P200 — Day 15

momentum + collisions:

O $\xrightarrow{v_0}$ O

m_1 $\quad$ m_2

perfectly elastic head-on (1-D)

$m = m_1 = m_2$

after 2 solutions:

$v_{1f} = 0$ and $v_{2f} = v_0$ → total transfer

— or —

$V_{1f} = v_0$ $\quad$ $v_{2f} = 0$ → they miss (tunneling)

init: at rest, (but spring is coiled)

Carts travels distances simultaniously. given: m_1 d_1 and m_2, compute what d_2 should be.

$$\sum \vec{P_0} = \sum \vec{P_f}$$

$$0 = +m_2 v_2 - m_1 v_1 = 0$$

$$m_2 v_2 = m_1 v_1$$

set $v_2 = \dfrac{d_2}{t}$ $\quad v_1 = \dfrac{d_1}{t}$

same t if they hit simultaniously.

$$m_2 \frac{d_2}{\cancel{t}} = m_1 \frac{d_1}{\cancel{t}} \qquad \boxed{d_2 = \frac{m_1}{m_2} d_1}$$

heavy one travels shorter distance.

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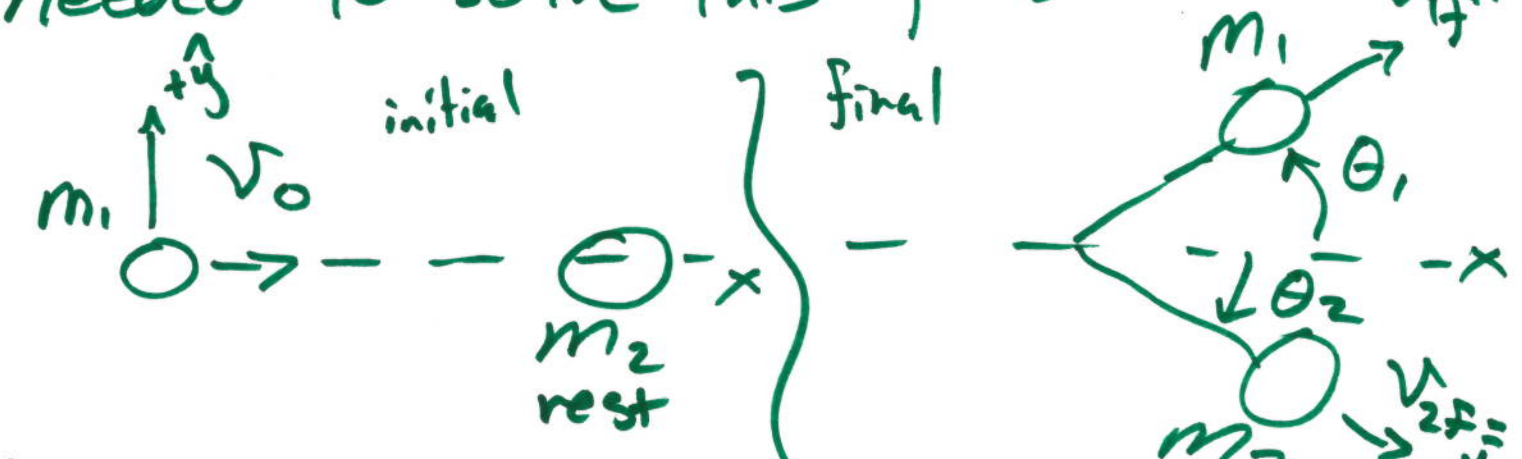
write down equations needed to solve this problem:



Know: $m_1, m_2\ v_0\ \theta_1\ \theta_2$

solve: $v_{1f}\ v_{2f}$

inelastic $\Rightarrow$ don't use E.

$$\sum \vec{P_0} = \sum \vec{P_f} \qquad \vec{p} = m\vec{v}$$

$$\hat{x}: \; m_1 v_0 + 0 = m_1 v_{1f} \underline{\cos} \theta_1 + m_2 v_{2f} \underline{\cos} \theta_2$$

$$\hat{y}: \; 0 + 0 = m_1 v_{1f} \sin\theta_1 - m_2 v_{2f} \sin\theta_2$$

If it is al<u>so</u> **elastic**:

Given: $m_1 \, v_0 \, m_2 \, \theta_1$

Find v_{1f} and $\vec{v_{2f}} = v_{2f}$ and θ_2 or in rectangular

$$\vec{v_{2f}} = (v_{2f_x}, v_{2f_y})$$

$$\sum \vec{P_0} = \sum \vec{P_F} \qquad \vec{p} = m\vec{v}$$

$\sum E_0 = \sum E_f$ if elastic or perfect

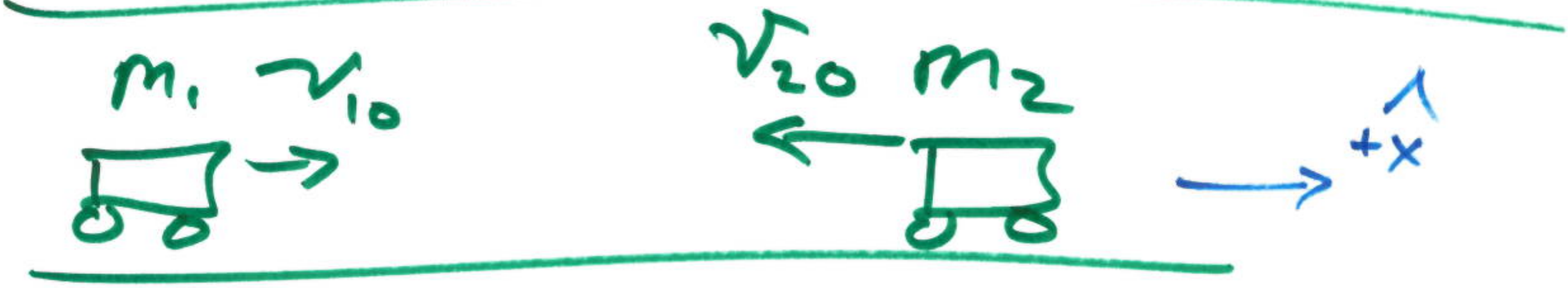
write down equ **needed**!

$$\frac{1}{2} m_1 v_0^2 + \underset{\text{easy}}{0} = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

★ ★

$\hat{p}_x$: $m_1 v_0 + 0 = m_1 v_{1f}^{*} \cos\theta_1 + m_2 v_{2f}^{*} \cos\theta_2^{*}$

$\hat{p}_y$: $0 + 0 = m_1 v_{1f}^{*} \sin\theta_1 - m_2 v_{2f}^{*} \sin\theta_2^{*}$



we see

what happened to can 1? What is $\vec{v}_{1f}$?

$$\sum \vec{P}_0 = \sum \vec{P}_f$$

$$m_1 v_{10} - m_2 v_{20} = + m_2 v_{2f} - m_1 v_{1f}$$

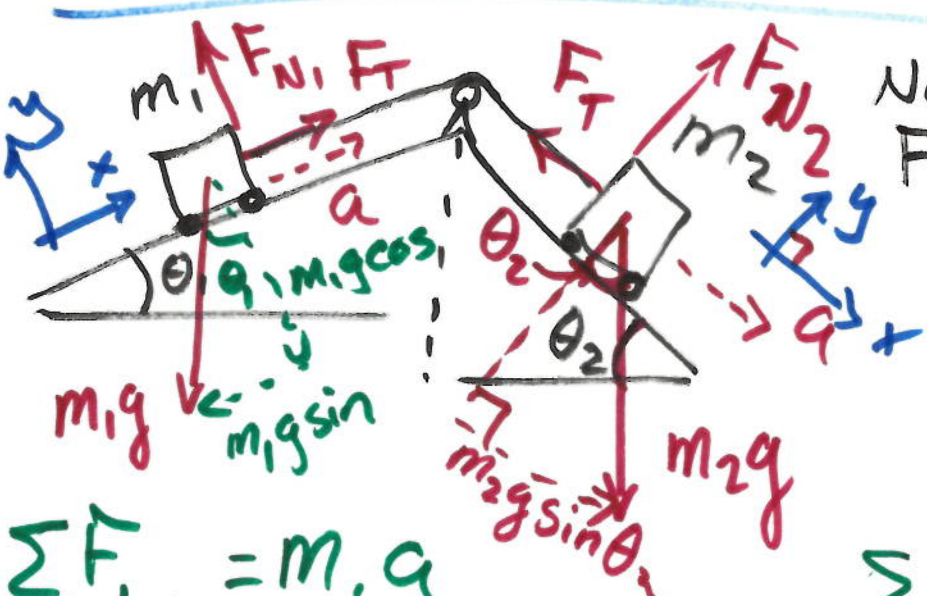
$$v_{1f} = \frac{m_2}{m_1} v_{2f} + \frac{m_2}{m_1} v_{20} - v_{10}$$

