Thurs Dec 7th: Exam 3 Review/Office Hours

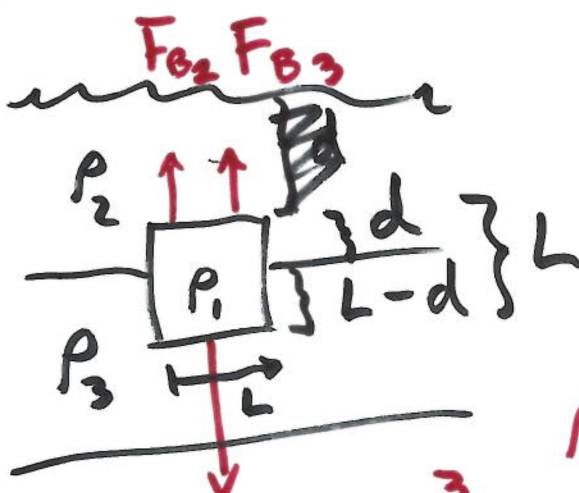
8 people came! I was not lonely. yay!

Topics:

- Gravity
- ★ Torque $\Sigma\tau = I\alpha$
- Moment of Inertia
- Constant α
- ★ Statics $\Sigma\tau = 0 \quad \Sigma F = 0$
- Oscillations
- Fluids: Bouyant Force / Pressure (depth)

$E_{K_{rot}}$

$L = I\omega = rp\sin\theta$



$\rho_3 > \rho_1 > \rho_2$

$\rho_1 = \frac{m}{L^3} \Rightarrow m = \rho_1 L^3$

$mg = \rho_1 L^3 g$

$F_B = \rho g V = \rho_{fluid}\, g\, V_{disp.}$

$F_{B_2} = \rho_2 L^2 d g$

$F_{B_3} = \rho_3 L^2 (L-d) g$

$F_{B_2} + F_{B_3} - mg = 0$

$\rho_2 \cancel{L^2} d \cancel{g} + \rho_3 \cancel{L^2} (L-d) \cancel{g} = \rho_1 L^{\cancel{3}\,1} \cancel{g}$

$$\rho_2 d_* + \rho_3 L - \rho_3 d_* = \rho_1 L$$

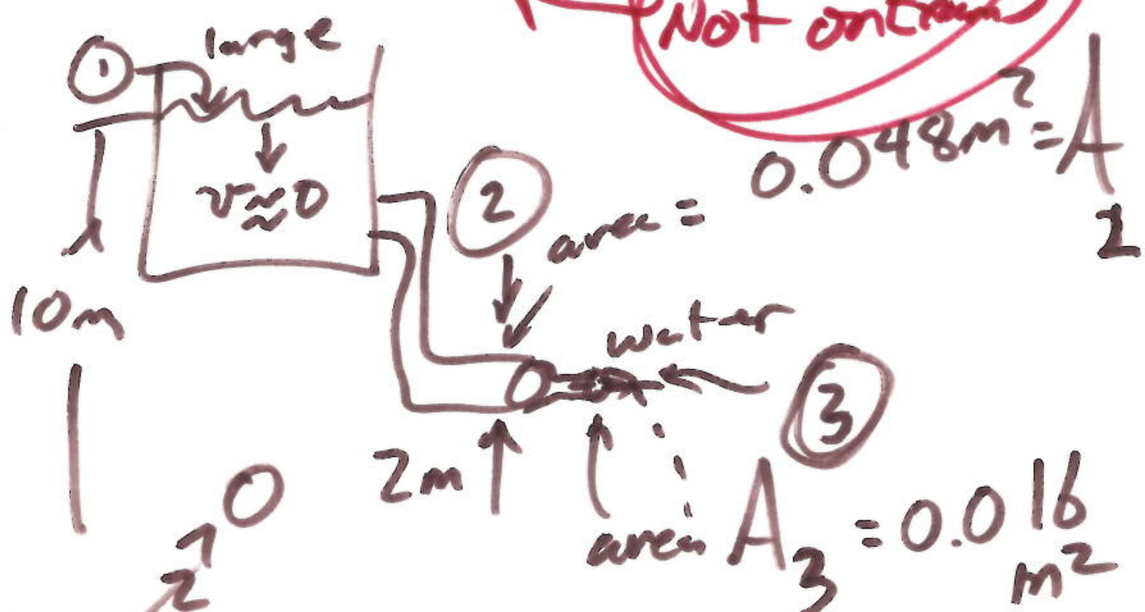
$$d(\rho_2 - \rho_3) = L(\rho_1 - \rho_3)$$

$$d = L\frac{(\rho_1 - \rho_3)}{\rho_2 - \rho_3}$$

Water Flowing from tank

Homework only Not on Exam 3

find $\frac{dV}{dt}$



$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho \cancel{v_1^2} = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$

