

Tribhuvan University
Institute of Science and Technology
 2065
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Bachelor Level/First Year/ Second Semester/ Science
Computer Science and Information Technology (STA. 159)
 (Calculus and Analytic Geometry)

Full Marks: 60
 Pass Marks: 24
 Time: 3 hours

Candidates are required to give their answers in their own words as far as practicable.
 The figures in the margin indicate full marks.

All notations have the usual meanings.

Group A

(2 x 10 = 20)

Answer any two questions

- Describe a situation where multistage sampling is an appropriate method of drawing a random sample. Clearly state the procedure of drawing random sample in two stage sampling plan. In two stage sampling with random sampling without replacement at both stages obtain an expression for an unbiased estimator of the population total and derive the expression for the unbiased estimator. Hence, in particular case (when $M_i = M$ and $m_i = m$ for all i), obtain the simplified version of unbiased estimator and variance of unbiased estimator.
- What do you mean by Latin Square Design (LSD)? Write and explain the statistical model for $m \times m$ LSD. Give the statistical analysis of $m \times m$ LSD with one observation per cell.
- (a) What is questionnaire? What are the requisites of a good questionnaire?
 (b) Explain the effect model $y_{ij} = \mu + \tau_i + e_{ij}$, $i = 1, 2, \dots, a$ and $j = 1, 2, \dots, n$ with the assumptions made on y_{ij} . Why the assumptions are required?

Group B

Answer any eight questions

(8 x 5 = 40)

- The following table summarizes population size (N_h) and population variance (S_h^2) of four strata. Compute the variance of the stratified estimator \bar{y}_{st} of the population mean for proportional allocation of a total sample of size 100.

h	1	2	3	4
N_h	14000	3000	1500	1500
S_h^2	34	94	175	319

- Describe the procedure of drawing a linear systematic sample of size n from a population consisting of N units when $N = n \times k$ where k is a positive integer. Write down the problem of drawing a linear systematic sample of size 4 from a population consisting of 17 units, numbered from 1 to 17.
- In pps with replacement sampling, show that an unbiased estimator of population total Y is $\hat{Y}_{pps} = \frac{1}{n} \sum_{i=1}^n \frac{y_i}{p_i}$. Derive the expression for the variance of \bar{Y}_{pps} .
- Clearly state the procedure of drawing a random sample in cluster sampling plan. In a simple random sampling without replacement of n clusters from a population of N clusters each containing M element, derive an unbiased estimator of the parameter $\bar{\bar{Y}}$, population mean per element.
- Describe census and sample survey. Write down the advantages of sample survey over the census. Write down the major steps involved in a sample survey.
- Explain the terms – factor, experimental units, treatment and experimental error with suitable examples.
- In a single factor model $y_{ij} = \mu + \tau_i + e_{ij}$, show that

$$\sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y})^2 = n \sum_{i=1}^a (\bar{y}_i - \bar{y})^2 + \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_i)^2.$$

What is significance of this result in experimental design?

- State the mathematical model with the hypothesis to be tested in a two way ANOVA. Write down the ANOVA table for a two way ANOVA.
- Consider the partially completed ANOVA table below. Complete the ANOVA table and answer the followings. What design was employed? How many treatments were compared? How many observations were analyzed? At the 0.05 level of significance can one conclude that the treatments have different effects? Why?

Source	SS	df	MS	F
Treatments	231.50	2		
Blocks		7		
Error	573.75			
Total	903.75	23		

- The results of 2^2 experiments with 36 replications are presented below. Estimate the main effects, interaction effects, SS_A , SS_B , and SS_{AB} . Which effects appear to be large?

Treatment Combination	Replication		
	I	II	III
(1)	22	30	25
A	32	42	29
b	35	33	50
ab	55	45	46

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Group A

(2 x 10 = 20)

Answer any two questions

1. Describe a situation where probability proportion to size (pps) sampling in an appropriate method for drawing a random sample. Clearly state the procedure of drawing a random sample in pps sampling plan. In pps sampling with replacement derive an unbiased estimator of the parameter Y , population total, and also derive the variance of the estimator.
2. What do you mean by Completely Randomized Design (CRD)? Write and explain the statistical model for CRD. Give the statistical analysis of CRD with one observation per cell.
3. (a) Write down the major steps of a sample survey and state the major sources of errors in a sample survey.
 (b) Write down the basic principles of experimental designs and explain the term experimental errors.

Group B

Answer any eight questions

(8 x 5 = 40)

4. The following table summarizes information related to four strata. Compute the variance of the stratified estimator \bar{y}_{st} of the population mean assuming proportional allocation of a total sample of size 100.

h	1	2	3	4
N_h	200	400	300	100
S_h^2	9	4	4	9

5. Describe the situation where the systematic sampling is useful. Write down the problems of drawing as linear systematic sample of size 4 from a population consisting of 17 units, numbered from 1 to 17.

