Conversion from NFA to DFA using subset construction method.

* RE is already given
* Form NFA using thompson construction method (Rule).
* Then convert ~~into~~ it into DFA using subset construction method.

Subset Construction Algorithm

Input : An NFA N.

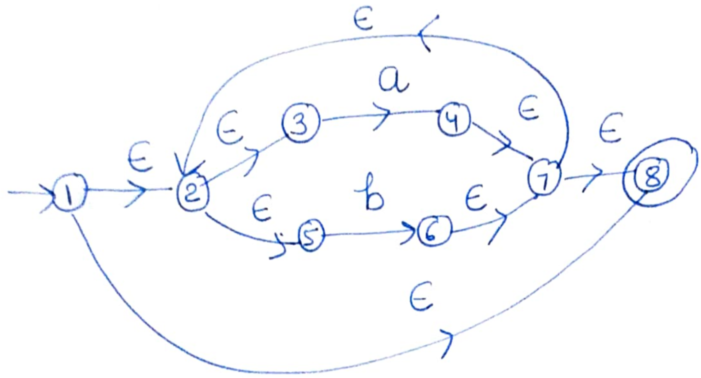
Output : A DFA D accepting the same language.

Method : Algorithm construct a transition table Dtran for D. We use the following operation.

Operation	Description
ε-closure(s)	Set of NFA states reachable from NFA state s on ε-transition alone.
ε-closure	Set of NFA states reachable from NFA state s in T on ε-transition alone.
δ(T, a)	Set of NFA state to which there is a transition on input symbol a from some NFA state s in T.

Ques-1 Convert regular expression to minimized DFA.
$(a|b)^*$

Step-1 Convert ϵ-NFA for RE $(a|b)^*$ using thompson construction method.



Step-2 Convert the given NFA to DFA using subset construction method.

(*) find ϵ-closure(s).
so, state with initial state (1).

ϵ-closure(1) $= \{1, 2, 3, 5\ 8\}$ --- (A)

$A = \{1, 2, 3, 5, 8\}$

→ To find the transition of (A) state of the input symbol (a, b).

transition $\delta(A, a)$ and $\delta(A, b)$

state $A = \{1, 2, 3, 5, 8\}$

Now, check the symbol (a) is leaving out from any state.

$\delta(A, a) = \{4\}$ | a on A $3 \xrightarrow{a} 4$

* find ϵ-closure(4) $= \{4, 7\ 8, 2, 3, 5\}$
$= \{2, 3, 4, 5, 7, 8\}$ --- (B)

Then check (b) symbol is leaving out from any state.

$\delta(A, b) = \{6\}$ | b on A $5 \xrightarrow{a} 6$.

* find ϵ-closure $(6) = \{6, 7, 8, 2, 3, 5\}$ --
$= \{2, 3, 5, 6, 7, 8\}$ --- C

→ To find the transition of B state of the input symbol (a, b).

Transition $\delta(B, a)$ and $\delta(B, b)$

state $B = \{2, 3, 4, 5, 7, 8\}$

Now, check the symbol (a) is leaving out from any state.

$\delta(B, a) = \{4\}$

I have already find the ϵ-closure 4 that is state B.

Then, check the symbol (b) is leaving out from any state.

$\delta(B, b) = \{6\}$

I have already find the ϵ-closure 6 that is state C.

→ To find the transition of C state of the input symbol (a, b).

Transition $\delta(C, a)$ and $\delta(C, b)$

state $C = \{2, 3, 5, 6, 7, 8\}$

Now check the symbol (a) is leaving out from any state.

$\delta(C, a) = \{4\}$

I have already find the ϵ-closure 4 that is state B

Then, check the symbol (b) is leaving out from any state.

$\delta(C, b) = \{6\}$

I have already find the ϵ-closure 6 that is state C.

