

*CSIT Questions Collections 2068*

## **First Year/ First Semester**

csitnepal

*CSIT Questions Collections 2068*

## **First Year/Second Semester**

## First Year/Second Semester

**Subject :Digital Logic (CSc-151)**  
**Time: 3 hours**

**FM: 60**  
**PM: 24**

*Candidates are required to give their answers in their own words as far as practicable.  
The figures in the margin indicate full marks.*

**Year: 2065**

### Long Answer Questions:

**Attempt any two questions. (2x10=20)**

1. Draw a block diagram, truth table and logical circuit of a 16 x 1 multiplexer and explain its working principle.
2. Explain the 4 bit ripple counter and also draw a timing diagram.
3. Design the full subtractor circuit with using Decoder and explain the working principle.

### Short Answer Questions:

**Attempt any eight questions. (8x5=40)**

4. Design a half adder logic circuit using only NOR gate.
5. Convert the following decimal numbers into hexadecimal and octal number.  
(a) 304 (b) 224
6. Describe the three variables K-map with example.
7. Design the Decoder using Universal gates.
8. What is combinational logic? What are its important features?
9. Describe the clocked RS flip-flop.
10. What do you mean by triggering of flip flop?
11. What are the shift Register operations?
12. Describe the Ripple counter.
13. Write short notes on:  
(a) Registers.(b) Digital. (c) EBCDIC.

**Year: 2066**

### Long Answer Questions:

**Attempt any two questions. (2x10=20)**

1. Design the 4 bit Synchronous updown counter with timing diagram, logic diagram and truth table.
2. Design a Full subtractor with truth table and logic gates.
3. Design a decimal adder with logical diagram and truth table.

### Short Answer Questions:

**Attempt any eight questions. (8x5=40)**

4. Differentiate between Analog and Digital System.
5. Convert the following octal numbers to hexadecimal.  
(a) 1760.46b (b) 6055.263
6. Which gates can be used as inverters in addition to the NOT gate and how?
7. Draw a logic gates that implements the following

$$a) A = (Y1 + Y2) (Y3 + Y4) + (Y5 + Y6 + Y7)$$

$$b) A = (X1 \text{ OX } 2) + (X3 \text{ OX } 4) + (X4 \text{ OX } 5) + (X6 \text{ OX } 7)$$

8. State and prove De Morgan's theorem 1 and 2 with logic gates and truth table.
9. Reduce the following expressions using K-map.  
a)  $A + B(A + B + D)(B + C)(B + C + D)$
10. Differentiate between a MUX and a DEMUX.
11. Explain the operation of Decoder,
12. What are the various types of shift registers?
13. What do you mean by Synchronous counter?

**Year: 2067**

### Long Answer Questions:

**Attempt any two questions. (2x10=20)**

1. What is magnitude comparator? Design logic circuits for 4 bit magnitude comparator and explain it.
2. What do you mean by full adder and full subtractor? Design a 3 to 8 line decoder using two 2 to 4 line decoder and explain it.
3. What is JK master slave flip-flop? Design its logic circuit, truth table and explain the working principle.

### Short Answer Questions:

**Attempt any eight questions. (8x5=40)**

4. Convert the following hexadecimal number to corresponding octal numbers.  
(a) 0FFF (b) 3FFF
5. Design a half adder logic circuit using NOR gates only.
6. Prove the 1st and 2<sup>nd</sup> law of De Morgan's theorems with logic gate and truth table.
7. What do you mean by universal gate? Realize the following logic gates using NOR gates.

**OR**

- (a) OR gate (b) AND gate.
8. Draw a logic circuit of 4X1 multiplexer.
9. What is a flip-flop? Mention the application of flip-flop.
10. Explain the ripple counter.
11. Design the Decimal adder.
12. What do you mean by shift registers? Explain.
13. Write short notes on (any two):  
(a) Decoder (b) Integrated circuit (c) PLA

**Year 2068**

### Long Answer Questions:

**Attempt any two questions. (2x10=20)**

1. Draw a block diagram truth table and logic circuit of 1X16 Demultiplexer and explain its working principle.
2. Design a 3 bit synchronous counter and explain it.
3. What is magnitude comparator? Design a logic circuit for 4 bit comparator and explain it.

### Short Answer Questions.

#### Attempt any eight questions. (8X5=40)

4. Design a half subtractor circuit using only NAND gates.
5. Convert the following decimal numbers into Hexadecimal and octal number.
  - a) 504
  - b) 250
6. Design an encoder using universal gates.
7. What do you mean by D-flip-flop?
8. What is sequential logic? What are the important features?
9. Simplify the Boolean function using K-maps.

$$F = X'yz + X'yz' + Xy'z + Xy'z'$$

10. Draw a parallel-in-parallel out shift register and explain it.

11. Explain the 4-bit ripple counter.

12. Explain the programmable logic array (PLA).

13. Write short notes on:

- a) Asynchronous counter
- b) Multiplexers
- c) State reduction table

## First Year/Second Semester

Subject :Discrete Structure (CSc-152)

Time: 3 hours

FM: 60

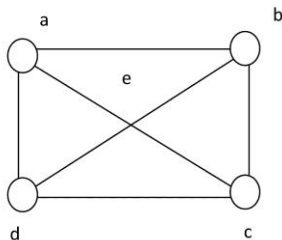
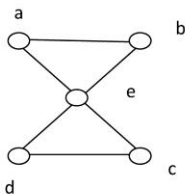
PM: 24

**Year: 2065**

**Attempt all questions:**

### Group A

1. Given propositions  $p$  and  $q$ , define conjunction and disjunction of them.
2. Define existential quantifications with suitable example. ,
3. State which rule of inference is basis of the following argument: “It is below freezing and raining now, therefore, it is below freezing now”.
4. State and prove the Pigeonhole principle.
5. Define linear homogeneous recurrence relation. .
6. Define the terms a language over a vocabulary and the phrase-structure grammar.
7. Distinguish between deterministic and nondeterministic finite state automaton.
8. Define the complete graph  $K_n$  on  $n$  vertices and the complete bipartite graph  $K_{m,n}$  with suitable examples.
9. Which of the undirected graphs in the following figure have an Euler circuit? Explain.



10. What is the chromatic number of the complete bipartite graph  $K_{m,n}$  where  $m$  and  $n$  are positive integers.

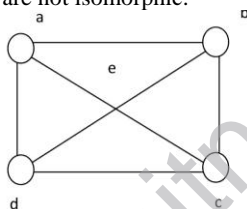
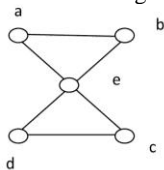
### Group B

11. Explain the 4 rules of inference for quantified statements.
12. Find an explicit formula for the Fibonacci numbers, with recursion relation  $f_{n-1} + f_{n-2}$  and  $f_0=0, f_1=1$ .
13. Define finite-state with output with suitable examples.

**OR**

Define deterministic finite state automata. When are two finite state automata equivalent? Give an example.

14. Show that the graphs in the following figure are not isomorphic.



What can you say about the complexity of graph isomorphism algorithms in terms of complexity?

15. Prove that an undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

### Group C

(5x8=40)

16. Explain Tautologies, contradictions and contingencies with suitable examples.

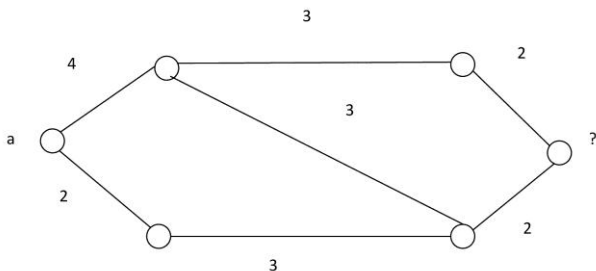
OR

Explain the method of proving theorems by direct, indirect, contradiction and by cases.

17. Define linear homogeneous recursion relation of degree K with constant coefficient with suitable examples. What is the solution of the recurrence relation  $a_n = a_{n-1} + 2a_{n-2}$  with  $a_0 = 2$  and  $a_1 = 7$ ?
18. Let G be the grammar with vocabulary  $V = \{S, 0, 1\}$ , set of terminals  $T = \{0, 1\}$ , starting symbol S, and productions  $P \{S \rightarrow 11S, S \rightarrow 0\}$ , what is  $L(G)$ , the language of this grammar?
19. Explain the concept of network flows and max-flow min-cut theorem with suitable examples.
20. Define Euler and Hamiltonian circuits and paths with examples illustrating the existence and nonexistence of them.

