EE3025 IDP Assignment-1

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Download all python codes from

```
https://github.com/csrs2000/EE3025_IDP/tree/main/Assignment-1/code
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and latex-tikz codes from

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https://github.com/csrs2000/EE3025_IDP/tree/main/Assignment-1
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1 Question

1.1. Let

$$x(n) = \left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\} \tag{1.1.1}$$

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{1.1.2}$$

Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{1.1.3}$$

and $H(k)$ using $h(n)$.

2 Solution

2.1. When Unit impulse signal is given as input to the LTI system then its impulse response is the ouput of the system. So,from equation (1.1.2) we can say that the Impulse response of the system is,

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2) \tag{2.1.1}$$

2.2. DFT of a Input Signal $x(n)$ is :

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.2.1}$$

Similarly, DFT of Impulse Response $h(n)$ is,

$$H(k) \triangleq \sum_{n=0}^{N-1} h(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.2.2}$$

2.3. Dft matrix multiplication method- DFT of x(n) and h(n) using Matrix multiplication method

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.3.1}$$

Lets take $W_N^{nk} = e^{-j2\pi kn/N}$, expressing $X(k)$ in terms of product of DFT matrix and $x(k)$:

$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1\\ 1 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5\\ 1 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10}\\ 1 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15}\\ 1 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20}\\ 1 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} \end{bmatrix} \begin{bmatrix} x(0)\\ x(1)\\ x(2)\\ x(3)\\ x(4)\\ x(5) \end{bmatrix} \tag{2.3.2}$$

given x(n) = $\left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\}$
,here N=6 so above equation becomes

$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5) \end{bmatrix} = \begin{bmatrix} 1+2+3+4+2+1\\ 1+(2)e^{-j\pi/3}+...+(1)e^{-j5\pi/3}\\ 1+(2)e^{-2j\pi/3}+...+(1)(e^{-2j5\pi/3}\\ 1+(2)e^{-3j\pi/3}+...+(1)e^{-3j5\pi/3}\\ 1+(2)e^{-4j\pi/3}+...+(1)e^{-4j5\pi/3}\\ 1+(2)e^{-5j\pi/3}+...+(1)e^{-5j5\pi/3} \end{bmatrix} \tag{2.3.3}$$

On solving we get,

$$\implies X(0) = 13 + 0j, \tag{2.3.4}$$
$$X(1) = -4 - 1.732j, \tag{2.3.5}$$
$$X(2) = 1 + 0j, \tag{2.3.6}$$
$$X(3) = -1 + 0j, \tag{2.3.7}$$
$$X(4) = 1 + 0j, \tag{2.3.8}$$
$$X(5) = -4 + 1.732j \tag{2.3.9}$$

in order to find $H(k)$ we need to find h(n) first. So we will first calculate h(n). we will find that using Y(Z),X(Z)For that we need to first find the Y(z) by applying Z-transform on equation

(1.1.2) i.e.,

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \quad (2.3.10)$$

$$\implies Y(z) = \frac{2(z^2+1)}{z(2z+1)}X(z) \quad (2.3.11)$$

Now we can find H(z) using Y(z) i.e.,

$$H(z) = \frac{Y(z)}{X(z)} \quad (2.3.12)$$

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \quad (2.3.13)$$

$$H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \quad (2.3.14)$$

h(n) is,

$$H(Z) = Z^{-1}\left[\frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}}\right] \quad (2.3.15)$$

$$h(n) = \left[\frac{-1}{2}\right]^n u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \quad (2.3.16)$$

lets assume that length of h(n) is 6,i.e, N = 6. using matrix multiplication method we obtain

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} h(0) + .. + h(5) \\ h(0) + h(1)e^{-j\pi/3} + .. + h(5)e^{-j5\pi/3} \\ h(0) + h(1)e^{-2j\pi/3} + .. + h(5)e^{-2j5\pi/3} \\ h(0) + h(1)e^{-3j\pi/3} + .. + h(5)e^{-3j5\pi/3} \\ h(0) + h(1)e^{-4j\pi/3} + ... + h(5)e^{-4j5\pi/3} \\ h(0) + h(1)e^{-5j\pi/3} + .. + h(5)e^{-5j5\pi/3} \end{bmatrix} \quad (2.3.17)$$

On calculating we get,

$$\implies H(0) = 1.28125 + 0j, \quad (2.3.18)$$
$$H(1) = 0.51625 - 0.5141875j, \quad (2.3.19)$$
$$H(2) = -0.078125 + 1.1095625j, \quad (2.3.20)$$
$$H(3) = 3.84375 + 0j, \quad (2.3.21)$$
$$H(4) = -0.071825 - 1.1095625j. \quad (2.3.22)$$
$$H(5) = 0.515625 + 0.5141875j \quad (2.3.23)$$

lets verify these values by using python code

2.4. Python Code to compute the DFT of $x(n)$ and $h(n)$ is given below

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https://github.com/csrs2000/EE3025_IDP/
    blob/main/Assignment−1/code/
    ee18btech11007.py
```

2.5. using the above code we get the following plots.

```
https://github.com/csrs2000/EE3025_IDP/tree
    /main/Assignment−1/figs
```

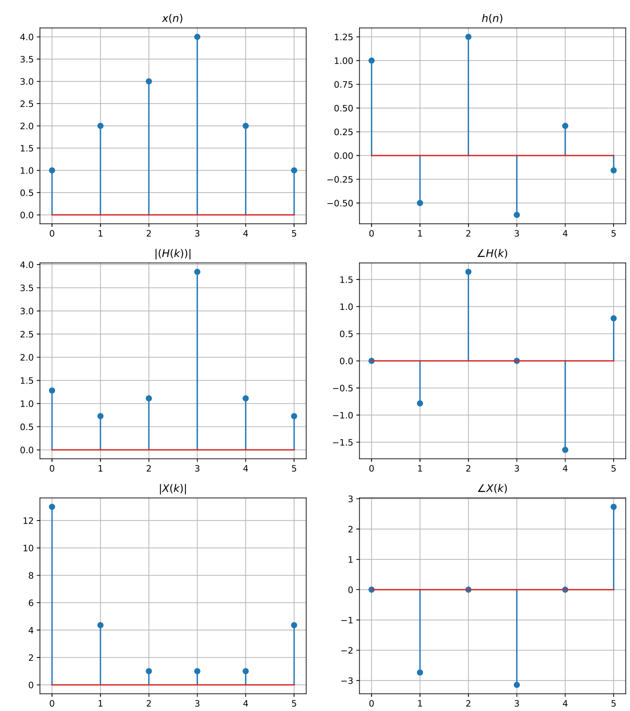

Fig. 2.5: Plots of x(n) and h(n),their DFTs

manually calculated values match with the values obtained from codes