Conversion from NFA to DFA using subset construction method.

* RE is already given
* Form NFA using Thompson construction method (Rule).
* Then convert it into DFA using subset construction method.

Subset Construction Algorithm

Input : An NFA N.

Output : A DFA D accepting the same language.

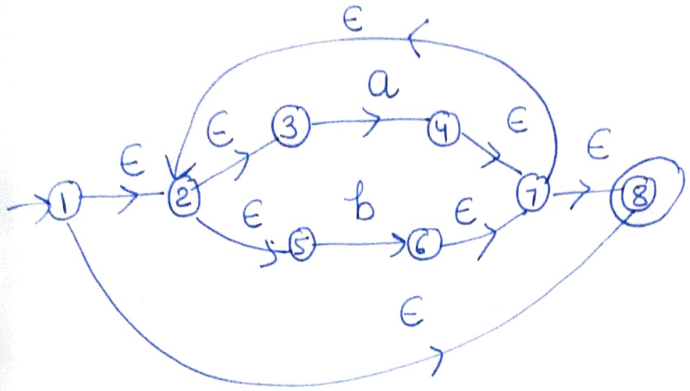
Method : Algorithm construct a transition table Dtran for D. We use the following operation.

operation	Description
ϵ-closure(s)	set of NFA states reachable from NFA state s on ϵ-transition alone.
ϵ-closure	set of NFA states reachable from NFA state s in T on ϵ-transition alone.
move(T, a)	set of NFA state to which there is a transition on input symbol a from some NFA state s in T.

1. Convert regular expression to minimized DFA.

$(a|b)^*$

Step-1 Convert ϵ-NFA for RE $(a|b)^*$ using thompson construction method.



Step-2 Convert the given NFA to DFA using subset construction method.

(*) find ϵ-closure(1).

so, state with initial state (1).

ϵ-closure(1) = {1, 2, 3, 5 8} ---(A)

A = {1, 2, 3, 5, 8}

> To find the transition of (A) state of the input symbol (a, b).

Transition $\delta(A, a)$ and $\delta(A, b)$

state A = {1, 2, 3, 5, 8}

Now, check the symbol (a) is leaving out from any state.

$\delta(A, a) = \{4\}$ | a on A $3 \xrightarrow{a} 4$

* find ϵ-closure(4) = {4, 7 8, 2, 3, 5}

= {2, 3 4, 5, 7, 8} ---(B)

Then check (b) symbol is leaving out from any state.

$S(A, b) = \{6\}$ | b on A, $5 \xrightarrow{a} 6$.

* find ϵ-closure (6) = $\{6, 7, 8, 2, 3, 5\}$ --
$= \{2, 3, 5, 6, 7, 8\}$ --- (C)

→ To find the transition of (B) state of the input symbol (a, b).

- Transition $S(B, a)$ and $S(B, b)$

state B = $\{2, 3, 4, 5, 7, 8\}$

Now, check the symbol (a) is leaving out from any state.

$S(B, a) = \{4\}$

I have already find the ϵ-closure (4) that is state (B).

Then, check the symbol (b) is leaving out from any state.

$S(B, b) = \{6\}$

I have already find the ϵ-closure (6) that is state (C).

→ To find the transition of (C) state of the input symbol (a, b).

- Transition $S(C, a)$ and $S(C, b)$

state C = $\{2, 3, 5, 6, 7, 8\}$

Now check the symbol (a) is leaving out from any state.

$S(C, a) = \{4\}$

I have already find the ϵ-closure (4) that is state (B)

Then, check the symbol (b) is leaving out from any state.

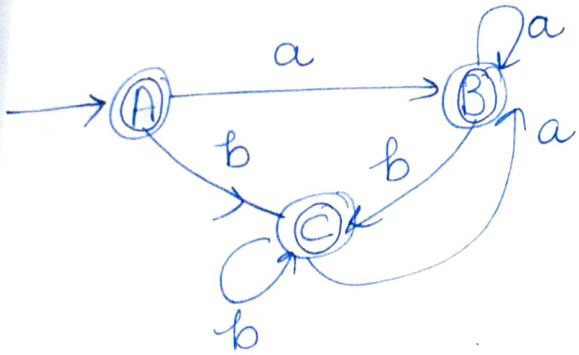
$S(C, b) = \{6\}$

I have already find the ϵ-closure (6) that is state (C).

