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A Two-Individual Based Evolutionary Algorithm for the Flexible Job Shop Scheduling Problem

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Outline

- 1 The Job Shop Scheduling Problem
- 2 The Flexible Job Shop Scheduling Problem
- 3 The Disjunctive Graph Model
- 4 Neighborhood Structure
- 5 Approximate Evaluation
- 6 Our Proposed MAE Algorithm

Job shop scheduling problem (JSP): Given a set of jobs $J = \{J_1, \dots, J_n\}$ that must be processed on a set $M = \{M_1, \dots, M_m\}$ of machines. Each job $J_i, i = 1, \dots, n$, consists of n_i operations $O_i = \{o_{i1}, \dots, o_{in_i}\}$ that should be sequentially processed. Besides, each operation o_{ij} requires uninterrupted and exclusive use of its assigned machine for its whole processing time.

- Job set $J = \{J_1, \dots, J_n\}$.
- Machine set $M = \{M_1, \dots, M_m\}$.
- Operation set $O_i = \{o_{i1}, \dots, o_{in_i}\}, i \in \{1, \dots, n\}$.
- Each operation o_{ij} can be only processed by one machine $m \in M$ and requires a processing time of t_{ij} .

The problem is to assign each operation to a machine and to order the operations on the machines, such that the maximum completion time of all jobs (i.e., makespan) is minimized.

An example

- Three machines M_1, M_2, M_3 .
- Three jobs J_1, J_2, J_3 .
- Each job has three operations, $J_{1,1}, J_{1,2}, J_{1,3}$; $J_{2,1}, J_{2,2}, J_{2,3}$; $J_{3,1}, J_{3,2}, J_{3,3}$.

Table 1: An instance

Job	Processing sequence (machine, time)		
J_1	(1,3)	(2,2)	(3,3)
J_2	(1,2)	(3,3)	(2,4)
J_3	(2,2)	(1,2)	(3,3)

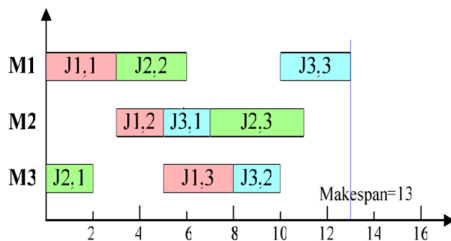


Figure 1: The gantt chat of a solution

The extension of JSP: FJSP

The flexible job shop scheduling problem (FJSP) is an extension of JSP by allowing an operation o_{ij} to be processed on one of a set of candidate machines $M(o_{ij}) \subseteq M$. The processing time of operation o_{ij} on machine $M_k \in M(o_{ij})$ is denoted by $t_{o_{ij}k}$.

- Each operation o_{ij} can be processed on a set of candidate machines $M(o_{ij}) \subseteq M$.
- The processing time of operation o_{ij} on machine $M_k \in M(o_{ij})$ is denoted by $t_{o_{ij}k}$.

Both JSP and FJSP are proven to be NP-hard.

The **disjunctive graph** is a directed acyclic graph, in which vertices (representing operations to be performed) may be connected by both directed and undirected edges (representing timing constraints between operations).

Table 2: An instance

Job	Processing sequence (machine, time)		
J_1	(1,3)	(2,2)	(3,3)
J_2	(1,2)	(3,3)	(2,4)
J_3	(2,2)	(1,2)	(3,3)

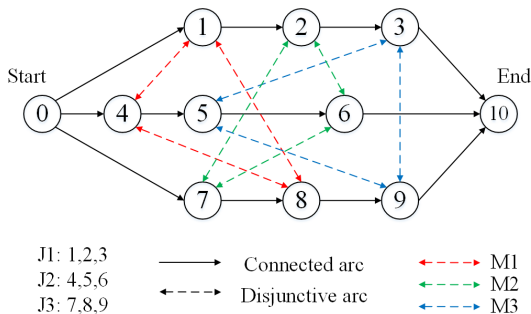


Figure 2: The disjunctive graph of the instance in Table 2

Critical path and operations

Critical path: the longest path from the start node to the end node, of which the length is the makespan.

Critical operations: the operations along the critical path.

When the processing sequence of the operations are determined on each machine, a solution is obtained.

