Digital Twins, the experimental - numerical dialog

Jumeaux numériques et dialogue calculs/essais

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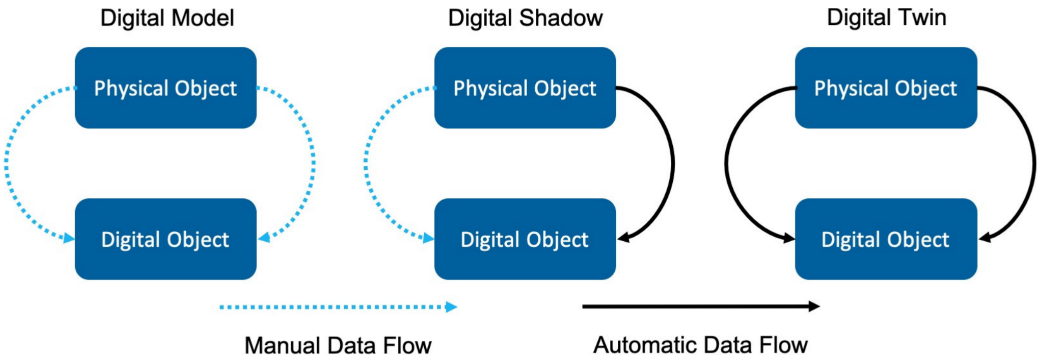
Laboratoire de Mécanique Paris-Saclay

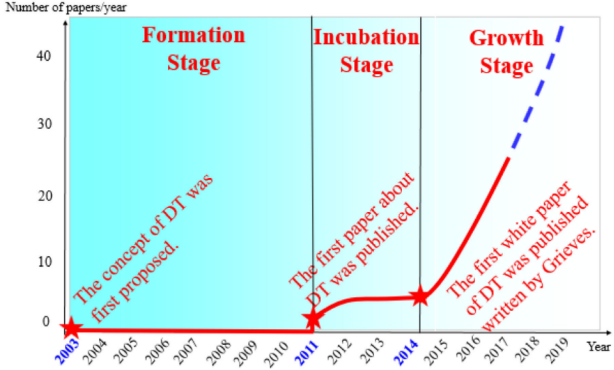
What is a digital twin



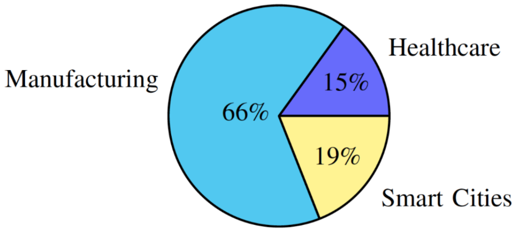
[1]

[1] Fuller2020.

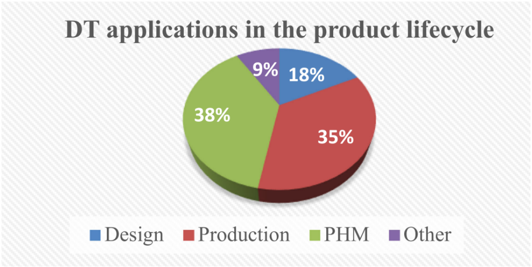
Literature activity



[2]



[3]



[4]

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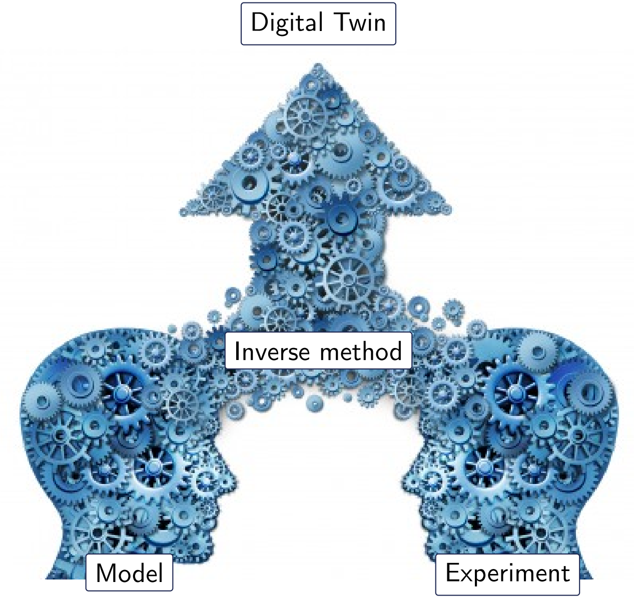
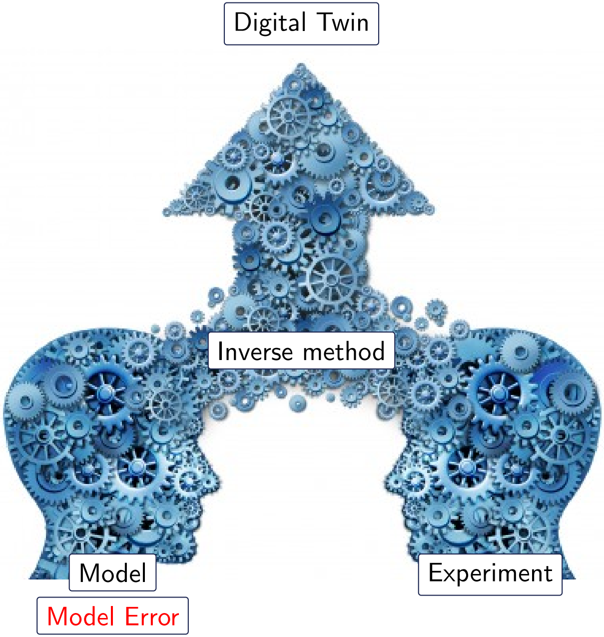
[2] Tao2019.
[3] Fuller2020.
[4] Tao2019.

The digital twin in material mechanics



The digital twin in material mechanics



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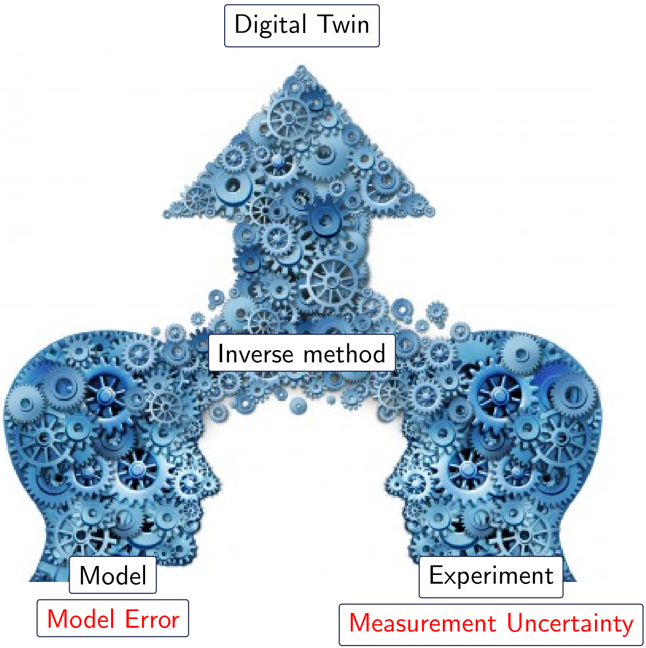
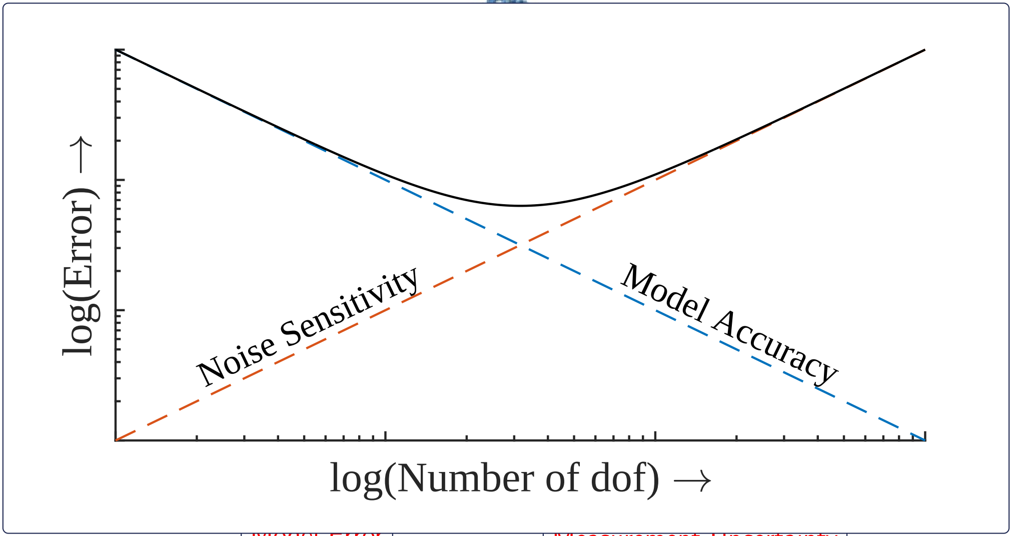


