



## Butterworth Filter

$$|H(j\omega)|^2 = \frac{(\omega_0)^{2n}}{\omega_0^{2n} + \omega^{2n}}$$

$$-\omega_0^{2n} = \omega^{2n}$$

$$A(\omega) = 10 \log_{10} \left[ 1 + \left( \frac{\omega}{\omega_0} \right)^{2n} \right]$$

$$A(\omega) = -10 \log_{10} (|H(j\omega)|^2)$$

\* attenuation

⇒ This shows us that attenuation is the negative gain

We can use this expression at various values to find significant points.

e.g.  $A_{max} = A(\omega_p)$

$$A_{min} = A(\omega_s)$$

We can use algebra to solve for

$$\omega_o : \omega_o = \omega_s / (10^{A_{min}/10} - 1)^{1/2n}$$

$$\omega_o = \omega_p / (10^{A_{max}/10} - 1)^{1/2n}$$

$\omega_o$  is the radius of every pole

If we set the 2  $\omega_o$  equations equal to each other, we can solve for  $n$  which is filter Order

$$n = \frac{\ln \left( \frac{10^{A_{min}/10} - 1}{10^{A_{max}/10} - 1} \right)}{2 \ln \left( \omega_s / \omega_p \right)}$$

\* in matlab,  
 $\ln$  is "log"

