Image Processing

Introduction (Chapter 1)

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Introduction

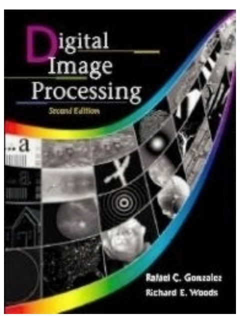
"One picture is worth more than ten thousand words"

Anonymous

References

Text / Reference books:

1. K. Castlemann, “*Digital Image Processing*”, Prentice Hall of India Ltd., 1996.
2. A. K. Jain, “*Fundamental of Digital Image Processing*”, Prentice Hall of India Pvt. Ltd., 1995.
3. R. C. Gonzalez and P. Wintz, “*Digital Image Processing*”, Addison-Wesley Publishing, 1987.
4. Sing_Tze Bow, M. Dekker, “*Pattern Recognition and Image Processing*”, 1992
5. M. James, “*Pattern Recognition*”, BSP professional books, 1987.
6. P. Monique and M. Dekker, “*Fundamentals of Pattern Recognition*”, 1989.



“Digital Image Processing”, Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002

– Much of the material that follows is taken from this book

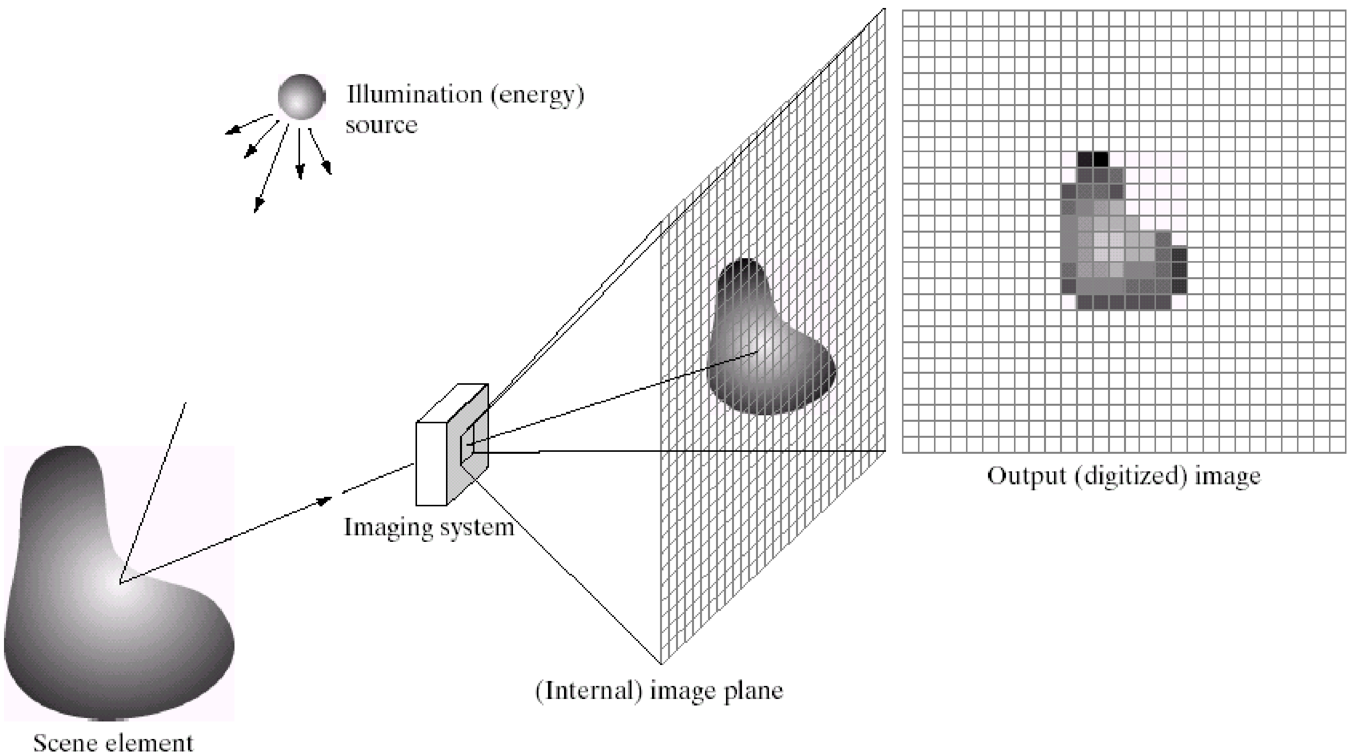
Contents

This lecture will cover:

- What is a digital image?
- What is digital image processing?
- History of digital image processing
- State of the art examples of digital image processing
- Key stages in digital image processing
- Sampling and Quantization
- Relationship between Pixels

What is a Digital Image?

A **digital image** is a representation of a two-dimensional image as a finite set of digital values, called picture elements or pixels

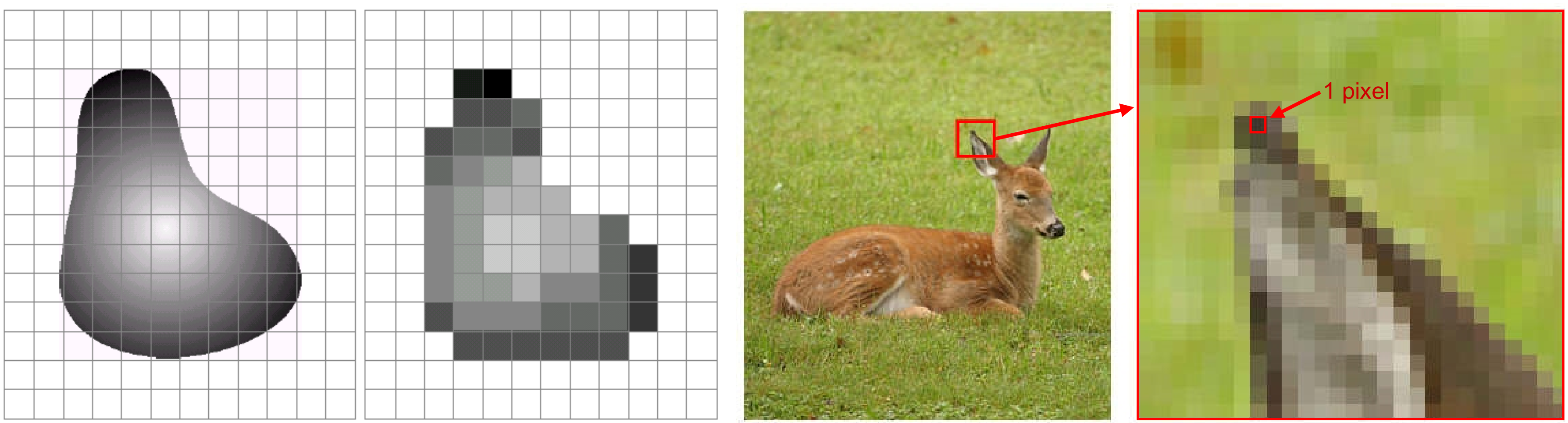


Images taken from Gonzalez & Woods, Digital Image Processing (2002)

What is a Digital Image? (cont…)

Pixel values typically represent gray levels, colors, heights, opacities etc.

Remember *digitization* implies that a digital image is an *approximation* of a real scene



What is a Digital Image? (cont…)

Common image formats include:

- 1 sample per point (B&W or Grayscale)
- 3 samples per point (Red, Green, and Blue)
- 4 samples per point (Red, Green, Blue, and "Alpha", aka Opacity)

For most of this course we will focus on grey-scale images

What is Digital Image Processing?

Digital image processing focuses on two major tasks

- Improvement of pictorial information for human interpretation
- Processing of image data for storage, transmission and representation for autonomous machine perception

Some argument about where image processing ends and fields such as image analysis and computer vision start

What is DIP? (cont…)

The continuum from image processing to computer vision can be broken up into low-, mid- and high-level processes

Low Level Process	Mid Level Process	High Level Process
Input: Image **Output:** Image	**Input:** Image **Output:** Attributes	**Input:** Attributes **Output:** Understanding
Examples: Noise removal, image sharpening	**Examples:** Object recognition, segmentation	**Examples:** Scene understanding, autonomous navigation

In this course we will stop here

History of Digital Image Processing

Early 1920s: One of the first applications of digital imaging was in the news-paper industry

Early digital image

- The Bartlane cable picture transmission service
- Images were transferred by submarine cable between London and New York
- Pictures were coded for cable transfer and reconstructed at the receiving end on a telegraph printer

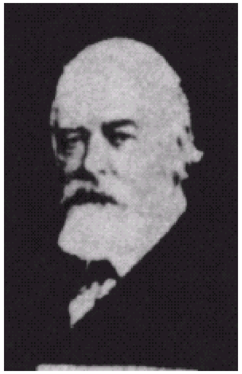
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

History of DIP (cont…)

Mid to late 1920s: Improvements to the Bartlane system resulted in higher quality images

– New reproduction processes based on photographic techniques

– Increased number of tones in reproduced images



Improved digital image

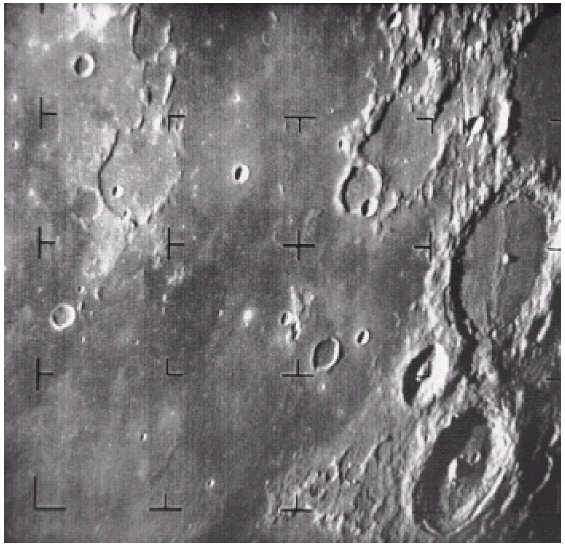
Early 15 tone digital image

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

History of DIP (cont…)

1960s: Improvements in computing technology and the onset of the space race led to a surge of work in digital image processing

- **1964:** Computers used to improve the quality of images of the moon taken by the *Ranger 7* probe
- Such techniques were used in other space missions including the Apollo landings



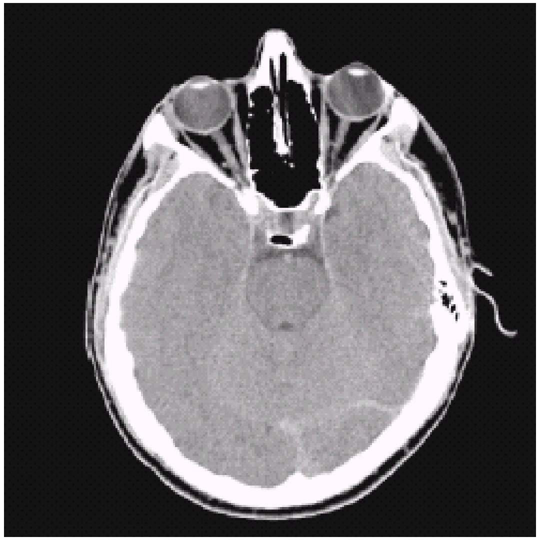
A picture of the moon taken by the Ranger 7 probe minutes before landing

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

History of DIP (cont…)

1970s: Digital image processing begins to be used in medical applications

- **1979:** Sir Godfrey N. Hounsfield & Prof. Allan M. Cormack share the Nobel Prize in medicine for the invention of tomography, the technology behind Computerised Axial Tomography (CAT) scans



Typical head slice CAT image

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

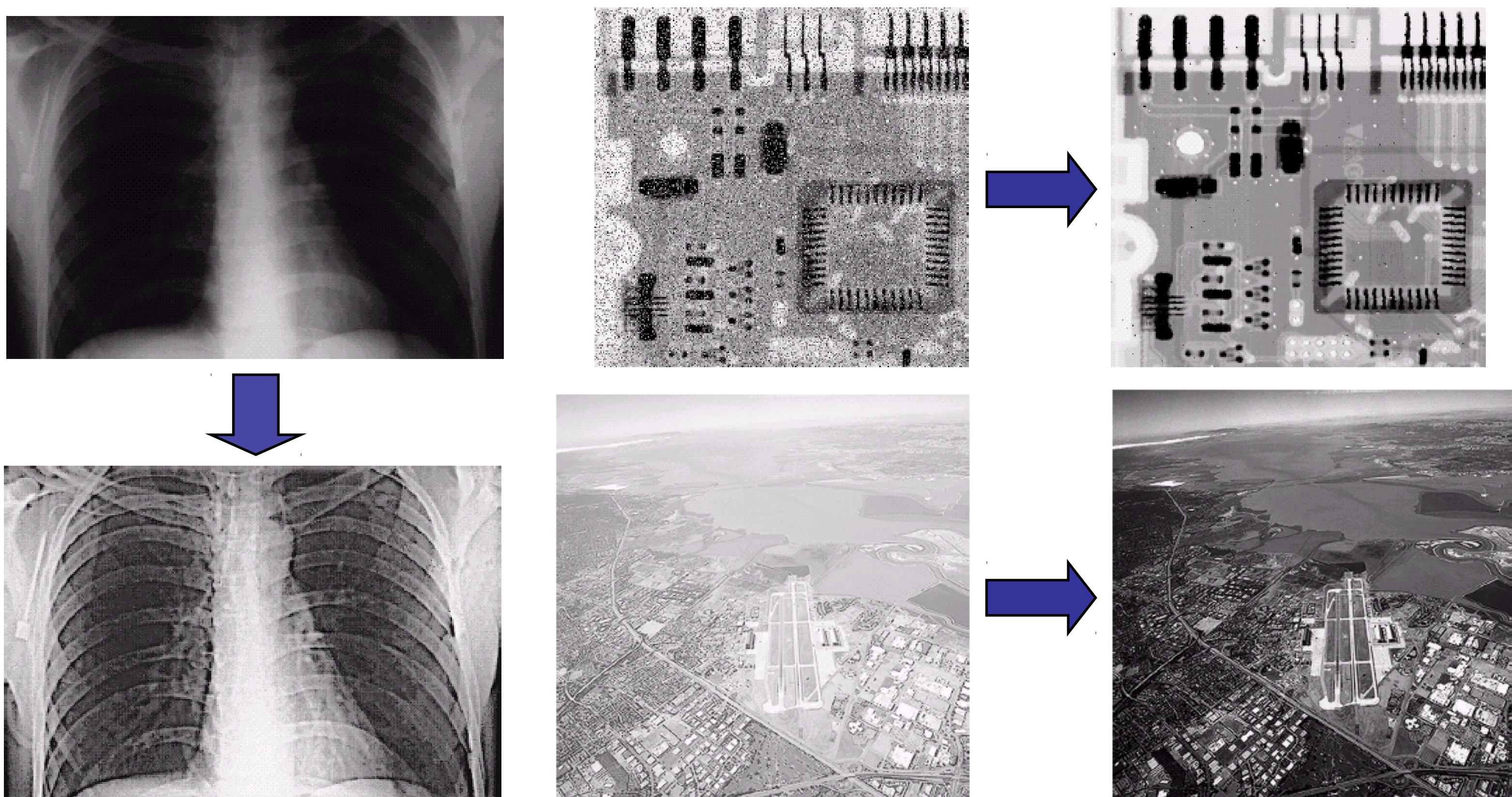
History of DIP (cont…)

1980s - Today: The use of digital image processing techniques has exploded and they are now used for all kinds of tasks in all kinds of areas

– Image enhancement/restoration
– Artistic effects
– Medical visualisation
– Industrial inspection
– Law enforcement
– Human computer interfaces

Examples: Image Enhancement

One of the most common uses of DIP techniques: improve quality, remove noise etc



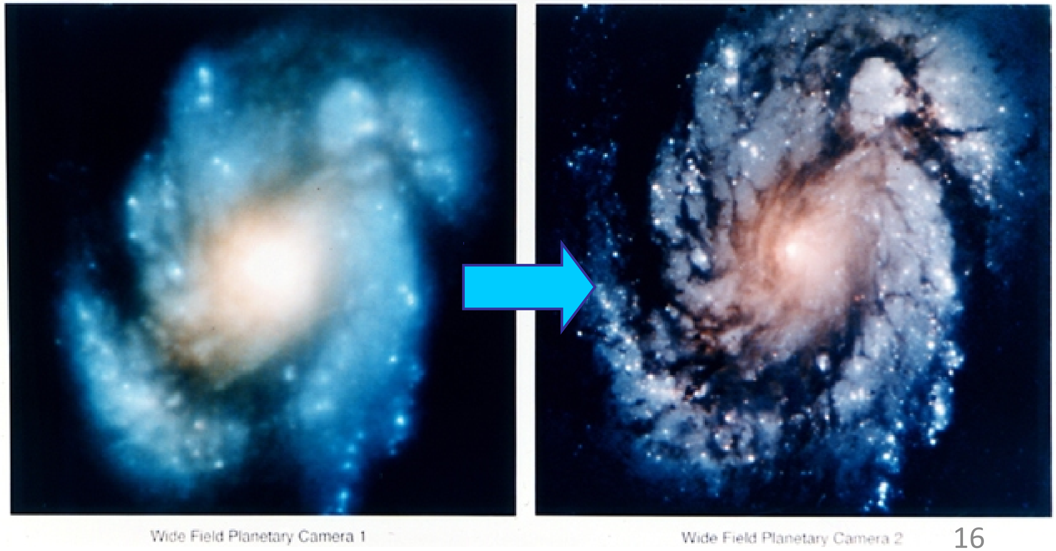
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Examples: The Hubble Telescope

Launched in 1990 the Hubble telescope can take images of very distant objects

However, an incorrect mirror made many of Hubble's images useless

Image processing techniques were used to fix this



Examples: Artistic Effects

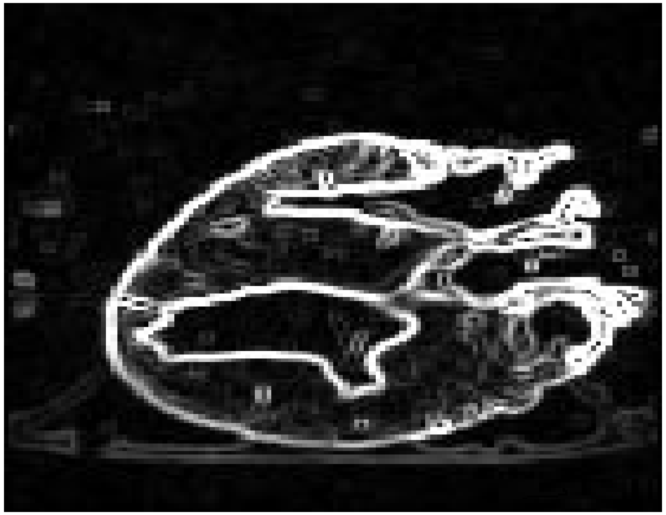
Artistic effects are used to make images more visually appealing, to add special effects and to make composite images

