Physics 200 Day 18

Last time: θ, ω, α

Const. α: speeding up / slowing of a rotating system.

ω = angular speed unit: $\frac{\text{"radians"}}{\text{second}}$

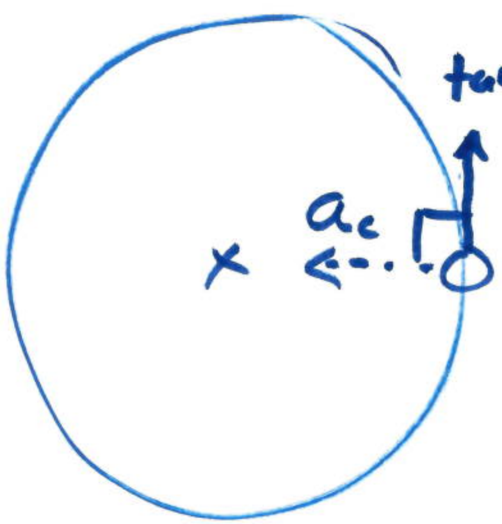
$$v = \frac{2\pi R}{T}$$

$$d = \theta \cdot R$$

"radians"

$$v = \omega \cdot R$$

$$a_t = \alpha \cdot R \quad \text{(tangential)}$$



$$a_c = \frac{v^2}{R} \text{ toward center of circle}$$

$$a_c = \frac{v^2}{R} = \frac{(\omega \cdot R)^2}{R} = \omega^2 R$$

Rotational 2nd law:

$$\sum \tau = I \, \alpha$$

next, later, know this person

Torque, τ, causes rotation.
Due to a force.

$\tau = RF\sin\theta$ $\left(\vec{\tau} = \vec{R} \times \vec{F}\right)$
later

R: distance from axis of rotation to where F applied

F: the Force

$\theta = \measuredangle$ between $\vec{R}$ and $\vec{F}$.

$\sin\theta$ is <u>max</u> at 90°.

To get larger τ (torque) you can:

1. Increase F
2. Increase R
3. Optimize θ toward 90°.

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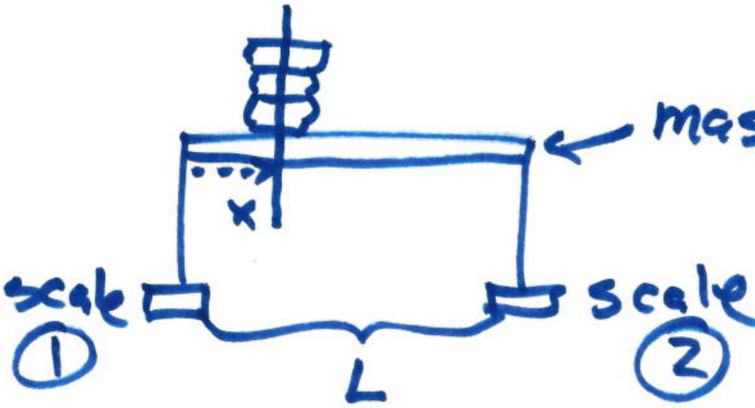
statics: $\sum \tau = 0$

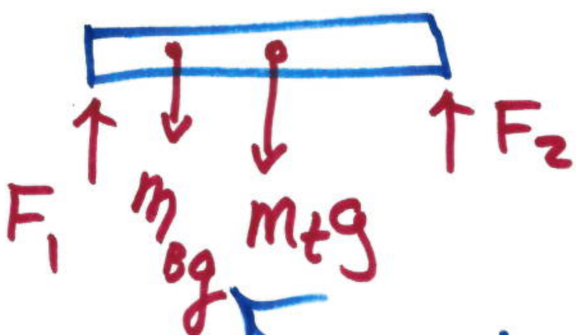
2-D table, uniform, mass M_T, length L and a stack of Books, M_B, at x.



mass m_T uniform

Assume light table legs.

$\vec{F}$ on table top:



$\Sigma F = 0$ on table

$F_1 - m_B g - m_t g + F_2 = 0$

$\underline{F_1 + F_2 = (m_B + m_t) g}$

Newton's 3rd Law

$\Sigma F = 0$... statics.

$F_N - F_g = 0$

$F_N = F_g = m_b \cdot g$

between books and table.