$P_1 = 1\text{atm}$ $\qquad$ $P_2 = 1\text{atm} + \rho g\, 8m \cancel{+ \tfrac{1}{2}\rho v_2^2}$

$h_1 = 10m$ $\qquad$ $h_2 = 2m$

$P_3 = 1atm \qquad h_3 = 2m \qquad \underline{v_3 = \star}$

$$\overset{1atm}{\cancel{P_1}} + \rho g h_1 + 0 = \overset{1atm}{\cancel{P_3}} + \rho g h_3 + \frac{1}{2}\rho v_3^2$$

$$\cancel{\rho} g (\underbrace{\overset{h_1}{10m} - \overset{h_3}{2m}}_{8m}) = \frac{1}{2}\cancel{\rho} v_3^2$$

$$\sqrt{2g8m} = \sqrt{v_3^2} = \sqrt{16 \cdot 9.8 \frac{m^2}{s^2}}$$

$$\boxed{v_3 = 12.5 \frac{m}{s}}$$

$$\frac{dV}{dt} = A_3 \cdot v_3 \qquad \circ \Rrightarrow v_3$$

area 3

$$\frac{dV}{dt} = 0.016 \times 12.5$$

B. Gague pressure at 2?

$$A_2 v_2 = A_3 v_3$$

$$v_2 = v_3 \frac{A_3}{A_2} = 12.5\frac{m}{s} \cdot \frac{0.016m^2}{0.048m^2}$$

$v_2 = 4.17 \frac{m}{s}$

$$P_2 + \cancel{\rho g h_2} + \frac{1}{2}\rho v_2^2 = P_3 + \cancel{\rho g h_3} + \frac{1}{2}\rho v_3^2$$

$$h_2 \simeq h_3 \qquad P_3 = 1atm$$

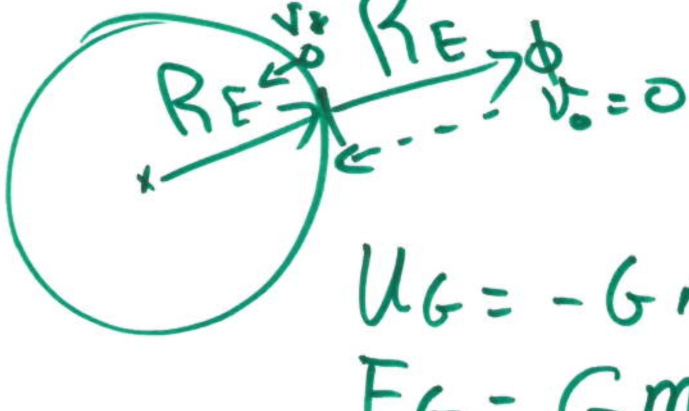
"gague pressure" at 2 : $P_2 - 1atm$

$$\underbrace{P_2 - P_3}_{1atm} = P_2 \text{ gague} = \frac{1}{2}\rho v_3^2 - \frac{1}{2}\rho v_2^2$$

$$= \frac{1}{2}\, 10^3 \frac{kg}{m^3}\left(\left(12.5\frac{m}{s}\right)^2 - \left(4.17\frac{m}{s}\right)^2\right)$$

$$= 69400 \text{ Pa}$$

If you drop 50kg rock from a height $= R_E$ above Earth, with what v should it hit?



$R_E = 6.4 \times 10^6 m$

$M_E = 6 \times 10^{24} kg$

$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

$$U_G = -G m_1 m_2 / r$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

→ orbits

→ Can find g on surface

$$g = G\frac{M_p}{r_p^2}$$

use U_G.

$$\Sigma E_o = \Sigma E_f$$ since $W_{N.C.} = 0$ No fric.