The digital twin in material mechanics

Digital Twin

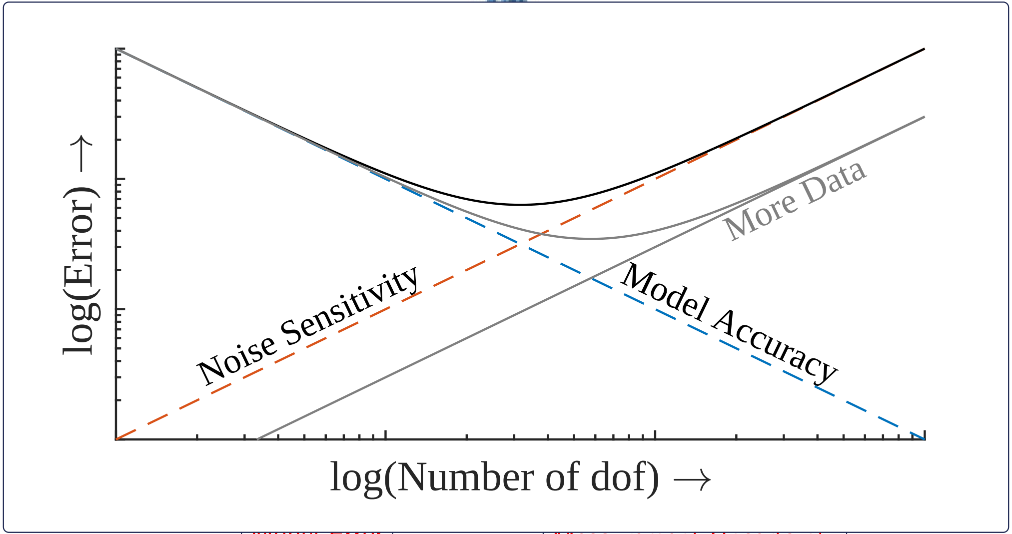


Model Error

Measurement Uncertainty

The digital twin in material mechanics

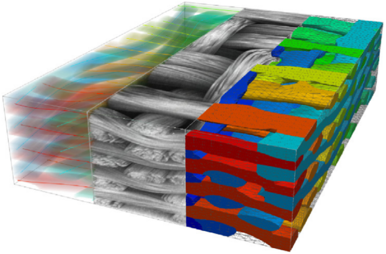
Digital Twin



Model Error

Measurement Uncertainty

Example 1: Woven composite segmentation

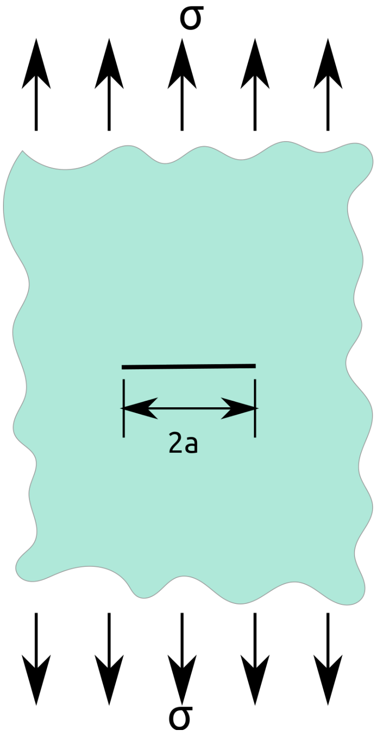


5

- Model: Yarn topology
- Experiment: Tomography (Gigavoxel)
- Dialog: Virtual image / yarn locations / shape

[5] **Mendoza2019a**.

Example 2: Measuring Stress Intensity Factors



DIC for Cracks - Williams Series [Williams 1951]

$$\vec{u}(\vec{z}) = \sum_{j=I}^{II} \sum_{n=n_{\mathbf{min}}}^{n_{\mathbf{max}}} p_n^j \varphi_n^j(\vec{z})$$

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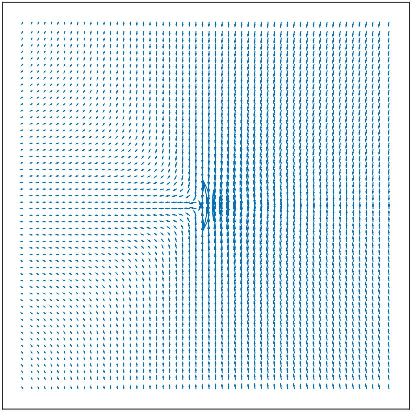
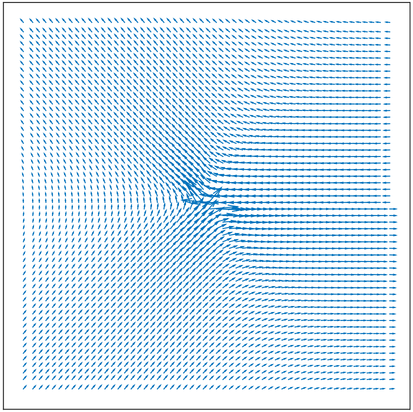


n = -1, Mode II,



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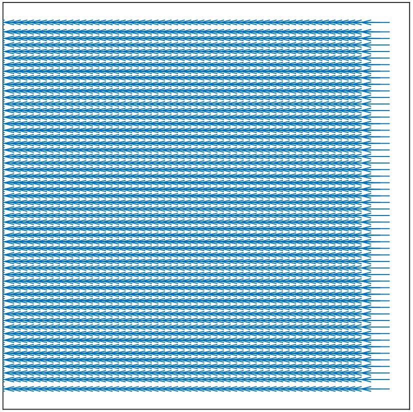

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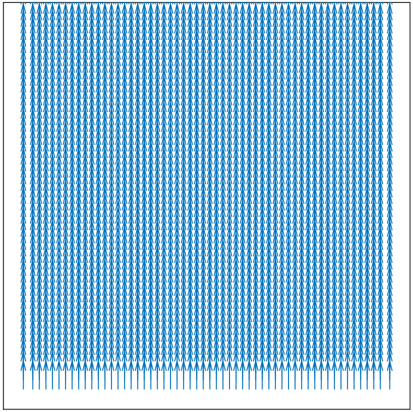

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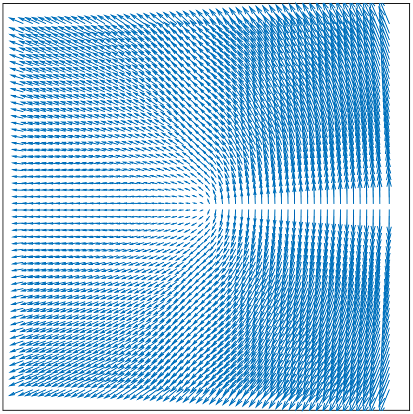

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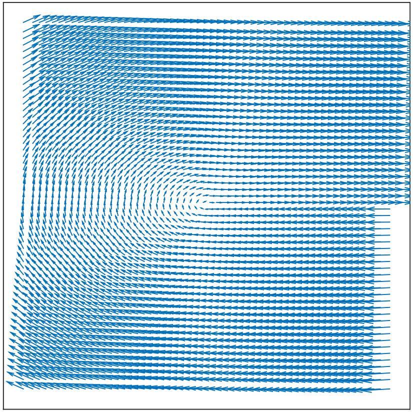

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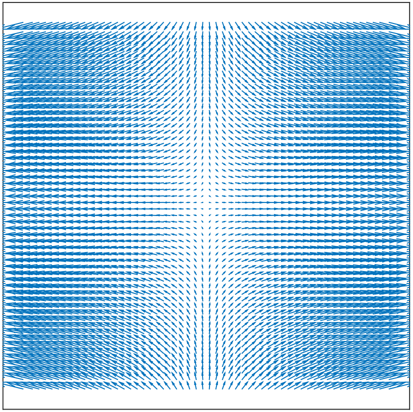
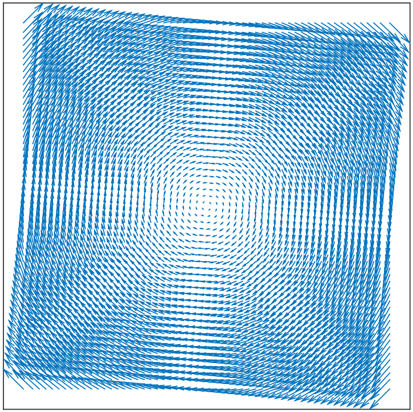
n = 2, Mode I,



n = 2, Mode II,



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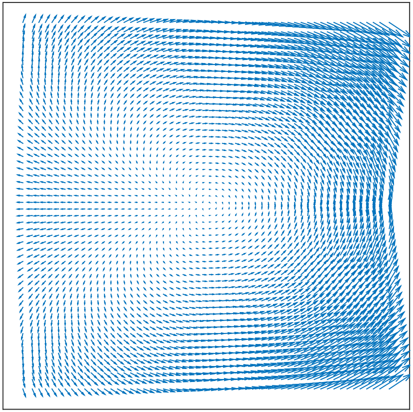
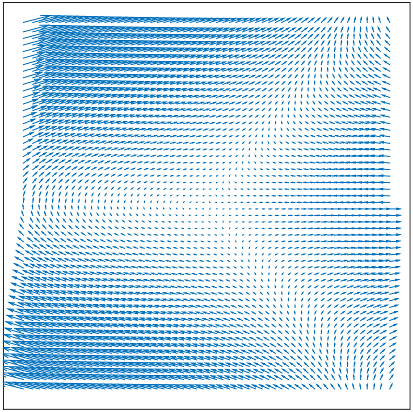
n = 3, Mode I,



n = 3, Mode II,



DIC for Cracks - Region of Interest

Williams Hypotheses:

- Semi-Infinite Medium
- Elasticity

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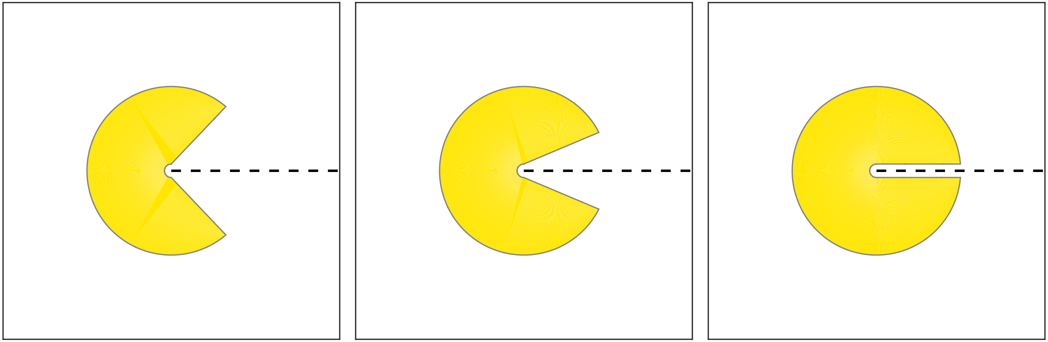

