

- This assignment is **due in Week 8 on Sunday 24 September, 11:59pm**.
- Submission is on [GradeScope](#).
- All work must be **done individually** without consulting anyone else's solutions in accordance with the University's "[Academic Dishonesty and Plagiarism](#)" policies.
- For clarifications and more details on all aspects of this assignment (e.g., level of justification expected, late penalties, repeated submissions, what to do if you are stuck, etc.) you are expected to regularly monitor the Ed Forum post "[Assignment FAQ](#)".

Problem 1. (10 marks) Fix $\Sigma = \{a, b\}$. For each of the following grammars, provide five strings that are in the language and five strings that are not (you score $\max(0, 5 - \text{num_errors})$ points per part).

1.

$$S \rightarrow aSbb \mid \epsilon$$

2.

$$\begin{aligned} S &\rightarrow aA \\ A &\rightarrow aA \mid bB \\ B &\rightarrow bB \mid \epsilon \end{aligned}$$

Problem 2. (10 marks) Consider the following context-free grammar G over $\Sigma = \{a, b\}$.

$$\begin{aligned} S &\rightarrow AA \\ A &\rightarrow AAA \mid bA \mid Ab \mid a \end{aligned}$$

1. Show the grammar is ambiguous by providing two leftmost derivations of the same string.
2. Provide a grammar in Chomsky Normal Form equivalent to G .

Problem 3. (20 marks) Fix $\Sigma = \{a, b\}$. Design context-free grammars for the following languages:

1. The set of strings of length 6 that have the same number of a and b .
2. The set of strings of the form uvw with $u, v, w \in \{a^n b^n \mid n \in \mathbb{Z}^+\}$.
3. The set of palindromes whose length is a multiple of 20. Use at most 100 rules.
4. The set of strings not in the set $\{a^n b^{2n+3} \mid n \in \mathbb{Z}^+\}$.

Problem 4. (20 marks) Fix $\Sigma = \{a, b\}$. A string is *extreme* if its amount of a is more than twice its amount of b plus one, or the other way around. For example, $aaaab$ and $babbabbbbba$ are both extreme, while $bbbbbaa$ and $ababab$ are not.

1. Prove that the set E of extreme strings is not regular (10 marks).
2. Prove that the set P of strings that have an extreme string as a prefix is not regular. Recall that a string x is a prefix of a string y if $y = xw$ for some string w . Thus $aaaabbbb$ is in P (since it has $aaaab$ as a prefix), while $ababab$ is not in P (since no prefix of it is extreme). (10 marks)

Problem 5. (10 marks) Fix $\Sigma = \{a, b\}$. Write a program that takes in a context-free grammar G in CNF and outputs a context-free grammar for the language of strings that are in $L(G)$ and whose number of a s is a multiple of 5.

For full marks, your solution should be able to take an arbitrary CNF grammar with 10 variables and output a grammar with at most 100 variables.

Problem 6. (20 marks) You're honing your skills for the world championship of popular card game *Magic: the Gathering*. Your goal is to draw through all the cards in your deck, and to this end, you've included lots of cards that help you draw more cards. However, if you get unlucky, with some deck orderings you've found that this is impossible. You're interested in determining which deck orderings can be won, and which can't.

At the start of the game, you draw seven cards from your deck into your hand. The cards in your deck are of the following types:

- *Ancestral Recall*. When played, you draw three cards.
- *Wheel of Fortune*. When played, you discard your hand, and then draw seven cards.
- *Cruel Bargain*. When played for the first time in a game, you draw four cards. It cannot be played more than once in a game.
- Other, useless cards.

Aside: Unlike in the actual game, you can play any number of cards from your hand in any order, without needing to pay mana.

Your goal is to draw all the cards in your deck, at which point you win. Note that "overdraw" (drawing more cards than exist in your deck) meets this condition, and so is also a win. We encode a possible ordering of your deck as a string over the alphabet $\Sigma = \{a, w, c, x\}$ where a, w, c correspond to the cards starting with those letters, and x corresponds to a useless card. We are interested in the language of strings for which it is possible for you to win.

For example, the following strings are in the language:

$aax^{11}, wx^{13}, cawx^{18}, awx^5wx^{13}$

while the following strings are not in the language:

$aa x^{12}, wx^{14}, cawx^{19}, awx^5wx^{14}$

Design a context-free grammar for the language described.

- 4 marks will be awarded for cases with strings containing only a, x .
- 4 marks will be awarded for cases with strings containing only a, c, x .
- 4 marks will be awarded for cases with strings containing only w, x .
- 4 marks will be awarded for cases with strings containing only a, w, x .
- The final 4 marks will be awarded for the general case, i.e., for strings over the alphabet a, w, c, x .