OR

Discuss the shortest path algorithm of Dijkstra for finding the shortest path between two vertices. Use this algorithm to find the length of the shortest path between a and z in the following weighted graph?



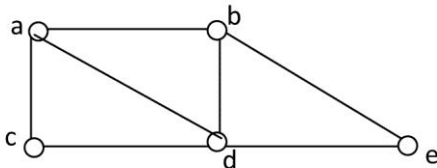
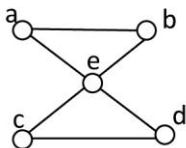
Give the idea of traveling salesman problem and the difficulties of solving it.

**Year: 2065**

### Group A

1. Define proposition and its negation with an example.
2. Show that  $\neg(p \vee q)$  and  $\neg p \wedge \neg q$  are logically equivalent.
3. State which rule of inference is the basis of the following argument; "It is below freezing now. Therefore, it is either below freezing or raining now."
4. State the Pigeonhole principle. How many students must be in a class to guarantee that at least two students receive the same score on the final exam is graded on a scale from 0 to 100?
5. Let  $\{a_n\}$  be a sequence that satisfies the recursion relation  $a_n = a_{n-1} - a_{n-2}$  for  $n \geq 2$  and suppose that  $a_0 = 3$  and  $a_1 = 5$ . Find the values  $a_2$  and  $a_3$ .

6. Let  $G$  be the grammar with vocabulary  $V = \{S, A, a, b\}$ ,  $t = \{a, b\}$ , starting symbol  $S$  and productions  $P = \{S \rightarrow a.A, S \rightarrow b, A \rightarrow aa\}$ . What is  $L(G)$ , the language of this grammar?
7. Determine the Kleene closures of the sets  $A = \{0\}$ ,  $B = \{0, 1\}$ ,  $C = \{11\}$ .
8. How many edges are there in a graph with ten vertices each of degree six?
9. Which of the undirected graphs in the following have an Euler path?

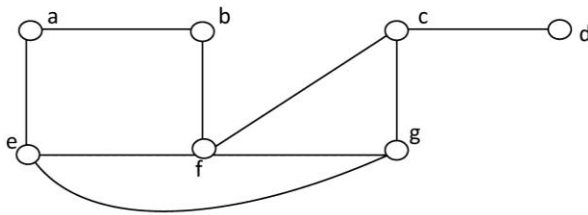


10. Determine the chromatic number  $K_n$ .
11. Differentiate between existential and universal quantifiers with suitable examples.
12. Find the solution of the recursion relation  
 $a_n = a_{n-1} + 2a_{n-2}$  with  $a_0 = 2$  and  $a_1 = 7$ ?

**OR**

Find an explicit formula for the Fibonacci numbers.

13. Define deterministic finite state automata. Construct a DFA whose language is the set of strings that ends with 111 and contains an odd number of one's.
14. Prove that an undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.
15. Find a spanning tree of the simple graph in the following graph, if it exists.



Can there be more possibilities?

### Group C

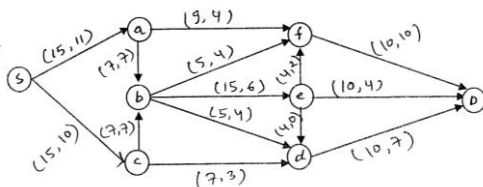
(5x840)

16. Discuss the techniques of proofs by contradiction and by cases with suitable examples.
17. Describe linear homogeneous and linear nonhomogeneous recurrence relations with suitable examples.
18. Explain nondeterministic finite automata and language of NFA with suitable example.
19. State and prove the maxflow and Mincut theorem.

**OR**

Find a maximum flow for the network in the figure below.



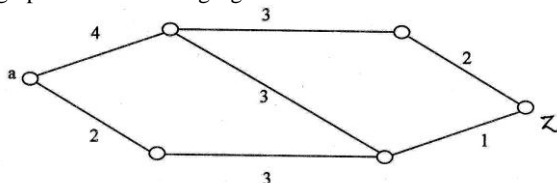


20. Define Hamiltonian paths and circuits with suitable example for the existence and nonexistence.

Show that  $K_n$  has a Hamilton circuit whenever  $n \geq 3$ .

OR

Write the shortest path algorithm of Dijkstra for finding the shortest path between two vertices. What is the length of shortest path between a and z in the weighted graph in the following figure?

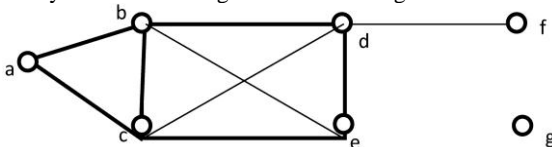


Apply the stated algorithm for finding the solution.

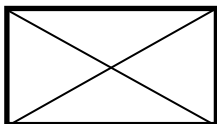
**Year: 2067**

**Attempt all questions:**

- What do you mean by proposition. Give example to justify your answer?
- How do you define logically equivalent propositions?
- Give examples of addition rule and simplification rule of inference.
- State and prove the Pigeonhole principle.
- How many ways are there to select a first, second and third —prize winners from 10 different people?
- Discuss the types of phrase-structure grammars and their relations,
- Give formal definition of regular expressions over a set I.
- Verify the Handshaking theorem in the figure.



9. Is the graph  $K_4$  planar? How?



10. Determine the chromatic number  $K_n$ .

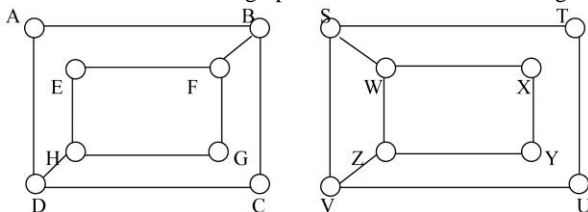
### Group B

11. Explain the 2 rules of inference for quantified statements and give suitable examples.

OR

Show that the propositions  $p \vee (q \wedge r)$  and  $(r \vee q) \wedge (p \vee r)$  are logically equivalent.

12. Define the binomial coefficient and give the general term of the binomial coefficient. Show that the sum of the binomial coefficient is  $2^n$ .
13. How do you distinguish deterministic and nondeterministic finite-state automation? Give suitable examples.
14. Determine whether the graphs shown in the following figure are isomorphic.



What can you say about the graph isomorphism algorithms in terms of efficiency?

15. Prove that a tree with  $n$ -vertices has  $n-1$  edges.

### Group C

(5x8=40)

16. Discuss the techniques of direct proof indirect proof and vacuous proof for proving implications with suitable examples.
17. Find the solution to the recursion relation  $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$  with initial conditions  $a_0=2, a_1=5$  and  $a_2=15$ .

OR

Suppose that a person deposits Rs. 10,000/- in a fixed account at a bank yielding 11% peryear with interest compounded annually. How much will be in the account after 10 years? Solve the problem with modeling it into recursion relations.

18. What do you mean by phase- structure grammar? Let  $C1$  be the grammar with vocabulary  $V = \{S, 0, 1\}$ ; set of terminals  $T = \{0, 1\}$ , starting symbol  $S$ , and productions  $P \{S \rightarrow 11S, S \rightarrow 0\}$ . Determine the language  $L(G)$  of this grammar.
19. Explain the concept of network flows and max-flow min-cut theorem with suitable examples.

OR

1. Define Euler circuit and Euler path with suitable examples. Give the multigraph model of the two of Koenigaberg state a necessary and sufficient condition for Euler circuit in connection to your definitions and model.
2. Discuss the Algorithm of Dijkstra for finding the shortest path in a weighted graph between two vertices with suitable example. Moreover, explain the traveling salesman problem and the efficiency of algorithm for solving this problem.