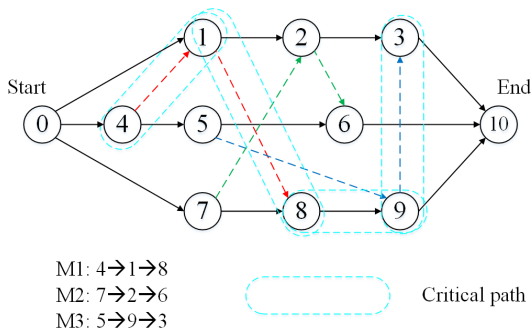


Figure 3: The disjunctive graph of a solution for the instance in Table 2

Critical block

Critical block: The consecutive critical operations on the same machine.

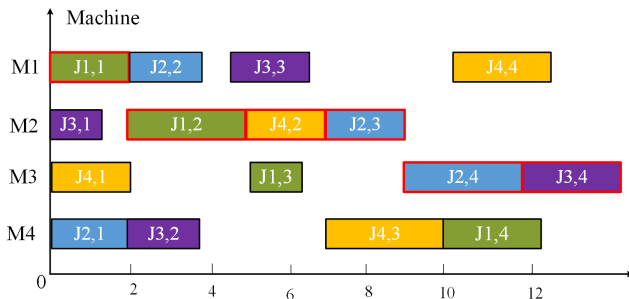


Figure 4: The illustration of critical block

Three critical blocks: (J1,1) (J1,2 J4,2 J2,3) (J2,4 J3,4).

The gantt chart of a solution

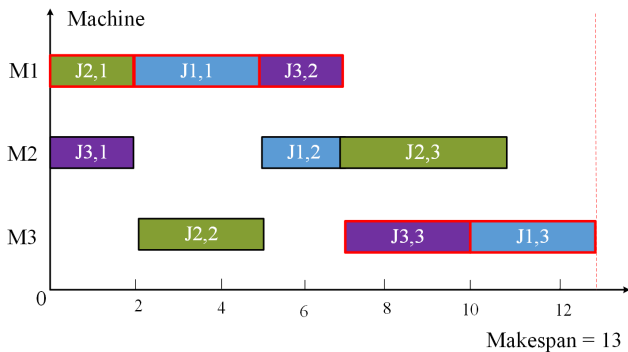


Figure 5: The gantt chart of the solution in Fig. 3

N5

- N5 was proposed in Nowicki and Smutnicki [1996].
- It only swaps the two consecutive operations on the front and rear of the critical block, in order to avoid moving the inner operations.
- N5 is very fast to search due to the small size, but it is less efficiency because potentiality of searching the promising areas is restricted.

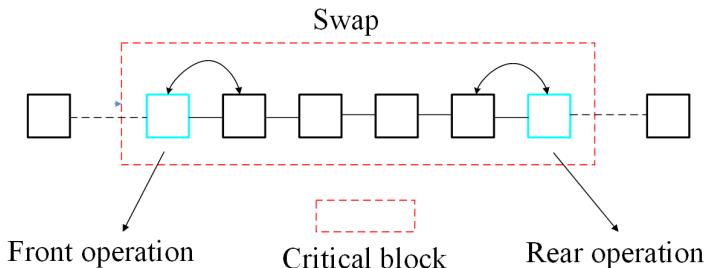


Figure 6: The illustration of N5

N6

- N6 was proposed in Balas and Vazacopoulos [1998].
- It moves the inner operations of a critical block to the front or rear operations of it.

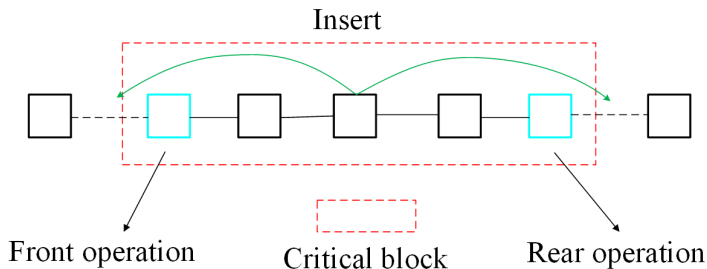


Figure 7: The illustration of N6

N7

- N7 was proposed in Zhang *et al.* [2007].
- Based on N6, N7 additionally moves the front and rear operations of a critical block to the inner positions of it.
- N7 is the most effective neighborhood structure used in local search methods.

