## Motion in 1-D

I. Constant $\vec{v}$

is simple

one example

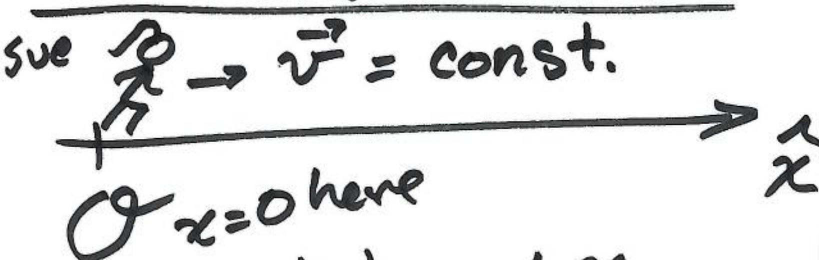
II. Constant $\vec{a}$

3 equations of

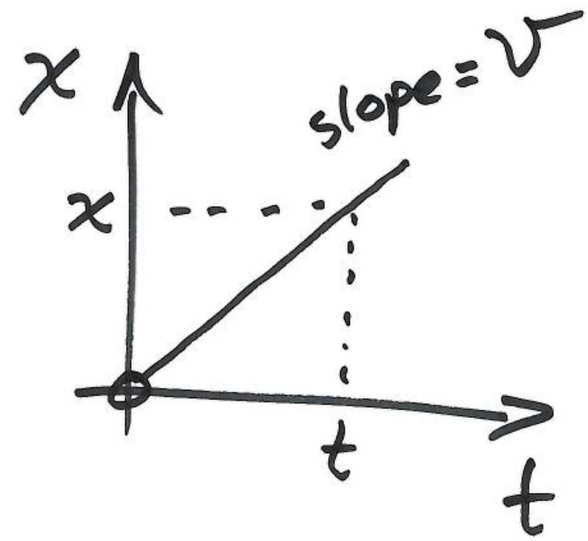
many examples

If lab1 you missed, worry not! Revisit it we shall after lab 2.

↑ Yoda, I can write like



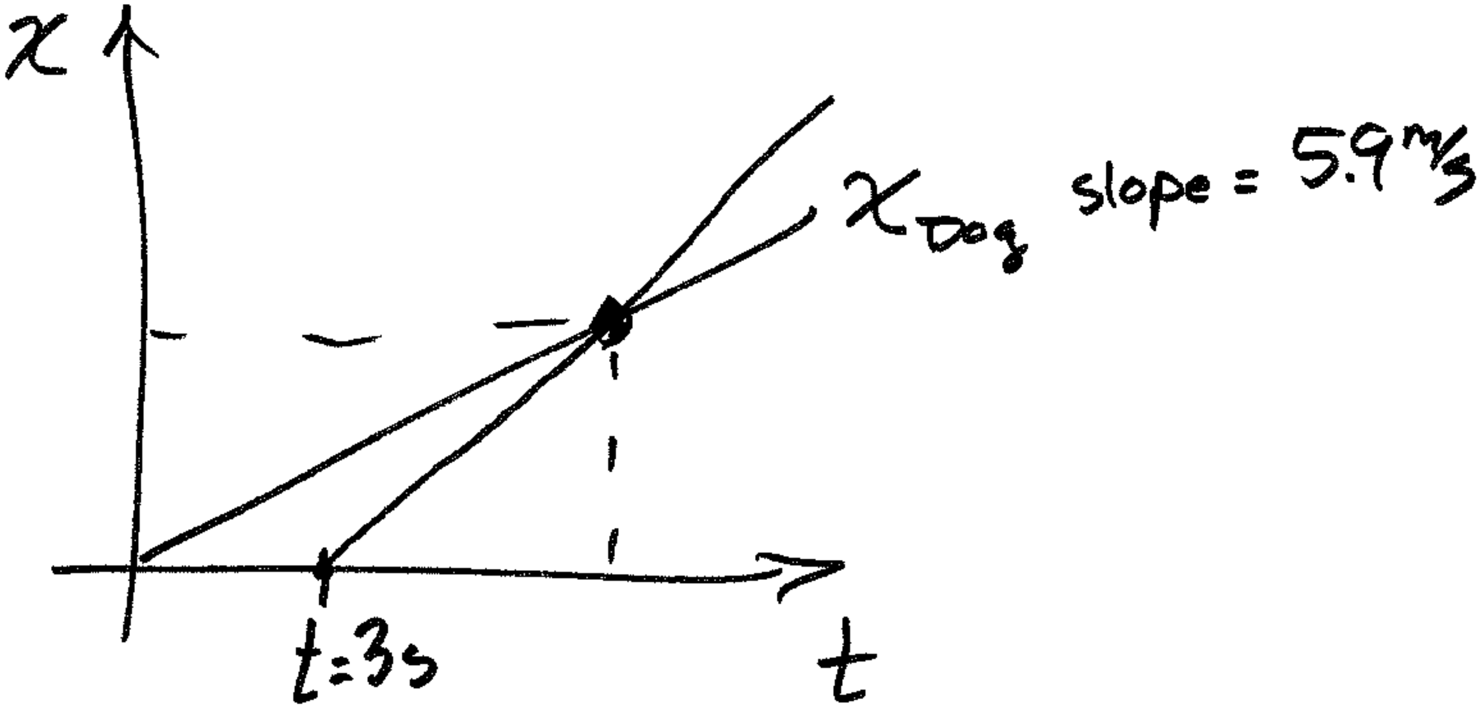

$$v = \frac{\text{dist}}{\text{time}} = \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$




