Polynomial chaos-based uncertainty quantification of material parameters

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This session focuses on the use of the polynomial chaos expansion for the uncertainty quantification of material parameters of a nonlinear constitutive behaviour. Three major steps of the uncertainty quantification field are illustrated for this problem: uncertainty propagation, sensitivity analysis, and inverse problem solving by Bayesian inference.

Physical model

The behaviour of the material of interest is modelled according to a Ramberg-Osgood nonlinear elastic constitutive equation. This behaviour satisfies a relation of the following form

$$\varepsilon = f(\sigma; X), \qquad X = (E, \nu, \sigma_0, \alpha, n),$$

where ε is the strain, σ the stress, E the Young modulus, ν the Poisson ratio, σ_0 the yield strength, α and n are numerical parameters. The material parameter $X = (E, \nu, \sigma_0, \alpha, n)$ is assumed to be a random vector drawn according to a prescribed distribution.

Given a sample of the material parameter, a tensile test and a simple shear test are simulated with the `MFront` material knowledge and the `mtest` unit mechanical behaviour testing softwares. Using these tools, we have at our disposal a numerical model that maps the material parameter to the stresses (strains are imposed) associated to the simulated experimental tests:

$$(E, \nu, \sigma_0, \alpha, n) = X \longmapsto Y = (\sigma_1, ..., \sigma_m).$$

Uncertainty quantification

The first step of the practical session will focus on the propagation of uncertainties. Indeed, sensitivity analysis and inference require a large number of model evaluations. In order to keep the computational cost low, the initial model is replaced by a metamodel that is inexpensive to evaluate. The metamodel is based here on the truncated polynomial chaos expansion. The parameters of the metamodel will be evaluated by a regression method commonly used in machine learning.

The second step will consist in performing a sensitivity analysis, i.e. quantifying the proportions of the model output uncertainties induced by the uncertainties of each input parameter. In particular, we will exploit the properties of polynomial chaos to efficiently evaluate sensitivity indices called Sobol indices.

The third and final step will aim at solving the inverse problem of finding the distribution of the material parameter given experimental data. We will be particularly interested in optimization-based Bayesian inference methods that yield approximate posterior distributions.

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