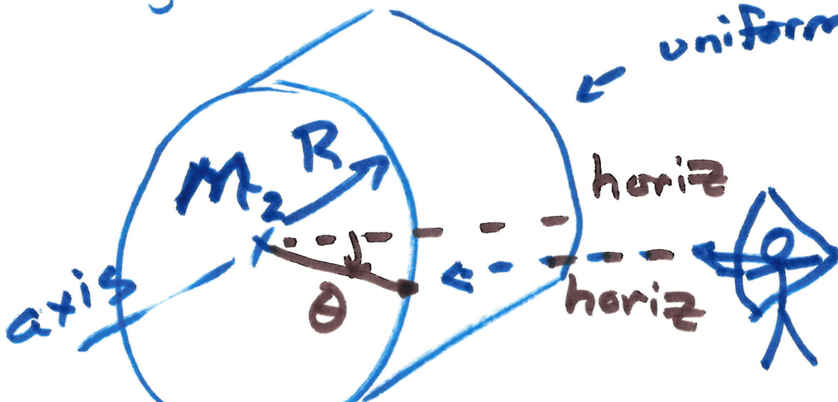
Physics 200

Kepler ← Day 21

angular collisions




arrow mass $m_1$

initially, at rest.

Quiz Monday
momentum
$\vec{p} = m\vec{v}$
$\sum \vec{p_0} = \sum \vec{p_f}$
vectors.
Review tomorrow
9AM - Noon
may move into a lab.

If arrow sticks into target, ~~by~~ measuring $\omega_f$ can give velocity of arrow.

Need: $m_1$, $m_2$, $R$, $\omega_f$
and exactly where arrow hits.
given by $\theta$ in picture.

Also, $I_{disk} = \frac{1}{2} m_2 R^2$ for "bale of hay"

spelled in protest with @ which means I don't care.

angular momentum $L$

$$\underbrace{L = I\omega}_{\text{final}} \qquad \underbrace{L = rp\sin\theta}_{\text{initial}}$$

or $\vec{L} = \vec{r} \times \vec{p}$

angle between $\vec{r}$ and $\vec{p}$?

$$\sum L_0 = \sum L_f$$

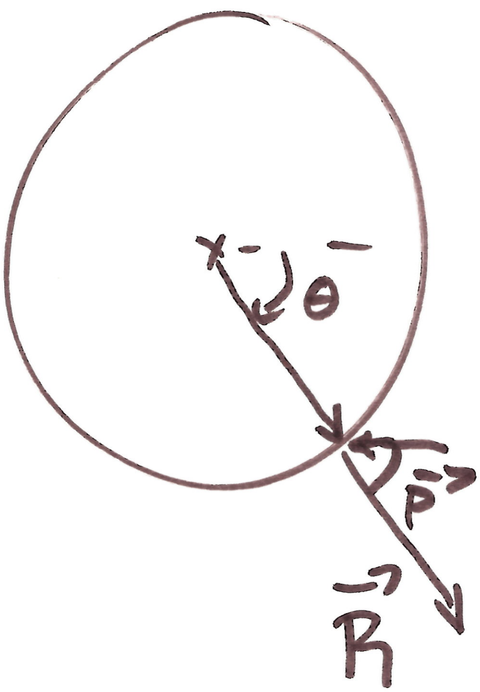
arrow + ~~bale~~ = arrow + bale

0

just before it hits

$$R(m_1 \overset{*}{v_0})\sin(\theta) = I\omega_f + \overbrace{m_1 R^2}\omega_f$$

$$= \underbrace{\tfrac{1}{2} m_2 R^2}_{I_{bale}}\omega_f + \underbrace{\qquad}_{\text{arrow}}$$

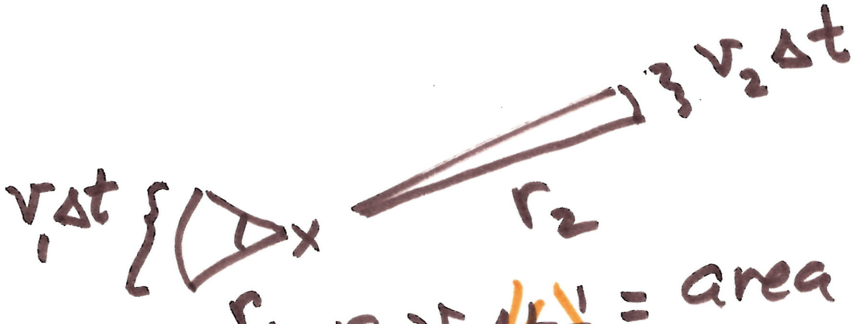



$$\overset{*}{v_0} = \frac{\tfrac{1}{2} m_2 R^2 \omega_f + m_1 R^2 \omega_f}{R m_1 \sin\theta}$$

"totally inelastic" → they stick together.