Examples: Medicine

Take slice from MRI scan of canine heart, and find boundaries between types of tissue

- Image with gray levels representing tissue density
- Use a suitable filter to highlight edges

Original MRI Image of a Dog Heart



Edge Detection Image

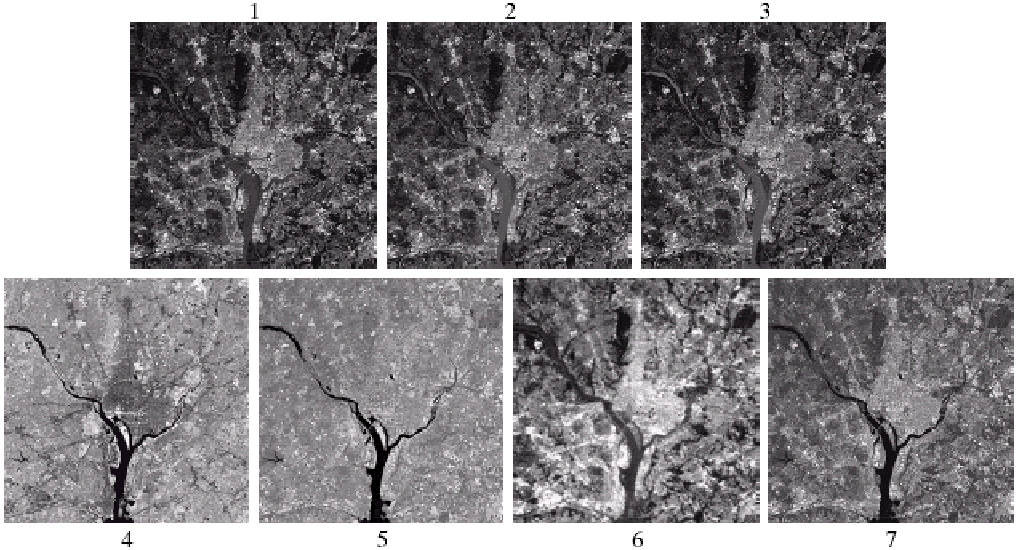
Examples: GIS

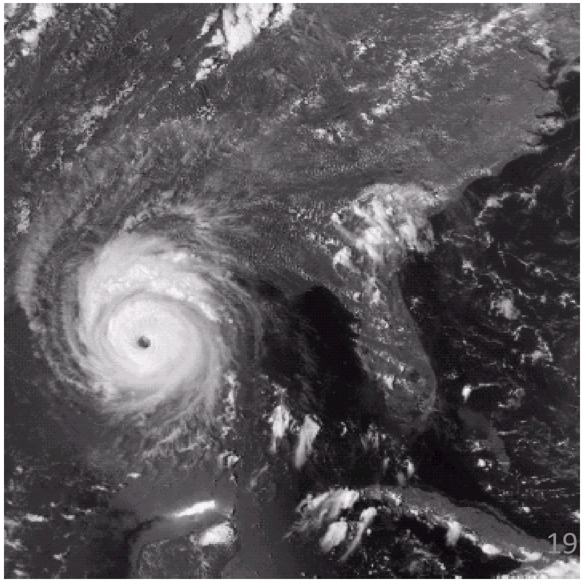
Geographic Information Systems

– Digital image processing techniques are used extensively to manipulate satellite imagery

– Terrain classification

– Meteorology





Examples: GIS (cont…)

Night-Time Lights of the World data set

- Global inventory of human settlement
- Not hard to imagine the kind of analysis that might be done using this data

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

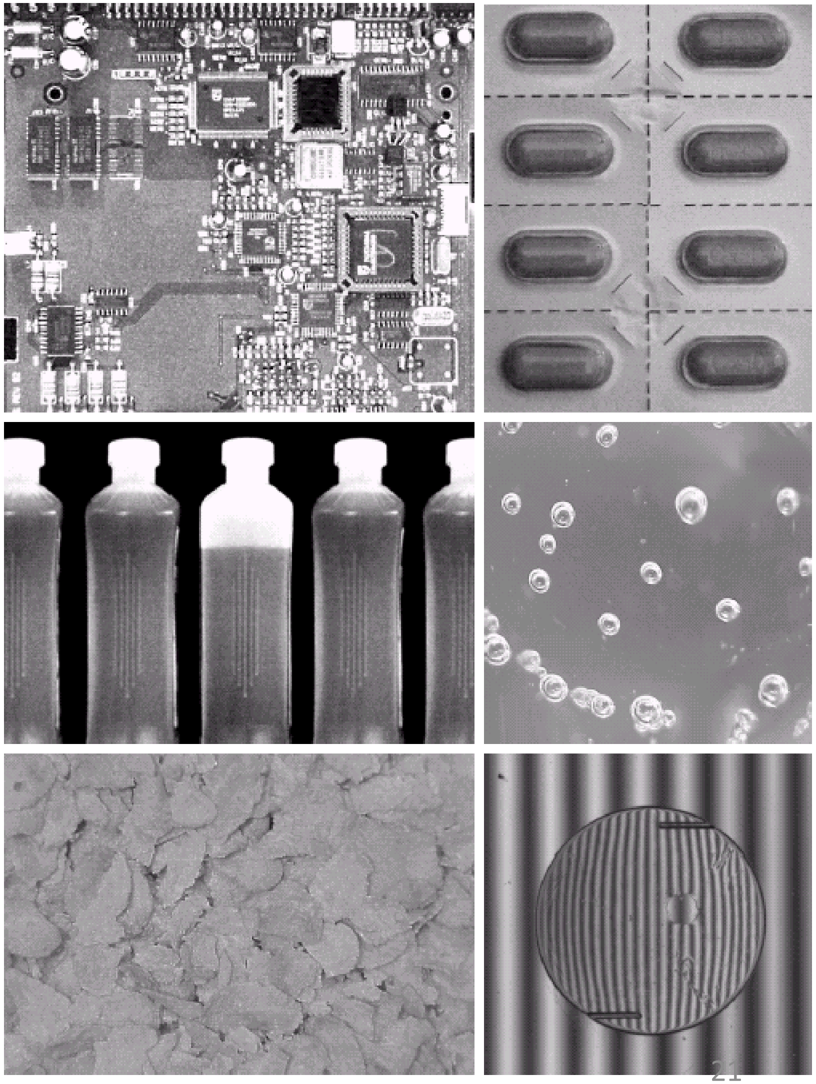
Examples: Industrial Inspection

Human operators are expensive, slow and unreliable

Make machines do the job instead

Industrial vision systems are used in all kinds of industries

Can we trust them?

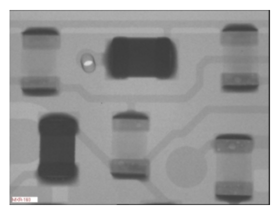


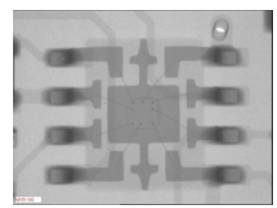
Images taken from Gonzalez & Woods, Digital Image Processing (2002)

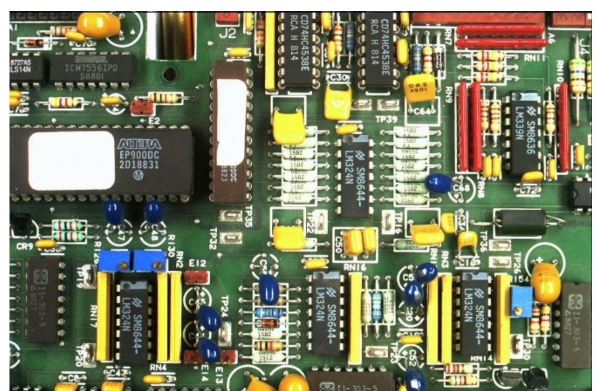
Examples: PCB Inspection

Printed Circuit Board (PCB) inspection

- Machine inspection is used to determine that all components are present and that all solder joints are acceptable
- Both conventional imaging and x-ray imaging



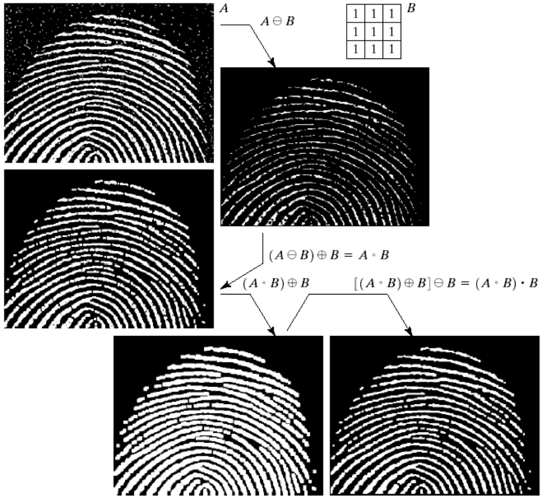




Examples: Law Enforcement

Image processing techniques are used extensively by law enforcers

- Number plate recognition for speed cameras/automated toll systems
- Fingerprint recognition
- Enhancement of CCTV images



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

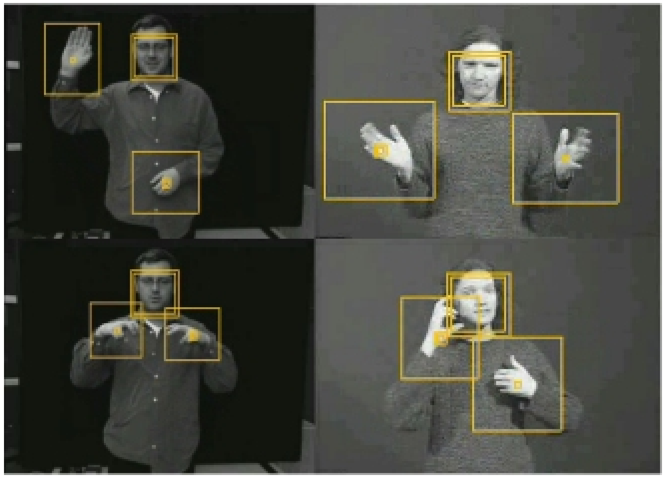
Examples: HCI

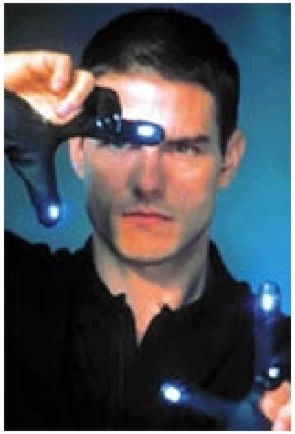
Try to make human computer interfaces more natural

- Face recognition
- Gesture recognition

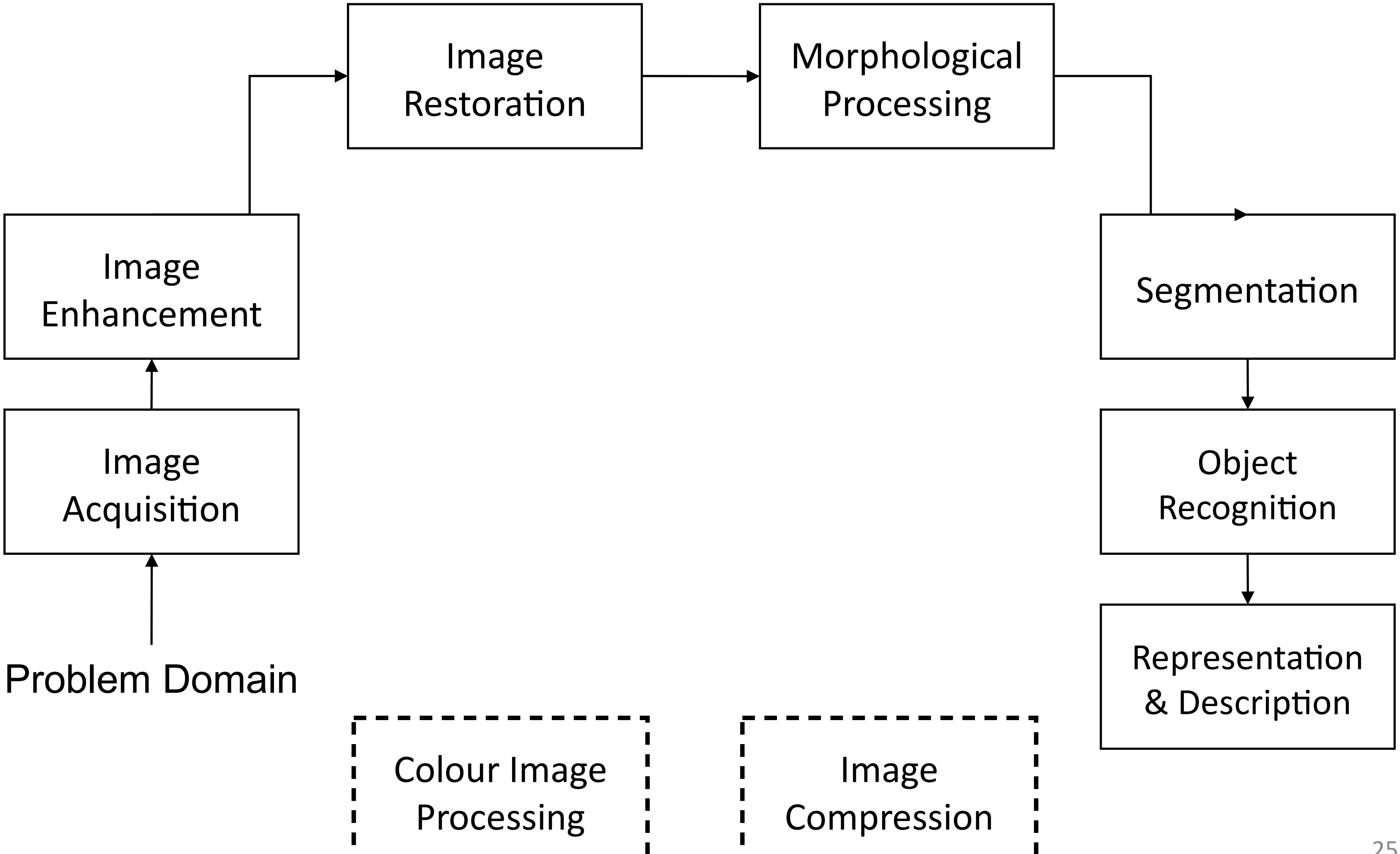
Does anyone remember the user interface from "Minority Report"?

These tasks can be extremely difficult





Key Stages in Digital Image Processing



Key Stages in Digital Image Processing: Image Aquisition

Image Restoration

Morphological Processing

Image Enhancement

Segmentation

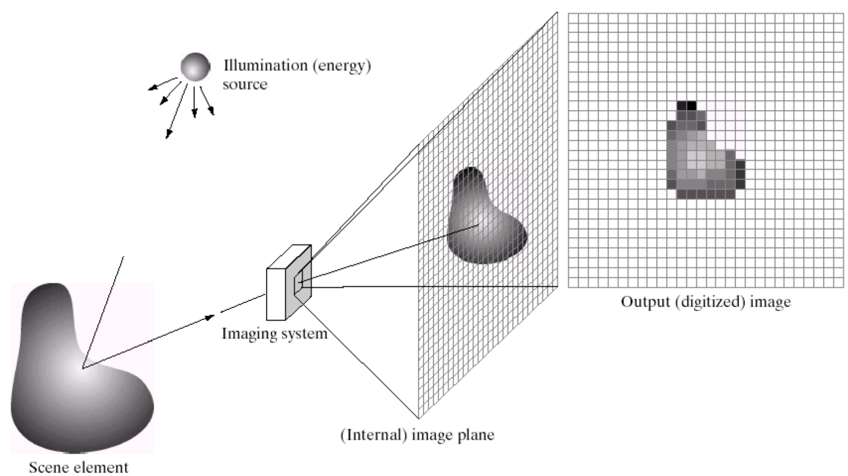


Image Acquisition

Object Recognition

Representation & Description

Problem Domain

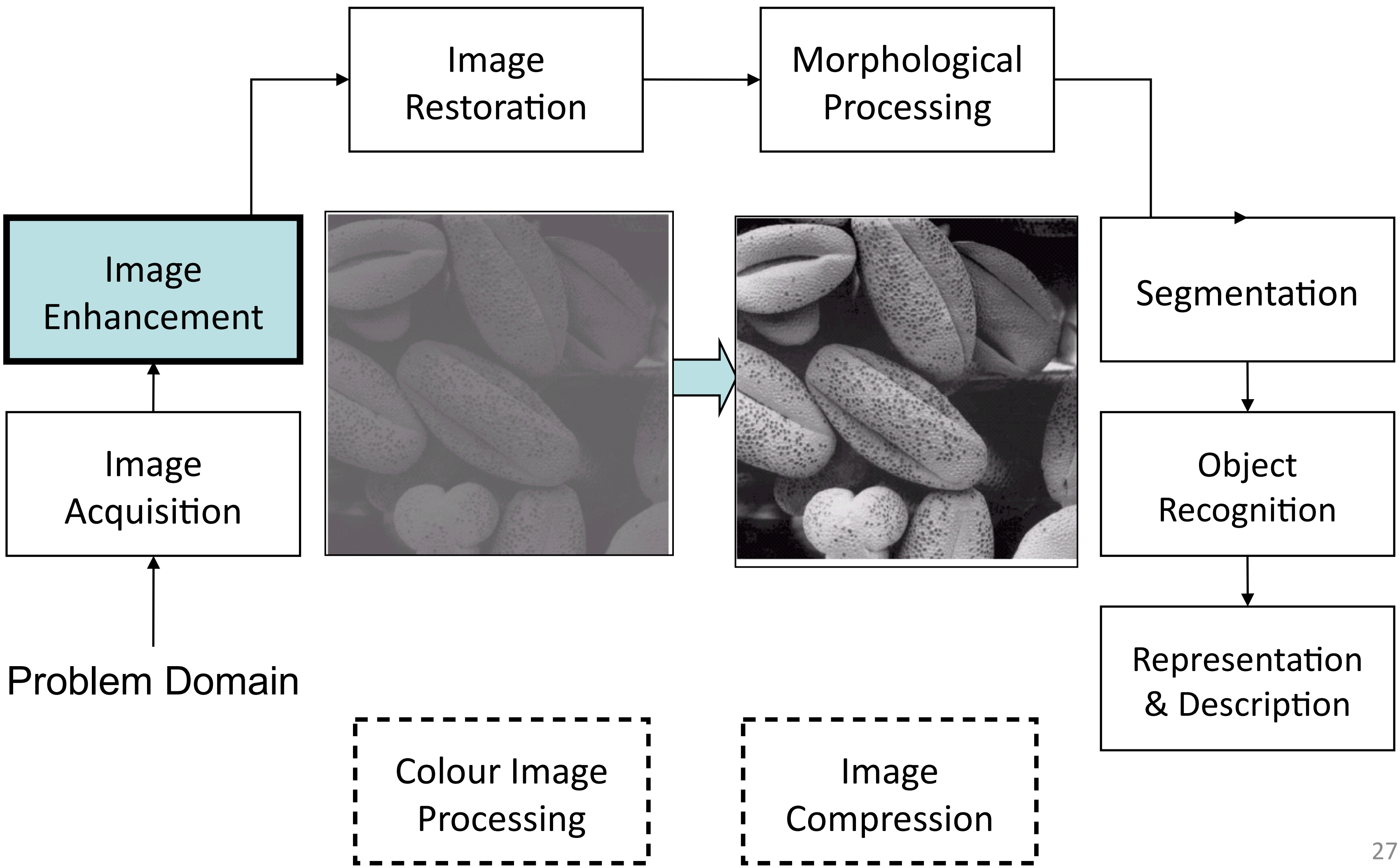
Colour Image Processing

Image Compression

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Key Stages in Digital Image Processing: Image Enhancement

