1 Proof 1

Claim: If n is an odd integer, then there is an integer m such that $n = 4m+1$ or $n = 4m + 3$

n is an odd integer. If n is an odd integer, by definition, $n = 2x+1$, $\forall x \in \mathbb{Z}$. Therefore, $2x + 1 = 4m + 1$ or $2x + 1 = 4m + 3$. This simplifies to $x = 2m$ or $x = 2m + 1$.

By definition, if x is an integer, then there is an integer m such that $x = 2m$ or $x = 2m + 1$. Within the set of integers, any individual number can either be even or odd, in other words, $\{2n : n \in \mathbb{Z}\} \cup \{2n + 1 : n \in \mathbb{Z}\} = \mathbb{Z}$.

This means that, since x is an integer, x is either even or odd. If x is even, $x = 2y$ where y is an integer, which implies that there is some integer m such that $2y = 2m \implies y = m$ which we know to be true. Similarly, if x is odd, $x = 2y + 1$ where y is again an integer. This implies that there is some integer m such that $2y + 1 = 2m + 1 \implies 2y = 2m \implies y = m$ which we also know to be true.

Therefore, if n is an odd integer, then there must be some integer m such that n either equals $4m + 1$ or $4m + 3$.

2 Proof 2

Claim: Every non-empty finite set of unique integers has a smallest number.

Given the set $S = \{s_1, s_2, ..., s_n\}$, consisting of n unique terms, we know that $n > 0$, and S contains at least one element. Since S consists of unique numbers, we know that $s_1 \neq s_2 \neq s_3 ... \neq s_n$, in other words, each value of S is distinct. Because of this, we can infer that each value is either larger or smaller than each other value in S. Because of this fact, we know that there must be some number s_x such that $s_x < S \setminus s_x$. Therefore, every non empty finite set of unique integers must contain a smallest number.

3 Proof 3

Claim: For any real number x, $x^2 - 4x + 3 > 0$

We can restate this claim; if x is real, then $x^2 - 4x + 3 > 0$. Solving for x with the quadratic formula yields zeros for this equation at $1 + \frac{\sqrt{-2}}{2}$ and $1 - \frac{\sqrt{-2}}{2}$. These are both imaginary roots, which means the only 2 points at which this equation is equal to zero are not real. We can infer from the fact that x does not cross the horizontal axis in the real domain, that it is never less than zero in said real domain. Therefore, while x is real $x^2 - 4x + 3 > 0$.