Physics 200

Day ~~8~~ 9?

Atwood's Machine
mechanical advantage.
Circular Motion:
- m on string
- top/bottom of a hill

on m_1

$$\Sigma F_1 = m_1 a.$$

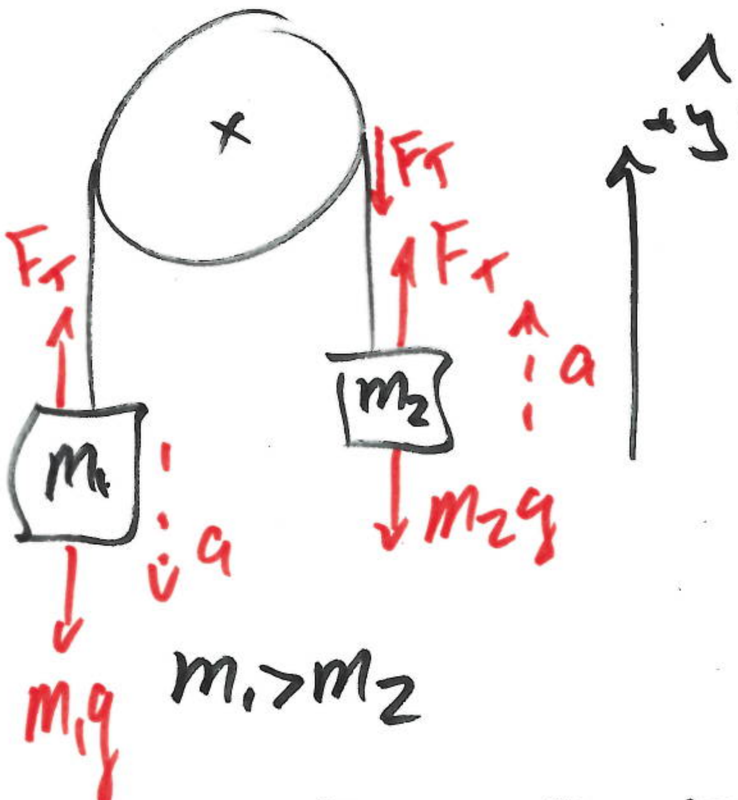
$$\boxed{F_T - m_1 g = -m_1 a}$$

$$\Sigma F_2 = m_2 a_2$$

$$\boxed{F_T - m_2 g = +m_2 a}$$

Solve for a
and F_T.

Given: $m_1, m_2\ g$ + picture
ideal string
ideal pulley



$m_1 > m_2$

Solve F_T in ①: $F_T = -m_1 a + m_1 g$

subb into ②:

$$\overbrace{-m_1 a + m_1 g}^{F_T} - m_2 g = +m_2 a$$

$$m_1 g - m_2 g = m_1 a + m_2 a$$

$$(m_1 - m_2) g = (m_1 + m_2) a$$

$$\left(\frac{m_1 - m_2}{m_1 + m_2}\right) g = a \qquad F_T = -\frac{m_1(m_1 - m_2)g}{m_1 + m_2} + m_1 g$$

$$F_T = \left(\frac{-m_1^2 + m_1 m_2}{m_1 + m_2}\right) g + m_1 g \cdot \frac{(m_1 + m_2)}{(m_1 + m_2)}$$

$$= \frac{g}{m_1 + m_2}\left(\cancel{-m_1^2} + m_1 m_2 + \cancel{m_1^2} + m_1 m_2\right)$$

$$\boxed{F_T = \frac{2 m_1 m_2 g}{m_1 + m_2}}$$

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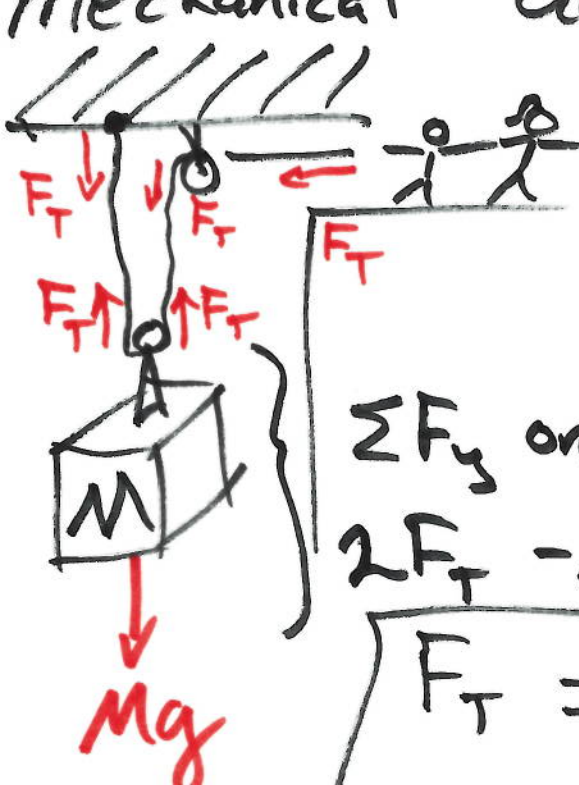
mechanical advantage:

