# A Master-Apprentice Evolutionary Algorithm for the Flexible Job Shop Scheduling Problem

#### Abstract

This paper presents an evolutionary algorithm based on two individuals (called master-apprentice evolutionary algorithm, MAE) for solving the flexible job shop scheduling problem, which distinguishes itself from both single solution-based and traditional population-based metaheuristics. MAE integrates a tabu search procedure, a path-relinking recombination operator, and an individual updating strategy. At each generation, a path relinking recombination operator is applied on the two individuals to generate two child solutions. The individual updating strategy is conducted once a given number of generations (called a cycle) is reached: one of the two individuals is updated at the end of a cycle with the best solution found in the previous cycle. The other individual is initialized when the two individuals are close to each other at each generation. Computational experiments on 178 public instances show that MAE achieves highly competitive results in terms of both solution quality and computational efficiency compared with the state-of-the-art algorithms in the literature. Specifically, it improves the previous best known results for 47 instances while matching the best known results on all except 5 of the remaining instances with short computational time.

*Keywords:* flexible job shop scheduling problem, master-apprentice evolutionary algorithm, path relinking.

### 1. The computational results of MAE

This paper is an appendix of the paper submitted to IJCAI-2018, which provides the overall computational results of MAE on the benchmark sets (*DPdata* (Dauzre-Prs and Paulli, 1997), *BCdata* (Barnes and Chambers, 1998), *BRdata* (Brandimarte, 1993), and *HUdata* (Hurink et al., 1994)) and make comparisons with the recent state-of-the-art algorithms (SSPR Gon-

zlez et al. (2015) and GRASP-mELS Kemmo-Tchomt et al. (2017)) in the literature.

## 1.1. The results of MAE on DPdata set

Table 1: Comparison between MAE and other reference algorithms on DPdata instance set

Ins.	Qunitiq		SSPR		GRASP-mELS	MAE
11101	LB	UB	LB	best	best	best
01a	2505*	2505	2505	2505	2505	2505
02a	2228*	2228	2228	2229	2229	2228
03a	2228*	2228	2228	<b>2228</b>	2228	2228
04a	2503*	2503	2503	2503	2503	2503
05a	2192	2204	2189	2211	2212	2203
06a	2163	2171	2162	2183	2195	2181
07a	2216	2264	2187	2274	2276	2254
08a	2061*	2061	2061	2064	2069	2062
09a	2061*	2061	2061	2062	2069	2062
10a	2212	2241	2178	2269	2263	2245
11a	2018	2037	2017	2051	2065	2045
12a	1969	1984	1969	2018	2039	2008
13a	2197	2239	2161	2248	2252	2236
14a	2161*	2161	2161	2163	2170	2162
15a	2161*	2161	2161	<b>2162</b>	2172	2162
16a	2193	2231	2148	2244	2243	2232
17a	2088	2105	2088	2130	2145	2121
18a	2057	2070	2057	2119	2146	2108

<sup>1.2.</sup> The results of MAE on BCdata set

#### References

Barnes, J. W., Chambers, J. B., June 1998. Flexible job shop scheduling by tabu search. Tech. rep., The University of Texas at Austin.

<sup>1.3.</sup> The results of MAE on BRdata set

<sup>1.4.</sup> The results of MAE on HUdata set

Table 2: Comparison between MAE and other reference algorithms on BCdata instance set

Ins.	Qunitiq		SSPR		GRASP-mELS	MAE
1110.	LB	UB	LB	best	best	best
mt10c1	927*	927	655	927	927	927
mt10cc	908*	908	655	908	908	908
mt10x	918*	918	655	918	918	918
mt10xx	918*	918	655	918	918	918
mt10xxx	918*	918	655	918	918	918
mt10xy	905*	905	655	905	905	905
mt10xyz	$847^{*}$	847	655	847	847	847
setb4c9	914*	914	857	914	914	914
setb4cc	$907^{*}$	907	857	907	907	907
setb4x	925*	925	846	$\boldsymbol{925}$	925	<b>925</b>
setb4xx	925*	925	846	$\boldsymbol{925}$	925	<b>925</b>
setb4xxx	925*	925	846	$\boldsymbol{925}$	925	<b>925</b>
setb4xy	910*	910	845	910	910	910
setb4xyz	902*	902	838	905	902	902
seti5c12	1169*	1169	1027	1170	1169	1170
seti5cc	$1135^{*}$	1135	955	1135	1135	1135
seti5x	1198*	1198	955	1198	1198	1198
seti5xx	1194*	1194	955	1197	1194	1197
seti5xxx	1194*	1194	955	1194	1194	1197
seti5xy	$1135^{*}$	1135	955	1135	1135	1135
seti5xyz	1125*	1125	955	1125	1125	1125

