Work and Energy

Note: Try the written Example Force Problems.

$$W = \int \vec{F}(x) \cdot d\vec{x}$$

$$\Sigma W = \Delta E_K$$

$$E_K = \frac{1}{2} m v^2$$

const $\vec{F}$ and direction of motion

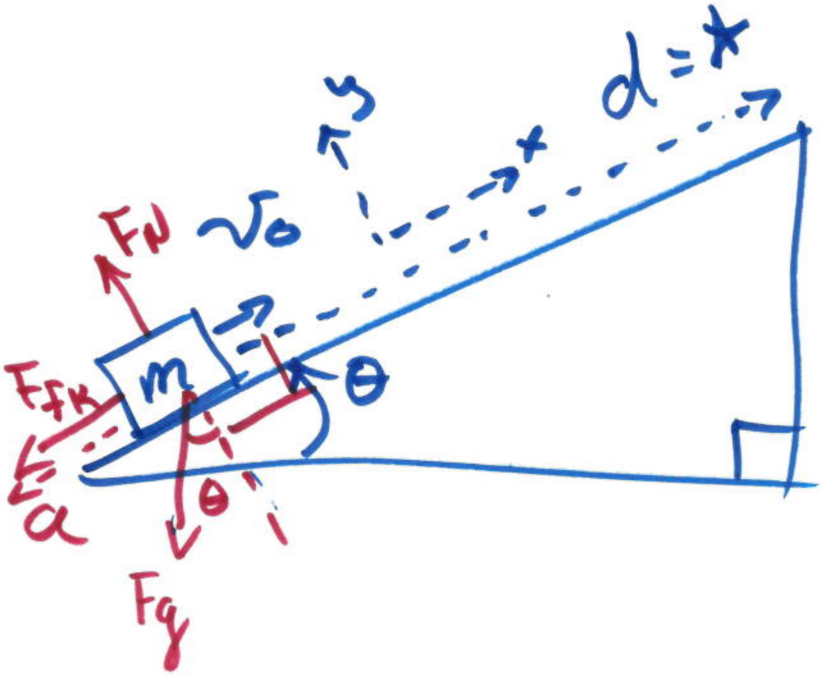
$$W = F d \cos\theta$$

unit

$$\text{Joule}(J) = N \cdot m = kg \cdot \frac{m^2}{s^2}$$

4.186 J = 1 calorie = heat to increase T of 1 gram of H_2O by 1°C

food calorie = Calorie = 1 kilocalorie



How far up the ramp does it go?

Given: $m = 2.25$ kg

$\theta = 29°$

$v_0 = 8.85$ m/s

$\mu_k = 0.12$

force	θ	$\cos\theta$
F_N	90°	0
F_{f_K}	180°	-1
F_g	$-90^\circ-\theta$ $= \theta+90^\circ$	$\cos(90^\circ+\theta)$

$$W = Fd\cos\theta$$

$$\Sigma W = \cancel{W_{F_N}}^{0} + W_{F_f} + W_{F_g}$$

$$W_{F_g} = mg\,d\,\overbrace{\cos(\cancel{180^\circ}\ 119^\circ)}^{\cancel{-1}} = +mgd\cos 119^\circ$$

