

A Two-Individual Based Evolutionary Algorithm for the Flexible Job Shop Scheduling Problem

Abstract

Population-based evolutionary algorithms usually manage a large number of individuals to maintain the diversity of the search, which is complex and time-consuming. In this paper, we propose an evolutionary algorithm using only two individuals, called master-apprentice evolutionary algorithm (MAE), for solving the flexible job shop scheduling problem (FJSP). In order to ensure the diversity and the quality of the evolution, MAE integrates a tabu search procedure, a recombination operator based on path relinking using a novel distance definition, and an effective individual updating strategy, taking into account the multiple complex constraints of FJSP. The proposed distance definition unifies the assignment difference and sequence difference in the same manner. Experiments on 313 public instances show that MAE achieves highly competitive results in terms of both solution quality and computational efficiency. Specifically, it improves the previous best known results for 47 instances while matching the best known results on all except 5 of the remaining instances with the same computational time compared with the state-of-the-art metaheuristics. Besides, MAE finds new world records for 10 hard instances compared with the well-known industrial solver.

ues marked with * denote the optimal solutions, and the best known solutions that can be obtained by each reference algorithm are indicated in bold. Rows #better, #even, and #worse give the number of instances for which the best solutions obtained by MAE within 5 minutes or 90 seconds are better, equal, and worse than each reference algorithm. The best known solutions obtained by MAE which are smaller than the UB values of Quintiq are marked in red. To the best of our knowledge, these solutions, which break the world records of Quintiq, are best in the world up to now.

From Table 1- Table6, one observes that MAE improves the best known solutions provided by Quintiq for 10 instances. The corresponding sequences of the improved solutions are reported in Section 2. The format of the sequences are as follows: the first line gives the instance name and makespan. The following lines are the operations assigned on each of the machines. For each of the lines, the first number is the machine index, the second number is the number of operations assigned on this machine. The rest numbers in the line are composed of pairs, each pair represents one operation which contains two numbers, the first number is the job index of the operation, the second number is the index of the operation in the job.

1 The computational results of MAE

This paper provides the overall computational results of MAE on the benchmark sets (*DPdata* [Dauzère-Pérès and Paulli, 1997], *BCdata* [Barnes and Chambers, 1998], *BRdata* [Brandimarte, 1993], and *HUdata* [Hurink *et al.*, 1994]) and make comparisons with the recent state-of-the-art algorithms in the literature.

We compare our MAE algorithm with the recent state-of-the-art algorithms (CDDS [Hmida *et al.*, 2010], HDE-N2 [Yuan and Xu, 2013a; 2013b], SSPR [González *et al.*, 2015], HGTS [Palacios *et al.*, 2015], HA [Li and Gao, 2016], and GRASP-mELS [Kemmoé-Tchomté *et al.*, 2017]) on the four benchmark sets. The comparative results are reported in Tables 1-6. Note that columns best (avg) and t(s) are the best (average) solutions obtained and average computational time in seconds required by each algorithm, the *LB* and *UB* are the lower and upper bounds provided in Quintiq¹, the *LB* val-

¹<http://www.quintiq.com/optimization/flexible-job-shop-scheduling-problem-results.html>

Table 1: Comparison between MAE and other reference algorithms on the *DPdata* instance set

Ins.	Quintiq		2010 CCDS	2013 HDE-N2		2015 SSPR		2015 HGTS		2016 HA		2017 GRASP-mELS		This paper MAE(5 min)		This paper MAE(1 hour)	
	LB	UB	best(avg)	best(avg)	t(s)	best(avg)	t(s)	best(avg)	t(s)	best	t(s)	best(avg)	t(s)	best(avg)	t(s)	best(avg)	t(s)
01a	2505*	2505	2518(2525)	2505 (2513)	838	2505 (2508)	68	2505 (2505)	122	2505	108	2505 (2505)	62	2505 (2505)	28.56	2505 (2505)	28.56
02a	2228*	2228	2231(2235)	2230(2231)	973	2229(2230)	100	2230(2234)	205	2230	133	2229(2231)	86	2228 (2230.7)	145.12	2228 (2229.9)	712.33
03a	2228*	2228	2229(2232)	2228 (2229)	1165	2228 (2228)	110	2228 (2230)	181	2229	97	2228 (2230)	94	2228 (2228)	55.8	2228 (2228)	55.8
04a	2503*	2503	2503 (2510)	2506(2506)	850	2503 (2504)	57	2503 (2503)	112	2503	87	2503 (2503)	31	2503 (2503)	8.62	2503 (2503)	8.62
05a	2192	2204	2216(2218)	2212(2215)	931	2211(2215)	112	2214(2218)	208	2212	116	2212(2215)	126	2208(2211.45)	125.69	2203 (2208.05)	834.49
06a	2163	2171	2196(2203)	2187(2192)	1167	2183(2192)	181	2193(2198)	260	2197	93	2195(2200)	181	2182(2188.85)	177.42	2181 (2184.3)	1867.14
07a	2216	2264	2283(2296)	2288(2303)	1547	2274(2285)	139	2270(2280)	344	2279	204	2276(2284)	127	2269(2274.6)	180.3	2254 (2273.85)	2316.78
08a	2061*	2061	2069(2069)	2067(2074)	1906	2064(2066)	181	2070(2074)	318	2067	184	2069(2072)	144	2063(2064.3)	122.58	2062 (2063.4)	1741.55
09a	2061*	2061	2066(2067)	2069(2073)	943	2062 (2063)	213	2067(2069)	376	2065	201	2069(2071)	170	2062 (2063.15)	176.44	2062 (2063.1)	472.62
10a	2212	2241	2291(2303)	2297(2302)	1590	2269(2287)	120	2247(2266)	369	2287	238	2263(2278)	110	2247(2266.4)	224.36	2245 (2266.15)	2428.7
11a	2018	2037	2063(2072)	2061(2067)	1826	2051(2058)	193	2064(2069)	294	2060	181	2065(2068)	170	2050(2051.8)	200.57	2045 (2049.75)	2865.49
12a	1969	1984	2031(2034)	2027(2036)	914	2018(2020)	280	2027(2033)	486	2027	151	2039(2045)	148	2016(2021.45)	215.64	2008 (2019.3)	1588.16
13a	2197	2239	2257(2260)	2263(2269)	2900	2248(2257)	119	2250(2264)	416	2248	293	2252(2263)	158	2247(2251.75)	116.55	2236 (2246.65)	2674.32
14a	2161*	2161	2167(2179)	2164(2168)	3238	2163(2164)	269	2170(2173)	396	2167	210	2170(2174)	191	2163(2163.9)	191.26	2162 (2163.2)	2915.81
15a	2161*	2161	2165(2170)	2163(2166)	2112	2162 (2163)	376	2168(2169)	523	2163	192	2172(2174)	173	2162 (2164.35)	203.2	2162 (2163.15)	568.14
16a	2193	2231	2256(2258)	2259(2266)	2802	2244(2253)	131	2246(2257)	384	2249	160	2243(2258)	151	2242(2251.65)	196.5	2232 (2245.45)	2135.66
17a	2088	2105	2140(2146)	2137(2141)	3096	2130(2134)	299	2142(2146)	483	2140	203	2145(2152)	190	2128(2132.7)	245.71	2121 (2129.3)	1682.54
18a	2057	2070	2127(2132)	2124(2128)	2489	2119(2123)	409	2129(2133)	650	2132	133	2146(2151)	164	2118(2124.85)	242.2	2108 (2114.6)	1752.68
RPD			1.55(1.8)	1.5(1.73)		1.18(1.4)		1.34(1.59)		1.43		1.49(1.73)		1.01(1.14)		0.85(1.17)	
CI-CPU			170	1181.96		139.88		214.45		1105.03		149.94		158.7		1480.52	
#better			17	16		12		14		16		15					
#even			1	2		6		4		2		3					
#worse			0	0		0		0		0		0					

Table 2: Comparison between MAE and other reference algorithms on the *BCdata* instance set