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

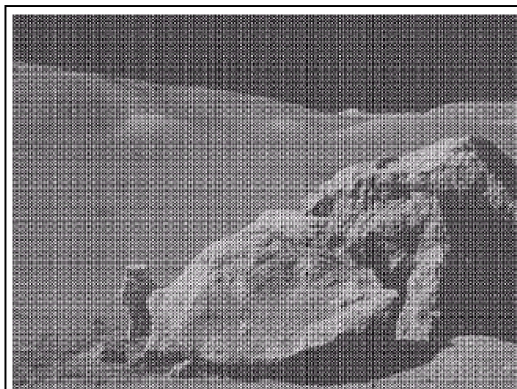


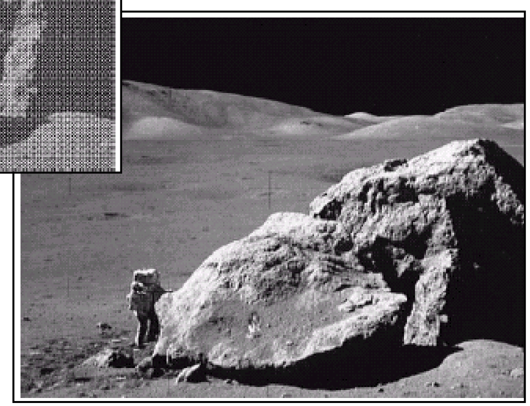
Key Stages in Digital Image Processing: Image Restoration

Image Restoration

Morphological Processing

Image Enhancement





Segmentation

Image Acquisition

Object Recognition

Problem Domain

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Key Stages in Digital Image Processing: Morphological Processing

Image Restoration

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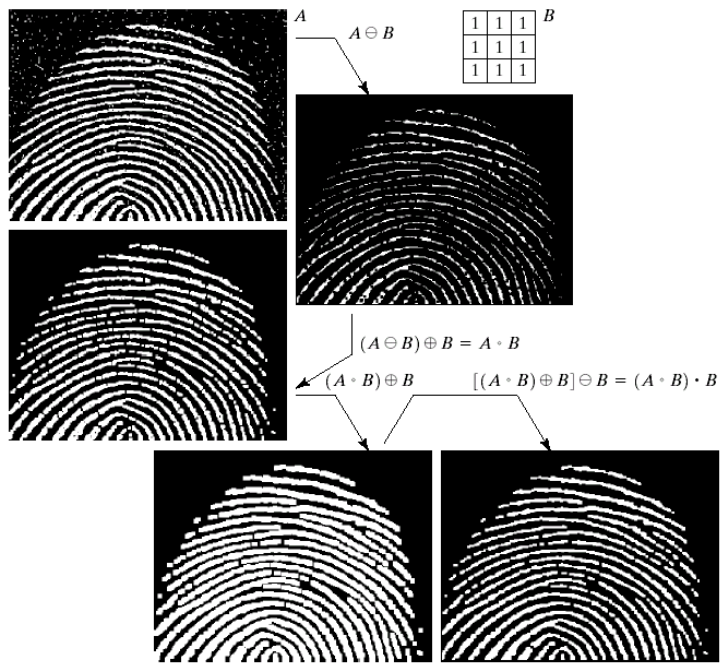

Colour Image Processing

Image Compression

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Key Stages in Digital Image Processing: Segmentation

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Image Restoration

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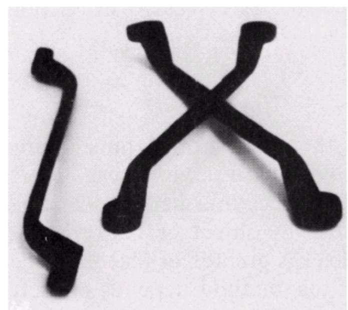
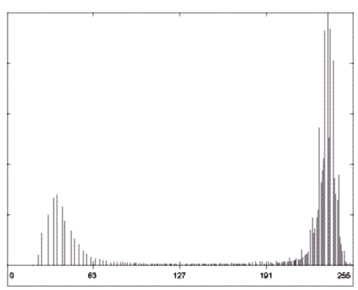
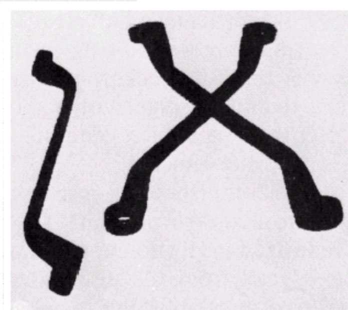
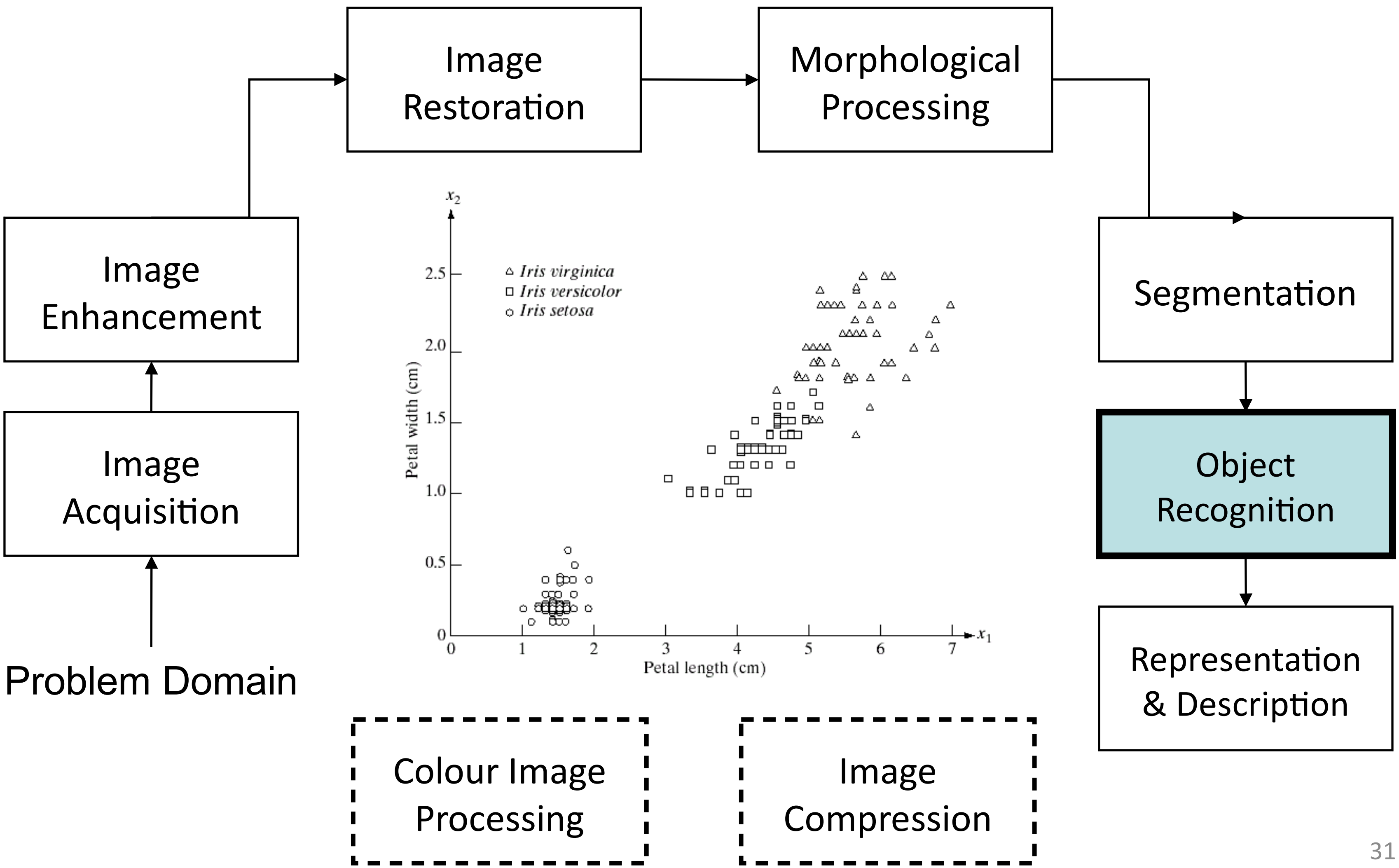




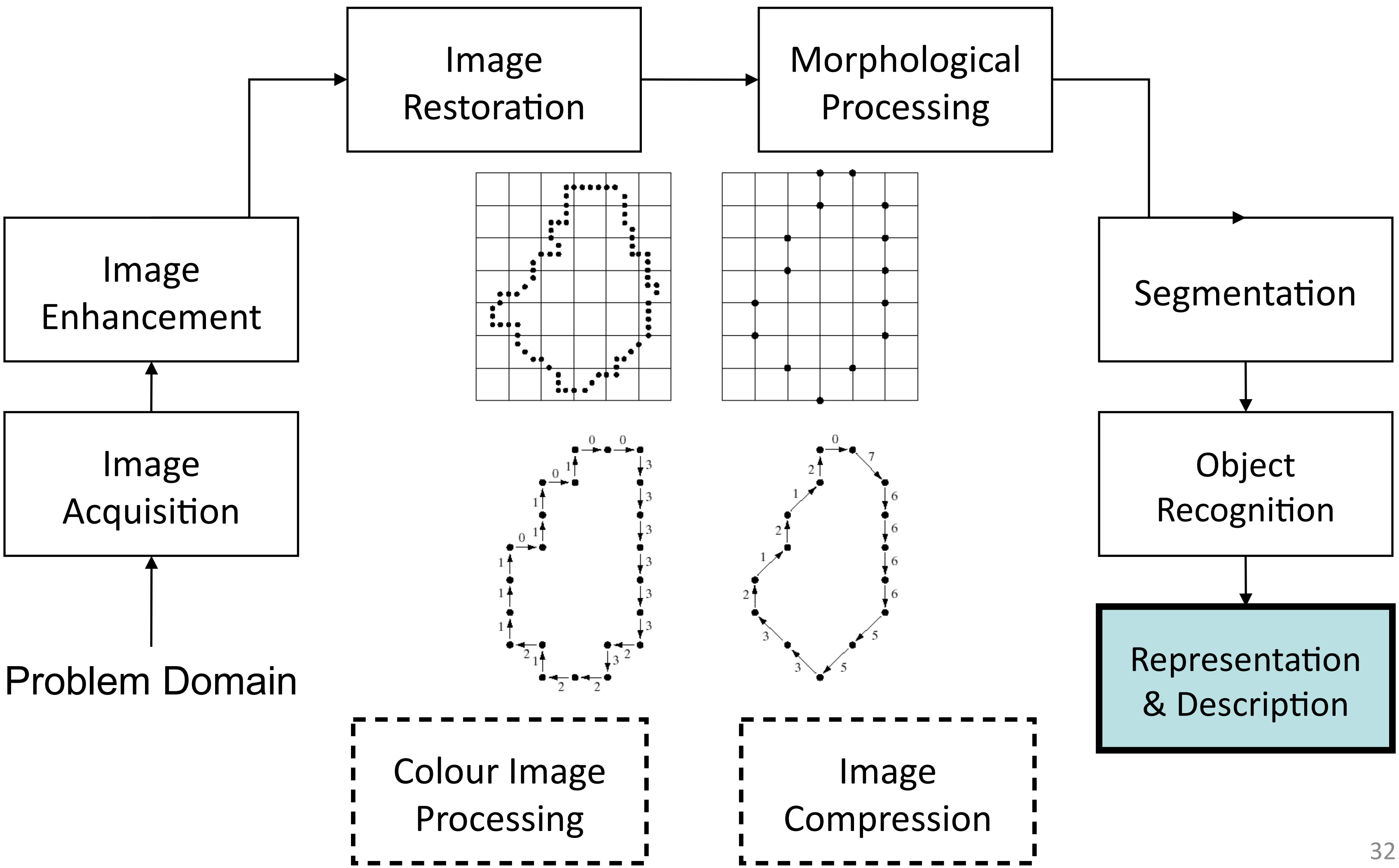


Key Stages in Digital Image Processing: Object Recognition



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Key Stages in Digital Image Processing: Representation & Description



Key Stages in Digital Image Processing: Image Compression

Image Restoration

Morphological Processing

Image Enhancement

Segmentation

Image Acquisition

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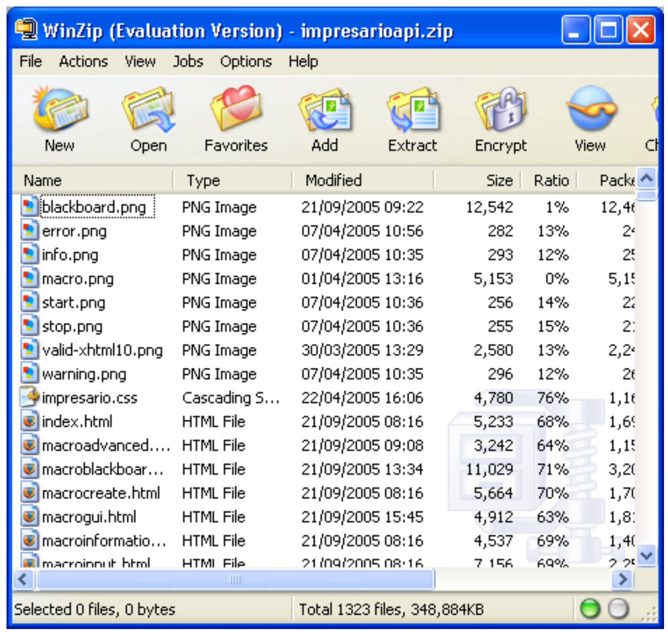
Representation & Description

Problem Domain



Colour Image Processing

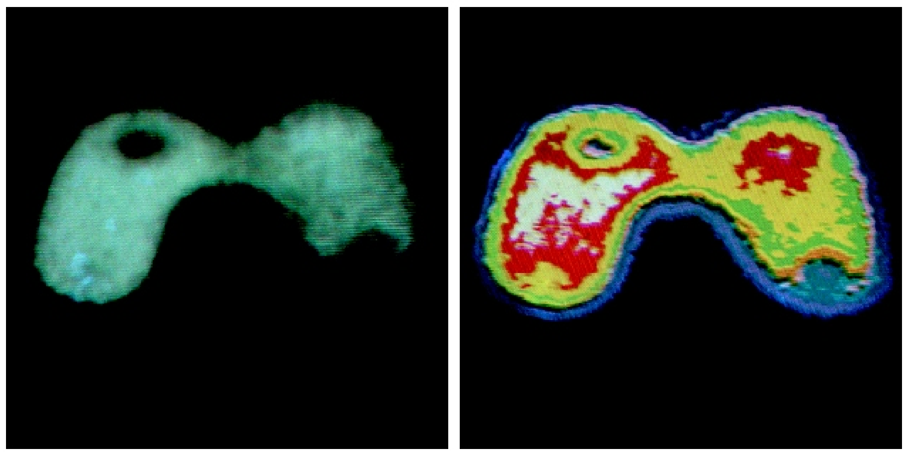
Image Compression

Key Stages in Digital Image Processing: Colour Image Processing

Image Restoration

Morphological Processing

Image Enhancement



Segmentation

Image Acquisition

Object Recognition

Problem Domain

Representation & Description

Colour Image Processing

Image Compression

Summary

We have looked at:

- What is a digital image?
- What is digital image processing?
- History of digital image processing
- State of the art examples of digital image processing
- Key stages in digital image processing

Contents

This lecture will cover:

Light and the electromagnetic spectrum

Image representation

Image sensing and acquisition

Sampling, quantisation and resolution

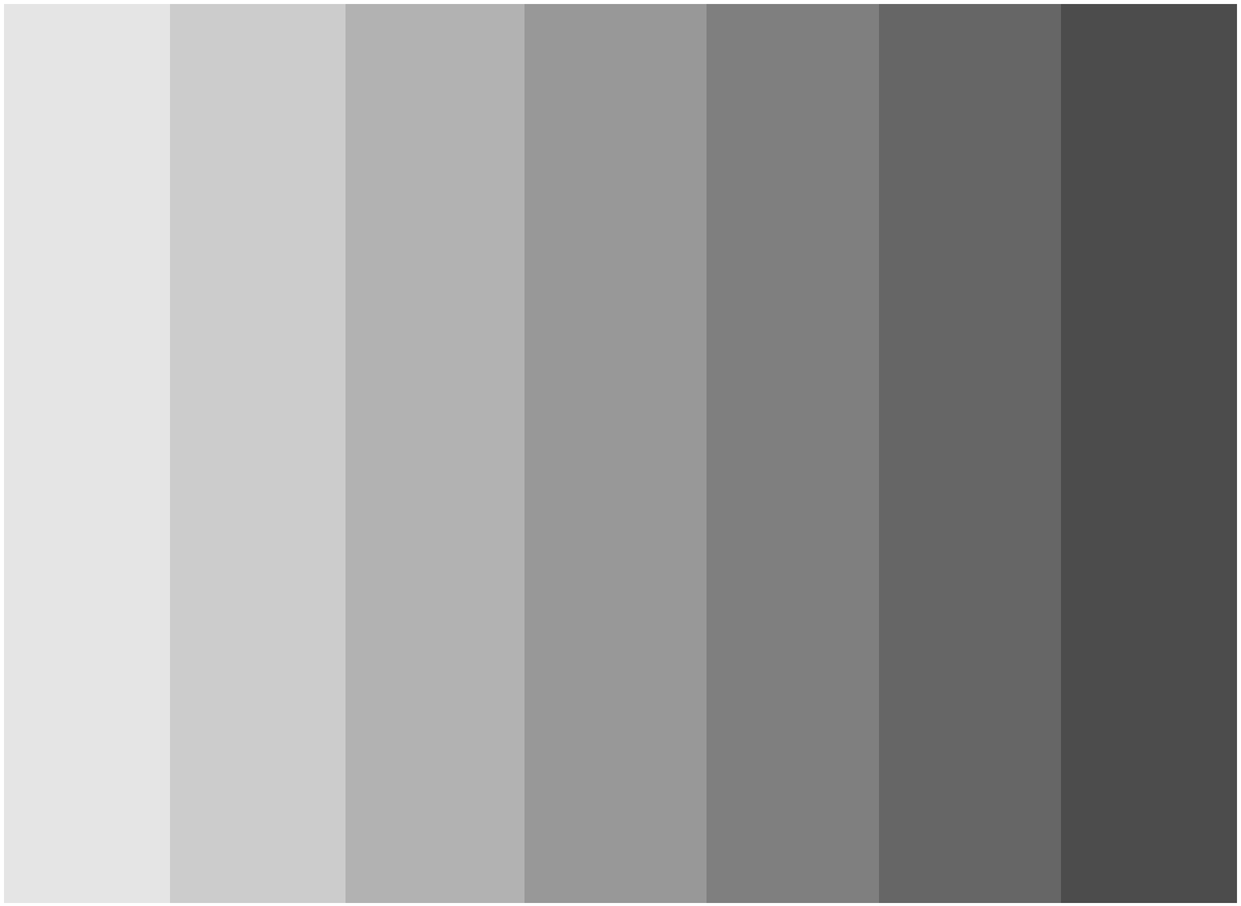
Brightness Adaptation & Discrimination

The human visual system can perceive approximately 10^{10} different light intensity levels

However, at any one time we can only discriminate between a much smaller number – *brightness adaptation*

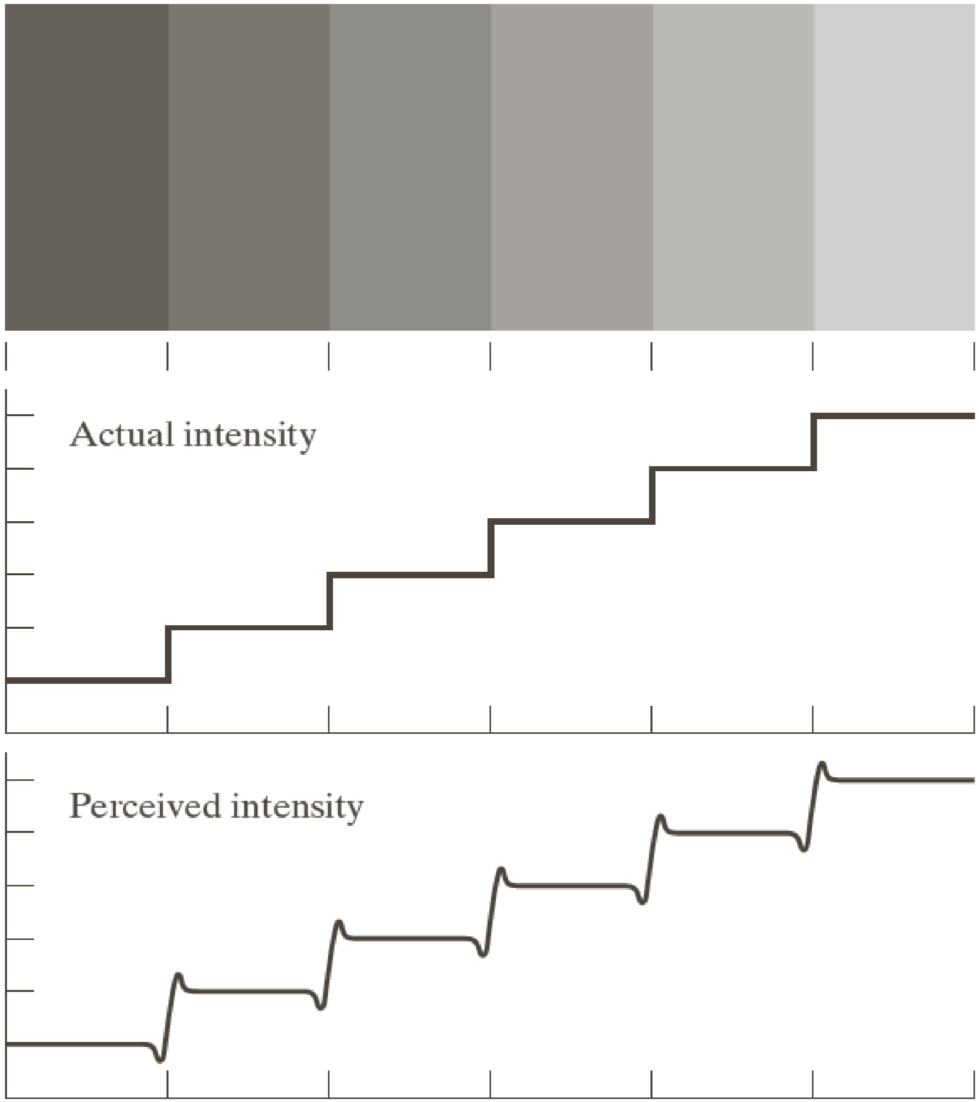
Similarly, the *perceived intensity* of a region is related to the light intensities of the regions surrounding it

Brightness Adaptation & Discrimination (cont…)



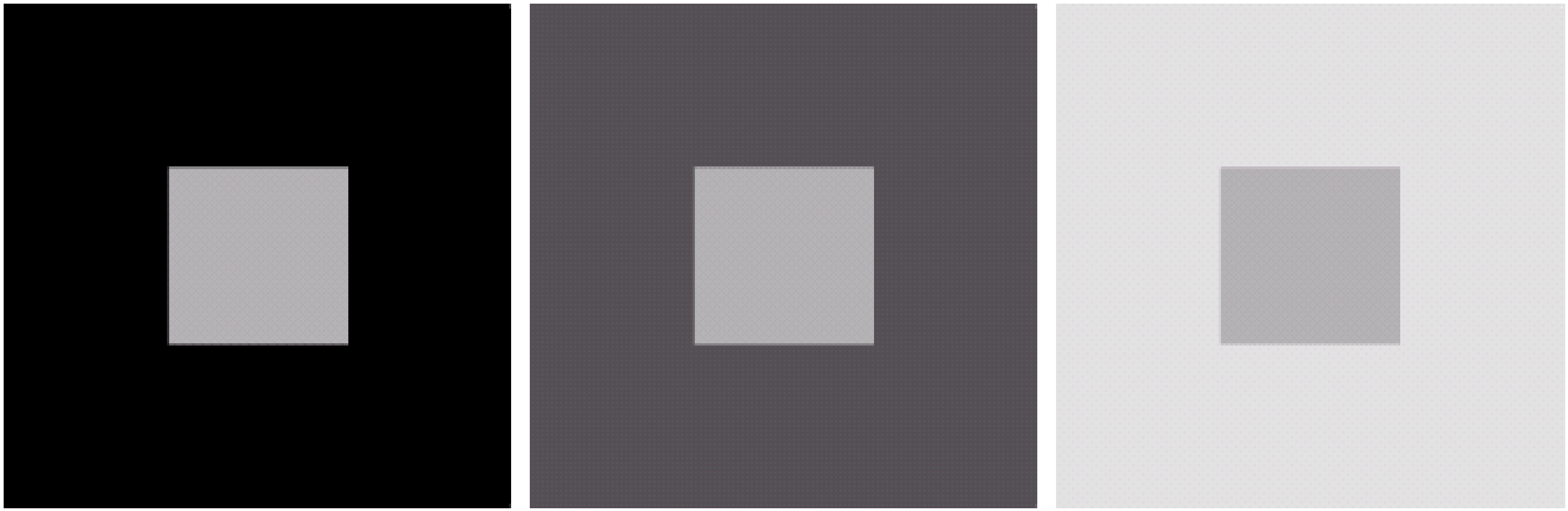
An example of Mach bands

Brightness Adaptation & Discrimination (cont…)



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Brightness Adaptation & Discrimination (cont…)

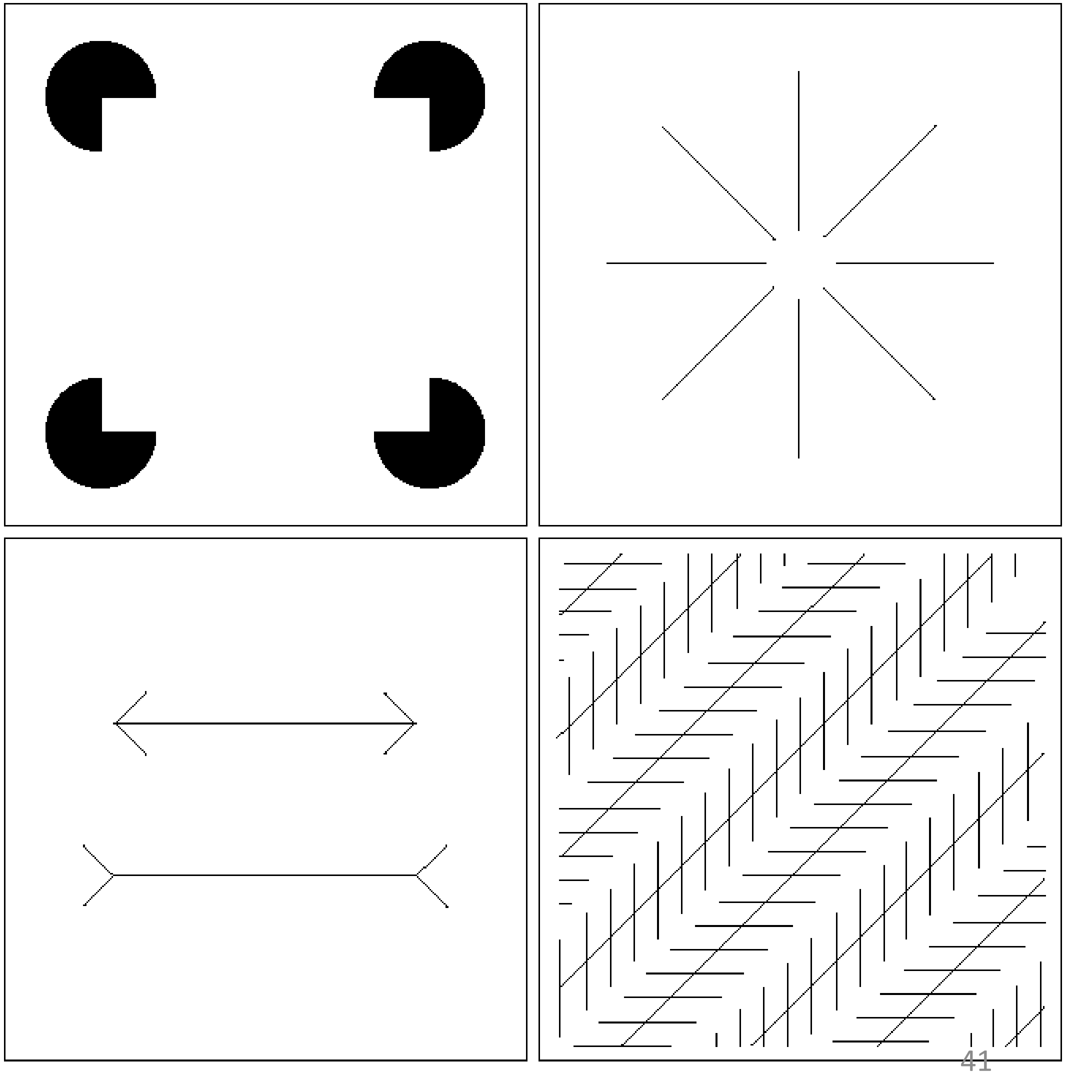


An example of *simultaneous contrast*

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

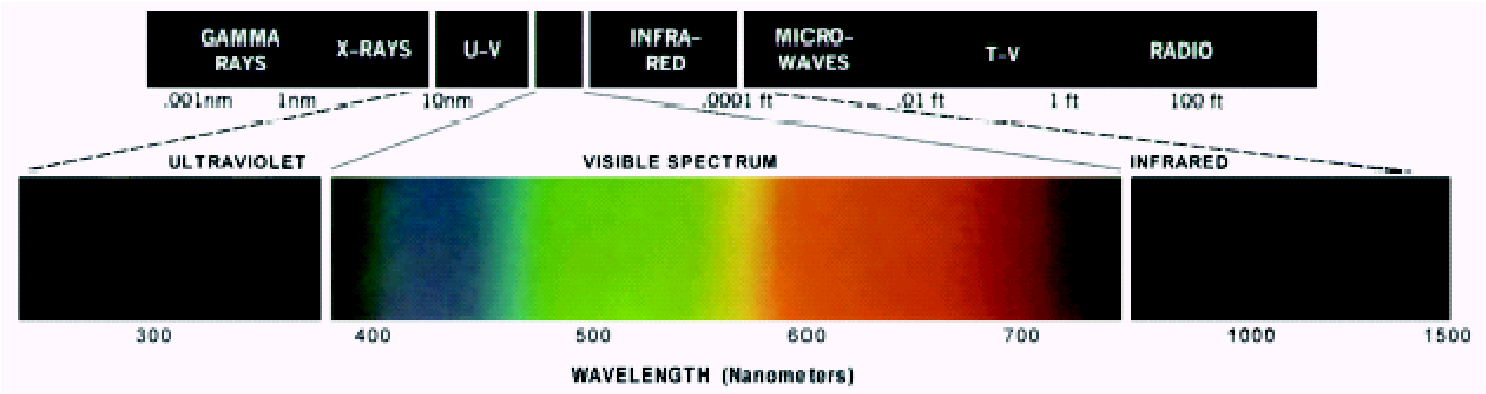
Optical Illusions

Our visual systems play lots of interesting tricks on us



Light And The Electromagnetic Spectrum

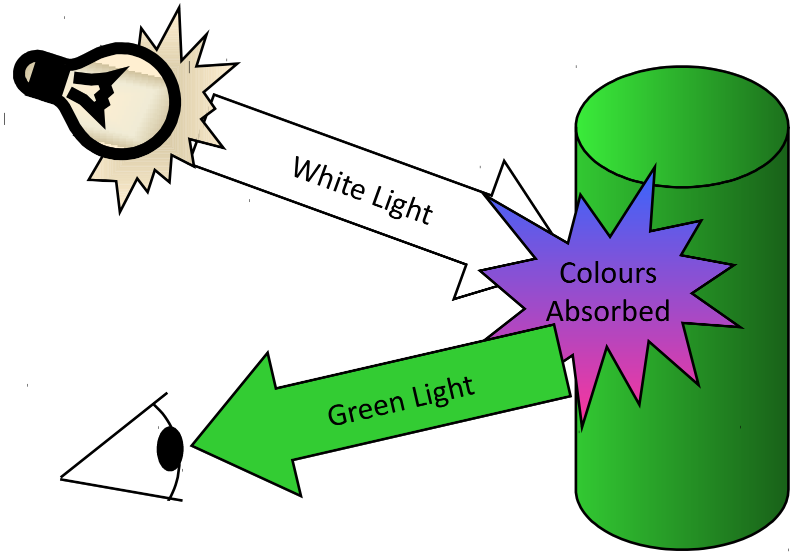
- Light is just a particular part of the electromagnetic spectrum that can be sensed by the human eye
- The electromagnetic spectrum is split up according to the wavelengths of different forms of energy



Reflected Light

The colours that we perceive are determined by the nature of the light reflected from an object

For example, if white light is shone onto a green object most wavelengths are absorbed, while green light is reflected from the object



Sampling, Quantisation And Resolution

In the following slides we will consider what is involved in capturing a digital image of a real-world scene

- Image sensing and representation
- Sampling and quantisation
- Resolution

Image Representation

Before we discuss image acquisition recall that a digital image is composed of M rows and N columns of pixels each storing a value

Pixel values are most often grey levels in the range 0-255(black-white)