$$\boxed{v_{1f} = \frac{m_2}{m_1}\left(v_{2f} + v_{20}\right) - v_{10}}$$

Physics 200 Day 16

Quiz 2.0: Forces
42 people + Me.
Quiz Solution
Example w/ pulley, tension

Tues: Review 5–7pm in lab.



No friction.
Find a.

$$\Sigma F_{1x} = m_1 a$$
$$F_T - m_1 g\sin\theta_1 = m_1 a$$
$$\Sigma F_{1y} = 0$$
$$F_{N_1} - m_1 g\cos\theta_1 = 0$$
$$F_T = m_1 a + m_1 g\sin\theta_1$$

$$\Sigma F_{2x} = m_2 a$$
$$m_2 g\sin\theta_2 - F_T = m_2 a$$
$$\Sigma F_{2y} = 0$$
$$F_{N_2} - m_2 g\cos\theta_2 = 0$$

$$m_2 g\sin\theta_2 - m_1 a - m_1 g\sin\theta_1 = m_2 a$$
$$g(m_2\sin\theta_2 - m_1\sin\theta_1) = m_2 a + m_1 a$$
$$= a(m_1 + m_2)$$

Physics 200 quiz 2.0 – Force - Fall 2017 Name On Both Sides

1. A 9.3 kg box slides down a ramp inclined at 14^0 above the horizontal and has a coefficient of kinetic friction of 0.12. Find the acceleration of the mass.