$$U_{G_o} + \cancel{E_{k_o}}^{0} = U_{G_f} + E_{kf}^{\star}$$

$$-\frac{G \cancel{m_{rock}} m_E}{2R_E} + 0 = -\frac{G \cancel{m_{rock}} m_E}{R_E} + \frac{1}{2}\cancel{m_{rock}} v_f^{2\,\star}$$

$$1.\ G\frac{m_E}{R_E} - \frac{G m_E}{2R_E} = \frac{1}{2} v_f^{2\,\star}$$

$$\sqrt{\frac{G m_E}{\cancel{2}R_E}} = \cancel{\tfrac{1}{2}}\sqrt{v_f^2} = \sqrt{\frac{6.67\times10^{-11}\ 6\times10^{24}}{6.4\times10^{6}}}$$

$$v_f = \sqrt{\frac{6.67\times6}{6.4}\times10^{7}}$$

$$v_f = 10^3 \frac{m}{s}\sqrt{\frac{6.67\times60}{6.4}} = 7900.\cancel{2}\ \frac{m}{s}$$

$$7.9\times10^{3}\frac{m}{s}$$

Oscillation #10 from sample exam

$$x(t) = A\sin(\omega t)$$

$$y(t) = 0.15\ \sin(134\,t)$$

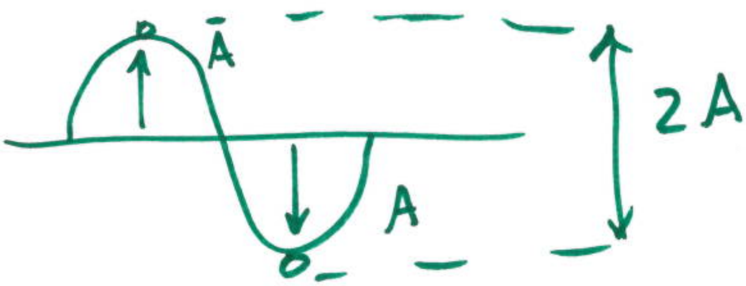
y unit of mm — t unit of s

$A = 0.15$ mm but wing moves 0.30 mm from top to bottom of motion.

$134 = \omega$ unit $\frac{1}{s}$ or $\frac{rad}{s}$

$2\pi f = \omega$

f: the frequency in Hz $= \frac{1}{s}$ or $\frac{cycles}{s}$



$\omega = \sqrt{\frac{k}{m}}$ for mass on spring

$\omega = \sqrt{\frac{g}{L}} \leftarrow g$ pendulum

$$f = \frac{\omega}{2\pi}$$

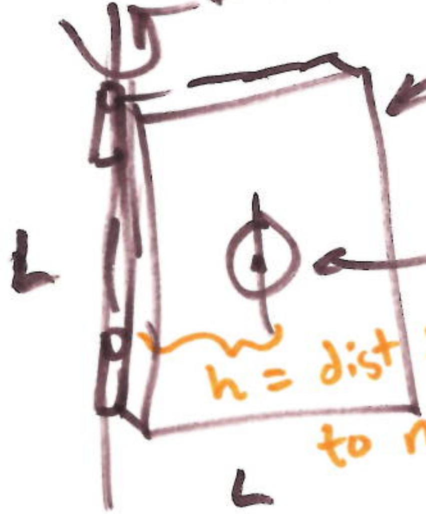
$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

I example: Hobbit Door

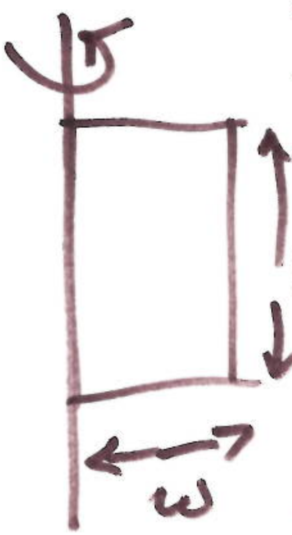
Door w/ big spherical knob



m_1

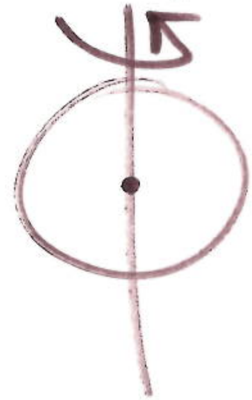
sphere in middle
radius r
mass m_2.

h = dist from cm
to new axis.



$$I = \frac{1}{3} m w^2$$

$$I_{sphere} = \frac{2}{5} m r^2$$



$$\underbrace{I_{\parallel}}_{} \quad I_{new} = I_{cm} + m h^2$$

$$I_{tot} = \frac{1}{3} m_1 L^2 + \frac{2}{5} m_2 r^2 + m_2 \left(\frac{L}{2}\right)^2$$

A 1.5m radius disk speeds up from rest to $\omega_f = 1.5 \frac{rad}{s}$ over 1 revolution.

Find α, t assuming α = const.