We will see later on that images can easily be represented as matrices

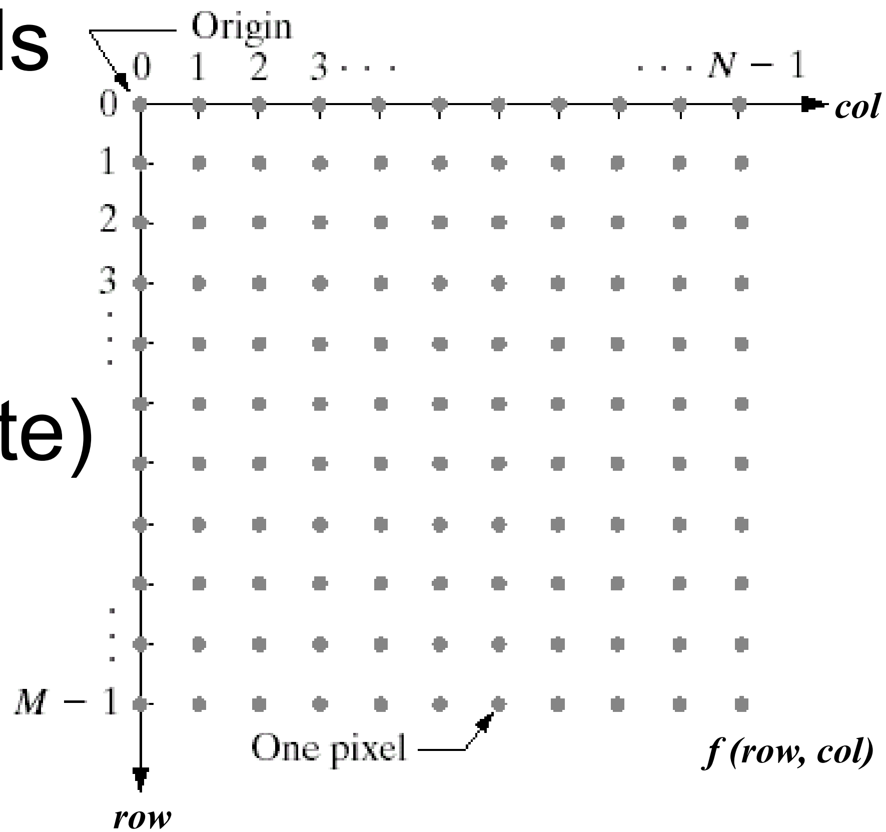
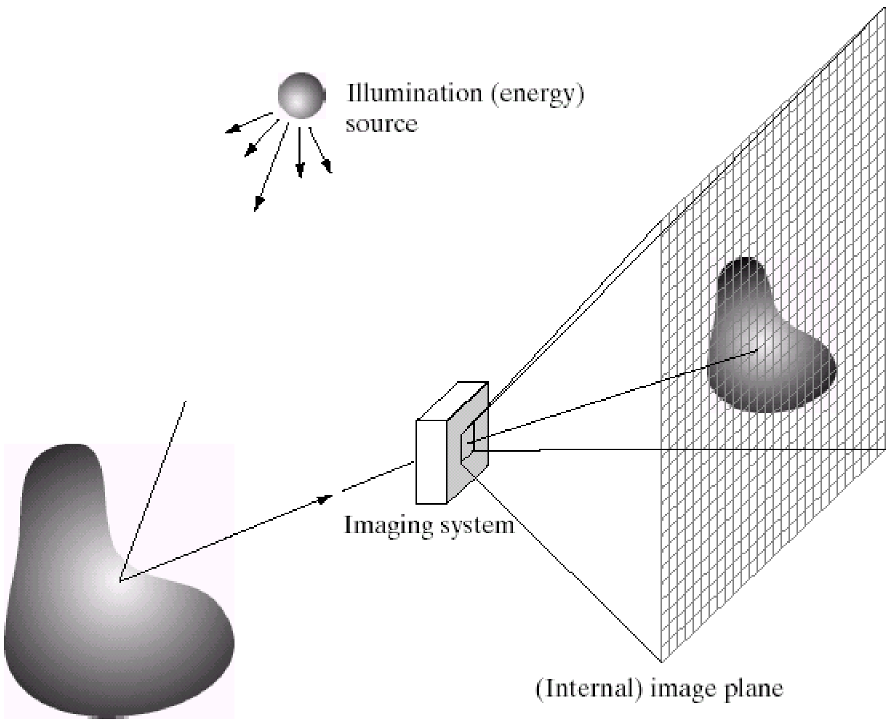


Image Acquisition

Images are typically generated by *illuminating* a *scene* and absorbing the energy reflected by the objects in that scene

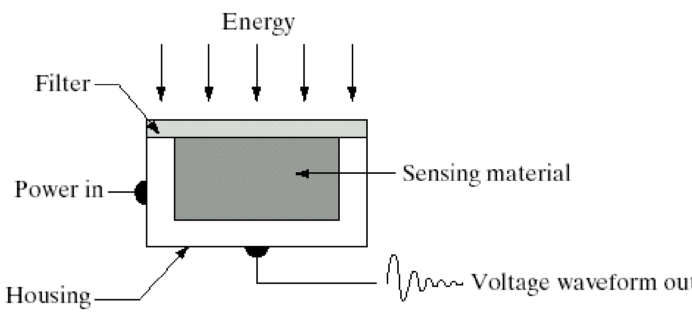


– Typical notions of illumination and scene can be way off:
  - X-rays of a skeleton
  - Ultrasound of an unborn baby
  - Electro-microscopic images of molecules

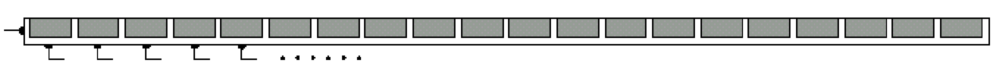
Image Sensing

Incoming energy lands on a sensor material responsive to that type of energy and this generates a voltage

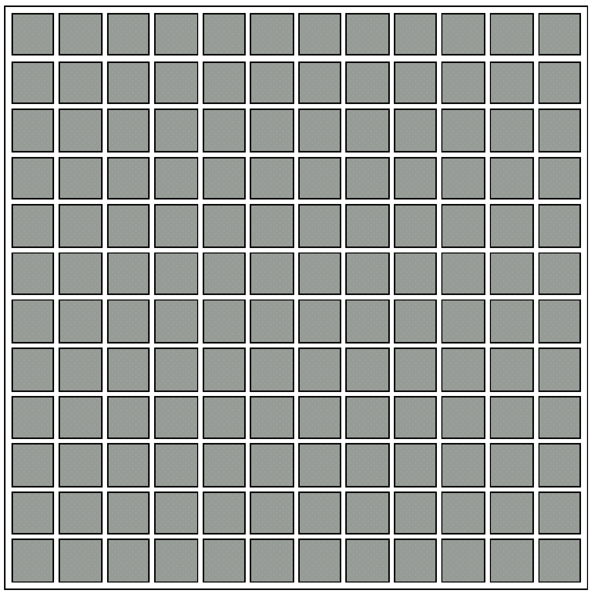
Collections of sensors are arranged to capture images



Imaging Sensor



Line of Image Sensors



Array of Image Sensors

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Image Sampling And Quantisation

A digital sensor can only measure a limited number of **samples** at a **discrete** set of energy levels

Quantisation is the process of converting a continuous **analogue** signal into a digital representation of this signal

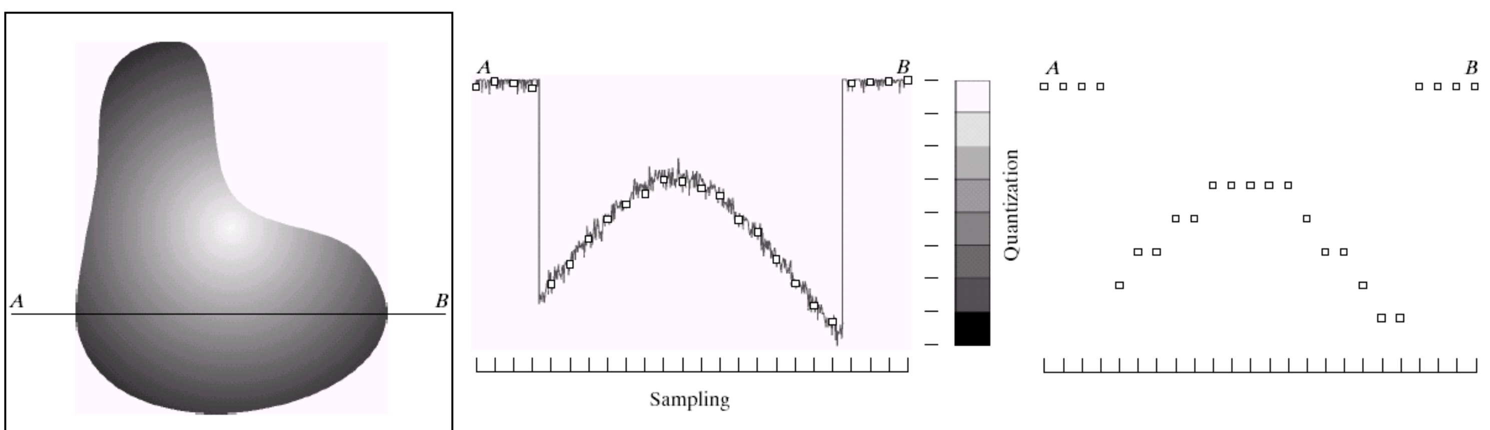
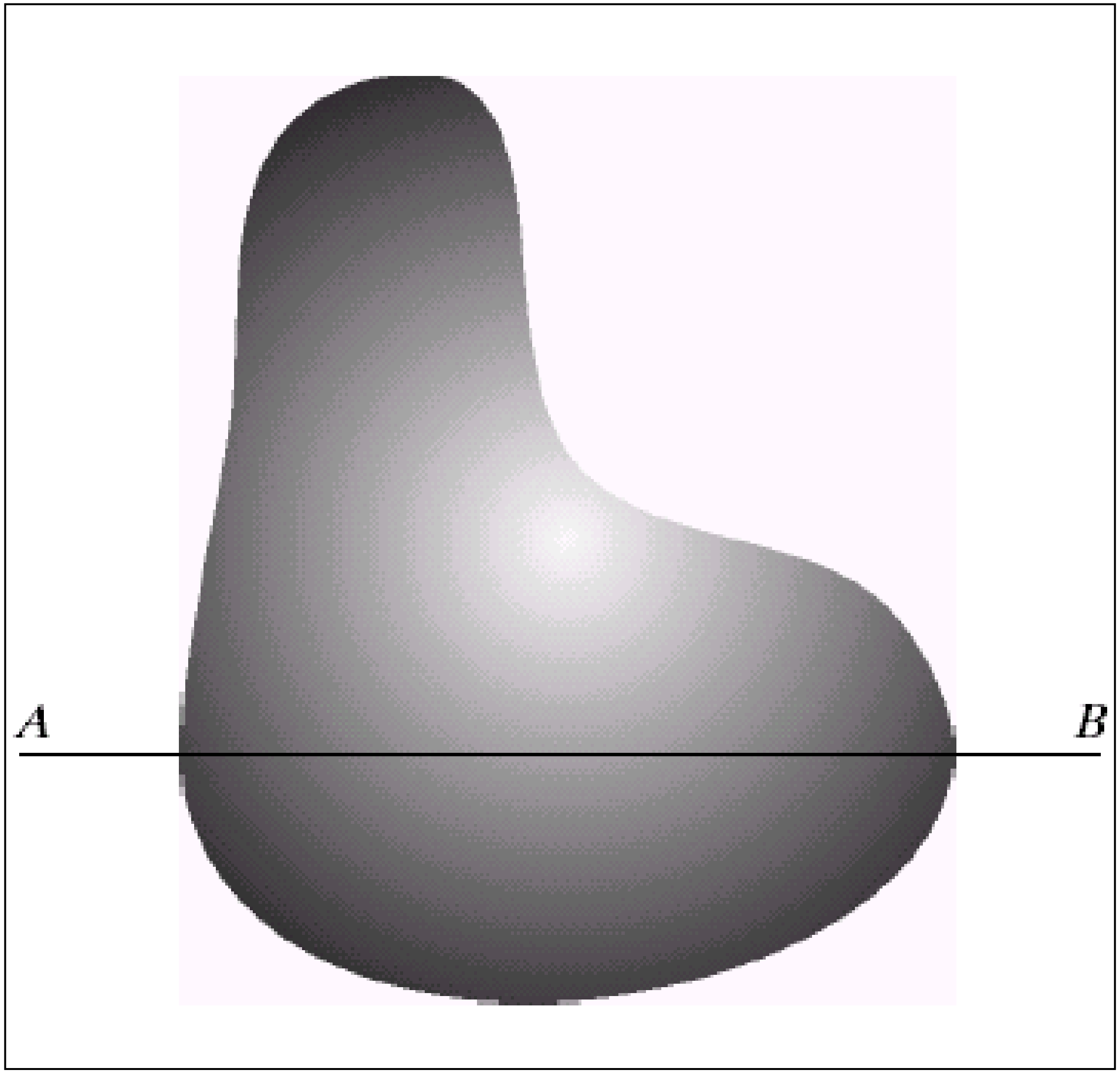


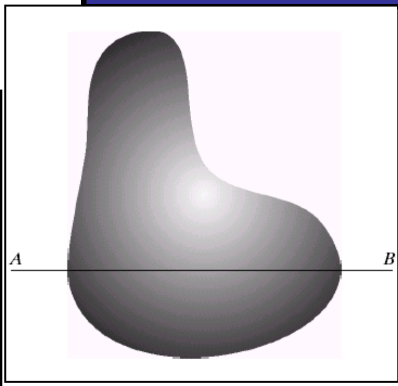
Image Sampling And Quantisation

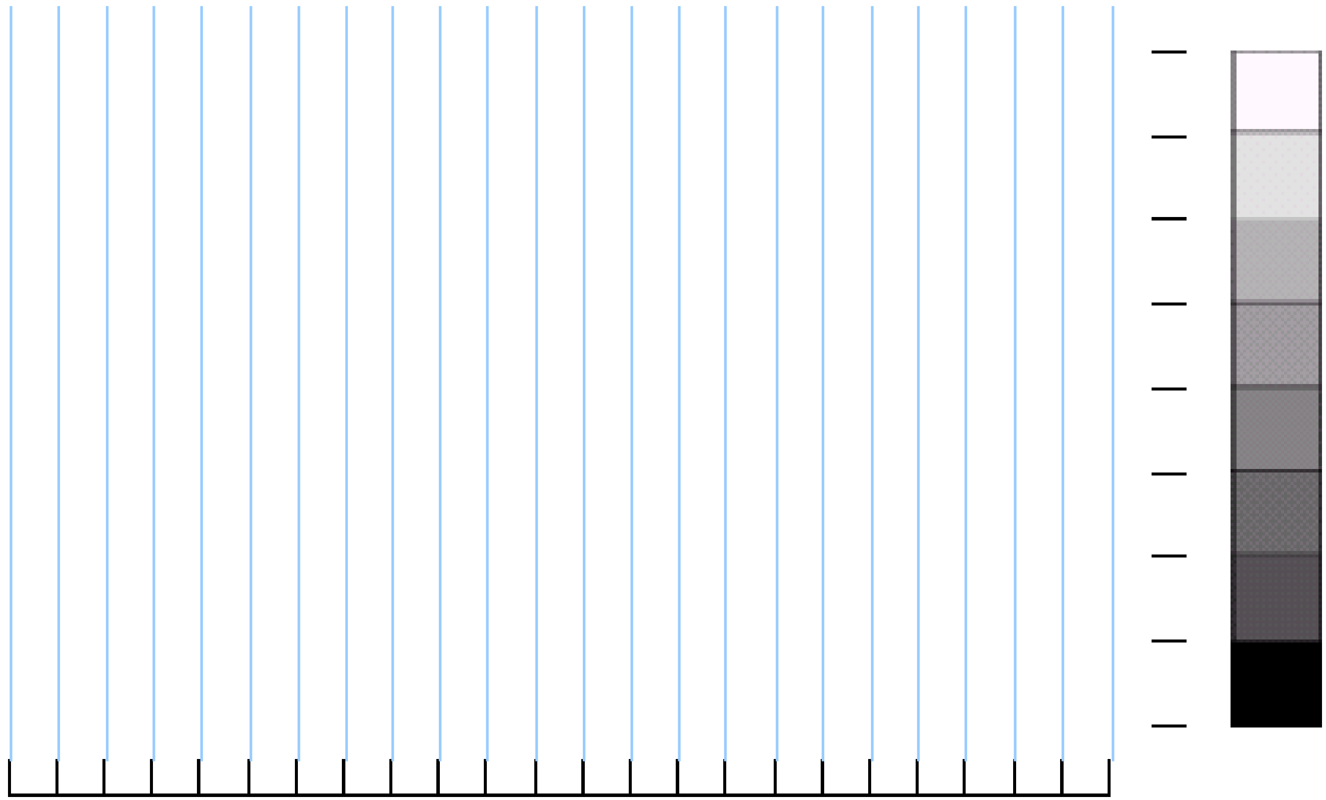


Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Image Sampling And Quantisation

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

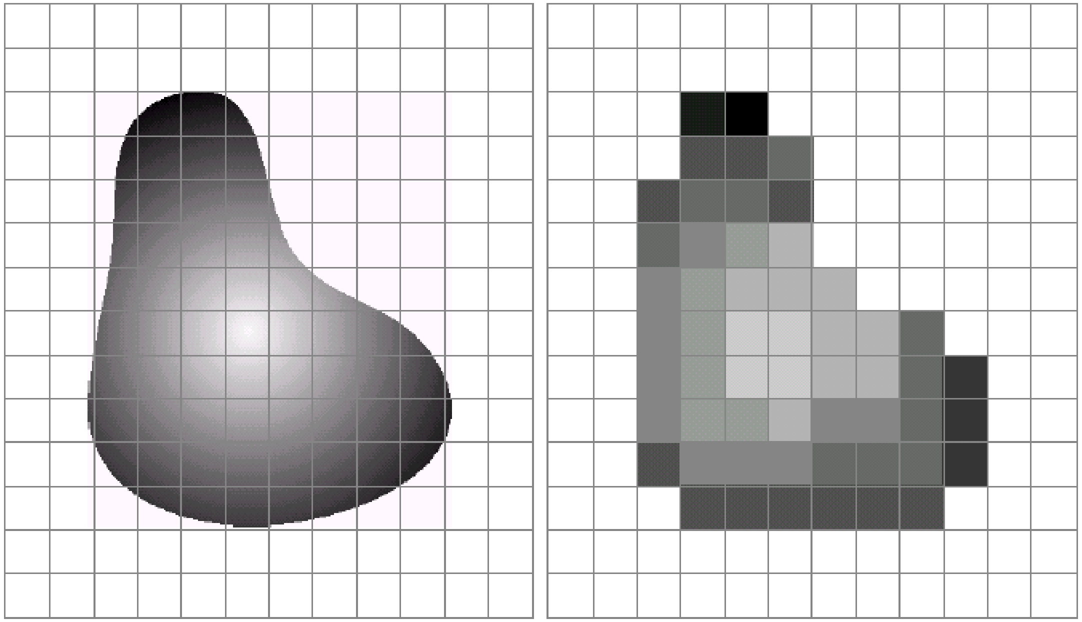




Sampling

Image Sampling And Quantisation (cont…)

Remember that a digital image is always only an **approximation** of a real world scene



Spatial Resolution

- *The spatial resolution* of an image is determined by how sampling was carried out
- Spatial resolution simply refers to the smallest discernable deta
- Vision specialists will often talk about pixel size
- Graphic designers will talk about *dots per inch* (DPI)

Spatial Resolution (cont…)

Spatial Resolution (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Spatial Resolution (cont…)

