

# **Variance of Different Probability Distributions' Formula Sheet**

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# 1 Chapter 1

$$P(A \cup B) = P(A) + P(B) - P(AB) \quad (1)$$

$$P(A - B) = P(A) - P(AB) \quad (2)$$

$$P(B|A) = \frac{P(AB)}{P(A)} \quad (3)$$

$$P(AB) = P(A|B) \times P(B) \quad (4)$$

$$P(A) = \sum_{i=1}^n [P(A|B_i) \times P(B_i)] \quad (5)$$

$$P(B_i|A) = \frac{P(B_i) \times P(A|B_i)}{\sum_{j=1}^n P(B_j) \times P(A|B_j)} \quad (6)$$

# 2 Chapter 2

$$F(x) = P\{X \leq x\}, (-\infty < x < +\infty) \quad (7)$$

$$\lim_{x \rightarrow -\infty} F(x) = 0 \quad (8)$$

$$\lim_{x \rightarrow +\infty} F(x) = 1 \quad (9)$$

$$P\{X = k\} = C_k^n \times p^k \times (1-p)^{n-k} \quad X \sim B(n, p) \quad (10)$$

$$P\{X = k\} = \frac{\lambda^k}{k!} \times e^{-\lambda} \quad X \sim \pi(n, p) \quad (11)$$

$$F(x) = \int_{-\infty}^x f(t) dt \quad (12)$$

$$f(x) = \frac{1}{2} \times e^{-|x|} \quad (13)$$

$$f(x) = \frac{1}{b-a}, \quad (a < x < b) \quad X \sim U(a, b) \quad (14)$$

$$f(x) = \frac{1}{\theta} \times e^{-\frac{x}{\theta}} \quad X \sim e(\theta) \quad (15)$$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \times e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad X \sim N(\mu, \sigma^2) \quad (16)$$

$$F_Y(y) = P\{Y \leq y\} = P\{g(x) \leq y\} = \int_{g(x) \leq y} f_X(t) dt \quad (17)$$

$$f_Y(y) = \frac{dF_Y(y)}{dy} \quad (18)$$

$$f_Y(y) = f_X[h(y)] \times |h'(y)| \quad (19)$$

### 3 Chapter 3

$$F(x, y) = \int_{-\infty}^y \int_{-\infty}^x f(u, v) du dv \quad (20)$$

$$f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y} \quad (21)$$

$$f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy \quad (22)$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx \quad (23)$$

$$f(x, y) = \frac{1}{S}, \quad (x, y) \in G \quad (X, Y) \sim U(G) \quad (24)$$

$$F_{X|Y}(x|y) = \int_{-\infty}^x \frac{f(u, y)}{f_Y(y)} du \quad (25)$$

$$f_{X|Y}(x|y) = \frac{f(x, y)}{f_Y(y)} \quad (26)$$

$$F(x, y) = F_X(x) \times F_Y(y) \quad (27)$$

$$f(x, y) = f_X(x) \times f_Y(y) \quad (28)$$

#### 3.1 Z=X+Y

$$F_Z(z) = P\{X + Y \leq Z\} = \iint_{x+y \leq z} f(x, y) dx dy = \int_{-\infty}^{+\infty} \int_{-\infty}^z f(u - y, y) du dy \quad (29)$$

$$f_Z(z) = \int_{-\infty}^{+\infty} f(z - y, y) dy \quad (30)$$

$$f_Z(z) = \int_{-\infty}^{+\infty} f(x, z - x) dx \quad (31)$$

$$f_X * f_Y = \int_{-\infty}^{+\infty} f_X(x) f_Y(z - x) dx = \int_{-\infty}^{+\infty} f_X(z - y) f_Y(y) dy \quad (32)$$

#### 3.2 $Z = \frac{Y}{X}, Z=XY$

$$U = \frac{Y}{X}, V = XY \quad (33)$$

$$f_U(u) = \int_{-\infty}^{\infty} |x| f(x, xu) dx \quad (34)$$

$$f_V(v) = \int_{-\infty}^{\infty} |x| f(x, \frac{v}{x}) dx \quad (35)$$

#### 3.3 M=max(X,Y), N=min(X,Y)

$$F_M(z) = F_X(z) F_Y(z) \quad (36)$$

$$F_N(z) = 1 - [1 - F_X(z)][1 - F_Y(z)] \quad (37)$$

## 4 Chapter 4

Table 1: Mathematical Expectation

Distribution Type	$E(X)$
(0-1) distribution	$p$
Binomial distribution	$np$
Poisson distribution	$\lambda$
Uniform distribution	$\frac{a+b}{2}$
Exponential distribution	$\theta$
normal distribution	$\mu$