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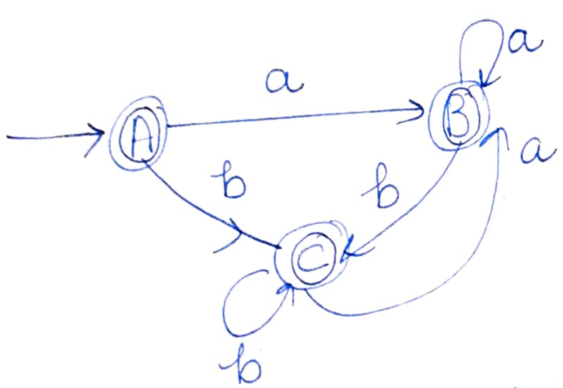
Transition Table

States Q.	Input Symbol a	Input Symbol b
→(A) = {1,2,3,5,8}	B	C
(B) = {2,3,4,5,7,8}	B	C
(C) = {2,3,5,6,7,8}	B	C

Transition Digram



* Note : Accepting state in NFA is 8.
8 is element of (A, B, C)

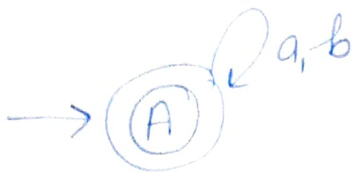
so, (A, B, C) are acceptance state in DFA.

Step-3 Minimization Process.

Optimized the DFA using transition table.
Total state = (A, B, C)
Seprate final non-accepting states final accepting states.

$\{A, B, C\}$

0 Equivalance	$\{A, B, C\}$

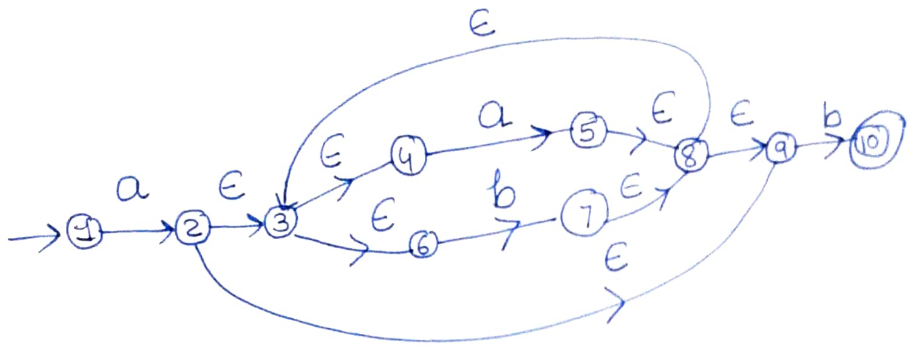


Minimised DFA

Transition Digram.

Q2 Convert regular expression to minimised DFA.
$a(a|b)^*b$

Step-1 Construct ε-NFA for RE $a(a|b)^*b$ using thompson construction method.



Step-2 Convert the given NFA to DFA using subset construction method.

(*) find ε-Closure(s)
So, start with initial state 1.

ε-closure(1) = {1} --- A

A = {1}

→ To find the transition of A state of the input symbol (a, b).

Transition S(A, a) and S(A, b).

state A = {1}

Now check the symbol (a) is leaving out from any state.

S(A, a) = {2} | a on A $1 \xrightarrow{a} 2$

(*) find ε-closure(2) = {2, 3, 4, 6, 9} --- B

Then ~~check the symbol (b) is~~

Then check (b) symbol is leaving out from any state.

$\delta(A, b) =$ no move of b | b on A =
$1 \xrightarrow{b} X$.

→ To find the transition of (B) state of the input symbol (a, b). $B = \{2, 3, 4, 6, 9\}$

Transition $\delta(B, a)$ and $\delta(B, b)$.

state $B = \{2, 3, 4, 6, 9\}$

Now check the symbol (a) is leaving out from any state.

$\delta(B, a) = \{5\}$ | $4 \xrightarrow{a} 5$

(*) find ϵ - closure(5) $= \{5, 8, 9, 3, 4, 6\}$
$= \{3, 4, 5, 6, 8, 9\}$ --- (C)

Then Check (b) symbol is leaving out from any state.

$\delta(B, b) = \{7, 10\}$

(*) find ϵ -closure (7, 10) $= \{7, 10, 8, 9, 3, 4, 6\}$
$= \{3, 4, 6, 7, 8, 9, 10\}$ --- (D)

→ To find the transition of (C) state of the input symbol (a, b).

Transition $\delta(C, a)$ and $\delta(C, b)$

(T)

state $C = \{3, 4, 5, 6, 8, 9\}$

Now, check the symbol (a) is leaving out from any state.

$\delta(C, a) = \{5\}$

I already find the ϵ- closure of (5) that is state (C)

Then, check (b) symbol is leaving out from any state.

