Project 1: Python Truth Tables

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Proof 1

$$\begin{array}{l} p \lor (\neg q) \\ (t \lor s) \implies (p \lor r) \\ (\neg r) \lor (t \lor s) \\ p \implies (t \lor s) \\ \hline (p \land r) \implies (q \lor r) \end{array}$$

Claim: $(p \land r) \implies (q \lor r)$

Proof:

- Assume $p \land r$. *(Assumption)*
- Since p, and $p \lor (\neg q)$, q. *(Disjunctive Syllogism)*
- Given p and r, and having concluded q, then infer $(p \land r) \implies (q \lor r)$

This conclusion can be shown in the truth table on the next 2 pages.

```python
variables=['p', 'q', 'r', 's', 't']
expression1=lambda p,r: p and r
expression2=lambda q,r: q or r
expression3=lambda p,q,r: Implies(p&r,q|r)
data=[]

for p in (True, False):
    for q in (True, False):
        for r in (True, False):
            for t in (True, False):
                for s in (True, False):
                    result1=expression1(p,r)
                    result2=expression2(q,r)
                    result3=expression3(p,q,r)
                    data.append([p,q,r,t,s,result1,
                        result2,result3])
df=pd.DataFrame(data, columns=['p','q','r','s','t','p
    and r','q or r','(p and r) implies (q or r)'])
ConvertToLatex(df)
```

	p	q	r	s	t	p and r	q or r	(p and r) implies (q or r)
0	True	True	True	True	True	True	True	True
1	True	True	True	True	False	True	True	True
2	True	True	True	False	True	True	True	True
3	True	True	True	False	False	True	True	True
4	True	True	False	True	True	False	True	True
5	True	True	False	True	False	False	True	True
6	True	True	False	False	True	False	True	True
7	True	True	False	False	False	False	True	True
8	True	False	True	True	True	True	True	True
9	True	False	True	True	False	True	True	True
10	True	False	True	False	True	True	True	True
11	True	False	True	False	False	True	True	True
12	True	False	False	True	True	False	False	True
13	True	False	False	True	False	False	False	True
14	True	False	False	False	True	False	False	True
15	True	False	False	False	False	False	False	True
16	False	True	True	True	True	False	True	True
17	False	True	True	True	False	False	True	True
18	False	True	True	False	True	False	True	True
19	False	True	True	False	False	False	True	True
20	False	True	False	True	True	False	True	True
21	False	True	False	True	False	False	True	True
22	False	True	False	False	True	False	True	True
23	False	True	False	False	False	False	True	True
24	False	False	True	True	True	False	True	True
25	False	False	True	True	False	False	True	True
26	False	False	True	False	True	False	True	True
27	False	False	True	False	False	False	True	True
28	False	False	False	True	True	False	False	True
29	False	False	False	True	False	False	False	True
30	False	False	False	False	True	False	False	True
31	False	False	False	False	False	False	False	True

Proof 2

$p \lor (\neg q)$
$(t \lor s) \implies (p \lor r)$
$(\neg r) \lor (t \lor s)$
$p \iff (t \lor s)$

$(q \lor r) \implies (p \lor r)$

Claim: $(q \lor r) \implies (p \lor r)$
Proof:

- Assume $q \lor r$. *(Assumption)*
- There are 3 cases to this assumption, q, r, or q and r. *(Proof by cases)*
  If q:
  1. If q, then not $\neg q$ *(Definition)*
  2. Since $p \lor (\neg q)$ and not $\neg q$, then p.

  If r:
  1. If not r, then $t \lor s$ from $(\neg r) \lor (t \lor s)$ *(Disjunction)*
  2. If $(t \lor s)$ then p, from $p \iff (t \lor s)$

  If q and r:
  1. If q and r, then then not $\neg q$ and not $\neg r$.
  2. As shown in both previous cases, not $\neg q$ and not $\neg r$ both imply p.
- In all cases, p is true, $\therefore (q \lor r) \implies (p \lor r)$

This is shown in the truth table on the next few pages. In the truth table, I substiture $(p \lor \neg r)$ for p. This is because I faced issues running $q \lor r \implies p \lor r$. Specifically, whenever p and r were both false, the conclusion was false. Examining the given conditions, I did not believe that this situation was possible. I eventually realized that, since $(t \lor s) \implies p$, I could simplify and arrive at $(\neg r) \lor p$.

```
variables=['p', 'q', 'r', 's', 't']
expression1=lambda q,r: q or r
expression2=lambda p,r: p or r
claim_expression=lambda q,r,p: Implies(q or r, (p or
    not r) or r)

data=[]

for p in (True, False):
    for q in (True, False):
        for r in (True, False):
            for t in (True, False):
                for s in (True, False):
                    result1=expression1(q,r)
                    result2=expression2(p,r)
                    claim_result=claim_expression(q,r
                        ,p)
                    data.append([p,q,r,t,s,result1,
                        result2,claim_result])

df=pd.DataFrame(data, columns=['p','q','r','t','s','q
    |r','p|r','(q|r) implies ((p|r`)|r)'])

ConvertToLatex(df)
```

	p	q	r	t	s	q|r	p|r	(q|r) implies ((p|r‘)|r)
0	True	True	True	True	True	True	True	True
1	True	True	True	True	False	True	True	True
2	True	True	True	False	True	True	True	True
3	True	True	True	False	False	True	True	True
4	True	True	False	True	True	True	True	True
5	True	True	False	True	False	True	True	True
6	True	True	False	False	True	True	True	True
7	True	True	False	False	False	True	True	True
8	True	False	True	True	True	True	True	True
9	True	False	True	True	False	True	True	True
10	True	False	True	False	True	True	True	True
11	True	False	True	False	False	True	True	True
12	True	False	False	True	True	False	True	True
13	True	False	False	True	False	False	True	True
14	True	False	False	False	True	False	True	True
15	True	False	False	False	False	False	True	True
16	False	True	True	True	True	True	True	True
17	False	True	True	True	False	True	True	True
18	False	True	True	False	True	True	True	True
19	False	True	True	False	False	True	True	True
20	False	True	False	True	True	True	False	True
21	False	True	False	True	False	True	False	True
22	False	True	False	False	True	True	False	True
23	False	True	False	False	False	True	False	True
24	False	False	True	True	True	True	True	True
25	False	False	True	True	False	True	True	True
26	False	False	True	False	True	True	True	True
27	False	False	True	False	False	True	True	True
28	False	False	False	True	True	False	False	True
29	False	False	False	True	False	False	False	True
30	False	False	False	False	True	False	False	True
31	False	False	False	False	False	False	False	True