**Group A**

**(10×2=20)**

1. Define disjunction and conjunction with suitable example.
2. Is the following argument valid?  
Smoking is healthy.  
If smoking is healthy, then cigarettes are prescribed by physicians.  
 $\therefore$  Cigarettes are prescribed by physicians.
3. State the rules for the strong form of mathematical induction with propositions.
4. State and prove “the extended pigeonhole principle”?
5. Define the term language over a vocabulary and the phrase-structure grammar.
6. Distinguish between binary tree and spanning tree with suitable examples.
7. Consider  $K_n$  the complete graph on  $n$  vertices. What is the degree of each vertex?
8. Explain the state transition function of the finite state machine with a suitable table.
9. Define regular expressions over a non-empty set  $A$ .
10. What is the chromatic number of the complete bipartite graph  $K_{m,n}$  where  $m$  and  $n$  are the positive integers.

**Group B**

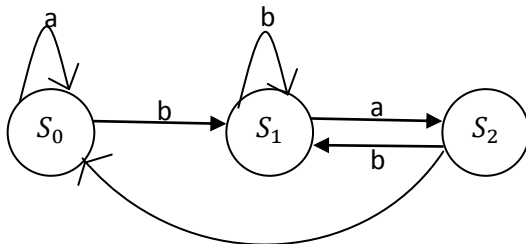
**(5×4=20)**

11. Explain the rule of inference for quantified statements.
12. Let  $A = \{p, q, r\}$ . Give the regular set corresponding to the regular expression given:
  - a)  $(p \vee q)r q^*$
  - b)  $p(q q)^* r$
13. Find and explicit formula for the Fibonacci sequence defined by  
 $f_n = f_{n-1} + f_{(n-2)}, f_1 = f_2 = 1$
14. Define finite-state machines with output.
15. Show that the maximum number of vertices in a binary tree of height  $n$  is  $2^{n+1} - 1$ .

OR

Draw all possible unordered trees on the set  $\{a, b, c\}$ .

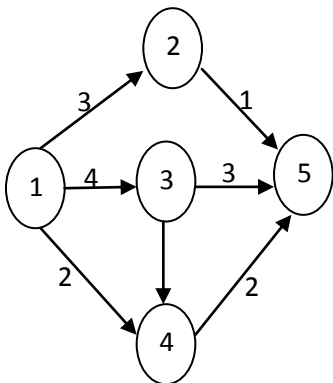
16. Construct the state transition table of the finite-state machine whose diagram is shown?



17. Let  $G = (V, S, v_0, \rightarrow)$  where  $V = \{v_0, x, y, z\}$ ,  $S = \{x, y, z\}$  and
- $$\rightarrow v_0 \rightarrow xv_0$$
- $$v_0 \rightarrow yv_0$$
- $$v_0 \rightarrow z$$

What is  $L(G)$ , the language of this grammar?

18. Find a maximum flow in the network shown in figure



19. Prove that a symmetric connected relation has a undirected spanning tree.

OR

Give a simple condition on the weight of a graph that will guarantee that there is a unique maximal spanning tree for the graph.

## First Year/Second Semester

**Subject :Microprocessor (CSc-153)**

**FM: 60**

**Time: 3 hours**

**PM: 24**

**Year: 2065**

### SectionA

**Attempt any two questions: (2x10=20)**

1. Draw the block diagram of basic microprocessor and explain it. Which block design is simple and explain it?
2. Why addressing modes are required in the microprocessor? Discuss different types of addressing modes with suitable examples.
3. Write a program in 8-bit Microprocessor to multiply two 16 bits numbers and store in the memory location starting from 3500h. Save the carry bits in the location starting from 3600h.

### Section B

**Attempt any eight questions: (8x5=40)**

4. Differentiate between PUSH and POP operations. Write a program to illustrate the use of PUSH operations.
5. Write an assembly language program to subtract two 16-bit numbers.
6. What do you understand by address decoding in the case of memory interfacing? Explain address decoding using 3 to 8 Decoder.
7. Which 110 interface is used in the 8-bit microprocessor? Explain different types of I/O instructions.
8. Why interrupt is required? Draw the block diagram of interrupt handler and explain it.
9. Explain the Basic DMA Operation with required timing diagram.
10. How can you interface 8086 microprocessor?, . . .
11. How can you achieve pipelining in the basic microprocessor? Explain any type of basic pipelining with suitable diagram.
12. Draw the timing diagram for ADD C and explain it.
13. Write an assembly language program to display a string "Assembly language coding is difficult" using 16 bit microprocessor code. Assume any necessary data.

**Year: 2066**

### SectionA

**Attempt any two questions: (2x10=20)**

1. Explain the SAPI architecture with suitable block diagram, Compare it with SAP2 architecture
2. Explain the application of flags in the microprocessor. Discuss different types of flags with suitable examples.
3. Write a program in 8-bit Microprocessor to store 60h, BAh, 7Ch and 10h in the memory location starting from 2000h. Add these data and store the result in 3000h and carry flag in 500h. Explain all the steps.

## **Section B**

**Attempt any eight questions: (8x5=40)**

4. Explain about fetch operation and timing diagram.
5. Write an assembly language program to multiply 05h and 06h. Explain all the steps.
6. What are macro assemblers? Explain it.
7. What are the functions of I/O interface? Explain it with suitable example.
8. What do you mean by interrupt? Explain in detail about software interrupt.
9. Explain the Basic DMA Operation with required timing diagram. What are the uses of the DMA transfers?
10. Explain about RS 232 interface with suitable example.
11. Write an assembly language program to display a string "I want to know more about microprocessor" using 16 bit microprocessor code. Assume any necessary data.
12. Why parallel communication is required? Explain with reference to 8-bit system.
13. Differentiate between PUSH and POP operations with suitable example.

**Year: 2067**

## **Section A**

**Attempt any two questions: (2x10=20)**

1. Draw the block diagram of SAP2 architecture and explain it. Compare it with SAP1 architecture.
2. Explain the importance of addressing modes in the microprocessor? Discuss different types of addressing modes with suitable examples.
3. Write a program in 8-bit Microprocessor to multiply two 16 bit numbers (ABCDh and 1234h) and store in the memory location starting from 3000h.

## **Section B**

**Attempt any eight questions: (8x5=40)**

4. Explain execute operation and timing diagram with suitable example.
5. Write an assembly language program to add two 16-bit numbers (3467h and ACDCh).
6. Differentiate between data and address bus with suitable example.
7. Explain different types of I/O instructions used in 8-bit microprocessor.
8. Why interrupt is required? Draw the block diagram of interrupt handler and explain it.
9. Explain the basic DMA Operation with required timing diagram.
10. Explain three types of flags with suitable examples.
11. Why do we require serial communication? Explain with suitable example.
12. Explain about keyboard and display controller.
13. Write an assembly language program to display a string "I like programming in the assembly language" using 16 bit microprocessor code. Assume any necessary data.

**Year: 2068**

## **Section A**

**Answer any two questions.**

1. Explain the operation of 8085 microprocessor using block diagram. Justify that design of control unit is more difficult.
2. What do you mean by addressing mode? Discuss different types of addressing modes with examples.
3. Write a program in 8-bit Microprocessor to store 68h, B3h, COh, and 11h in the memory location starting from 3000h. Move these data and store in the memory location starting from 3200h.

### **Section B**

**Attempt any eight questions. (8X5=40)**

4. What do you understand by PUSH operation? Explain the use of push operation in the stack.
5. Write an assembly language program to add 16-bit numbers.
6. What do you understand by address decoding in the case of memory interfacing? Explain address decoding using simple NAND gates Decoder.
7. What do you understand by I/O interface? Explain different types of I/O instructions.
8. What do you mean by interrupt? Explain in detail about basic interrupt processing.
9. Explain the Basic DMA Operation with required timing diagram.
10. How can you interface 80286SX microprocessor? Explain.
11. How can you implement pipelining in the basic microprocessor? Explain it with diagram.
12. Draw the timing diagram for MVIB and explain it.
13. Write an assembly language program to display a string "Microprocessor programming is a fun" using 16 bit microprocessor code. Assume any necessary data.

<b>First Year/Second Semester</b>
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**Subject :Data Structure and Algorithm(CSC-154)**  
**Time: 3 hours**

**FM: 60**  
**PM: 24**

**Year: 2065**

### **SectionA**

**Attempt any two questions: (2x10=20)**

1. What do you mean by binary tree? Explain the binary search tree with example.
2. What do you mean by Recursion? Explain the implementation of factorial and fibonacci sequences with example.
3. Explain the implementation of stack and queue with example.