$\delta(C, b) = \{7, 10\}$

I already found the ϵ - closure of (7, 10), that is state D.

→ To find the transition of D state of the input symbol (a, b).

Transition $\delta(D, a)$ and $\delta(D, b)$.

state $D = \{3, 4, 6, 7, 8, 9, 10\}$

Now, check the symbol (a) is leaving out from any state.

$\delta(D, a) = \{5\}$

I already find the ϵ - closure of 5, that is state C.

Then, check (b) symbol is leaving out from any state.

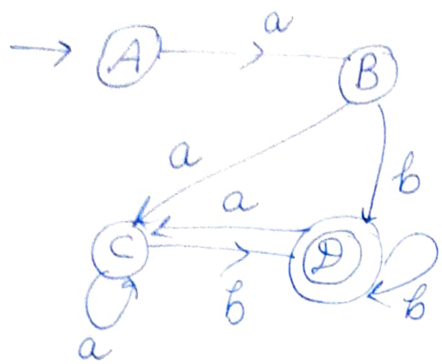
$\delta(D, b) = \{7, 10\}$

I already found the ϵ - closure of (7, 10) that is state D.

Transition Table

States Q.	Input Symbol a	b
*A = {1}	B	—
B = {2, 3, 4, 6, 9}	C	D
C = {3, 4, 5, 6, 8, 9}	C	D
D {3, 4, 6, 7, 8, 9, 10}	C	D

Transition ~~Table~~ Digram



Note: Accepting state in NFA was 10. And 10 ~~was~~ is element of state D.

So, D state is accepting state. That's final state.

step-3 : Minimization Process.

- Remove unreachable state.
- Separate the final and non-final state.

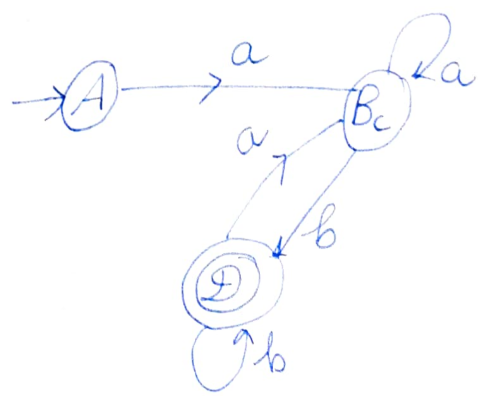
0 Equivalance $(\Pi_0) = \{A, B, C\}, \{D\}$

1 Equivalance $(\Pi_1) = \{A\}, \{B, C\}, \{D\}$

2 Equivalance $(\Pi_2) = \{A\}, \{B, C\}, \{D\}$

Transition Table

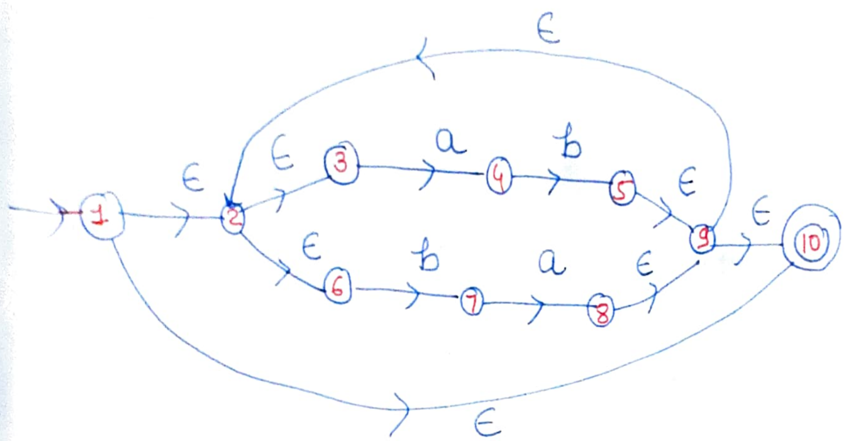
State Q	Input a	b
A	B_C	ϕ
B_C	B_C	D
D	B_C	D



DFA [Transition Digram]

Ques-3 Convert Regular Expression $(ab|ba)^*$ to minimised DFA.

Step-1 Convert Regular Expression $(ab|ba)^*$ using Thompson construction method.



Step-2 Convert the given NFA to DFA using subset construction method.

