Physics 200 — Day 22

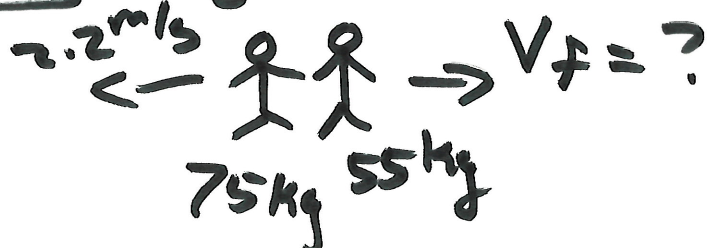
on which we take a momentum ~superquiz.

At end o' class: put chairs in columns, please :)

or else it takes me through Mordor to reach you.

SOH CAH TOA

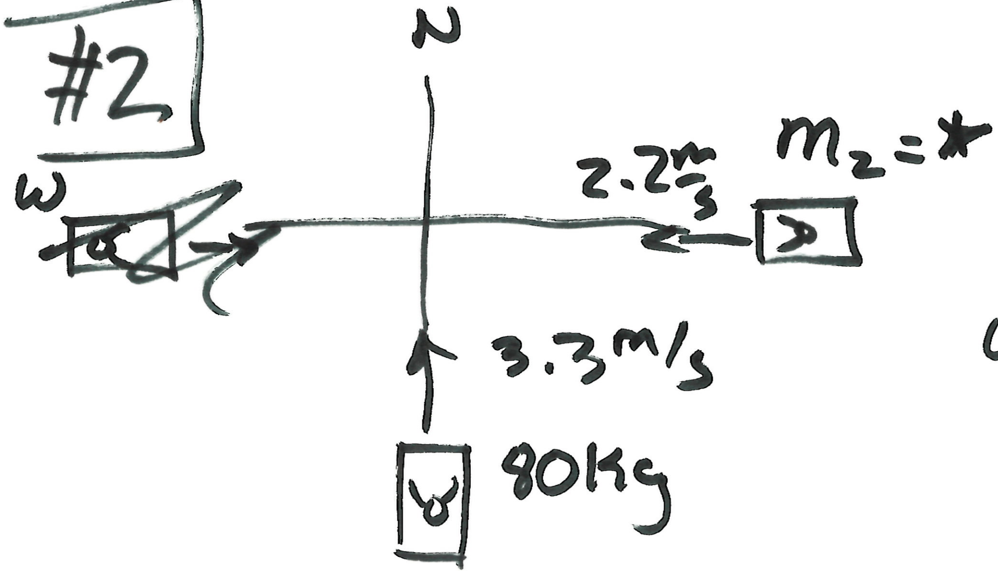
#1.)

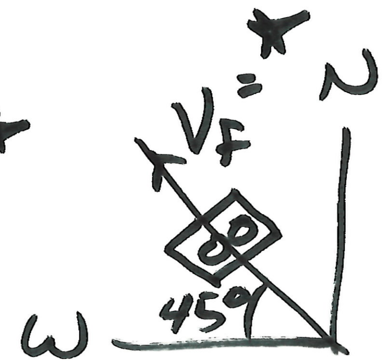


$$0 = 55\,\text{kg}\,V_f - 75\,\text{kg}\,2.2\tfrac{m}{s}$$

$$\frac{75}{55} \times 2.2\frac{m}{s} = \boxed{V_f = 3.0\frac{m}{s}}$$

#2





North direction:

$$\Sigma P_{oy} = \Sigma P_{fy}$$

I $80\,\text{Kg} \cdot 3.3\,\tfrac{m}{s} + m_2 \cdot 0 = (80\,\text{Kg} + \overset{*}{m_2})\overset{*}{V_f} \sin 45^\circ$

$$\Sigma P_{ox} = \Sigma P_{fx}$$

II $80\,\text{Kg} \cdot 0 + m_2 \cdot 2.2\,\tfrac{m}{s} = (80\,\text{Kg} + m_2) V_f \cos 45^\circ$

I $V_f = \dfrac{80\,\text{Kg} \cdot 3.3\,\tfrac{m}{s}}{(80\,\text{Kg} + m_2) \sin 45^\circ}$

$$\overset{*}{m_2} \cdot 2.2\,\tfrac{m}{s} = \cancel{(80\,\text{Kg} + m_2)} \cdot \overbrace{\frac{80\,\text{Kg}\ 3.3\,\tfrac{m}{s}}{\cancel{(80\,\text{Kg} + m_2)}}}^{V_f} \cdot \overbrace{\frac{\cos 45^\circ}{\sin 45^\circ}}^{1}$$

$$m_2 = 80\,\text{Kg} \times \frac{3.3\,\tfrac{m}{s}}{2.2\,\tfrac{m}{s}} = 120\,\text{Kg}$$

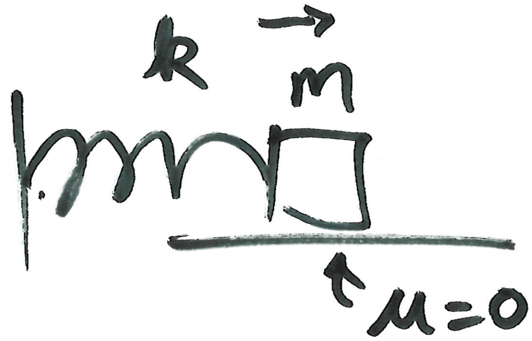
(I)

$$80\,\text{Kg} \cdot 3.3\,\tfrac{m}{s} + 0 = (200\,\text{Kg}) \frac{V_f}{\sqrt{2}}$$

$$\frac{80\,\text{Kg} \cdot 3.3\,\tfrac{m}{s} \sqrt{2}}{200\,\text{Kg}} = \boxed{V_f = 1.87\,\tfrac{m}{s}}$$

