CS130 - Sets Summary

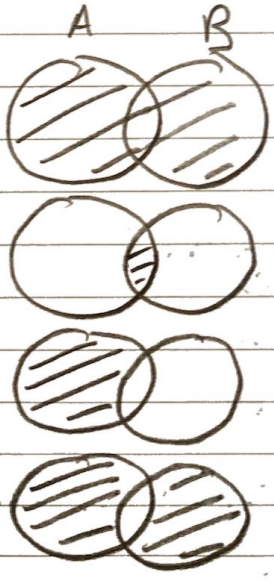
↳ Sets are an unordered collection of objects, with no multiplicity

↳ Notation:

- $A = \{ \dots \}$, Set definition
- $|A|$, Set Cardinality (length)
- $A \subseteq B$, A is a subset of B (every element in A is also an element of B)
- $a \in A$, a is an element of A
- $A \not\subseteq B$, A is not a subset of B, (A contains at least one element not in B)
- $A \subsetneq B$, A is a subset of, but not equal to B
- $\emptyset$ $(= \{\})$, the empty set.

↓

Subset of all other sets, but has no subsets itself



- $A \cup B$, A union B, the set of elements in ~~both~~ either A or B
- $A \cap B$, A intersection B, the set of elements in both A and B
- $A \setminus B$, A set difference B, the set of elements in A but not B
- $A \Delta B$, A symmetric difference B, the set of elements in A or B, but not both.

↳ Sets also have laws like boolean algebra, but negation is only well defined if there is a universal set.

→ Like Σ & Π:

- $\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \dots \cup A_n$
- $\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \dots \cap A_n$

→ n can be infinit

↳ Power Sets are the Sets of all the Subsets of a given set, e.g.

$S = \{a, b\}$, $2^S = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

$2^\emptyset = \{\emptyset\}$

↳ $|2^S| = 2^{|S|}$ for all finite sets S

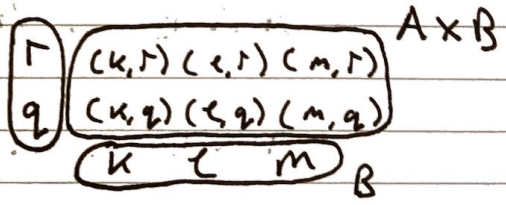
↳ Because you can make 2^n sets of n things.

↳ Cartesian products

↳ the Set of all the ordered pairs that can be formed from two sets, e.g.

$A = \{k, \ell, m\}$

$B = \{q, r\}$



$\therefore A \times B = \{(k, r), (\ell, r), (m, r), (k, q), (\ell, q), (m, q)\}$

↳ If it were $B \times A$, it would be $\{(r, k), (r, \ell), \dots\}$

↳ $A \times \emptyset = \emptyset$ (think about there being no columns in the diagram, so it is empty.

↳ $(A \times B) \times C \neq A \times (B \times C)$

↳ Sets are not associative under a cartesian product.

↳ e.g. $\{((a,b),c),\dots\}$

$\neq \{(a,(b,c)),\dots\}$

↳ Sets are easy to intuitively understand, but hard to thoroughly define, since problems like Russell's paradox (consider the set of all sets which don't contain themselves) occur.

↳ other set builder notation.

↳ Intervals:

'[' endpoints included (called closed)

'(' endpoints excluded (called open).

↳ Examples:

$[a;\infty) = \{x \in \mathbb{R} : a \leq x\}$

$(= \{x \in \mathbb{R} : a \leq x < \infty\})$

$(-\infty;b) = \{x \in \mathbb{R} : x < b\}$