DIC for Cracks - Region of Interest

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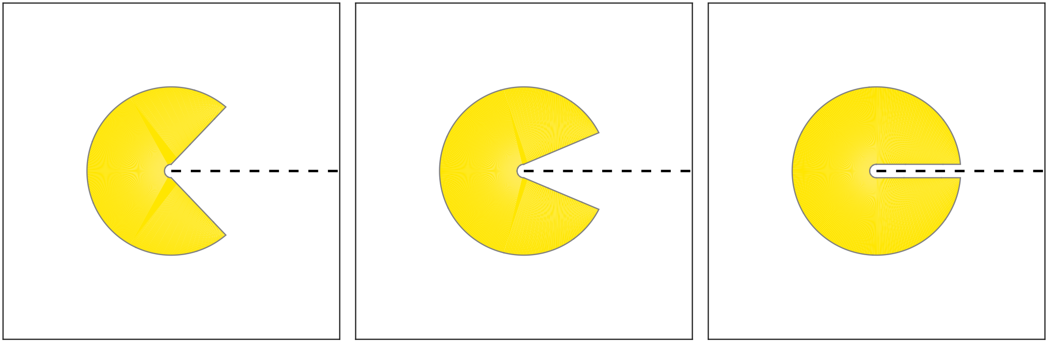


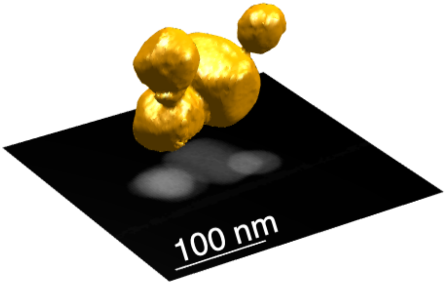
- Exclude the crack
- Exclude the Fracture Process Zone
- Exclude the border effects

Digital Twin

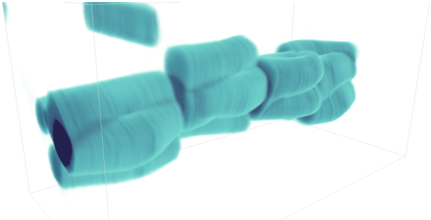
- Model: Williams series
- Experiment: images
- Dialogue: Williams amplitudes / Crack tip position / Stress Intensity Factor

Example 3: TEM Tomograpy

Experiment



Sinogram



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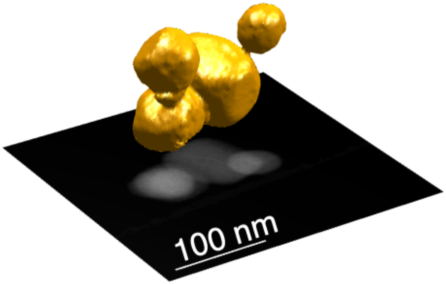

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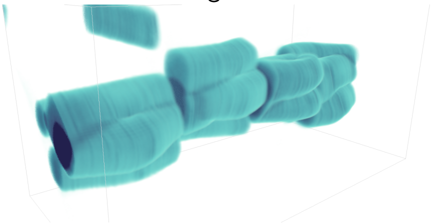

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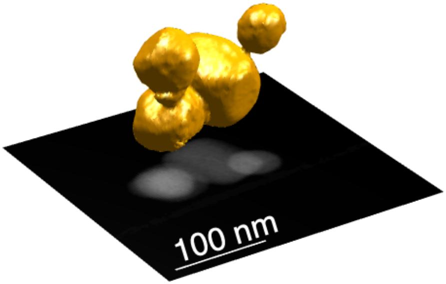

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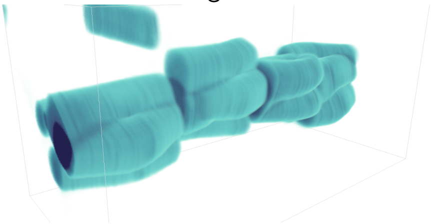

Tomography

$R = F - \Pi(\vec{p})\ V$
$F =$ Sinogram
$V =$ Volume (unknown)
$\Pi =$ Projection matrix

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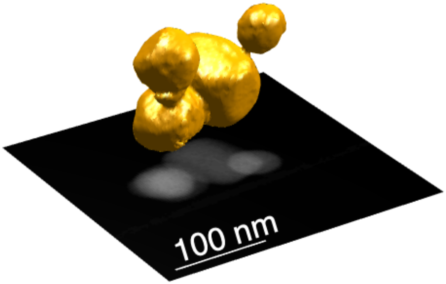

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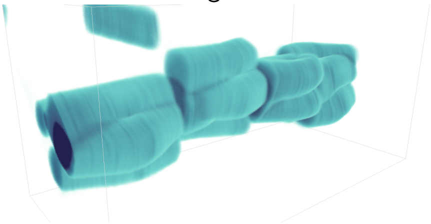

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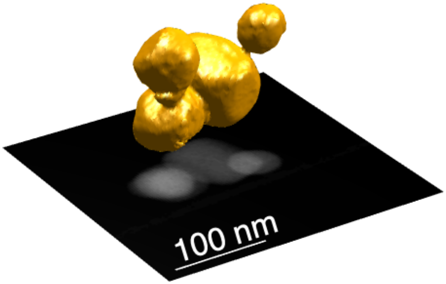

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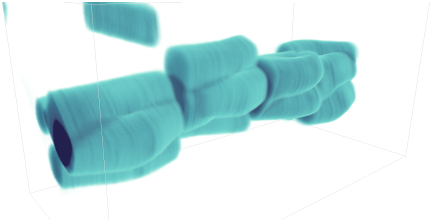

Tomography

$R = F - \Pi(\color{red}{\vec{p}}\color{black})\, V$

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Π = Projection matrix

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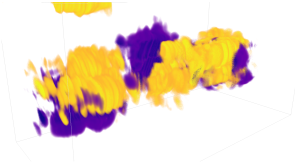

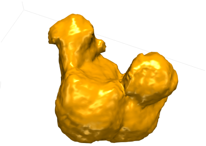

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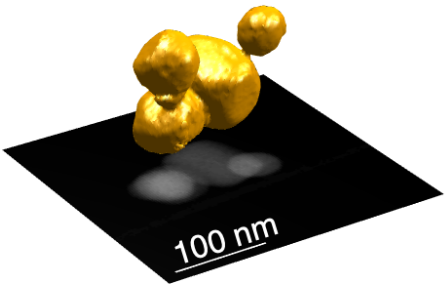

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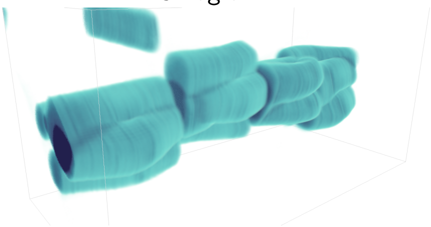

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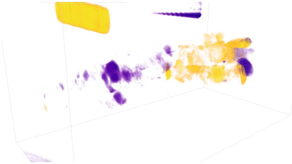

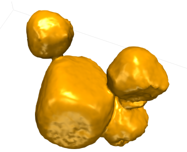

Example Case



- AA2219
- Notched sample
- 2 mm thickness

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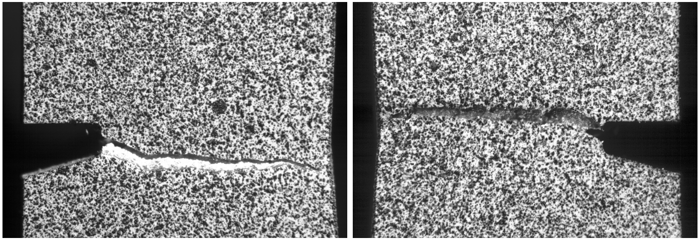
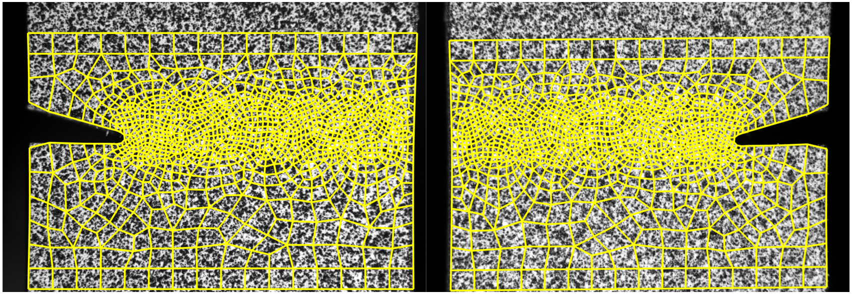
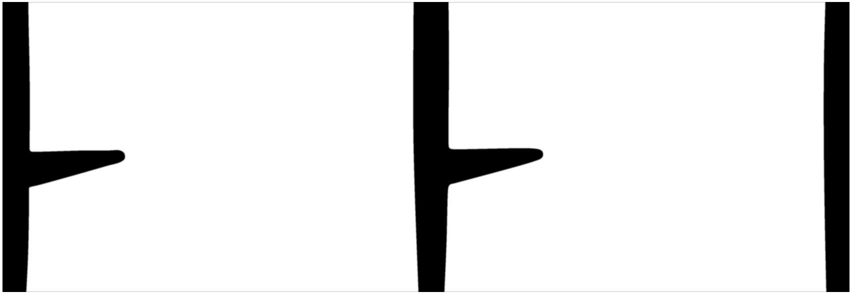