6. In two stage sampling with simple random sampling without replacement at both stages, an unbiased estimator of Y is

$$\bar{Y} = \frac{N}{n} \sum_{i=1}^n \frac{M_i}{j=1} y_{ij}$$

Derive the variance of the above estimator.

7. Clearly state the procedure of drawing a sample random sampling without replacement of n clusters from a population of N clusters each containing M elements, derive an unbiased estimator of the parameter \bar{Y} , population mean per element.
8. Describe census and sample survey. Write down the advantages of sample survey over census. Write down the major steps involved in a sample survey.
9. Explain the terms – factor, experimental units, treatment and experimental error with suitable examples.
10. Write down (a) layout of two way ANOVA with its assumptions, (b) effect model and (c) ANOVA table.
11. In a single factor model $y_{ij} = \mu + \tau_i + e_{ij}$, show that

$$\sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y})^2 = n \sum_{i=1}^a (\bar{y}_i - \bar{y})^2 + \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_i)^2.$$

What is the significance of this result in experimental design?

12. Fill in the (_____) in the following ANOVA table of Latin Square Design.

Source of variation	Sum of squares	Degrees of freedom	Mean square	F-Value
Rows	_____	72	_____	2
Columns	_____	_____	36	_____
Treatments	_____	180	_____	_____
Error	6	_____	12	_____
Total	_____	_____	_____	_____

13. The results of 2^2 experiment with 3 replications are presented below. Estimate the main effects, interaction effects, SS_A , SS_B and SS_{AB} . Which effects appear to be large?

Treatment Combination	Replication		
	I	II	III
(1)	22	30	25
A	32	42	29
B	35	33	50
ab	55	45	46

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Group A

(2 x 10 = 20)

Answer any two questions

- Describe a situation where cluster sampling is appropriate for drawing random sample. Clearly state the procedure of drawing a random sampling in cluster sampling plan. In a sampling without replacement of n clusters from a population of N clusters each containing M elements, derive an unbiased estimator of the parameter $\bar{\bar{Y}}$, population mean per element and also derive the variance of the estimator.
- (a) Explain the terms: a random sample, sampling frame, sampling error and non-sampling error.
 (b) Explain the terms: factor, experimental units, treatment and experimental error.
- What do you mean by Randomized Block Design (RBD)? Write and explain the statistical model for RBD. Give the statistical analysis of RBD with one observation per cell.

Group B

Answer any eight questions

(8 x 5 = 40)

- The following table summarizes population size (N_h) and population variance (S_h^2) related to four strata. If the required sample size is 4000, what are the sample sizes that would be drawn from each stratum for (a) proportional allocation and (b) optimum allocation assuming the survey cost per unit is same in each stratum.

h	1	2	3	4
N_h	14000	3000	1500	1500
S_h^2	34	94	175	319

- Describe the procedure of drawing a linear systematic sample of size n from a population consisting of N units when $N = n \times k$ where k is a positive integer. Write down the problem of drawing a linear systematic sample of size 4 from a population consisting of 17 units, numbered from 1 to 17.

6. In pps with replacement sampling, show that an unbiased estimator of population total Y is

$$\hat{Y}_{pps} = \frac{1}{n} \sum_{i=1}^n \frac{y_i}{p_i}.$$

Derive the expression for the variance of \hat{Y}_{pps} .

7. In two stage sampling with simple random sampling without replacement at both stages, show that an unbiased estimator of Y is

$$\hat{Y} = \frac{N}{n} \sum_{i=1}^n \frac{M_i}{m_i} \sum_{j=1}^{m_i} y_{ij}.$$

What would be the above expression if $M_i = M$ and $m_i = m$ for all i ?

8. What is questionnaire? Explain. Write down the pre-requisites of a good questionnaire?

9. Write down the principles of experimental design.

10. In a single factor model $y_{ij} = \mu + \tau_i + e_{ij}$, show that

$$\sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y})^2 = n \sum_{i=1}^a (\bar{y}_i - \bar{y})^2 + \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_i)^2.$$

What is the significance of this result in experimental design?

11. Consider the partially completed ANOVA table below. Complete the ANOVA table and answer the followings. What design was employed? How many treatments were compared? How many observations were analyzed? At the 0.05 level of significance, can one conclude that the treatments have different effects? Why?

Source	SS	df	MS	F
Treatments	231.50	2		
Blocks		7		
Error	573.75			
Total	903.75	23		

12. Write down statistical model for a Latin Square Design (LSD) and explain it. Also, write down the ANOVA table for LSD.
13. Write down the four treatment combinations of 2^2 experiment using standard notations. Write down the expressions for computing main effects, interaction effect, SS_A , SS_B and SS_{AB} if experiment is replicated r times.