Transition Table

States Q.	Input Symbol a	Input Symbol b
→ A = {1,2,3,5,8}	B	C
(B) = {2,3,4,5,7,8}	B	C
(C) = {2,3,5,6,7,8}	B	C

Transition Digram



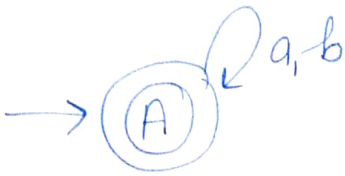
* Note : Accepting state in NFA is 8.
8 is element of (A, B, C)
so, (A, B, C) are acceptance state in DFA.

Step 3 Minimization Process.

Optimized the DFA using transition table.
Total state = (A, B, C)
Seprate final non-accepting states final accepting states.

{A, B, C}

OEquivalance	{A, B, C}



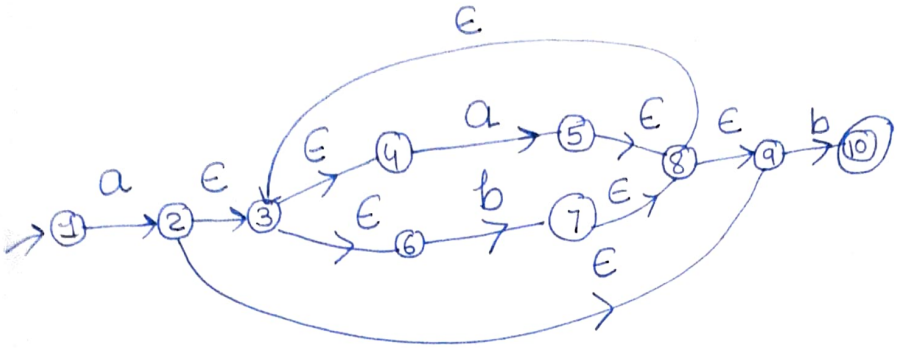
Minimised DFA

Transition Digram.

Convert regular expression to minimised DFA.

$a(a|b)^*b$

Step-1 Construct E-NFA for RE $a(a|b)^*b$ using thompson construction method.



Step-2 Convert the given NFA to DFA using subset construction method.

(*) find ϵ-Closure(2)

So, start with initial state (1).

ϵ-closure(1) = {1} --- (A)

A = {1}

→ To find the transition of (A) state of the input symbol (a, b).

Transition S(A, a) and S(A, b).

state A = {1}

Now check the symbol (a) is leaving out from any state.

S(A, a) = {2} | a on A

1 —a→ 2

find ϵ-closure(2) = {2, 3, 4, 6, 9} --- (B)

~~Then check the symbol (b) is~~

Then check (b) symbol is leaving out from any state.

$S(A, b)$ = no move of b | b on A = $1 \xrightarrow{b} X$.

→ To find the transition of (B) state of the input symbol (a, b).

$B = \{2, 3, 4, 6, 9\}$

- Transition $S(B, a)$ and $S(B, b)$.

state $B = \{2, 3, 4, 6, 9\}$

Now check the symbol (a) is leaving out from any state.

$S(B, a) = \{5\}$ | $4 \xrightarrow{a} 5$

(*) find ϵ - closure(5) $= \{5, 8, 9, 3, 4, 6\}$

$= \{3, 4, 5, 6, 8, 9\}$ --- (C)

Then check (b) symbol is leaving out from any state.

$S(B, b) = \{7, 10\}$

(*) find ϵ - closure(7, 10) $= \{7, 10, 8, 9, 3, 4, 6\}$

$= \{3, 4, 6, 7, 8, 9, 10\}$ --- (D)

→ To find the transition of (C) state of the input symbol (a, b).

- Transition $S(C, a)$ and $S(C, b)$

state $C = \{3, 4, 5, 6, 8, 9\}$ (1)

Now, check the symbol (a) is leaving out from any state.

$S(C, a) = \{5\}$

I already find the ϵ - closure of (5) that is state (C)

Then, check (b) symbol is leaving out from any state.

$\delta(C, b) = \{7, 10\}$

I already found the ϵ-closure of (7,10), that is state (D).