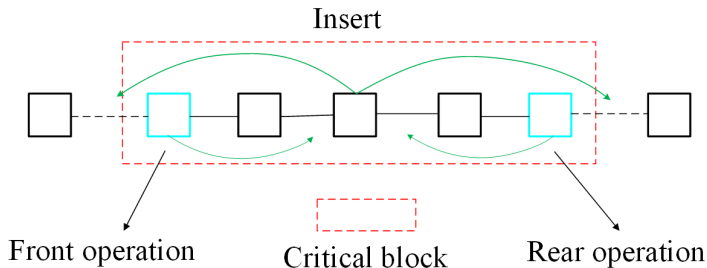


Figure 8: The illustration of N7

Definition of JP , JS , MP , and MS

- $JP[i]$: The predecessor of operation i on the same job.
- $JS[i]$: The successor of operation i on the same job.
- $MP[i]$: The predecessor of operation i on the same machine.
- $MS[i]$: The successor of operation i on the same machine.

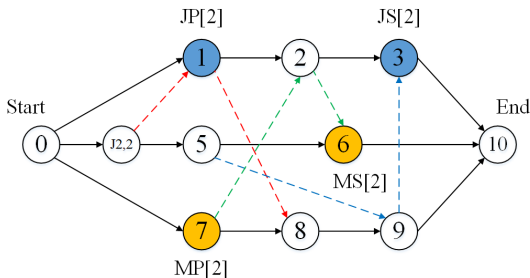


Figure 9: The illustration of JP , JS , MP , and MS

Definition of R and Q

- $R[i]$: The longest path from the start node to operation i .
- $Q[i]$: The longest path from operation i to the end node.
- $R[i] = \max\{R[JP[i]] + t[JP[i]], R[MP[i]] + t[MP[i]]\}$.
- $Q[i] = \max\{Q[JS[i]] + t[JS[i]], Q[MS[i]] + t[MS[i]]\}$.
- For critical operation i , $makespan = R[i] + t[i] + Q[i]$.

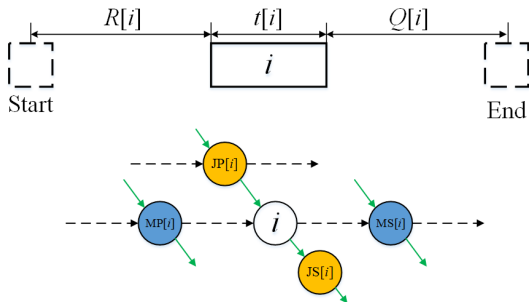


Figure 10: The illustration of R and Q

Estimation of insert move

Suppose u and v are the operations on the same machine, and u is on the left of v , after insert u into the rear of v , the makespan can be estimated as:

$$makespan^{u,v} = \max\{R^{u,v}(w) + t[w] + Q^{u,v}(w)\}, w \in \{u, L_1, \dots, L_k, v\}$$

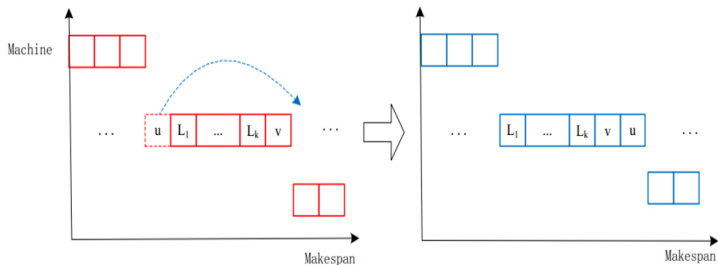


Figure 11: The illustration of insert move estimation

An example of unfeasible move

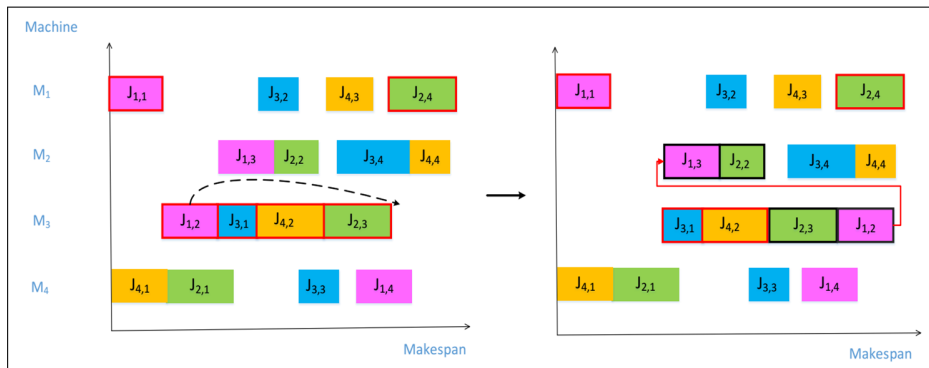


Figure 12: The illustration of unfeasible move

A cycle is encountered: $J_{2,3} \rightarrow J_{1,2} \rightarrow J_{1,3} \rightarrow J_{2,2} \rightarrow J_{2,3}$