Harmonic Motion:

Consider a mass on a horizontal, frictionless surface:



v=5.3

$$\Sigma F_x = ma$$

$$-k\,x(t) = m\,a(t) \quad \text{and} \quad a(t) = \frac{d^2x(t)}{dt^2} = \ddot{x}$$

$$-\frac{k}{m}\,x(t) = \frac{d^2x(t)}{dt^2} = \ddot{x}$$

<u>try:</u> $x(t) = A\cos(\omega t)$

A = amplitude unit meter

ω = angular frequency unit: $\dfrac{\text{"radian"}}{\text{second}}$

$$\dot{x} = \frac{dx}{dt} = -A\sin(\omega t)\cdot\omega$$

$$\ddot{x} = \frac{d^2x}{dt^2} = -A\cos(\omega t)\cdot\omega^2$$

We are testing

$$-\frac{k}{m}\cancel{A}\cancel{\cos(\omega t)} \stackrel{?}{=} -\cancel{A}\cancel{\cos(\omega t)}\cdot\omega^2$$

"works" if $-\frac{k}{m} = -\omega^2$

$\omega = \sqrt{k/m}$. yes. It works.

$$\omega = \frac{2\pi}{T}$$

Simple Harmonic Motion:

one frequency, goes forever.

k = spring constant

m = mass

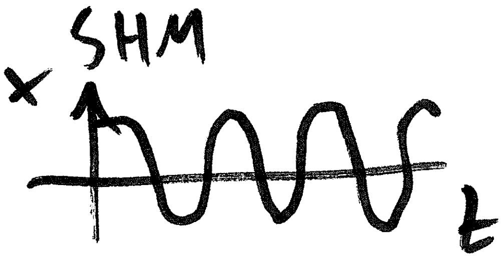
T = the period = time one cycle

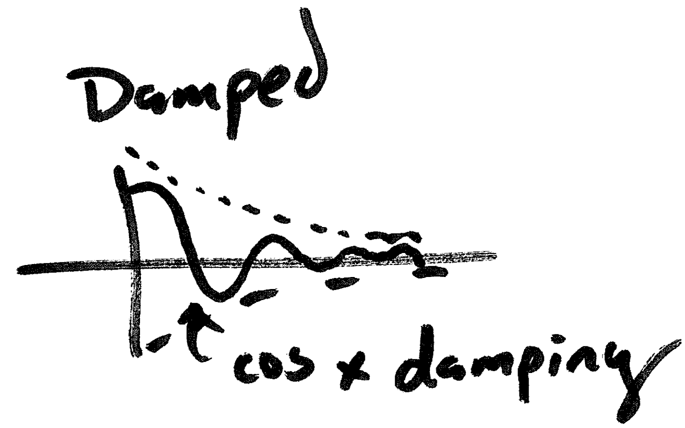
Damped Harmonic Motion:

$$F_{damp} = -bv = -b\dot{x}$$

$$\Sigma F = ma = m\ddot{x}$$

$$-kx - b\dot{x} = m\ddot{x}$$





try:

$$x(t) = A\cos(\omega t)\, e^{-\alpha t}$$

$$\dot{x} = -A\sin(\omega t)\cdot\omega\, e^{-\alpha t} + A\cos(\omega t)\, e^{-\alpha t}(-\alpha)$$

$$\ddot{x} = -A\cos(\omega t)\cdot\omega^2 e^{-\alpha t} - A\sin(\omega t)\cdot\omega\, e^{-\alpha t}(-\alpha)$$
$$+ A(-1)\sin(\omega t)\cdot\omega\, e^{-\alpha t}(-\alpha) + A\cos(\omega t)\, e^{-\alpha t}(-\alpha)^2$$

get: new ω in terms of k m b
and α

$-kA\cos e - b(-A)\sin\cdot\omega\cdot e - bA\cos e(-\alpha) =$

$-Am\cos\omega^2 e \quad -mA\sin\cdot\omega\, e(-\alpha) + -mA\sin\omega\, e(-\alpha)$

$+Am\cos()\, e\,(-\alpha)^2$

cos terms only: with $\cos(\omega t)$ and $e^{-\alpha t}$ div. out:

(I) $-k\cancel{A} - b\cancel{A}(-\alpha) = -\cancel{A}m\omega^2 + \cancel{A}m(-\alpha)^2$

(I) $-k + b\alpha = -m\omega^2 + m\alpha^2$

sin terms only with $\sin(\omega t)$ $e^{-\alpha t}$ A divided out:

(II) $b\cancel{\omega} = m\cancel{\omega}\alpha + m\cancel{\omega}\alpha$

(II) $b = 2m\alpha$ $\boxed{\alpha = \frac{b}{2m}}$ plug into (I) above

(I) solve for ω^2: $m\omega^2 = m\alpha^2 - b\alpha + k$

$\omega^2 = \alpha^2 - \frac{b}{m}\alpha + \frac{k}{m}$

$\omega^2 = \frac{b^2}{4m^2} - \frac{b^2}{2m^2} + \frac{k}{m} = \frac{b^2}{2m^2} + \frac{k}{m}$

$$\boxed{\omega = \sqrt{\frac{k}{m} + \frac{b^2}{2m^2}}}$$