Physics 200 Day 10

Work and Energy

Work $W = \int \vec{F}(\vec{x}) \cdot d\vec{x}$

The dot product: mult. two vectors gives a scalar

if direction of motion is constant and force constant,

$$\int \vec{F}(\vec{x}) \cdot d\vec{x} \longrightarrow \vec{F} \cdot \vec{d} = W$$

in general, $\vec{F} \cdot \vec{d} = |\vec{F}|\,|\vec{d}| \cos\theta = W$

W is scalar, but <u>can</u> be +, − or 0.

$\vec{F} \cdot \vec{d} = F_{//} d$ ⇒ $F_{//}$ is the component of $\vec{F}$ // to $\vec{d}$.

$= F d_{//}$

↳ $d_{//}$ is the $\vec{d}$ component // to $\vec{F}$.

Work - Energy Theorm

$$\sum W = \Delta E_K$$
$$= E_{K_f} - E_{K_0}$$

(Note: $E_K := K$)

$$E_K = \frac{1}{2} m v^2$$

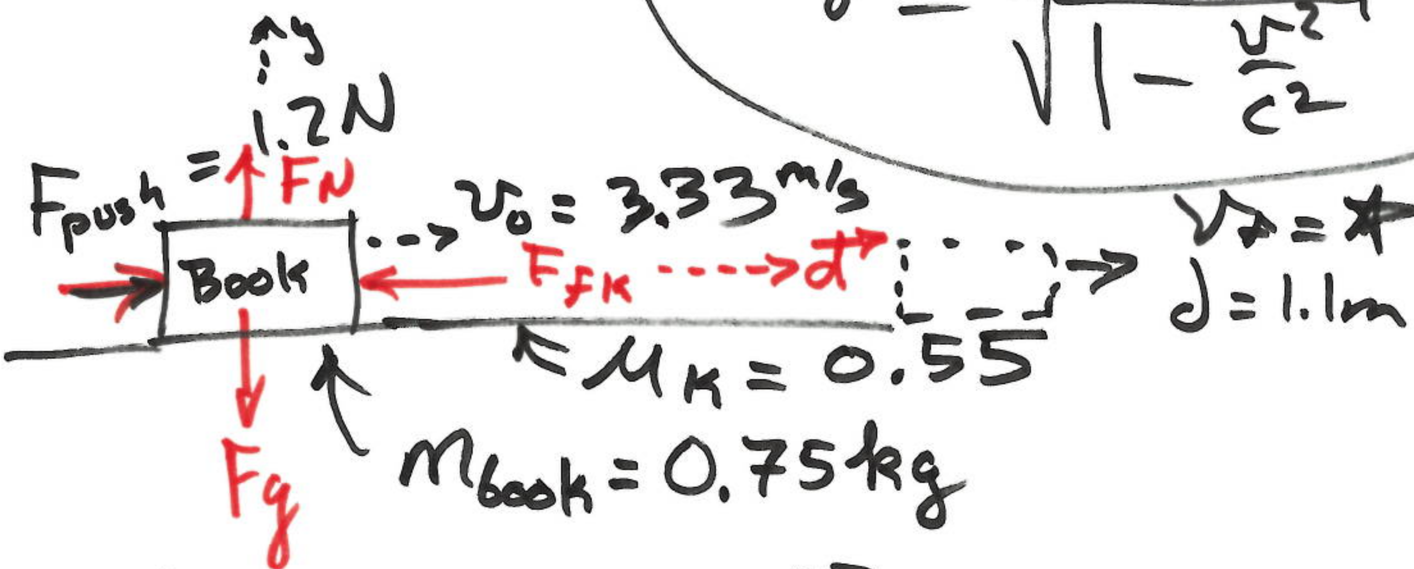
($v << 3 \times 10^8 \frac{m}{s}$ speed of light)

