Homework 3 Solutions: Projectile Motion

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1. You kick a ball at $v_O = 7.6$m/s at 39^O above the horizontal toward a wall, 12.3 m away. Does it hit? Where?

Let us first find the time when the ball would be at the location of the wall. Pick the origin to be the starting spot of the ball, so $x_0 = 0$ and $y_0 = 0$. This requires equation II in the x direction:

$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

but $a_x = 0$, so this simplifies greatly.

In the x direction, $v_{0x} = 7.6\cos 39 = 5.91$m/s.

Set $x(t) = 12.3m$. Solving for t, we find: $t = 12.3m/5.91m/s = 2.08$s.

Now, let us find $y(t)$ at this time also using equation II, but in the y:

$y(t) = y_O + v_{oy}t + \frac{1}{2}a_y t^2$.

$y(t) = 0 + 7.6\sin(39) \times (2.08) - 9.8/2 \times (2.08)^2$

$y(t) = 9.94m - 21.2m$, so this is clearly under the ground! The ball must have already landed, before reaching the wall.

To find the location of the ball landing, we set $y(t) = 0$, and find the time, then find x at that time.

Again, using II in the y:

$0 = 0 + v_{0y}t - g/2t^2$

This has two solutions. We toss out the $t = 0$ solution as unphysical and solve for:

$v_{0y} = g/2t$ thus: $t = v_{0y} \times 2/g = 4.78 \times 2/9.8 = 0.976$s.

Now $x(t) = 0 + 5.91m/s \times 0.976s = 5.77$m.

The ball lands at x=5.77 meters, so, to answer the question: How close to the wall does the ball get?

We subtract 12.3 - 5.77 = 6.53m. The ball lands 6.53 meters from the wall. Kick harder next time.

2. You toss a dart 1.5 m off the ground, horizontally, and it lands 6.1 m away. Find the initial speed.

Set y=0 on the ground, and x=0 where the dart begins, so $x_0 = 0$ and $y_0 = +1.5m$, and $y(t) = 0$ and $x(t) = +6.1$m. Other choices are possible.

We can use equation II in the y to find the time:

$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$.

Since the dart is initially going horizontally, $v_{0y} = 0$.

$0 = 1.5 + 0 - 4.9t^2$.

Thus: $t^2 = -1.5/-4.9 = 0.306s^2$ thus time $t = 0.553$s.

Now using the x-equation II, we can find v_0, which points entirely in the x-direction:

$6.1m = 0 + v_0 0.553s$ Thus $v_0 = 6.1/0.553\text{m/s} = 11.02\text{m/s}$, which is pretty fast, about 20 miles per hour, so probably someone can toss a dart this fast. Supposedly, according to dartbase.com, the average speed of a dart hitting is 40 miles per hour, but according to another article, the average time of flight of a dart, when players are focusing on speed only, is 400 ms, and the dartboard is about 2.4 m away, leading to a much slower speed.

3. Ball is launched at 30 degrees from a table 0.85 m above the floor. It lands 2.15 m away, horizontally. Find the initial speed (direction is given).

Let us set y=0 on the floor, so the initial y position is +0.85m, and y(t)=0. But set the initial x to zero. Thus, the origin is at a point on the floor directly below the release point of the ball. This way, $x_0 = 0$ and $x(t) = +2.15\text{m}$.

Since there is only one good equation in the x-direction, let us use this one first:

$x(t) = x_0 + v_{0x}t + \frac{1}{2}t^2$.

$2.15 = 0 + v_0 \cos 30t + 0$

Both v_0 and t are unknown. We need a y-equation to solve something. Try either I or II. But I uses the unknown final y-velocity, so try II:

$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$0 = 0.85 + v_0 \sin 30t - 4.9t^2$.

Staring at these two equations for a while, I realize they both contain $v_0 t$. Let me show you what we can do with this: from the x equation:

$2.15/\cos 30 = v_0 t$

Now substitute this into the y-equation for $v_0 t$. This is a trick. For some of you, it might be handy someday, so I am showing it to you.

$0 = 0.85 + \sin 30 2.15/\cos 30 - 4.9t^2$.

Note the only unknown left is t^2, and that sine over cosine is tangent.

$\frac{0.85+2.15\tan 30}{4.9} = t^2$

Thus: $t = 0.653$ s.

Now return to our earlier equation to solve for v_0:

$\frac{2.15}{\cos 30t} = v_0$

$v_0 = 3.80\text{m/s}$.