$$\sum F = ma$$

$$F_g = mg$$
$$F_{fk} = \mu_K F_N$$

$\mu_K = 0.12$

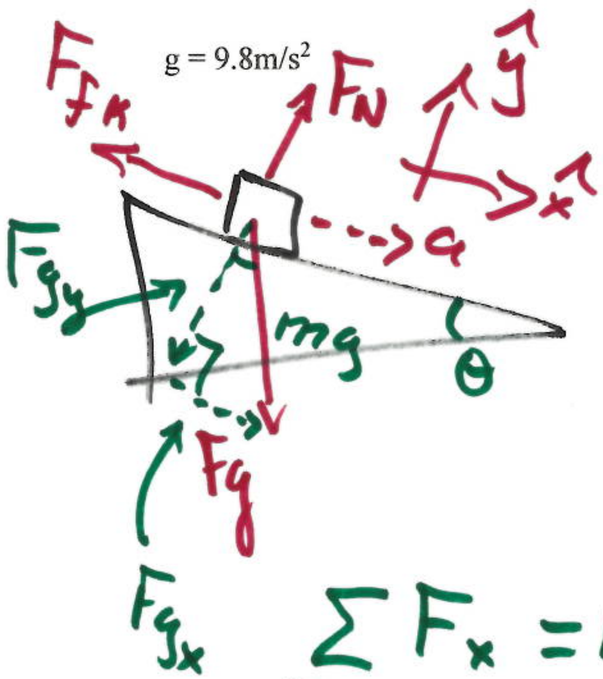
$\sum \vec{F} = m\vec{a}$

$F_g = mg$

$F_{fk} = \mu_k F_N$

$g = 9.8\,m/s^2$

SOH CAH TOA



$\sum F_x = ma$

$F_{gx} - F_{fK} = ma \rightarrow mg\sin\theta - \mu_K F_N = ma$

$\sum F_y = 0$

$F_N - F_{gy} = 0$

$F_N = mg\cos\theta$

$\cancel{m}g\sin\theta - \mu_K \cancel{m}g\cos\theta = \cancel{m}a$

$g(\sin\theta - \mu_K\cos\theta) = a$

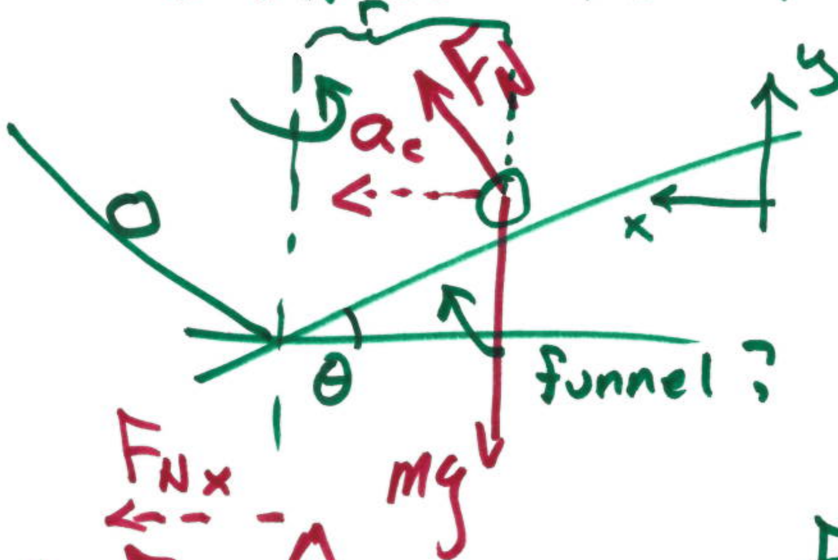
$9.8\frac{m}{s^2}(\sin 14^\circ - 0.12\cos 14^\circ) = a$

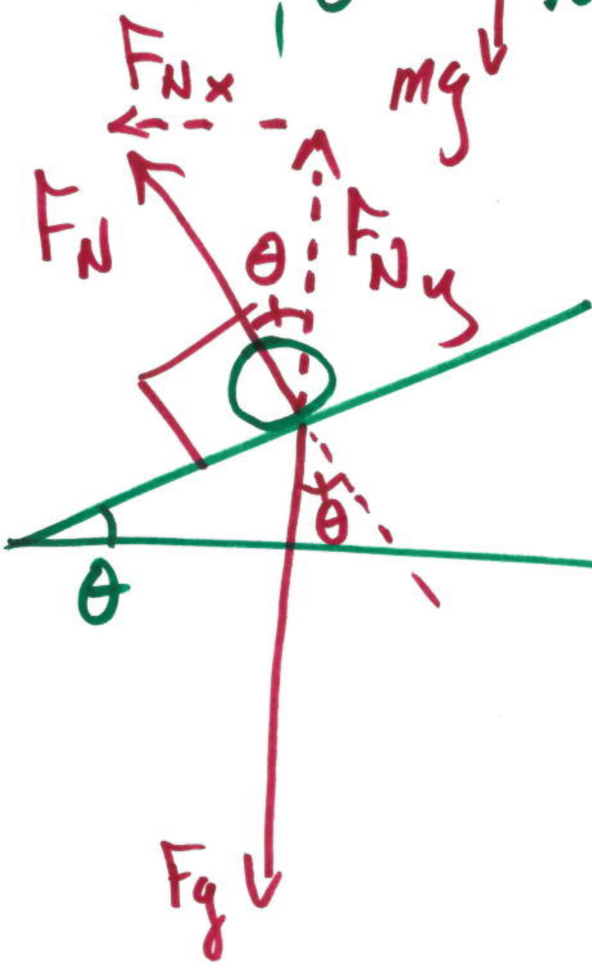
$\boxed{1.23\frac{m}{s^2} = a}$

$$a = \frac{(m_2 \sin\theta_2 - m_1 \sin\theta_1)\,g}{m_2 + m_1}$$

a circ. motion example:

Banked Turn





$$\Sigma F = ma$$

$$\Sigma F_x = ma_c$$

$$\Sigma F_y = 0$$

$$F_N \cos\theta - mg = 0$$

$$F_N = \frac{mg}{\cos\theta}$$

$$\Sigma F_x = ma_c$$

$$F_{Nx} = ma_c$$

$$F_N \sin\theta = ma_c$$

$$\cancel{m}g \frac{\sin\theta}{\cos\theta} = \cancel{m}a_c$$

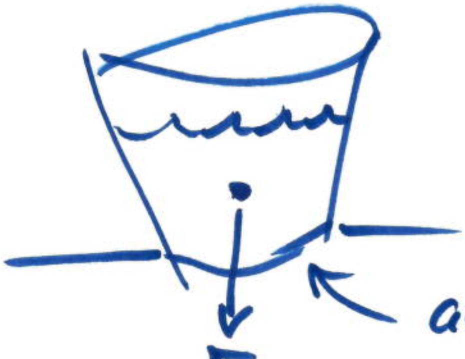
$$= \frac{v^2}{r}$$

$$rg\tan\theta = v^2$$

Center of Mass = where force gravity acts

where does F_g act?

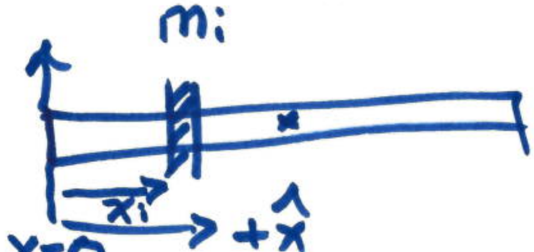
Why can my cup o' water tip over?



any contact point could be F_N

~~F_g~~

F_f



"uniform" thin rod

length L mass m

show cm is in middle.

$$X_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i = m} = \frac{\int x\, dm}{m}$$

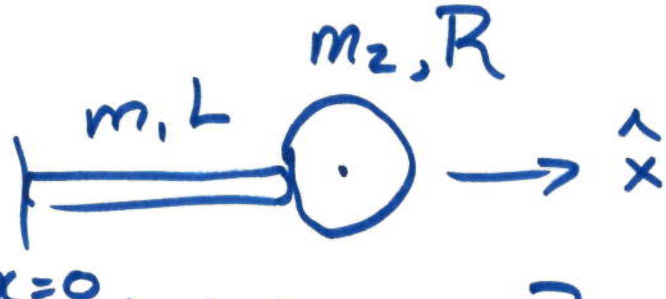
m ← total mass

$$dm = \underbrace{\frac{\text{mass}}{\text{length}}}_{\lambda} \times dx \qquad \lambda = \frac{m}{L}$$

$$x_{cm} = \frac{1}{m}\int x \underbrace{\lambda\, dx}_{dm}$$

$$x_{cm} = \frac{\lambda}{m}\int_{x=0}^{x=L} x\,dx = \frac{\lambda}{m}\left[\frac{x^2}{2}\right]_{x=0}^{x=L}$$

$$x_{cm} = \frac{\lambda}{m}\left[\frac{L^2}{2} - \frac{0}{2}\right] = \frac{\lambda L^2}{2m} = \frac{m}{\cancel{L}}\,\frac{L^{\cancel{2}}}{2\cancel{m}} = \frac{L}{2}$$



$$x_{cm} = \frac{\sum x_i m_i}{M_{tot}}$$

What is x_{cm}?

$$x_{cm} = \frac{1}{m_1+m_2}\left[\underbrace{\left(\sum x_i m_i\right)}_{\text{rod}} + \underbrace{\left(\sum x_i m_i\right)}_{\text{ball}}\right]$$

$$x_{cm} = \frac{1}{m_1+m_2}\left[\left(m_1\frac{L}{2}\right) + \left(m_2(L+R)\right)\right]$$

$$= \frac{m_1\frac{L}{2} + m_2(L+R)}{m_1+m_2}$$

Frames of Reference

"Inertial" = non-acceleration = good

non inertial = acceleration = bad

↳ Coriolis forces

> Can change between any inertial frames easily.

add $\Delta \vec{v}$ to everything in problem/universe.