Given Mg + picture and $a \to 0$ find min. force to lift M.

answer: $F_T = \frac{Mg}{2}$



ΣF_y on M

$$2F_T - Mg = M\cancel{a}^{\,0}$$

$$\boxed{F_T = \frac{1}{2} Mg}$$

Circular Motion:

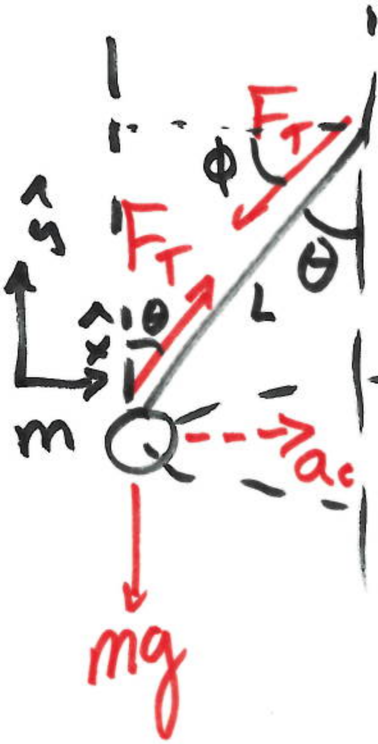
Uniform: Const. speed.

$$a_c = \frac{v^2}{r}$$

Points toward Center of circle.

Centripetal

$$\sum \vec{F} = m\vec{a}$$



given: $m\ g\ \theta\ L$

find: v, F_T to maintain uniform circular motion.

horiz. circle.

$$\sum F_x = ma_x$$

$$F_{T_x} = ma_c$$

$$\sum F_y = ma_y$$

$$F_{T_y} - mg = 0 \quad \text{horizontal}$$

$$F_T\cos\theta - mg = 0$$

$$\boxed{F_T = \frac{mg}{\cos\theta}}$$

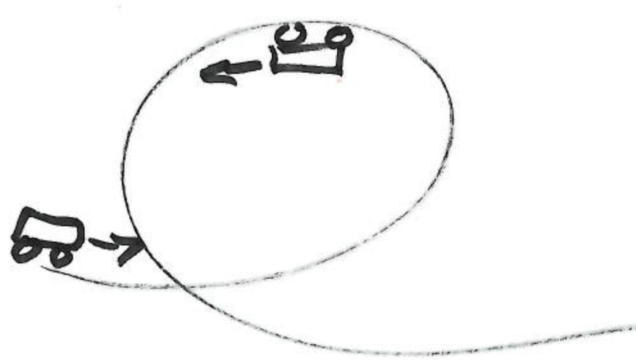
$$F_T\sin\theta = \cancel{m}a_c = \frac{\cancel{m}g}{\cos\theta}\cdot\sin\theta = g\tan\theta$$