### **Section B**

**Attempt any eight questions: (8x5=40)**

4. What are the differences between two dimension array and multidimensional array?
5. What are the major characteristics of algorithms?
6. How can you convert from infix to postfix notation?
7. How can you use Queue as ADT?
8. What is Post-order traversal?

9. What is sorting? Describe the insertion.
10. Explain the binary searching.
11. Differentiate between Pre-order and In order traversal.
12. Explain the tower of Hanoi algorithm.
13. Explain the Kruskal's algorithm.

**Year: 2066**

### **Section A**

**Attempt any two questions: (2x10=20)**

1. Write a Menu program to demonstrate the simulation of stack operations in array implementation.
2. State relative merits and demerits of contiguous list and Linked list. Explain the steps involved in inserting and deleting a node in singly linked list.
3. A binary tree T has 12 nodes. The in-order and pre-order traversals of T yield the following sequence of nodes:

In-order :	VPNAQRSOKBTM
Pre-order :	SPVONARTOKBM

Construct the Binary tree T showing each step. Explain, how you arrive at solution in brief?

### **Section B**

**Attempt any eight questions: (8x5=40)**

4. Consider the function:  

```
Void transfer (int n, char from, char to, char temp)
{ if(n>0)
{ Transfer (n — 1, from, temp, to);
Prinif ("n Move Disk %d from %C to %C", N, from, to);
Transfer (n — 1, temp, to, from);
}
Trace the output with the function call!
Transfer (3, 'R', 'L', 'C');
```
5. "To write an efficient program, we should know about data structures." Explain the above statement.
6. Write C function to display all items in a circular queue n array implementation. Write assumptions, you need.
7. Divide and conquer algorithm taking reference to Merge Sort.
8. Trace Binary Search algorithm for the data:  
21, 36, 56, 79, 101, 123, 142, 203  
And Search for the values 123 and 153.
9. Differentiate between tree and graph. What are spanning forest and spanning tree. Explain MST (Minimum cost Spanning Tree) problem.
10. A file containing 100 symbols in which following alphabets with their probability of occurrence. Build a Huffman tree according to Greedy strategy.
11. Explain the use of Big-on notation in analyzing algorithms. Compare sorting time, efficiencies of Quick-Sort and Merge-Sort.
12. Explain CLL, DLL, DCLL (Circular, Doubly, Doubly Circular Linked List).

**3 Write short notes on (Any Two):**

- a) Hash function
- b) External sorting



e) ADT.

**Year: 2067**

**Section A**

**Attempt any two questions: (2x10=20)**

1. Define stack as ADT. Describe its primitive operations on Array implementation and linked list implementation. .
2. Describe properties of Binary search tree. Write recursive algorithms for constructing BST and its traversals. Illustrate them with an example.
3. What is external and internal sorting? Explain partition strategies of Merge sort and Quicksort. Trace these sort algorithms for following data:  
11, 45, 61, 33, 55, 9, 83, 25.

**Section B**

**Attempt any eight questions: (8x5=40)**

4. Write recursive algorithm to get Fibonacci term. Illustrate it drawing recursion tree.
5. Instruct expression tree from the following postfix:  $AB + C * DC - FG + \$$ .
6. Differentiate between Singly linked list, DLL, CLL and DCLL.
7. Describe circular Queue operations in array implementation.
8. Compare and Contrast between Binary searching and Binary tree searching.
9. State collision resolution techniques in hashing. Explain Double hashing and Quadratic probing techniques.
10. State MST (Minimum cost spanning tree) problem and shortest path (single source and all other destination) problem. Name the algorithms for solving these problems.
11. Justify the statement: "To write an efficient program, we should know about data structures and Algorithms".
12. Discuss the merits and demerits of contiguous list and linked list.
13. What is priority queue? How it is best implemented?

**Year: 2068**

**Attempt any Two questions: (2X10=20)**

1. Define Queue as an ADT . Write a program for basic operations in Linear queues in array implementation.
2. Why recursion is required? Explain with Tower –of-Hanoi example. How recursive algorithm makes program effective ? Write the merits and demerits of recursion in Programming.
3. Explain In-fix to Postfix Conversion Algorithm. Illustrate it with an example. What changes should be made for converting fix to prefix.

**Section B**

**Attempt any Eight Questions (8X5=40)**

4. Explain Kruskal's algorithm with example.
5. Write a program in C for bubble sorting.
6. Differentiate between contiguous list and linked list with examples.
7. Explain binary search. Illustrate it with example.
8. Explain hashing with example.
9. Explain why linked list is called dynamic list? Write the algorithm for deleting a new node before a node.
10. Explain the characteristics of Huffman's algorithm and its application.
11. Write merits and demerits of recursive function over non-recursive function.
12. Write the steps involved in deleting a node in an Binary selection tree.
13. Discuss merge sort. How you rate this sorting from selection sort?

**First Year/Second Semester**

**Subject :Linear Algebra(CSC-154)**

**FM : 60**

**Time : 3 hours**

**PM : 24**

**Year: 2065**

**Group A**

**(10x2=20)**

1. Illustrate by an example that a system of linear equations has either exactly one solution or infinitely many solutions.
2. When is a linear transformation invertible?
3. Solve the system  $3x_1 + 4x_2 = 3$ ,  $5x_1 + 6x_2 = 7$  by using the inverse of the matrix  $A =$

$$\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}.$$

4. State the numerical importance of determinant calculation by row operation.
5. State Cramer's Rule for an invertible  $n \times n$  matrix  $A$  and vector  $b \in \mathbb{R}^n$  to solve the system  $Ax = b$ . Is this method efficient from computational point of view?
6. Determine if  $\{v_1, v_2, v_3\}$  is a basis for  $\mathbb{R}^3$ , where  $V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ ,  $V_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ ,  $V_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ .

7. Determine if  $W = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$  is a Nul (A) for  $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$ .
8. Show that 7 is an eigenvalue of  $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$ .
9. If  $S = \{ \mathbf{u}_1, \dots, \mathbf{u}_p \}$  is an orthogonal set of nonzero vectors in  $\mathbb{R}^2$ , then S is linearly independent and hence is a basis for the subspace spanned by S.
10. Let  $W = \text{span} \{ \mathbf{x}_1, \mathbf{x}_2 \}$  Where  $X_1 = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$  and  $X_2 = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$ . Then Construct orthogonal basis for W.

### Group B

(5x4=20)

11. Determine if the given set is linearly dependent:

a)  $\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$ .

12. Find the 3 x 3 matrix that corresponds to the composite transformation of a scaling by 0.3, a rotation of  $90^\circ$ , and finally a translation that adds  $(-0.5, 2)$  to each point of a figure.

OR

Describe the Leontief Input-Output model for certain economy and derive formula for  $(I - C)^{-1}$ , where the symbols have their usual meanings.

13. Find the coordinate vector  $[X]_B$  of  $x$  relative to the given basis  $B = \{b_1, b_2\}$ , where  $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ ,  $b_2 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$ ,  $X = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ .

Let  $A = b_1 = \begin{bmatrix} 4 & -9 \\ 4 & 8 \end{bmatrix}$ ,  $b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ ,  $b_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ , and basis  $B = \{b_1, b_2\}$ . Find the B-

matrix for the transformation  $x \rightarrow Ax$  with  $P = \{b_1, b_2\}$ .

15. Let  $u$  and  $v$  be nonzero vectors in  $\mathbb{R}^3$  and the angle between them  $\Phi$ . Then prove that  $u \cdot v = \|u\| \|v\| \cos \Phi$  where the symbols have their usual meanings.

### Group C

(5x8=40)

16. Let  $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a linear transformation. Then T is one-to-one if and only if the equation  $T(x) = 0$  has only the trivial solution, prove the statement.

Let  $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$ ,  $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ ,  $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$ ,  $c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$

And define  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  by  $T(x) = Ax$ . Then

(a) Find  $T(u)$ .

(b) Find an  $x \in \mathbb{R}^2$  whose image under  $T$  is  $b$ .

(c) Is there more than one  $x$  whose image under  $T$  is  $b$ ?

(d) Determine if  $c$  is the range of  $T$ .