Camera Alignment

Global DIC with Q4 mesh:

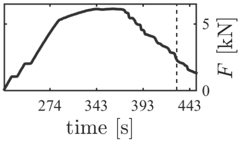


Aligning Cameras:



Correcting: Translation, Rotation, Magnification

Example Case - Measurement



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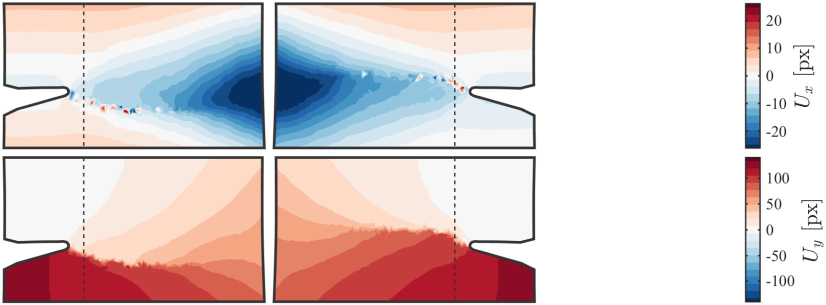

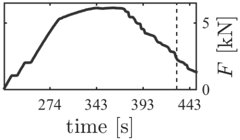

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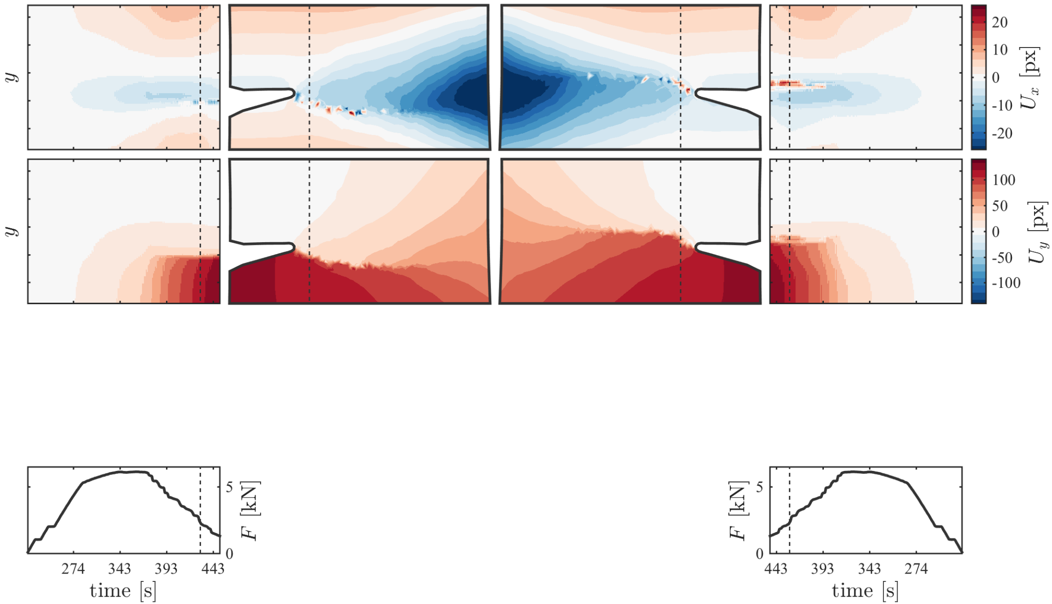

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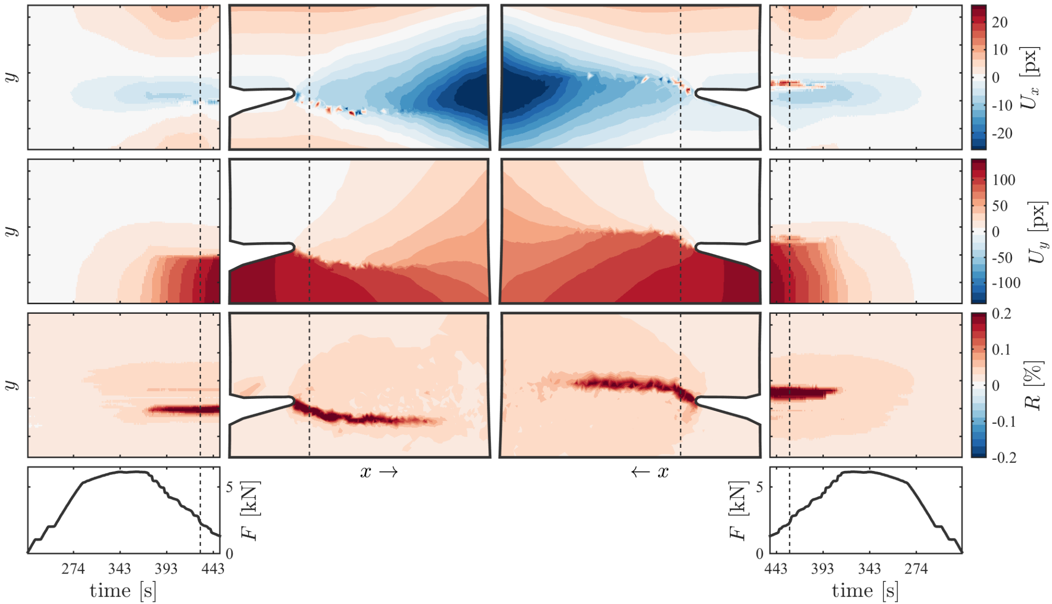

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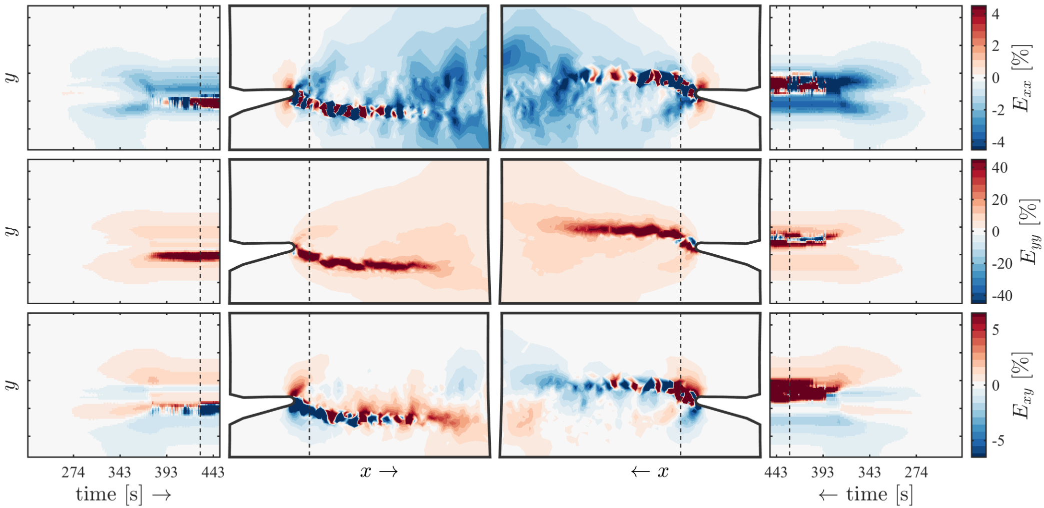
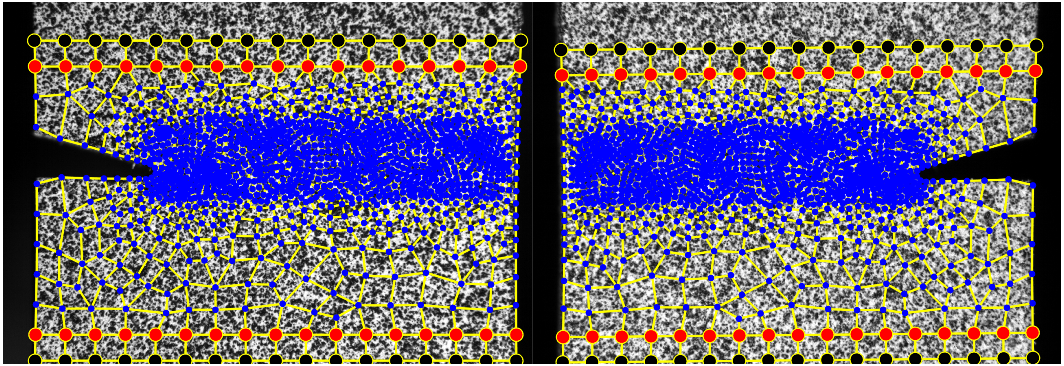


Calibrating the digital twin

Slanted Crack

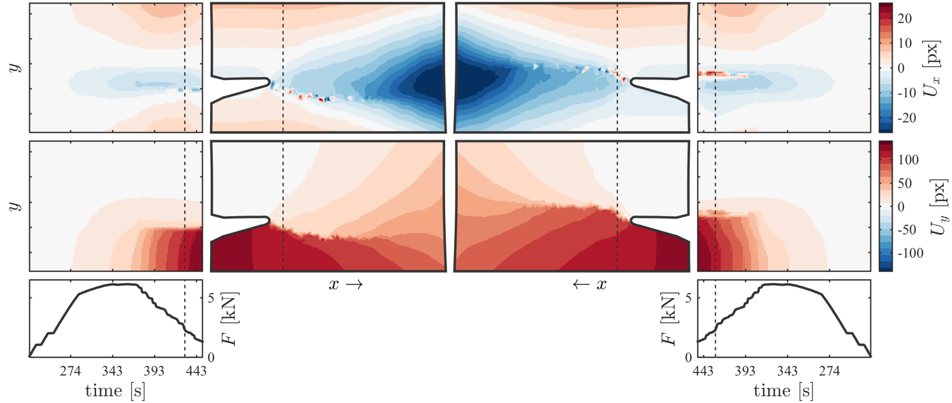
- Experimentally behavior is known
- Apply displacement boundary conditions everywhere
- Optimize on internal forces $\rightarrow$ Equilibrium Gap Method[6]



[6]Claire2004.