Ins.	Quintiq		2010 CCDS	2013 HDE-N2	t(s)	2015 SSPR	2015 HGTS	t(s)	2016 HA	t(s)	2017 GRASP-mELS	t(s)	This paper MAE(5 min)		This paper MAE(1 hour)		
	LB	UB	best(avg)	best(avg)		best(avg)	best(avg)		best		best(avg)		best(avg)	t(s)	best(avg)	t(s)	
mt10c1	927*	927	928(929)	927 (928)	179	927 (928)	26	927 (927)	13	927	12	927 (927)	8	927 (927.3)	45.72	927 (927)	61.69
mt10cc	908*	908	910(911)	908 (911)	180	908 (908)	20	908 (910)	13	908	10	908 (909)	17	908 (909.85)	14.25	908 (909.4)	125.5
mt10x	918*	918	918 (918)	918 (918)	179	918 (918)	23	918 (918)	15	918	11	918 (918)	2	918 (918)	25.67	918 (918)	25.67
mt10xx	918*	918	918 (918)	918 (918)	170	918 (918)	19	918 (918)	12	918	11	918 (918)	2	918 (918)	4.5	918 (918)	4.5
mt10xxx	918*	918	918 (918)	918 (918)	160	918 (918)	20	918 (918)	12	918	11	918 (918)	2	918 (918)	6.78	918 (918)	6.78
mt10xy	905*	905	906(906)	905 (906)	174	905 (906)	21	905 (905)	13	905	11	905 (905)	26	905 (905)	34.42	905 (905)	34.42
mt10xyz	847*	847	849(851)	847 (851)	166	847 (847)	20	847 (850)	18	847	9	847 (847)	26	847 (847.65)	35.46	847 (847)	256.38
setb4c9	914*	914	919(919)	914 (917)	338	914 (916)	28	914 (914)	16	914	15	914 (914)	11	914 (918.25)	39.78	914 (914)	302.61
setb4cc	907*	907	909(911)	907 (910)	336	907 (907)	21	907 (908)	15	907	15	907 (907)	29	907 (907)	12.54	907 (907)	12.54
setb4x	925*	925	925 (925)	925 (926)	354	925 (925)	19	925 (925)	15	925	13	925 (925)	4	925 (925)	16.42	925 (925)	16.42
setb4xx	925*	925	925 (925)	925 (926)	330	925 (925)	21	925 (925)	14	925	5	925 (925)	2	925 (925)	7.7	925 (925)	7.7
setb4xxx	925*	925	925 (925)	925 (926)	315	925 (925)	22	925 (925)	15	925	9	925 (925)	3	925 (925)	8.45	925 (925)	8.45
setb4xy	910*	910	916(916)	910 (914)	313	910 (912)	32	910 (910)	19	910	12	910 (910)	18	910 (910)	58.79	910 (910)	58.79
setb4xyz	902*	902	905(907)	903(905)	317	905(905)	21	905(905)	15	905	14	902 (904)	11	902 (905.6)	34.6	902 (903.55)	956.86
seti5c12	1169*	1169	1174(1175)	1171(1175)	1113	1170(1173)	25	1170(1171)	41	1170	31	1169 (1172)	39	1170(1174.4)	64.13	1170(1173.2)	205.68
seti5cc	1135*	1135	1136(1137)	1136(1138)	1079	1135 (1136)	29	1136(1137)	34	1136	17	1135 (1136)	24	1136(1136)	30.95	1135 (1135.65)	243.52
seti5x	1198*	1198	1201(1202)	1200(1206)	1087	1198 (1199)	41	1199(1201)	38	1198	27	1198 (1199)	36	1198 (1201.6)	75.48	1198 (1199.35)	341.4
seti5xx	1194*	1194	1199(1199)	1197(1203)	1251	1197(1199)	37	1197(1198)	34	1197	29	1194 (1197)	26	1197(1198.5)	45.76	1197(1197)	473.48
seti5xxx	1194*	1194	1197(1198)	1197(1202)	1244	1194 (1198)	38	1197(1198)	31	1197	19	1194 (1197)	27	1197(1198.45)	35.5	1194 (1196.7)	612.57
seti5xy	1135*	1135	1136(1138)	1136(1138)	1141	1135 (1136)	29	1136(1137)	34	1136	17	1135 (1136)	28	1136(1136.1)	23.42	1135 (1136)	227.91
seti5xyz	1125*	1125	1125 (1125)	1125 (1130)	1223	1125 (1126)	35	1125 (1126)	43	1125	33	1125 (1127)	42	1125 (1128.75)	32.96	1125 (1125.65)	336.27
RPD			0.19(0.26)	0.05(0.3)		0.03(0.12)		0.07(0.13)		0.05		0(0.07)		0.04(0.16)		0.02(0.08)	
CI-CPU			12.75	377.2		19.54		13.8		7.88		19.88		31.11		205.67	
#better			11	3		1		2		1		0					
#even			10	18		17		19		20		16					
#worse			0	0		3		0		0		5					

Table 3: Comparison between MAE and other reference algorithms on the *BRdata* instance set

Ins.	Quintiq		2010 CCDS	2013 HDE-N2	t(s)	2015 SSPR	2015 HGTS	t(s)	2016 HA	t(s)	2017 GRASP-mELS	t(s)	This paper MAE(5 min)		This paper MAE(1 hour)		
	LB	UB	best(avg)	best(avg)		best(avg)	best(avg)		best		best(avg)		best(avg)	t(s)	best(avg)	t(s)	
Mk01	40*	40	40 (40)	40 (40)	4	40 (40)	11	40 (40)	5	40	0	40 (40)	0	40 (40)	0.2	40 (40)	0.2
Mk02	26*	26	26 (26)	26 (26)	6	26 (26)	15	26 (26)	15	26	1	26 (26)	10	26 (26)	0.55	26 (26)	0.55
Mk03	204*	204	204 (204)	204 (204)	31	204 (204)	24	204 (204)	2	204	0	204 (204)	0	204 (204)	0.16	204 (204)	0.16
Mk04	60*	60	60 (60)	60 (60)	13	60 (60)	19	60 (60)	10	60	0	60 (60)	0	60 (60)	0.47	60 (60)	0.47
Mk05	172*	172	173(174)	172 (173)	38	172 (172)	57	172 (172)	18	172	5	172 (173)	15	172 (172)	1.46	172 (172)	1.46
Mk06	57*	57	58(59)	57 (59)	98	57 (58)	40	57 (58)	63	57	54	58(58)	36	57 (58.15)	30.4	57 (57.25)	268.54
Mk07	139*	139	139 (139)	139 (139)	26	139 (141)	84	139 (139)	33	139	20	139 (140)	32	139 (139.7)	61.58	139 (139)	481.27
Mk08	523*	523	523 (523)	523 (523)	189	523 (523)	83	523 (523)	3	523	0	523 (523)	0	523 (523)	0.36	523 (523)	0.36
Mk09	307*	307	307 (307)	307 (307)	123	307 (307)	52	307 (307)	24	307	1	307 (307)	0	307 (307)	1.13	307 (307)	1.13
Mk10	189	193	197(198)	198(202)	266	196(197)	94	198(199)	104	197	33	197(199)	59	195(195.95)	36.78	193 (194.6)	827.34
RPD			0.66(0.94)	0.48(1.1)		0.37(0.74)		0.48(0.71)		0.42		0.6(0.83)		0.35(0.43)		0.23(0.34)	
CI-CPU			12.75	53.99		35.93		17.45		5.7		16.57		13.31		158.15	
#better			3	1		1		1		1		2					
#even			7	9		9		9		9		8					
#worse			0	0		0		0		0		0					

Table 4: Comparison between MAE and other reference algorithms on the *edata* of *HUdata* instance set