Kepler's 2nd law: equal areas in equal time?

$\Delta t$ equal.




$r_2 v_2 \cancel{\Delta t} \cancel{\tfrac{1}{2}} = \text{area} = r_1 v_1 \cancel{\Delta t} \times \cancel{\tfrac{1}{2}}$

$v_1 = r_1 \omega_1$

$v_2 = r_2 \omega_2$

L of planet conserved thus:

$\cancel{L = I}$

$L = \vec{r} \times \vec{p}$

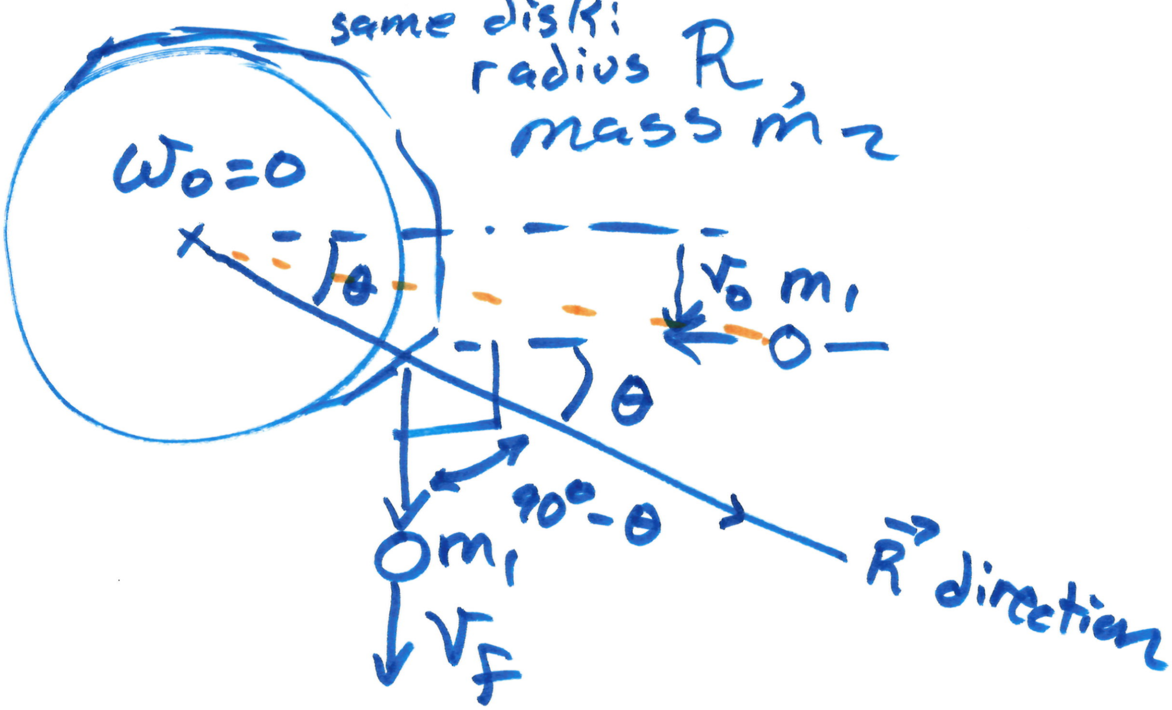
$= r m v$

$r_1 \cancel{m} v_1 = r_2 \cancel{m} v_2$

$L_1 = L_2$

Can get Kepler's 3rd law from uni. circ. motion

$T^2 \propto R^3$ later with $F_G = G \frac{m_1 m_2}{r^2}$




Given: $m_1$ $v_0$ $v_F$ $m_2 R$ $I_{disk}$,

find $\omega_F$

$L = rp\sin\theta$

$L = I\omega$

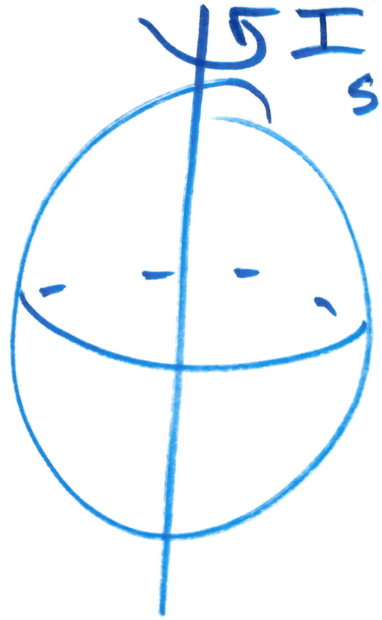
$\Sigma L_0 = \Sigma L_F$

$R m_1 v_0 + \cancel{I \cdot 0} = I\omega_F + \underline{R m_1 v_F \sin(90^\circ - \theta)}$

$\sin\theta$ $\quad 0$

$= \frac{1}{2} m_2 R^2 \omega_F + \underline{R m_1 v_F \sin(90^\circ - \theta)}$

$R m_1 v_0 \sin\theta =$



$I_s = \frac{2}{5} m R^2$ $\qquad R_E = 6.4 \times 10^6 \, m$

$\omega_{\text{Earth on axis}} = \frac{2\pi}{T}$ $\qquad m_E = 5.97 \times 10^{24} \, kg$

$T = 1 \text{day} \times \frac{24 \text{ hour}}{1 \text{day}} \times \frac{3600 \, s}{1 \text{hour}}$

$= 86400 \, s$

