Course Title**: Numerical Method**
Course No.: CSC-204
Credit Hours: 3

Full Marks: 60+20+20
Pass Marks: 24+8+8

**Nature of the Course:** Theory (3hrs.) + Lab (3hrs.)

**Course Synopsis:** This course contains the concept of numerical techniques of solving the differential equations and algebraic equations.

**Goal:** To be familiar with the theory of numerical analysis for solving algebraic equations, solution of ordinary and partial differential equations related to engineering, computer science and mathematics.

**Course Contents:**

**Unit 1: Solution of Nonlinear Equations (10 hrs.)**

1.1 Review of Calculus and Taylor's Theorem – Definition of Big O notation, statement of Taylor's Theorem, meaning of solution of non-linear equations.
1.2 Errors in numerical Calculations – Truncation, round-off, errors in original data, blunders, propagated error and floating point arithmetic, errors in converting values, relative-absolute errors.
1.3 Trail and Error Method
1.4 Half-Interval Method – the bisection method, algorithm, implementation and convergence.
1.5 Secant Method – Secant method, algorithms, implementation and convergence.
1.6 Newton's Method – the Newton's method, algorithm, relating Newton's method to other methods, complex roots, implementation and convergence.
1.7 Fixed-point Iteration – the method, different rearrangements, the algorithm, implementation and convergence.
1.8 Newton's Method for Polynomials – the method, synthetic division algorithm and remainder theorem, Horner's method and algorithm, implementation, convergence.

(Relations between different methods, their advantages/disadvantages and comparison between different tests are essential).

**Unit 2: Interpolation and Approximation (8 hrs.)**

1.1 Interpolation – definition, application, definition of extrapolation.
2.2 Lagrang's Interpolation – Lagrangian polynomials, errors, algorithms, numerical calculations and implementations.
2.3 Newton's Interpolation – divided difference, algorithm for constructing divided difference table, divided difference for a polynomial, error of interpolation, evenly spaced data, differences vs. divided differences, algorithms and implementations.
2.4 Cubic Spline Interpolation - definitions, derivation, algorithm, examples illustrating cubic spline interpolation.
2.5 Least Squares Approximations – definition and application, derivations, algorithm and implementation of least square approximation for linear and non-linear (exponential and polynomial) data.

(Each method should be implemented and compared with each other drawing conclusions of their advantages, accuracy and error).

**Unit 3: Numerical Differentiation and Integration (5 hrs.)**

3.1 Numerical Differentiation – definition, application. Derivatives from divided difference table, error term, algorithm. Derivatives for evenly spaced data, forward difference formula, central difference formula, error terms. Second order derivatives. Maxima and minima of tabulated function.

3.2 Numerical Integration – definition, application. Newton-cotes quadrature formulas, trapezoidal rule, Simpson's 1/3 rule, Simpson's 3/8 rule. Composite formulas for trapezoidal & Simpson's rules, algorithms and implementation, Gaussian integration algorithm, derivation, Romberg integration formula, algorithm.

**Unit 4: Solution of Linear Algebraic Equations (10hrs.)**

4.1 Review – existence and uniqueness of solution of systems of linear equations, properties of matrices.

4.2 Gaussian elimination method and algorithm, pivoting, ill-conditioning, Gauss-Jordan method and algorithm, matrix inversion.

4.3 Matrix factorization - Dolittle algorithm, Cholesky's factorization.

4.4 Iterative Methods – Jacobi method and Gauss-Seidel method.

4.5 Eigenvalues and eigenvectors problems, solving eigenvalue problems using power method.

**Unit 5: Solution of Ordinary Differential Equations (7 hrs.)**

5.1 Review of differential equations, definition of ordinary differential equations and examples, order and degree, initial value problems.

5.2 Taylor's series method and error term, Picard's method, Euler's method and its accuracy, Heun's method, Runge-Kutta methods ($4^{th}$ order method: formula and problem solution), algorithms and implementation.

5.3 Solutions of higher-order equations – definition and examples of higher order difference equations, solution of system of differential equations, representation of higher order equations into system of equations.

5.4 Boundary Value Problems – definitions and examples, shooting method and its algorithm.

**Unit 6: Solution of Partial Differential Equations (5hrs.)**

Review of partial differential equations, deriving difference equations, Laplacian equation and Poisson's equations, their solution techniques, algorithms and examples.

**References :**

1 C.F. Gerald and P.O. Wheatley, Applied Numerical Analysis, Addison-Wesley Publishing Company, New York.
2 E. Balagurusamy, Numerical Methods, Tata McGraw-Hill
3 W. Cheney and D. Kincaid, Numerical Mathematics and Computing, Brooks/Cole Publishing Co.
4 S.Yakowitz and F. Szidarovszky, An Introduction to Numerical Computations, Macmillan Publishing Co. New York.
5 W.H. Press, B.P. Flannery et al. Numerical Recipes in C, Cambridge Press.