(*) find ε-closure(s).

so, start with initial state (1)

ε-closure(1) = {1, 2, 3, 6, 10} ---- (A)

A = {1, 2, 3, 6, 10}

→ To find the transition of (A) state of the input symbol (a, b)

transition $\delta(A, a)$ and $\delta(A\,b)$ | state A = {1,2,3,6,10}

Now, check the symbol (a) is leaving out from any state.

$\delta(A, a) = \{4\}$ | a on A $3 \xrightarrow{a} 4$

(10)

* find ε-closure(4) = {4} --- (B)

B = {4}

Then, check the symbol $\delta(A, b)$ is leaving out from any state.

$\delta(A, b) = \{7\}$ --- | b on A $6 \xrightarrow{b} 7$

* find ε-closure(7) = {7} ---- (C)

→ To find the transition of (B) state of the input symbol (a, b)

transition δ(B, a) and δ(B, b)

state B = {4}

Now check the symbol (a) is leaving out from any state.

δ(B, a) = no move | a on B

$4 \xrightarrow{a} X$

Then, check the symbol (b) is leaving out from any state.

δ(B, b) = {5}

* find ε-closure(5) = {5, 9, 10, 2, 3, 6}

= {2, 3, 5, 6, 9, 10} --- (D)

→ To find the transition of (C) state of the input symbol (a, b)

transition δ(C, a) and δ(C, b)

state C = {7}

Now, check the symbol (a) is leaving out from any state.

δ(C, a) = {8} | a on C

$7 \xrightarrow{a} 8$

* find the ε-closure (8) = {8, 9, 10, 2, 3, 6}

= {2, 3, 6, 8, 9, 10} --- (E)

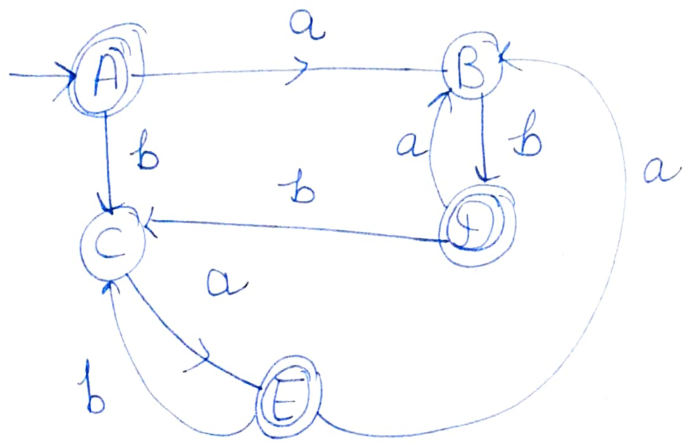
Then check the symbol (b) is leaving out from any state.

δ(C, b) = no move | b on C

$7 \rightarrow X$

Transition Table

State Q	Input a	b
→A = {1,2,3,6,10}	B	C
B = {4}	–	D
C = {7}	E	–
D = {2,3,5,6,9,10}	B	C
E = {2,3,6,8,9,10}	B	C



Transition Diagram.

* Note: Accepting state in NFA is 10

10 is element of {A, D, E}

so, {A, D, E} are accepting state in DFA

Step-3 Minimization Process.

0 Equivalance = {A, D E}, {B, C}

1 Equivalance = {A, D E}, {B}, {C}

name new state (A, D E) as Z.

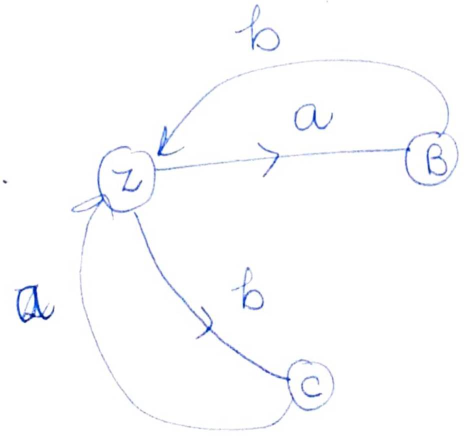
State Q	Input a	Input b
A	B	C
B	—	D
C	E	—
D	B	C
E	B	C

Z.

Relace (A D E) as Z in Table.

State Q	Input a	Input b
→ Z	B	C
B	—	Z
C	Z	—

Transition ~~Digram~~ Table



Minimized DFA

Transition Digram