Tribhuvan University

Institute of Science and Technology

2065

✡

Bachelor Level/First Year/ Second Semester/ Science | Full Marks: 80

Computer Science and Information Technology | Pass Marks: 32

(MTH.155 – Linear Algebra) | Time: 3hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

Group A **(10 x 2 = 20)**

1. Illustrate by an example that a system of linear equations has either equations has either exactly one solution or infinitely many solutions.

2. When is a linear transformation invertible?

3. Solve the system
$$3x_1 + 4x_2 = 3, \qquad 5x_1 + 6x_2 = 7$$
by using the inverse of the matrix $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$.

4. State the numerical importance of determinant calculation by row operation.

5. State Cramer's rule for an invertible n x n matrix A and vector $b \epsilon R^n$ to solve the system $Ax = b$. Is this method efficient from computational point of view?

6. Determine if $\{v_1, v_2 v_3\}$ is basis for R^3, where $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

7. Determine if $W = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is a Nul(A) for $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$.

8. Show that 7 is an eigen value of $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$.

9. If $S = \{u_1, \cdots\cdots\cdots, u_p\}$ is an orthogonal set of nonzero vectors in R^2, show S is linearly independent and hence is a basis for the subspace spanned by S.

10. Let $W = span\{x_1,\ x_2\}$ where $x_1 = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$. Their construct orthogonal basis for W.

Group B (5 x 4 = 20)

11. Determine if the given set is linearly dependent:

    a) $\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$ b) $\begin{bmatrix} -2 \\ 4 \\ 6 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ -9 \\ 15 \end{bmatrix}$

12. Find the 3 x 3 matrix that corresponds to the composite transformation of a scaling by 0.3, a rotation of 90^0, and finally a translation that adds (-0.5, 2) to each point of a figure.

OR

Describe the Leontief Input-Output model for certain economy and derive formula for $(I\text{-}C)^{-1}$, where symbols have their usual meanings.

13. Find the coordinate vector $[X]_B$ of a x relative to the given basis $B = \{b_1,\ b_2\}$, where $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, $x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

14. Let $A = \begin{bmatrix} 4 & -8 \\ 4 & 8 \end{bmatrix}$, $b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and basis $B = \{b_1,\ b_2\}$. Find the B-matrix for the transformation $x \rightarrow Ax$ with $P = \{b_1,\ b_2\}$.

15. Let u and v be non-zero vectors in R^3 and the angle between them be ϕ. Then prove that
$$u.v = \|u\|\|v\| \cos \emptyset,$$
where the symbols have their usual meanings.

Group C (5 x 8 = 40)

16. Let $T: R^n \rightarrow R^m$ be a linear transformation. Then T is one-to-one if and only if the equation T(x) = 0 has only the trivial solution, prove the statement.

OR

Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$, $c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ and define $T: R^2 \rightarrow R^3$ by $T(x) = Ax$. Then

a) Find T(u)
b) Find an $x \in R^2$ whose image under T is b.
c) Is there more than one x whose image under T is b?
d) Determine if c is the range of T.

17. Compute the multiplication of partitioned matrices for

$$A = \left[\begin{array}{ccc|cc} 2 & -3 & 1 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ \hline 0 & 4 & -2 & 7 & -1 \end{array}\right] \quad and \quad B = \left[\begin{array}{cc} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ \hline -1 & 3 \\ 5 & 2 \end{array}\right].$$

18. What do you mean by change of basis in R^n? Let $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $b_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, $c_1 = \begin{bmatrix} -7 \\ 9 \end{bmatrix}$, $c_2 = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$, and consider the bases for R^2 given by $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$.
    a) Find the change of coordinate matrix from C to B.
    b) Find the change of coordinate matrix from B to C.

OR

Define vector spaces, subspaces, basis of vector space with suitable examples. What do you mean by linearly independent set and linearly dependent set of vectors?

19. Diagonalize the matrix $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$, if possible.

20. Find the equation $y = \beta_0 + \beta_1 x$ of the least squares line that best fits the data points (2, 1), (5, 2), (7, 3), (8, 3). What do you mean by least squares lines?