Table 3: Comparison between MAE and other reference algorithms on BRdata instance set

Ins.	Qunitiq		SSPR		GRASP-mELS	MAE
1110.	LB	UB	LB	best	best	best
Mk01	40*	40	36	40	40	40
Mk02	26*	26	24	<b>26</b>	26	<b>26</b>
Mk03	204*	204	204	204	204	204
Mk04	60*	60	48	60	60	60
Mk05	172*	172	168	172	172	172
Mk06	57*	57	33	<b>57</b>	58	<b>57</b>
Mk07	139*	139	133	139	139	139
Mk08	523*	523	523	523	523	523
Mk09	307*	307	299	307	307	307
Mk10	189	193	165	196	197	193

Brandimarte, P., 1993. Routing and scheduling in a flexible job shop by tabu search. Annals of Operations Research 41 (3), 157–183.

Dauzre-Prs, S., Paulli, J., 1997. An integrated approach for modeling and solving the general multiprocessor job-shop scheduling problem using tabu search. Annals of Operations Research 70 (1), 281–306.

Gonzlez, M. n., Vela, C. R., Varela, R., 2015. Scatter search with path relinking for the flexible job shop scheduling problem. European Journal of Operational Research 245 (1), 35–45.

Hurink, J., Jurisch, B., Thole, M., 1994. Tabu search for the job-shop scheduling problem with multi-purpose machines. Operations-Research-Spektrum 15 (4), 205–215.

Kemmo-Tchomt, S., Lamy, D., Tchernev, N., 2017. An effective multi-start multi-level evolutionary local search for the flexible job-shop problem. Engineering Applications of Artificial Intelligence 62, 80–95.

 $\underline{\text{Table 4: Comparison between MAE and other reference algorithms on } \textit{edata} \text{ instance set}}$ 

Ins.	Qunitiq		SSPR		GRASP-mELS	MAE
	LB	UB	LB	best	best	best
edata-abz5	1167*	1167	_	_	_	1167
edata-abz6	925*	925	_	-	_	<b>925</b>
edata-abz7	604	610	_	_	_	610
edata-abz8	625	636	_	_	_	636
edata-abz9	644*	644				646
			-	-	-	
edata-car1	6176*	6176	-	-	-	6176
edata-car2	6327*	6327	-	-	-	6327
edata-car3	6856*	6856	-	-	-	6856
edata-car4	7789*	7789	-	-	-	7789
edata-car5	7229*	7229	-	-	-	7229
edata-car6	7990*	7990	-	-	_	7990
edata-car7	6123*	6123	_	-	_	6123
data-car8	7689*	7689	_	_	_	7689
data-la01	609*	609	609	609	609	609
edata-la02	655*	655	655	655	655	655
	550*					550
data-la03		550	550	550	550	
data-la04	568*	568	568	568	568	568
data-la05	503*	503	503	503	503	503
data-la06	833*	833	833	833	833	833
data-la07	762*	762	762	<b>762</b>	<b>762</b>	762
data-la08	845*	845	845	$\bf 845$	845	$\bf 845$
data-la09	878*	878	878	878	878	878
edata-la10	866*	866	866	866	866	866
data-la11	1103*	1103	1087	1103	1103	1103
data-la12	960*	960	960	960	960	960
data-la13	1053*	1053	1053	1053	1053	1053
data-la14	1123*	1123	1123	1123	1123	1123
data-la15	1111*	1111	1111	1111	1111	1111
data-la16	892*	892	892	$\bf 892$	$\bf 892$	$\bf 892$
data-la17	707*	707	707	707	707	707
data-la18	842*	842	842	$\bf 842$	842	$\bf 842$
data-la19	796*	796	796	796	796	796
edata-la20	857*	857	857	857	857	857
edata-la21	1009*	1009	895	1010	1009	1009
	880*	880				
edata-la22			832	880	880	880
data-la23	950*	950	950	950	950	950
edata-la24	908*	908	881	908	908	908
edata-la25	936*	936	894	939	936	<b>936</b>
data-la26	1106*	1106	1089	1109	1107	1111
data-la27	1181*	1181	1181	1181	1181	1181
data-la28	1142*	1142	1116	1144	1144	1142
data-la29	1107*	1107	1058	1111	1113	1112
data-la30	1188*	1188	1147	1204	1198	1193
data-la31	1532*	1532	1523	1533	1536	1532
data-la32	1698*	1698	1698	1698	1698	1698
edata-la33	1547*	1547	1547	1547	1547	1547
data-la34	1599*	1599	1592	1599	1599	1599
data-la35	1736*	1736	1736	1736	1736	1736
data-la36	1160*	1160	1006	1160	1160	1160
data-la37	1397*	1397	1355	1397	1397	1397
edata-la38	1141*	1141	1019	1141	1141	1141
data-la39	1184*	1184	1151	1184	1184	1184
data-la40	1144*	1144	1034	1144	1144	1144
	55*					
data-mt06		55	55	55 571	55	55 0 <b>5</b> 1
data-mt10	871*	871	871	871	871	871
data-mt20	1088*	1088	1088	1088	1088	1088
data-orb1	977*	977	-	-	-	<b>977</b>
data-orb10	933*	933	-	-	-	<b>933</b>
edata-orb2	865*	865	-	-	_	865
data-orb3	951*	951	_	_	_	951
edata-orb4	984*	984				984
			-	-	-	
data-orb5	842*	842	-	-	-	842
edata-orb6	958*	958	-	-	-	958
edata-orb7	389*	389	-	-	-	389
edata-orb8	894*	894	-	-	-	$\bf 894$
	933*	933	_	_	_	933

