y \ x	0	1	2	
0	$3/28$	$9/28$	$3/28$	$15/28$
1	$6/28$	$6/28$	0	$12/28$
2	$1/28$	0	0	$1/28$
	$10/28$	$15/28$	$3/28$	1

$$P(x=1 \mid y=0) = \frac{9/28}{15/28}$$

$$= \frac{P(x=1, y=0)}{P(y=0)}$$

$f(x \mid y=0)$ conditional for $y=0$

$$\begin{cases} 3/15 & x=0 \\ 9/15 & x=1 \\ 3/15 & x=2 \end{cases}$$

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ex) $f(x,y) = \begin{cases} 6e^{-(2x+3y)} & x>0,\ y>0 \\ 0 & \text{o.w.} \end{cases}$

$$f(x) = \int_{y=0}^{\infty} 6e^{-(2x+3y)}\,dy = \begin{cases} 2e^{-2x} & x>0 \\ 0 & \text{else} \end{cases}$$

$$f(y) = \int_{x=0}^{\infty} 6e^{-(2x+3y)}\,dx = \begin{cases} 3e^{-3y} & y>0 \\ 0 & \text{else} \end{cases}$$

$$f(x \mid y) = \frac{f(x,y)}{f(y)} = \frac{6e^{-(2x+3y)}}{3e^{-3y}} = \begin{cases} 2e^{-2x} & x>0 \\ 0 & \text{else} \end{cases}$$

$$f(x \mid y=1) = \begin{cases} 2e^{-2x} & x>0 \\ 0 & \text{else} \end{cases} \qquad f(y \mid x) = \begin{cases} 3e^{-3y} & y>0 \\ 0 & \text{else} \end{cases}$$

---

ex) $f(x,y) = \begin{cases} 2/7\,(5x+2y) & 0<x<1;\ 0<y<1 \\ 0 & \text{o.w.} \end{cases}$

else

find $f(x/y) = \frac{f(x,y)}{f(y)} = \frac{\frac{2}{7}(5x+2y)}{\frac{2}{7}(\frac{5}{2}+2y)} = \frac{5x+2y}{\frac{5}{2}+2y}$

$f(y) = \int_{x=0}^{1} \frac{2}{7}(5x+2y)\,dx = \frac{2}{7}(\frac{5}{2}+2y) \quad 0<y<1$

$f(x/y=\frac{1}{2}) = \frac{5x+1}{\frac{5}{2}+1} = \frac{10x+2}{7} \quad 0<x<1$

$P(x>\frac{1}{3} / y=\frac{1}{2}) = \int_{x=\frac{1}{3}}^{1} \frac{10x+2}{7}\,dx \Rightarrow$ Maths

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x \ y	5	10
1	1/10	3/10
2	4/10	2/10

$P(x>\frac{3}{2} / y=5)$

$f(x/y)$

$f(x/y=3)$

$f(x_1, x_2, x_3, x_4, x_5)$

$f(x_5 / x_1, x_2, x_3, x_4) = \frac{f(x_1, x_2, x_3, x_4, x_5)}{f(x_1, x_2, x_3, x_4)}$

$f(x_2, x_3 / x_1, x_4, x_5) = \frac{f(x_1, x_2, x_3, x_4, x_5)}{f(x_1, x_4, x_5)}$

$f(x_1, x_2, x_3, x_4, x_5) = f(x_1) f(x_2/x_1) f(x_3/x_1, x_2) f(x_4/x_1, x_2, x_3) f(x_5/x_1, x_2, x_3, x_4)$

ex) find $f(x_1, x_2)$ for:

x_1 = # heads on 1 flip of balanced coin

x_2 = # on roll of 6 sided die

x_1 and x_2 are independent

$f(x_1, x_2) = f(x_1) f(x_2)$

$f(x_1) = \begin{cases} 1/2 & x_1 = 0, 1 \end{cases}$ $\quad f(x_2) = \begin{cases} 1/6 & x_2 = 1,2,3,4,5,6 \end{cases}$

$\Rightarrow \begin{cases} 1/12 & x_1 = 0, 1 \\ & x_2 = 1,2,3,4,5,6 \end{cases}$

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	y = 1	y = 2	
x = 0	1/12	1/6	3/12
x = 1	1/6	1/3	6/12
	1/12	1/6	3/12
	4/12	8/12	

1.) $f(x)$

2.) $f(y)$

3.) $P(x \geq .5)$

4.) $P(y^2 < 3)$

5.) are x & y independent?

6.) $f(x/y=2)$

7.) $P(x > .5 \mid y=2)$

① ✓

$f(x) = \begin{cases} 3/12 & x = 0 \\ 6/12 & x = 1 \\ 3/12 & \end{cases}$

② ✓

$$f(x) = \begin{cases} 6/12 & x=1 \\ 3/12 & x=2 \end{cases}$$

② $$f(y) = \begin{cases} 4/12 & y=1 \\ 8/12 & y=2 \end{cases}$$

③ $P(x \geq 0.5) = 9/12$

④ $P(y^2 < 3) = 4/12$

⑤ Yes. Why? multiply marginals. if we get the joint in every case, it independent

⑥ $f(x \mid y=2) \Rightarrow 2^{nd}$ colum, all

$$\begin{cases} 1/4 & x=0 \\ 1/2 & x=1 \\ 1/4 & x=2 \end{cases}$$

⑦ $P(x > 0.5 \mid y=2)$
$= 3/4$