I $\omega_f = \omega_0 + \alpha t$

II $\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

III $\omega_f^2 = \omega_0^2 + 2\alpha(\theta(t) - \theta_0)$

<u>Know:</u>

$\omega_f = 1.5 \frac{rad}{s}$

$\omega_0 = 0$ (rest)

$\theta_0 = 0$

$\theta(t) = 2\pi$

<u>Want:</u>

α

t

(III) $\left(1.5 \frac{rad}{s}\right)^2 = 0 + 2\alpha(2\pi \text{ rad})$

$\boxed{0.179 \frac{rad}{s^2} = \alpha}$

(I) $1.5 \frac{rad}{s} = 0 \frac{rad}{s} + \alpha t$

$\frac{1.5 \frac{rad}{s}}{0.179 \frac{rad}{s^2}} = \boxed{t = 8.37 s}$

Part B. Now you slow down at $\alpha = -0.123 \frac{rad}{s^2}$ How long does it take? How many revolutions? To slow down to <u>rest</u>.

<u>Know:</u>

$\omega_f = 0$

$\alpha = -0.123 \frac{rad}{s^2}$

$\omega_0 = 1.5 \frac{rad}{s}$

Can set $\theta_0 = 0$

<u>Want:</u>

t

$\theta(t)$

then get $\#_{rev.} = \frac{\theta(t)}{2\pi}$

(I) $\omega(t) = \omega_0 + \alpha t$

$$0 = 1.5 \frac{rad}{s} - 0.123 \frac{rad}{s^2} t$$

$$\frac{-1.5 \frac{rad}{s}}{-0.123 \frac{rad}{s^2}} = \boxed{t = 12.2\,s}$$

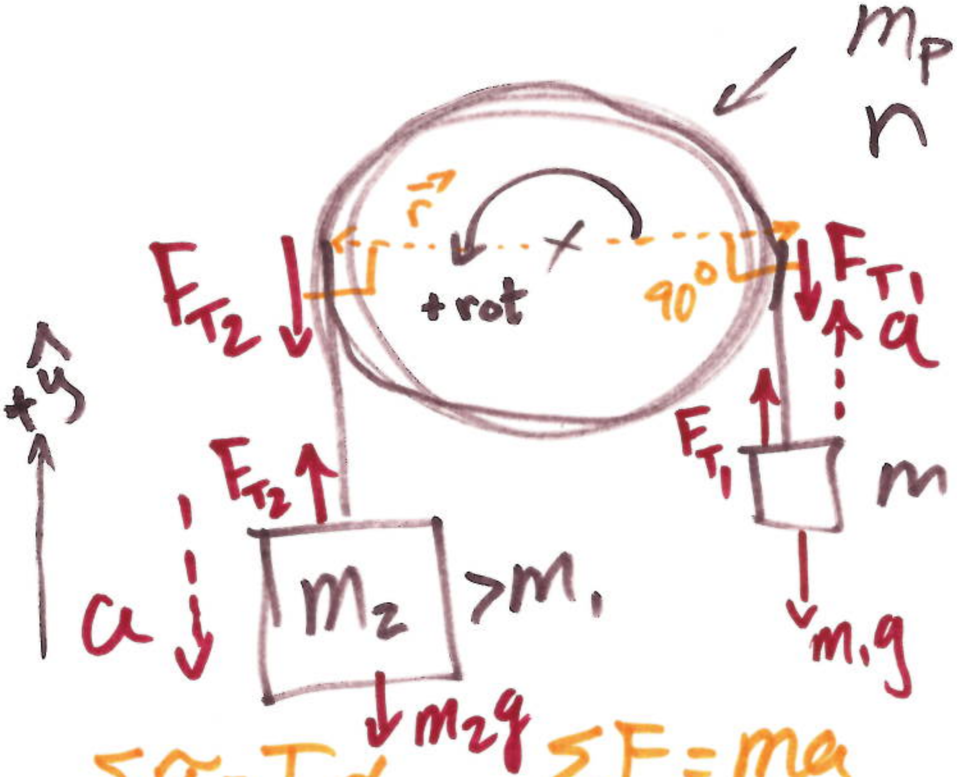
(II) $\theta(t) = \cancelto{0}{\theta_0} + \omega_0 t + \frac{1}{2}\alpha t^2$

$$= 0 + 1.5 \frac{rad}{s}(12.2s) + \frac{1}{2}\left(-0.123 \frac{rad}{s^2}\right)(12.2s)^2$$

$$= 0 + 18.3\,rad - 9.15\,rad$$

$\theta(t) = 9.15 \text{ rad}$

$n_{rev} = \dfrac{\theta}{2\pi} = 1.46 \text{ revolutions}$



disk

$I_{disk} = \frac{1}{2} m r^2$

find α of pulley + a of each mass.