$$W_{F_f} = F_f\cdot d\cdot \overbrace{\cos(180^\circ)}^{-1} = -F_f\cdot d$$

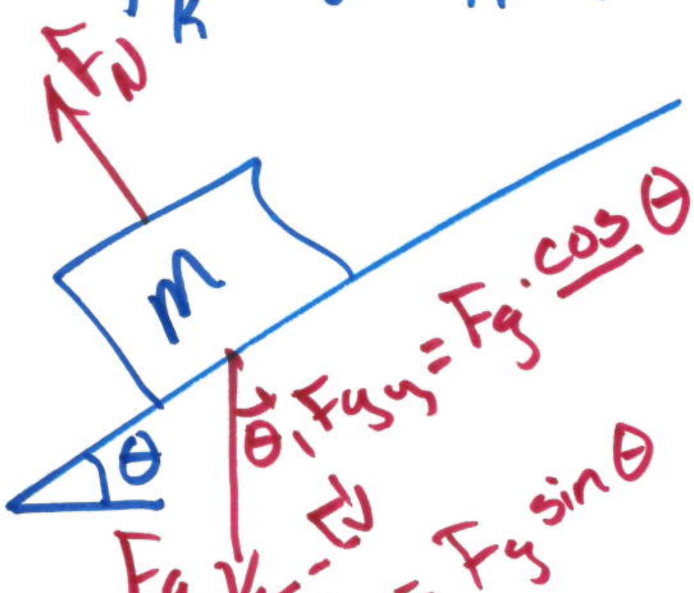
$$F_{f_K} = \mu_K F_N$$

$$\Sigma F_y = m\cancel{a_y}^{0}$$

$$F_N - F_{g_y} = 0$$

$$F_N = F_{g_y}$$

$$F_N = mg\cos\theta$$



$$W_{F_f} = \overbrace{-}^{\cos\theta}\overbrace{\mu_K\underbrace{mg\cos\theta}_{F_N}}^{F_{f_K}}\overbrace{d}^{d}$$

$$\Sigma W = \Delta E_K = \frac{1}{2}m\cancel{v_f}^2 - \frac{1}{2}mv_0^2$$

$$W_{F_g} + W_{F_f} = -\frac{1}{2}mv_0^2$$

$$\cancel{m}gd^*\cos 119^\circ - \mu_K \cancel{m}g\cos\theta\, d^* = -\frac{1}{2}\cancel{m}v_0^2$$

$$\underbrace{g d^* \left(\cos 119^\circ - \overset{0.12}{\mu_K}\cos\underset{29^\circ}{\theta}\right)}_{\substack{-0.5897 \\ -0.590}} = -\frac{1}{2}v_0^2$$

$v_0 = 8.85\,\frac{m}{s}$

$$d = \frac{-v_0^2}{2g(-0.59)} = \frac{-\left(8.85\frac{m}{s}\right)^2}{2\left(9.8\frac{m}{s^2}\right)(-0.59)}$$