Lets do simple examples and verify that Work - Energy agrees with Newton's Laws + Const. accel

$$E_K - mc^2 = \gamma mc^2$$

rated +

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



W_F by	θ between $\vec{F}$ and $\vec{d}$	$\cos\theta$	
F_{push}	0	1	
F_g	90°	0	w=0 X
F_N	90°	0	w=0
F_{fK}	180°	-1	

$$\sum W = \Delta E_K = E_{Kf} - E_{K0}$$

$$W_{F_p} + \cancel{W_{F_g}} + \cancel{W_{F_N}} + W_{F_{fK}}$$

$$F_p \cdot d \cdot \underset{1}{\cos 0^\circ} + F_{fK} \cdot d \cdot \underbrace{\cos 180^\circ}_{-1} = E_{Kf} - E_{K0}$$

?

$$F_{fK} = \mu_K \cdot \overset{?}{F_N} \qquad f = \mu N$$

$$\sum F_y = m\cancel{a_y}^{\,0}$$

$$F_N - mg = 0 \rightarrow F_N = mg$$

$$F_p d - \mu_K m g d = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_0^2$$

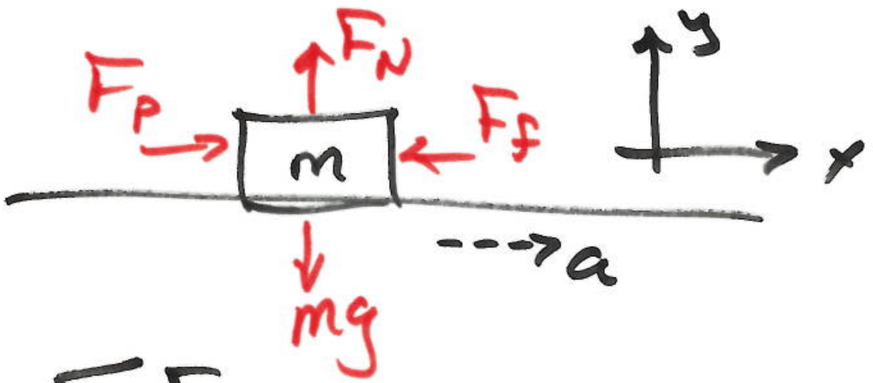
$$1.2\,N(1.1\,m) - 0.55(0.75\,kg)\left(9.8\,\tfrac{m}{s^2}\right)(1.1\,m) =$$

$$-3.13\,\underbrace{N \cdot m}_{\text{Joule (J)}} = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_0^2$$

$$-3.13\,J = \tfrac{1}{2}(0.75\,kg)\,v_f^2 - \underbrace{\tfrac{1}{2}\underset{0.75\,kg}{m}\, v_0^2}_{4.16\,J} \qquad (3.33\,m/s)^2$$

$$4.16\,J - 3.13\,J = 1.03\,J = \tfrac{1}{2}\underset{0.75\,kg}{m} v_f^2 \qquad v_f^2 = 2.75\,\tfrac{m^2}{s^2}$$

$$\boxed{v_f = 1.66\,m/s}$$



$$\sum F_x = ma$$

$$F_p - F_{fk} = ma \rightarrow F_p - \overbrace{\mu_k \overbrace{mg}^{F_N}}^{F_{fk}} = ma^{\star}$$

$$\sum F_y = 0$$

$$F_N - mg = 0 \rightarrow F_N = mg$$

$$\frac{F_p}{m} - \frac{\mu_k \cancel{m} g}{\cancel{m}} = a$$

$$\frac{1.2\,N}{0.75\,kg} - 0.55 \times 9.8\,\frac{m}{s^2} = a$$

$$1.6\,\frac{m}{s^2} - 5.39\,\frac{m}{s^2} = -3.79\,\frac{m}{s^2} = a$$

Const. accel: $a = -3.79\,m/s^2$

$v_0 = +3.33\,m/s$

$d = x(t) - x_0 = 1.1\,m$

$v_f = \star$

$$v_f^2 = v_0^2 + 2a(\overbrace{x - x_0}^{d}) = (3.33)^2\,\frac{m^2}{s^2} - 2(3.79)(1.1)\,\frac{m^2}{s^2}$$

$$v_f^2 = 11.1\frac{m^2}{s^2} - 8.338$$

$$= 2.762\frac{m^2}{s^2}$$

$$\boxed{v_f = 1.66\frac{m}{s}}$$