Need τ to tell how F distributed.

But $\tau = RF\sin\theta$ . . . where is axis of rotation? R = dist. from axis to $\vec{F}$.

each $\tau = RF\sin\theta$

all θ in this problem are $90°$.

θ is $\measuredangle$ between $\vec{R}$ and $\vec{F}$?

$\Sigma\tau = 0$ about any axis. choose left side.

$$\pm \cancel{0 \cdot F_1 \sin(\;)}^{\,0} - x\, m_B g \sin(90°) - \frac{L}{2} m_t g \sin 90°$$

$$+ L F_2 \sin(90°) = 0$$

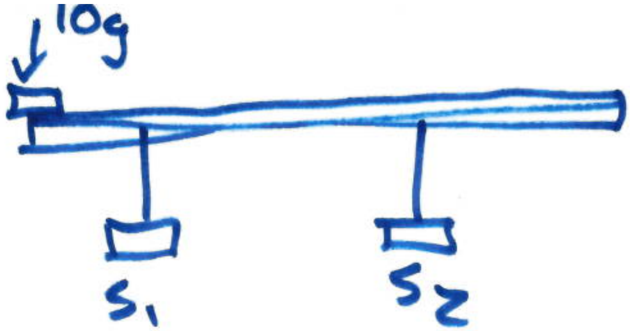
$$-x\, m_B g - \frac{L}{2} m_t g + L \overset{\star}{F_2} = 0$$

