

KEY POINTS

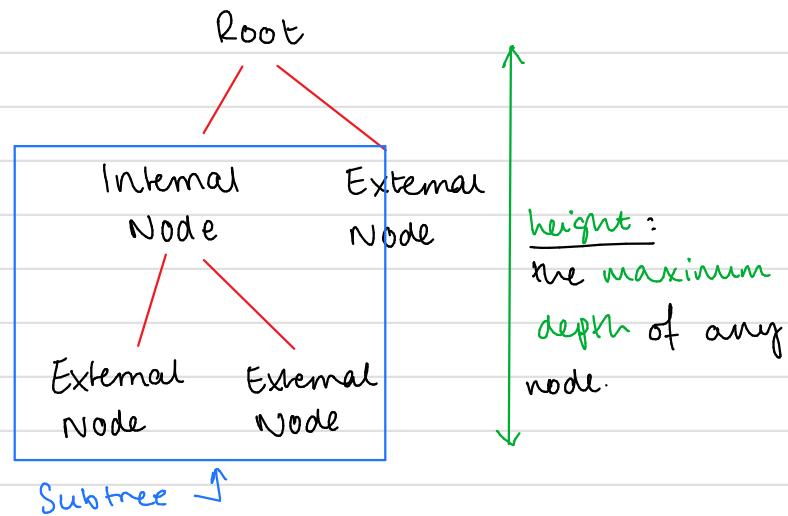
Trees - an **abstract model** of a **hierarchical structure**.

- A tree will have **generic**, **accessor**, **query** methods.
- One can use **preorder** and **postorder** traversal.

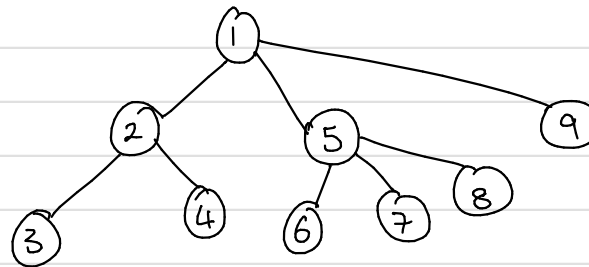
NAME/DATE/SUBJECT

Trees Introduction.

NOTES

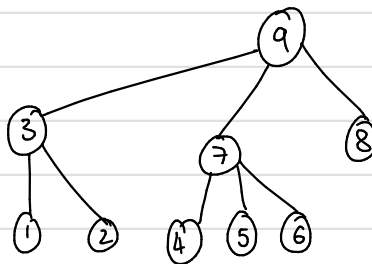


Postorder traversal



Print the node **before** children.

Preorder Traversal:



Print the node **after** its children.

SUMMARY

KEY POINTS

- Each node has at most two children.
- ⇒ A proper binary tree will have exactly 2 children per node.
- E.g. arithmetic or decision trees.

ADT Functions

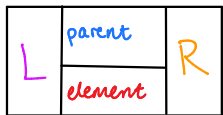
All return position

left(p) right(p)
sibling(p) } null if failure

Traversal

- Inorder traversal means that a node is printed after the left subtree and before the right subtree.
- Useful for printing arithmetic expressions - use postorder trav. to evaluate the tree.

Linked Structure:



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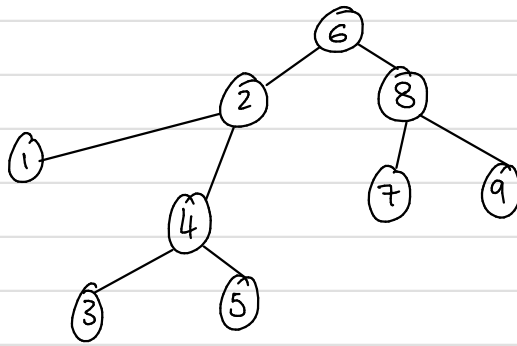
Binary Trees

NOTES

Properties of proper binary trees:

n	# nodes	$e = i + 1$	$e \leq 2^h$
e	# external nodes	$n = 2e - 1$	
i	# internal nodes	$h \leq i$	$h \geq \log_2 e$
h	height	$h \leq \frac{(n-1)}{2}$	$h \geq \log_2 (n+1) - 1$

Inorder traversal :



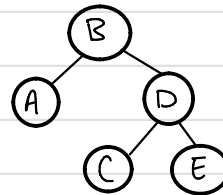
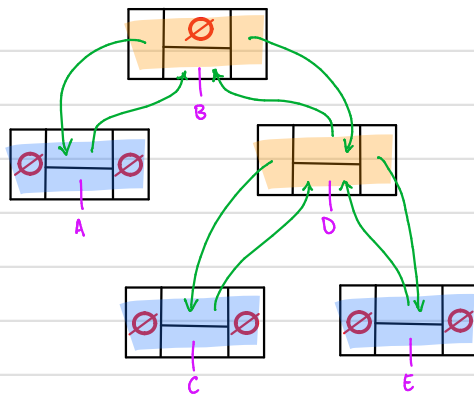
Algo inorder :

```

if left(v) != null
    inorder(left(v))
print(v)
if right(v) != null
    inorder(right(v))

```

Linked Structure



SUMMARY

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You can store binary trees in an **array** by having $f(p)$ store the node p .

$f(0) = \text{root}$

p is the **left child** of q :

$$f(p) = 2 \cdot f(q) + 1$$

p is the **right child** of q :

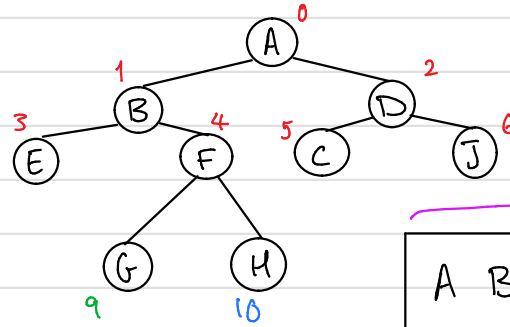
$$f(p) = 2 \cdot f(q) + 2$$

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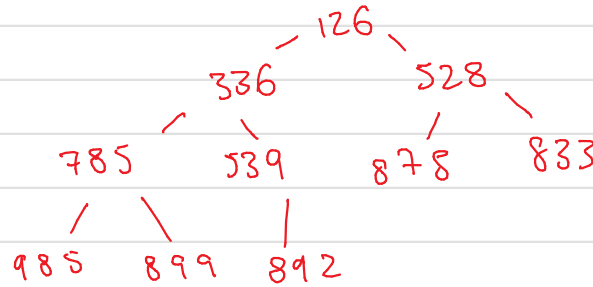
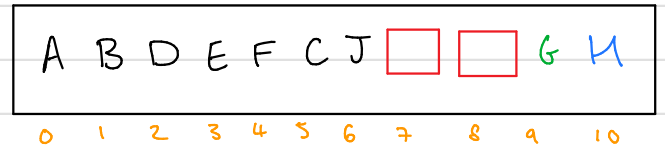
Array based tree

NOTES

Binary tree in an array:



In the worst case we need $2^n - 1$ space.



SUMMARY