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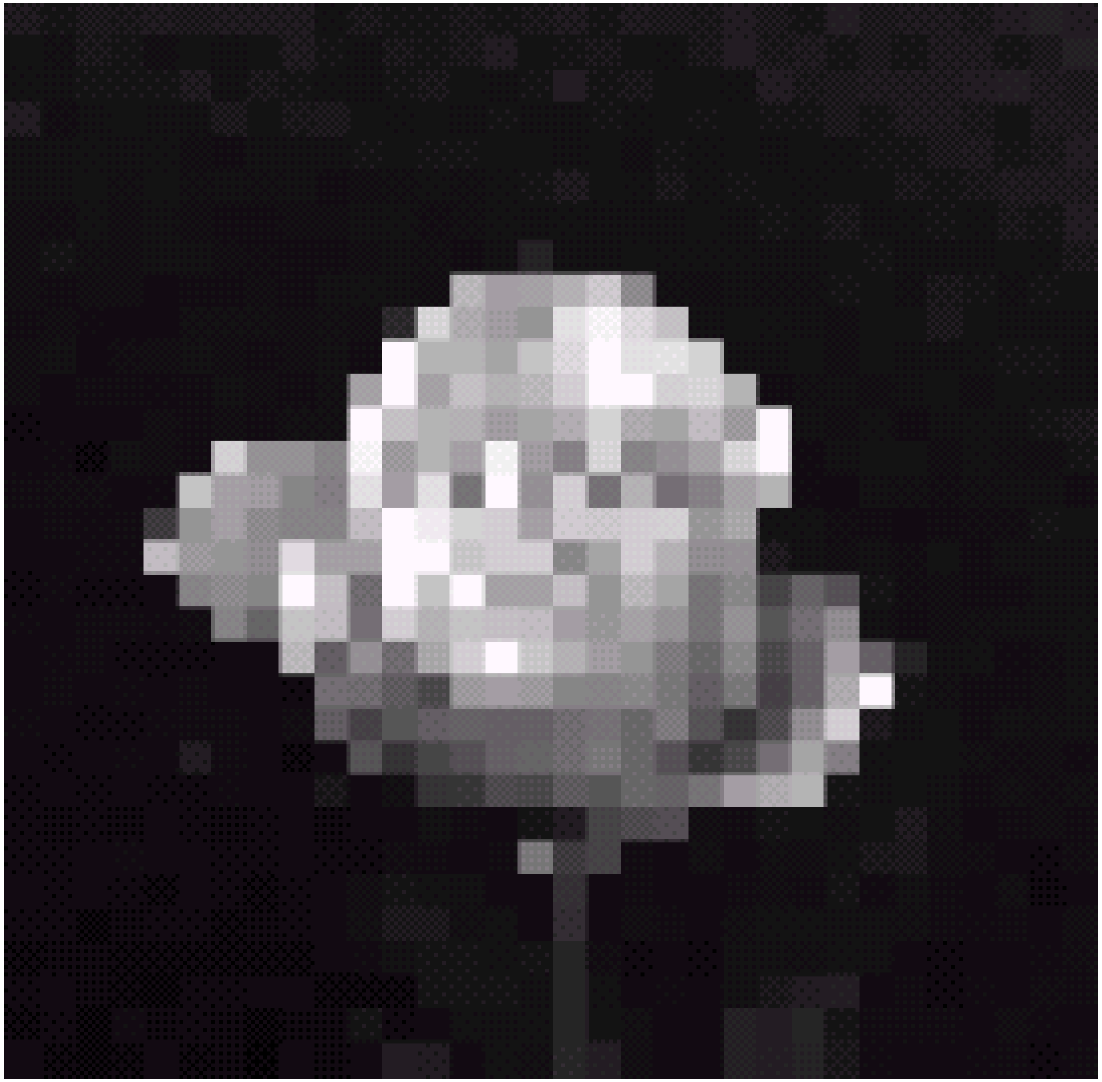


Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Intensity Level Resolution

Intensity level resolution refers to the number of intensity levels used to represent the image

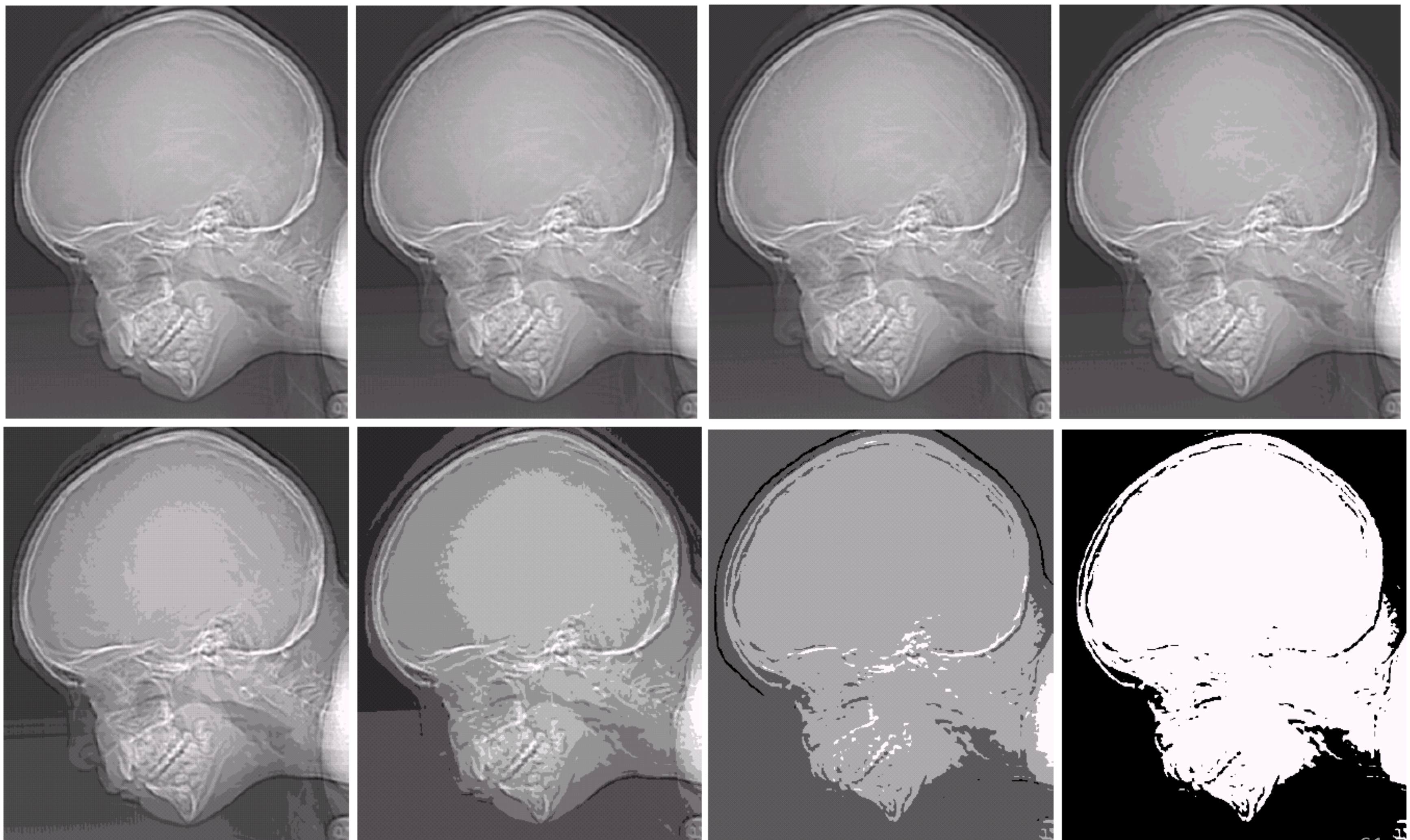
The more intensity levels used, the finer the level of detail discernable in an image

Intensity level resolution is usually given in terms of the number of bits used to store each intensity level

Number of Bits	Number of Intensity Levels	Examples
1	2	0, 1
2	4	00, 01, 10, 11
4	16	0000, 0101, 1111
8	256	00110011, 01010101
16	65,536	1010101010101010

Intensity Level Resolution (cont…)

256 grey levels (8 bits per pixel) | 128 grey levels (7 bpp) | 64 grey levels (6 bpp) | 32 grey levels (5 bpp)



16 grey levels (4 bpp) | 8 grey levels (3 bpp) | 4 grey levels (2 bpp) | 2 grey levels (1 bpp)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Intensity Level Resolution (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Intensity Level Resolution (cont…)

Intensity Level Resolution (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Intensity Level Resolution (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

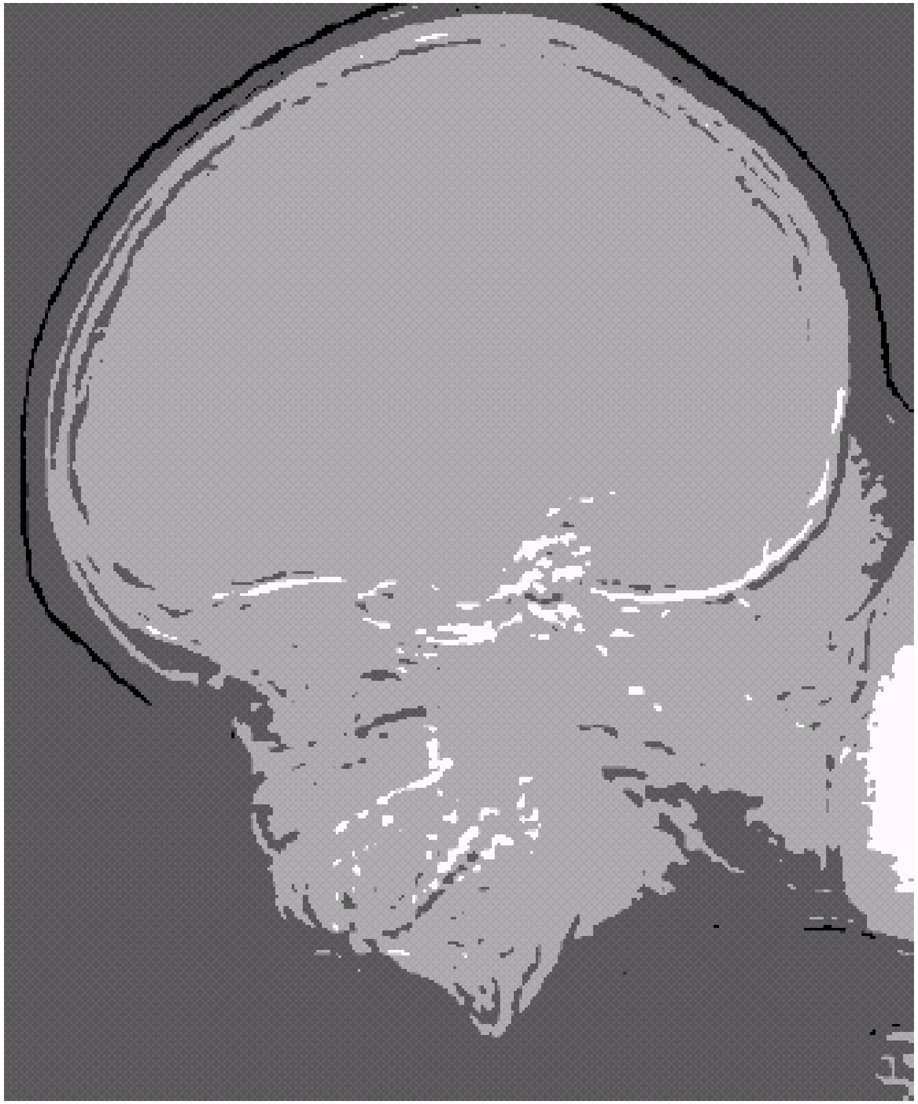
Intensity Level Resolution (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Intensity Level Resolution (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Intensity Level Resolution (cont…)

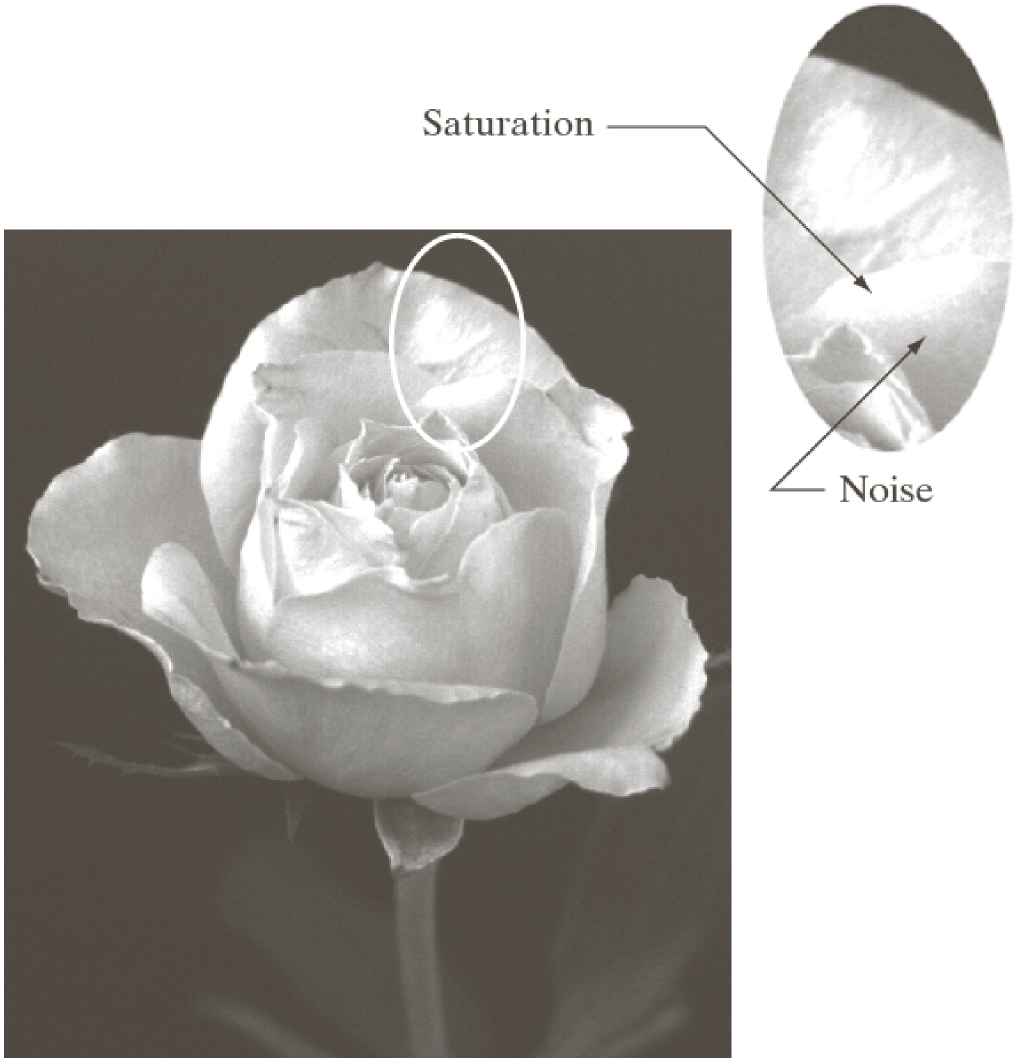


Intensity Level Resolution (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Saturation & Noise

Images taken from Gonzalez & Woods, Digital Image Processing (2002)



Resolution: How Much Is Enough?

The big question with resolution is always *how much is enough*?

This all depends on what is in the image and what you would like to do with it

Key questions include

- Does the image look aesthetically pleasing?
- Can you see what you need to see within the image?

Resolution: How Much Is Enough? (cont…)

The picture on the right is fine for counting the number of cars, but not for reading the number plate

Intensity Level Resolution (cont…)

Low Detail

Medium Detail

High Detail

Intensity Level Resolution (cont…)

Intensity Level Resolution (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Intensity Level Resolution (cont…)

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Summary

We have looked at:

Light and the electromagnetic spectrum

Image representation

Image sensing and acquisition

Sampling, quantisation and resolution

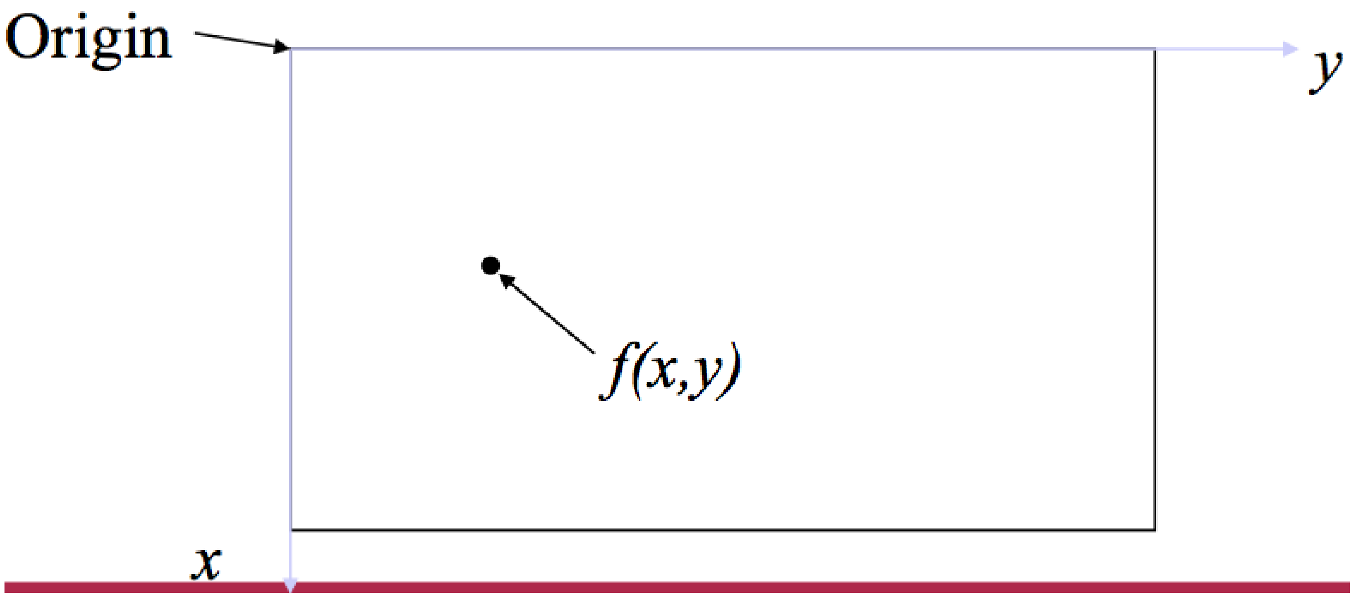
Next time we start to look at techniques for image enhancement

Digital image representation

Monochrome image (or simply image) refers to a 2- dimensional light intensity function $f(x,y)$

– x and y denote spatial coordinates

– the value of $f(x,y)$ at (x,y) is proportional to the brightness (or *gray level*) of the image at that point



Digital image representation

A digital image is an image *f(x,y)* that has been discretized both in spatial coordinates and in brightness

- Considered as a matrix whose row and column indices represent a point in the image
- The corresponding matrix element value represents the gray level at that point
- The elements of such an array are referred to as:
  - image elements
  - picture elements (pixels or pels)

Steps in image processing

The *problem domain* in this example consists of pieces of mail and the objective is to read the address on each piece