$$x = v \cdot t + x_0$$

↑ initial $x$-position

Sue's dog, Buttercup, runs at 5.9 m/s and has a 3 second head start. Sue runs at 6.5 m/s. When and Where does Sue catch Buttercup?
(Note: Sue & Dog begin at same point)




$$x_{Dog} = 5.9\,\tfrac{m}{s} \cdot t + 0$$

$$x_{Sue} = 6.5\,\tfrac{m}{s}\,(t - 3s) + 0$$

Set $x_{Dog} = x_{Sue}$ and solve for t.

$$5.9\,\tfrac{m}{s}\,t = 6.5\,\tfrac{m}{s}\,(t - 3s)$$

$$6.5\,\tfrac{m}{s} \cdot 3s = 6.5\,\tfrac{m}{s}\,t - 5.9\,\tfrac{m}{s}\,t$$

$$19.5\,m = 0.6\,\tfrac{m}{s}\,t$$

$$\frac{19.5\,m}{0.6\,\frac{m}{s}} = t = 32.5\,s$$ time dog runs.

Sun runs 3 s less.

Where are they? $x(t)$.

$$x_{Dog} = 5.9\tfrac{m}{s} \cdot 32.5s = 192m \checkmark$$

$$x_{Sue} = 6.5\tfrac{m}{s}(32.5s - 3s) = 192m \checkmark$$

yay!

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Acceleration $a = \frac{dv}{dt}$ speed up, slow down, (or change direction).

if $a$ = constant, ...

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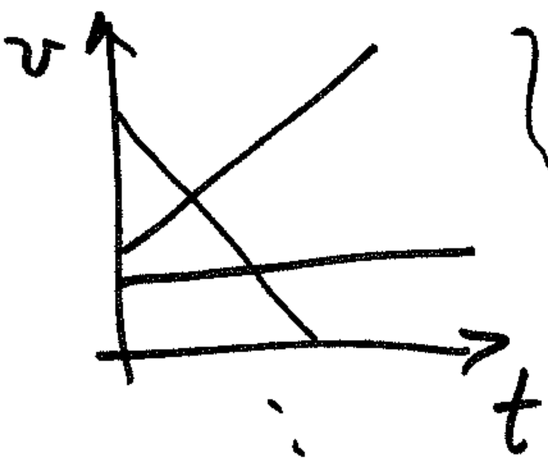
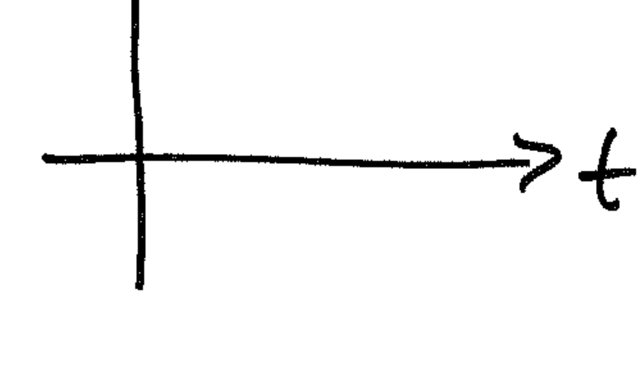
Why? $a$ caused by Forces (later) and sometimes Forces = constant.



all 3 are "constant a"

$a$ = slope $\Rightarrow$ straight line.

$$\text{I} \quad \underbrace{v(t)}_{\text{at any time... could be "final".}} = at + v_0 \quad \text{of constant } a.$$



$$\frac{dx}{dt} = v$$ ... if we know $v(t)$ how get $x(t)$?

$$\int_0^t \frac{dx(t)}{dt}\,dt = \int_0^t v(t)\,dt$$

$$x(t) - \underbrace{x(t=0)}_{x_0} = \int_0^t (at + v_0)\,dt$$

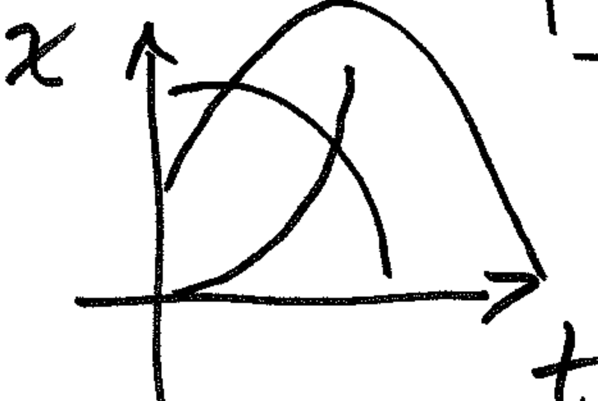
$$= \int_0^t at\,dt + \int_0^t v_0\,dt$$

$$= a\int_0^t t\,dt + v_0 \int_0^t dt$$

$$x(t) - x_0 = \frac{1}{2}at^2 + v_0 t$$

II $\quad x(t) = x_0 + v_0 t + \frac{1}{2}at^2$

parabola




To get III, eliminate $t$ from I and II.