Table 5: Comparison between MAE and other reference algorithms on rdata instance set

Ins.	Qunitiq		SSPR		GRASP-mELS	MAE	
	LB	UB	LB	best	best	best	
rdata-abz5	954*	954	-	-	-	954	
data-abz6	807*	807	-	-	-	807	
data-abz7	493	524	-	-	_	522	
data-abz8	507	536	_	_	_	535	
data-abz9	517	536	_	-	-	536	
data-car1	5034*	5034	_	_	_	5034	
data-car2	5985*	5985	_	_	_	5985	
data-car3	5622*	5622	_	_	_	$\bf 5622$	
data-car4	6514*	6514	_	_	_	6514	
data-car5	5615*	5615	_	-	-	5615	
data-car6	6147*	6147	_	-	-	6147	
data-car7	4425*	4425	_	-	-	4425	
data-car8	5692*	5692	_	_	_	5692	
data-la01	570*	570	570	571	570	570	
data-la02	529*	529	529	530	529	529	
data-la03	477*	477	477	477	477	477	
data-la04	502*	502	502	502	502	502	
data-la05	457*	457	457	457	457	457	
data-la06	799*	799	799	799	799	799	
data-la07	749*	749	749	749	749	749	
data-la08	765*	765	765	765	765	765	
data-la09	853*	853	853	853	853	853	
data-la10	804*	804	804	804	804	804	
	1071*	1071	1071	1071	1071	1071	
data-la11	936*	936	936	936	936	936	
data-la12	1038*	1038	1038				
data-la13	1070*	1070		1038	$1038 \\ 1070$	1038	
data-la14			1070	1070		1070	
data-la15	1089*	1089	1089	1089	1089	1089	
data-la16	717*	717	717	717	717	717	
data-la17	646*	646	646	646	646	646	
data-la18	666*	666	666	666	666	666	
data-la19	700*	700	647	700	700	700	
data-la20	756*	756	756	<b>756</b>	756	756	
data-la21	808	825	808	830	832	825	
data-la22	741	755	737	756	757	753	
rdata-la23	816	832	816	835	836	831	
rdata-la24	775	796	775	802	802	795	
rdata-la25	768	783	752	784	784	779	
data-la26	1056	1057	1056	1059	1060	1057	
data-la27	1085*	1085	1085	1089	1089	1086	
data-la28	1075	1076	1075	1078	1077	1076	
data-la29	993	994	993	996	996	994	
data-la30	1068	1071	1068	1074	1074	1071	
data-la31	1520*	1520	1520	1520	1521	1520	
data-la32	$1657^*$	1657	1657	1658	1658	1657	
data-la33	1497*	1497	1497	1498	1498	1497	
data-la34	1535*	1535	1535	1535	1535	1535	
data-la35	1549*	1549	1549	1550	1550	1549	
data-la36	1023*	1023	1016	1023	1023	1023	
data-la37	1062*	1062	989	1069	1066	1062	
data-la38	954*	954	943	961	958	954	
data-la39	1011*	1011	966	1024	1018	1011	
data-la40	955*	955	955	961	958	955	
data-mt06	47*	47	47	47	47	47	
data-mt10	686*	686	679	686	686	686	
data-mt20	1022*	1022	1022	${\color{red}1022}$	<b>1022</b>	1022	
data-mt20	746*	746	-	-	-	746	
data-orb10	740*	$740 \\ 742$	-	-	-	740	
data-orb2	696*	696	-	-	-	696	
			-	-	-		
data-orb3	712*	712	-	-	-	712	
data-orb4	753*	753	-	-	-	753	
data-orb5	639*	639	-	-	-	639	
rdata-orb6	754*	754	-	-	-	754	
rdata-orb7	302*	302	-	-	-	302	
rdata-orb8	639*	639	-	-	-	639	
rdata-orb9	694*	694	_	_	_	<b>694</b>	

