CS130 - Partial Orders Summary

Consider a set P and a relation $R_{\sim}$

$(P, R_{\sim})$ is a partially ordered set if the relation is reflexive, antisymmetric, and transitive.

↳ Relation called partial ordering
Set called poset.

A subcategory of this is total orders, which are when every element relates to every other element:

$$\forall x, y \in P \;.\; x \sim y \;\vee\; y \sim x$$

Two elements are called incomparable if there is no relation either way between them:

$$x, y \in P \;.\; x \not\sim y \;\wedge\; y \not\sim x$$

An element is called least if it is related to every other element

$$\forall y \in P \quad x \sim y \qquad (x \leq y)$$

An element is called greatest if it is related to by every other element

$$\forall y \in P \quad y \sim x \qquad (y \leq x)$$

An element is called minimal if it is related to no other element

$$\forall y \in P \qquad y \sim x \Rightarrow y = x$$

$(y \leq x)$

An element is called maximal if it relates to no other element

$(x \leq y)$

$$\forall y \in P \qquad x \sim y \Rightarrow x = y$$

Hasse diagrams are representations of partial orders, defined as the directed graph:

$$G = (P, \{(x,y) : x \sim y\}).$$

For example: $(\mathbb{Z}, \leq)$