17. Compute the multiplication of partitioned matrices for

$$A = \begin{bmatrix} 2 & -3 & 1 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ 0 & 4 & -2 & 7 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ -1 & 3 \\ 5 & 2 \end{bmatrix}$$

18. What do you mean by change of basis in  $\mathbb{R}^n$ ? Let  $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ ,  $b_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ ,  $c_1 = \begin{bmatrix} -7 \\ 9 \end{bmatrix}$ ,

$$c_2 = \begin{bmatrix} -5 \\ 7 \end{bmatrix}, \text{ and}$$

Consider the bases for  $\mathbb{R}^2$  given by  $B = \{b_1, b_2\}$  and  $C = \{c_1, c_2\}$ .

(a) Find the change of coordinate matrix from  $C$  to  $B$ .

(b) Find the change of coordinate matrix from  $B$  to  $C$ .

19. Diagonalize the matrix  $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$ , if possible.

20. Find the equation  $y = \beta_0 + \beta_1 x$  of the least squares line that best fits the data points  $(2, 1)$ ,  $(5, 2)$ ,  $(7, 3)$ ,  $(8, 3)$ . What do you mean by least squares lines?

### First Year/Second Semester

**Subject : Linear Algebra(CSc-155)**

**FM : 60**

**Time : 3 hours**

**PM : 24**

**Year: 2066**

#### Group A

**(10x2=20)**

- When is a system of linear equations consistent and inconsistent?
- Write numerical importance of partitioning matrices.
- How do you distinguish singular and nonsingular matrices?
- If  $A$  and  $B$  are  $n \times n$  matrices, then verify with an example that  $\det(AB) = \det(A)\det(B)$ .
- Calculate the area of the parallelogram determined by the columns of  $A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \end{bmatrix}$ .
- Determine if  $w = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$  is in  $\text{Nul}(A)$ , where  $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$ .

7. Determine if  $\{v_1, v_2, v_3\}$  is a basis for  $\mathbb{R}^3$ , where  $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ ,  $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ ,  $v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ .
8. Find the characteristics polynomial and eigen values of the matrix  $\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$ .
9. Let  $\vec{v} = (1, -2, 2, 0)$ . Find a unit vector  $\vec{u}$  in the same direction as  $\vec{v}$ .
10. let  $\{u_1, \dots, u_p\}$  be an orthogonal basis for a subspace  $W$  of  $\mathbb{R}^n$ . Then prove that for each  $y \in W$ , the weights in  $y = c_1 u_1 + \dots + c_p u_p$  are given by  $c_j = \frac{y \cdot u_j}{u_j \cdot u_j}$ .
11. prove that any set  $\{v_1, \dots, v_p\}$  in  $\mathbb{R}^n$  is linearly if  $p > n$ .
12. Consider the Leontief input-output model equation  $x = cx + d$  where the consumption matrix is  $C = \begin{bmatrix} .50 & .40 & .20 \\ .20 & .30 & .10 \\ .10 & .10 & .30 \end{bmatrix}$

Suppose the final demand is 50 units for manufacturing 30 units for agriculture, 20 units for services. Find the production level  $x$  that will satisfy this demand.

13. What do you mean by basis of a vector space? Find the basis for the row space of

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$

OR

State the prove the unique representation theorem for coordinate systems.

14. What do you mean by eigen values, eigen vectors and characteristics polynomial of a matrix? Explain with suitable examples.
15. Define the Gram-Schmidt process. Let  $W = \text{span}\{x_1, x_2\}$ , where  $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$ ,  $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

Then construct an orthogonal basis  $\{v_1, v_2\}$  for  $W$ .

### Group C

(5x8=40)

16. Given the matrix  $\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$ , discuss the word phase and backward phase of the row reduction algorithm.
17. Find the inverse of  $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$ , if exists; by using elementary row reduce the augmented matrix.
18. What do you mean by change of basis in  $\mathbb{R}^n$ ? Let  $b_1 = \begin{bmatrix} -9 \\ 1 \end{bmatrix}$ ,  $b_2 = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$ ,  $b_3 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$ ,  $c_2 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$  and consider the bases for  $\mathbb{R}^2$  given by  $B = \{b_1, b_2\}$  and  $C = \{c_1, c_2\}$ . Find the change of coordinate's matrix from  $B$  to  $C$ .

19. Diagonalize the matrix  $\begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$ , if possible.

OR

Find the eigen values of  $A = \begin{bmatrix} 0.50 & -0.60 \\ 0.75 & 1.1 \end{bmatrix}$ , and find a basis for each eigen space.

20. Find a least-squares solution for  $Ax=b$  with  $A = \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 1 & 1 \\ 1 & 7 \end{bmatrix}$   $b = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 6 \end{bmatrix}$

What do you mean by least-squares problems?

OR

Define a least-squares solution of  $Ax=b$ , prove that the set of least-squares solutions of  $Ax=b$  coincides with the nonempty set of solution of the normal equations  $A^T Ax = A^T b$

**Year: 2067**

### Group A

**Attempt All Questions (10x2=20)**

1. Illustrate by an example that a system of linear equations has either no solution or exactly one solution.
2. Define singular and nonsingular matrices.
3. Using the Invertible matrix Theorem or otherwise, show that  $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$  is invertible.
4. What is numerical drawback of the direct calculation of the determinants?
5. Verify with an example that  $\det(AB) = \det(A) \det(B)$ . Let  $A$  and  $B$  be any  $n \times n$  matrices.
6. Find a matrix  $A$  such that  $w \in \text{col}(A)$ .

$$w = \left\{ \begin{bmatrix} 6a-b \\ a+b \\ -7a \end{bmatrix} : a, b \in \mathbb{R} \right\}.$$

7. Define subspace of a vector space with an example.
8. Are the vectors  $u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$  and  $v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$  eigenvectors of  $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$ ?
9. Find the distance between the vectors  $u = (7, 1)$  and  $y = (3, 2)$ . Define the distance between two vectors in  $\mathbb{R}^n$ .
10. Let  $w = \text{span} \{x_1, x_2\}$  where  $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$ ,  $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ .

Then construct orthogonal basis for  $w$ .

### Group B

**Attempt All Questions(5x4 =20)**

11. If a set  $s = \{v_1 \dots v_p\}$  in  $\mathbb{R}^n$  contains the zero vector, then prove that the set is linearly dependent. Determine if the set  $\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix}$  is linearly dependent.
12. Given the Leontief input-output model  $x=Cx + d$ , where the symbols have their usual meanings, consider any economy whose consumption matrix is given by  $C = \begin{bmatrix} .50 & .40 & .20 \\ .20 & .30 & .10 \\ .10 & .10 & .30 \end{bmatrix}$
- Suppose the final demand is 50 units for manufacturing 30 units for agriculture, 20 units for services. Find the production level  $x$  that will satisfy this demand.
13. Define rank of a matrix and state Rank Theorem. If  $A$  is a  $7 \times 9$  matrix with a two dimensional null space, find the rank of  $A$ .
14. Determine the eigenvalues and eigenvectors of  $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$  in complex numbers.
- Let  $A = \begin{bmatrix} 4 & -9 \\ 4 & 8 \end{bmatrix}$ ,  $b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ ,  $b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$  and basis  $B = \{b_1, b_2\}$ . Find the  $B$ -matrix for the transformation  $Ax \rightarrow x$  with  $P = [b_1, b_2]$ .
15. Let  $u$  and  $v$  be nonzero vectors in  $\mathbb{R}^2$  and the angle between them be  $\theta$  then prove that  $u \cdot v = \|u\| \|v\| \cos \theta$ , where the symbols have their usual meanings.
16. Line if the following homogeneous system has a nontrivial solution. Then describe the solution set.  $3x_1 + 5x_2 - 4x_3 = 0$ ,  $-3x_1 - 2x_2 + 4x_3 = 0$ ,  $6x_1 + x_2 - 8x_3 = 0$ .
17. An  $n \times n$  matrix  $A$  is invertible if and only if  $A$  is row equivalent to  $I_n$ , and in this case, any sequence of elementary row operations that reduces  $A$  to  $I_n$  also transforms  $I_n$  into  $A^{-1}$ . Use this statement to find the inverse of the matrix  $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$ , if exists.

18. What do you mean by basis change? Consider two bases  $B = \{b_1, b_2\}$  and  $c = \{c_1, c_2\}$  for a vector space  $V$ , such that  $b_1 = 4c_1 + c_2$  and  $b_2 = 6c_1 + c_2$ . Suppose  $[X]_B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$  i.e.,  $x = 3b_1 + b_2$ . Find  $[x]_C$ .