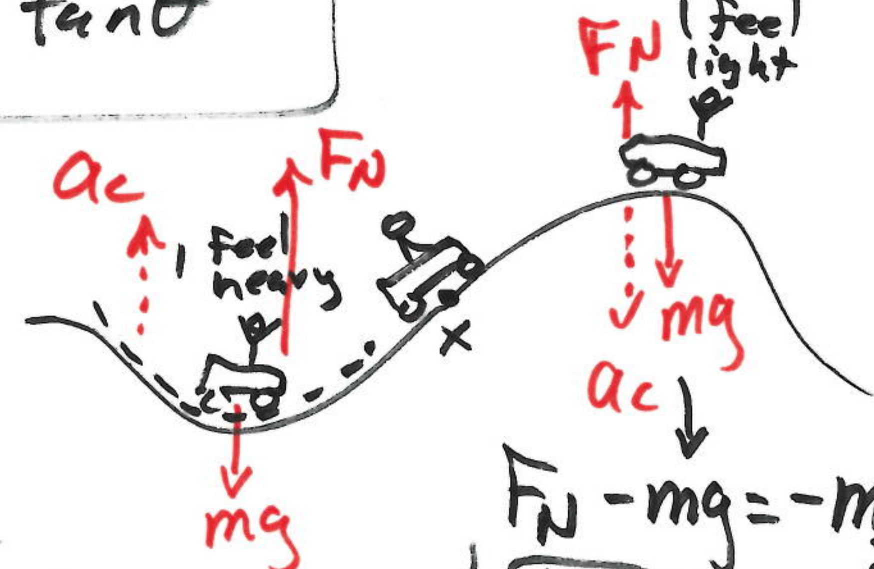
$$a_c = g\tan\theta$$

$$\frac{v^2}{r} = \frac{v^2}{L} = g\tan\theta$$

L ← length of string: given.

$$\boxed{v = \sqrt{Lg\tan\theta}}$$





$$F_N - mg = -m\frac{v^2}{r}$$

$$\boxed{F_N = mg - m\frac{v^2}{r}}$$

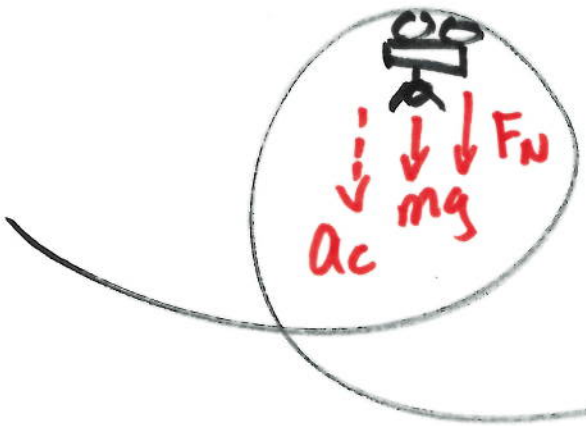
Given:
r, m, g, v

$$\Sigma F = ma$$

$$F_N - mg = +m\frac{v^2}{r}$$

$$\boxed{F_N = mg + m\frac{v^2}{r}}$$

Want:
F_N at each point & compare to mg.

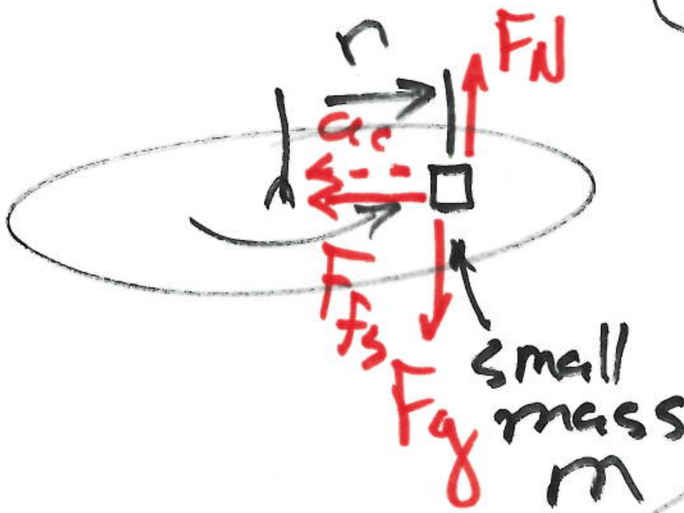


$$\Sigma F_y = ma$$

$$-mg - F_N = -\frac{mv^2}{r}$$

$$mg + F_N = \frac{mv^2}{r}$$

$$\boxed{F_N = -mg + \frac{mv^2}{r}}$$



$$\Sigma F_x = ma_c$$

$$F_{fs} = ma_c$$

$$\Sigma F_y = 0$$

$$F_N - mg = 0$$

$$F_N = mg$$

$$\mu_s F_N \geq F_{fs}$$

$$\mu_s F_N \geq ma_c = m\frac{v^2}{r}$$

$$\mu_s \cancel{m} g \geq \cancel{m}\frac{v^2}{r} \qquad \boxed{\mu_s g r \geq v^2}$$

if not, it flies off