Variance of Different Probability Distributions'Formula Sheet

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January 10, 2024

Contents

1 Chapter 1

$$P(A \cup B) = P(A) + P(B) - P(AB) \tag{1}$$
$$P(A - B) = P(A) - P(AB) \tag{2}$$
$$P(B|A) = \frac{P(AB)}{P(A)} \tag{3}$$
$$P(AB) = P(A|B) \times P(B) \tag{4}$$
$$P(A) = \sum_{i=1}^{n} [P(A|B_i) \times P(B_i)] \tag{5}$$
$$P(B_i|A) = \frac{P(B_i) \times P(A|B_i)}{\sum_{j=1}^{n} P(B_j) \times P(A|B_j)} \tag{6}$$

2 Chapter 2

$$F(x) = P\{X \leqslant x\}, (-\infty < x < +\infty) \tag{7}$$
$$\lim_{x \to -\infty} F(x) = 1 \tag{8}$$
$$\lim_{x \to +\infty} F(x) = 0 \tag{9}$$
$$P\{X = k\} = C_k^n \times p^k \times (1-p)^{n-k} \quad X \sim B(n,p) \tag{10}$$
$$P\{X = k\} = \frac{\lambda^k}{k!} \times e^{-\lambda} \quad X \sim \pi(n,p) \tag{11}$$
$$F(x) = \int_{-\infty}^{x} f(t)\, dt \tag{12}$$
$$f(x) = \frac{1}{2} \times e^{-|x|} \tag{13}$$
$$f(x) = \frac{1}{b-a}, \quad (a < x < b) \quad X \sim U(a,b) \tag{14}$$
$$f(x) = \lambda \times e^{x\lambda} \quad X \sim E(\lambda) \tag{15}$$
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \times e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad X \sim N(\mu, \sigma^2) \tag{16}$$
$$F_Y(y) = P\{Y \leqslant y\} = P\{g(x) \leqslant y\} = \int_{g(x) \leqslant y} f_X(t)\, dt \tag{17}$$
$$f_Y(y) = \frac{dF_Y(y)}{dy} \tag{18}$$
$$f_Y(y) = f_X[h(y)] \times |h'(y)| \tag{19}$$

3 Chapter 3

$$F(x,y)=\int_{-\infty}^{y}\int_{-\infty}^{x}f(u,v)\,dudv \tag{20}$$

$$f(x,y)=\frac{\partial^2 F(x,y)}{\partial x\partial y} \tag{21}$$

$$f_X(x)=\int_{-\infty}^{+\infty}f(x,y)\,dy \tag{22}$$

$$f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)\,dx \tag{23}$$

$$f(x,y)=\frac{1}{S},\quad (x,y)\in G\quad (X,Y)\sim U(G) \tag{24}$$

$$F_{X|Y}(x|y)=\int_{-\infty}^{x}\frac{f(u,y)}{f_Y(y)}\,du \tag{25}$$

$$f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)} \tag{26}$$

$$F(x,y)=F_X(x)\times F_Y(y) \tag{27}$$

$$f(x,y)=f_X(x)\times f_Y(y) \tag{28}$$

3.1 Z=X+Y

$$F_Z(z)=P\{X+Y\leqslant Z\}=\iint_{x+y\leqslant z}f(x,y)\,dxdy=\int_{-\infty}^{+\infty}\int_{-\infty}^{z}f(u-y,y)\,dudy \tag{29}$$

$$f_Z(z)=\int_{-\infty}^{+\infty}f(z-y,y)\,dy \tag{30}$$

$$f_Z(z)=\int_{-\infty}^{+\infty}f(x,z-x)\,dx \tag{31}$$

$$f_X*f_Y=\int_{-\infty}^{+\infty}f_X(x)f_Y(z-x)\,dx=\int_{-\infty}^{+\infty}f_X(z-y)f_Y(y)\,dy \tag{32}$$

3.2 $Z=\frac{Y}{X}$, Z=XY

$$U=\frac{Y}{X},V=XY \tag{33}$$

$$f_U(u)=\int_{-\infty}^{\infty}|x|\,f(x,xu)\,dx \tag{34}$$

$$f_V(v)=\int_{-\infty}^{\infty}|x|\,f(x,\frac{v}{x})\,dx \tag{35}$$

3.3 M=max(X,Y), N=min(X,Y)

$$F_M(z)=F_X(z)F_Y(z) \tag{36}$$

$$F_N(z)=1-[1-F_X(z)][1-F_Y(z)] \tag{37}$$

4 Chapter 4

Table 1: Mathematical Expectation

Distribution Type	**E(X)**
(0-1) distribution	p
Binomial distribution	np
Poisson distribution	λ
Uniform distribution	$\frac{a+b}{2}$
Exponential distribution	$\frac{1}{\lambda}$
normal distribution	μ