 $\underline{\text{Table 6: Comparison between MAE and other reference algorithms on } \textit{vdata} \text{ instance set}$ 

Ins.	Qunitiq		SSPR		GRASP-mELS	MAE
	LB	UB	LB	best	best	best
vdata-abz5	859*	859	_	_	_	859
vdata-abz6	742*	742	_	_	_	$\bf 742$
vdata-abz7	492*	492	-	-	_	498
vdata-abz8	506	507	_	_	_	512
vdata-abz9	497*	497	_	_	_	503
vdata-car1	5005*	5005	_	_	_	5005
vdata-car2	5929*	5929	_	_	_	<b>5929</b>
vdata-car3	5597*	5597	_	-	_	5597
vdata-car4	6514*	6514	_	-	_	6514
vdata-car5	4909	4911	_	_	_	4910
vdata-car6	5486*	5486	_	_	_	5486
vdata-car7	4281*	4281	_	-	_	4281
vdata-car8	4613*	4613	_	-	_	4613
vdata-la01	570*	570	570	570	<b>57</b> 0	570
vdata-la02	529*	529	529	<b>529</b>	<b>529</b>	<b>529</b>
vdata-la03	477*	477	477	477	477	477
vdata-la04	502*	502	502	<b>502</b>	502	502
vdata-la05	457*	457	457	457	457	457
vdata-la06	799*	799	799	799	799	799
vdata-la07	749*	749	749	<b>749</b>	749	749
vdata-la08	765*	765	765	765	765	765
vdata-la09	853*	853	853	853	853	853
vdata-la10	804*	804	804	804	804	804
vdata-la11	1071*	1071	1071	1071	1071	1071
vdata-la11 vdata-la12	936*	936	936	936	936	936
vdata-la12 vdata-la13	1038*	1038	1038	1038	1038	1038
vdata-la14	1070*	1070	1070	1070	1070	1070
vdata-la14 vdata-la15	1089*	1089	1089	1089	1070	1089
vdata-la16	717*	717	717	717	717	717
vdata-la10 vdata-la17	646*	646	646	646	646	646
vdata-la17 vdata-la18	663*	663	663	663	663	663
	617*	617	617	617	617	617
vdata-la19	756*	756	756	756	756	756
vdata-la20	800*	800	800	802	804	801
vdata-la21 vdata-la22	733*	733	733	734	737	<b>733</b>
	809*	809	809	811	813	810
vdata-la23	773*	773	773	775		774
vdata-la24	751*	773 751	773 751	753	776	$\frac{774}{752}$
vdata-la25	1052*	1052			755	
vdata-la26			1052	1053	1054	1052
vdata-la27	1084*	1084	1084	$1084 \\ 1069$	1086	1084
vdata-la28	1069*	1069	1069		1070	1069
vdata-la29	993*	993	993	994	995	993
vdata-la30	1068*	1068	1068	1069	1070	1068
vdata-la31	1520*	1520	1520	1520	1521	1520
vdata-la32	1657*	1657	1657	1658	1658	1657
vdata-la33	1497*	1497	1497	1497	1498	1497
vdata-la34	1535*	1535	1535	1535	1535	1535
vdata-la35	1549*	1549	1549	1549	1549	1549
vdata-la36	948*	948	948	<b>948</b>	948	<b>948</b>
vdata-la37	986*	986	986	986	986	<b>986</b>
vdata-la38	943*	943	943	943	943	<b>943</b>
vdata-la39	922*	922	922	$\bf 922$	$\boldsymbol{922}$	$\bf 922$
vdata-la40	955*	955	955	<b>955</b>	955	955
vdata-mt06	$47^{*}$	47	47	<b>47</b>	47	47
vdata-mt10	655*	655	655	<b>655</b>	655	655
vdata-mt20	1022*	1022	1022	$\boldsymbol{1022}$	1022	$\boldsymbol{1022}$
vdata-orb1	695*	695	-	-	-	695
vdata-orb10	681*	681	-	-	-	681
vdata-orb2	620*	620	-	-	-	620
vdata-orb3	648*	648	-	-	-	648
vdata-orb4	753*	753	-	-	_	753
vdata-orb5	584*	584	-	-	-	584
vdata-orb6	715*	715	-	-	-	715
vdata-orb7	275*	275	_	_	_	275
vdata-orb8	573*	573	_	_	_	573
vdata-orb9	659*	659		_	_	659