Ins.	Quintiq		2015 SSPR			2017 GRASP-mELS			This paper MAE(5 min)			This paper MAE(1 hour)		
	LB	UB	best	avg	t(s)	best	avg	t(s)	best	avg	t(s)	best	avg	t(s)
edata-abz5	1167*	1167	-	-	-	-	-	-	1167	1167	29.54	1167	1167	29.54
edata-abz6	925*	925	-	-	-	-	-	-	925	925	18.45	925	925	18.45
edata-abz7	604	610	-	-	-	-	-	-	613	614.85	64.5	613	613.45	548.16
edata-abz8	625	636	-	-	-	-	-	-	643	647.5	82.33	643	646.3	681.79
edata-abz9	644*	644	-	-	-	-	-	-	648	649.05	127.4	646	648.7	259.38
edata-car1	6176*	6176	-	-	-	-	-	-	6176	6191.75	129.25	6176	6180.6	1025.34
edata-car2	6327*	6327	-	-	-	-	-	-	6335	6414.25	102.61	6335	6370.25	1152.74
edata-car3	6856*	6856	-	-	-	-	-	-	6856	6875.75	126.34	6856	6858.25	856.36
edata-car4	7789*	7789	-	-	-	-	-	-	7789	7789	0.71	7789	7789	0.71
edata-car5	7229*	7229	-	-	-	-	-	-	7229	7229	8.26	7229	7229	8.26
edata-car6	7990*	7990	-	-	-	-	-	-	7990	7990.5	90.21	7990	7990	114.58
edata-car7	6123*	6123	-	-	-	-	-	-	6123	6123	7.98	6123	6123	7.98
edata-car8	7689*	7689	-	-	-	-	-	-	7689	7689	6.91	7689	7689	6.91
edata-la01	609*	609	609	609	13.3	609	609	0	609	609	0.37	609	609	0.37
edata-la02	655*	655	655	655	16.5	655	655	0	655	655	0.35	655	655	0.35
edata-la03	550*	550	550	550	16	550	550	0.2	550	550	12.68	550	550	12.68
edata-la04	568*	568	568	568	19.6	568	568	0	568	568	2.21	568	568	2.21
edata-la05	503*	503	503	503	13.1	503	503	0	503	503	0.25	503	503	0.25
edata-la06	833*	833	833	833	23.5	833	833	0	833	833	0.46	833	833	0.46
edata-la07	762*	762	762	762	28.1	762	762	1.95	762	762	21.2	762	762	21.2
edata-la08	845*	845	845	845	29.2	845	845	0	845	845	0.76	845	845	0.76
edata-la09	878*	878	878	878	25.9	878	878	0	878	878	0.69	878	878	0.69
edata-la10	866*	866	866	866	24.8	866	866	0	866	866	0.56	866	866	0.56
edata-la11	1103*	1103	1103	1103	43.4	1103	1103	0.7	1103	1103	49	1103	1103	49
edata-la12	960*	960	960	960	44.6	960	960	0	960	960	0.33	960	960	0.33
edata-la13	1053*	1053	1053	1053	43	1053	1053	0	1053	1053	0.45	1053	1053	0.45
edata-la14	1123*	1123	1123	1123	42.8	1123	1123	0	1123	1123	1.25	1123	1123	1.25
edata-la15	1111*	1111	1111	1111	49.5	1111	1111	0	1111	1111	1.68	1111	1111	1.68
edata-la16	892*	892	892	892	16	892	892	6	892	892	2.21	892	892	2.21
edata-la17	707*	707	707	707	14	707	707	0.05	707	707	3.7	707	707	3.7
edata-la18	842*	842	842	842	17.5	842	842	1.1	842	842	6.82	842	842	6.82
edata-la19	796*	796	796	796	16.9	796	796	3.55	796	796	26.14	796	796	26.14
edata-la20	857*	857	857	857	17.1	857	857	0.95	857	857	1.63	857	857	1.63
edata-la21	1009*	1009	1010	1014.4	36.5	1009	1016.7	90.55	1009	1014.4	31.15	1009	1014.15	358.45
edata-la22	880*	880	880	880.1	32.3	880	880	7.75	880	880.15	84.21	880	880	225.69
edata-la23	950*	950	950	950	23.4	950	950	2.7	950	950	17.8	950	950	17.8
edata-la24	908*	908	908	908.9	21.4	908	908	52.65	908	908.2	17.4	908	908	86.19
edata-la25	936*	936	939	940.7	28.9	936	940	74.6	936	941	22.36	936	940.7	238.58
edata-la26	1106*	1106	1109	1112.8	48.3	1107	1111.2	109.45	1107	1116.05	32.8	1107	1115.85	684.15
edata-la27	1181*	1181	1181	1182.4	48.5	1181	1181.35	55.9	1181	1182	70.76	1181	1181.95	121.02
edata-la28	1142*	1142	1144	1145.9	56.7	1144	1146.55	94.6	1142	1146.9	22.64	1142	1146.5	269.16
edata-la29	1107*	1107	1111	1114.5	53.1	1113	1116.45	84.25	1111	1117.6	28.74	1111	1116.2	894.18
edata-la30	1188*	1188	1204	1206.5	62.4	1198	1203.5	90.45	1195	1203.2	91.91	1193	1201.2	513.84
edata-la31	1532*	1532	1533	1537.1	111.3	1536	1540.45	19.55	1532	1533.1	32.12	1532	1532.95	331.78
edata-la32	1698*	1698	1698	1698	82.8	1698	1698	0.85	1698	1698	5.5	1698	1698	5.5
edata-la33	1547*	1547	1547	1547	78.7	1547	1547	0.35	1547	1547	6.02	1547	1547	6.02
edata-la34	1599*	1599	1599	1599	70	1599	1599	5.5	1599	1599	7.36	1599	1599	7.36
edata-la35	1736*	1736	1736	1736	66.2	1736	1736	0	1736	1736	2.68	1736	1736	2.68
edata-la36	1160*	1160	1160	1160.2	31.8	1160	1160	20.1	1160	1160.6	42.25	1160	1160.45	658.45
edata-la37	1397*	1397	1397	1397	30.4	1397	1397	7.65	1397	1397	5.78	1397	1397	5.78
edata-la38	1141*	1141	1141	1143.1	36.6	1141	1142.85	85.4	1141	1141.2	30.33	1141	1141.1	224.3
edata-la39	1184*	1184	1184	1184.4	31.3	1184	1184	8.2	1184	1184.1	46.2	1184	1184	112.51
edata-la40	1144*	1144	1144	1144	27.7	1144	1144.3	65.65	1144	1144.35	31.8	1144	1144.2	415.4
edata-mt06	55*	55	55	55	30.8	55	55	0	55	55	0.17	55	55	0.17
edata-mt10	871*	871	871	872	25.9	871	871	10.2	871	872.7	28.66	871	871.7	592.5
edata-mt20	1088*	1088	1088	1088	48.8	1088	1088	5.9	1088	1088	6.8	1088	1088	6.8
edata-orb1	977*	977	-	-	-	-	-	-	977	977	49.1	977	977	49.1
edata-orb10	933*	933	-	-	-	-	-	-	933	933.45	70.25	933	933	159.48
edata-orb2	865*	865	-	-	-	-	-	-	865	865	7.67	865	865	7.67
edata-orb3	951*	951	-	-	-	-	-	-	951	951.8	112.5	951	951	1025.89
edata-orb4	984*	984	-	-	-	-	-	-	984	984.15	94.56	984	984	193.1
edata-orb5	842*	842	-	-	-	-	-	-	842	842	25.63	842	842	25.63
edata-orb6	958*	958	-	-	-	-	-	-	958	959.1	70.8	958	958.65	378.15
edata-orb7	389*	389	-	-	-	-	-	-	389	389.4	67.91	389	389	261.5
edata-orb8	894*	894	-	-	-	-	-	-	894	894	1.59	894	894	1.59
edata-orb9	933*	933	-	-	-	-	-	-	933	933.4	74.56	933	933	315.98
CI-CPU			37.18			21.09			19.17			137.03		
#better			5			4								
#even			38			39								
#worse			0			0								
#opt			36			38			59			59		

Table 5: Comparison between MAE and other reference algorithms on the *rdata* of *HUdata* instance set