Define basis of a subspace of a vector space.

Let  $V_1 = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$ ,  $V_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$ ,  $V_3 = \begin{bmatrix} 6 \\ 16 \\ -5 \end{bmatrix}$ , where  $v_3 = 5v_1 + 3v_2$ , and let  $H$  span  $\{v_1, v_2, v_3\}$ .

Show that  $\text{span}\{v_1, v_2, v_3\} = \text{span}\{v_1, v_2\}$  and find a basis for the subspace  $H$ .

19. Diagonalize the matrix  $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$ , if possible.
20. What do you mean by least — squares lines? Find the equation  $y = \beta_0 + \beta_1 x$  of the least squares line that fits the data points  $(2,1), (5,2), (7,3), (8,3)$ .
- OR**
- Find the least — squares solution of  $Ax = b$  for

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix} \quad b = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}$$

**Year 2068**

**Attempt all question:**

**Group A**

**(10 × 2 = 20)**

1. Write down the conditions for consistent of non-homogenous system of linear equations.
2. What is meant by independent of vectors?
3. What is normal form of a matrix?
4. Define nonsingular linear transformation with suitable example.
5. Consider the matrix  $A = \begin{pmatrix} 2 & 5 \\ 1 & 7 \end{pmatrix}$  as a linear mapping. Write the corresponding co-ordinate equations.
6. State the numerical importance of determinant calculation by row operation.
7. Show that  $\{(1,1), (-1,0)\}$  form a basis for  $R^2$ .
8. Let  $T: R^2 \rightarrow R^2$  be a linear transformation defined by  
 $T(x, y) = (x + y, y)$ . Find Ker T.
9. If  $\lambda$  is an eigen value of matrix A, find the eigen values of  $A^{-1}$ .
10. Let  $u = (1, 2, -1, 3)$  and  $v = (3, 0, 2, -2)$ . Compute the inner product  $(u, u + v)$ .

**Group B**

**(5 × 4 = 20)**

11. Determine whether the following vectors in  $R^3$  are linearly dependent:
  - a.  $(1, 0, 1), (1, 1, 0), (-1, 0, -1),$
  - b.  $(2, 1, 1), (3, -2, 2), (-1, 2, -1).$
12. Investigate and interpret geometrically the transformation of the unit square whose vertices are  $O(0,0,1), A(1,0,1), B(0,1,1)$  and  $C(1,1,1)$  effected by the  $3 \times 3$  matrix:

$$\begin{bmatrix} 1 & 1 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$$

OR



Is the set of vectors  $\{(1,0,1), (0,1,0), (-1,0,1)\}$  orthogonal? Obtain the corresponding orthonormal set in  $R^3$ .

13. In the vector space  $R^2$ , express the given vector  $(1,2)$  as a linear combination of the vectors  $(1, -1)$  and  $(0,1)$
14. Find the matrix representation of the linear transformation  $T: R^2 \rightarrow R^2$  defined by  $T(x, y) = (x, x + 2y)$  relative to the basis  $(1,0)$  and  $(1,1)$
15. Let  $u$  and  $v$  be nonzero vector in  $R^n$  and the angle between them be  $\theta$ . Then prove that

$$u \cdot v = \|u\| \|v\| \cos \theta$$

Where the symbol have their usual meanings.

### Group C

(5×8=40)

16. Test for consistency and solve:
 
$$\begin{aligned} 2x - 3y + 7z &= 5 \\ 3x + y - 3z &= 13 \\ 2x + 19y - 47z &= 32 \end{aligned}$$
17. Let  $U$  and  $V$  be vector spaces over a field and assume that  $\dim U = \dim V$ . If  $T: U \rightarrow V$  is a linear transformation, then prove that the following are equivalent;
  - i.  $T$  is invertable
  - ii.  $T$  is one-one and onto, and
  - iii.  $T$  is non-singular

OR

Verify that the set of matrices of the form  $\begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}$  is a subspace of the vector space of  $3 \times 3$  matrices.

18. Verify Cayley-Hamilton Theorem for matrix:

$$A = \begin{bmatrix} 6 & 2 & -1 \\ -6 & -1 & 2 \\ 7 & 2 & -2 \end{bmatrix}$$

19. Diagonalize the matrix  $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$

OR

Compute the multiplication of partitioned matrices for

$$A = \begin{bmatrix} 1 & 2 & 4 & 6 & 7 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 2 & 3 & 6 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -12 \\ 2 & 3 & 1 \\ 1 & 4 & 5 \\ 2 & 2 & 0 \\ 0 & 7 & 6 \end{bmatrix}$$

20. Find the equation  $y = \beta_0 + \beta_1 x$  for the least squares line that best fits the data points (2,0), (3,4), (4,10), (5,16).

### First Year/Second Semester

Subject : STA. 159 - Statistics II

FM : 60

Time : 3 hours

PM : 24

**Year: 2065**

#### Group A

**Answer any two questions (10x2=20)**

- Describe a situation where multistage sampling is an appropriate method of drawing a random sample. Clearly state the procedure of drawing random sample in two-stage sampling plan. In two-stage sampling with random sampling without replacement at both stages obtain an expression for an unbiased estimator, of the population, total, and derive the expression for the unbiased estimator. Hence, in particular case (when  $M_i = M$  and  $m_i = m$  for all  $i$ ), obtain the simplified version of the unbiased estimator and variance of the unbiased estimator.
- What do you mean by Latin Square Design (LSD)? Write and explain the statistical model for  $m \times m$  LSD. Give the statistical analysis of  $m \times m$  LSD with one observation per cell.
- What is questionnaire? What are the requisites of a good questionnaire?
  - Explain the effect model  $y_{ij} = \mu + \tau_i + e_{ij}$ ,  $i = 1, 2, \dots, a$  and  $j = 1, 2, \dots, n$  with the assumptions made on  $y_{ij}$ . Why the assumptions are required?

#### Group B

**Answer any eight questions (8x5=40)**

- The following table summarizes population size ( $N_h$ ) and population variance ( $S_h^2$ ) of four strata. Compute the variance of the stratified estimator  $\bar{y}_{st}$  of the population mean for proportional allocation of a total sample of size 100.

H	1	2	3	4
N	14000	3000	1500	1500
S	34	94	175	319

- Describe the procedure of drawing a linear systematic sample of size  $n$  from a population consisting of  $N$  units when  $N = n \times k$  where  $k$  is a positive integer. Write

down the problem of drawing a linear systematic sample of size 4 from a population consisting of 17 units, numbered from 1 to 17.

3. In pps with replacement sampling, show that an unbiased estimator of Population total Y is  $\hat{Y}_{pps} = \frac{1}{n} \sum_{i=1}^n \frac{y_i}{p_i}$ . Derive the expression for the variance  $\hat{V}_{pps}$ .
4. Clearly state the procedure of drawing a random sample in cluster sampling plan. In a simple random sampling without replacement of n clusters from a population of N clusters each containing M elements, derive an unbiased estimator of the parameter  $\bar{Y}$ , population mean per element.
5. Describe census and sample survey. Write down the advantages of sample survey over census. Write down the major steps involved in a sample survey.
6. Explain the terms: factor, experimental units, treatment, and experimental error with suitable examples.
7. In a single factor model  $y_{ij} = \mu + \tau_i + e_{ij}$ , show that

$$\sum_{i=1}^n \sum_{j=1}^m (y_{ij} - \bar{y})^2 = n \sum_{i=1}^n (y_i - \bar{y})^2 + \sum_{i=1}^n \sum_{j=1}^m (y_{ij} - \bar{y}_i)^2$$

What is significance of this result in experimental design?

8. State the mathematical model with the hypotheses to be tested in a two-way ANOVA. Write down the ANOVA table for a two-way ANOVA.
9. Consider the partially completed ANOVA table below. Complete the ANOVA table and answer the followings. What design was employed? How many treatments were compared? How many observations were analyzed? At the 0.05 level of significance, can one conclude that the treatments have different effects? Why?