Theorem of feasible move

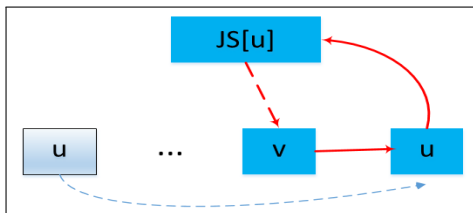


Figure 13: The illustration of feasible move

Theorem 1: Suppose u and v are the critical operations on the same machine of a feasible solution, and u is on the left of v . If $Q[v] \geq Q[JS[u]]$, therefore, insert u after v results in a feasible solution.

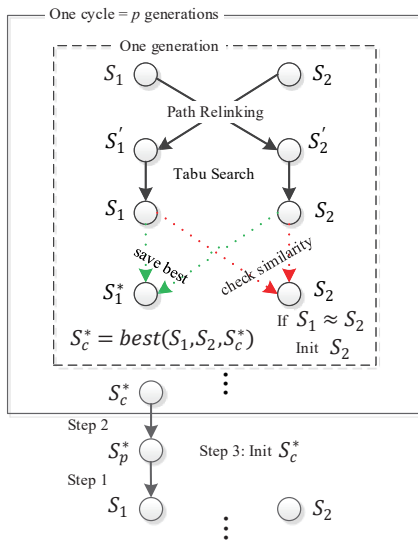


Figure 14: Diagram of MAE

MAE for FJSP

Algorithm 1 MAE, a two-individual based evolutionary algorithm for FJSP

```

1: Input: Problem instance
2: Output: The best solution  $S^*$  found
3:  $gen \leftarrow 0$ ,  $S_1, S_2, S_c^*, S_p^*, S^* \leftarrow \text{Init}()$ 
4: while stopping condition is not reached do
5:    $S_1' \leftarrow \text{PR}(S_1, S_2)$ ,  $S_2' \leftarrow \text{PR}(S_2, S_1)$ 
6:    $S_1 \leftarrow \text{TS}(S_1')$ ,  $S_2 \leftarrow \text{TS}(S_2')$ 
7:    $S_c^* \leftarrow \text{save\_best}(S_1, S_2, S_c^*)$ 
8:    $S^* \leftarrow \text{save\_best}(S_c^*, S^*)$ 
9:   if  $gen$  is equal to an integer parameter  $p$  then
10:     $S_1 \leftarrow S_p^*$ ,  $S_p^* \leftarrow S_c^*$ ,  $S_c^* \leftarrow \text{Init}()$ ,  $gen \leftarrow 0$ 
11:   end if
12:   if  $S_1 \approx S_2$  then
13:     $S_2 \leftarrow \text{Init}()$ 
14:   end if
15:    $gen \leftarrow gen + 1$ 
16: end while
17: return  $S^*$ 

```

Distance definition

- Distance on the same machine: $d_o(S_1, S_2) = |P_o^{S_1} - P_o^{S_2}|$

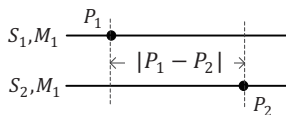


Figure 15: The illustration of the distance of the operation on the same machine.

Distance definition

- Distance on difference machines:

$$d_o(S_1, S_2) = \min\{P_o^{S_1} + P_o^{S_2}, (L_{M_o^{S_1}}^{S_1} - P_o^{S_1}) + (L_{M_o^{S_2}}^{S_2} - P_o^{S_2})\}$$

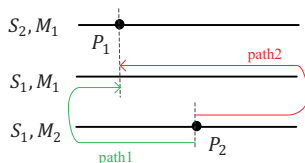


Figure 16: The illustration of the distance of the operation on the different machine.