Rotation:

motion: Kinematics

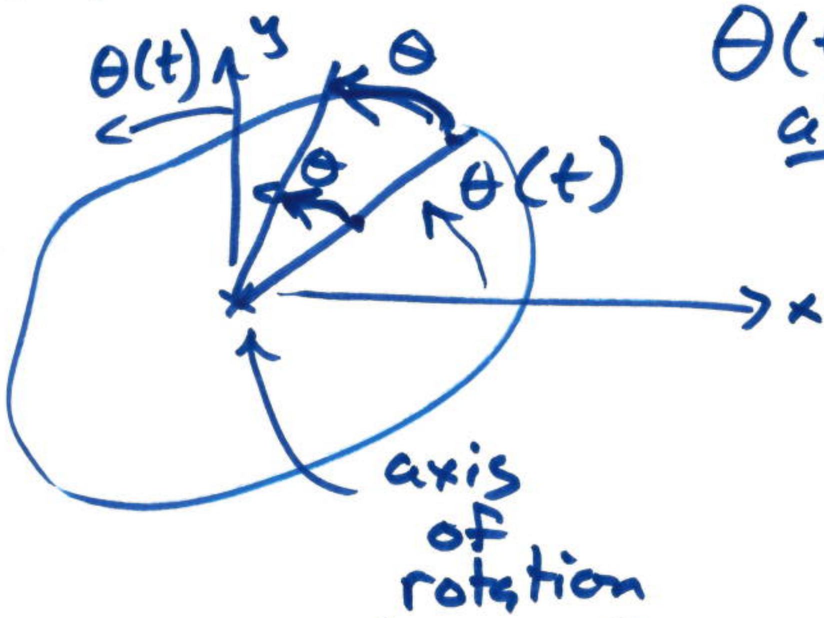
Newton's Law of Rotation

Energy, Rotation

Angular Momentum

will take a while!

Rotational Motion



$\theta(t)$ same for <u>all</u> $\Rightarrow$ rigid

angle travelled : θ

angular speed : $\omega = \frac{d\theta}{dt}$

angular acceleration: $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

Caused by Torque

so at <u>times</u> either: ω or α is <u>const.</u>

<u>If ω = Const</u> $\Leftrightarrow$ uniform circular motion

$$\theta(t) = \theta_0 + \omega \cdot t$$

usually in radians, thus

$$\#\text{ revolutions} = \frac{\theta}{2\pi}$$ if θ in radians.

If $\alpha = \text{const}$ (speeding up or slowing down)

$$\alpha = \frac{d\omega}{dt} \qquad \cancel{\theta = \int dt} \qquad \omega = \frac{d\theta}{dt} \rightarrow \boxed{\Delta\theta = \int \omega(t)\,dt}$$

$$\omega(t) = \omega_0 + \alpha t \quad \text{I}$$

$$\int_0^t dt \underbrace{\omega(t)}_{\frac{d\theta}{dt}} = \int_0^t dt\,(\omega_0 + \alpha t)$$

$$\int_0^t \frac{d\theta}{dt}\,dt = \int_0^t \omega_0 \underset{t^0 = 1}{dt} + \int_0^t \alpha t'\,dt$$

$$\theta(t) - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 \quad \text{II}$$

$$\text{III} \quad \omega^2(t) = \omega_0^2 + 2\left(\theta(t) - \theta_0\right)\alpha$$

Ex: Your CD-ROM speeds up from rest to 4500 RPM in 1.1 s. Find: α, $\theta(t)$ if $\alpha \simeq$ const.

$\omega_0 = 0$ t

$$4500 \frac{\text{revol}}{\text{min}} = \omega(t)$$

$$4500\ \frac{\text{revol}}{\text{min}} \times \frac{2\pi\ \text{radians}}{1\ \text{revol.}} \times \frac{\text{min}}{60\ s} = \omega(t)$$

$$\omega(t) = 471\ \frac{\text{rad}}{s}$$

I $\omega(t) = \cancel{\omega_0} + \alpha t$

$$\frac{\omega(t)}{t} = \alpha = \frac{471\ \text{rad}/s}{1.1\ s}$$

$$\boxed{\alpha = 428\ \frac{\text{rad}}{s^2}}$$