Ins.	Quintiq		2015 SSPR			2017 GRASP-mELS			This paper MAE(5 min)			This paper MAE(1 hour)		
	LB	UB	best	avg	t(s)	best	avg	t(s)	best	avg	t(s)	best	avg	t(s)
rdata-abz5	954*	954	-	-	-	-	-	-	954	954	7.68	954	954	7.68
rdata-abz6	807*	807	-	-	-	-	-	-	807	807	0.45	807	807	0.45
rdata-abz7	493	524	-	-	-	-	-	-	527	529.9	112.35	522	528.45	541.56
rdata-abz8	507	536	-	-	-	-	-	-	540	542	113.41	535	540.05	1128.15
rdata-abz9	517	536	-	-	-	-	-	-	541	542.8	164.72	541	541	765.45
rdata-car1	5034*	5034	-	-	-	-	-	-	5035	5040.6	133.65	5035	5036.65	841.37
rdata-car2	5985*	5985	-	-	-	-	-	-	5985	5985.85	24.33	5985	5985.15	687.41
rdata-car3	5622*	5622	-	-	-	-	-	-	5623	5625.1	123.25	5623	5623.65	549.48
rdata-car4	6514*	6514	-	-	-	-	-	-	6514	6514.15	118.45	6514	6514	122.25
rdata-car5	5615*	5615	-	-	-	-	-	-	5621	5640.35	142.54	5621	5630.9	1023.82
rdata-car6	6147*	6147	-	-	-	-	-	-	6147	6147	6.12	6147	6147	6.12
rdata-car7	4425*	4425	-	-	-	-	-	-	4425	4429.55	44.34	4425	4426.05	612.24
rdata-car8	5692*	5692	-	-	-	-	-	-	5692	5692	84.15	5692	5692	84.15
rdata-la01	570*	570	571	571	16.1	570	570.8	38.2	570	570.9	10.54	570	570.3	412.59
rdata-la02	529*	529	530	530	17	529	529.95	33.2	529	529.95	15.2	529	529.25	841.52
rdata-la03	477*	477	477	477.1	19.3	477	477.1	73.6	477	477	5.54	477	477	5.54
rdata-la04	502*	502	502	502.3	19.6	502	502	43.95	502	502	23.77	502	502	23.77
rdata-la05	457*	457	457	457	17.3	457	457	20.2	457	457	1.42	457	457	1.42
rdata-la06	799*	799	799	799	27.4	799	799	5.2	799	799	1.25	799	799	1.25
rdata-la07	749*	749	749	749.3	35.4	749	749.1	47.85	749	749	4.71	749	749	4.71
rdata-la08	765*	765	765	765	37.8	765	765	37.75	765	765	2.34	765	765	2.34
rdata-la09	853*	853	853	853	31.5	853	853	8.3	853	853	2.35	853	853	2.35
rdata-la10	804*	804	804	804	33	804	804.05	52.35	804	804	3.12	804	804	3.12
rdata-la11	1071*	1071	1071	1071	52.5	1071	1071	0.95	1071	1071	1.23	1071	1071	1.23
rdata-la12	936*	936	936	936	50.6	936	936	0	936	936	1.12	936	936	1.12
rdata-la13	1038*	1038	1038	1038	49.3	1038	1038	0.5	1038	1038	1.35	1038	1038	1.35
rdata-la14	1070*	1070	1070	1070	49.5	1070	1070	0.4	1070	1070	1.45	1070	1070	1.45
rdata-la15	1089*	1089	1089	1089.2	71.5	1089	1089	71.15	1089	1089	25.87	1089	1089	25.87
rdata-la16	717*	717	717	717	13.8	717	717	0.55	717	717	1.24	717	717	1.24
rdata-la17	646*	646	646	646	9.4	646	646	0	646	646	0.77	646	646	0.77
rdata-la18	666*	666	666	666	10.2	666	666	3.4	666	666	1.58	666	666	1.58
rdata-la19	700*	700	700	700.9	19.8	700	700	49.5	700	700	2.87	700	700	2.87
rdata-la20	756*	756	756	756	9.7	756	756	0	756	756	1.25	756	756	1.25
rdata-la21	808	825	830	833.8	44.4	832	837.75	92.75	825	827.25	95.36	825	826.15	635.15
rdata-la22	741	755	756	760.1	35.5	757	761.2	79	755	757.05	115.34	753	756.8	512.02
rdata-la23	816	832	835	837.6	42.1	836	840.3	63.15	833	833.1	83.74	831	832.85	658.15
rdata-la24	775	796	802	804.1	42.3	802	806.85	88.75	799	799.2	104.13	795	798.1	325.76
rdata-la25	768	783	784	786.4	44	784	789.75	93.15	783	783.1	105.84	779	783.05	512.17
rdata-la26	1056	1057	1059	1060.7	79.5	1060	1063.15	92.9	1057	1058.5	88.5	1057	1058.3	448.1
rdata-la27	1085*	1085	1089	1089.8	70.6	1089	1091.8	76.95	1086	1087.4	105.87	1086	1087.15	671.05
rdata-la28	1075	1076	1078	1078.9	65.8	1077	1080.1	67.5	1076	1077.25	88.3	1076	1077.1	353.33
rdata-la29	993	994	996	996.7	71.3	996	998.15	64.25	994	995.35	90.46	994	995.2	312.36
rdata-la30	1068	1071	1074	1076.1	72.1	1074	1081.45	90.95	1071	1072.55	61.58	1071	1072.1	744.58
rdata-la31	1520*	1520	1520	1520.6	114.7	1521	1522.45	78.9	1520	1520	56.67	1520	1520	56.67
rdata-la32	1657*	1657	1658	1658	113.8	1658	1659.25	84	1657	1657.95	110.25	1657	1657.65	333.51
rdata-la33	1497*	1497	1498	1498	109.8	1498	1499.15	66.65	1497	1497.95	78.24	1497	1497.3	425.09
rdata-la34	1535*	1535	1535	1535.4	125.1	1535	1535.95	57	1535	1535	35.02	1535	1535	35.02
rdata-la35	1549*	1549	1550	1550	105.6	1550	1550.4	56.65	1549	1549.1	71.4	1549	1549	71.4
rdata-la36	1023*	1023	1023	1027.1	30.4	1023	1028.5	78.55	1023	1023	53.46	1023	1023	53.46
rdata-la37	1062*	1062	1069	1072	44.3	1066	1076.7	83.45	1062	1063.4	60.58	1062	1063.2	548.15
rdata-la38	954*	954	961	962.4	44.9	958	970.95	46.7	954	954.45	89.63	954	954.3	1158.25
rdata-la39	1011*	1011	1024	1024	26.4	1018	1023.9	88.2	1011	1013.45	178.58	1011	1012.85	789.13
rdata-la40	955*	955	961	964.3	44.3	958	965.85	87.4	955	955.95	121.1	955	955.8	164.51
rdata-mt06	47*	47	47	47	4.4	47	47	0	47	47	0.15	47	47	0.15
rdata-mt10	686*	686	686	686	11.9	686	686	6.4	686	686	2.57	686	686	2.57
rdata-mt20	1022*	1022	1022	1022	56.9	1022	1022	10.55	1022	1022	15.68	1022	1022	15.68
rdata-orb1	746*	746	-	-	-	-	-	-	746	746	1.25	746	746	1.25
rdata-orb10	742*	742	-	-	-	-	-	-	742	742	32.57	742	742	32.57
rdata-orb2	696*	696	-	-	-	-	-	-	696	696	6.42	696	696	6.42
rdata-orb3	712*	712	-	-	-	-	-	-	712	712	16.48	712	712	16.48
rdata-orb4	753*	753	-	-	-	-	-	-	753	753	1.12	753	753	1.12
rdata-orb5	639*	639	-	-	-	-	-	-	639	639	6.74	639	639	6.74
rdata-orb6	754*	754	-	-	-	-	-	-	754	754	1.95	754	754	1.95
rdata-orb7	302*	302	-	-	-	-	-	-	302	302	9.37	302	302	9.37
rdata-orb8	639*	639	-	-	-	-	-	-	639	639	2.58	639	639	2.58
rdata-orb9	694*	694	-	-	-	-	-	-	694	694	1.36	694	694	1.36
CI-CPU			46.3			47.46			41.95			239.05		
#better			19			18								
#even			24			25								
#worse			0			0								
#opt			24			25			50			50		

Table 6: Comparison between MAE and other reference algorithms on the *vdata* of *HUdata* instance set