Step 1: image acquisition

– Acquire a digital image using an image sensor

- a monochrome or color TV camera: produces an entire image of the problem domain every 1/30 second
- a line-scan camera: produces a single image line at a time, motion past the camera produces a 2-dimensional image

– If not digital, an *analog-to-digital* conversion process is required

– The nature of the image sensor (and the produced image) are determined by the application

- Mail reading applications rely greatly on line-scan cameras
- CCD and CMOS imaging sensors are very common in many applications

Steps in image processing

- Step 2: preprocessing
  - Key function: improve the image in ways that increase the chance for success of the other processes
  - In the mail example, may deal with contrast enhancement, removing noise, and isolating regions whose texture indicates a likelihood of alphanumeric information

Steps in image processing

- Step 3: segmentation
  - Broadly defined: breaking an image into its constituent parts
  - In general, one of the most difficult tasks in image processing
    - Good segmentation simplifies the rest of the problem
    - Poor segmentation make the task impossible
  - Output is usually raw pixel data: may represent region boundaries, points in the region itself, etc.
    - Boundary representation can be useful when the focus is on external shape characteristics (e.g. corners, rounded edges, etc.)
    - Region representation is appropriate when the focus is on internal properties (e.g. texture or skeletal shape)
  - For the mail problem (character recognition) both representations can be necessary

Steps in image processing

- Step 4: representation & description
  - Representation: transforming raw data into a form suitable for computer processing
  - Description (also called feature extraction) deals with extracting features that result in some quantitative information of interest or features which are basic for differentiating one class of objects from another
  - In terms of character recognition, *descriptors* such as lakes (holes) and bays help differentiate one part of the alphabet from another

Steps in image processing

- Step 5: recognition & interpretation

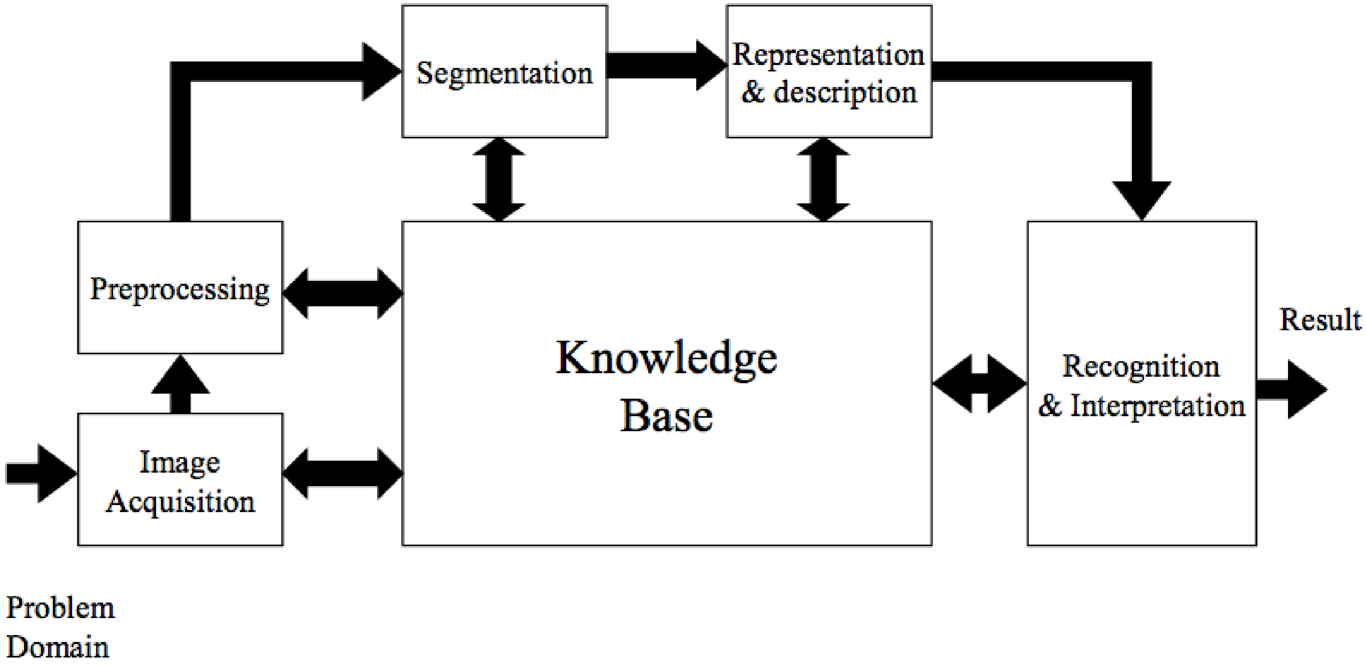
  – Recognition: The process which assigns a label to an object based on the information provided by its descriptors

  A may be the alphanumeric character A

  – Interpretation: Assigning meaning to an ensemble of recognized objects

  35487-0286 may be a ZIP code

Steps in image processing



A Knowledge Base

- Knowledge about a problem domain is coded into an image processing system in the form of a *knowledge database*
  - May be simple:
    - detailing areas of an image expected to be of interest
  - May be complex
    - A list of all possible defects of a material in a vision inspection system
  - Guides operation of each processing module
  - Controls interaction between modules
  - Provides feedback through the system

Sampling and Quantization

- To be suitable for computer processing an image, f(x,y) must be digitized both spatially and in amplitude
- Digitizing the spatial coordinates is called *image sampling*
- Amplitude digitization is called gray-level quantization
- f(x,y) is approximated by equally spaced samples in the form of an NxM array where each element is a discrete quantity

$$f(x,y) \approx \begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,M-1) \\ f(1,0) & f(1,1) & \dots & f(1,M-1) \\ . & & & \\ f(N-1,0) & f(N-1,1) & \dots & f(N-1,M-1) \end{bmatrix}$$

Sampling and Quantization

- Common practice is to let N and M be powers of two; N=2^n and M=2^k
- And G=2^m where G denotes the number of gray levels
- The assumption here is that gray levels are equally space in the interval [0,L]
- The number of bits, b, necessary to store the image is then

$$b = NxMxm$$

$$or\ if\ N = M$$

$$b = N^2 m$$

- For example, a 128x128 image with 64 gray levels would require 98,304 bits of storage

Sampling and Quantization

Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

- How many samples and gray levels are required for a "good" approximation?
- The resolution (the degree of discernible detail) depends strongly on these two parameters

Effects of reducing spatial resolution

256x256

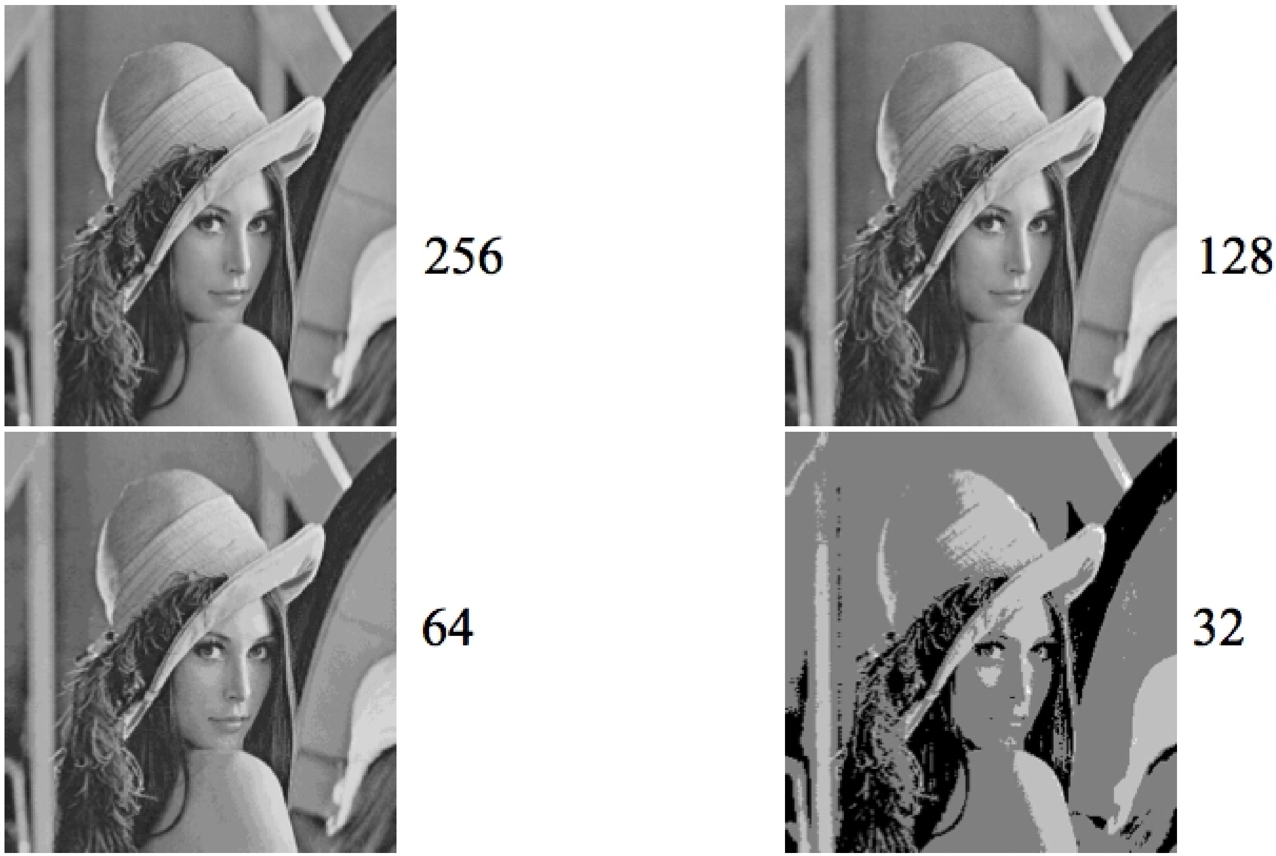
128x128

64x64

32x32

Pixel replication occurs as resolution is decreased

Effects of reducing gray levels



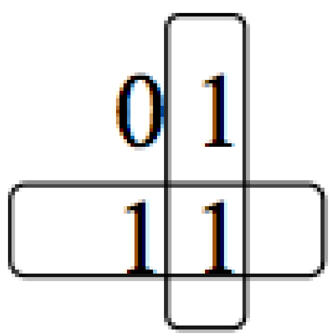
Ridgelike structures develop as gray level is decreased: false contours

Basic relationships between pixels

- An image is denoted by: $f(x,y)$
- Lowercase letters (e.g. *p, q*) will denote individual pixels
- A subset of $f(x,y)$ is denoted by *S*
- Neighbors of a pixel:
  - A pixel *p* at (x,y) has 4 horizontal/vertical neighbors at
    - $(x+1,y),(x-1,y),(x,y+1)$ and $(x,y-1)$
    - called the *4-neighbors* of *p*: $N_4(p)$
  - A pixel *p* at (x,y) has 4 diagonal neighbors at
    - $(x+1,y+1),(x+1,y-1),(x-1,y+1)$ and $(x-1,y-1)$
    - called the *diagonal–neighbors* of *p*: $N_D(p)$
  - The *4-neighbors* and the *diagonal-neighbors* of *p* are called the *8-neighbors* of *p*: $N_8(p)$

Connectivity between pixels

- Connectivity is an important concept in establishing boundaries of object and components of regions in an image
- When are two pixels connected?
  - If they are adjacent in some sense (say they are 4-neighbors)
  - and, if their gray levels satisfy a specified criterion of similarity (say they are equal)
- Example: given a binary image (e.g. gray scale = [0,1]), two pixels may be 4-neighbors but are not considered connected unless they have [illegible]ue



Connectivity between pixels

- Let V be the set of values used to determine connectivity
  - For example, in a binary image, $V=\{1\}$ for the connectivity of pixels with a value of 1
  - In a gray scale image, for the connectivity of pixels with a range of intensity values of, say, 32 to 64, it follows that $V=\{32,33,\ldots,63,64\}$
  - Consider three types of connectivity
    - *4-connectivity*: Pixels p and q with values from V are *4-connected* if q is in the set $N_4(p)$
    - *8-connectivity*: Pixels p and q with values from V are *8-connected* if q is in the set $N_8(p)$
    - *m-connectivity (mixed)*: Pixels p and q with values from V are *m-connected* if
      - q is in the set $N_4(p)$, *or*
      - q is in the set $N_D(p)$ *and* the set $N_4(p) \cap N_4(q)$ is empty (This is the set of pixels that are 4-neighbors of p and q and whose values are from V)

Connectivity between pixels

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

An arrangement
of pixels

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-connectivity of
the pixels
V={1}

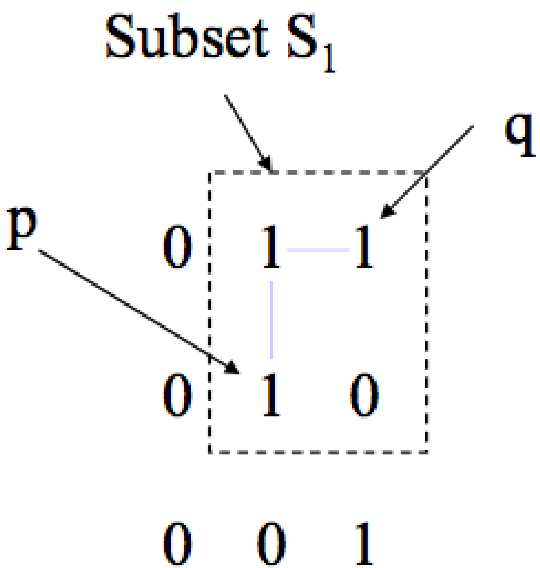
$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

m-connectivity of
the pixels
V={1}

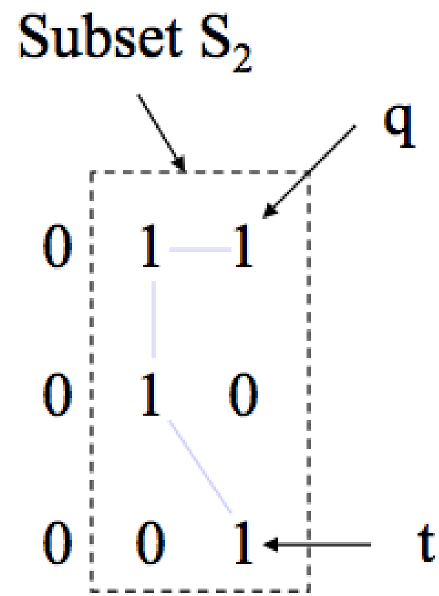
Pixel adjacencies and paths

- Pixel *p* is adjacent to *q* if they are connected
  - We can define 4-, 8-, or m-adjacency depending on the specified type of connectivity
- Two image subsets S_1 and S_2 are adjacent if some pixel in S_1 is adjacent to S_2
- A path from *p* at (x,y) to *q* at (s,t) is a sequence of distinct pixels with coordinates (x_0,y_0), (x_1,y_1),....., (x_n,y_n)
  - Where $(x_0,y_0)=(x,y)$ and $(x_n,y_n)=(s,t)$ and
  - (x_i,y_i) is adjacent to (x_{i-1},y_{i-1}) for $1 <= i <= n$
  - n is the *length* of the path
- If *p* and *q* are in *S*, then *p* is connected to *q* in *S* if there is a path from *p* to *q* consisting entirely of pixels in *S*

Example paths



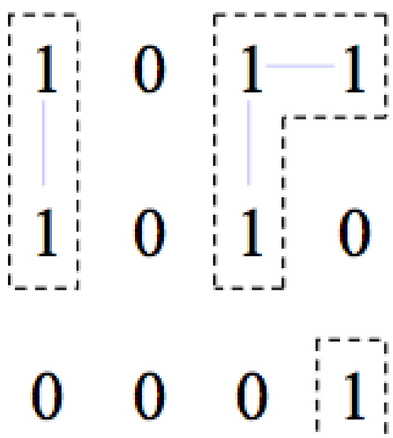
A 4-connected path from p to q (n=2). p and q are connected in S_1



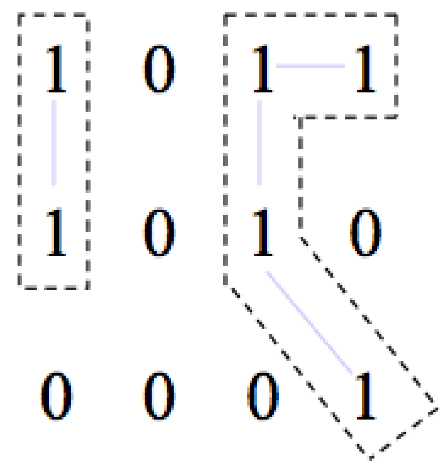
An m-connected path from t to q (n=3). t and q are connected in S_2

Connected components

- For any pixel *p* in *S*, the set of pixels connected to *p* form a connected component of *S*
- Distinct connected components in *S* are said to be *disjoint*



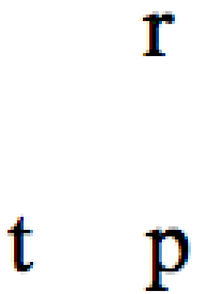
3 4-connected components of S



2 m-connected components of S

Labeling 4-connected components

- Consider scanning an image pixel by pixel from left to right and top to bottom



  - Assume, for the moment, we are interested in 4-connected components
  - Let *p* denote the pixel of interest, and *r* and *t* denote the upper and left neighbors of *p*, respectively
  - The nature of the scanning process assures that *r* and *t* have been encountered (and labeled if 1) by the time *p* is encountered

Labeling 4-connected components

- Consider the following procedure

if p=0 continue to the next position

if r=t=0 assign a new label to p (Ln)