Calibrating the digital twin

Measurement:



Calibrating the digital twin

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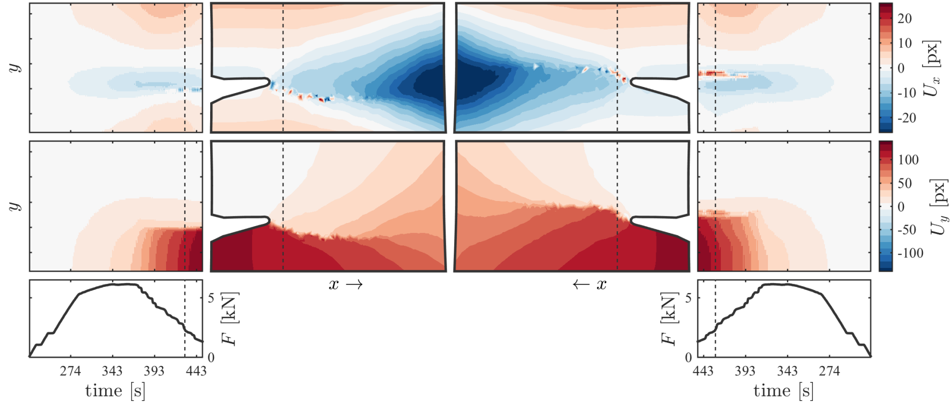
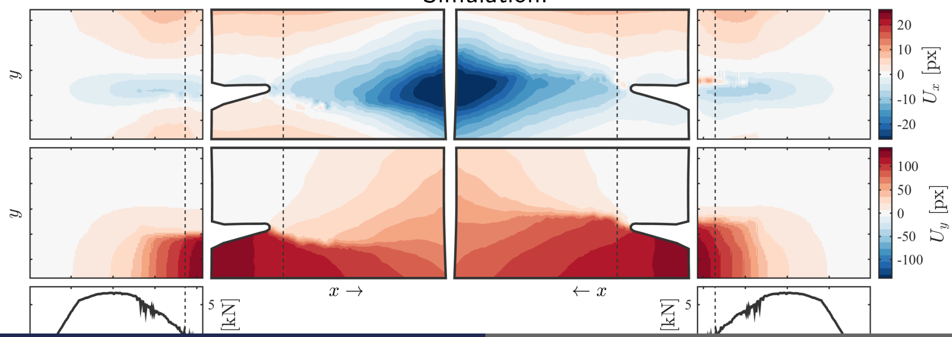
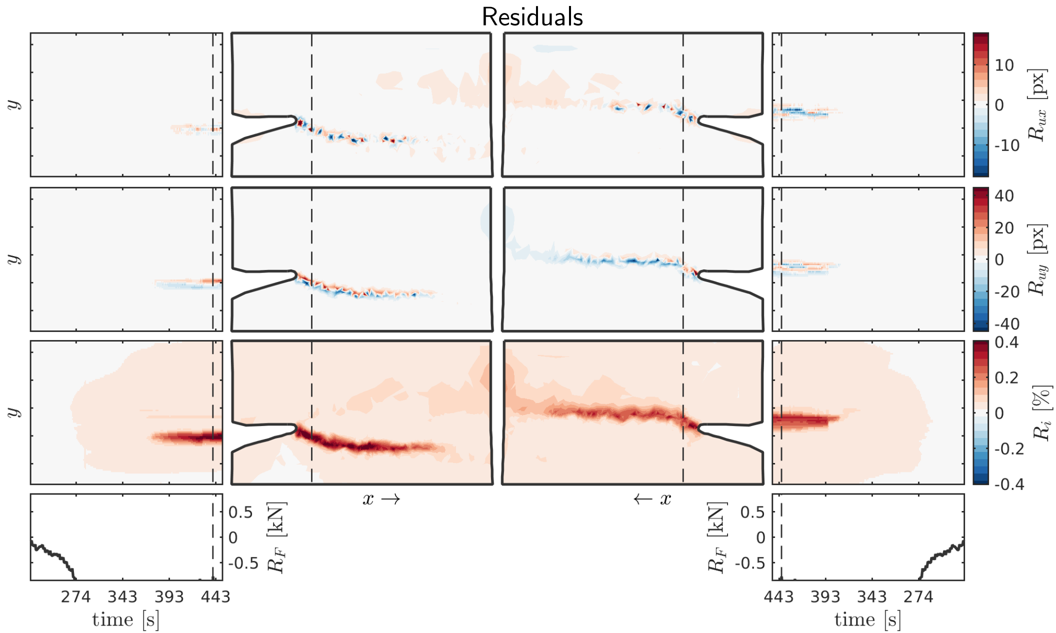


Simulation:



Calibrating the digital twin



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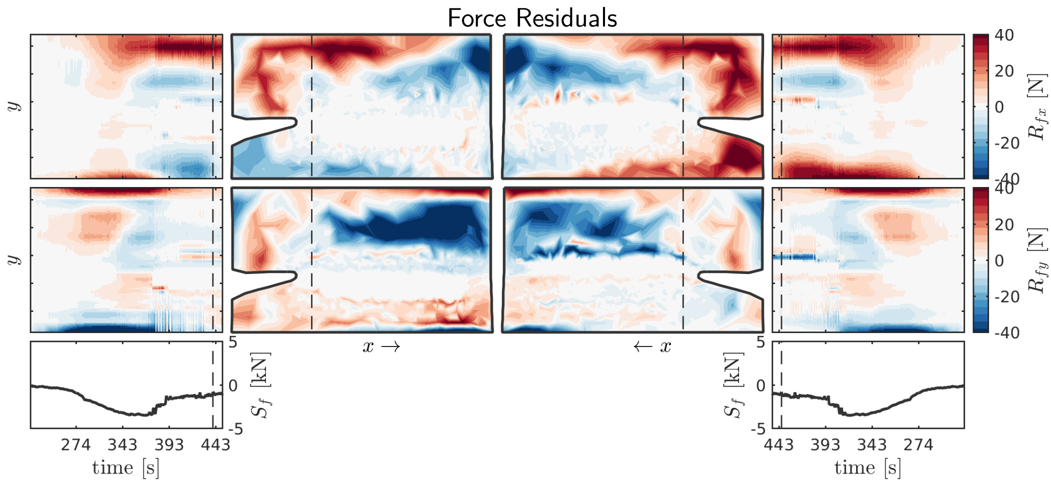
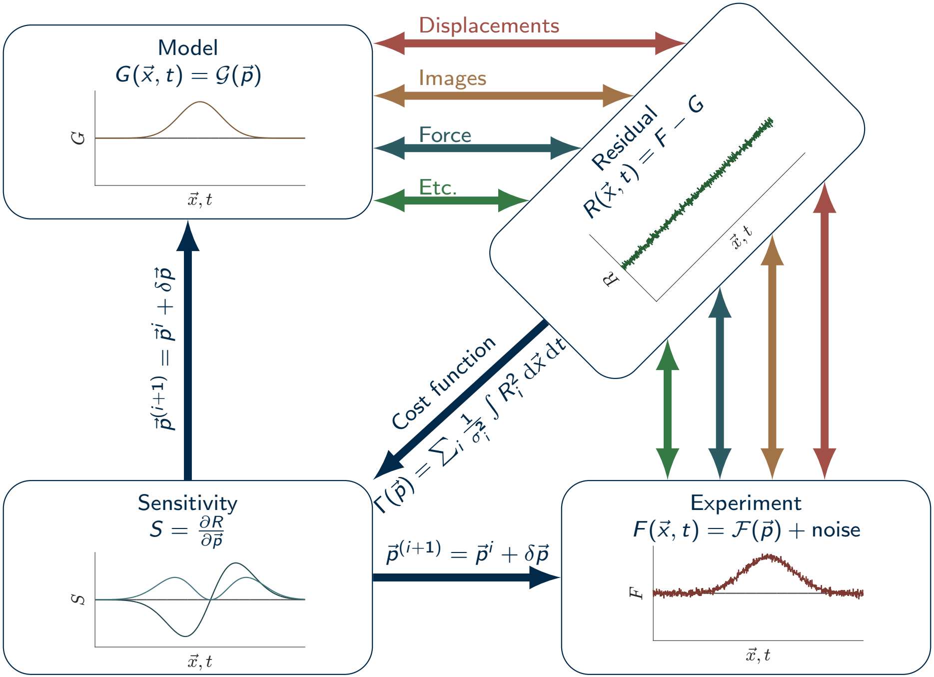


Measuring with Models



Defining the distance

Goals

- Use all available data,
- Consider uncertainty
- Weight each data source

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Consider N observables

$$x_i = \mathcal{G}_i(\{p\})$$

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$$\hat{x}_i = x_i + \zeta_i$$

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Probability of one observable being x_i

$$P_i = \frac{1}{(2\pi)^{1/2}\gamma_i} \exp\left(-\frac{(\hat{x}_i - x_i)^2}{2\gamma_i^2}\right),$$

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$$\eta^2 = \sum_{i=1}^{N}\frac{(\hat{x}_i - x_i)^2}{\gamma_i^2}$$

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Different observables $N = N_1 + N_2$

$$\eta^2 = \frac{1}{\gamma_1^2}\sum_{i=1}^{\boldsymbol{N_1}}(\hat{x}_i - x_i)^2 + \frac{1}{\gamma_2^2}\sum_{j=\boldsymbol{N_1}+1}^{N}(\hat{x}_j - x_j)^2$$