Ins.	Quintiq		2015 SSPR			2017 GRASP-mELS			This paper MAE(5 min)			This paper MAE(1 hour)		
	LB	UB	best	avg	t(s)	best	avg	t(s)	best	avg	t(s)	best	avg	t(s)
vdata-abz5	859*	859	-	-	-	-	-	-	859	859	1.45	859	859	1.45
vdata-abz6	742*	742	-	-	-	-	-	-	742	742	0.54	742	742	0.54
vdata-abz7	492*	492	-	-	-	-	-	-	499	499.85	98.45	499	499.6	345.84
vdata-abz8	506	507	-	-	-	-	-	-	513	514.4	151.2	513	513.9	541.02
vdata-abz9	497*	497	-	-	-	-	-	-	504	504.5	121.68	503	503.9	1025.15
vdata-car1	5005*	5005	-	-	-	-	-	-	5005	5005.9	40.88	5005	5005.2	891.25
vdata-car2	5929*	5929	-	-	-	-	-	-	5929	5929	8.42	5929	5929	8.42
vdata-car3	5597*	5597	-	-	-	-	-	-	5597	5597.8	58.17	5597	5597.3	358.15
vdata-car4	6514*	6514	-	-	-	-	-	-	6514	6514	36.58	6514	6514	36.58
vdata-car5	4909	4911	-	-	-	-	-	-	4912	4915.05	130.15	4910	4912.65	1176.45
vdata-car6	5486*	5486	-	-	-	-	-	-	5486	5486	0.28	5486	5486	0.28
vdata-car7	4281*	4281	-	-	-	-	-	-	4281	4281	0.24	4281	4281	0.24
vdata-car8	4613*	4613	-	-	-	-	-	-	4613	4613	0.47	4613	4613	0.47
vdata-la01	570*	570	570	570.1	23.2	570	570	15.2	570	570	2.32	570	570	2.32
vdata-la02	529*	529	529	529	17.2	529	529	9.9	529	529	1.2	529	529	1.2
vdata-la03	477*	477	477	477	18.6	477	477	22	477	477	1.65	477	477	1.65
vdata-la04	502*	502	502	502	15.5	502	502	3.3	502	502	1.01	502	502	1.01
vdata-la05	457*	457	457	457	19.7	457	457.05	24.4	457	457	7.33	457	457	7.33
vdata-la06	799*	799	799	799	31.1	799	799	1.8	799	799	1.02	799	799	1.02
vdata-la07	749*	749	749	749	42.8	749	749	35.5	749	749	2.75	749	749	2.75
vdata-la08	765*	765	765	765	31.2	765	765	10.65	765	765	1.52	765	765	1.52
vdata-la09	853*	853	853	853	30.8	853	853	2.1	853	853	1.38	853	853	1.38
vdata-la10	804*	804	804	804	29.9	804	804	3.85	804	804	1.65	804	804	1.65
vdata-la11	1071*	1071	1071	1071	54.4	1071	1071	0.35	1071	1071	1.5	1071	1071	1.5
vdata-la12	936*	936	936	936	56.3	936	936	0.15	936	936	1.24	936	936	1.24
vdata-la13	1038*	1038	1038	1038	56.5	1038	1038	0.45	1038	1038	1.33	1038	1038	1.33
vdata-la14	1070*	1070	1070	1070	53.4	1070	1070	0.05	1070	1070	1.58	1070	1070	1.58
vdata-la15	1089*	1089	1089	1089	50.3	1089	1089	5.7	1089	1089	3.54	1089	1089	3.54
vdata-la16	717*	717	717	717	13.3	717	717	0	717	717	0.2	717	717	0.2
vdata-la17	646*	646	646	646	12.4	646	646	0	646	646	0.21	646	646	0.21
vdata-la18	663*	663	663	663	15.4	663	663	0	663	663	0.48	663	663	0.48
vdata-la19	617*	617	617	617	23	617	617	0.55	617	617	1.29	617	617	1.29
vdata-la20	756*	756	756	756	13.6	756	756	0	756	756	0.11	756	756	0.11
vdata-la21	800*	800	802	803.3	63.6	804	807.2	87.7	801	802.35	101.68	801	801.95	302.54
vdata-la22	733*	733	734	735.5	70.3	737	739.2	87.55	733	734.1	112.45	733	733.95	214.69
vdata-la23	809*	809	811	811.5	62.1	813	815.8	77	810	810.85	84.21	810	810.25	548.77
vdata-la24	773*	773	775	775.6	61.7	776	778.95	78.2	774	774.8	75.36	774	774.3	525.51
vdata-la25	751*	751	753	753.8	66	755	758.2	100.55	752	753.2	98.34	752	752.9	270.57
vdata-la26	1052*	1052	1053	1053.1	108.6	1054	1055.1	71.25	1052	1052.15	117.24	1052	1052	192.35
vdata-la27	1084*	1084	1084	1084.4	100.5	1086	1086.25	78.7	1084	1084	31.91	1084	1084	31.91
vdata-la28	1069*	1069	1069	1069.2	101.9	1070	1071.1	62.75	1069	1069	36.54	1069	1069	36.54
vdata-la29	993*	993	994	994	89.7	995	995.8	70.55	993	993.85	58.15	993	993.05	658.58
vdata-la30	1068*	1068	1069	1069.3	98.9	1070	1071.25	80.35	1068	1068.8	49.61	1068	1068.3	414.45
vdata-la31	1520*	1520	1520	1520	169.2	1521	1521	40.7	1520	1520	17.12	1520	1520	17.12
vdata-la32	1657*	1657	1658	1658	174.4	1658	1658.65	70.8	1657	1657.05	79.27	1657	1657	106.48
vdata-la33	1497*	1497	1497	1497.9	185.6	1498	1498.35	63.35	1497	1497.05	116.02	1497	1497	135.6
vdata-la34	1535*	1535	1535	1535	174.1	1535	1535.95	35.45	1535	1535	28.45	1535	1535	28.45
vdata-la35	1549*	1549	1549	1549	181.8	1549	1549.95	40.3	1549	1549	46.12	1549	1549	46.12
vdata-la36	948*	948	948	948	69.9	948	948	1.1	948	948	4.33	948	948	4.33
vdata-la37	986*	986	986	986	75.3	986	986	2.55	986	986	5.61	986	986	5.61
vdata-la38	943*	943	943	943	61.7	943	943	0.05	943	943	2.23	943	943	2.23
vdata-la39	922*	922	922	922	69.2	922	922	3.35	922	922	4.68	922	922	4.68
vdata-la40	955*	955	955	955	62.8	955	955	0.05	955	955	2.35	955	955	2.35
vdata-mt06	47*	47	47	47	4.2	47	47	0	47	47	0.08	47	47	0.08
vdata-mt10	655*	655	655	655	14.9	655	655	0	655	655	0.41	655	655	0.41
vdata-mt20	1022*	1022	1022	1022	48.8	1022	1022	1.3	1022	1022	2.19	1022	1022	2.19
vdata-orb1	695*	695	-	-	-	-	-	-	695	695	0.45	695	695	0.45
vdata-orb10	681*	681	-	-	-	-	-	-	681	681	0.68	681	681	0.68
vdata-orb2	620*	620	-	-	-	-	-	-	620	620	1.12	620	620	1.12
vdata-orb3	648*	648	-	-	-	-	-	-	648	648	0.5	648	648	0.5
vdata-orb4	753*	753	-	-	-	-	-	-	753	753	0.25	753	753	0.25
vdata-orb5	584*	584	-	-	-	-	-	-	584	584	0.81	584	584	0.81
vdata-orb6	715*	715	-	-	-	-	-	-	715	715	0.39	715	715	0.39
vdata-orb7	275*	275	-	-	-	-	-	-	275	275	1.24	275	275	1.24
vdata-orb8	573*	573	-	-	-	-	-	-	573	573	0.43	573	573	0.43
vdata-orb9	659*	659	-	-	-	-	-	-	659	659	0.56	659	659	0.56
CI-CPU			63.34			27.66			26.34			84.1		
#better			9			13								
#even			34			30								
#worse			0			0								
#opt			34			30			58			58		

Table 7: Comparison between MAE and other reference algorithms on the *sdata* of *HUdata* instance set

