α. Θέτω $\log_2(\sqrt[3]{2}) = x$ άρα

$$\log_2(\sqrt[3]{2}) = x \Rightarrow 2^x = \sqrt[3]{2} \Rightarrow$$
$$\Rightarrow 2^x = 2^{\frac{1}{3}} \Rightarrow x = \frac{1}{3}$$

άρα $\log_2(\sqrt[3]{2}) = \frac{1}{3}$.

β. Θέτω $\log_2 \sqrt[4]{8} = x$

$$\log_2 \sqrt[4]{8} = x \Rightarrow 2^x = \sqrt[4]{8} \Rightarrow 2^x = \sqrt[4]{2^3} \Rightarrow 2^x = 2^{\frac{3}{4}} \Rightarrow x = \frac{3}{4}$$

άρα $\log_2 \sqrt[4]{8} = \frac{3}{4}$.

γ. Θέτω $\log_3 \sqrt[3]{9} = x$

$$\log_3 \sqrt[3]{9} = x \Rightarrow 3^x = \sqrt[3]{9} \Rightarrow 3^x = \sqrt[3]{3^2} \Rightarrow 3^x = 3^{\frac{2}{3}} \Rightarrow x = \frac{2}{3}$$

άρα $\log_3 \sqrt[3]{9} = \frac{2}{3}$.

δ. Θέτω $\log_4 \frac{1}{2\sqrt[5]{16}} = x$

$$\log_4 \frac{1}{2\sqrt[5]{16}} = x \Rightarrow 4^x = 2\sqrt[5]{16} \Rightarrow$$
$$\Rightarrow 4^x = 2\sqrt[5]{2^4} \Rightarrow$$
$$\Rightarrow (2^2)^x = 2 \cdot 2^{\frac{5}{4}} \Rightarrow$$
$$\Rightarrow 2^{2x} = 2^{\frac{9}{4}} \Rightarrow 2x = \frac{9}{4} \Rightarrow x = \frac{9}{8}$$

άρα $\log_4 \frac{1}{2\sqrt[5]{16}} = \frac{9}{8}$.

ε. Θέτω $\log_5 \sqrt[3]{5} = x$

$$\log_5 \sqrt[3]{5} = x \Rightarrow 5^x = \sqrt[3]{5} \Rightarrow 5^x = 5^{\frac{1}{3}} \Rightarrow x = \frac{1}{3}$$

άρα $\log_5 \sqrt[3]{5} = \frac{1}{3}$.

στ. Θέτω $\log_5 \frac{1}{\sqrt{125}} = x$

$$\log_5 \frac{1}{\sqrt{125}} = x \Rightarrow 5^x = \frac{1}{\sqrt{125}} \Rightarrow 5^x = \frac{1}{\sqrt{5^3}} \Rightarrow 5^x = 5^{-\frac{3}{2}} \Rightarrow x = -\frac{3}{2}$$

άρα $\log_5 \frac{1}{\sqrt{125}} = -\frac{3}{2}$.