$\Sigma \tau = I\alpha$ $\quad \Sigma F = ma \quad F_g = mg$

$$F_{T_2} - m_2 g = -m_2 a$$

$$F_{T_1} - m_1 g = +m_1 a$$

$\Sigma \tau = I\alpha$ on pulley each $\underline{\tau = rF\sin\theta}$

$$rF_{T_2}\sin 90° - rF_{T_1}\sin 90° = \underbrace{I}_{\frac{1}{2}m_p r^2}\alpha$$

$$\left.\begin{aligned} rF_{T_2} - rF_{T_1} &= \tfrac{1}{2} m_p r^2 \alpha \\ F_{T_2} - m_2 g &= -m_2 a \\ F_{T_1} - m_1 g &= +m_1 a \end{aligned}\right\}$$

3 equ. $\underline{4}$ unknowns

F_{T_1} F_{T_2} a α

need $4^{\underline{th}}$ equ.

$a = r \cdot \alpha$

$$\cancel{r}(F_{T_2}-F_{T_1}) = \frac{1}{2}m_p \cancel{r^2}\underbrace{\left(\frac{a}{\cancel{r}}\right)}_{\alpha}$$

$$F_{T_2}-F_{T_1} = \frac{1}{\cancel{2}}m_p a$$

$$F_{T_2} = m_2 g - m_2 a$$

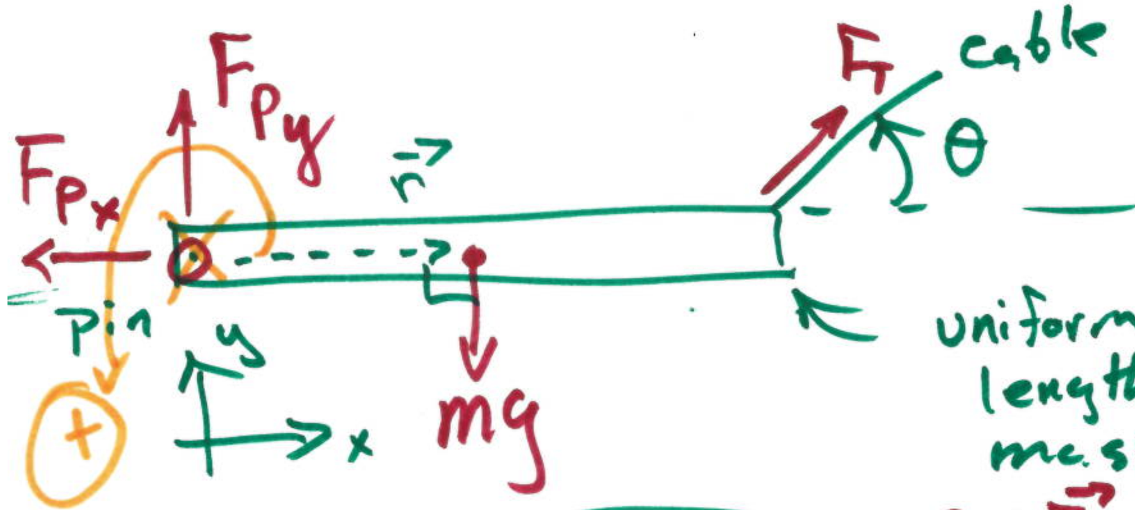
$$F_{T_1} = m_1 g + m_1 a$$

$$m_2 g - m_2 a - (m_1 g + m_1 a) = \frac{1}{2}m_p a$$

$$m_2 g - m_1 g - m_2 a - m_1 a = \frac{1}{2}m_p a$$

$$g(m_2 - m_1) = a\left(\frac{1}{2}m_p + m_1 + m_2\right)$$

$$\boxed{\frac{(m_2 - m_1)g}{\frac{1}{2}m_p + m_1 + m_2} = a}$$



uniform beam
length L
mass m
find $\vec{F_p}$ and F_T in cable.

$\Sigma F_x = 0$

$+F_T \underline{\cos}\,\theta - F_{px} = 0$

$\Sigma F_y = 0$

$F_{py} - mg + F_T \underline{\sin}\,\theta = 0$

each $\tau = rF\sin\theta$

$\Sigma \tau = 0$ about pin we shall sum. could chose elsewhere.

$$0F_{px} + 0F_{py} - \frac{L}{2} mg \sin(90°) + LF_T \sin(\theta) = 0$$

$$-\frac{\cancel{L}}{2} mg + \cancel{L} F_T \sin\theta = 0$$

$$\boxed{F_T = \frac{1}{2}\frac{mg}{\sin\theta}}$$