Ins.	Quintiq		2015 SSPR			2017 GRASP-mELS			This paper MAE(5 min)			This paper MAE(1 hour)		
	LB	UB	best	avg	t(s)	best	avg	t(s)	best	avg	t(s)	best	avg	t(s)
sdata-abz5	1234*	1234	-	-	-	-	-	-	1234	1234.2	89.15	1234	1234	215.57
sdata-abz6	943*	943	-	-	-	-	-	-	943	943	4.78	943	943	4.78
sdata-abz7	656*	656	-	-	-	-	-	-	664	666	68.57	661	664.75	546.87
sdata-abz8	653	667	-	-	-	-	-	-	675	678.4	102.38	675	676.3	1158.25
sdata-abz9	678*	678	-	-	-	-	-	-	688	688.25	115.74	688	688	215.76
sdata-car1	7038*	7038	-	-	-	-	-	-	7038	7038	2.15	7038	7038	2.15
sdata-car2	7166*	7166	-	-	-	-	-	-	7166	7166	2.6	7166	7166	2.6
sdata-car3	7312*	7312	-	-	-	-	-	-	7312	7362.7	125.6	7312	7316.4	784.15
sdata-car4	8003*	8003	-	-	-	-	-	-	8003	8003	1.2	8003	8003	1.2
sdata-car5	7702*	7702	-	-	-	-	-	-	7702	7705.6	117.48	7702	7702	358.61
sdata-car6	8313*	8313	-	-	-	-	-	-	8313	8313	42.32	8313	8313	42.32
sdata-car7	6558*	6558	-	-	-	-	-	-	6558	6558.75	64.8	6558	6558	94.57
sdata-car8	8264*	8264	-	-	-	-	-	-	8264	8295.05	129.23	8264	8271.5	1536.54
sdata-la01	666*	666	666	666	13.3	-	-	-	666	666	0.26	666	666	0.26
sdata-la02	655*	655	655	655	18.4	-	-	-	655	655	0.51	655	655	0.51
sdata-la03	597*	597	597	597	24.6	-	-	-	597	597	3.24	597	597	3.24
sdata-la04	590*	590	590	590	17.7	-	-	-	590	590	0.87	590	590	0.87
sdata-la05	593*	593	593	593	17.3	-	-	-	593	593	0.21	593	593	0.21
sdata-la06	926*	926	926	926	32.2	-	-	-	926	926	0.23	926	926	0.23
sdata-la07	890*	890	890	890	27.4	-	-	-	890	890	0.68	890	890	0.68
sdata-la08	863*	863	863	863	29.3	-	-	-	863	863	0.43	863	863	0.43
sdata-la09	951*	951	951	951	32.9	-	-	-	951	951	0.33	951	951	0.33
sdata-la10	958*	958	958	958	35	-	-	-	958	958	0.2	958	958	0.2
sdata-la11	1222*	1222	1222	1222	59.6	-	-	-	1222	1222	0.36	1222	1222	0.36
sdata-la12	1039*	1039	1039	1039	54	-	-	-	1039	1039	0.31	1039	1039	0.31
sdata-la13	1150*	1150	1150	1150	59.1	-	-	-	1150	1150	0.25	1150	1150	0.25
sdata-la14	1292*	1292	1292	1292	70.1	-	-	-	1292	1292	0.19	1292	1292	0.19
sdata-la15	1207*	1207	1207	1207	47.6	-	-	-	1207	1207	1.24	1207	1207	1.24
sdata-la16	945*	945	945	945	22.1	-	-	-	945	945	54.78	945	945	54.78
sdata-la17	784*	784	784	784	22.9	-	-	-	784	784	2.71	784	784	2.71
sdata-la18	848*	848	848	848	17.9	-	-	-	848	848	3.92	848	848	3.92
sdata-la19	842*	842	842	842	14.8	-	-	-	842	842	79.23	842	842	79.23
sdata-la20	902*	902	902	902	17.9	-	-	-	902	902	66.9	902	902	66.9
sdata-la21	1046*	1046	1046	1046.4	35.2	-	-	-	1046	1046.7	113.26	1046	1046.4	315.25
sdata-la22	927*	927	927	927.6	33.7	-	-	-	927	927.5	105.5	927	927.15	511.91
sdata-la23	1032*	1032	1032	1032	21.5	-	-	-	1032	1032	1.53	1032	1032	1.53
sdata-la24	935*	935	935	935.9	30.8	-	-	-	935	936.2	122.09	935	935.8	412.56
sdata-la25	977*	977	977	978.1	36	-	-	-	977	978.4	115.34	977	978.2	1357.16
sdata-la26	1218*	1218	1218	1218	40.2	-	-	-	1218	1218	2.62	1218	1218	2.62
sdata-la27	1235*	1235	1235	1235	41.6	-	-	-	1235	1235.35	78.7	1235	1235	109.4
sdata-la28	1216*	1216	1216	1216	41.3	-	-	-	1216	1216	4.19	1216	1216	4.19
sdata-la29	1152*	1152	1160	1163.2	67.9	-	-	-	1160	1164.85	123.25	1156	1161.4	1487.62
sdata-la30	1355*	1355	1355	1355	42.2	-	-	-	1355	1355	2.3	1355	1355	2.3
sdata-la31	1784*	1784	1784	1784	68.1	-	-	-	1784	1784	1.54	1784	1784	1.54
sdata-la32	1850*	1850	1850	1850	64.9	-	-	-	1850	1850	1.81	1850	1850	1.81
sdata-la33	1719*	1719	1719	1719	64.6	-	-	-	1719	1719	1.72	1719	1719	1.72
sdata-la34	1721*	1721	1721	1721	66.6	-	-	-	1721	1721	2.9	1721	1721	2.9
sdata-la35	1888*	1888	1888	1888	74.3	-	-	-	1888	1888	2.55	1888	1888	2.55
sdata-la36	1268*	1268	1268	1268.2	30.2	-	-	-	1268	1268.4	114.46	1268	1268.2	789.8
sdata-la37	1397*	1397	1397	1398.6	37.7	-	-	-	1397	1398.9	138.22	1397	1398.7	635.48
sdata-la38	1196*	1196	1198	1200.7	33.3	-	-	-	1196	1200.4	130.12	1196	1200.3	301.28
sdata-la39	1233*	1233	1233	1233.2	35	-	-	-	1233	1233.75	102.5	1233	1233.1	1248.69
sdata-la40	1222*	1222	1224	1224.8	36.9	-	-	-	1224	1225.2	96.41	1224	1224.6	500.12
sdata-mt06	55*	55	55	55	75.2	-	-	-	55	55	0.32	55	55	0.32
sdata-mt10	930*	930	930	931.8	33.9	-	-	-	930	930.2	98.81	930	930	157.1
sdata-mt20	1165*	1165	1165	1165	66.7	-	-	-	1165	1165	2.02	1165	1165	2.02
sdata-orb1	1059*	1059	-	-	-	-	-	-	1059	1063	93.83	1059	1059	465.45
sdata-orb10	944*	944	-	-	-	-	-	-	944	944	9.53	944	944	9.53
sdata-orb2	888*	888	-	-	-	-	-	-	888	888.15	123.52	888	888	224.73
sdata-orb3	1005*	1005	-	-	-	-	-	-	1005	1005	21.17	1005	1005	21.17
sdata-orb4	1005*	1005	-	-	-	-	-	-	1005	1011.75	121.36	1005	1009.3	981.25
sdata-orb5	887*	887	-	-	-	-	-	-	887	888.5	66.28	887	887.8	435.75
sdata-orb6	1010*	1010	-	-	-	-	-	-	1010	1010.3	78.41	1010	1010	139.13
sdata-orb7	397*	397	-	-	-	-	-	-	397	397	35.65	397	397	35.65
sdata-orb8	899*	899	-	-	-	-	-	-	899	903.9	121.27	899	899.25	846.32
sdata-orb9	934*	934	-	-	-	-	-	-	934	934	6.18	934	934	6.18
CI-CPU			39.81						37.35			157.63		
#better			1											
#even			42											
#worse			0											
#opt			40						61			61		

2 The sequences of improved solutions

- 05a 2203

1 41 4 1 1 1 5 2 2 3 10 2 9 5 10 3 6 2 2 6 1 5 4 7 4 8 3 6 3 7 10 9 8 8 8 9 7 5 8 11
10 11 5 10 4 11 8 13 3 11 5 12 8 15 10 13 4 13 3 14 5 15 2 18 9 21 7 12 6 12 2
20 3 19 10 17 7 15 8 25 7 16 10 19
2 36 5 1 2 2 8 2 9 4 6 1 2 5 9 7 9 8 3 4 8 6 6 4 6 5 10 8 5 8 4 9 9 12 9 13 2 11 2
12 1 9 5 11 4 12 9 17 9 18 10 14 1 12 8 18 8 19 8 20 8 21 6 13 9 22 8 23 8 24 10
18 6 15
3 42 9 1 9 2 9 3 1 2 8 3 2 4 5 4 9 6 5 6 10 6 10 7 5 7 8 7 6 6 6 7 9 11 2 9 5 9 4 10
3 9 7 6 3 10 9 15 1 10 9 16 8 14 2 15 5 13 9 19 8 17 6 11 10 15 4 15 2 19 4 16
10 16 4 17 4 18 1 14 3 22 5 21 7 17
4 39 2 1 8 1 10 1 3 2 4 3 1 4 4 4 8 4 4 5 4 6 2 7 6 3 7 2 7 3 2 8 6 8 3 8 6 9 9 14 8
12 2 13 2 14 3 12 7 9 7 10 8 16 2 17 5 14 9 20 3 15 3 16 3 17 3 18 7 13 7 14 3
20 3 21 9 23 3 23
5 38 3 1 4 2 7 1 5 3 1 3 3 3 5 5 10 4 10 5 8 5 9 9 3 5 9 10 1 6 7 4 1 7 2 10 10 10
8 10 1 8 6 10 10 12 7 7 7 8 1 11 3 13 2 16 7 11 4 14 1 13 5 16 5 17 5 18 8 22 5
19 6 14 5 20 1 15