Tribhuvan University

Institute of Science and Technology

2066

✡

Bachelor Level/First Year/ Second Semester/ Science — Full Marks: 80

Computer Science and Information Technology — Pass Marks: 32

(MTH.155 – Linear Algebra) — Time: 3hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

Group A **(10 x 2 = 20)**

1. When is system of linear equation consistent or inconsistent?

2. Write numerical importance of partitioning matrices.

3. How do you distinguish singular and non-singular matrices?

4. If A and B are n x n matrices, then verify with an example that det(AB) = det(A)det(B).

5. Calculate the area of the parallelogram determined by the columns of
$$A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \end{bmatrix}.$$

6. Determine if $w = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is Nul(A), where, $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$.

7. Determine if $\{v_1, v_2, v_3\}$ is a basis for λ^3, where $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

8. Find the characteristic polynomial for the eigen values of the matrix $\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$.

9. Let $\vec{v} = (1, -2, 2, 0)$. Find a unit vector $\vec{u}$ in the same direction as $\vec{v}$.

10. Let $\{u_1, \cdots\cdots\cdots, u_p\}$ be an orthogonal basis for a subspace W of R^n. Then prove that for each $y \epsilon W$, the weights in $y = c_1 u_1 + \cdots\cdots\cdots + c_p u_p$ are given by
$$c_j = \frac{y.u_j}{u_j.u_j} \qquad (j = 1, \cdots\cdots\cdots, p)$$

Group B **(5 x 4 = 20)**

11. Prove that any set $\{v_1, \cdots\cdots\cdots, v_p\}$ in R^n is linearly dependent if $p > n$.

12. Consider the Leontief input – output model equation $x = cx + d$, where the consumption matrix is

$$C = \begin{bmatrix} .50 & .40 & .20 \\ .20 & .30 & .10 \\ .10 & .10 & .30 \end{bmatrix}.$$

Suppose the final demand is 50 units of manufacturing, 30 units of agriculture, 20 units for services. Find the production level x that will satisfy the demand.

13. What do you mean by basis of a vector space? Find the basis for the row space of

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$

OR

State and prove the unique representation theorem for coordinate systems.

14. What do you mean by eigen values, eigen vectors and characteristic polynomial of a matrix? Explain with suitable examples.

15. Define the Gram-Schmidt process. Let W=span{x_1, x_2}, where $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Then construct an orthogonal basis {v_1, v_2} for w.

Group C **(5 x 8 = 40)**

16. Given the matrix

$$\begin{bmatrix} 0 & 3 & -6 & 6 & -5 \\ 3 & -7 & 8 & -5 & 9 \\ 3 & -9 & 12 & -9 & 15 \end{bmatrix},$$

discuss the for word phase and backward phase of the row reduction algorithm.

17. Find the inverse of $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$, if it exists, by using elementary row reduce the augmented matrix.

18. What do you mean by change of basis in R^n? Let $b_1 = \begin{bmatrix} -9 \\ 1 \end{bmatrix}$, $b_2 = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$, $c_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, $c_2 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$, and consider the bases for R^2 given by B={b_1, b_2} and C={c_1, c_2}. Find the change of coordinates matrix from B to C.

19. Diagonalize the matrix $\begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$, if possible

OR

Find the eigen value of $A = \begin{bmatrix} 0.50 & -0.60 \\ 0.75 & 1.1 \end{bmatrix}$, and find a basis for each eigen space.

20. Find a least-square solution for Ax = b with $A = \begin{bmatrix} 1 & -6 \\ 1 & -2 \\ 1 & 1 \\ 1 & 7 \end{bmatrix}$, $b = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 6 \end{bmatrix}$. What do you mean by least squares problems?

OR

Define a least-squares solution of Ax = b, prove that the set of least squares solutions of Ax = b coincides with the non-empty set of solutions of the normal equations $A^TAx = A^Tb$.

Tribhuvan University

Institute of Science and Technology

2067

✡

Bachelor Level/First Year/ Second Semester/ Science — Full Marks: 80

Computer Science and Information Technology — Pass Marks: 32

(MTH.155 – Linear Algebra) — Time: 3hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

Group A **(10 x 2 = 20)**

1. Illustrate by an example that a system of linear equation has either no solution or exactly one solution.

2. Define singular and nonsingular matrices.

3. Using the Invertible matrix theorem of otherwise, show that $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$ is invertible.