**Laboratory works:** The laboratory experiments will consist of program development and testing of non-linear equations, interpolation, numerical integration and differentiation, linear algebraic equations and differential equations. Each algorithm should be implemented in an appropriate language (C, C++). However, software like Matlab, Mathematica and Octave are also recommended for the supplements.

## List of Lab Exercises:

1. Solution of non-linear equations using Bisection method and Secant Method

2. Solution of non-linear equations using Newton-Raphson Method and Fixed Point iteration Method
3. Solution of polynomial using Newton's method, use Hornor's rule to evaluate polynomial
4. Polynomial interpolation using Lagrange's Interpolation and Newton's Divided Difference Interpolation.
5. Fitting a linear and non-linear function using least square method
   i. Linear (straight line) i.e. $y= ax +b$
   ii. Non-linear functions
      a) Exponential function i.e. $y=a e^{bx}$
      b) Quadratic function i.e. $y=ax^2+bx+c$
6. Derivative from divided difference table
7. Integration using Trapezoidal rule, Simpson's 1/3 Rule and Simpson's 3/8 Rule.
8. Solution of system of linear equations using Gauss Elimination method (using partial pivoting)
9. Gauss-Jordan Method
10. Gauss Seidel method, Power method.
11. Solution of ordinary differential equation using Euler's Method, Heun's Method and 4th Order Runge-Kutta method
12. Boundary value problems using shooting method.

**Remarks:**

1. This detail course is based on the prescribed course of study. The respective teacher should follow the prescribed course of BScCSIT for the teaching.
2. Following teachers from following colleges have been involved to prepare this document.

| Name of the Campus | Name of Teacher | E-mail | Signature |
|---|---|---|---|
| CDCSIT, TU | Dr. Tanka Nath Dhamala | | |
| Patan Multiple Campus | Sharad Kumar Ghimire,<br>Modick Bahadur Basnet | | |
| Amrit Science Campus | Urmila Pyakurel | | |
| Mahendranagar | ----------------------- | | |
| New Summit | Urmila Pyakurel | | |
| St.Xavier's Campus | Rajiv Nakarmi | | |
| Model College of Management, Janakpur | Dhirendra Kumar Jha | | |
| Kathford Int'l College | Rajesh Prasad Shrestha<br>Urmila Pyakurel<br>Ram Kaji Budhathoki | | |

**Model Question Paper**
**Bachelors of Science in Computer Science and Information Technology (BScCSIT)**
**Institute of Science and Technology**
**Tribhuvan University**

Course Title**:** Numerical Method
Course No.: CSC-204

Full Marks: 60
Pass Marks: 24
Time: 3 hrs

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

**Attempt all questions**.

1. Derive the formula for Secant Method using an illustrative figure. Find a real root of following equation using secant method correct up to two decimal places
$\sin x - 2x + 1 = 0$ [3+5]

**OR**

Derive the equation of Newton Raphson's method, and find a real root of $x^3 + x^2 - 3x - 3 = 0$ in the interval [1,2] correct up to three significant digits. [3+5]

2. Derive the equation for Lagrange's interpolating polynomial and find the value of f(x) at x = 0 for following function. [4+4]

| | | | | |
|---|---|---|---|---|
| x | -1 | -2 | 2 | 4 |
| f(x) | -1 | -9 | 11 | 69 |

3. a) Evaluate $\int_0^1 e^{-2x}\,dx$ using Gaussian integration three point formula. [4]

b) Calculate the integral value of following function from x=0 to x=1.6 using Simpson's 1/3 rule. [4]

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| x | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 |
| f(x) | 0 | 0.24 | 0.55 | 0.92 | 1.63 | 1.84 | 2.37 | 2.95 | 3.56 |

4. What is pivoting? Why is it necessary? Solve the following system of linear equation using Gauss-Jordan method (use partial pivoting if necessary) or Gauss-Seidel method. [2+6]

$$x_2 + 3x_3 + 2x_4 = 19$$
$$2x_1 - 2x_2 - x_3 - x_4 = -9$$
$$3x_2 + 2x_3 + 2x_4 = 20$$
$$x_1 + 4x_2 + 2x_4 = 17$$

5. Solve the following differential equation $\frac{dy}{dx} = 3x + \frac{y}{2}, \quad with\ y(0) = 1$

for $0 \le x \le 0.2$ using, [3+4+1]

a) Euler's method

b) Heun's method
Also compare the results.

6. Derive a difference equation to represent a Poisson's equation. Solve the Poison's equation $\nabla^2 f = 2x^2y^2$ over the square domain 0≤x≤3 and 0≤y≤3 with f=0 on the boundary and h=1. [3+5]

7. Write an algorithm and program to solve system of linear equation using Gauss elimination method. [5+7]