- 07a 2254

1 37 13 1 13 3 9 2 3 1 13 5 14 2 12 4 3 2 12 5 2 6 10 9 10 10 6 4 9 12 14 9 1 3 8
5 4 12 9 14 4 13 11 12 2 15 15 20 1 6 4 14 13 12 8 16 7 10 8 18 12 16 9 21 1 11
13 14 3 21 8 23 9 23 1 15
2 34 13 2 13 4 15 4 7 3 15 7 9 4 4 6 4 7 13 7 15 10 14 6 11 7 15 13 15 14 10 13
15 16 10 16 6 5 2 14 3 12 5 9 14 14 15 23 12 13 6 9 12 15 3 16 3 17 8 19 3 19 3
20 5 19 4 17 8 25
3 33 7 2 10 2 12 3 15 6 2 3 2 5 10 6 13 8 15 11 13 10 10 11 3 6 12 10 10 14 15
17 2 13 14 13 9 16 10 19 3 13 15 22 8 14 7 9 15 24 13 13 11 16 6 11 6 12 2 20 8
22 13 16 11 19 3 23
4 38 15 1 9 1 12 1 10 1 1 1 14 1 11 3 4 5 6 1 6 2 11 4 14 5 10 8 13 9 12 6 12 7
11 8 9 11 11 9 8 4 2 11 7 6 8 7 8 8 7 7 1 4 8 11 13 11 6 8 3 14 9 19 14 15 11 17
12 17 7 14 5 18 7 16 1 14
5 38 7 1 5 1 15 3 9 3 4 4 8 1 10 5 15 9 3 3 3 4 3 5 5 4 2 9 10 12 4 11 5 5 8 6 14
12 5 7 3 11 15 19 7 8 11 14 8 13 6 7 5 12 5 13 1 8 3 15 8 17 4 15 14 16 7 12 1
12 7 15 1 13 7 17 5 21
6 39 2 1 4 2 11 2 15 5 2 2 7 4 10 4 13 6 7 5 9 6 4 8 11 6 4 9 14 8 12 9 3 7 14 10
14 11 10 15 3 9 10 18 5 8 11 13 15 21 2 16 12 12 9 18 12 14 2 18 5 14 9 20 7 11
1 10 5 17 6 13 11 18 4 16 12 20 6 15
7 39 4 1 15 2 4 3 10 3 15 8 14 3 5 3 6 3 10 7 8 2 2 7 9 8 9 9 12 8 4 10 2 10 9 13
2 12 10 17 11 11 15 18 12 11 1 5 5 10 9 17 2 17 11 15 15 25 2 19 3 18 5 16 8 20
8 21 13 15 9 22 6 14 3 22 5 20 4 18
8 35 11 1 12 2 5 2 1 2 9 5 2 4 14 1 11 5 9 7 14 7 15 12 2 8 9 10 8 3 15 15 11 10
3 8 5 6 9 15 3 10 8 9 8 10 6 6 8 12 5 11 1 7 8 15 6 10 1 9 5 15 7 13 12 18 12 19
8 24 12 21

- 13a 2236

1 41 3 1 16 2 20 2 17 2 13 3 9 4 9 5 20 6 17 7 15 7 2 7 20 8 17 9 19 5 13 8 12 6
10 7 7 5 8 14 14 6 17 13 3 12 4 9 4 10 10 13 13 11 1 15 9 18 20 15 12 13 8 19
19 15 17 18 8 21 11 16 4 17 15 23 8 24 2 20 19 22 19 23
2 40 9 1 10 1 15 2 13 2 5 3 20 4 4 3 17 5 6 7 17 8 5 8 9 9 1 9 11 4 11 5 8 13 19 8
5 13 10 10 8 16 9 14 19 9 14 8 5 15 7 10 14 10 1 14 4 13 17 17 15 19 18 15 3 18
19 16 19 17 12 17 7 15 11 17 19 19 10 18 16 17
3 33 5 1 8 2 10 3 9 3 4 2 7 1 13 5 17 6 18 4 11 2 14 1 8 11 18 8 9 12 1 11 15 12
16 9 12 8 3 13 1 13 4 11 10 14 5 17 9 19 16 15 14 14 18 16 10 17 17 20 7 17 2
19 19 21 11 19
4 42 2 1 20 1 15 3 19 2 12 4 3 2 5 3 3 16 4 10 4 5 7 16 6 11 3 5 9 4 5 20 9 7 4
18 9 4 7 12 7 8 15 18 10 7 9 15 14 16 11 11 9 11 10 15 17 3 15 15 18 4 14 14 13
19 14 11 14 8 20 10 16 14 15 3 20 2 18 9 22 12 20 10 19
5 36 18 2 8 1 17 1 1 3 6 4 12 3 16 3 5 4 6 6 3 4 8 9 3 7 14 3 14 4 5 11 13 9 13 10
2 9 3 11 6 10 16 10 12 9 18 12 15 16 3 14 9 17 2 13 9 20 9 21 12 15 7 14 13 15
3 21 19 20 6 15 3 23
6 37 16 1 18 3 6 2 5 2 20 3 1 5 8 5 4 4 9 8 13 7 2 8 9 10 9 11 20 10 17 11 11 6 3
10 9 13 14 7 18 11 8 17 12 10 16 12 18 14 11 12 19 13 11 13 6 11 2 14 5 20 13
14 19 18 6 14 5 21 15 24 14 16 8 25
7 34 1 1 19 1 9 2 15 4 8 4 20 5 19 3 13 6 15 8 18 5 10 5 7 3 1 10 19 7 3 9 10 8
10 9 7 8 5 14 20 12 10 12 8 18 17 16 4 12 20 14 16 14 3 16 12 14 4 15 2 16 4 16
12 18 8 23 11 18
8 40 18 1 12 1 6 3 14 17 3 17 4 13 4 2 5 9 7 16 5 20 7 3 6 15 9 18 6 19 6 17 10
8 12 16 8 14 5 5 12 7 6 4 8 15 13 17 14 15 15 20 13 18 13 11 11 16 13 14 11 13
12 13 13 7 12 7 13 12 16 17 19 15 21 18 17 17 21 15 25
9 40 13 1 4 1 10 2 11 1 2 3 15 5 12 5 9 6 5 6 1 6 8 8 1 7 19 4 1 8 14 2 18 7 5 10
15 10 15 11 17 12 20 11 1 12 11 8 17 15 14 9 9 16 5 16 12 11 19 12 12 12 7 11
10 15 11 15 6 12 3 19 20 16 15 22 12 19 4 18 13 16
10 44 6 1 15 1 1 2 12 2 2 8 3 6 5 2 4 15 6 8 6 8 7 2 6 3 5 7 2 8 10 16 7 10 6 3 8
4 6 6 8 6 9 7 7 11 7 2 10 10 11 9 15 19 10 2 11 19 11 2 12 14 12 5 18 5 19 3 17
15 20 2 15 6 13 8 22 2 17 7 16 16 16 3 22 9 23 12 21

- rdata-abz7 522

1 22 12 1 7 1 18 3 17 3 1 5 16 5 13 3 10 6 5 6 18 8 19 9 14 9 13 7 9 8 20 9 2 11
5 12 3 13 7 14 17 13 8 15 17 15

2 20 14 2 5 2 5 3 20 4 18 6 2 5 5 5 17 6 15 5 16 9 4 6 7 10 3 10 16 12 7 12 7 13
20 11 13 12 12 12 15 15

3 20 1 1 8 1 1 3 18 4 10 3 9 4 3 4 3 5 7 7 2 8 19 10 16 11 8 10 5 11 10 12 10 13
2 13 11 14 20 14 4 15

4 23 17 1 1 2 2 2 7 2 15 3 11 3 7 5 8 5 20 6 3 6 10 7 19 8 17 8 5 9 18 10 6 10 6
11 20 10 9 11 18 13 15 14 12 13 11 15

5 20 16 2 1 4 16 4 12 5 14 5 7 6 16 8 6 7 9 6 4 7 10 11 17 10 15 11 4 10 14 13
20 12 13 13 4 13 5 15 12 15