4. What is numerical drawback of the direct calculation of the determinants?

5. Verify with an example that det (AB) = det(A)det(B) for any n x n matrices A and B.

6. Find a matrix A such that w = col (A).
$$w = \left\{ \begin{bmatrix} 6a - b \\ a + b \\ -7a \end{bmatrix} : a, b \in R \right\}.$$

7. Define subspace of a vector with an example.

8. Are the vectors; $u = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ eigen vectors of $= \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$?

9. Find the distance between vector u = (7, 1) and v = (3, 2). Define the distance between two vectors in x R.

10. Let w = span {x_1, x_2}, where
$$x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}, \qquad x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$
Then construct orthogonal basis for w.

Group B **(5 x 4 = 20)**

11. If a set $s = \{v_1 \cdots\cdots\cdots v_p\}$ in R^n contains the zero vector, then prove that the set is linearly dependent. Determine if the set

$$\begin{bmatrix}2\\3\\5\end{bmatrix}, \quad \begin{bmatrix}0\\0\\0\end{bmatrix}, \quad \begin{bmatrix}1\\1\\8\end{bmatrix}$$

is linearly independent

12. Given the Leontief input-output model x = Cx + d, where the symbols have their usual meanings, consider any economy whose consumption matrix is given by

$$C = \begin{bmatrix}.50 & .40 & .20\\ .20 & .30 & .10\\ .10 & .10 & .30\end{bmatrix}.$$

Suppose the final demand is 50 units for manufacturing 30 units for agriculture, 20 units for services. Find the production level x that will satisfy this demand.

13. Define rank of a matrix and state Rank Theorem. If A is 7 x 9 matrix with a two dimensional null space, find the rank of A.

14. Determine the eigen values and eigen vectors of $A = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ in complex numbers.

OR

Let $A = \begin{bmatrix}4 & -9\\ 4 & 8\end{bmatrix}$, $\quad b_1 = \begin{bmatrix}3\\2\end{bmatrix}$, $\quad b_2 = \begin{bmatrix}2\\1\end{bmatrix}$ and basis B = $\{b_1, b_2\}$.

Find the B-matrix for the transformation $x \to [A]x$ with P = $[b_1, b_2]$.

15. Let u and v be nonzero vectors in R^2 and the angle between the m be θ then prove that

$$u, v = \|u\|\ \|v\| \cos\theta,$$

where the symbols have their usual meanings.

Group C **(5 x 8 = 40)**

16. Determine if the following homogeneous system has a nontrivial solution. Then describe the solution set.

$$3x_1 + 5x_2 - 4x_3 = 0, \quad -3x_1 - 2x_2 + 4x_3 = 0, \quad 6x_1 + x_2 - 8x_3 = 0.$$

17. An n x n matrix A is invertible if and only if A is row equivalent to In, and in this case, any sequence of elementary row operation that reduces A to In also transform I_n into A^{-1}. Use this statement to find the inverse of the matrix $A = \begin{bmatrix}0 & 1 & 2\\ 1 & 0 & 3\\ 4 & -3 & 8\end{bmatrix}$, if exists.

18. What do you mean by basis change? Consider two bases B = {b_1, b_2} and c = {c_1, c_2} for a vector space V, such that $b_1 = 4c_1 + c_2$ and $b_2 = 6c_1 + c_2$. Suppose $[x]_B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ i.e., $x = 3b_1 + b_2$. Find $[x]_C$.

OR

Define basis of a subspace of a vector space.

Let $v_1 = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 6 \\ 16 \\ -5 \end{bmatrix}$, where $v_3 = 5v_1 + 3v_2$, and let H = span {v_1, v_2, v_3}. Show that span {v_1, v_2, v_3} = span {v_1, v_2} and find a basis for a subspace H.

19. Diagonalize the matrix $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$, if possible.

20. What do you mean by least squares lines? Find the equation $y = \beta_0 + \beta_1 x$ of the least squares line that fits the data points (2, 1), (5, 2), (7, 3), (8, 3).

OR

Find the least square solution of Ax = b for

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}, \qquad b = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}.$$