Table 2: Variance of Different Probability Distributions

Distribution Type	**Variance, D(X)**
(0-1) distribution	$p(1-p)$
Binomial distribution	$np(1-p)$
Poisson distribution	λ
Uniform distribution	$\frac{(b-a)^2}{12}$
Exponential distribution	$\frac{1}{\lambda^2}$
Normal distribution	σ^2

$$E(X) = \int_{-\infty}^{+\infty} xf(x)\,dx \tag{38}$$

$$E(Y) = E(g(X)) = \int_{-\infty}^{+\infty} g(x)f(x)\,dx \tag{39}$$

$$E(CX) = CE(X) \tag{40}$$

$$E(X+Y) = E(X) + E(Y) \tag{41}$$

$$E(XY) = E(X)E(Y) \tag{42}$$

$$E(X \pm C) = E(X) \pm C \tag{43}$$

$$D(X) = \sum_{k=1}^{n} [x_k - E(X)]^2 p_k \tag{44}$$

$$D(X) = \int_{-\infty}^{+\infty} [x - E(X)]^2 f(x)\,dx \tag{45}$$

$$D(X) = E(X^2) - [E(X)]^2 \tag{46}$$

$$D(X \pm C) = D(X) \tag{47}$$
$$D(CX) = C^2 D(X) \tag{48}$$
$$Cov(X,Y) = E(XY) - E(X) \times E(Y) \tag{49}$$
$$Cov(X,Y) = Cov(Y,X) \tag{50}$$
$$Cov(aX,bY) = abCov(X,Y) \tag{51}$$
$$Cov(X_1+X_2,Y) = Cov(X_1,Y) + Cov(X_2,Y) \tag{52}$$
$$Cov(X,Y) = E(XY) - E(X)E(Y) \tag{53}$$
$$X^* = \frac{X - E(X)}{\sqrt{D(X)}} \tag{54}$$
$$Y^* = \frac{Y - E(Y)}{\sqrt{D(Y)}} \tag{55}$$
$$\rho_{xy} = Cov(X^*,Y^*) = \frac{Cov(X,Y)}{\sqrt{(D(X))}\sqrt{(D(Y))}} \tag{56}$$

$$P(|X-\mu| \geq \epsilon) \leq \frac{\sigma^2}{\epsilon^2} \tag{57}$$
$$P(|X-\mu| < \epsilon) \geq 1 - \frac{\sigma^2}{\epsilon^2} \tag{58}$$
$$\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} = N(0,1) \tag{59}$$

5 Chapter 5

$$\text{Mean:} \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{60}$$
$$\text{Sample Variance:} \quad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \tag{61}$$
$$\text{Sample Standard Deviation:} \quad s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{62}$$
$$\text{Population Variance:} \quad \sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2 \tag{63}$$
$$\text{Population Standard Deviation:} \quad \sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2} \tag{64}$$
$$\text{k-th Raw Moment:} \quad \mu_k = E(X^k) = \int_{-\infty}^{\infty} x^k f(x)dx \tag{65}$$
$$\text{k-th Central Moment:} \quad \mu_k' = E\left[(X-\mu)^k\right] = \int_{-\infty}^{\infty} (x-\mu)^k f(x)dx \tag{66}$$

5.1 Distribution

$$\chi^2 = X_1^2 + X_2^2 + \cdots + X_n^2 \quad \chi^2 \sim \chi^2(n) \tag{67}$$

$$X \sim N(0,1), \quad Y \sim \chi^2(n) \tag{68}$$

$$T = \frac{X}{\sqrt{\frac{Y}{n}}} \quad T \sim t(n) \tag{69}$$

$$U \sim \chi^2(m), \quad V \sim \chi^2(n) \tag{70}$$

$$F = \frac{\frac{U}{m}}{\frac{V}{N}} \quad F \sim F(m,n) \tag{71}$$

$$\frac{1}{F} \sim F(n,m) \tag{72}$$

$$F_{1-\alpha}(n,m) = \frac{1}{F_\alpha(m,n)} \tag{73}$$

$$\tag{74}$$

5.2 Sampling distribution

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1) \tag{75}$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1) \tag{76}$$

$$\frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t(n-1) \tag{77}$$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}} \sim N(0,1) \tag{78}$$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{S_\omega \sqrt{\frac{1}{m} + \frac{1}{n}}} \sim t(m+n-2) \tag{79}$$

$$S_\omega = \sqrt{\frac{(m-1)S_X^2 + (n-1)S_Y^2}{m+n-2}} \tag{80}$$

$$\frac{\frac{S_X^2}{S_Y^2}}{\frac{\sigma_1^2}{\sigma_2^2}} \sim F(m-1, n-1) \tag{81}$$

6 Chapter 6

$$\bar{X} = E(X) = \int_{-\infty}^{+\infty} x f(x)\, dx \tag{82}$$

$$L(\theta) = \prod_{i=1}^{n} f(x_i; \theta) \tag{83}$$

$$\frac{dL}{d\theta} = 0 \tag{84}$$

Table 3: Test Statistics and Confidence Intervals for Normally Distributed Population Parameters

Parameter	**Statistic**	**Estimator**	**Confidence Interval**
μ, σ^2 known	Z	$\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$	$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
	χ^2	$\frac{(n-1)s^2}{\sigma^2}$	$\left[\frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}, \frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2},n-1}}\right]$
μ, σ^2 unknown	T	$\frac{\bar{X}-\mu}{\frac{s}{\sqrt{n}}}$	$\bar{X} \pm t_{\frac{\alpha}{2},n-1} \frac{s}{\sqrt{n}}$
	χ^2	$\frac{(n-1)s^2}{\sigma^2}$	$\left[\frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}, \frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2},n-1}}\right]$