

# Mechanics Equations

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Version 5.1

Units:

Joule J = kg m<sup>2</sup>/s<sup>2</sup>

Newton N = kg m/s<sup>2</sup>

Pascal Pa = N/m<sup>2</sup>

Watt W = J/s

1 J = 1 N m

Some Metric Prefixes:

kilo k = 10<sup>3</sup>

centi c = 10<sup>-2</sup>

mili m = 10<sup>-3</sup>

micro  $\mu$  = 10<sup>-6</sup>

nano n = 10<sup>-9</sup>

pico p = 10<sup>-12</sup>

Constant Acceleration:

$v(t) = v_0 + at$

$x(t) = x_0 + v_0t + \frac{1}{2}at^2$

$v(t)^2 = v_0^2 + 2a(x(t) - x_0)$

Forces:

$\sum \vec{F} = m\vec{a}$

$\vec{F}_{12} = -\vec{F}_{21}$

$F_{f(k)} = \mu_k F_N$

$F_{f(s)} \leq \mu_s F_N$

$F_g = mg$

$F_G = \frac{Gm_1m_2}{R^2}$

$\vec{F}_e = -k\Delta\vec{x}$

$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$

$g = 9.8 \text{ m/s}^2$

$a_c = v^2/R = \omega^2 R$

$\sum \vec{F} = \frac{d\vec{p}}{dt}$

Work, Energy and Momentum:

$W = Fd \cos(\theta)$

$W = Fd_{\parallel} = F_{\parallel}d$

$W = \int \vec{F}(t)dt$

$W = \vec{F} \cdot \vec{d}$

$E_k = \frac{1}{2}mv^2$

$\Sigma W = \Delta E_k$

$U_g = mg\Delta h$

$U_G = \frac{-Gm_1m_2}{R}$

$U_e = \frac{1}{2}k\Delta x^2$

$\Sigma E_f - \Sigma E_o = \Sigma W_{NC}$

$\vec{p} = m\vec{v}$

$\Sigma \vec{p}_0 = \Sigma \vec{p}_f$

$\vec{J} = \Delta \vec{p}$

$\vec{J} = \int \vec{F}(t)dt$

$x_{CM} = \sum m_i x_i$

Power  $P = W/t$

Angular Quantities:

$R\theta = d$

$R\omega = v_t$

$R\alpha = a_t$

Constant Angular Acceleration:

$\omega(t) = \omega_0 + \alpha t$

$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega(t)^2 = \omega_0^2 + 2\alpha(\theta(t) - \theta_0)$

Torque, Angular Momentum:

$\tau = rF \sin(\theta) = rF_{\perp} = r_{\perp}F$

$\vec{\tau} = \vec{r} \times \vec{F}$

$I = \sum m_i r_i^2$

$I_{new} = I_{cm} + mh^2$

$I = \int r^2 dm$

$\Sigma \tau = I\alpha$

$L = I\omega = rp \sin \theta$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$\Sigma \vec{L}_0 = \Sigma \vec{L}_f$

$E_{krot} = \frac{1}{2}I\omega^2$

$T = 1/f$

$\omega = 2\pi f$

$v = \frac{2\pi r}{T}$

Pressure/Fluids:

1 atm = 1.013  $\times 10^5$  Pa

1 torr = 1 mm Hg

760 torr = 1 atm

1 bar = 10<sup>5</sup> Pa

$$\rho = m/V$$

$$P = F/A$$

$$P(d) = P_0 + \rho g d$$

$$P + 1/2 \rho v^2 + \rho g h = \text{const}$$

$$A_1 v_1 = A_2 v_2$$

$$F_B = \rho_{fluid} V_{disp} g$$

Oscillations:

$$x(t) = A \sin(2\pi t/T) \text{ or}$$

$$x(t) = A \cos(2\pi t/T)$$

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

$$T_p = 2\pi \sqrt{\frac{l}{g}}$$

Waves:

$$y(x, t) = A \sin(kx \pm \omega t) \text{ or}$$

$$x(t) = A \cos(kx \pm \omega t)$$

$$T = 1/f = 2\pi/\omega$$

$$k = 2\pi/\lambda \text{ and } \omega = 2\pi f = 2\pi/T$$

$$v = f\lambda$$

Sound:

$$v_{sound} = 343 \text{ m/s}$$

$$\beta = 10 \log_{10}(I/I_0)$$

$$I_0 = 1 \times 10^{-12} \text{ W/m}^2$$

$$f' = \frac{f_0}{1 \pm \frac{v_o}{v}}$$

$$f' = f_0(1 \pm \frac{v_o}{v})$$

$$f_b = |f_1 - f_2|$$

Temperature:

$$T(K) = T(C) + 273.15 \text{ K}$$

$$\Delta L = \alpha L_0 \Delta T$$

$$L = L_0(1 + \alpha \Delta T)$$

$$PV = nRT$$

$$PV = Nk_B T$$

$$\bar{E}_k = E_{Kavg} = \frac{1}{2} m \bar{v}^2 = \frac{3}{2} k_B T$$

$$\frac{1}{2} m \bar{v}_x^2 = \frac{1}{2} k_B T$$

$$R = 8.315 \text{ J/mol K} = 1.99 \text{ cal/mol K} = 0.0821$$

$$\text{atm liter/mol K}$$

$$k_B = 1.38 \times 10^{-23} \text{ J/K}$$

$$N_A = 6.022 \times 10^{23}$$

$$R = N_A k_B$$

$$N = n N_A$$

Heat:

$$Q = cm\Delta T = C\Delta T$$

$$Q = Lm$$

$$0 = \Delta Q_1 + \Delta Q_2$$

Thermodynamics:

$$\Delta U = Q - W$$

$$W = P\Delta V$$

$$W = \int P(V) dV$$

$$\Delta S = Q/T$$

$$\Delta S = \int Q(T)/T dT$$

Trig:

$$\sin \theta = \text{opp/hyp}$$

$$\cos \theta = \text{adj/hyp}$$

$$\tan \theta = \text{opp/adj}$$

Quadratic:

$$\text{Given: } ax^2 + bx + c = 0$$

$$\text{Then: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Areas, Volumes:

Sphere:

$$\text{Volume} = 4/3 \pi r^3, \text{ Area} = 4\pi r^2$$

Cylinder:

$$\text{Volume} = \pi r^2 h, \text{ Area} = 2\pi r h + 2\pi r^2$$

Circle:

$$\text{Area} = \pi r^2, \text{ circumference} = 2\pi r$$

Moments of Inertia

Passing Through Center of Mass:

Thin Disk Perpendicular to Plane

$$I = \frac{1}{2} m r^2$$

Sphere

$$I = \frac{2}{5} m r^2$$

Thin Rod Perpendicular to Length:

$$I = \frac{1}{12} m L^2$$

Thin Plate Perpendicular to Plane:

$$I = \frac{1}{12} m (w^2 + L^2)$$