$$L = I\omega = \frac{\frac{2}{5} \cdot 5.97 \times 10^{24} kg \, (6.4 \times 10^6 m)^2 \cdot 2\pi}{8.64 \times 10^4 \, s}$$

$$L = 71.1 \times 10^{32} = 7.11 \times 10^{33} \, \frac{kg \, m^2}{s}$$

$$\tau = \frac{d}{dt}(L) = I \cancel{\omega} \alpha = \frac{d}{dt}(\underbrace{I\omega}_{L})$$

If $\tau = \text{const.}$

$$\tau \cdot (\Delta t) = L$$

$F = \frac{dp}{dt}$

$\tau = R \cdot F \cdot 2$

$F_{big} = 52 \, MN$ ← $10^6$

$$2 \times 6.4\times10^{6}\,m \times 5.2\times10^{7}\,N \cdot \Delta t =$$

$$L_E = 7.11\times10^{33}\ \frac{kg\,m^2}{s}$$

$$\Delta t = \frac{7.11\times10^{33}}{2\cdot 6.4\times10^{6}\,m\cdot 5.2\times10^{7}} = 10^{19}\ \text{sec.}$$

$3\times10^{7}\,\frac{s}{year}$

$10^{12}$ years

$10^{10}$ years $\approx$ age universe

axis

drop ring at rest. $m_2 R_2$ "thin"

rot disk $\omega_0$

$m_1 R_1$

$I_{disk} = \frac{mR^2}{2}$

$I_{\text{thin ring}} = mR^2$

$\omega_f = \star$

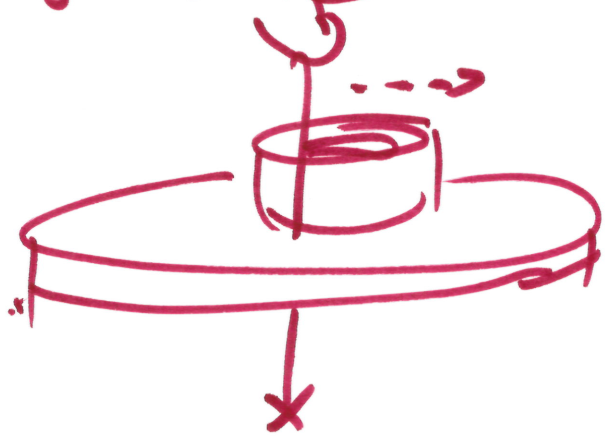
$$\Sigma L_0 = \Sigma L_f$$

disk only = (disk + ring)

$$\frac{1}{2} m_1 R_1^2 \omega_0 = \frac{1}{2} m_1 R_1^2 \omega_f + m_2 R_2^2 \omega_f$$

$$\frac{\frac{1}{2}m_1 R_1^2}{\frac{1}{2}m_1 R_1^2 + M_2 R_2^2} \qquad \omega_0 = \omega_F$$

If drop ring away from center



$I_{new} = I_{cm} + mh^2$