explicit

$$\text{I} \quad v(t) = v_0 + a t$$

$$\text{II} \quad x(t) = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$t = \frac{v(t) - v_0}{a}$$

$$x(t) = x_0 + v_0\left(\frac{v(t) - v_0}{a}\right) + \frac{1}{2}\cancel{a}\,\frac{\overbrace{\left(v(t) - v_0\right)^2}}{a^{\cancel{2}}}$$

foiled

$$= x_0 + \frac{v_0 v(t)}{a} - \frac{v_0^2}{a} + \frac{1}{2}\frac{1}{a}\left(v^2(t) - 2v_0 v(t) + v_0^2\right)$$

$$= x_0 + \cancel{\frac{v_0 v(t)}{a}} - \frac{v_0^2}{a} + \frac{v^2(t)}{2a} - \cancel{\frac{2 v_0 v(t)}{2a}} + \frac{v_0^2}{2a}$$

$$= x_0 + -\frac{1}{2}\frac{v_0^2}{a} + \frac{v^2(t)}{2a} = x(t)$$

$$x(t) - x_0 = \frac{\left(v^2(t) - v_0^2\right)}{2a}$$

$$\boxed{2a\left(x(t) - x_0\right) + v_0^2 = v^2(t)} \quad \text{III}$$

3 equ: of const. $\vec{a}$.

(I) $v(t) = v_0 + at$

(II) $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$

(III) $v^2(t) = v_0^2 + 2a(x(t) - x_0)$

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A cheetah can accelerate from rest to $28\frac{m}{s}$ over $12m$.

Let's assume $a$ = const. (This will be $\bar{a}$)

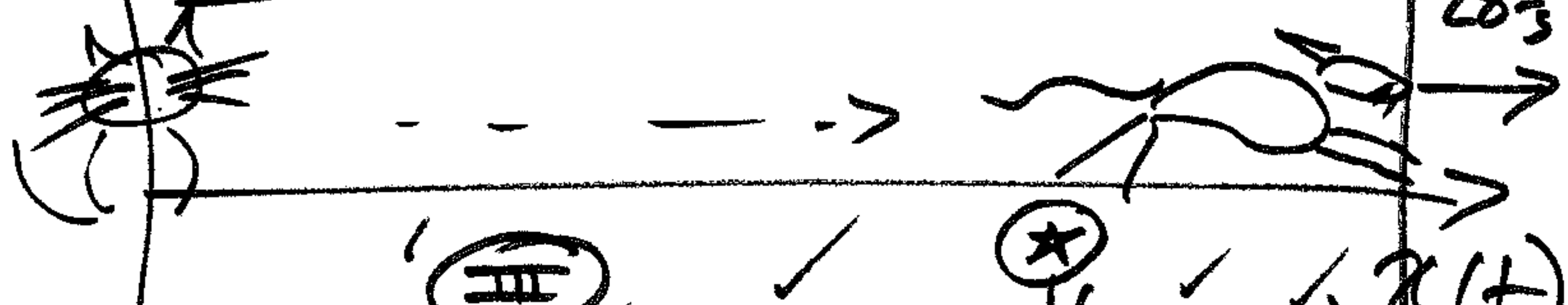
average

| Know | want | Don't care |
|---|---|---|
| $v_0 = 0$ | $a$ | $t$ |
| $x(t) = 12_m$ | | |
| $v(t) = 28\frac{m}{s}$ | | |
| $x_0 = 0$ | | |

$x(t) = 12\,m$

$v_0 = 0$

$28\frac{m}{s}$



$x(t)$

$x_0 = 0$

(III) $v^2(t) = v_0^2 + 2a(x(t) - x_0)$

I want

$$\left(28\tfrac{m}{s}\right)^2 = \left(0\tfrac{m}{s}\right)^2 + 2a(12m - 0m)$$

$$\frac{784\frac{m^2}{s^2}}{24m} = \frac{a \cdot 24m}{24m}$$

$$33\tfrac{m}{s^2} = a$$ over $\underline{3g\text{'s}}$!

for \$100,000 ! OK most of us cannot buy...

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A car you can ~~buy~~ buy (for comparison):
goes from $0\frac{m}{s} \to 28\frac{m}{s}$ in ~~7.0s~~
Lets find $a$ (assuming its constant).

Tesla model "s" in 2.8s

**Know**

$v_0 = 0$

$v(t) = 28\frac{m}{s}$

$t = 2.8s$

**Want** (II)

$a$

$$x(t) = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$\text{(I)}\quad v(t) = v_0 + at$$

$$28\tfrac{m}{s} = 0 + a(2.8s)$$

$$\frac{28\frac{m}{s}}{2.8s} = \boxed{a = 10\tfrac{m}{s^2}}$$

Cheetah is quicker by 300%