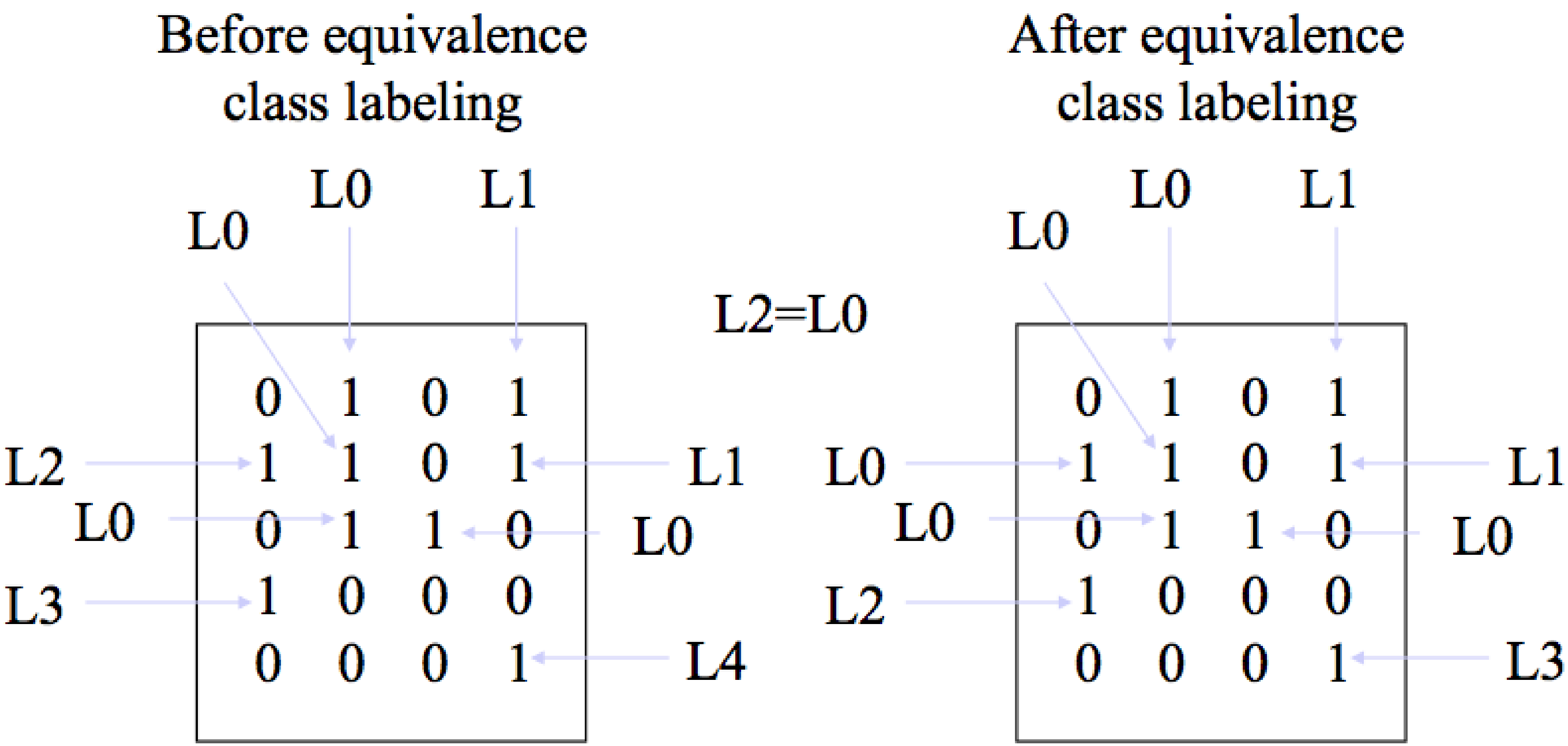
if r=t=1 and they have the same label, assign that label to p

if only one of r and t are 1, assign its label to p

if r=t=1 and they have different labels, assign one label to p and note that the two labels are equivalent (that is r and t are connected through p)

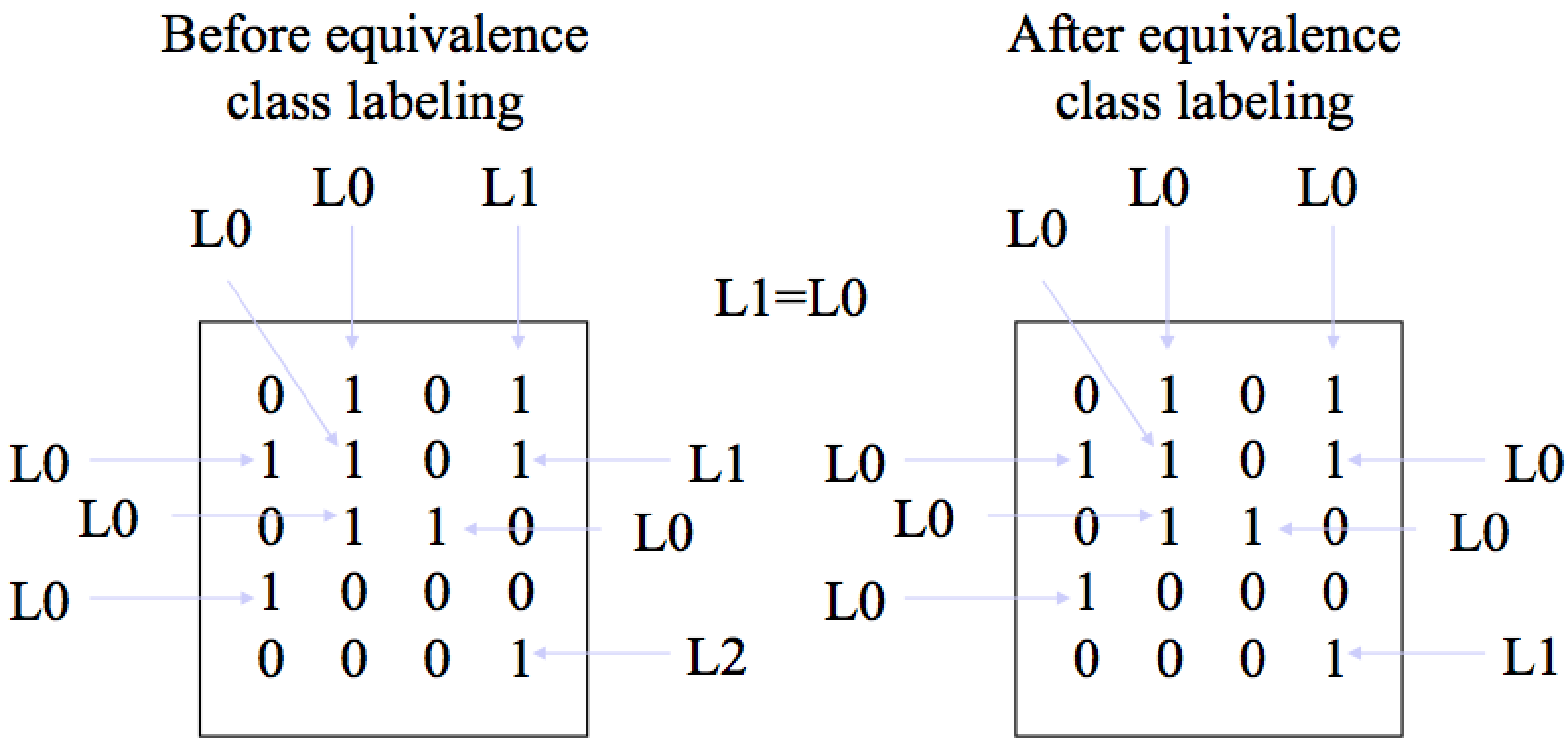
At the end of the scan, sort pairs of equivalent labels into equivalence classes and assign a different label to each class

Labeling 4-connected components (example)



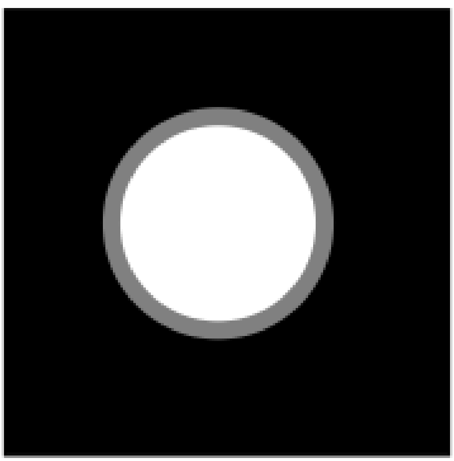
Labeling 8-connected components

- Proceed as in the 4-connected component labeling case, but also examine two upper diagonal neighbors (*q* and *s*) of *p*



Labeling connected components in non-binary images

- The 4-connected or 8-connected labeling schemes can be extended to gray level images
- The set *V* may be used to connect into a component only those pixels within a specified range of pixel values



000	000	000	000	000
000	000	128	000	000
000	128	255	128	000
000	128	255	128	000
000	000	128	000	000
000	000	000	000	000

	L0	
L0	L1	L0
L0	L1	L0
	L0	

Distance measures

- Given pixels *p*, *q*, and *z* at (*x,y*), (*s,t*) and (*u,v*) respectively,
- *D* is a *distance function* (or *metric*) if:
  - $D(p,q) \geq 0$ ($D(p,q)=0$ iff $p=q$),
  - $D(p,q) = D(q,p)$, and
  - $D(p,z) \leq D(p,q) + D(q,z)$.
- The *Euclidean distance* between *p* and *q* is given by:

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

- The pixels having distance less than or equal to some value *r* from (x,y) are the points contained in a disk of radius *r* centered at (x,y)

Distance measures

- The D_4 distance (also called the *city block distance*) between *p* and *q* is given by:

$$D_4(p,q) = |x - s| + |y - t|$$

- The pixels having a D_4 distance less than some *r* from (x,y) form a diamond centered at (x,y)
- Example: pixels where $D_4 \le 2$

$$\begin{array}{ccccc} & & 2 & & \\ & 2 & 1 & 2 & \\ 2 & 1 & 0 & 1 & 2 \\ & 2 & 1 & 2 & \\ & & 2 & & \end{array}$$

Note: Pixels with D_4=1 are the 4-neighbors of (x,y)

Distance measures

- The D_8 distance (also called the *chessboard distance*) between *p* and *q* is given by:

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

- The pixels having a D_8 distance less than some *r* from (x,y) form a square centered at (x,y)
- Example: pixels where $D_8 \leq 2$

$$\begin{matrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 2 & 2 & 2 & 2 \end{matrix}$$

Note: Pixels with $D_8=1$ are the 8-neighbors of (x,y)

Distance measures and connectivity

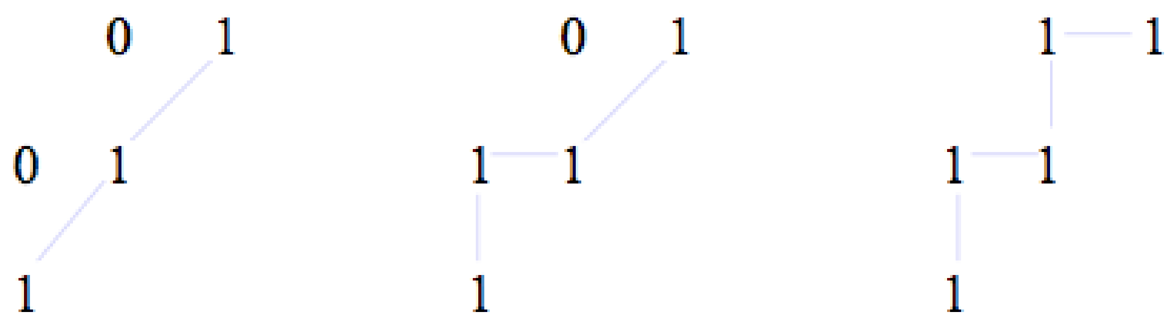
- The D_4 distance between two points *p* and *q* is the shortest 4-path between the two points
- The D_8 distance between two points *p* and *q* is the shortest 8-path between the two points
- D_4 and D_8 may be considered, regardless of whether a connected path exists between them, because the definition of these distances involves only the pixel coordinates
- For *m-connectivity*, the value of the distance (the length of the path) between two points depends on the values of the pixels along the path

Distance measures and m-connectivity

	p_3	p_4
p_1	p_2	
p		

- Consider the given arrangement of pixels and assume
  - p, p_2 and p_4 =1
  - p_1 and p_3 can be 0 or 1
- If V={1} and p_1 and p_3 are 0, the m-distance (p, p_4) is 2 If either p_1 or p_3 are 1, the m-distance (p, p_4) is 3 If p_1 and p_3 are 1, the m-distance (p, p_4) is 4

M-connectivity example



m-distance(p,p_4)=2 *m-distance*(p,p_4)=3 *m-distance*(p,p_4)=4

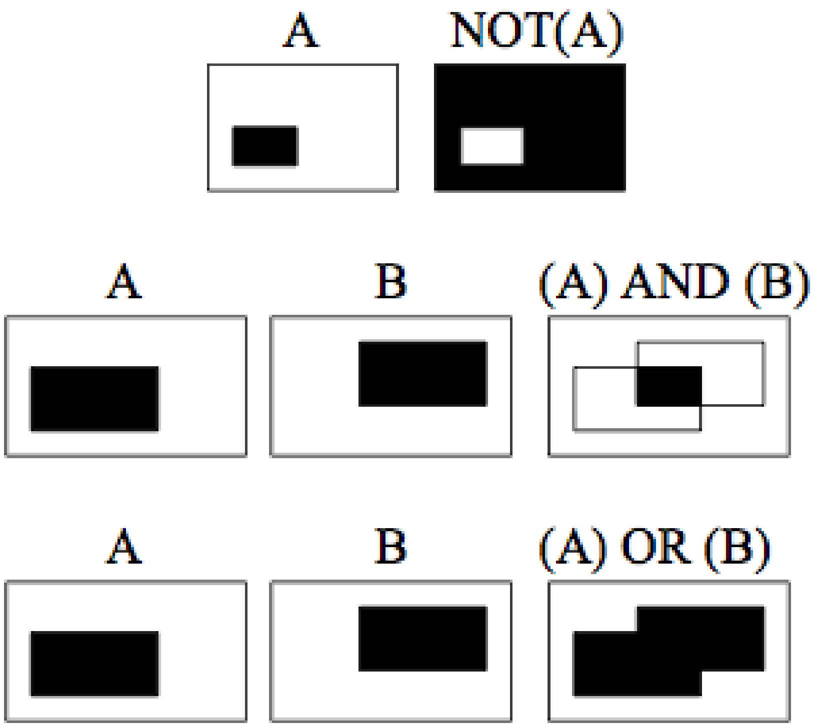
Arithmetic & logic operations

- Arithmetic & logic operations on images used extensively in most image processing applications
  - May cover the entire image or a subset Arithmetic operation between pixels *p* and *q* are defined as:
  - Addition: ($p+q$)
    - Used often for image averaging to reduce noise
  - Subtraction: ($p-q$)
    - Used often for static background removal
  - Multiplication: ($p*q$) (or pq, $p\times q$)
    - Used to correct gray-level shading
  - Division: ($p\div q$) (or p/q)
    - As in multiplication

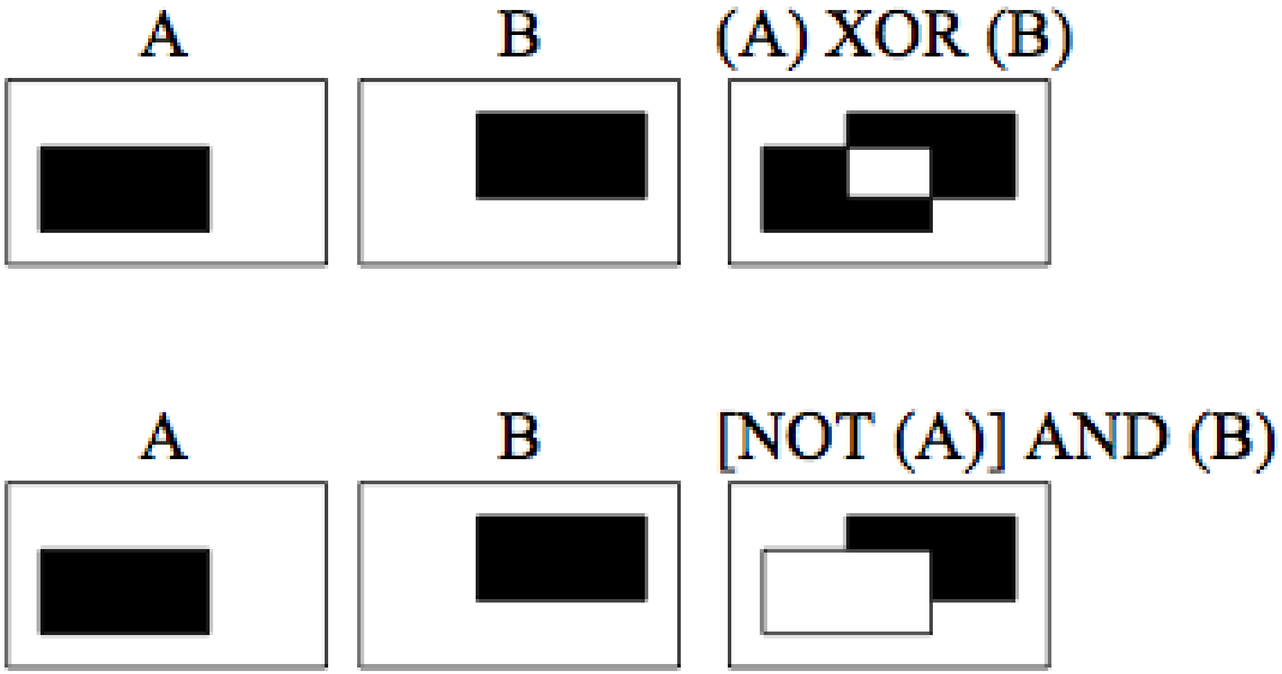
Logic operations

- Arithmetic operation between pixels *p* and *q* are defined as:
  - AND: *p* AND *q* (also p·q)
  - OR: *p* OR *q* (also p+q)
  - COMPLEMENT: NOT*q* (also *q'*)
- Form a *functionally complete* set
- Applicable to binary images
- Basic tools in binary image processing, used for:
  - Masking
  - Feature detection
  - Shape analysis

Examples of logic operations



Examples of logic operations



Neighborhood-oriented operations

- Arithmetic and logical operations may take place on a subset of the image
  - Typically neighborhood oriented
- Formulated in the context of *mask* operations (also called *template*, *window* or *filter* operations)
- Basic concept:let the value of a pixel be a function of its (current) gray level and the gray level of its neighbors (in some sense)

Neighborhood-oriented operations

- Consider the following subset of pixels in an image
- Suppose we want to filter the image by replacing the value at Z_5 with the average value of the pixels in a 3x3 region centered around Z_5
- Perform an operation of the form:

Z_1	Z_2	Z_3
Z_4	Z_5	Z_6
Z_7	Z_8	Z_9

$$z = \frac{1}{9}(z_1 + z_2 + \ldots + z_9) = \frac{1}{9}\sum_{i=1}^{9} z_i$$

- and assign to z_5 the value of z

Neighborhood-oriented operations

- In the more general form, the operation may look like:

$$z = (w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9) = \sum_{i=1}^{9} w_i z_i$$

- This equation is widely used in image processing
- Proper selection of coefficients (weights) allows for operations such as
  - noise reduction
  - region thinning
  - edge detection

Z_1	Z_2	Z_3
Z_4	Z_5	Z_6
Z_7	Z_8	Z_9

W_1	W_2	W_3
W_4	W_5	W_6
W_7	W_8	W_9