory	address
---------------	---------

binary	decimal	Memory content
0000000000	0	101101011101110:
000000001	1	1010000110000110
000000010	2	001001110111000
:	:	:
:	:	:
1111111101	1021	1110010101010010
1111111110	1022	0011111010101111
1111111111	1023	101100011001010





- The Write operation:
 - Transfers the address of the desired word to the address lines
 - Transfers the data bits (the word) to be stored in memory to the data input lines
 - Activates the Write control line (set Read/Write to 0)
- The Read operation:
 - Transfers the address of the desired word to the address lines
 - Activates the Read control line (set Read/Write to 1)

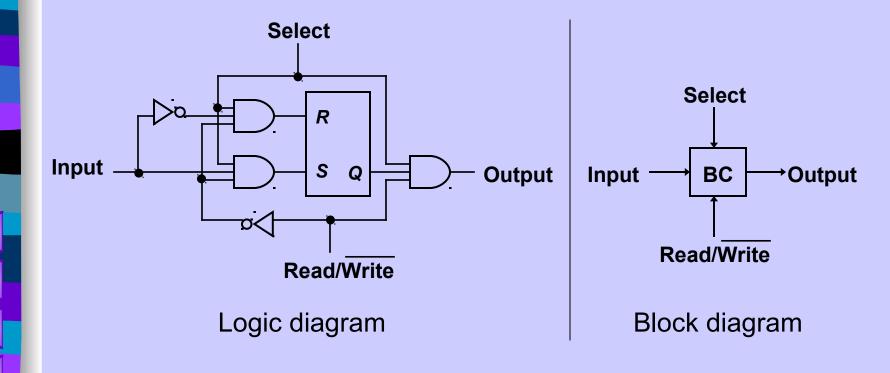


The Read/Write operation:

Memory Enable	Read/Write	Memory Operation
0	X	None
1	0	Write to selected word
1	1	Read from selected word

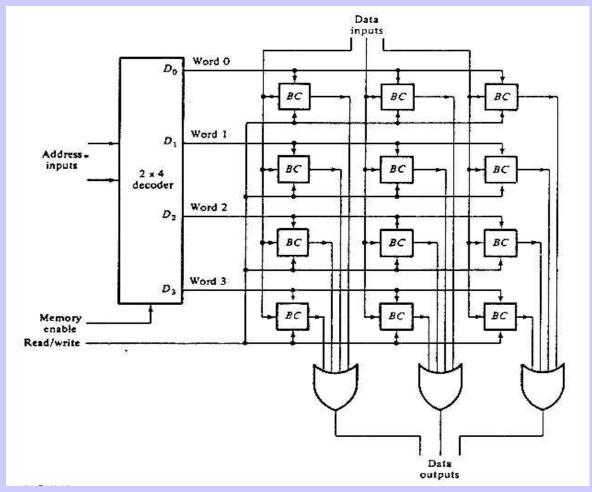
- Two types of RAM: Static and dynamic.
 - Static RAMs use flip-flops as the memory cells.
 - Dynamic RAMs use capacitor charges to represent data. Though simpler in circuitry, they have to be constantly refreshed.

 A single memory cell of the static RAM has the following logic and block diagrams.



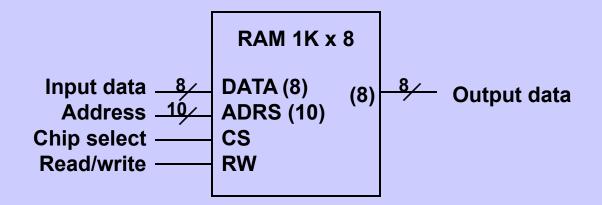
Logic construction of a 4 x 3 RAM (with decoder and

OR gates):

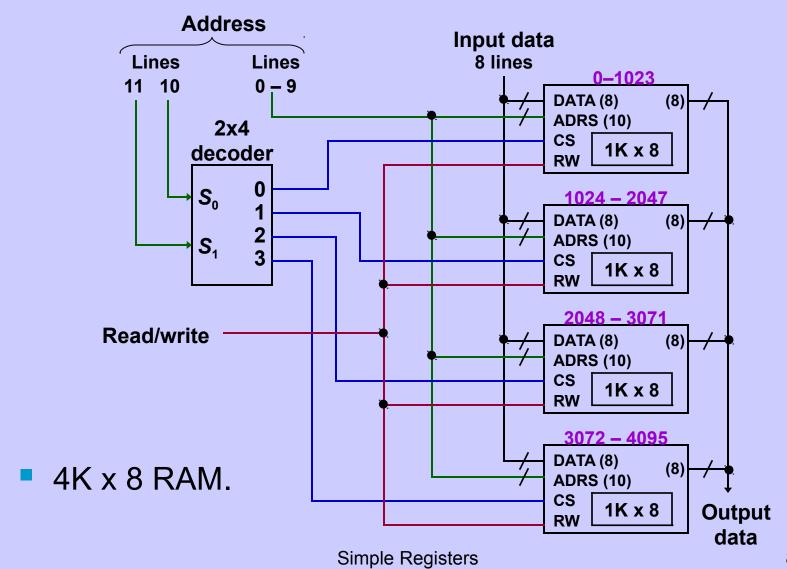




- An array of RAM chips: memory chips are combined to form larger memory.
- A 1K x 8-bit RAM chip:



Block diagram of a 1K x 8 RAM chip



End of segment