$$F_2 = \frac{1}{2} m_t g + \frac{x}{2} m_B g$$

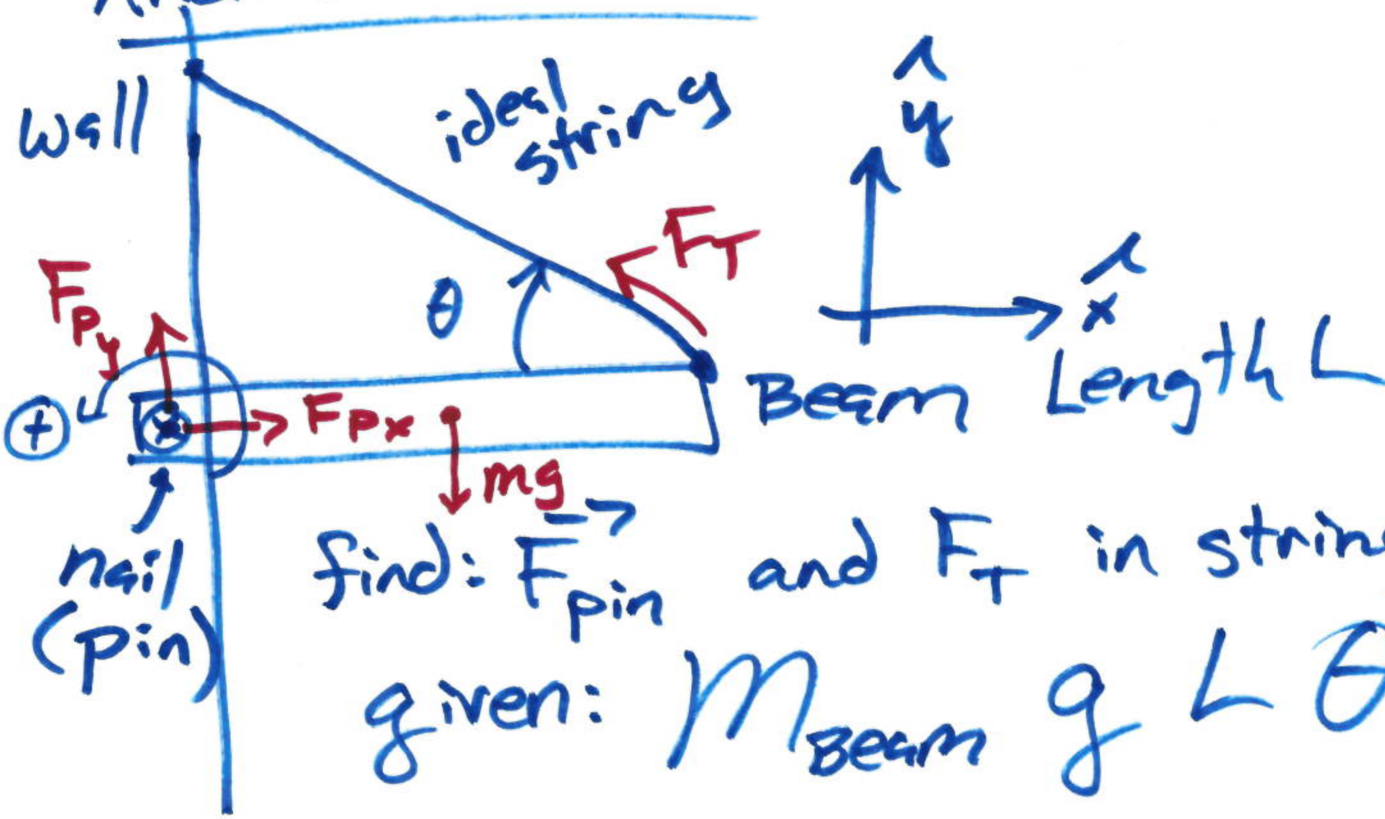
$$F_1 + F_2 = m_t g + m_B g$$

solve F_1.

$$F_1 = \frac{1}{2} m_t g + \left(1 - \frac{x}{L}\right) m_B g$$



Another Static:



find: $\vec{F}_{pin}$ and F_T in string

given: M_{Beam} g L θ

$$\Sigma F_y = 0$$

$$F_{py}^* - mg + F_T^* \underline{\sin}\,\theta = 0$$

$$\Sigma F_x = 0 = +F_{px}^* - F_T^* \underline{\cos}\,\theta = 0$$

$\Sigma \tau = 0$ about the pin. CCW is $\oplus$

each $\tau = RF\sin\theta$ $\otimes \circlearrowleft \oplus$

$$0 \cdot F_{py} + 0 F_{px} - \frac{L}{2} mg \sin 90^\circ + L F_T \sin\theta = 0$$

$$-\frac{\cancel{L}}{2} mg + \cancel{L} F_T \sin\theta = 0$$

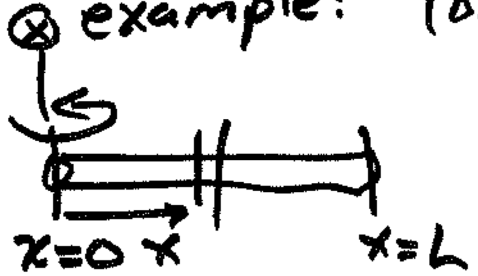
$$\boxed{F_T = \frac{mg}{2\sin\theta}}$$

Moment of Inertia:
How hard is something to rotate?

$$I = \int r^2\, dm$$

↑ wee bit o' mass

example: long thin rod, mass M
length L



$$\frac{\text{mass}}{\text{unit length}} = \lambda = \frac{M}{L}$$

↑ lambda λ

$$dm = \lambda\, dx$$

$$I_{\text{end rod}} = \int_{x=0}^{L} x^2 (\lambda dx) = \lambda \int_0^L x^2 dx = \lambda \left[\frac{x^3}{3}\right]_0^L$$

$$= \lambda \frac{L^3}{3} = \frac{M}{L}\frac{L^3}{3} = \frac{1}{3} M L^2$$

unit: $kg \cdot m^2$

I takes the place of mass in rotational formuli.

$$\Sigma \vec{F} = m\vec{a} \qquad \Sigma \tau = I\alpha$$

$$\vec{p} = m\vec{v} \qquad \vec{L} = I\vec{\omega}$$

↑ angular momentum

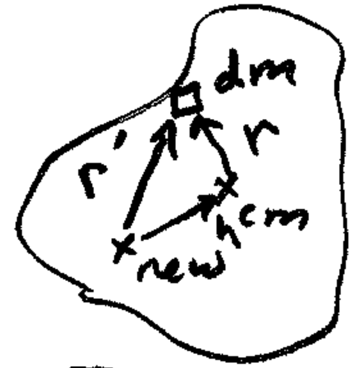
Parallel Axis Theorm:

$$I_{new} = I_{cm} + mh^2$$

↑ distance from $\underline{cm}$ to new axis.

cm = center of mass.