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Measurement aggregation

$$\eta^2 = \eta_1^2 + \eta_2^2$$

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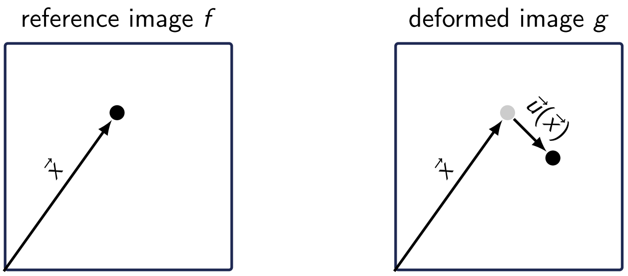
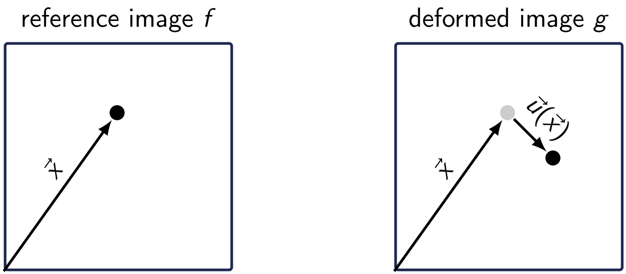
Weight factors

$$\omega_i = \frac{1}{\gamma_i^2}$$

Digital Image Correlation



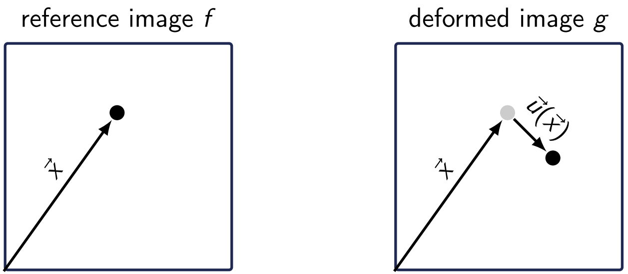
Digital Image Correlation



Brightness Conservation

$$f(\vec{x}) \approx g(\vec{x} + \vec{u}(\vec{x}, \{a\}))$$

Digital Image Correlation



Brightness Conservation

$$f(\vec{x}) \approx g(\vec{x} + \vec{u}(\vec{x}, \{a\}))$$

Least Squares cost function

$$\eta^2 = \frac{1}{2\gamma^2} \sum_{k=1}^{N_k} \Big(f(\vec{x}_k) - g(\vec{x}_k + \vec{u}(\vec{x}_k, \{a\})) \Big)^2$$

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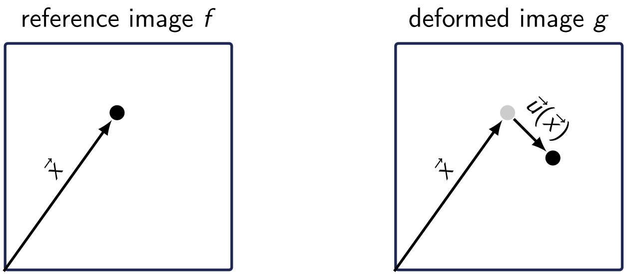

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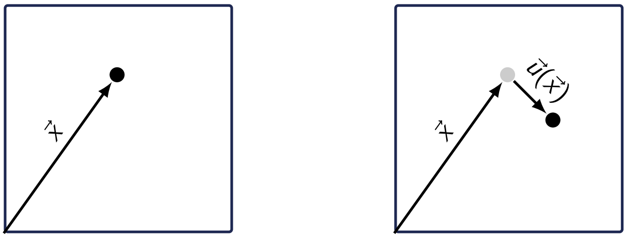
Iterative Optimization

$$\{a\}^{(\text{it}+1)} = \{a\}^{\text{it}} + \{\delta a\}$$

$$[M]\{\delta a\} = \{b\}$$

Digital Image Correlation

reference image f deformed image g



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Basis functions

$$\vec{\varphi}_i = \frac{\partial \vec{u}}{\partial a_i}$$

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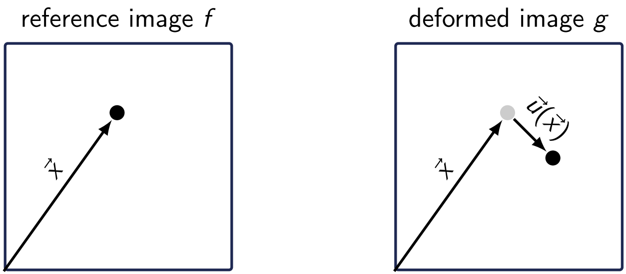
$$\{a\}^{(\text{it}+1)} = \{a\}^{\text{it}} + \{\delta a\}$$

$$[M]\{\delta a\} = \{b\}$$

$$M_{ij} = \frac{1}{2\gamma^2} \sum_{k=1}^{N_k} \vec{\nabla} f_k \vec{\varphi}_{ki} \vec{\varphi}_{kj} \vec{\nabla} f_k$$

$$b_i = \frac{1}{2\gamma^2} \sum_{k=1}^{N_k} \vec{\nabla} f_k \vec{\varphi}_{ki} r_k$$

Digital Image Correlation



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Global DIC:
$\vec{\varphi} \rightarrow$ FE-shapefunctions
$\{a\} \rightarrow$ nodes

Implementing DIC

$$M_{ij} = \frac{1}{2\gamma^2} \sum_{k=1}^{N_k} \vec{\nabla} f_k \vec{\varphi}_{ki} \vec{\varphi}_{kj} \vec{\nabla} f_k$$

$$b_i = \frac{1}{2\gamma^2} \sum_{k=1}^{N_k} \vec{\nabla} f_k \vec{\varphi}_{ki} r_k$$

$$r_k = f(\vec{x}_k) - \tilde{g}(\vec{x}_k)$$

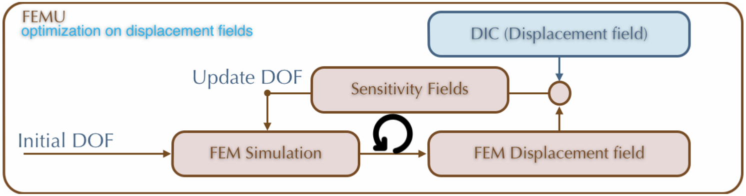
$$\tilde{g} = g(\vec{x}_k + \vec{u}(\vec{x}_k, \{a\}))$$

$$[M]\{\delta a\} = \{b\}$$

$$\{a\}^{(\mathrm{it}+1)} = \{a\}^{\mathrm{it}} + \{\delta a\}$$

```
% image gradient
[fx, fy] = gradient(f);

% shapefunctions
phi = TriangleShapefunGridded(coor, conn, size(f));

% back-deformed image
gt = interp2(X,Y,g,X+Ux,Y+Uy)

% residual
r = f - gt;

% The shapefunction gradient image product
Lx = phi .* repmat( fx(:), 1, Nn);
Ly = phi .* repmat( fy(:), 1, Nn);

% Hessian
M = zeros( 2*Nn, 2*Nn );
M(Ix,Ix) = transpose(Lx) * Lx;
M(Ix,Iy) = transpose(Lx) * Ly;
M(Iy,Ix) = transpose(Ly) * Lx;
M(Iy,Iy) = transpose(Ly) * Ly;

% the Jacobian matrix
b = zeros(2*Nn,1);
b(Ix,1) = transpose(Lx) * res(:);
b(Iy,1) = transpose(Ly) * res(:);

% solve the system
da = M \ b;

% update the dof
a = a + da;
```

Finite Element Method Updating



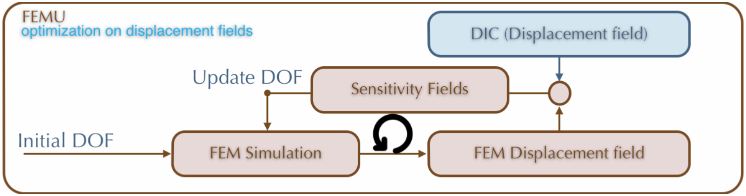
FEMU-U displacement cost function

$$\eta_U^2(\{p\}) = \frac{1}{2\gamma_U^2} \sum_{t=1}^{N_t} \sum_{k=1}^{N_k} \left(U_{tk}^{\text{exp}} - U_{tk}(\{p\}) \right)^2$$

FEMU-U system of equations

$$\begin{aligned} [H_U]\{\delta p\} &= \{J_U\} \\ [H_U]_{ij} &= [S_U]_i [S_U]_j \\ [J_U]_i &= [S_U]_i \{R_U\} \end{aligned}$$

Finite Element Method Updating



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FEMU-F cost function

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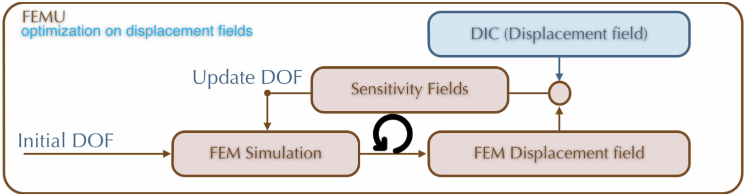
Finite Element Method Updating



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FEMU-F cost function

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FEMU-UF cost function

$$\eta_e^2 = \eta_I^2 + \eta_F^2$$

FEMU-U system of equations

$$\begin{aligned} [H_U]\{\delta p\} &= \{J_U\} \\ [H_U]_{ij} &= [S_U]_i [S_U]_j \\ [J_U]_i &= [S_U]_i \{R_U\} \end{aligned}$$