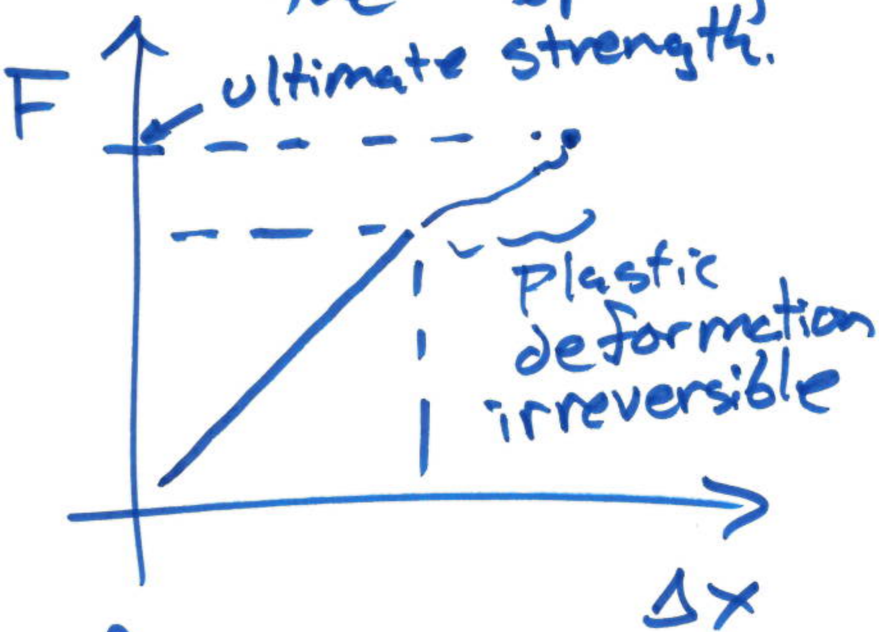
$$\boxed{d = 6.78\,m}$$

work done by elastic (spring) force

$$\vec{F_e} = -k \, \Delta \vec{x}$$

Hook's Law

k ← spring constant, stiffness of the spring

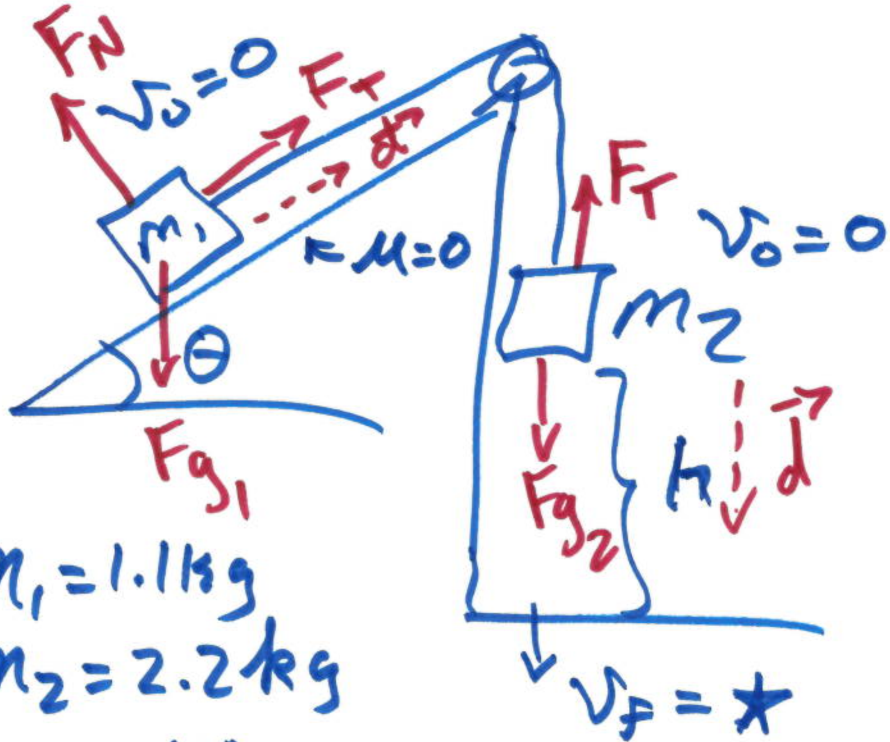


$$W = \int F(x)\,dx$$

$$= -\int k \, x \, dx$$

$$= -\frac{1}{2} k x^2$$

used soon for energy stored in a spring



on m_1:

F	θ	$\cos\theta$
F_T	0	1
F_N	$90°$	0
F_{g_1}	$\theta + 90°$	

$m_1 = 1.1\text{kg}$

$m_2 = 2.2\text{kg}$

$\theta = 41°$

$h = 3.3\text{m}$

$v_f = \star$

$\Sigma W = \Delta E_K$

on m_2:

F	θ	$\cos\theta$
F_T	$180°$	-1
F_{g_2}	$0°$	1

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mass 1

$$W_{F_T} + \cancel{W_{F_N}}^{0} + W_{F_{g_1}} = E_{K_{1f}} - \cancel{E_{K_{1o}}}^{0 \text{ rest}}$$

$$= \frac{1}{2} m_1 v_f^2$$

mass 2

$$W_{F_T} + W_{F_{g_2}} = \frac{1}{2} m_2 v_f^2$$

$$\vec{F}_T \cdot h \cdot \underbrace{\cos 0}_{1} + m_1 g h \cos(90^\circ + \theta) = \frac{1}{2} m_1 v_f^2$$

$$F_T \cdot h \cdot \underbrace{\cos 180^\circ}_{-1} + m_2 g h \underbrace{\cos 0^\circ}_{1} = \frac{1}{2} m_2 v_f^2$$

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add

$$m_1 g h \cos(90^\circ + \theta) + m_2 g h = \frac{1}{2}(m_1 + m_2) v_f^2$$

$$\boxed{\frac{2gh\left(m_1 \cos(90^\circ + \theta) + m_2\right)}{(m_1 + m_2)} = v_f^2}$$