CS130 - Induction Summary.

A proof technique to show that a statement holds for all cases, consisting of a base case, and an inductive step.

Format of an inductive proof:

1) Base case: Prove that the proposition holds for the first element(s) of the set of cases

2) Inductive Step: Assume that the proposition holds for an arbitrary case (the inductive hypothesis), then prove that this being true implies the next case is true.

3) Completeness Statement: State that since the base case and the inductive step hold, the statement holds for all cases

Example

Proving the proposition $P(n)$ is true $\forall n \in \mathbb{Z}, n > 1$

Base case: Prove $P(1)$ is true

Inductive step: ~~[illegible]~~ Assume $P(k)$ is true, then prove $P(k+1)$ is hence true

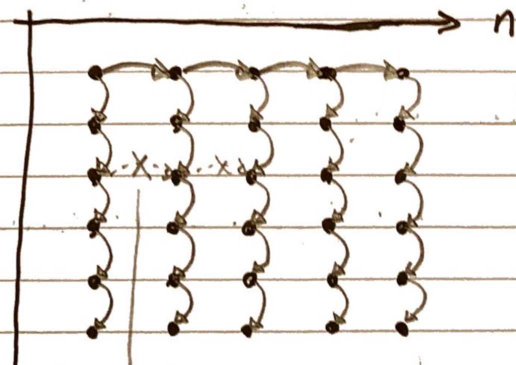
Completeness statement: Since the base case and inductive step hold, by mathematical induction the proposition holds for all $P(n)$, $n > 1$.

Weak induction uses $P(k) \rightarrow P(k+1)$, i.e. only using the predecessor

Strong induction uses $P(1) \wedge P(2) \wedge \ldots \wedge P(k) \rightarrow P(k+1)$, i.e. using <u>all</u> predecessors

Weak induction is simpler, but sometimes does not provide enough information.

For two dimensional induction, we needn't prove induction for all values in all directions, instead, we can prove only the base case along in one variable, then arbitrary value in the other



<u>Prove:</u>

$P(n,m)\ \forall n, m=1$

$P(n,m)\ \forall m, \exists m.$

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