Table 7: Comparison between MAE and other reference algorithms on sdata instance set

Ins.	Qunitiq		SSPR		GRASP-mELS	MAE
	LB	UB	LB	best	best	best
sdata-abz5	1234*	1234	_	_	_	1234
sdata-abz6	943*	943	_	_	_	<b>943</b>
sdata-abz7	656*	656	_	_	_	658
sdata-abz8	653	667	_	_	_	667
sdata-abz9	678*	678	_	_	_	678
	7038*	7038	-	-	-	
sdata-car1			-	-	-	7038
sdata-car2	7166*	7166	-	-	-	7166
sdata-car3	7312*	7312	-	-	-	7312
sdata-car4	8003*	8003	-	-	-	8003
sdata-car5	7702*	7702	-	-	-	7702
sdata-car6	8313*	8313	-	-	-	8313
sdata-car7	6558*	6558	-	-	-	<b>6558</b>
sdata-car8	8264*	8264	-	-	-	8264
sdata-la01	666*	666	666	666	_	666
sdata-la02	655*	655	655	655	_	655
sdata-la03	597*	597	597	597	_	597
sdata-la05 sdata-la04	590*	590	590	590		590
					-	
sdata-la05	593*	593	593	593	-	593
sdata-la06	926*	926	926	926	-	926
sdata-la07	890*	890	890	890	-	890
sdata-la08	863*	863	863	863	-	$\bf 863$
sdata-la09	951*	951	951	951	-	951
sdata-la10	958*	958	958	<b>958</b>	-	<b>958</b>
sdata-la11	1222*	1222	1222	$\boldsymbol{1222}$	_	1222
sdata-la12	1039*	1039	1039	1039	_	1039
sdata-la13	1150*	1150	1150	1150	_	1150
sdata-la14	1292*	1292	1292	1292		1292
	1207*	1207	1207	1207	-	$\begin{array}{c} 1292 \\ 1207 \end{array}$
sdata-la15					-	
sdata-la16	945*	945	945	945	-	945
sdata-la17	784*	784	784	<b>784</b>	-	<b>784</b>
sdata-la18	848*	848	848	848	-	848
sdata-la19	842*	842	842	$\bf 842$	-	$\bf 842$
sdata-la20	902*	902	902	<b>902</b>	-	<b>902</b>
sdata-la21	1046*	1046	1040	1046	_	1046
sdata-la22	927*	927	927	<b>927</b>	_	<b>927</b>
sdata-la23	1032*	1032	1032	1032	_	1032
sdata-la24	935*	935	935	935		935
	933 977*	933 977	933 977	933 977	-	933 977
sdata-la25					-	
sdata-la26	1218*	1218	1218	1218	-	1218
sdata-la27	1235*	1235	1235	1235	-	1235
sdata-la28	1216*	1216	1216	1216	-	1216
sdata-la29	1152*	1152	1120	1160	-	1153
sdata-la30	1355*	1355	1355	1355	-	1355
sdata-la31	1784*	1784	1784	1784	_	1784
sdata-la32	1850*	1850	1850	1850	_	1850
sdata-la33	1719*	1719	1719	1719	_	1719
sdata-la34	1721*	1721	1721	$\begin{array}{c} 1713 \\ 1721 \end{array}$	_	1721
sdata-la35	1888*	1888	1888	1888	_	1888
					-	
sdata-la36	1268*	1268	1268	1268	-	1268
sdata-la37	1397*	1397	1397	1397	-	1397
sdata-la38	1196*	1196	1184	1198	-	1196
sdata-la39	1233*	1233	1233	1233	-	1233
sdata-la40	1222*	1222	1222	1224	-	1224
sdata-mt06	55*	55	55	55	-	55
sdata-mt10	930*	930	930	930	-	930
sdata-mt20	1165*	1165	1165	1165	_	1165
sdata-mt20 sdata-orb1	1059*	1059	-	-	_	1059
					-	
sdata-orb10	944*	944	-	-	-	944
sdata-orb2	888*	888	-	-	-	888
sdata-orb3	1005*	1005	-	-	-	1005
sdata-orb4	1005*	1005	-	-	-	1005
sdata-orb5	887*	887	-	-	-	887
sdata-orb6	1010*	1010	-	-	-	1010
sdata-orb7	397*	397	_	-	_	397
sdata-orb8	899*	899	_	_	_	899
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