$I_{end} = \frac{1}{3}ML^2$



$dm\, r^2 = I_{cm}$

$dm\, r'^2 = I_{new}$

$\vec{r}' = \vec{h} + \vec{r} \ldots$

prove this somehow.

$$\frac{1}{3}ML^2 = I_{cm} + m\left(\frac{L}{2}\right)^2$$

$$\frac{1}{3}ML^2 = I_{cm} + \frac{1}{4}mL^2$$

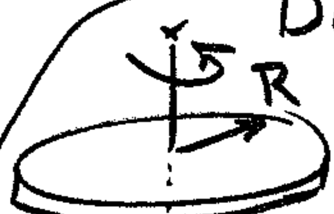
$$\left(\frac{1}{3} - \frac{1}{4}\right)mL^2 = I_{cm}$$

$$\left(\frac{4}{12} - \frac{3}{12}\right)mL^2$$

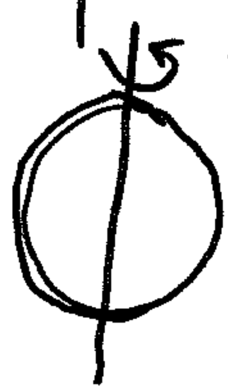
$$\boxed{\frac{1}{12}mL^2 = I_{cm}}$$

long thin rod.

Disk



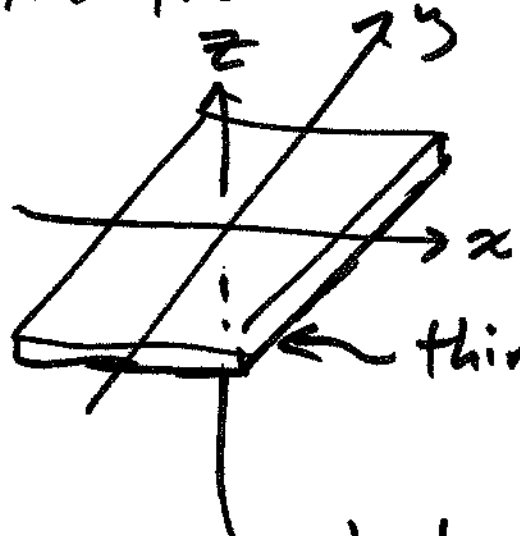
$I = \frac{1}{2}mR^2$ ✓



$I = \frac{1}{4}mR^2$

for "thin" objects, you can use $\perp$ axis theorem in x, y plane

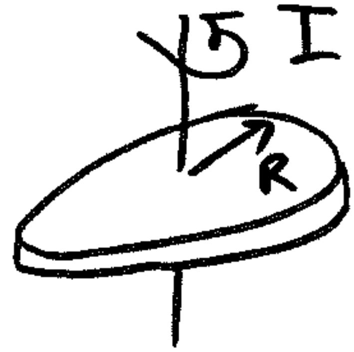
$\uparrow$ perpendicular



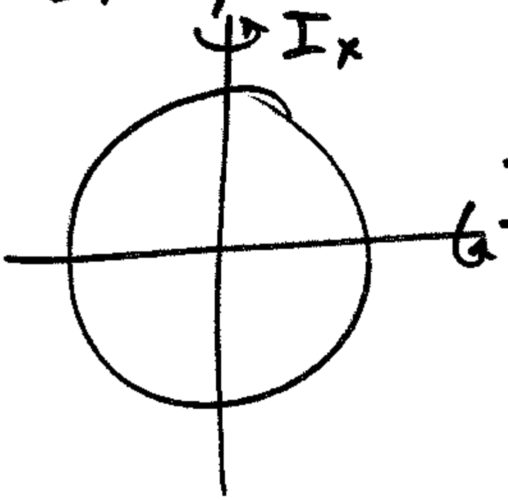
$$I_z = I_x + I_y$$

thin in $\hat{z}$ direction.

$$I = \tfrac{1}{2}mR^2$$



Example: disk:



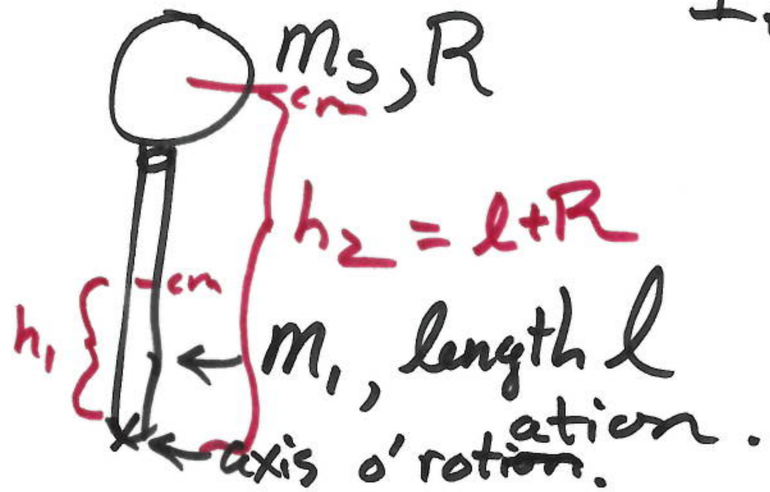
$I_x = I_y$ by symetry.

$$I_z = I_x + I_y$$

$$\tfrac{1}{2}mR^2 = 2I_x$$

$$\boxed{\tfrac{1}{4}mR^2 = I_x}$$

moments of inertia add.



$$I_{tot} = I_{rod} + I_{sphere}$$

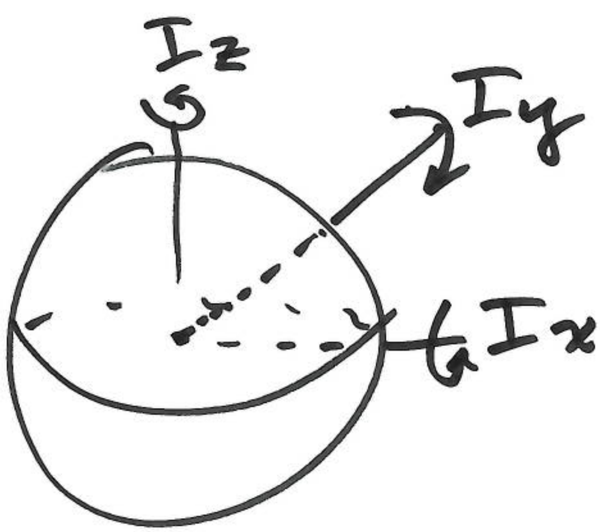
$$=$$

given:

$$I_{\substack{rod \\ (cm)}} = \frac{1}{12} m L^2 \qquad I_{\substack{sphere \\ (cm)}} = \frac{2}{5} m R^2$$

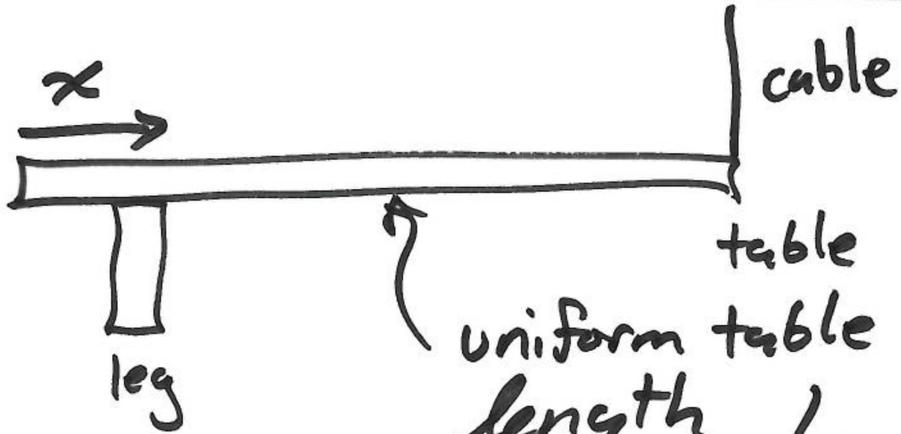
$$I_{new} = I_{cm} + m h^2$$

$$I_{tot} = \frac{1}{12} m_1 l^2 + m_1 \Big(\underbrace{\tfrac{l}{2}}_{h_1}\Big)^2 + \frac{2}{5} m_s R^2 + m_s \big(\underbrace{l+R}_{h_2}\big)^2$$