6 20 19 1 17 2 12 3 11 2 18 5 20 5 18 7 14 6 9 5 1 10 2 9 6 9 3 11 1 14 14 12 8
12 5 13 4 12 10 15 13 15

7 18 2 1 3 1 4 2 8 3 1 6 12 6 1 8 6 6 3 7 16 10 17 9 13 8 11 11 9 10 11 13 16 14
9 14 18 15

8 18 5 1 16 3 19 3 10 4 10 5 4 4 11 5 4 5 8 8 1 11 14 10 4 8 2 10 3 12 6 12 9 13
7 15 6 15

9 20 14 1 6 1 13 1 9 3 7 4 5 4 13 4 16 7 11 6 8 7 5 7 10 9 15 9 15 10 18 11 2 12
19 13 3 14 8 14 4 14

10 23 16 1 18 1 4 1 8 2 10 2 7 3 3 2 6 4 12 7 13 5 19 7 7 8 5 8 3 9 9 7 11 10 4 9
19 12 18 12 10 14 2 14 13 14 20 15

11 20 11 1 18 2 19 2 13 2 14 4 11 4 8 6 2 7 10 8 3 8 6 8 5 10 13 9 17 11 12 11 9
12 15 13 5 14 3 15 9 15

12 21 12 2 9 2 6 2 4 3 8 4 17 5 19 6 20 7 15 6 14 8 20 8 10 10 7 11 14 11 13 10
11 12 16 13 8 13 20 13 18 14 14 15

13 18 15 1 10 1 2 3 20 3 17 4 1 7 15 4 17 7 15 7 11 8 11 9 19 11 12 10 1 15 14
14 16 15 6 14 12 14

14 17 9 1 20 2 12 4 2 4 3 3 19 5 6 5 1 9 13 6 15 8 8 9 1 13 8 11 15 12 6 13 17 14
19 15

15 20 20 1 15 2 14 3 6 3 19 4 16 6 2 6 12 8 14 7 11 7 7 9 18 9 1 12 12 9 9 9 13
11 4 11 17 12 19 14 2 15

- rdata-abz8 535

1 20 1 1 15 1 15 2 17 3 5 5 18 5 2 7 16 8 4 6 12 7 9 6 3 9 5 9 13 10 17 11 4 12
17 13 20 14 6 14 19 15

2 22 8 1 10 1 3 2 5 3 15 4 18 3 16 6 10 4 12 6 5 7 4 7 18 9 16 10 3 10 2 11 8 12
1 11 7 15 20 12 13 14 14 14 9 15

3 21 20 1 19 2 12 1 16 3 1 3 4 3 13 5 13 6 7 7 8 8 8 9 9 7 12 10 15 11 14 9 3 12
17 12 18 14 9 13 10 14 10 15

4 17 3 1 18 2 4 2 16 5 9 4 14 5 11 6 19 8 11 8 20 9 10 8 10 9 2 12 5 11 6 12 1 13
17 15

5 21 16 1 5 2 17 2 20 4 2 5 15 6 12 5 4 5 19 7 7 8 5 8 6 8 14 8 9 8 14 10 18 12 1
12 14 12 11 14 15 15 14 15

6 22 7 1 11 2 6 3 15 3 2 4 15 5 12 4 17 5 8 7 10 5 6 7 14 7 19 9 3 11 7 13 16 13
13 12 11 12 4 13 5 13 1 14 6 15

7 19 17 1 6 2 12 2 3 4 18 4 8 5 16 7 6 6 1 7 17 8 8 10 6 9 19 10 15 13 12 13 2 13
14 13 3 14 2 15

8 21 11 1 20 2 10 2 7 3 5 4 3 5 19 6 17 6 2 8 10 6 4 9 11 9 7 12 18 11 7 14 20 11
19 12 9 12 18 15 12 15 1 15

9 20 14 1 9 1 7 2 8 4 13 4 2 6 11 5 1 6 13 8 4 8 12 9 10 7 11 10 5 10 11 11 9 10
8 14 11 13 4 14 3 15

10 19 2 1 1 2 20 3 19 4 14 4 7 6 6 5 13 7 18 8 3 8 7 9 16 11 17 10 10 10 18 13
12 14 8 15 5 14 5 15

11 17 6 1 14 2 8 3 9 3 17 4 7 5 1 5 18 7 16 9 13 9 18 10 12 11 4 11 19 11 13 13
5 12 20 15

12 22 19 1 8 2 16 2 3 3 14 3 12 3 19 5 5 6 15 8 17 7 20 8 15 10 2 10 4 10 6 10
12 12 16 14 9 11 20 13 10 13 13 15 4 15

13 19 5 1 19 3 13 3 7 4 11 4 4 4 15 7 18 6 14 6 15 9 2 9 17 9 1 10 13 11 9 9 10
11 10 12 17 14 11 15

14 19 2 2 13 2 9 2 16 4 11 3 1 4 20 6 9 5 3 7 12 8 7 10 1 9 8 11 20 10 6 11 15 14
6 13 19 14 9 14

15 21 18 1 13 1 4 1 2 3 6 4 20 5 10 3 8 6 3 6 20 7 11 7 1 8 7 11 16 12 15 12 8 13
14 11 3 13 19 13 2 14 16 15

- rdata-la22 753

1 19 10 1 10 2 11 2 11 3 3 4 13 5 5 2 2 5 2 6 5 5 5 6 5 7 4 9 15 9 15 10 7 8 1 10
2 10 7 10

2 21 4 1 9 1 2 3 8 2 9 2 14 1 6 3 13 4 11 4 5 3 7 4 15 5 14 5 6 7 10 9 14 7 3 8 1 7
3 9 4 10 5 10

3 16 2 1 2 2 4 2 3 2 3 3 1 3 15 3 4 5 13 6 10 6 9 8 7 5 13 9 6 9 13 10 7 9

4 12 11 1 13 2 12 2 13 3 9 4 6 4 15 4 10 5 8 8 1 6 5 9 11 10

5 12 8 1 15 1 5 1 4 4 6 5 14 4 11 8 10 7 12 8 6 8 7 7 3 10

6 14 12 1 1 2 8 4 2 4 1 4 8 7 4 7 3 6 10 8 14 6 5 8 2 8 9 10 12 10

7 14 6 1 10 3 4 3 7 3 8 6 11 6 9 7 15 6 2 7 3 7 7 6 1 8 12 9 14 9

8 14 13 1 6 2 12 3 12 4 8 5 14 3 11 7 1 5 12 6 15 7 9 9 15 8 2 9 14 10

9 14 3 1 8 3 9 3 15 2 14 2 9 5 9 6 4 6 6 6 12 7 13 7 11 9 14 8 10 10

10 14 1 1 7 1 7 2 10 4 3 5 11 5 5 4 12 5 4 8 8 9 13 8 8 10 19 6 10

- rdata-la23 831

1 1531911538212311415648135281186741019910
 2 1515122102112635484145761178813814898310
 3 1321152449313475861577713712859510
 4 16814323242515510535116146108781729210610
 5 171011315242624515474144551587661471591391210
 6 1541321421211364141061071361589768129710
 7 146114153143104261251261611918101079110
 8 1471119210313346124115275638898101410
 9 151117273331347948595965758111015101310
 10 1651121132122833465363749127109391496999

- rdata-la24 795

1 145192621212515546861171385878210110
 2 183110242132105145746614714847149761191285919
 7 10
 3 131425214313441078536972989410610
 4 1441103231331121225596157571271010141069
 5 181419122431541069411334261512312448109151013
 10 510
 6 141311112483455411435277511815913979
 7 1521717214413414115116563788779968810
 8 141511115282326484951461371715867129
 9 16816110463735313513616125126983811109101210
 10 14101121539333651562810887184939310

- rdata-la25 779

1 1651435333155512610327138105771104101291410
 2 131211429572134145985786288811101010
 3 174113162231314641188515637106107128610310710
 4 113122811165473155661949149
 5 15151152931211445117135146137354767148210
 6 177112212344115241257465995887127381595101310
 7 1614111213214312425843410418157687869109810
 8 1410191113631338246561617369102979
 9 18111116110292153941541449713675764815813989
 15 10
 10 132142523296832614711959108391210

- vdata-car5 4910

1 9116171927273148496
 2 1121224252821038335659566
 3 1131513223335363932526106
 4 1010110224641049474368586
 5 941816234445557556
 6 10911243135410515164676

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