→ To find the transition of (D) state of the input symbol (a, b).

Transition $\delta(D, a)$ and $\delta(D, b)$.

state $D = \{3, 4, 6, 7, 8, 9, 10\}$

Now, check the symbol (a) is leaving out from any state.

$\delta(D, a) = \{5\}$

I already find the ϵ-closure of (5), that is state (E).

Then, check (b) symbol is leaving out from any state.

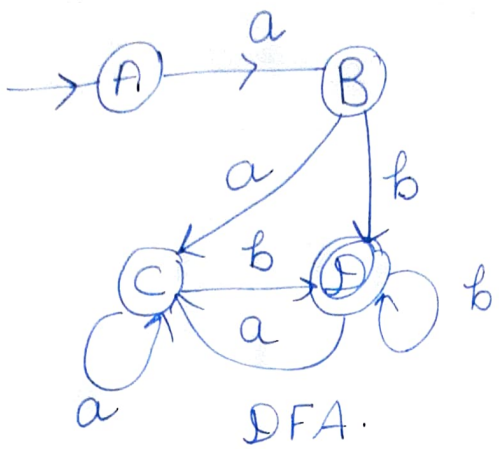
$\delta(D, b) = \{7, 10\}$

I already found the ϵ-closure of (7,10) that is state D.

Transition Table.

States Q.	Input Symbol a	Input Symbol b
$\{1\}$	B	—
$\{2, 3, 4, 6, 9\}$	C	(D)
$\{3, 4, 5, 6, 8, 9\}$	C	(D)
$\{3, 4, 6, 7, 8, 9, 10\}$	C	(D)

Transition Diagram



DFA.

* Note: Accepting state in NFA is 10.
10 is element of (D).
So, D is acceptance state in DFA.

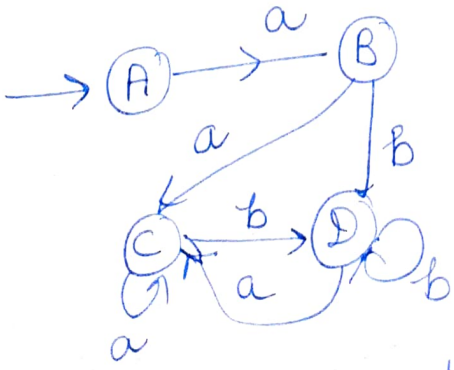
Step -3 Minimization Process

Seprate the final and non-final state in 0 Equalence.

0 Equ. = {A, B, C}, {D}

1 Equ = {A}, {C}, {B}, {D}

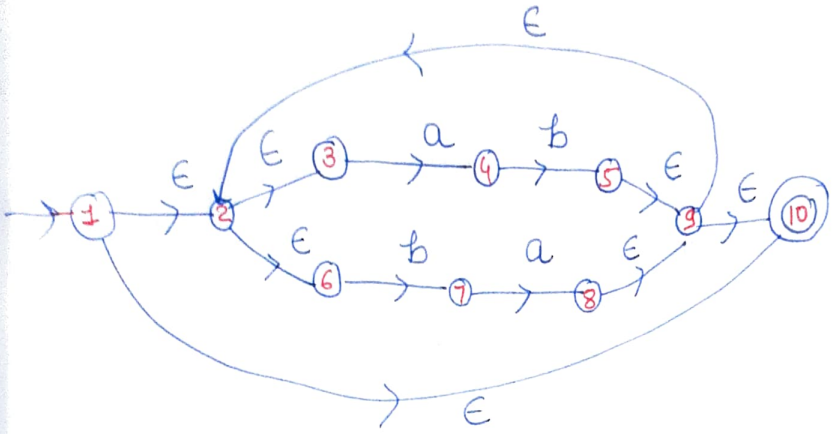
all states are sperate from others.
Means the constructed DFA is already minimised.



optimized/ minimised DFA.

Eg-3 Convert Regular Expression $(a\,b|ba)^*$ to minimised DFA.

Step-1 Convert Regular Expression $(a\,b|ba)^*$ using thompson construction method.



Step-2 Convert the given NFA to DFA using subset construction method.