Algorithm 2 A path relinking based recombination operator

```

1: Input: Initial solution  $S_i$  and guiding solution  $S_g$ 
2: Output: A reference solution  $S_r$ 
3:  $S_c \leftarrow S_i, PathSet \leftarrow \emptyset, N \leftarrow \emptyset$ 
4: while  $d(S_c, S_g) > \alpha \times d(S_i, S_g)$  do
5:   for each operation  $o$  in  $S_c$  do
6:     if  $M_o^{S_c} \neq M_o^{S_g}$  then
7:        $N \leftarrow N \cup N_g^k(S_c, o)$ 
8:     else if  $M_o^{S_c} = M_o^{S_g}$  and  $P_o^{S_c} \neq P_o^{S_g}$  then
9:        $N \leftarrow N \cup N_g^\pi(S_c, o)$ 
10:    end if
11:  end for
12:  for each solution  $S \in N$  do
13:    if  $d(S, S_g) > d(S_c, S_g)$  then  $N \leftarrow N \setminus \{S\}$ 
14:    else estimate makespan  $obj(S)$  end if
15:  end for
16:  for each solution  $S \in N$  do
17:     $indexDis(S) \leftarrow |\{T \in N | d(T, S_g) < d(S, S_g)\}|$ 
18:     $indexObj(S) \leftarrow |\{T \in N | obj(T) < obj(S)\}|$ 
19:  end for
20:  sort  $N$  in increasing order of  $indexDis(S) + indexObj(S)$ , breaking ties randomly
21:   $k \leftarrow rand\{0, 1, \dots, \min\{\gamma, |N| - 1\}\}$ 
22:   $S_c \leftarrow N(k); N \leftarrow \emptyset$ 
23:  if  $d(S_c, S_g) < \beta \times d(S_i, S_g)$  then
24:     $PathSet \leftarrow PathSet \cup \{S_c\}$ 
25:  end if
26: end while
27:  $S_r = \arg \min\{f(S), S \in PathSet\},$ 
28: return  $S_r$ 

```

Benchmark sets

Table 3: The descriptions of the benchmark sets

Set	n	m	$flex.$	Size	$\#opt$
$DPdata$	[10, 20]	[5, 10]	[1.13, 5.02]	18	8
$BCdata$	[10, 15]	[11, 18]	[1.07, 1.30]	21	21
$BRdata$	[10, 20]	[5, 15]	[1.43, 4.10]	10	9
$HUdata$	$edata$	[6, 30]	1.15	66	64
	$rdata$	[6, 30]	2.00	66	54
	$vdata$	[6, 30]	[2.50, 7.50]	66	65
	$sdata$	[6, 30]	1.00	66	64
Total				313	285

Comparison with the-state-of-the-art metaheuristics

- SSPR [González *et al.*, 2015] and GRASP-mELS [Kemmoé-Tchomté *et al.*, 2017] are the recent state-of-the-art metaheuristics for FJSP.
- Comparison are made in the same time limit.
- MAE improves the previous best results obtained by for 47 out of 178 benchmark instances while matching the best known results for others.
- MAE improves the previous best known results obtained by GRASP-mELS for 52 out of 178 benchmark instances and matches the best known results on all except 5 of the remaining instances.

Comparison with exact methods: summary

Table 4: Summary of MAE compared with CPO and Quintiq

Set	MAE(1 hour) vs CPO(8 hours)			MAE(1 hour) vs Quintiq		
	#better	#even	#worse	#better	#even	#worse
<i>DPdata</i>	13	5	0	3	2	10
<i>BCdata</i>	0	18	3	0	0	0
<i>BRdata</i>	2	8	0	0	2	0
<i>edata</i>	2	60	4	0	20	0
<i>HUdata rdata</i>	18	48	0	6	30	1
<i>vdata</i>	10	53	3	1	22	0
<i>sdata</i>	0	63	3	0	18	6
Total	45	255	13	10	94	17

Comparison with the exact method: CPO

Table 5: The improved results of MAE compared with CPO on 45 instances

Ins.	CPO		MAE	Ins.	CPO		MAE
	LB	UB	UB		LB	UB	UB
Mk05	168	173	172	rdata-la23	816	832	831
Mk10	183	195	193	rdata-la24	775	805	795
02a	2228	2234	2228	rdata-la25	752	787	779
05a	2189	2213	2203	rdata-la26	1056	1066	1057
06a	2162	2191	2181	rdata-la27	1085	1099	1086
07a	2206	2277	2254	rdata-la28	1075	1079	1076
08a	2061	2066	2062	rdata-la29	993	1001	994
10a	2197	2263	2245	rdata-la30	1068	1089	1071
11a	2017	2067	2045	rdata-la31	1520	1522	1520
12a	1969	2013	2008	rdata-la32	1657	1658	1657
13a	2161	2258	2236	rdata-la33	1497	1498	1497
14a	2161	2163	2162	rdata-la34	1535	1536	1535
16a	2148	2240	2232	vdata-car1	5005	5006	5005
17a	2088	2140	2121	vdata-car3	5597	5599	5597
18a	2057	2125	2103	vdata-car5	4909	4912	4910
edata-abz7	564	620	610	vdata-la22	733	734	733
edata-abz8	586	639	636	vdata-la25	751	753	752
rdata-abz7	492	535	522	vdata-la29	993	994	993
rdata-abz8	506	558	535	vdata-la30	1068	1069	1068
rdata-abz9	497	553	536	vdata-la32	1657	1658	1657
rdata-car3	5597	5623	5622	vdata-la33	1497	1498	1497
rdata-la21	808	838	825	vdata-la35	1549	1550	1549
rdata-la22	741	757	755				