FEMU-F system of equations

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FEMU-UF system of equations

$$([H_U] + [H_F])\{\delta p\} = \{J_U\} + \{J_F\}$$

Implementing FEMU

$$[H_U]\{\delta p\} = \{J_U\}$$
$$[H_U]_{ij} = [S_U]_i[S_U]_j$$
$$[J_U]_i = [S_U]_i\{R_U\}$$

$$[H_F]\{\delta p\} = \{J_F\}$$
$$[H_F]_{ij} = [S_F]_i[S_F]_j$$
$$[J_F]_i = [S_F]_i\{R_F\}$$

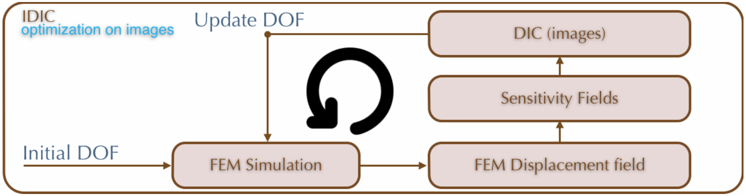
$$[S_U]_i = \frac{\partial U_{tk}(\{p\})}{\partial p_i} \approx \frac{U_{tk}(\{p\}^{\text{per}}) - U_{tk}(\{p\}^{\text{ref}})}{p_i^{\text{per}} - p_i^{\text{ref}}}$$
$$[S_F]_i = \frac{\partial F_t(\{p\})}{\partial p_i} \approx \frac{F_{tk}(\{p\}^{\text{per}}) - F_{tk}(\{p\}^{\text{ref}})}{p_i^{\text{per}} - p_i^{\text{ref}}}$$
$$p_i^{\text{per}} = p_i^{\text{ref}} + \epsilon p_i^{\text{ref}}$$

Relative sensitivity

$$[S_U]_i^* \approx \frac{U_{tk}(\{p\}^{\text{per}}) - U_{tk}(\{p\}^{\text{ref}})}{\epsilon} = [S_U]_i p_i^{\text{ref}}$$
$$[S_F]_i^* \approx \frac{F_{tk}(\{p\}^{\text{per}}) - F_{tk}(\{p\}^{\text{ref}})}{\epsilon} = [S_F]_i p_i^{\text{ref}}$$

```
% run abaqus
[Uref, Fref, Su, Sf] = abaqus(p, ABQ);

% compute the residual
Ru = Uexp - Uref;
Rf = Fexp - Fref;

% compute the Hessian
Hu = transpose(Su) * Su;
Hf = transpose(Sf) * Sf;

% compute the Jacobian matrix
Ju = transpose(Su) * Ru;
Jf = transpose(Sf) * Rf;

% combine the two systems
H = Wu * Hu + Wf * Hf;
J = Wu * Ju + Wf * Jf;

% solve for the update in the parametesr:
dp = H \ J;

% update the parameters
p = p + dp .* p;
```

Integrated DIC



I-DIC cost function

$$\eta_I^2(\{p\}) = \frac{1}{2\gamma_I^2} \sum_{t=1}^{N_t} \sum_{k=1}^{N_k} \left(f_k - \tilde{g}_{kt}(\{p\}) \right)^2$$

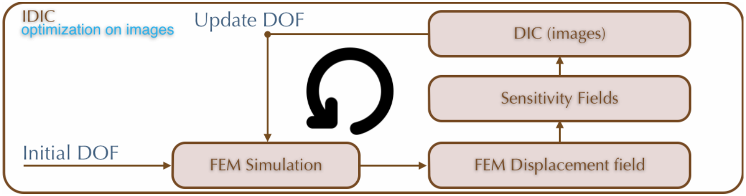
IDIC system of equations

$$[H_I]\{\delta p\} = \{J_I\}$$

$$[H_I]_{ij} = [S_U]_i \{\vec{\nabla} f\} [\vec{\varphi}]^T [\vec{\varphi}] \{\vec{\nabla} f\} [S_I]_j$$

$$[J_I]_i = [S_U]_i \{\vec{\nabla} f\} [\vec{\varphi}]^T \{r\}$$

Integrated DIC



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Force cost function

$$\eta_F^2(\{p\}) = \frac{1}{\gamma_F^2}\sum_{t=1}^{N_t}(F_t^{\exp} - F_t(\{p\}))^2$$

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$$[H_I]\{\delta p\} = \{J_I\}$$

$$[H_I]_{ij} = [S_U]_i\{\vec{\nabla} f\}[\vec{\varphi}]^T[\vec{\varphi}]\{\vec{\nabla} f\}[S_I]_j$$

$$[J_I]_i = [S_U]_i\{\vec{\nabla} f\}[\vec{\varphi}]^T\{r\}$$

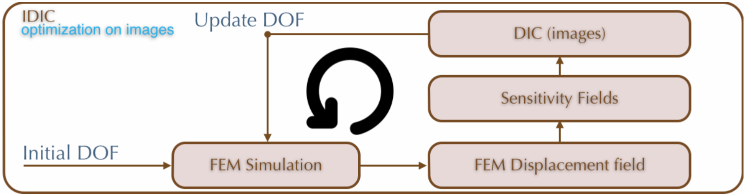
Force system of equations

$$[H_F]\{\delta p\} = \{J_F\}$$

$$[H_F]_{ij} = [S_F]_i[S_F]_j$$

$$[J_F]_i = [S_F]_i\{R_F\}$$

Integrated DIC



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Single experiment cost function

$$\eta_e^2 = \eta_I^2 + \eta_F^2$$

IDIC system of equations

$$[H_I]\{\delta p\} = \{J_I\}$$
$$[H_I]_{ij} = [S_U]_i \{\vec{\nabla} f\}[\vec{\varphi}]^T[\vec{\varphi}]\{\vec{\nabla} f\}[S_I]_j$$
$$[J_I]_i = [S_U]_i \{\vec{\nabla} f\}[\vec{\varphi}]^T\{r\}$$

Force system of equations

$$[H_F]\{\delta p\} = \{J_F\}$$
$$[H_F]_{ij} = [S_F]_i [S_F]_j$$
$$[J_F]_i = [S_F]_i \{R_F\}$$

Total system of equations

$$([H_I] + [H_F])\{\delta p\} = \{J_U\} + \{J_F\}$$

Implementing IDIC

$$[H_I]\{\delta p\} = \{J_I\}$$

$$[H_I]_{ij} = [S_U]_i\{\vec{\nabla} f\}[\vec{\varphi}]^T[\vec{\varphi}]\{\vec{\nabla} f\}[S_I]_j$$

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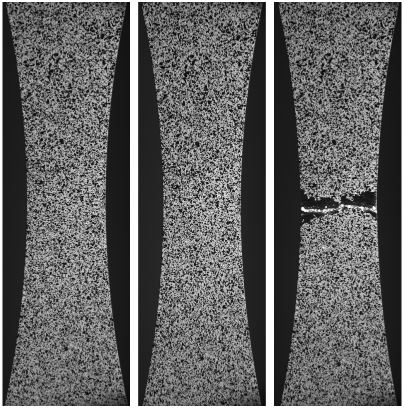
```
% compute the Hessian
Hu = transpose(Su) * M * Su;

% compute the Jacobian Matrix
Ju = transpose(Su) * b;
```

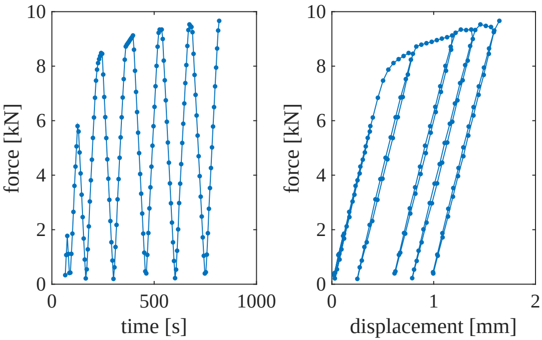
Implementing IDIC

$$
\begin{aligned}
[H_I]\{\delta p\} &= \{J_I\} \\
[H_I]_{ij} &= [S_U]_i\{\vec{\nabla} f\}[\vec{\varphi}]^T[\vec{\varphi}]\{\vec{\nabla} f\}[S_I]_j \\
[J_I]_i &= [S_U]_i\{\vec{\nabla} f\}[\vec{\varphi}]^T\{r\} \\
&\approx [\vec{S_U}]_i[M]\delta\{a\} \rightarrow \text{FEMU}
\end{aligned}
$$

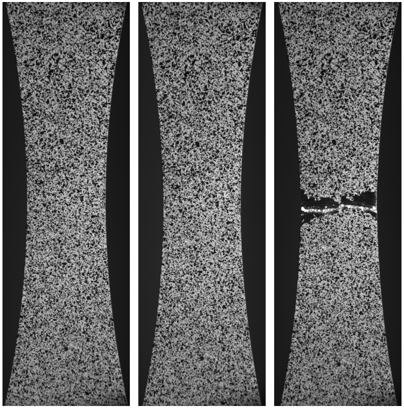
```
 % compute the Hessian
Hu = transpose(Su) * M * Su;

% compute the Jacobian Matrix
Ju = transpose(Su) * b;
```

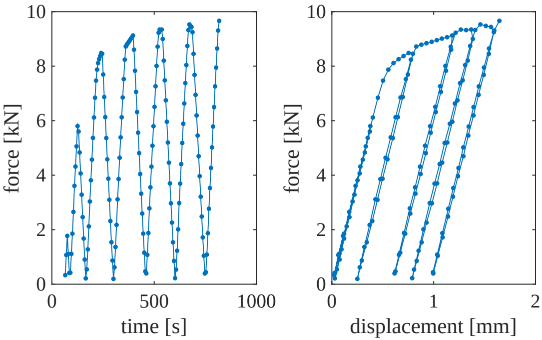
Workshop Case - AA2219





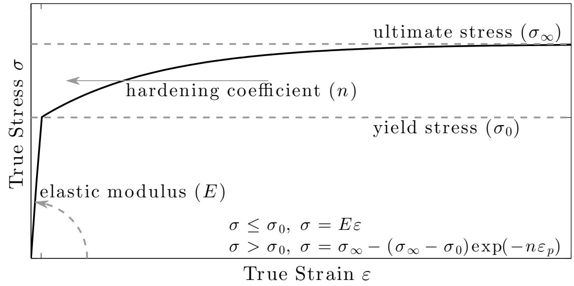
Workshop Case - AA2219





Example Case - Model

- Aluminum Alloy 2219
- 2 Elastic parameters E, ν
- 3 Isotropic Exponential Hardening parameters σ_0, σ_∞, n



Voce hardening

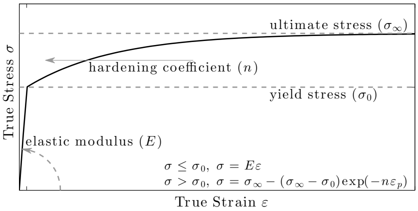
$$
\begin{aligned}
\sigma &= E\varepsilon, && \sigma \leq \sigma_0 \\
\sigma &= \sigma_\infty - (\sigma_\infty - \sigma_0)\exp(-n\varepsilon_p), && \sigma > \sigma_0
\end{aligned}
$$

[a]

[a] **Voce1955**.