* find ϵ-closure(s).

so, state with initial state (1)

ϵ-closure(1) = {1, 2, 3, 6, 10} ---- (A)

A = {1, 2, 3, 6, 10}

→ To find the transition of (A) state of the input symbol (a, b)
Transition $S(A,a)$ and $S(A,b)$ | state A = {1, 2, 3, 6, 10}

Now, check the symbol (a) is leaving out from any state.

$S(A,a)$ = {4} | a on A
3 —a→ 4

* find ϵ-closure(4) = {4} --- (B)

B = {4}

Then, check the symbol $S(A,b)$ is leaving out from any state.

$S(A,b)$ = {7} --- | b on A
6 —b→ 7

* find ϵ- closure (7) = {7} ---- (C)

→ To find the transition of (B) state of the input symbol (a, b)

Transition S(B, a) and S(B, b)

state B = {4}

Now, check the symbol (a) is leaving out from any state.

S(B, a) = no move | a on B
4 → a X

Then, check the symbol (b) is leaving out from any state.

S(B, b) = {5}

* find ϵ- closure (5) = {5, 9, 10, 2, 3, 6}
= {2, 3, 5, 6, 9, 10} --- (D)

→ To find the transition of (C) state of the input symbol (a, b)

Transition S(C, a) and S(C, b)

state C = {7}

Now, check the symbol (a) is leaving out from any state.

S(C, a) = {8} | a on C
7 → a 8

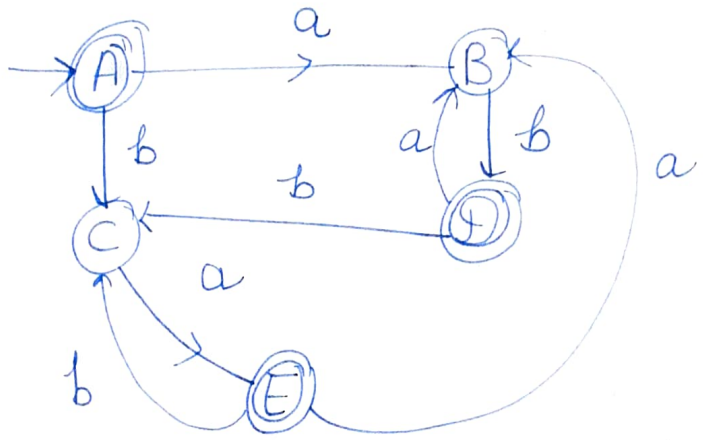
* find the ϵ- closure (8) = {8, 9, 10, 2, 3, 6}
= {2, 3, 6, 8, 9, 10} --- (E)

Then, check the symbol (b) is leaving out from any state.

S(C, b) = no move | b on C
7 → X

Transition Table

State Q	Input a	Input b
→ (A) = {1, 2, 3, 6, 10}	B	C
B = {4}	–	(D)
C = {7}	E	–
(D) = {2, 3, 5, 6, 9, 10}	B	C
(E) = {2, 3, 6, 8, 9, 10}	B	C



Transition Digram.

Note: Accepting state in NFA is 10

10 is element of {A, D, E}

so, {A, D, E} are accepting state in DFA

Step-3 Minimization Process.

0 Equivalance = {A, DE}, {B, C}

1 Equivalance = {A, DE}, {B}, {C}

name new state (A, DE) as Z.

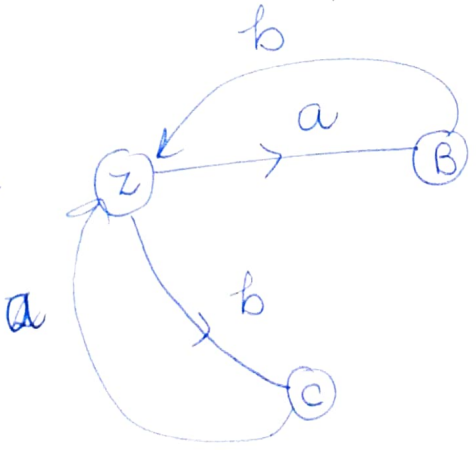
State Q	Input a	Input b
A	B	C
B	—	D
C	E	—
D	B	C
E	B	C

Z

Relace (ADE) as Z in Table.

State Q	Input a	Input b
→ Z	B	C
B	—	Z
C	Z	—

Transition ~~Digram~~ Table



Minimized DFA

Transition Digram