Source	SS	Df	MS	F
Treatments	231.50	2		
Blocks		7		
Error	573.75			
Total	903.75	23		

10. The results of  $2^2$  experiments with 3 replications are presented below. Estimate the main effects, interaction effect,  $SS_A$ ,  $SS_B$  and  $SS_{AB}$ . Which effects appear to be large?

Treatment Combination	Replication		
	I	II	III
1	22	30	25
A	32	42	29
	35	33	50
Ab	55	45	46

## Bachelor Level / First Year/Second Semester/Science

Subject : Statistics II STA159  
Time : 3 hours

FM : 60  
PM : 24

**Year: 2066**

### Group A

**Answer any two questions (10x2=20)**

- Describe a situation where probability proportion to size (pps) sampling is an appropriate method for drawing a random sample. Clearly state the procedure of drawing a random sample in pps sampling plan. In pps sampling with replacement derive an unbiased estimator of the parameter  $Y$ , population total, and, also, derive the variance of the estimator.
- What do you mean by Completely Randomized Design (CRD)? Write and explain the statistical model for CRD. Give the statistical analysis of CRD with one observation per cell.
- Write down the major steps of a sample survey and state the major sources of errors in a sample survey.
  - Write down the basic principles of experimental designs and explain the term Experimental errors.

### Group B

**Answer any eight questions (8x5=40)**

- Following table summarizes information related to four strata. Compute the variance of the stratified estimator  $\bar{y}_{st}$  of the population mean assuming proportional allocation of a total sample of size 100.

h	1	2	3	4
$N_h$	200	400	300	100
$S^2_h$	9	4	4	9

- Describe the situation where the systematic sampling is useful. Write down the problem of drawing a linear systematic sample of size 4 from a population consisting of 17 units, numbered from 1 to 17.
- In two stage sampling with simple random sampling without replacement at both stages, an unbiased estimator of  $Y$  is  $\bar{Y} = J$ . Derive the variance of the above estimator.
- Clearly state the procedure of drawing a random sample in cluster sampling plan, in a simple random sampling without replacement of  $n$  clusters from a population of  $N$  clusters each containing  $M$  elements, derive an unbiased estimator of the parameter  $\bar{Y}$ , population mean per element.
- Describe census and sample survey. Write down the advantages of sample survey over census. Write down the major steps involved in a sample survey.
- Explain the terms: factor, experimental units, treatment, and experimental error-with suitable examples.
- Write down (a) layout of two-way ANOVA with its assumptions, (b) effect model and (c) ANOVA table.

11. In a single factor model  $y_{ij} = \mu + \tau_i + e_{ij}$ , show that:

$$\sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_{..})^2 = n \sum_{i=1}^a (y_i - \bar{y}_{..})^2 + \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_{i.})^2$$

What is significance of this result in experimental design?

12. Fill in the ( ) in the following ANOVA table of Latin Square Design.

Source Of Variation	Sum Of Squares	Degrees of Freedom	Mean Square	F-Value
Rows				
Columns			36	
Treatments		180		
Error	6		12	
Total				

13. The results of 22 experiments with 3 replications are presented below. Estimate the main effects, interaction effect, SSA, SSB and SSP. Which effects appear to be large?

EXPERIMENT	Replication		
	I	II	III
	22	30	25
	32	42	29
	35	33	50
	55	45	46

**Year: 2067**

### Group A

**Answer any two questions (10x2=20)**

- Describe a situation where cluster sampling is appropriate for drawing random sample. Clearly state the procedure of drawing a random sample in cluster sampling plan. In a sampling without replacement of  $n$  clusters from a population of  $N$  clusters each containing  $M$  elements, derive an unbiased estimator of the parameter  $\bar{Y}$ , population mean per element, and, also, derive the variance of the estimator.
- Explain the terms: a random sample, sampling frame, sampling error and non-sampling error.
  - Explain the terms: factor, experimental units, treatment, and experimental error.
- What do you mean by Randomized Block Design (RBD)? Write and explain the statistical model for RBD. Give the statistical analysis of RBD with one observation per cell.

### Group B

**Answer any eight questions (8x5=40)**

- The following table summarizes population size ( $N_h$ ) and population variance ( $S_h^2$ ) related to four strata. If the required sample size is 4000, what are the sample sizes that would be drawn from each stratum for (a) proportional allocation, and (b) optimum allocation assuming the survey cost per unit is same in each stratum.

H	1	2	3	4
N <sub>h</sub>	14000	3000	1500	1500
S <sup>2</sup> <sub>h</sub>	34	94	175	319

14. Describe the procedure of drawing a linear systematic sample of size n from a population consisting of N units when  $N = n \times k$  where k is a positive integer. Write down the problem of drawing a linear systematic sample of size 4 from a population consisting of 17 units, numbered from 1 to 17.
6. In pps with replacement sampling, show that an unbiased estimator Of Population total Y is  $\hat{Y}_{pps} = \frac{1}{n} \sum_{i=1}^n \frac{y_i}{p_i}$ . Derive the expression for the Variance,  $\hat{V}_{pps}$ .
7. In two-stage sampling with simple random sampling without replacement at both stages, show that an unbiased estimator of Y is  $\hat{Y} = \frac{N}{n} \sum_{i=1}^n \frac{M_i}{m_i} \sum_{j=1}^{m_i} y_{ij}$   
What would be the above expression if  $M_i = M$  and  $m_i = m$  for all i?
8. What is questionnaire? Explain. Write down the prerequisites of a good questionnaire?
9. Write down the principles of experimental design.
10. In a single factor model  $y_{ij} = \mu + \tau_i + e_{ij}$ , show that

$$\sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y})^2 = n \sum_{i=1}^a (y_{i.} - \bar{y})^2 + \sum_{i=1}^a \sum_{j=1}^n (y_{.ij} - \bar{y}_{i.})^2$$

What is significance of this result in experimental design?

11. Consider the partially completed ANOVA table below. Complete the ANOVA table and answer the followings. What design was employed? How many treatments were compared? How many observations were analyzed? At the 0.05 level of significance, can one conclude that the treatments have different effects? Why?

Source	SS	Df	MS	F
Treatments	231.50	2		
Blocks		7		
Error	573.75			
Total	903.75	23		

12. Write down statistical model for a Latin Square Design (LSD) and explain it. Also, write down the ANOVA table for LSD.
13. Write down the four treatment combinations of  $2^2$  experiment using standard notations. Write down the expressions for computing main effects, interaction effect,  $SS_A$ ,  $SS_B$  and  $SS_{AB}$  if experiment is replicated r times.

All notations have the usual meanings.

**Group A**

**Attempt any two questions.**

**(2X10=20)**

- Under which situation probability proportion to size (pps) sampling is an appropriate method for drawing a random sample. Explain the procedure of drawing a random sample in pps sampling plan. In pps sampling with replacement, derive an unbiased estimator of the parameter Y, population total and also derive the variance of the estimator.
- What do you mean by factorial design? Discuss its role in design of experiment. Obtain main effect and interaction effect in  $2^2$  factorial design.
- What do you mean by ANOVA . Explain the underlying assumptions of ANOVA
  - Explain the term-sampling error and non-sampling error.

**Group B**

**Attempt any eight questions. (8X5=40)**

- Suppose it is required to estimate the average value of output of a group of 5000 factories in an industrial area so that one sample estimate lies within 10% of the true value with a confidence of 95%. Determine the minimum sample size required. It is also known that the population coefficient of variation is 60%.
- Derive the expression of the sample mean in case of cluster sampling, each cluster containing equal number of element.
- The following table summarizes population size ( $N_h$ ) and population variance ( $S_h^2$ ) of four strata. Calculate the variance of the stratified estimator  $\bar{y}_{st}$  of the population mean for proportional allocation of a total sample size 100.

H	1	2	3	4
$N_h$	14000	3000	1500	1500
$S_h$	34	94	175	319

- In two stage sampling with simple random sampling without replacement (srsWOR) at both stages, an unbiased estimator of Y is

$$Y^{\wedge} = \frac{N}{n} \sum_{t=1}^n \frac{M_t}{m_t} \sum_{j=1}^{m_t} y_{ij} = \frac{N}{n} \sum_{t=1}^n M_t \bar{y}_t$$

Also derive the variance of the above estimator.