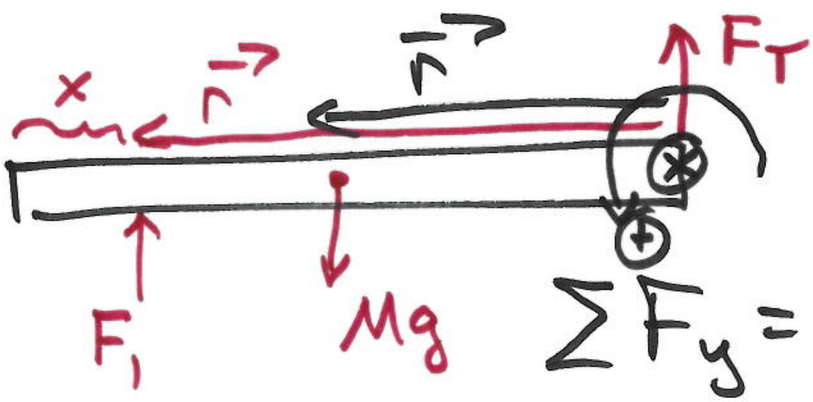
$I_x = I_y = I_z$
for sphere
by symetry

$$\sum \check{\tau} = \check{I}\check{\alpha} \quad \text{yay!}$$



uniform table mass M
length L

<u>Find</u>: tension in cable, force in leg.



Given: $M\,L\,x\,g$
+ picture.
cable, leg are
<u>vertical</u>.

$$\sum F_y = 0$$

$$F_1 + F_T - Mg = 0 \checkmark$$

$$\sum \tau = 0 \qquad \tau = rF\sin\theta$$

$$0 \cdot F_T + \frac{L}{2} Mg \sin(90°) - (L-x) F_1 \sin(90°) = 0$$

$$\frac{L}{2} Mg - (L-x) F_1^{\star} = 0$$

$$\boxed{\frac{LMg}{2(L-x)} = F_1} \qquad F_T = Mg - \frac{LMg}{2(L-x)}$$

Physics 200 Day 20

Exam 2 returned $\bar{x} = 30.5/40$

for Exam 1 $\bar{x} \simeq 33/40$

$2\frac{1}{2}$ points lower... OK

Could do quiz on uniform α 1

or torque (statics 2 or dynamics) 0

or ... moment of inertia ($\frac{1}{2}$)

or ... momentum and collisions $\vec{p}$? ✓ yep.

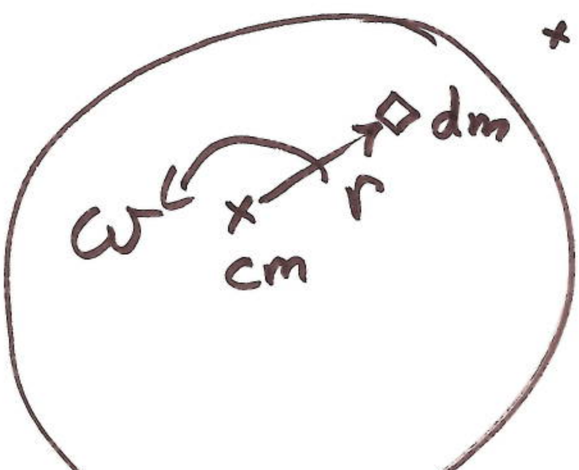
Date: next Mon 15

next Wed 11

following Mon 2

This Thurs: only Office Hours moved to 9-Noon.

Rotational Kinetic Energy:



$$E_K = \sum_i \frac{1}{2} m_i v_i^2$$

$$v_i = \omega \cdot r_i$$

$$E_K = \frac{1}{2} \cdot \sum m_i (\omega r_i)^2$$

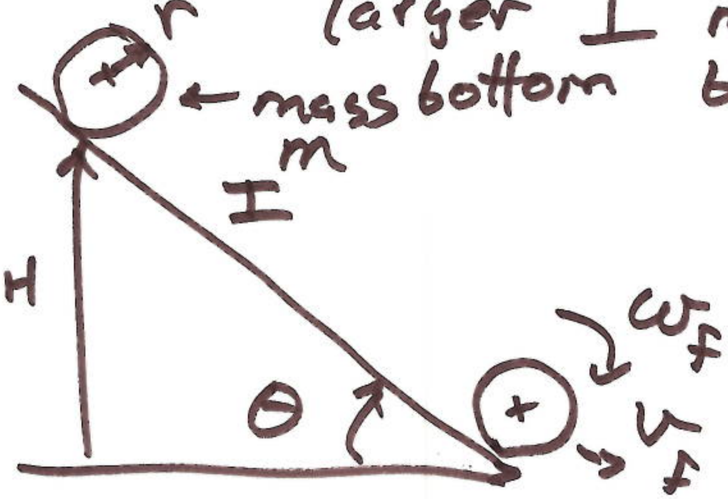
$$E_K = \frac{1}{2}\omega^2 \left(\sum_i m_i r_i^2\right)$$

