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Dear Editors,

I am sending you our research article entitled *"Intrusive acceleration strategies for Uncertainty Quantification for hyperbolic systems of conservation laws"*. Uncertainty Quantification for hyperbolic problems becomes a challenging task, especially when the solution shows shocks in the random space. To enable the use of adaptivity, we use intrusive methods, which provide a set of equations describing the time evolution of the polynomial chaos (PC) coefficients. The Intrusive Polynomial Moment (IPM) method possesses a large number of desirable quantities that are violated by standard intrusive methods such as stochastic-Galerkin. However, the IPM system requires solving a convex optimization problem which computes the entropy variables corresponding to a given set of PC coefficients. This optimization problem needs to be solved in every spatial cell in each time step, yielding high computational costs.

In our paper, we propose different methods to accelerate the IPM algorithm. First, we introduce a framework using adaptivity in the random space, which allows refining the solution in spatial cells that show shocks or complex structures in the uncertain domain. Furthermore, to accelerate the IPM optimization problem for steady problems, we propose to not solve the IPM optimization problem exactly. Thereby, we converge the entropy variables and PC coefficients to their steady state simultaneously, which significantly speeds up the calculation. We can show that the proposed method has local convergence, which is the same convergence result as provided by the standard IPM algorithm. The effectiveness of the proposed methods is shown by studying an uncertain NACA0012 as well as a pipe test case for uncertainties up to dimension three. The results are compared with Stochastic Collocation methods, showing that the use of the proposed acceleration techniques enables intrusive methods to compete with non-intrusive methods.

The presented theoretical findings and numerical methods could have a considerable impact on the uncertainty quantification community. While non-intrusive techniques are gaining popularity, we believe that the proposed adaptive algorithm makes IPM (and intrusive methods in general) an important tool to quantify uncertainties for hyperbolic problems, which often show uncertain shocks in small portions of the spatial domain. A further novelty of this paper is the IPM code, which can be run for two dimensional domains and high-dimensional uncertainties. The code is made publicly available to guarantee reproducibility of the presented results.

We confirm that this manuscript has not been published elsewhere and is not under consideration by another journal. All authors have approved the manuscript and agree with its submission to the "Methods and Algorithms for Scientific Computing" category of the SIAM Journal on Scientific Computing.

Yours sincerely,
(on behalf of the authors)

J. Kusch

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