- In simple random sampling without replacement ( $N, n$ ), show that the bias of the ratio estimator  $\hat{R}$  is approximately equal to :

$$B(\hat{R}) \sim \frac{1-f}{n\bar{X}^2} (RS_x^2 - \rho S_y S_x) = Rcv(\bar{X})[cv(\bar{X}) - \rho cv(\bar{y})].$$

- State the mathematical model with the hypothesis to be tested in a two way ANOVA and prepare ANOVA table.
- Carry out the statistical analysis of  $m \times m$  latin Square Design (LSD) with one observation per cell.

11. Explain the terms – experimental units, treatments, blocking in design of experiment.
12. Introduce Randomized Complete Block Design (RCBD). Prepare an Analysis of Variance (ANOVA) table for RCBD.
13. The following is partially completed ANOVA table.

Source of Variation	Sum of squares	Degrees of freedom	Mean Square	F
Treatments	901.9	5		
Blocks	219.43	3		
Error	229.63	-		
Total	1350.25	23		

Complete the ANOVA table and answer the followings.

What design was employed? How many treatments were compared? What about the total number of observations? At 5% level of significance, can we conclude that the treatments have different effects? Are the blocks homogeneous? Explain.



## First Year/Second Semester

**Subject : Physics II (CSc-156)**  
**Time : 3 hours**

**FM : 60**  
**PM : 24**

**Year: 2065**

### Long Answer Questions:

**Attempt any four questions: (7x4=28)**

1. Write Maxwell-Boltzmann distribution for number of particles of gas with energy and apply it to calculate the average energy of monoatomic ideal gas. Show the average energy per molecule is  $3/2 kT$ .
2. Consider a particle of mass  $m$  in a one dimensional rigid box of length  $L$ . Find the energy and wave function of the particle.
3. Electrons in a metal can be considered as an ideal gas. Their random velocity is caused by thermal motion. Show that electrical resistivity of this electron gas increases as  $T^{3/2}$ . Treat the electron gas classically.
4. (a)  $\psi(x)$  is the wave function of a quantum mechanical particle. Derive the expressions for probability density and probability current density in terms of  $\psi(x)$ .  
(b) A simple cubic structure is given by a unit cell, which is a cube of side 'a' and one atom sits at each corner of the cell. How many atoms are there in a unit cell and what is the number of nearest neighbour for any given atom? Write your arguments and show all your work.
5. What do you mean by intrinsic and extrinsic semiconductors? Show that Fermi level in an intrinsic semiconductor lies in the middle of the forbidden gap.

### Group B

#### Short Answer Questions:

**Attempt any eight questions: (8x4=32)**

6. Write the formula for Fermi-Dirac and Bose-Einstein distribution laws and discuss their salient features.
7. What are de Broglie waves associated with a material particle? An electron is moving with kinetic energy 1 KeV. Determine the de Broglie wave length of the electron. [ $m_e = 9.1 \times 10^{-31} \text{ kg}$ ,  $h = 6.63 \times 10^{-34} \text{ Js}$ ,  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ ].
8. An electron is confined in a layer of thickness 15 Å. Calculate the uncertainty in its momentum.

9. Express the momentum operators in differential form and calculate its commutator with  $x$ , i.e. calculate  $[p, x]$ .
10. The crystal of a lattice of a crystal is given by a set of three vectors  $\mathbf{a}_1, \mathbf{a}_2$  and  $\mathbf{a}_3$ . Define the reciprocal lattice vectors and calculate the volume of the reciprocal lattice unit cell. (2+2)
11. The number of electrons per unit volume available for conduction in sodium metal is  $2.5 \times 10^{28}$  electrons per  $\text{m}^3$ . Calculate the Fermi energy and Fermi temperature. ( $h = 1.05 \times 10^{-34} \text{ Js}^{-1}$ , mass of electron  $= 9.11 \times 10^{-31} \text{ Kg}$ , Boltzmann constant  $k = 1.381 \times 10^{-23} \text{ JK}^{-1} = 8.617 \times 10^{-5} \text{ eKV}^{-1}$ ). (2+2)
12. What are Schottky junctions? Discuss their characteristics and advantages over conventional p-n junction diodes.
13. In an extrinsic semiconductor there are  $10^{14}$  donors per  $\text{cm}^3$ . Their ionization energy is  $E_d = 0.005 \text{ eV}$  and effective mass is  $0.01 m_e$  ( $m_e$  is the electronic mass). Estimate the density of the conduction electrons at  $T = 100 \text{ K}$ .
14. What is effective mass of carriers in a solid? Derive an expression for the effective mass of carriers in a semiconductor.
15. The gap energy in germanium crystal is  $0.70 \text{ eV}$ . Calculate the temperature at which the number of conduction electrons increases 10 fold over the same number at the room temperature.

**Year: 2066**

### Long Answer Questions:

**Attempt any four questions: (7x4=28)**

1. Derive Bose-Einstein Distribution Law.
2. Establish Schrodinger's equation for linear harmonic oscillator and solve it to obtain its eigen values.
3. Explain Miller indices. Draw the following indices in a cubic unit cell: (200), (110), (111), (010).
4. What are holes and electrons in a semiconductor? Derive an expression for their concentration with temperature.
5. a) Write the postulate of quantum mechanics. Explain the physical significance of wave function.  
b) Explain entropy and probability relate between them.

### Group B

#### Short Answer Questions:

**Attempt any eight questions: (8x4=32)**

6. If the Fermi energy of a metal is  $10 \text{ eV}$ , what is the corresponding classical temperature.
7. Calculate the de-Broglie wavelength of a ball of mass  $10^{-3} \text{ Kg}$  moving with a velocity of  $10^3 \text{ ms}^{-1}$ . ( $h = 6.6 \times 10^{-34} \text{ Js}$ )
8. Calculate the first two energy levels of an electron in a box  $10^{-8} \text{ m}$  wide.
9. Calculate the inter planar spacing for (110) plane in simple cubic lattice whose lattice constant is  $4.2 \times 10^{-10} \text{ m}$ .

- Write the seven systems of crystals with suitable diagram.
- Find the uncertainty in position of an electron moving with a velocity  $3 \times 10^7 \text{ ms}^{-1}$ .
- Compare M – B, F – D, B – E statics.
- Write Hamiltonian of hydrogen atom in cartesian coordinate system.
- Calculate wave length of electron. Which is accelerated through a potential of 10 volts?
- Given  $\psi(x) = Ce^{ikx}$ . By using normalization find the value of C.

**Year: 2067**

### Long Answer Questions:

**Attempt any four questions: (7x4=28)**

- Show that  $S = -k \log W$  where S is entropy, W is probability and K is a constant.
- Calculate the values of the energy of a particle in one dimensional box. Indicate graphically the first three wave function for such a particle.
- The random velocity of the electron was due to the thermal motion of electron gas then show that the electricity resistivity would increase with temperature as  $T^{3/2}$ .
- Discuss the formation of n-type and p-type semiconductor. Derive an expression for the concentration of the hole and electron in these semiconductors.
- What is significance of wave function? Derive Schrodinger wave equation.

### Group B

#### Short Answer Questions:

**Attempt any eight questions: (8x4=32)**

- Calculate the wavelength of an electron ( $m = 9.1 \times 10^{-31} \text{ kg}$ ) accelerated through a potential of 100 volts. (4)
- The wave function  $\psi = A \sin(n\pi x/L)$  is given. By using normalization show that  $A = \sqrt{2/L}$ .
- Write Hamiltonian for the hydrogen atom in terms of spherical polar coordinate.
- Calculate the inter planer spacing for a (321) plane in simple cubic lattice whose lattice constant is  $4.2 \times 10^{-10} \text{ m}$ .
- Calculate the concentration of conduction electron and Fermi energy of Copper. Given that copper has a mass density  $\rho_m = 3.95 \text{ g cm}^{-3}$ .
- If the position of electron ( $m = 9.1 \times 10^{-31} \text{ kg}$ ) in H atom could be determined with an accuracy of 0.01mm, what would be the uncertainty in its velocity.
- If the Fermi energy of metal is 10 eV, what is the corresponding classical temperature.
- Make a comparison of Maxwell-Boltzmann, Fermi-Dirac and Bose-Einstein statistics.
- Discuss Bravais lattices and give different type of crystal structure.
- Diamond and Silicon have similar electronic configuration and crystal structure. Discuss why the former an insulator while the latter is a semiconductor.