New world records obtained by MAE

Table 6: New world records obtained by MAE

Ins.	Previous world record						MAE
	LB	LB Ref.	LB Date	UB	UB Ref.	UB Date	UB
05a	2192	[Q]	Jan. 2014	2204	[Q]	Nov. 2015	2203
07a	2216	[CPO]	Dec. 2014	2264	[Q]	Nov. 2015	2254
13a	2197	[CPO]	Dec. 2014	2239	[Q]	Jan. 2016	2236
rdata-abz7	493	[Q]	Mar. 2013	524	[Q]	Jan. 2016	522
rdata-abz8	507	[Q]	Mar. 2013	536	[Q]	Jan. 2016	535
rdata-la22	741	[CPO]	Dec. 2014	755	[CPO]	Nov. 2013	753
rdata-la23	816	[Q]	Feb. 2000	832	[CPO]	Mar. 2013	831
rdata-la24	775	[Q]	Feb. 2000	796	[Q]	Nov. 2015	795
rdata-la25	768	[CPO]	Dec. 2014	783	[Q]	Jan. 2016	779
vdata-car5	4909	[Q]	Mar. 2013	4911	[Q]	Nov. 2015	4910

Comparison between MAE and the trajectory method

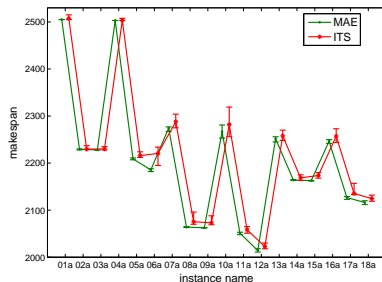


Figure 17: Comparison between MAE and ITS on *DPdata*.

The impact of the parameters: p

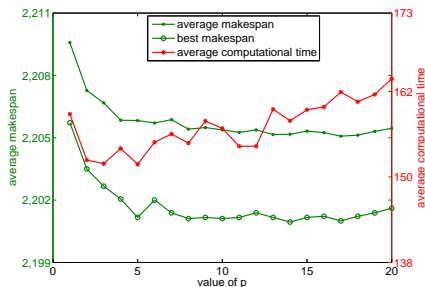


Figure 18: The average makespan and computational time corresponding to different values of parameter p on *DPdata*.

The impact of the parameters: α, β, γ

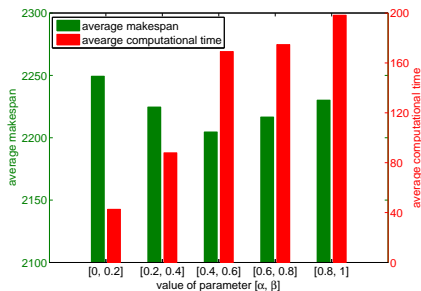


Figure 19: The average makespan and computational time corresponding to different values of parameter $[\alpha, \beta]$ on *DPdata*.

The impact of the parameters: γ

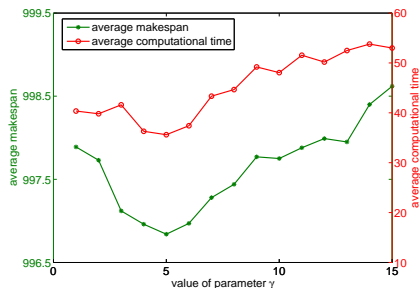


Figure 20: The average makespan and computational time corresponding to different values of parameter γ on *BCdata*.

The evolution process of the two individuals

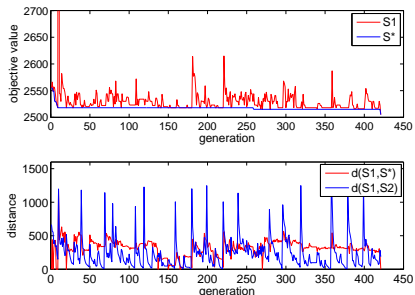


Figure 21: The evolution of objective values and distances when solving instance 01a.

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