Table 2: Variance of Different Probability Distributions

Distribution Type	Variance, $D(X)$
(0-1) distribution	$p(1 - p)$
Binomial distribution	$np(1 - p)$
Poisson distribution	$\lambda$
Uniform distribution	$\frac{(b-a)^2}{12}$
Exponential distribution	$\theta^2$
Normal distribution	$\sigma^2$

$$E(X) = \int_{-\infty}^{+\infty} x f(x) dx \quad (38)$$

$$E(Y) = E(g(X)) = \int_{-\infty}^{+\infty} g(x) f(x) dx \quad (39)$$

$$E(CX) = CE(X) \quad (40)$$

$$E(X + Y) = E(X) + E(Y) \quad (41)$$

$$E(XY) = E(X)E(Y) \quad (42)$$

$$E(X \pm C) = E(X) \pm C \quad (43)$$

$$D(X) = \sum_{k=1}^n [x_k - E(X)]^2 p_k \quad (44)$$

$$D(X) = \int_{-\infty}^{+\infty} [x - E(X)]^2 f(x) dx \quad (45)$$

$$D(X) = E(X^2) - [E(X)]^2 \quad (46)$$

$$D(X \pm C) = D(X) \quad (47)$$

$$D(CX) = C^2 D(X) \quad (48)$$

$$\text{Cov}(X, Y) = E([X - E(X)][Y - E(Y)]) \quad (49)$$

$$\text{Cov}(X, Y) = \text{Cov}(Y, X) \quad (50)$$

$$\text{Cov}(aX, bY) = ab\text{Cov}(X, Y) \quad (51)$$

$$\text{Cov}(X_1 + X_2, Y) = \text{Cov}(X_1, Y) + \text{Cov}(X_2, Y) \quad (52)$$

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y) \quad (53)$$

$$X^* = \frac{X - E(X)}{\sqrt{D(X)}} \quad (54)$$

$$Y^* = \frac{Y - E(Y)}{\sqrt{D(Y)}} \quad (55)$$

$$\rho_{xy} = \text{Cov}(X^*, Y^*) = \frac{\text{Cov}(X, Y)}{\sqrt{D(X)}\sqrt{D(Y)}} \quad (56)$$

$$P(|X - \mu| \geq \epsilon) \leq \frac{\sigma^2}{\epsilon^2} \quad (57)$$

$$P(|X - \mu| < \epsilon) \geq 1 - \frac{\sigma^2}{\epsilon^2} \quad (58)$$

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = N(0, 1) \quad (59)$$

## 5 Chapter 5

$$\text{Mean: } \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (60)$$

$$\text{Sample Variance: } s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \quad (61)$$

$$\text{Sample Standard Deviation: } s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2} \quad (62)$$

$$\text{Population Variance: } \sigma^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2 \quad (63)$$

$$\text{Population Standard Deviation: } \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2} \quad (64)$$

$$\text{k-th Raw Moment: } \mu_k = E(X^k) = \int_{-\infty}^{\infty} x^k f(x) dx \quad (65)$$

$$\text{k-th Central Moment: } \mu'_k = E[(X - \mu)^k] = \int_{-\infty}^{\infty} (x - \mu)^k f(x) dx \quad (66)$$

## 5.1 Distribution

$$\chi^2 = X_1^2 + X_2^2 + \cdots + X_n^2 \quad \chi^2 \sim \chi^2(n) \quad (67)$$

$$X \sim N(0, 1), \quad Y \sim \chi^2(n) \quad (68)$$

$$T = \frac{X}{\sqrt{\frac{Y}{n}}} \quad T \sim t(n) \quad (69)$$

$$U \sim \chi^2(m), \quad V \sim \chi^2(n) \quad (70)$$

$$F = \frac{\frac{U}{m}}{\frac{V}{n}} \quad F \sim F(m, n) \quad (71)$$

$$\frac{1}{F} \sim F(n, m) \quad (72)$$

$$F_{1-\alpha}(n, m) = \frac{1}{F_{\alpha}(m, n)} \quad (73)$$

$$(74)$$

## 5.2 Sampling distribution

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1) \quad (75)$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1) \quad (76)$$

$$\frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t(n-1) \quad (77)$$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}} \sim N(0, 1) \quad (78)$$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{S_{\omega} \sqrt{\frac{1}{m} + \frac{1}{n}}} \sim t(m+n-2) \quad (79)$$

$$S_{\omega} = \sqrt{\frac{(m-1)S_X^2 + (n-1)S_Y^2}{m+n-2}} \quad (80)$$

$$\frac{\frac{S_X^2}{m-1}}{\frac{S_Y^2}{n-1}} \sim F(m-1, n-1) \quad (81)$$

## 6 Chapter 6

$$\bar{X} = E(X) = \int_{-\infty}^{+\infty} x f(x) dx \quad (82)$$

$$L(\theta) = \prod_{i=1}^n f(x_i; \theta) \quad (83)$$

$$\frac{dL}{d\theta} = 0 \quad (84)$$

Table 3: Test Statistics and Confidence Intervals for Normally Distributed Population Parameters

Parameter	Statistic	Estimator	Confidence Interval
$\mu, \sigma^2$ known	Z	$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$	$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
	$\chi^2$	$\frac{(n-1)s^2}{\sigma^2}$	$\left[ \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}, \frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}, n-1}} \right]$
$\mu, \sigma^2$ unknown	T	$\frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$	$\bar{X} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}$
	$\chi^2$	$\frac{(n-1)s^2}{\sigma^2}$	$\left[ \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}, \frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}, n-1}} \right]$