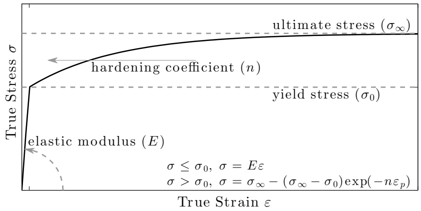
Stress-strain identification

- Extract Stress and Strain



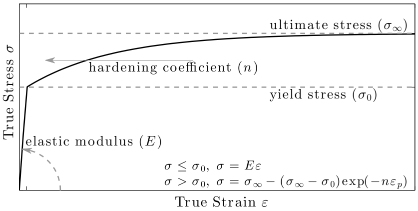
Stress-strain identification

- Extract Stress and Strain
- *DIC* $\rightarrow \varepsilon$
- $\sigma = F/A = F(1+\varepsilon)/A_0$



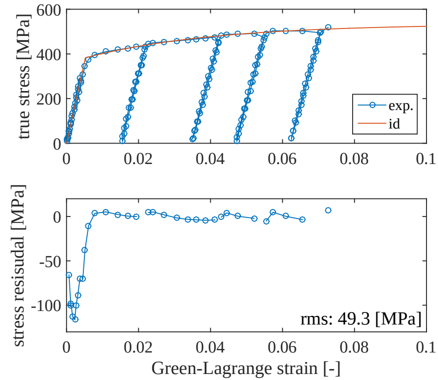
Stress-strain identification

- Extract Stress and Strain
- $DIC \rightarrow \varepsilon$
- $\sigma = F/A = F(1+\varepsilon)/A_0$
- 49 MPa stress residual



Parameters

p_1 E [GPa]	p_2 ν [-]	p_3 σ_0 [MPa]	p_4 σ_∞ [MPa]	p_5 n [-]
73.2	0.3*	382	534	28.2



Stress-strain identification

- Extract Stress and Strain
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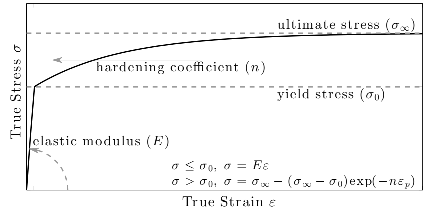
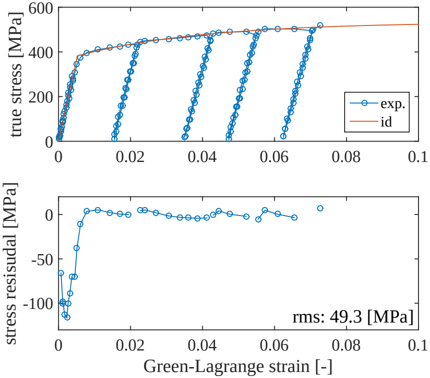
- $\sigma = F/A = F(1+\varepsilon)/A_0$
- 49 MPa stress residual

Parameters

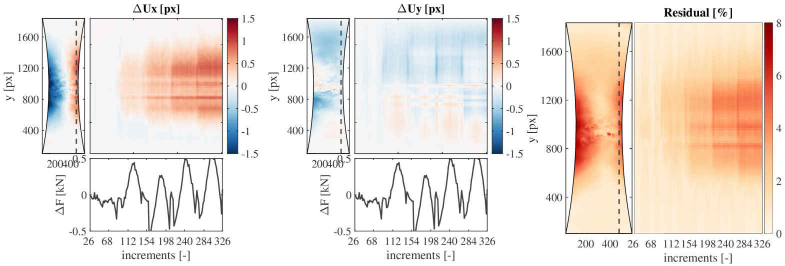
p_1	p_2	p_3	p_4	p_5
E	ν	σ_0	σ_∞	n
[GPa]	[-]	[MPa]	[MPa]	[-]
73.2	0.3*	382	534	28.2

- Is this a good identification?

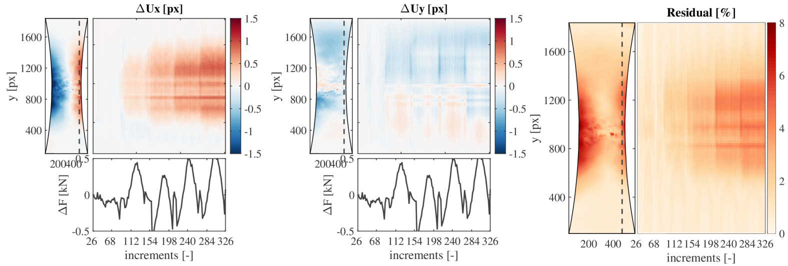




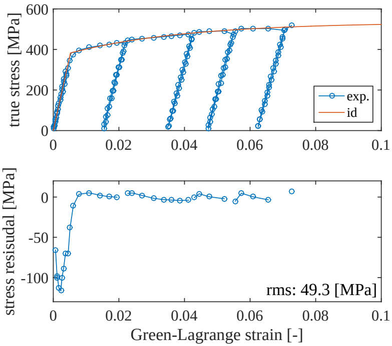
Stress-strain identification - Residuals



Stress-strain identification - Residuals

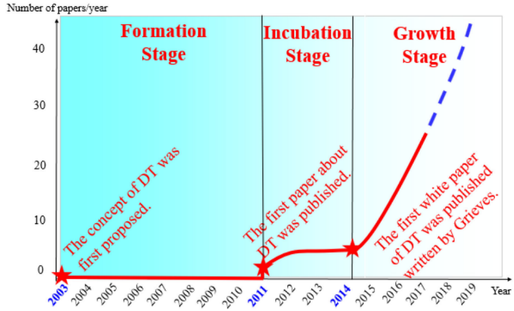


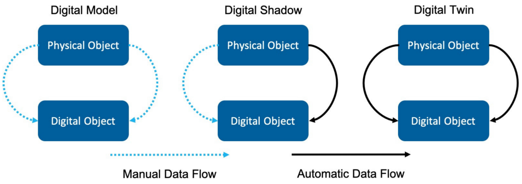
- Is this a good twin?



Conclusions

- Digital Twin
  - Fast-growing subject
  - A model that updates based on experimental data
  - Keep track of the physical state
  - Test virtual scenarios
  - Extract internal states
- Mini Symposium MS5
  - Wednessday morning
  - Julien RÉTHORÉ and Jean-Charles PASSIEUX





Exercises

three exercises:

1. DIC
2. FEMU
3. IDIC

at three different levels of programming difficulty:

1. easy
2. normal
3. hard

unzip the .zip you prefer $\rightarrow$ put the files in the main folder (together with /lib and the .mat files)

Exercise 01 - DIC

Easy

1. inspect the image series, what type of experiment is this?
2. what data type are the images after reading?
3. why convert them to floating point?
4. how is the DIC initialized? Does it matter? Can you think of a smarter choice?

Normal

1. what is the size of the support of a node?
2. each nodes has two degrees of freedom, how are they arranged in M, a and b?
3. how are the index vectors I_x and I_y used to achieve this?
4. is the DIC solution dependent on the initialization?
5. what influences the DIC accuracy?

Hard

1. are the shapefunctions φ vector fields or scalar fields?
2. how would you define the spatial resolution of GDIC?
3. is there a relation between M and the accuracy of the solution?

Exercise 02 - FEMU

Easy

1. what do the five parameters in $\{p\}$ represent?
2. what do the sensitivity fields represent?

Normal

1. are each of the parameters equally sensitive? How can you tell?
2. look at the help of abaqus.m, it discusses relative sensitivity fields. What is the consequence?
3. what is the relative weight of each of the nodes in the cost function?
4. the Levenberg-Marquardt implementation slows the convergences but improves robustness, try some different LM strengths. Which parameter becomes unstable first?

Hard

1. plot the 5 sensitivity fields, do you understand what they mean?
2. what are the eigenvectors and eigenvalues of H, do you understand what they mean?
3. improve the code by proposing a correction for the weight of each node
4. explain how the Levenberg-Marquardt works, what is the added cost function?

Exercise 03 - IDIC

Easy

1. run both the FEMU version of the code and the IDIC version of the code, how are they different?

Normal

1. this time the displacement fields are scaled with the pixelsize, why?
2. similarly, we use the gray value uncertainty γ_i to weight each cost function, why?
3. under which conditions would IDIC and FEMU-M be equal? and when would they be different
4. after having calibrated our model, what does the digital twin allow you to do?

Hard

1. $[S_U]^T[M][S_U]$ is easier to compute in a loop over the increments, why?
2. explain why the two Jacobian matrices $\{J_U\}$ (FEMU-M and IDIC) are approximate each other.