$$I = \sum_i m_i r_i^2 \qquad \boxed{E_{K_{rot}} = \frac{1}{2} I \omega^2}$$

so: I replaces m in rotational formuli.

Rolling without slipping:

larger I means slower v at bottom by conserv. of energy.



Given: m, I, r

H g

find: V_f and ω_f

Rolling $\underbrace{\text{without}}_{\text{w/o}}$ slipping means

$$V_{c.m.} = r \cdot \omega$$

1 revolution

Use Energy

$$V_{cm} = \frac{2\pi r}{\underbrace{T}_{\omega}} = r \cdot \omega$$

$$\sum E_f - \sum E_0 = W_{n.c.}$$

$$\frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 - mgH = 0$$

$$\frac{1}{2} m v^{*2} + \frac{1}{2} I \underbrace{\left(\frac{v^*}{r}\right)^2}_{\omega = \frac{v}{r}} = mgH$$

roll w/o slip

$$v^2 \left(\frac{m}{2} + \frac{1}{2}\frac{I}{r^2}\right) = mgH$$

$$\sqrt{v^2} = \sqrt{\frac{mgH}{\frac{m}{2} + \frac{I}{2r^2}}}$$

$$\omega = v/r$$

If cylinder, $I = \frac{1}{2} m r^2$

solid, uniform

answer simplifies:

$$\frac{I}{2r^2} = \frac{1}{4} m$$

$$v = \sqrt{\frac{\cancel{m} gH}{\frac{1}{2}\cancel{m} + \frac{1}{4}\cancel{m}}} = \sqrt{\frac{4}{3} gH}$$

angular momentum

$$L = I\omega = \underline{\vec{r} \times \vec{p}} = rp\sin\theta$$

is conserved.

4 quantum numbers

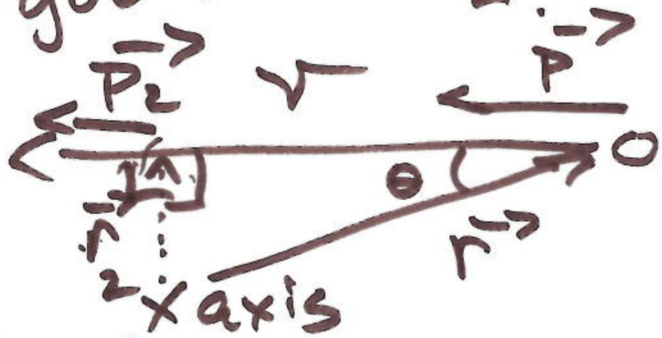
n l m s

n → energy

l → angular momentum

m → z-compon of angular momentum

L depends on axis of rotation. If you pass by the axis, you have $\vec{L}$



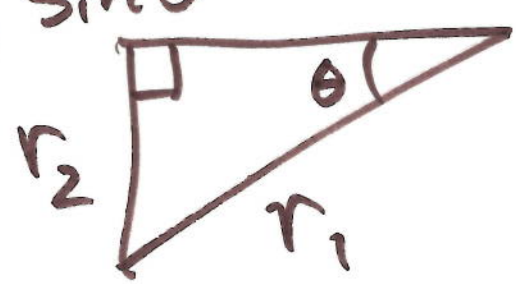
$\theta_2 = 90^\circ$

$$\vec{r_2} \times \vec{P_2} = \vec{r_1} \times \vec{P_1}$$

$$r_2 P_2 \sin\theta_2 = r_1 P_1 \sin\theta_1$$

$P_2 = P_1$ if no force on particle

$$r_2 = r_1 \sin\theta$$



$$\sin\theta = \frac{opp}{hyp} = \frac{r_2}{r_1}$$

$$r_1 \sin\theta = r_2 \checkmark$$

<u>yes</u>.